Les polynômes-Indications

Exercice 1   1. $\mathrm{pgcd}(X^5 + 3X^4 + X^3 + X^2 + 3X + 1, X^4 + 2X^3 + X + 2) = X^3 + 1$.

  2. $\mathrm{pgcd}(X^4 + X^3 - 3X^2 - 4X - 1, X^3 + X^2 - X - 1) = X + 1$

  3. $\mathrm{pgcd}(X^3 - X^2 - X - 2, X^5 - 2X^4 + X^2 - X - 2) = X - 2$.

  4. $\mathrm{pgcd}(X^4 + X^3 - 2X + 1, X^3 + X + 1) = 1$.

  5. $\mathrm{pgcd}(X^5 + 5X^4 + 9X^3 + 7X^2 + 5X + 1, X^4 + 2X^3 + 2X^2 + X + 1) = 1$.

Exercice 2   1. $D = X^2 + 3X + 2 = A(\frac{1}{18}X - \frac{1}{6}) + B(-\frac{1}{18}X^2 + \frac{1}{9}X + \frac{5}{18})$.

  2. $D = 1 = A(-X^3) + B(X^5 + X^3 + X + 1)$.

Exercice 3
$$\left(x^2 + \sqrt{2}x + 1\right)\left(x^2 - \sqrt{2}x + 1\right)$$

Exercice 4 L'ordre de multiplicité est 2.

Exercice 5 Pour $a = \frac{1}{64}$; la racine multiple est $-\frac{1}{2}$.

Exercice 6   1. $\begin{cases} X^3 - 3 &= (X - \sqrt[3]{3})(X^2 + \sqrt[3]{3}\,X + \sqrt[3]{9}) \\ &= (X - \sqrt[3]{3})(X + \frac{\sqrt[3]{3}}{2} - i\frac{\sqrt{3}\sqrt[3]{3}}{2})(X + \frac{\sqrt[3]{3}}{2} + i\frac{\sqrt{3}\sqrt[3]{3}}{2}). \end{cases}$

  2. $\begin{cases} X^{12} - 1 &= (X - 1)(X + 1)(X^2 + 1)(X^2 - X + 1)(X^2 + X + 1) \times \\ & \quad (X^2 - \sqrt{3}\,X + 1)(X^2 + \sqrt{3}\,X + 1) \\ &= (X - 1)(X + 1)(X - i)(X + i) \times \\ & \quad \left(X - \frac{1+i\sqrt{3}}{2}\right)\left(X - \frac{1-i\sqrt{3}}{2}\right)\left(X - \frac{-1+i\sqrt{3}}{2}\right)\left(X - \frac{-1-i\sqrt{3}}{2}\right) \times \\ & \quad \left(X - \frac{\sqrt{3}+i}{2}\right)\left(X - \frac{\sqrt{3}-i}{2}\right)\left(X - \frac{-\sqrt{3}+i}{2}\right)\left(X - \frac{-\sqrt{3}-i}{2}\right). \end{cases}$

Exercice 7

Exercice 8

Exercice 9