

40-414 Compiler Design



Introduction to Parsing

Lecture 4

Languages and Automata

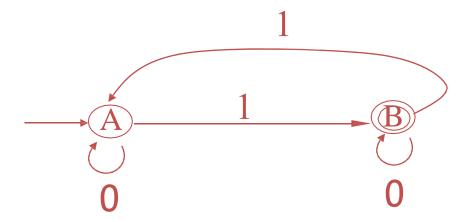
- Formal languages are very important in CS
 - Especially in programming languages
- Regular languages
 - The weakest formal languages widely used
 - Many applications
- We will also study context-free languages

Beyond Regular Languages

- Many languages are not regular
- · Strings of balanced parentheses are not regular:

$$\left\{ (^i)^i \mid i \geq 0 \right\}$$

What Can Regular Languages Express?



What Can Regular Languages Express?

Languages requiring counting modulo a fixed integer

 Intuition: A finite automaton that runs long enough must repeat states

 Finite automaton can't remember # of times it has visited a particular state

The Functionality of the Parser

· Input: sequence of tokens from lexer

• Output: parse tree of the program (But some parsers never produce a parse tree . . .)

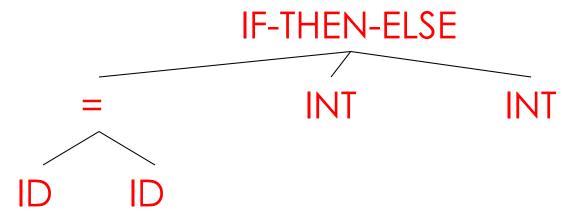
Example

· Cool

if
$$x = y$$
 then 1 else 2 fi

Parser input

Parser output



Comparison with Lexical Analysis

Phase	Input	Output
Lexer	String of characters	String of tokens
Parser	String of tokens	Parse tree (may be implicit)

The Role of the Parser

- Not all strings of tokens are programs . . .
- . . . parser must distinguish between valid and invalid strings of tokens
- · We need
 - A language for describing valid strings of tokens
 - A method for distinguishing valid from invalid strings of tokens

Chomsky Hierarchy

O Unrestricted

$$\alpha A\beta \rightarrow \alpha \gamma \beta$$

1 Context-Sensitive

| LHS | ≤ | RHS |

2 Context-Free

|LHS | = 1

3 Regular

|RHS| = 1 or 2, $A \rightarrow a \mid aB$, or $A \rightarrow a \mid Ba$

Context-Free Grammars

 Programming language constructs have recursive structure

An STMT is
 if EXPR then STMT else STMT
 while EXPR do STMT end

• • •

 Context-free grammars are a natural notation for this recursive structure

CFGs (Cont.)

- A CFG consists of
 - A set of terminals T
 - A set of non-terminals N
 - A start symbol 5 (a non-terminal)
 - A set of productions

$$X \rightarrow Y_1 Y_2 \dots Y_n$$

where $X \in N$ and $Y_i \in T \cup N \cup \{\epsilon\}$

Notational Conventions

- In these lecture notes
 - Non-terminals are written upper-case
 - Terminals are written lower-case
 - The start symbol is the left-hand side of the first production

Example of CFGs

Simple arithmetic expressions:

$$E \rightarrow E * E$$

$$| E + E$$

$$| (E)$$

$$| id$$

The Language of a CFG

Read productions as replacement rules:

$$X \rightarrow Y_1 \dots Y_n$$

Means X can be replaced by $Y_1...Y_n$

Key Idea

- 1. Begin with a string consisting of the start symbol "5"
- 2. Replace any non-terminal X in the string by a the right-hand side of some production

$$X \rightarrow Y_1 \dots Y_n$$

3. Repeat (2) until there are no non-terminals in the string

The Language of a CFG (Cont.)

More formally, write

$$X_{1} ... X_{i-1} X_{i} X_{i+1} ... X_{n} \rightarrow X_{1} ... X_{i-1} Y_{1} ... Y_{m} X_{i+1} ... X_{n}$$

if there is a production

$$X_i \rightarrow Y_1 \dots Y_m$$

The Language of a CFG (Cont.)

Write

$$X_1 \dots X_n \rightarrow^* Y_1 \dots Y_m$$

if

$$X_1 \dots X_n \rightarrow \dots \rightarrow M_1 \dots M_m$$

in 0 or more steps

The Language of a CFG

Let G be a context-free grammar with start symbol S. Then the language L(G) of G is:

$$\{a_1 \dots a_n \mid S \rightarrow^* a_1 \dots a_n \text{ and every } a_i \text{ is a terminal}\}$$

The sentential forms SF(G) of G is:

$$\{X_1 ... X_n \mid S \rightarrow *X_1 ... X_n \text{ and every } X_i \text{ is a terminal or non-terminal}\}$$

Therefore:
$$L(G) \subset SF(G)$$

Terminals

 Terminals are so-called because there are no rules for replacing them

Once generated, terminals are permanent

· Terminals ought to be tokens of the language

Examples

L(G) is the language of CFG G

Strings of balanced parentheses $\{(i)^i \mid i \geq 0\}$

Two grammars:

Arithmetic Example

Simple arithmetic expressions:

$$E \rightarrow E + E \mid E * E \mid (E) \mid id$$

Some elements of the language:

Notes

The idea of a CFG is a big step. But:

 Membership in a language is "yes" or "no"; also need parse tree of the input

Must handle errors gracefully

Need an implementation of CFG's (e.g., bison)

Derivations and Parse Trees

A derivation is a sequence of productions

$$S \rightarrow ... \rightarrow ... \rightarrow ... \rightarrow ... \rightarrow ...$$

A derivation can be drawn as a tree

- Start symbol is the tree's root
- For a production $X \to Y_1...Y_n$ add children $Y_1...Y_n$ to node X

Derivation Example

Grammar

$$E \rightarrow E + E \mid E * E \mid (E) \mid id$$

String

$$id * id + id$$

Derivation Example (Cont.)

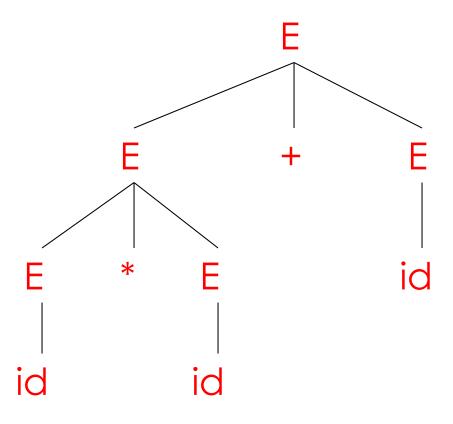
$$\rightarrow$$
 E+E

$$\rightarrow$$
 E * E+E

$$\rightarrow$$
 id * E + E

$$\rightarrow$$
 id * id + E

$$\rightarrow$$
 id * id + id

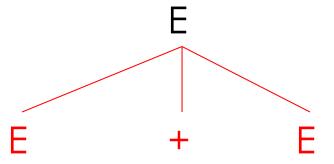


Derivation in Detail (1)

E

E

Derivation in Detail (2)



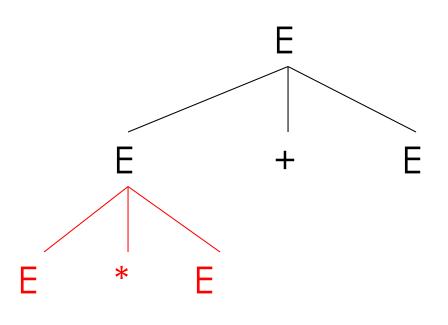
$$\rightarrow$$
 E+E

Derivation in Detail (3)

$$E$$

$$\rightarrow E+E$$

$$\rightarrow E*E+E$$



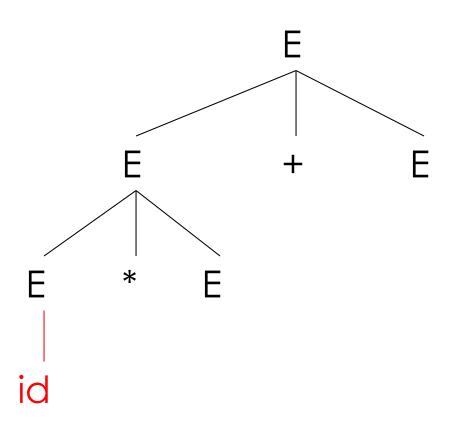
Derivation in Detail (4)

$$E$$

$$\rightarrow E+E$$

$$\rightarrow E*E+E$$

$$\rightarrow id*E+E$$



Derivation in Detail (5)

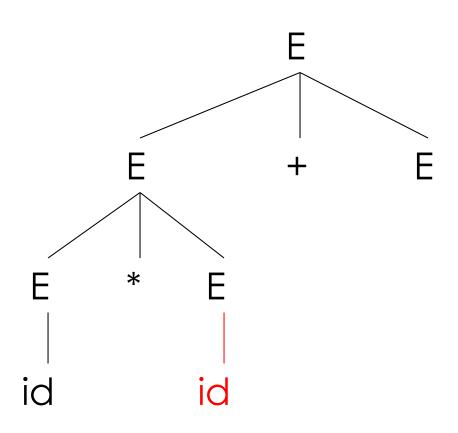
$$E$$

$$\rightarrow E+E$$

$$\rightarrow E*E+E$$

$$\rightarrow id*E+E$$

$$\rightarrow id*id+E$$



Derivation in Detail (6)

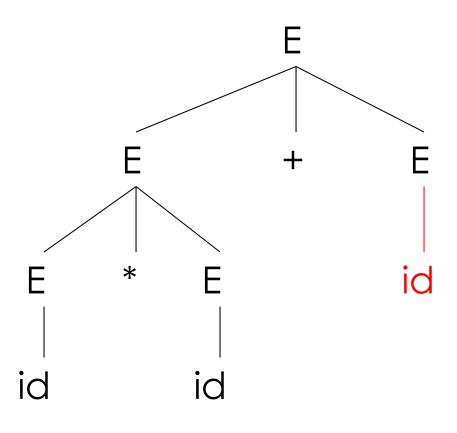
$$\rightarrow$$
 E+E

$$\rightarrow$$
 E * E+E

$$\rightarrow$$
 id * E + E

$$\rightarrow$$
 id * id + E

$$\rightarrow$$
 id * id + id



Notes on Derivations

- A parse tree has
 - Terminals at the leaves
 - Non-terminals at the interior nodes
- An in-order traversal of the leaves is the original input
- The parse tree shows the association of operations, the input string does not

Left-most and Right-most Derivations

- The example was a leftmost derivation
 - At each step, replaced the left-most non-terminal
- There is an equivalent notion of a right-most derivation

$$\rightarrow$$
 E+E

$$\rightarrow$$
 E+id

$$\rightarrow$$
 E * E + id

$$\rightarrow$$
 E * id + id

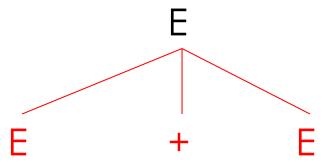
$$\rightarrow$$
 id * id + id

Right-most Derivation in Detail (1)

E

E

Right-most Derivation in Detail (2)



E

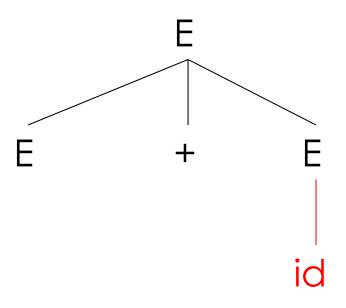
$$\rightarrow$$
 E+E

Right-most Derivation in Detail (3)

$$E$$

$$\rightarrow E+E$$

$$\rightarrow E+id$$



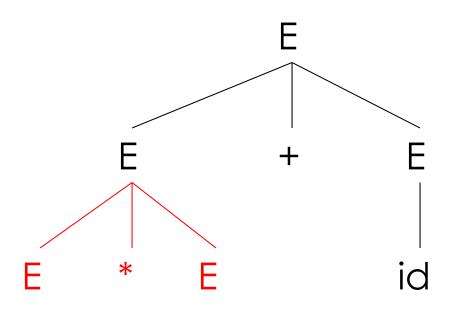
Right-most Derivation in Detail (4)

$$E$$

$$\rightarrow E+E$$

$$\rightarrow E+id$$

$$\rightarrow$$
 E*E + id



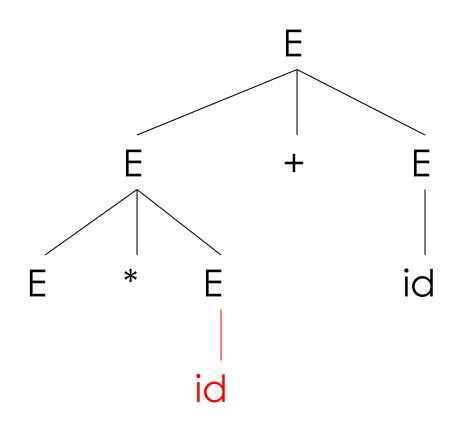
Right-most Derivation in Detail (5)

$$\rightarrow$$
 E+E

$$\rightarrow$$
 E+id

$$\rightarrow$$
 E * E + id

$$\rightarrow$$
 E*id + id



Right-most Derivation in Detail (6)

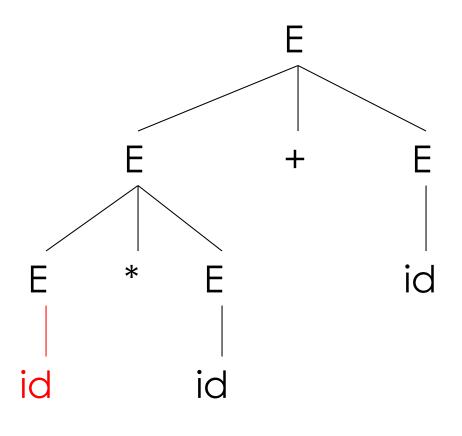
$$\rightarrow$$
 E+E

$$\rightarrow$$
 E+id

$$\rightarrow$$
 E * E + id

$$\rightarrow$$
 E * id + id

$$\rightarrow$$
 id * id + id



Derivations and Parse Trees

 Note that right-most and left-most derivations have the same parse tree

 The difference is the order in which branches are added

Summary of Derivations

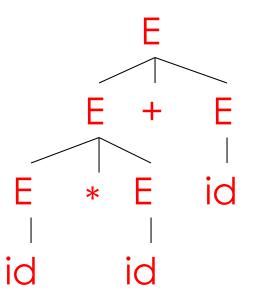
- We are not just interested in whether $s \in L(G)$
 - We need a parse tree for s
- A derivation defines a parse tree
 - But one parse tree may have many derivations
- Left-most and right-most derivations are important in parser implementation

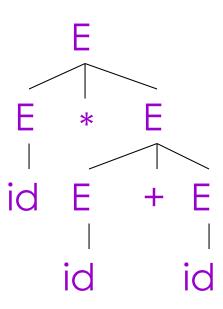
Ambiguity

- Grammar $E \rightarrow E + E \mid E * E \mid (E) \mid id$
- String id * id + id

Ambiguity (Cont.)

This string has two parse trees





Ambiguity (Cont.)

- A grammar is ambiguous if it has more than one parse tree for some string
 - Equivalently, there is more than one right-most or left-most derivation for some string
- Ambiguity is BAD
 - Leaves meaning of some programs ill-defined

Dealing with Ambiguity

- There are several ways to handle ambiguity
- Most direct method is to rewrite grammar unambiguously

$$E \rightarrow E + T \mid T$$

 $T \rightarrow T * F \mid F$
 $F \rightarrow (E) \mid id$

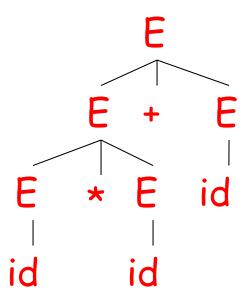
Enforces precedence of * over +

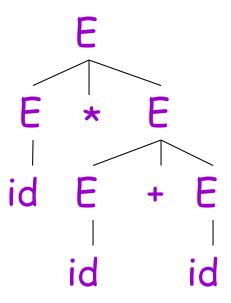
Ambiguity in Arithmetic Expressions

Recall the grammar

$$E \rightarrow E + E \mid E * E \mid (E) \mid id$$

The string id * id + id has two parse trees:



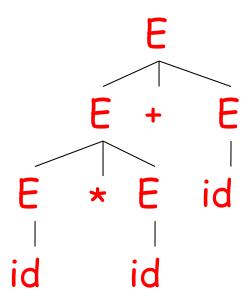


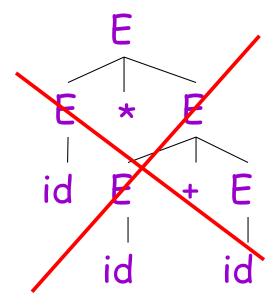
Ambiguity in Arithmetic Expressions

Recall the grammar

$$E \rightarrow E + E \mid E * E \mid (E) \mid id$$

The string id * id + id has two parse trees:





Ambiguity: The Dangling Else

Consider the grammar

```
S \rightarrow \text{if E then S}
| if E then S else S
| OTHER
```

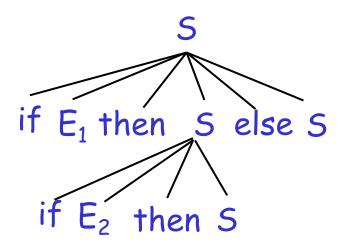
This grammar is also ambiguous

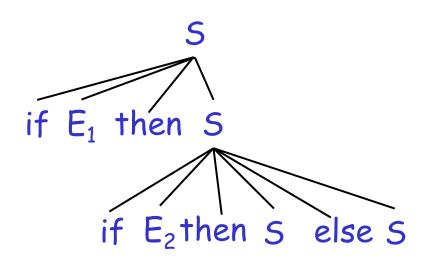
The Dangling Else: Example

The expression

if
$$E_1$$
 then if E_2 then S else S

has two parse trees





Typically we want the second form

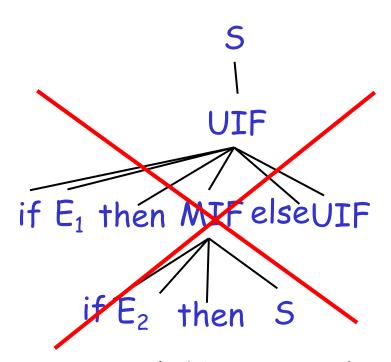
The Dangling Else: A Fix

- · else matches the closest unmatched then
- We can describe this in the grammar

Describes the same set of strings

The Dangling Else: Example Revisited

The expression if E₁ then if E₂ then S else S



 Not valid because the then expression is not a MIF $\begin{array}{c} S \\ \\ UIF \\ \\ \text{if } E_1 \text{ then } S \\ \\ \text{if } E_2 \text{ then } MIF \text{ else } MIF \end{array}$

A valid parse tree
 (for a UIF)

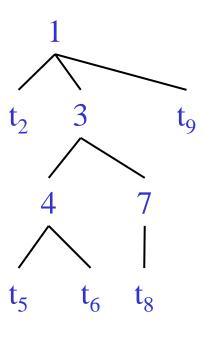
Ambiguity

- No general techniques for handling ambiguity
- Impossible to convert automatically an ambiguous grammar to an unambiguous one
- Used with care, ambiguity can simplify the grammar
 - Sometimes allows more natural definitions
 - We need disambiguation mechanisms

Introduction to Top-Down Parsing Recursive Descent

Intro to Top-Down Parsing: The Idea

- The parse tree is constructed
 - From the top
 - From left to right
- Terminals are seen in order of appearance in the token stream:



Consider the grammar

```
E \rightarrow T \mid T + E

T \rightarrow int \mid int * T \mid (E)
```

- Token stream is: (int)
- Start with top-level non-terminal E
 - Try the rules for E in order

```
E \rightarrow T \mid T + E

T \rightarrow int \mid int * T \mid (E)
```

E

```
( int )
↑
```

```
E \rightarrow T \mid T + E

T \rightarrow int \mid int * T \mid (E)
```

E | |

```
( int )
↑
```

```
E \rightarrow T \mid T + E

T \rightarrow int \mid int * T \mid (E)
```

```
E
T
Mismatch: int is not (!
int Backtrack ...
```

(int)
↑

```
E \rightarrow T \mid T + E

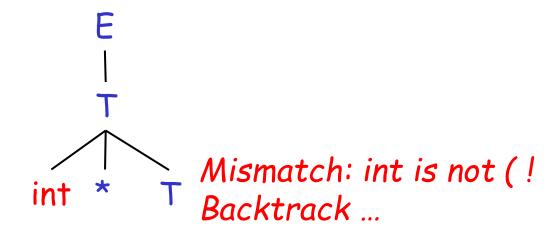
T \rightarrow int \mid int * T \mid (E)
```

E | T

(int)

$$E \rightarrow T \mid T + E$$

 $T \rightarrow int \mid int * T \mid (E)$



(int)
↑

$$E \rightarrow T \mid T + E$$

 $T \rightarrow int \mid int * T \mid (E)$

E | T

(int)

$$E \rightarrow T \mid T + E$$

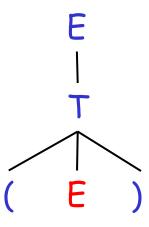
 $T \rightarrow int \mid int * T \mid (E)$



(int)
↑

$$E \rightarrow T \mid T + E$$

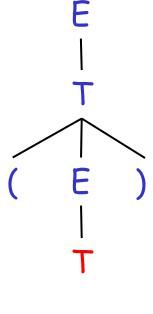
 $T \rightarrow int \mid int * T \mid (E)$





$$E \rightarrow T \mid T + E$$

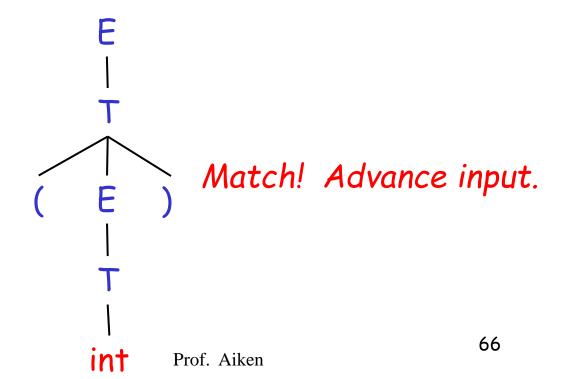
 $T \rightarrow int \mid int * T \mid (E)$



(int)
↑

$$E \rightarrow T \mid T + E$$

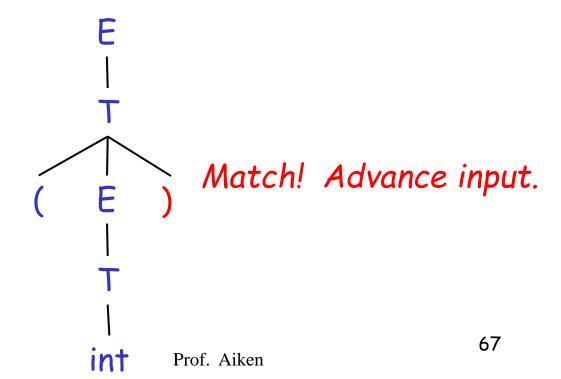
 $T \rightarrow int \mid int * T \mid (E)$





$$E \rightarrow T \mid T + E$$

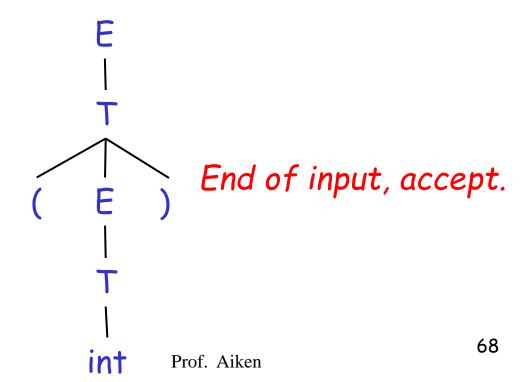
 $T \rightarrow int \mid int * T \mid (E)$



(int)

$$E \rightarrow T \mid T + E$$

 $T \rightarrow int \mid int * T \mid (E)$



(int)

Problems with Top Down Parsing

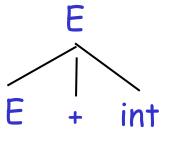
- Left Recursion in CFG may cause parser to loop forever!
- In there is a production of form $A \rightarrow A\alpha$, we say the grammar has left recursion

$$E \rightarrow E + int \mid int$$

- Solution: Remove Left Recursion...
 - without changing the Language defined by the Grammar.

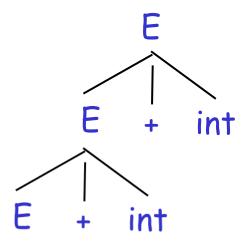
Problems with Top Down Parsing (Example)

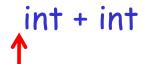
$$E \rightarrow E + int \mid int$$



Problems with Top Down Parsing (Example)

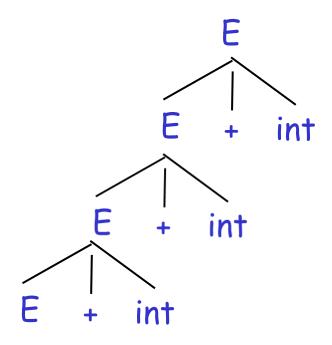
$$E \rightarrow E + int \mid int$$





Problems with Top Down Parsing (Example)

$$E \rightarrow E + int \mid int$$



Elimination of Left Recursion

· Consider the left-recursive grammar

$$S \rightarrow S \alpha \mid \beta$$

- 5 generates all strings starting with a β and followed by a number of α
- · Can rewrite using right-recursion

$$S \rightarrow \beta S'$$

 $S' \rightarrow \alpha S' \mid \epsilon$

More Elimination of Left-Recursion

In general

$$S \rightarrow S \alpha_1 \mid ... \mid S \alpha_n \mid \beta_1 \mid ... \mid \beta_m$$

- All strings derived from 5 start with one of $\beta_1,...,\beta_m$ and continue with several instances of $\alpha_1,...,\alpha_n$
- Rewrite as

$$S \rightarrow \beta_1 S' \mid \dots \mid \beta_m S'$$

 $S' \rightarrow \alpha_1 S' \mid \dots \mid \alpha_n S' \mid \epsilon$

General Left Recursion

The grammar

$$S \rightarrow A \alpha \mid \delta$$

$$A \rightarrow S \beta$$
 is also left-recursive because

$$S \rightarrow^+ S \beta \alpha$$

- · This left-recursion can also be eliminated
- See Dragon Book for general algorithm
 - Section 4.3.3

Predictive Top-Down Parsing

Parser Never Backtracks

For Example, Consider:

```
Type → Simple Start symbol

| ↑ id

| array [ Simple ] of Type

Simple → integer

| char

| num dotdot num
```

Suppose input is:

array [num dotdot num] of integer

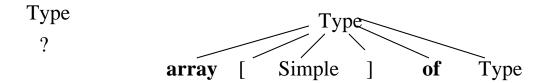
Parsing would begin with

$$Type \rightarrow ???$$

Predictive Parsing Example

Lookahead symbol

Input: array [num dotdot num] of integer



Start symbol

Type → simple

| ↑ id

| array [Simple] of Type

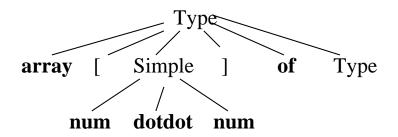
Simple → integer

| char

num dotdot num

Lookahead symbol

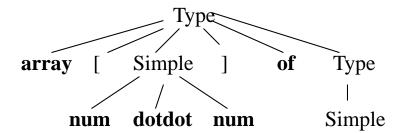
Input: array [num dotdot num] of integer



Predictive Parsing Example

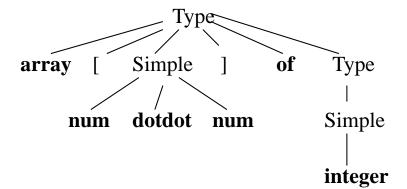
Lookahead symbol

Input: array [num dotdot num] of integer



Lookahead symbol

Input: array [num dotdot num] of integer



Start symbol Type → Simple | ↑ id | array [Simple] of Type Simple → integer | char | num dotdot num

Predictive Recursive Descent

 Parser is implemented by N + 1 subroutines, where N is the number grammar non-terminals

 There is one subroutine for attempting to Match tokens in the input stream

- There is also one subroutine for each nonterminal with two tasks:
 - 1. Deciding on the next production to use
 - 2. Applying the selected production

Procedure "Match" checks if the token matches the expected input

```
procedure Match ( expected_token );
{
    if lookahead = expected_token then
        lookahead := get_next_token
    else error
}
```

- The subroutine for each non-terminals has two tasks:
 - 1. Selecting the appropriate production
 - 2. Applying the chosen production

- Selection is done based on the result of a number of if-then-else statements
- Applying a production is implemented by calling the match procedure or other subroutines, based on the rhs of the production

Subroutine "Simple" for the given example:

```
Type \rightarrow Simple
        array [ Simple ] of Type
Simple \rightarrow integer
           char
           num dotdot num
```

```
procedure Simple;
  { if lookahead = integer then call Match (integer);
     else if lookahead = char then call Match (char);
          else if lookahead = num
                then { call Match ( num ); call Match ( dotdot );
                       call Match ( num ) }
               else error
```

Subroutine "Type" for the given example:

```
Type → Simple

| ↑ id
| array [ Simple ] of Type

Simple → integer
| char
| num dotdot num
```

How to write tests for selecting the appropriate production rule?

Basic Tools:

First: Let α be a string of grammar symbols. First(α) is the set that includes every terminal that appears leftmost in α or in any string originating from α . NOTE: If $\alpha \Rightarrow \in$, then \in is First(α).

Follow: Let A be a non-terminal. Follow(A) is the set of terminals a that can appear directly to the right of A in some sentential form. ($S \Rightarrow \alpha Aa\beta$, for some α and β). NOTE: If $S \Rightarrow \alpha A$, then \$ is Follow(A).

Computing First Sets

Definition

First(X) = {
$$t \mid X \rightarrow^* t\alpha$$
} \cup { $\epsilon \mid X \rightarrow^* \epsilon$ }

Algorithm sketch:

- 1. First(t) = { t }
- 2. $\varepsilon \in First(X)$
 - if $X \to \varepsilon$
 - if $X \to A_1 \dots A_n$ and $\varepsilon \in First(A_i)$ for $1 \le i \le n$
- 3. First(α) \subseteq First(X) if X \rightarrow $A_1 ... A_n <math>\alpha$
 - and $\varepsilon \in First(A_i)$ for $1 \le i \le n$

Computing First(X) for all Grammar Symbols

- 1. If X is a terminal, First(X) = {X}
- 2. If $X \rightarrow \in$ is a production rule, add \in to First(X)
- 3. If X is a non-terminal, and $X \rightarrow Y_1 Y_2 ... Y_k$ is a production rule

```
Place First(Y<sub>1</sub>) - \in in First(X)

if Y<sub>1</sub>\Rightarrow \in, Place First(Y<sub>2</sub>) - \in in First(X)

if Y<sub>2</sub>\Rightarrow \in, Place First(Y<sub>3</sub>) - \in in First(X)

...

if Y<sub>k-1</sub>\Rightarrow \in, Place First(Y<sub>k</sub>) in First(X)
```

NOTE: As soon as $Y_i \stackrel{*}{\Rightarrow} \in$, Stop.

Repeat above steps until no more elements are added to any First set.

First Sets. Example

Recall the grammar

$$E \rightarrow TX$$

 $T \rightarrow (E) \mid int Y$

$$X \rightarrow + E \mid \varepsilon$$

 $Y \rightarrow * T \mid \varepsilon$

First sets

```
First(() = {(}
First()) = {)}
First(int) = {int}
First(+) = {+}
First(*) = {*}
```

Computing Follow Sets

· Definition:

Follow(X) = {
$$t \mid S \rightarrow^* \beta X + \delta$$
 }

- Intuition
 - If $X \rightarrow A$ B then First(B) \subseteq Follow(A) and Follow(X) \subseteq Follow(B)
 - if $B \to^* \varepsilon$ then $Follow(X) \subseteq Follow(A)$
 - If S is the start symbol then \$ ∈ Follow(S)

Computing Follow Sets (Cont.)

Algorithm sketch:

- 1. $\$ \in Follow(S)$
- 2. First(β) { ϵ } \subseteq Follow(X)
 - For each production $A \rightarrow \alpha \times \beta$
- 3. $Follow(A) \subseteq Follow(X)$
 - For each production $A \rightarrow \alpha \times \beta$ where $\epsilon \in \text{First}(\beta)$

Follow Sets. Example

Recall the grammar

$$E \rightarrow TX$$
 $X \rightarrow + E \mid \varepsilon$
 $T \rightarrow (E) \mid int Y$ $Y \rightarrow * T \mid \varepsilon$

Follow sets

```
Follow( E ) = {), $}
Follow( T ) = {+, ), $}
Follow( Y ) = {+, ), $}
Follow( X ) = {), $}
```

Conditions for R.D. Parsing to be Predictive

R.D. parsing is Predictive when for all $A \rightarrow \alpha \mid \beta$

- 1. First(α) \cap First(β) = \emptyset ; besides, only one of α or β can derive ϵ
- 2. if α derives ε , then Follow(A) \cap First(β) = \emptyset

Predictive Parsing and Left Factoring

Consider the grammar

```
E \rightarrow T + E \mid T

T \rightarrow int \mid int * T \mid (E)
```

- Hard to predict because
 - For T two productions start with int
 - For E it is not clear how to predict
- We need to <u>left-factor</u> the grammar

Left-Factoring Example

Recall the grammar

$$E \rightarrow T + E \mid T$$

 $T \rightarrow int \mid int * T \mid (E)$

Factor out common prefixes of productions

$$E \rightarrow T X$$

 $X \rightarrow + E \mid \varepsilon$
 $T \rightarrow (E) \mid \text{int } Y$
 $Y \rightarrow * T \mid \varepsilon$

Error Handling

- Purpose of the compiler is
 - To detect non-valid programs
 - To translate the valid ones
- Many kinds of possible errors (e.g. in C)

Error kind	Example	Detected by
Lexical	\$	Lexer
Syntax	× *%	Parser
Semantic	int x; $y = x(3)$;	Type checker
Correctness	your favorite program	Tester/User

Syntax Error Handling

- Error handler should
 - Report errors accurately and clearly
 - Recover from an error quickly
 - Not slow down compilation of valid code

Good error handling is not easy to achieve

Approaches to Syntax Error Recovery

- From simple to complex
 - Panic mode
 - Error productions
 - Automatic local or global correction

Error Recovery: Panic Mode

- Simplest, most popular method
- When an error is detected:
 - Discard tokens until one with a clear role is found
 - Continue from there

- · Such tokens are called synchronizing tokens
 - Typically the statement or expression terminators

Syntax Error Recovery: Error Productions

- · Idea: specify in the grammar known common mistakes
- Essentially promotes common errors to alternative syntax
- Example:
 - Write 5 x instead of 5 * x
 - Add the production $E \rightarrow ... \mid E \mid E$
- · Disadvantage
 - Complicates the grammar

Error Recovery: Local and Global Correction

- · Idea: find a correct "nearby" program
 - Try token insertions and deletions
 - Exhaustive search

- Disadvantages:
 - Hard to implement
 - Slows down parsing of correct programs
 - "Nearby" is not necessarily "the intended" program

Syntax Error Recovery: Past and Present

Past

- Slow recompilation cycle (even once a day)
- Find as many errors in one cycle as possible

Present

- Quick recompilation cycle
- Users tend to correct one error/cycle
- Complex error recovery is less compelling
- Panic-mode seems enough

How many strings does the following grammar generate?

- 0 7
- \bigcirc 15
- 0 2
- 0 8
- 0 16
- O_4

- $A \rightarrow BB$
- $B \rightarrow CC$
- $C \rightarrow 1 \mid 2$

How many strings does the following grammar generate?

- 0 16
- \bigcirc 31
- 015
- \bigcirc 12
- 0 64
- \bigcirc 63
- \bigcirc 32
- 0 11

$$A \rightarrow BB$$

$$B \rightarrow C C$$

$$C \to 1 \, | \, 2 \, | \, \epsilon$$

Which of the following is a valid derivation of the given grammar?

S

aXa

abYa

acXca

acca

S

aXa

abYa

abcXcaabcbYcaabcbdca

S

aXa

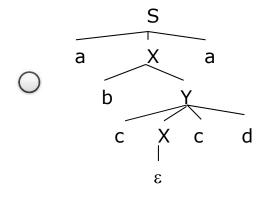
abYa abcXcda abccda $S \rightarrow aXa$

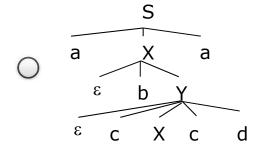
 $X \rightarrow \varepsilon \mid bY$

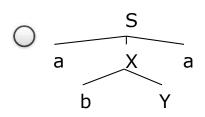
 $Y \rightarrow \varepsilon \mid cXc \mid d$

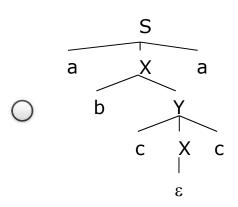
Which of the following is a valid parse tree for the given grammar?











 $X \rightarrow \varepsilon \mid bY$ $Y \rightarrow \varepsilon \mid cXc \mid d$

Choose the grammar that correctly eliminates left recursion from the given grammar: $E \rightarrow E + T \mid T$

$$\begin{array}{c}
\mathsf{E} \to \mathsf{E} + \mathsf{id} \mid \mathsf{E} + (\mathsf{E}) \\
\mid \mathsf{id} \mid (\mathsf{E})
\end{array}$$

$$E \rightarrow E' + T \mid T$$

$$\bigcirc E' \rightarrow id \mid (E)$$

$$T \rightarrow id \mid (E)$$

$$T \rightarrow id \mid (E)$$

$$E \rightarrow TE'$$

$$E' \rightarrow + TE' \mid \varepsilon$$

$$T \rightarrow id \mid (E)$$

$$\bigcirc \begin{tabular}{l} E \to id + E \mid E + T \mid T \\ T \to id \mid (E) \end{tabular}$$

Consider the following grammar. Adding which one of the following rules will cause the grammar to be left-recursive? [Choose all that apply]

$$OD \rightarrow A$$

$$OA \rightarrow D$$

$$\circ B \rightarrow C$$

$$OD \rightarrow B$$

$$0 \text{ C} \rightarrow 1 \text{ C}$$

$$S \rightarrow A$$

$$A \rightarrow B \mid C$$

$$B \rightarrow (C)$$

$$C \rightarrow B + C \mid D$$

$$D \rightarrow 1 \mid 0$$

Which of the following grammars are ambiguous?

- \square S \rightarrow SS| a| b
- \square E \rightarrow E+E| id
- \square S \rightarrow Sa| Sb
- $\square E \rightarrow E' \mid E' + E$ $E' \rightarrow -E' \mid id \mid (E)$

Choose the unambiguous version of the given ambiguous grammar: $S \rightarrow SS|a|b|\epsilon$

$$\circ$$
 S \rightarrow Sa Sb ϵ

$$\begin{array}{ccc}
S \rightarrow SS' \\
S' \rightarrow a \mid b
\end{array}$$

$$\begin{array}{cc} S \rightarrow S \mid S' \\ O & S' \rightarrow a \mid b \end{array}$$

$$\circ$$
 S \rightarrow Sa | Sb

Consider the following grammar. How many unique parse trees are there for the string 5 * 3 + (2 * 7) + 4?

- O_{2}
- 01
- 07
- \bigcirc 0
- 05
- O_4

$$E \rightarrow E * E | E + E | (E) | int$$

Which of the following statements are true about the given grammar?

```
S \rightarrow a T U b | \epsilon

T \rightarrow c U c | b U b | a U a

U \rightarrow S b | c c
```

Choose all that are correct.

- The follow set of S is {\$, b}
- The first set of U is {a, b, c}
- O The first set of S is $\{\epsilon, a, b\}$
- The follow set of T is { a, b, c }

Consider the following grammar:

$$S \rightarrow A (S)B \mid \varepsilon$$

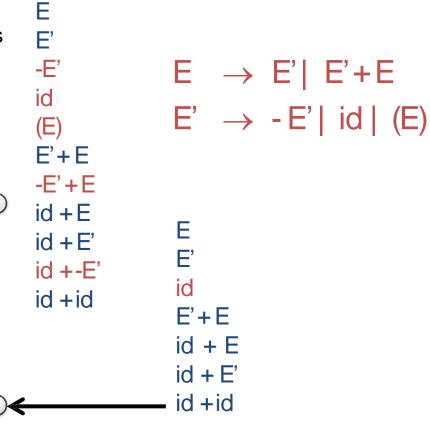
 $A \rightarrow S \mid SBx \mid \varepsilon$
 $B \rightarrow SB \mid y$

What are the first and follow sets of S

- O First: $\{x, y, (, \epsilon\}$ Follow: $\{y, x, (,)\}$
- $\bigcirc \quad \mathsf{First:} \{ \, \mathsf{x}, \, \varepsilon \} \qquad \qquad \mathsf{Follow:} \{ \, \mathsf{y}, \, \mathsf{x}, \, (,) \}$
- O First: $\{y, (, \epsilon)\}$ Follow: $\{\$, y, (,)\}$
- O First: $\{x, y, (, \epsilon\}$ Follow: $\{\$, y, x, (,)\}$
- \bigcirc First: $\{x, y, (\}$ Follow: $\{\$, y, x, (,)\}$
- O First: $\{x, (\}$ Follow: $\{\$, y, x\}$

Choose the derivation that is a valid recursive descent parse for the string id + id in the given grammar. Moves that are followed by backtracking are given in red.

```
E
E'
E'+E
id + E
id + E'
id + id
```



Choose the grammar that correctly eliminates left recursion from the given grammar: $E \rightarrow E + T \mid T$

$$T \rightarrow id \mid (E)$$

$$\begin{array}{c}
\mathsf{E} \to \mathsf{E} + \mathsf{id} \mid \mathsf{E} + \mathsf{(E)} \\
& \mid \mathsf{id} \mid \mathsf{(E)}
\end{array}$$

$$E \rightarrow TE'$$

$$\bigcirc E' \rightarrow + TE' \mid \varepsilon$$

$$T \rightarrow id \mid (E)$$

$$E \rightarrow E' + T \mid T$$

$$\bigcirc E' \rightarrow id \mid (E)$$

$$T \rightarrow id \mid (E)$$

$$\bigcirc \begin{array}{l}
\mathsf{E} \to \mathsf{id} + \mathsf{E} \mid \mathsf{E} + \mathsf{T} \mid \mathsf{T} \\
\mathsf{T} \to \mathsf{id} \mid (\mathsf{E})
\end{array}$$

Choose the alternative that correctly left factors "if" statements in the given grammar

```
EXPR → if BOOLthen { EXPR}

| if BOOLthen { EXPR} else { EXPR}

| ...

BOOL→ true | false
```