DL HW2

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1 Dropout vs. Batch Normalization

1.1

$$\mathbb{E}_D[P]_{ij} = \mathbb{E}_D[(D \odot X)_{ij}] = X_{ij}\mathbb{E}_D[D_{ij}] = X_{ij}((1-p)(0) + p(1)) = pX_{ij}$$

$$\mathbb{E}_D[(P^TP)]_{ij} = \begin{cases} \sum_{k=1}^{N} \mathbb{E}_D[D_{ki}D_{kj}X_{ki}X_{kj}] \overset{D_{ki},\, D_{kj} \text{ are independent}}{=} \sum_{k=1}^{N} \mathbb{E}_D[D_[ki]]\mathbb{E}_D[D_{kj}]X_{ki}X_{kj} \\ \overset{\text{Bernoulli expectation is p}}{=} p^2(X^TX)_{ij} & \text{if } i \neq j \\ \\ \sum_{k=1}^{N} \mathbb{E}_D[D_{ki}^2X_{ki}X_{kj}] = \sum_{k=1}^{N} \mathbb{E}_D[D_{ki}^2]X_{ki}X_{kj} = ((p)(1^2) + (1-p)(0^2))(X^TX)_{ij} \\ = p(X^TX)_{ij} & \text{if } i = j \end{cases}$$

1.2

$$\begin{aligned} \mathbb{E}_D[||y - P\hat{w}||_2^2] &= \mathbb{E}_D[(y - P\hat{w})^T(y - P\hat{w})] \\ &= \mathbb{E}_D[(y^T - \hat{w}^TP^T)(y - P\hat{w})] \\ &= \mathbb{E}_D[y^Ty - y^TP\hat{w} - \hat{w}^TP^Ty + \hat{w}^TP^TP\hat{w}] \\ &= y^Ty - y^TpX\hat{w} - \hat{w}^TpX^Ty + \hat{w}^Tp^2X^TX\hat{w} + (-p^2 + P)\hat{w}^T\hat{\Gamma}^2\hat{w} \\ &= (y^T - p\hat{w}^TX^T)(y - pX\hat{w}) + p(1-p)\hat{w}^T\hat{\Gamma}^2\hat{w} \\ &= ||y - pX\hat{w}||_2^2 + p(1-p)||\hat{\Gamma}\hat{w}||_2^2 \end{aligned}$$

1.3

$$\begin{aligned} w = p\hat{w} \Rightarrow &\mathbb{E}_D[||y - P\hat{w}||_2^2] \\ &= ||y - Xw||_2^2 + \frac{1-p}{p}||\hat{\Gamma}w||_2^2 \\ &= ||y - Xw||_2^2 + ||\Gamma w||_2^2 \\ &\Rightarrow \Gamma = \sqrt{\frac{1-p}{p}}\hat{\Gamma} \end{aligned}$$

1.4

$$\Gamma w = \sqrt{\lambda}\tilde{w} \Rightarrow w = \sqrt{\lambda}\Gamma^{-1}\tilde{w}$$
$$\Rightarrow \mathbf{L}(\tilde{w}) = ||y - X\sqrt{\lambda}\Gamma^{-1}\tilde{w}||_2^2 + \lambda||\tilde{w}||_2^2$$
$$= ||y - \tilde{X}\tilde{w}||_2^2 + \lambda||\tilde{w}||_2^2 \quad ; \quad \tilde{X} = X\sqrt{\lambda}\Gamma^{-1}$$

$$\tilde{w}_i = \alpha_i w_i \quad ; \quad \alpha_i = \lambda^{-0.5}\Gamma_{ii} \quad i = 1 \dots N$$
$$\tilde{X}_{ij} = \beta_j X_{ij} \quad ; \quad \beta_i = \lambda^{0.5}\Gamma_{ii}$$

1.5

As mentioned in the previous subsection, each element of the jth column of matrix X is divided by the jth element of the diagonal of gamma, which is the sum of all elements in X multiplied by a constant. This means that we are normalizing the batch.

In conclusion, using dropout is like to employing batch normalization with ridge regularization.

1.6

$$\frac{\partial J}{\partial w_j} = (y_d - \sum_{k=1}^{n}(w_k + \delta_k)x_k)x_j$$
$$\mathbb{E}_{\delta_k}[\frac{\partial J}{\partial w_j}] = (y_d - \sum_{k=1}^{n} w_k x_k)x_j$$

1.7

$$J = \frac{1}{2}(y_d - \sum_k (w_k - \delta_k)x_k)^2$$
$$= \frac{1}{2}(y_d - sum_k w_k x_k + \sum_k \delta_k x_k)^2$$
$$= \frac{1}{2}(y_d - sum_k w_k x_k)^2 + \frac{1}{2}(\sum_k \delta_k x_k)^2 + (\sum_k \delta_k x_k)(y_d - \sum_k w_k x_k)$$
$$= \frac{1}{2}(y_d - sum_k w_k x_k)^2 + \sum_i \sum_j \delta_i \delta_j x_i x_j + (\sum_k \delta_k x_k)(y_d - \sum_k w_k x_k)$$
$$\Rightarrow \mathbb{E}_\delta[J] = \frac{1}{2}(y_d - \sum_k w_k x_k)^2 + \sum_k \alpha w_k^2 x_k^2$$

The final term resembles a regularization component, and it is a specific instance of Tikhonov regularization, where the i-th diagonal element is equal to $\sqrt{\alpha}x_i$.

1.8

Spatial dropout sets an entire feature to zero, whereas this dropout technique makes some pixels zero. Consequently, the spatial filter will compel the model to not rely on a single filter, and the other force it to not depend on a specific pixel, which could correspond to a pattern in a particular image region.

In contrast, spatial dropout will lead to redundant features across different filters, while Gaussian dropout results in spatial redundancy.

2 Backpropagation of CNN

$$\frac{\partial L}{\partial W_j} = \sum_{i=0}^{N-j-1} \frac{\partial L}{\partial Z_i} X_{i+j}$$

This is the same as the convolution formula presented in formula 8, so we can derive the following formula:

$$\frac{\partial L}{\partial W} = \frac{\partial L}{\partial Z} * X$$

3 Batch Normalization

3.1

$$\begin{aligned}\frac{\partial L}{\partial x} &= \frac{\partial \hat{x}}{\partial x}\frac{\partial y}{\partial \hat{x}}\frac{\partial L}{\partial y} + \frac{\partial \sigma_B^2}{\partial x}\frac{\partial \hat{x}}{\partial \sigma_B^2}\frac{\partial y}{\partial \hat{x}}\frac{\partial L}{\partial y} + \frac{\partial \mu}{\partial x}[\frac{\partial \hat{x}}{\partial \mu} + \frac{\partial \sigma_B^2}{\partial \mu}\frac{\partial \hat{x}}{\partial \sigma_B^2}]\frac{\partial y}{\partial \hat{x}}\frac{\partial L}{\partial y} \\ &= \frac{1}{\sigma_B}\gamma\frac{\partial L}{\partial y} + \frac{2}{m}(x-\mu_B)^T\frac{1}{\sigma_B}\gamma\frac{\partial L}{\partial y} + \frac{-1}{m\sigma_B}\gamma[1 + \frac{2}{m}\sum_{i=1}^{m}(x_i - \mu_B)]\frac{\partial L}{\partial y}\end{aligned}$$

$$\frac{\partial L}{\partial \gamma} = x^T\frac{\partial L}{\partial y}$$

$$\frac{\partial L}{\partial \beta} = \sum_{i=1}^{N}\frac{\partial L}{\partial y_i}$$

3.2

1. Improves gradient flow through the network; by reducing the dependence of gradients on the scale (or initialization) of the parameters
2. Speedup the learning process because reduces the strong dependence on the initialization
3. Acts as a form of regularization in a way that shown in first question.

3.3

Since we typically use batch normalization before the activation function (e.g., ReLU), if we do not properly locate the batch, half of the values will be deleted, and this inactive bias will reduce the power of the model and may not be consistent with the purpose of the model. Additionally, if we remove the scale and shift, the output of all layers will become very similar, and this will prevent progress in extracting good features from the data.

3.4

Each batch is standardized in training time by subtracting its mean and dividing by its standard deviation. However, we perform the same calculation for test time using the running average of the mean and standard deviation.

We expect to see approximately the same numbers during test time and train time, which would happen if the data was well-shuffled. In this case, since each batch contains only a single class, the means can be very different from batch to batch.

As a result, the overall mean, which is more representative of the last batch, can be very different. It may be closer to the mean of class A, but we are testing a batch that contains class B, so the result is quite different from the train time.

4 CNN Basic

4.1

No. Because this bais will be removed in shifting process of batch normalization.

4.2

No. Because the output of convolution layer will scale to variance one in batch normalization.

4.3

(i)
For each dot product: k^2c
Number of dot products: $(\frac{w-k}{s}+1)(\frac{h-k}{s}+1)$
Total: $k^2c(\frac{w-k}{s}+1)(\frac{h-k}{s}+1)$

(ii)
For each pool average: k^2c
Number of pool averages: $(\frac{w-k}{s}+1)(\frac{h-k}{s}+1)$
Total: $k^2c(\frac{w-k}{s}+1)(\frac{h-k}{s}+1)$

(iii)
Total: whc

4.4

Reducing width of result tensor.(make more compacte features)

4.5

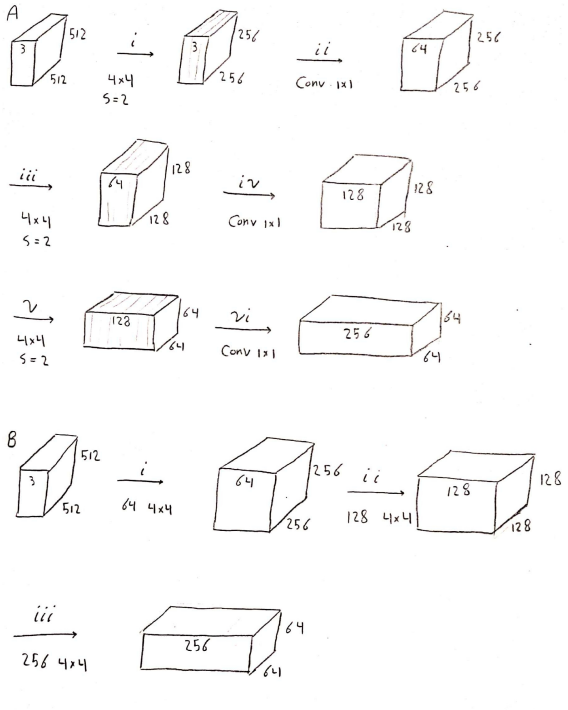
Losing some information will lead to a more generalized model because it can not memorize all inputs.

4.6

$$\text{receptive field} = 2(2(\overbrace{2(\underbrace{1\times 2}_{pool}-1)+3}^{conv})-1)+3 = 21$$

5 MobileNet

5.1



5.2

A:

$$\#params = (3 * 4 * 4) + (64 * 3) + (64 * 4 * 4) + (128 * 64) + (128 * 4 * 4) + (128 * 256) = 44272$$

B:

$$\#params = (64 * 3 * 4 * 4) + (128 * 64 * 4 * 4) + (256 * 128 * 4 * 4) = 658432$$

5.3

A:

$$\begin{aligned}\#ops = &(256 * 256 * 3 * 4 * 4) + (256 * 256 * 64 * 3) \\ &+ (128 * 128 * 64 * 4 * 4) + (128 * 128 * 128 * 64) \\ &+ (64 * 64 * 128 * 4 * 4) + (64 * 64 * 256 * 128) \\ &= 309,329,920\end{aligned}$$

B:

$$\begin{aligned}\#ops = &(256 * 256 * 64 * 4 * 4 * 3) \\ &+ (128 * 128 * 128 * 4 * 4 * 64) \\ &+ (64 * 64 * 256 * 4 * 4 * 128) \\ &= 4,496,293,888\end{aligned}$$

5.4

Using depthwise separable convolutions compared to full convolutions for generating same size output is 15 times smaller and 14.5 times less compute intensive.

5.5

The MobileNet model is based on depthwise separable convolutions which is a form of factorized convolutions which factorize standard convolution into a depthwise convolution and a 1×1 convolution called pointwise convolution (As showed in previos sections). This factorization has the effect of drastically reducing compution and model size.

Standard convolution have the computional cost of:

$$D_K \cdot D_k \cdot M \cdot N \cdot D_F \cdot D_F$$

where the computional cost depends multiplicativeky on the number of input channels M, the number of output channels N the kernel size $D_k \times D_k$ and the feature map size $D_F \times D_F$. MobileNet uses depthwise separable convolutions to break the interaction between the number of output channels and the size of the kernel.

Depthwise separable convolution cost is:

$$D_K \cdot D_K \cdot M \cdot D_F \cdot D_F + M \cdot N \cdot D_F \cdot D_F$$

which is the sum of the depthwise and 1×1 pointwise convolutions. MobileNet get a reduction in computation of:

$$\frac{D_K \cdot D_K \cdot M \cdot D_F \cdot D_F + M \cdot N \cdot D_F \cdot D_F}{D_K \cdot D_k \cdot M \cdot N \cdot D_F \cdot D_F} = \frac{1}{N} + \frac{1}{D_K^2}$$

Result of expremints show that, for example, MobileNet is nearly as accurate as VGG16 while being 32 times smaller and 27 times less compute intensive. It is more accurate than GoogleNet while being smaller and more than 2.5 times less computation.

In conclusion, it is not necessary to do two steps of standard convolution in one step It is more efficient to only filters input channels, and then combine them linearly to create new features.

5.6

1. **Width Multiplier: Thinner Models**
   In order to construct smaller and less computationally expensive models the paper introduce a very simple parameter α called width multiplier. For a given layer and width multiplier α, the number of input channels M becomes αM and the number of output channels N becomes αN. where $\alpha in(0, 1])$. $\alpha = 1$ is the baseline MobileNet and $\alpha < 1$ are reduced MobileNet. Width multiplier has the effect of reducing computional cost and the number of parameters quadratically by roughly α^2.

2. **Resolution Multiplier: Reduced Representation**
   The second hyper-parameter to reduce the computional cost of a neural network is resolution multiplier ρ. This multiplier will cahnge feature map size $D_F \times D_F$ to $\rho D_F \times \rho D_F$. Resolution multiplier has the effect of reducing computational cost by ρ^2.

Note that both parameters will reduce the accuracy for sake of less computional cost.

6 Loss Function: Center Loss

6.1

In clustering, we should predict y_i, The loss function for this part of NN can be:

$$y_i = \underset{k}{argmin}||x_i - C_k||_2^2$$

This loss function tries to find optimal class for a sample and center loss will make each class more compact, means decreasing intra-class distance.

Actually this is same as k-means but we are in the space of deep features.

6.2

$$\begin{aligned}
x_i &= W\hat{x_i} \\
\frac{\partial L}{\partial x_i} &= x_i - C_{y_i} \\
\frac{\partial L}{\partial W} &= \sum_{i=1}^{m} \frac{\partial x_i}{\partial W}\frac{\partial L}{\partial x_i} = \sum_{i=1}^{m}(x_i - C_{y_i})\hat{x}_i^T \\
\frac{\partial L}{\partial C_j} &= C_j - \sum_{i:y_i=j}^{m} x_i
\end{aligned}$$

6.3

Center loss decrease the intra-class distance but do not Increasing the inter-class distance This leads to misclassification. This paper make use of predefined evenly distribution class centroids, which makes the distance of inter-class be fixed and seprated from each other maximally.

The method of generating the predefined class center in the paper is based on the physical model with the lowest isotropic charge energy on the sphere, that is, it is assumed that the charge points on the hypersphere have repulsive force with each other, and the repulsive force decreases as the distance between the points increases. At the end of the movement, the point on the hypersphere stops moving. Due to the repulsive force, the points will be evenly distributed on the hypersphere. When the equilibrium state is reached, the n points on the hypersphere can be farthest apart.