

Le hors d'œuvre

$$U_x = \begin{bmatrix} -\frac{\alpha}{2} & 0 & \frac{\alpha}{2} \\ -(1-\alpha) & 0 & (1-\alpha) \\ -\frac{\alpha}{2} & 0 & \frac{\alpha}{2} \end{bmatrix} \quad \alpha \geq 0$$

1°/ pour obtenir masque Sobel, Prewitt:

Prewitt: $U_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \times \frac{1}{3}$

$$-\frac{\alpha}{2} = -\frac{1}{3} \Rightarrow \alpha = \frac{2}{3}$$

$$1-\alpha = \frac{1}{3} \Rightarrow \alpha = 1 - \frac{1}{3} = \frac{2}{3}$$

$$\boxed{\alpha = \frac{2}{3}}$$

Sobel: $U_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \times \frac{1}{4}$

$$-\frac{\alpha}{2} = -\frac{1}{4} \Rightarrow \alpha = \frac{1}{2}$$

$$1-\alpha = \frac{1}{4} \Rightarrow \alpha = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\boxed{\alpha = \frac{1}{2}}$$

2°/ $I = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

$J = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

a- Produit de convolution:

$$I_x = I * U_x$$

$$= -\frac{\alpha}{2} - (1-\alpha) - \frac{\alpha}{2}$$

$$I_x = -\alpha - 1 + \alpha = -1$$

$$U_y = \begin{bmatrix} -\frac{\alpha}{2} & -(1-\alpha) & -\frac{\alpha}{2} \\ 0 & 0 & 0 \\ \frac{\alpha}{2} & 1-\alpha & \frac{\alpha}{2} \end{bmatrix}$$

$$I_y = I * U_y$$

$$= -\frac{\alpha}{2} - (1-\alpha) + \frac{\alpha}{2} + 1 - \alpha$$

$$I_y = 0$$

b. Module de gradient du bloc I au centre

$$G_I = \sqrt{(-1)^2 + 0^2} = 1$$

c. Module de gradient du bloc J

$$J_x = J * U_x$$

$$= -\frac{\alpha}{2} - (1-\alpha) - \frac{\alpha}{2} + \frac{\alpha}{2}$$

$$= -\frac{\alpha}{2} - 1 + \alpha = -1 + \frac{\alpha}{2}$$

$$J_x = \frac{\alpha}{2} - 1$$

$$J_y = J * U_y$$

$$= -\frac{\alpha}{2} + \frac{\alpha}{2} + 1 - \alpha + \frac{\alpha}{2}$$

$$= 1 - \frac{\alpha}{2}$$

$$J_y = 1 - \frac{\alpha}{2}$$

$$G_J = \sqrt{J_x^2 + J_y^2}$$

$$= \sqrt{\left(\frac{\alpha}{2} - 1\right)^2 + \left(1 - \frac{\alpha}{2}\right)^2}$$

$$= \sqrt{\frac{\alpha^2}{4} - \alpha + 1 + 1 - \alpha + \frac{\alpha^2}{4}}$$

$$= \sqrt{\frac{\alpha^2}{2} - \alpha + 2}$$

d. Pour quelle α les 2 modules du gradient sont identiques

$$G_{\pm} = G_{\mp}$$

$$\frac{1}{2} = \sqrt{\frac{\alpha^2}{2} - 2\alpha + 2}$$

$$\frac{\alpha^2}{2} - 2\alpha + 2 = \frac{1}{4}$$

$$\alpha^2 - 4\alpha + 4 = \frac{1}{2}$$

$$\alpha^2 - 4\alpha + \frac{7}{2} = 0$$

$$\Delta = b^2 - 4ac$$

$$= 16 - 4 \times 1 \times \frac{7}{2} = 8 = (2\sqrt{2})^2$$

$$\alpha_1 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{4 - 2\sqrt{2}}{2} = 2 - \sqrt{2}$$

$$\alpha_2 = 2 + \sqrt{2}$$

\Rightarrow On choisit la plus petite valeur $\{ \Rightarrow \alpha = 2 - \sqrt{2}$

Prq? On a $-(1-\alpha) = -2 < 0$ d'après Sobol et $1/x$

$$\begin{cases} 1-\alpha > 0 \\ \alpha < 1 \end{cases}$$

D'où $\boxed{\alpha = 2 - \sqrt{2}}$

Le plat principal

$$P(C_1) = 1/3 \quad P(C_2) = 2/3$$

① Formule de Bayes

$$P(A/B) = \frac{P(A) \cdot P(B/A)}{P(B)}$$

$$P(X/C_i) = \frac{P(X) \cdot P(C_i/X)}{P(C_i)} \quad \text{avec } i=1, 2$$

$$P(X/C_i) \cdot P(C_i) = P(X) \cdot P(C_i/X)$$

② Distribution de probabilité $p(x)$

$$P(X) = \sum_{i=1}^2 P(X/C_i) \cdot P(C_i) = P(X/C_1) \cdot P(C_1) + P(X/C_2) \cdot P(C_2)$$

$$P(X/C_1) = ax + b \quad (\text{selon graphique})$$

$$x=0 \Rightarrow b=1$$

$$a = \frac{0-1}{2-0} = -\frac{1}{2} \Rightarrow a = -\frac{1}{2}$$

$$P(X/C_1) = -\frac{x}{2} + 1 = \frac{2-x}{2}$$

$$P(X/C_2) = ax + b$$

$$x=0 \Rightarrow b=0$$

$$a = \frac{1-0}{2-0} = \frac{1}{2}$$

$$\Rightarrow P(X/C_2) = \frac{x}{2}$$

$$P(X) = P(X/C_1) \cdot P(C_1) + P(X/C_2) \cdot P(C_2)$$

$$P(X) = \frac{2-x}{2} \times \frac{1}{3} + \frac{x}{2} \times \frac{1}{3} = \frac{2-x}{6} + \frac{x}{6} = \frac{2-x+x}{6} = \frac{x+2}{6}$$

$$P(X) = \frac{x+2}{6}$$

3° Expression des probas à posteriori

$$P(C_1/X) = \frac{P(C_1) \cdot P(X/C_1)}{P(X)}$$

$$= \frac{\frac{1}{3} \cdot \frac{2-x}{2}}{\frac{x+2}{6}} = \frac{\frac{2-x}{6}}{\frac{x+2}{6}} = \frac{2-x}{x+2}$$

$$P(C_2/X) = \frac{P(C_2) \cdot P(X/C_2)}{P(X)}$$

$$= \frac{\frac{1}{3} \cdot \frac{x}{2}}{\frac{x+2}{6}} = \frac{\frac{x}{6}}{\frac{x+2}{6}} = \frac{x}{x+2}$$

4° Règle de décision pour un pixel donné x :

$$P(C_1/X) \geq P(C_2/X)$$

$$\frac{2-x}{x+2} \geq \frac{x}{x+2}$$

$$2-x \geq x \Rightarrow 0 \geq 3x-2$$

5° Valeur du seuil où il y a indécision

$$P(C_1/X) = P(C_2/X)$$

$$\Leftrightarrow 3x-2=0$$

$$x = \frac{2}{3}$$

6-7° Les expressions du risque $\lambda(a_i/x)$ pour $i=1,2,3$ $\lambda(a_i/x) = \sum_{j=1}^2 P(a_i/C_j) \cdot P(C_j/X)$

$$\lambda(a_1/x) = \sum_{j=1}^2 P(a_1/C_j) \cdot P(C_j/X) = P(a_1/C_1) \cdot P(C_1/X) + P(a_1/C_2) \cdot P(C_2/X)$$

$$= 0 + 1 \cdot \frac{x}{x+2}$$

$$\begin{aligned} \lambda(a_1/x) &= P(a_1/G_1) \cdot P(G_1/x) + P(a_1/G_2) \cdot P(G_2/x) \\ &= 1 \cdot \frac{2-x}{2+x} + 0 \Rightarrow \lambda(a_1/x) = \frac{2-x}{2+x} \end{aligned}$$

$$\begin{aligned} \lambda(a_3/x) &= P(a_3/G_1) \cdot P(G_1/x) + P(a_3/G_2) \cdot P(G_2/x) \\ &= \frac{1}{4} \cdot \frac{2-x}{2+x} + \frac{1}{4} \cdot \frac{2x}{2+x} \\ &= \frac{2-x+2x}{4(2+x)} = \frac{2+x}{4(2+x)} = \frac{1}{4} \end{aligned}$$

8° Déterminer les domaines de valeurs de x vérifiant l'action optimale.

$$a^*(x) = \min \{ \lambda(a_i/x) \}, i=(1, 2, 3)$$

$$a^*(x) = \min \left\{ \frac{2-x}{2+x}, \frac{2x}{2+x}, \frac{1}{4} \right\}$$

Les intervalles :

$$\frac{2-x}{2+x} = \frac{1}{4}$$

$$\frac{2x}{2+x} = \frac{1}{4}$$

$$8-4x = 2+x$$

$$8x = 2+x$$

$$x = \frac{2}{7} \approx 0,3$$

$$x = \frac{6}{5} \approx 1,2$$

D'où $[0, \frac{2}{7}]$, $[\frac{2}{7}, \frac{6}{5}]$, $[\frac{6}{5}, 1]$

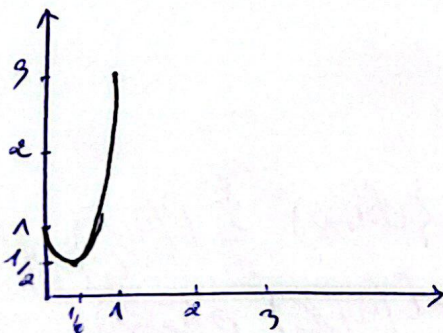
$$\text{le Risque totale} = R(a_i) = \int_0^1 \lambda(a_i/x) \cdot P(x) \cdot dx \approx 0,1714$$

le dessin (7pts)

$$R(x) = -4x + 6x^2 + 1, \quad x \in [0, 1]$$

a° Histogramme :

x	0	1	$\frac{1}{2}$
$R(x)$	1	3	$\frac{1}{2}$



b° valeur de λ définissant le minimum de k'

(5)

Minimum $\Rightarrow k'(\lambda) = 0$

$$-4 + 12\lambda = 0$$

$$\lambda = \frac{4}{12} = \frac{1}{3} \Rightarrow \boxed{\lambda = \frac{1}{3}}$$

c° Densité de probabilité: $\int_0^1 k(\lambda) d\lambda = 1$

$$\int_0^1 k(\lambda) d\lambda = \int_0^1 -4\lambda + 6\lambda^2 + 1 d\lambda$$

$$= -4 \left[\frac{\lambda^2}{2} \right]_0^1 + 6 \left[\frac{\lambda^3}{3} \right]_0^1 + [\lambda]_0^1$$

$$= -\frac{4}{2} + \frac{6}{3} + 1 = -2 + 2 + 1 = 1$$

d° La moyenne de l'image:

$$\int_0^1 \lambda k(\lambda) d\lambda = \int_0^1 -4\lambda^2 + 6\lambda^3 + \lambda d\lambda$$

$$= -4 \left[\frac{\lambda^3}{3} \right]_0^1 + 6 \left[\frac{\lambda^4}{4} \right]_0^1 + \left[\frac{\lambda^2}{2} \right]_0^1$$

$$= -\frac{4}{3} + \frac{6}{4} + \frac{1}{2} = \frac{2}{3}$$

e° Transformation $s = g(\lambda)$ permettant d'égaliser l'image:

$$g(\lambda) = \int_0^m k(\lambda) d\lambda = (1-0) \int_0^m k(\lambda) d\lambda$$

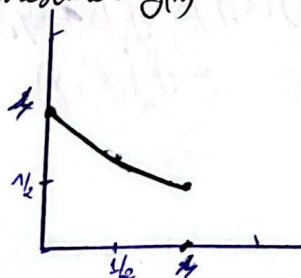
$$= \int_0^m -4\lambda^2 + 6\lambda^3 + \lambda d\lambda$$

$$= -4 \left[\frac{\lambda^3}{3} \right]_0^m + 6 \left[\frac{\lambda^4}{4} \right]_0^m + [\lambda]_0^m$$

$$= -\frac{4}{3} m^3 + \frac{6}{4} m^4 + m = -\frac{2}{3} m^3 + \frac{3}{2} m^4 + m$$

8°

Représenter $g(\lambda)$ $m = g(\lambda) = \frac{1}{\lambda+1}$



λ	0	1	1/2
$g(\lambda)$	1	1/2	2/3

9° Expression de l'histogramme $f(m)$ obtenu après l'application de g sur k

$$f(m) = \frac{k(\lambda)}{|g'(\lambda)|} = k(\lambda) \cdot \frac{d\lambda}{dm}$$

$$\lambda = g^{-1}(m)$$

$$g(\lambda) = m$$

$$\frac{1}{\lambda+1} = m \Rightarrow 1 = m(\lambda+1) \Rightarrow m\lambda + m = 1 \Rightarrow m\lambda = 1 - m \Rightarrow \lambda = \frac{1-m}{m}$$

(6)

$$\lambda = \frac{1-m}{m}$$

$$\begin{aligned} R(\lambda) &= -4 \left(\frac{1-m}{m} \right) + 6 \left(\frac{1-m}{m} \right)^2 + 1 \\ &= \frac{-4(1-m)}{m} + \frac{6(1-m)^2}{m^2} + \frac{m^2}{m^2} \\ &= \frac{-4m(1-m) + 6(1-m)^2 + m^2}{m^2} = \frac{11m^2 - 16m + 6}{m^2} \end{aligned}$$

$$\vartheta'(\lambda) = \left(\frac{1}{\lambda+1} \right)' = \frac{-1}{(\lambda+1)^2} = \frac{-1}{\left(\frac{1-m}{m} \right)^2} = - \left(\frac{m}{1-m} \right)^2$$

$$f(m) = \frac{R(\lambda)}{\vartheta'(\lambda)} = \frac{11m^2 - 16m + 6}{m^2} \times - \frac{m^2}{(1-m)^2} = \frac{-11m^2 + 16m + 6}{(1-m)^2}$$

f est fausse

i/ Supposons $f(m) = 5m^2 - 10m + 6$

Peut-on considérer f comme une densité de proba.

$$\begin{aligned} \int_0^1 f(m) dm &= \int_0^1 5m^2 - 10m + 6 \\ &= 5 \left[\frac{m^3}{3} \right]_0^1 - 10 \left[\frac{m^2}{2} \right]_0^1 + 6 \left[m \right]_0^1 \\ &= \frac{5}{3} - \frac{10}{2} + 6 = \frac{8}{3} \neq 1 \end{aligned}$$

J'ai plus faim!

1° Définition de TF $F(u, v)$ de $f(x, y)$

$$F(u, v) = \frac{1}{HN} \sum_{x=0}^{N-1} \sum_{y=0}^{H-1} f(x, y) \cdot \exp \left(-2\pi j \left(\frac{ux}{N} + \frac{vy}{H} \right) \right)$$

2°

On pose $\begin{cases} x' = x - my \\ x = x' + my \end{cases} \Rightarrow dx' = dx$

$$\begin{aligned} F(u, v) &= \frac{1}{HN} \sum \sum \vartheta(x, y) \exp \left(-2\pi j \left(\frac{ux}{N} + \frac{vy}{H} \right) \right) \\ &= \frac{1}{HN} \sum \sum f(x', y) \exp \left(-2\pi j \left(\frac{(x' + my)u}{N} + \frac{vy}{H} \right) \right) \\ &= \frac{1}{HN} \sum \sum f(x', y) \exp \left(-2\pi j \left(\frac{ux'}{N} + \frac{vy}{H} + \frac{umy}{N} \right) \right) \\ &= \frac{1}{HN} \sum \sum f(x', y) \exp \left(-2\pi j x' \left(\frac{u}{N} \right) \right) \cdot \exp \left(-2\pi j y \left(\frac{v}{H} + \frac{um}{N} \right) \right) \end{aligned}$$

$$\begin{aligned} \vartheta(x, y) &= f(x + my, y) \\ &= f(x', y) \end{aligned}$$