EXAM

Ex 1 :

a. $h(r;\alpha;\beta)$ est une densité de probabilité $\int_0^1 h(r) = 1$

$$\int_0^1 h(r)\,dr \int_0^1 \alpha r - \beta r^2\,dr = \left[\frac{\alpha}{2}r^2 - \frac{\beta}{3}r^3\right]_0^1 = 1$$

$$= \frac{\alpha}{2} - \frac{\beta}{3} = 1$$

$$= 3\alpha - 2\beta = 6$$

si $\alpha = 4$: $3 \times 4 - 2\beta = 6$

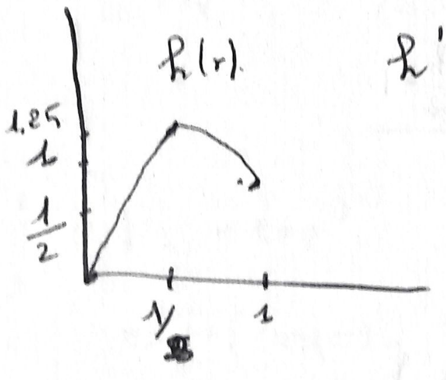
$$\beta = \frac{6-12}{-2} = 3$$

$$\boxed{\alpha = 4}$$
$$\boxed{\beta = 3}$$

b. $h(0) = 0$

$h(1) = 4 - 3 = 1$

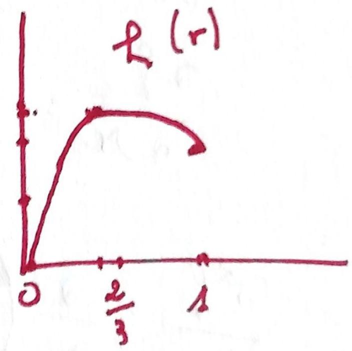
$\times \left[h\left(\frac{1}{2}\right) = 4 \times \frac{1}{2} - 3 \times \left(\frac{1}{2}\right)^2 = 2 - \frac{3}{4} = \cancel{0.5} = 1.25\right]$



$h'(r) = 4 - 6r = 0$

$r = \frac{2}{3}$

$h\left(\frac{2}{3}\right) = \frac{4}{3}$



c. $h'(r) = 4 - 6r = 0$

$$r = \frac{4}{6} = \frac{2}{3}$$

donc $h(r)$ atteint sa valeur maximal en $\frac{2}{3}$ et valeur minimal en 0.

d. $m = \int_0^1 r\, h(r) = \int_0^1 4r^2 - 3r^3 dr$

$$= \left[\frac{4}{3} r^3 - \frac{3}{4} r^4 \right]_0^1$$

$$= \frac{4}{3} - \frac{3}{4} = \frac{16-9}{12} = \frac{7}{12}$$

e. transformation $s = g(r)$

on applique $g(r) = \int_0^r h(t) dt$

fct de répartition :

$$g(r) = \int_0^r 4t - 3t^2 \, dt = \left[\frac{4}{2} t^2 - \frac{3}{3} t^3 \right]_0^r$$

$$= 2r^2 - r^3$$

Exercice 2

a. produit de convolution : $f(x) * \delta(x) = \int_{-\infty}^{+\infty} f(z)\, \delta(x-z)\, dz$

b. on a le différencier central :

$$\frac{\partial f}{\partial x} = \frac{f(x+\frac{h}{2}) - f(x-\frac{h}{2})}{h}$$

$$\frac{\partial^2 f}{\partial x} = \frac{\partial}{\partial x} \left(\frac{f(x+\frac{h}{2}) - f(x-\frac{h}{2})}{h} \right)$$

$$= \frac{1}{h} \left(f'(x+\frac{h}{2}) - f'(x-\frac{h}{2}) \right)$$

$$= \frac{1}{h} \left(\frac{f(x+\frac{h}{2}+\frac{h}{2}) - f(x)}{2 \times \frac{h}{2}} - \frac{f(x) - f(x-\frac{h}{2}-\frac{h}{2})}{2 \times \frac{h}{2}} \right)$$

$=$

$$\frac{\partial^2 f}{\partial x} = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

$\boxed{\text{Masque } (1, -2, 1)}$

(3

$$f(x) * S(x) - K \frac{\partial^2 f(x)}{\partial x^2}$$

$$f(x) * S(x) = (0, 1, 0) * \frac{1}{3} (0, 3, 0) = (0, 1, 0)$$

$$K = 1 \; ; \; \frac{\partial^2 f(x)}{\partial x^2} = (1, -2, 1)$$

$$f(x) * S(x) - K \frac{\partial^2 f(x)}{\partial x^2} = (0, 1, 0) - (1, -2, 1)$$
$$= (-1, 3, -1)$$

d. $$F(U) = \sum_{x=-1}^{1} \underbrace{\left(f(x) - K \frac{\partial^2 f(x)}{\partial x^2} \right)}_{h(x)} \exp(-2\pi j U x)$$

$$= h(-1) \exp(+2\pi j U) + h(0) \exp(0) + h(1) \exp^{(-2\pi j U)}$$

$$= -\exp^{(2\pi j U)} + 3 - \exp^{(-2\pi j U)}$$

on pose $w = 2\pi w$

$$\begin{cases} \exp^{jw} = \cos(w) - j \sin(w) \\ \exp^{-jw} = \cos(w) + j \sin(w) \end{cases}$$

$$F(U) = -\exp^{jw} - \exp^{-jw} + 3$$

$$= -(\cos(w) - j \sin(w)) - (\cos(w) + j \sin w) + 3$$

$$= -2 \cos w + 3.$$

e. TF de $\frac{\partial^2 f(x)}{\partial 2x} = -w^2 F(U)$ cas continu

$$= -(2(1 - \cos(w)) F(U)$$

$$H(x) = f(x) - K \frac{\partial^2 f(x)}{\partial x^2} \; ; \; K = 1$$

$$H(w) = F(w) - (-2(1 - \cos w)) F(w) = F(w)(1 + 2 - 2\cos(w))$$
$$= F(w)(3 - 2\cos(w))$$

spectre d'amplitude

$|H(w)| = |3 - 2\cos(w)|$

f- $f \to 0$.
$f(x)$ c a d on neglige interaction avec ~~f~~ signal

$F(v) = 3 - 2\cos(w)$
$= H(w) = 3 - 2\cos(w)$

g- $f(x) + K\dfrac{\partial^2 f(x)}{\partial x^2} = (0, 1, 0) + K(1, -2, 1)$

$= (K, 1 - 2K, K)$.

→ f. filtre moyenneur Σ de coefficient 1.

met 1: donc $K + 1 - 2K + K = 1$

$2K - 2K + 1 = 1$

pour tout valeur de K, on a filtre moyenneur.

met 2. $K \geq 0$

$1 - 2K \geq 0 \Rightarrow -2K \geq -1$

$K \leq \frac{1}{2}$

donc $\boxed{0 \leq K \leq 1/2}$

→ g. masque de prewitt en y: $\begin{pmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}$

en x: $\begin{pmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{pmatrix}$

le masque en x $(-1, 0, 1)$; $(K, 1 - 2K, K)$

$1 - 2K = 0 \to \boxed{K = \frac{1}{2}}$

ex 3: 5

1 - $P(x/C_1) = ax + b$

$p(0) = 1 \Rightarrow b = 1$

$p(2) = 0 \Rightarrow 0 = 2a + 1 \Rightarrow a = \frac{-1}{2}$.

$$p(x/C_1) = \boxed{\frac{-1}{2} x + 1}$$

$P(x/C_2) = ax^2 + bx + c$

$p(0) = 0 \Rightarrow \boxed{c = 0}$

$p(1) = \frac{3}{8} \Rightarrow \frac{3}{8} = a + b$

$p(2) = \frac{3}{2} \Rightarrow \frac{3}{2} = 4a + 2b$

$$\begin{cases} a + b = \frac{3}{8} \\ 4a + 2b = \frac{3}{2} \end{cases} \Rightarrow \begin{cases} 4a + 4b = \frac{3}{2} \\ 4a + 2b = \frac{3}{2} \end{cases}$$

$2b = 0 \Rightarrow \boxed{b = 0}$

$a + b = \frac{3}{8}$

$a + 0 = \frac{3}{8} \Rightarrow \boxed{a = \frac{3}{8}}$

$$\boxed{p(x/C_2) = \frac{3}{8} x^2}$$

2 - distribution de probabilité de x.

p

seuil :

si $P(C_1) = P(C_2)$

$\frac{p(x/C_1)}{p(x/C_2)} \gtrless 1$

$\Rightarrow p(x/C_1) \gtrless p(x/C_2)$

on résoudre l'équation

règle de Bayes : ~~P(C_1/x) . P(x~~

$P(C_1/x) . P(x) = P(x/C_i) . P(C_i)$

règle de décision :

$\Rightarrow p(C_1/x) \gtrless p(C_2/x)$

$\Rightarrow \frac{p(x/C_1) . P(C_1)}{\cancel{P(x)}} \gtrless \frac{P(x/C_2) . P(C_2)}{\cancel{P(x)}}$

$\Rightarrow \frac{p(x/C_1)}{p(x/C_2)} \gtrless \frac{P(C_2)}{P(C_1)}$

(6

$$P(x) = \sum P(x/c_i)\, P(c_i) = P(x/c_1)\, P(c_1) + P(x/c_2) . P(c_2)$$
$$= \left(-\frac{1}{2} x + 1\right) \frac{2}{3} + \left(\frac{3}{8} x^2\right) \times \frac{1}{3}$$
$$= \frac{1}{8} x^2 - \frac{1}{3} x + \frac{2}{3}$$

3 - probabilité à posteriori

$$P(C_1/x) = \frac{P(x/C_1)\, P(C_1)}{P(x)} = \frac{\left(-\frac{1}{2} x + 1\right) \frac{2}{3}}{\frac{1}{8} x^2 - \frac{1}{3} x + \frac{2}{3}} = \frac{-\frac{1}{3} x + \frac{2}{3}}{\frac{1}{8} x^2 - \frac{1}{3} x + \frac{2}{3}}$$

$$P(C_2/x) = \frac{P(x/C_2) . P(C_2)}{P(x)} = \frac{\frac{3}{8} x^2 \times \frac{1}{3}}{\frac{1}{8} x^2 - \frac{1}{3} x + \frac{2}{3}} = \frac{\frac{x^2}{8}}{\frac{1}{8} x^2 - \frac{1}{3} x + \frac{2}{3}}$$

4- $$P(C_1/x) \underset{}{\overset{x \in C_1}{\gtrless}} P(C_2/x)$$

$$\frac{P(x/C_1) . P(C_1)}{\cancel{P(x)}} \gtrless \frac{P(x/C_2) . P(C_2)}{\cancel{P(x)}}$$

$$\frac{P(x/C_1)}{P(x/C_2)} \underset{x \in C_2}{\overset{x \in C_1}{\gtrless}} \frac{P(C_2)}{P(C_1)}$$

$$\frac{-\frac{1}{2} x + 1}{\frac{3}{8} x^2} \gtrless \frac{\frac{1}{3}}{\frac{2}{3}}$$

5. seuil

si $P(C_1) = P(C_2)$

$$\frac{P(x/C_1)}{P(x/C_2)} \gtrless 1 \Rightarrow P(x/C_2) \gtrless P(x/C_1)$$
$$\Rightarrow \frac{3}{8} x^2 = -\frac{1}{2} x + 1$$

$\frac{3}{8}x^2 + \frac{1}{2}x - 1$

$\Delta = b^2 - 4ac = \left(\frac{1}{2}\right)^2 - 4 \times \frac{3}{8} \times (-1) = \frac{7}{4}$

$x_1 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{-\frac{1}{2} - \sqrt{\frac{7}{4}}}{2 \times \frac{3}{8}} = -2.43$

$x_2 = 1.09$

seuil c'est $\boxed{x = 1.09}$

donc si $x < 1,09 \Rightarrow x \in C_1$

~~Sinon~~ si $x > 1,09 \Rightarrow x \in C_2$

6- $x = 1,5$

$p(x/C_1) = \frac{-1}{2}x + 1 = 0,25$

$p(x/C_2) = \frac{3}{8} \times (1,5) = 0,84$

donc $x \in C_2$ $\left\{ \frac{p(x/C_1)}{p(x/C_2)} = \frac{0.25}{0.84} = 0,29 < 1 \right.$ donc $x \notin C_1$

Ex 4:

1- combien de bit

$2^m \geq 14$

$2^4 = 16 \geq 14$

$m = 4$

dynamique $= 2^4 = 16$

b -

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
h(k)	0	16	20	16	12	8	4	0	4	0	0	0	8	0	12	0
H(k)	0	16	36	52	64	72	76	76	80	80	80	80	88	88	100	100

c - bruit de type impulsionnel. = bruit sel et poivre

↳ pixels noirs et blancs apparaissant de manière ponctuelle dans l'image.

- filtre moyenneur: $\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

pixels (1,1) : $\frac{1}{9}(6+5+4+5+0+3+4+3+2) = \cancel{10,66}$

(3,6) : $\frac{1}{9}(14+1+2+12+0+1+8+12+14)$ $= 3,55$
$= 7,11$

(4,4) : $\frac{1}{9}(14+12+12+12+0+8+12+8+15) = 10,33$

(4,8) :

(5,1) :

(5,5) : $\frac{1}{9}(0+8+12+8+15+12+12+12+14) = 10,33$

(6,3) :

(8,8) :

(1,4) :

(1,8) :

(8,2) :

(8,5) :

d - $\frac{1}{12}\left((4-3,55)^2+(8-7,11)^2+\dots\right)$

e - (1,1) :

$\underbrace{0,2,3}\ 3,\underbrace{4,4,5,5,6}$

$\frac{N-1}{2} = 4$

le median = 4.

$(3,6)$: $\underbrace{0, 1, 1, 2}$, $\boxed{8}$, $\underbrace{12, 12, 14, 14}$ ⇒ le median = 8.

$(4,4)$: $\underbrace{0, 8, 8, 12}$, $\boxed{12}$, $\underbrace{12, 12, 14, 15}$

le median : 12.