Alma Mater Studiorum - Università di Bologna

TAKE HOME EXAM

Economics of financial markets

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1.1 Exercise 2

This paper by Bjorn, Shaliastovich and Wang(2015) shows that **high expected inflation has a negative and persistent impact on real long-term economic growth rates**, the magnitude of this effect varies across economics sectors and they proved that **it is significantly more pronounced for durable goods sectors, relative to non-durable goods and that there is a strong connection between the dividends and consumptions in the corresponding sectors.**

In our study we will try to explain how they obtained such results and the meaning behind it.

1.1.1 Macro econometric model:

A **VARMA(1,1)** model of the macroeconomic series has been used on data with **the growth rates of durable and non-durable real consumption, inflation, and dividend growth rates for durable- and non-durable- goods-producing firms as observable variables.**
As in **Bansal and Yaron (2004)**, the model also includes unobservable factors that represent the conditional expectations of the respective macro factors.

Although the scalar components methodology used to build VARMA models is rather difficult, the VAR models application being easier in practice, the forecasts based on the first models **have a higher degree of accuracy that's why it has been used in this paper.**

We denote by g_t a vector of the observed macroeconomic variables (non- durable consumption growth, non-durable inflation, and the growth rates of durable good stocks), **note that The use of the natural log permits to obtain approximations of percentage changes instead of unit changes.**

$$g_t = \begin{bmatrix} \Delta c_t \\ \pi_t \\ \Delta s_t \end{bmatrix} \text{ with } \Delta c_t = log(\frac{C_t}{C_{t-1}}),\ \Delta s_t = log(\frac{S_t}{S_{t-1}}) \text{ and } \pi_t = log(\frac{{P_t}^{\$}}{P^{\$}_{t-1}})$$

We denote x_t as Var(1) process of expected growth rates **it's like we are saying the expected growth in 2019 is likely to be or depend on the level of growth in 2018, of course we have**

some errors. That's why the model is such as :

$$\begin{cases} g_{t+1} = \mu_g + x_t + \Sigma_g \psi_{t+1} \\ x_{t+1} = \Pi x_t + \Sigma_x u_{t+1} \end{cases}$$

with $\mu_g = E(g_t)$, Σ_g and Σ_x are the co-variance matrices, ψ_{t+1} and u_{t+1} are vectors of independent Gaussian innovations. To have an idea about the persistence and potential feedback between the expected growth states the matrix Π is used.

The construction of the model in this way allows to show the effect of macro economic variables on each other and also on future aggregate growth.

In the equity side we also consider the effect on real dividend growth rates in both economies as shown in the paper, the same technique used in **Bansal and Yaron (2004)** which is that the dividend growth rates load only on the corresponding expected consumption growth rates.In the other side the dividend leverage parameters that they used are larger than the ones in the literature (other model restrictions are detailed in the paper).

The model confirms the results stated in the first paragraph and allows to formulate a **long-run risk model aimed at explaining a number of asset pricing puzzles** which is the main subject of our study.

Instead of working with the classical model in Bansal and Yaron (2004), Piazzesi and Schneider (2006), Eraker (2006), Hasseltoft (2012), Yang (2011), and Bansal and Shaliastovich (2013) **a nominal two-good economy** has been used.

This allows them to formally quantify the impact of expected inflation on aggregate consumption of durable and non-durable goods and because dividend cash-flows are exposed to the aggregate consumption risks through the dividend leverage channel, we can have an idea on the exposures of equity dividends to this type of risks.

1.1.2 Model setup:

They considered an infinite-horizon, discrete-time, endowment economy where investors consume durable and non-durable goods. **An endowment economy is a fancy term for an economy in which there is no endogenous production – the amount of income/output is exogenously given. With fixed quantities, it becomes particularly clear how price adjustment results in equilibrium**

The investors preferences over future consumption are described by the Kreps-Porteus, Epstein-Zin recursive utility function **which is a very important modification of utility function that shows up in a lot of work in asset pricing, the key is that it is non separable across states of nature : consumption (the biggest concepts in economics) in one state of nature will effect marginal utility in another state of nature** . We define it as :

$$U_t = [(1-\beta)\mu_t^{1-\frac{1}{\psi}} + \beta(E_t U_{t+1}^{1-\alpha})^{\frac{1-\frac{1}{\psi}}{1-\alpha}}]^{\frac{1}{1-\frac{1}{\psi}}}$$

where U_t is the lifetime utility function, μ_t is the intra-period consumption aggregator, β is the subjective discount factor, ψ is the elasticity of intertemporal substitution (IES measures the willingness on the part of the consumer to substitute future consumption for present consumption) , and α is the relative risk aversion coefficient (if we note w as wealth we have $\alpha = \frac{-U_t''(w)}{U_t'(w)}.w = ARA.w$) .

If consumption is deterministic: we have the usual standard time-separable expected discounted utility with discount factor β and IES $=\frac{1}{\alpha}$ with α risk aversion coefficient.The utility function can be solved forward to yield the familiar time-separable constant relative risk aversion (CRRA) power utility model.

Proof

If no uncertainty, then $E_t U_{t+1}^{1-\alpha} = U_{t+1}$, , we recover CRRA preferences:

$$U_t = [(1-\beta)\mu_t^{1-\alpha} + \beta(U_{t+1}^{1-\alpha})]^{\frac{1}{1-\alpha}}$$

so

$$W_t = [(1-\beta)\mu_t^{1-\alpha} + \beta(W_{t+1})] = (1-\beta)\sum_{j=0}^{+\infty}\beta^j\mu_{t+j}^{1-\alpha}$$

with

$$W_t = U_t^{1-\alpha}$$

In this economy agents have time-separable preferences defined over streams of consumption of non-durable goods C_t and service flows of durable goods , which are assumed to be proportional to its stock S_t.The stock of durable goods evolve according to the following law of motion:

$C_t = (1-\delta)S_{t-1} + E_t$ where $(1-\delta)$ is the fraction of the end-of-period stock at $t1$ that remains providing utility at t. A CRRA preferences is assumed over a CES aggregator of non-durable consumption and services from durable goods:

$$U(C_t, S_t) = [(1-\gamma)C_t^{1-\frac{1}{\epsilon}} + \gamma S_t^{1-\frac{1}{\epsilon}}]^{\frac{1}{1-\frac{1}{\epsilon}}}$$

$0 < \gamma < 1$ captures the weight of each type of consumption in agents preferences, $\epsilon < 1$ measures the elasticity of substitution between the two goods, high values indicate that the two goods can be easily substituted by the agent, while small values capture the complementarity between the two goods.

Notice that this utility function depends on the end-of-period stock of durables, S_t, after period t purchases and sales E_t. Durable goods have a relative price of P, with non-durable goods acting as the numeraire.

The marginal rate of substitution (MRIS) is the gain (loss) in future consumption needed to offset the loss (gain) in current consumption.

The intertemporal marginal rate of substitution (IMRS) is the gain (loss) in future consumption needed to offset the loss (gain) in current consumption and it's equal to the SLOPE of the indifference curve (slope of the tangent to the indifference curve). As shown by Yogo (2006), the equilibrium stochastic discount factor, valued in the units of non-durable consumption is given by :

$$M_{t+1} = [\delta(\frac{C_{t+1}}{C_t})^{-\frac{1}{\psi}} * \frac{v(\frac{S_{t+1}}{C_{t+1}})^{\frac{1}{\epsilon}-\frac{1}{\psi}}}{v(\frac{S_t}{D_t})}(R^w_{t+1})^{1-\frac{1}{k}}]^k$$

with

$$\begin{cases} k = \frac{1-\alpha}{1-\frac{1}{\psi}} \\ v(S/C) = [1-\gamma+\gamma*(\frac{S}{C})^{1-\frac{1}{\epsilon}}]^{\frac{1}{1-\frac{1}{\epsilon}}} \end{cases}$$

R^w is the return on the market portfolio.

(You can find in the paper that IMRS is written in another form which allows to expalain the meaning behind each term).

Epstein and Zin (1989, 1991) show that the IMRS can be expressed in an alternate form (equation 11 in the paper) if we consider a one good economy. **A difficulty with this approach is that R^w may not be well proxied by observable asset market returns, especially if human wealth and other nontradable assets are quantitatively important fractions of aggregate wealth**.

Thus, relative to a one-good economy, the specification with two goods incorporates fluctuations in the relative share of the goods, and further, the wealth portfolio of the agent is now composed of both the non-durable and durable consumption.

As we will show the agent's first-order conditions (FOC) for the consumption and portfolio choice problem imply the Euler equation.

$$E_t[M_{t+1}R_{i,t+1}] = 1$$

Proof :

There are $N+1$ tradeable assets in the economy, indexed by i = 0,...,N. In period t,the agent invests Bit units of wealth W_t in asset i, which realizes the gross rate of return $Ri,t+1$ in period $t+1$. The agent's total saving in assets satisfies the intraperiod identity:

$$\sum_{i=0}^{N} B_{it} = W_t - Ct - P_t E_t$$

With P_t is the price of the durable good in units of the nondurable good.

The agent's wealth in the subsequent period is given by the intertemporal budget constraint

$$W_{t+1} = \sum_{i=0}^{N} B_{it} R_{i,t+1}$$

Following Bansal, Tallarini, and Yaron (2004) and Cuoco and Liu (2000), We first simplify the consumption and portfolio choice problem with a durable consumption good through a change of variables.

We define :

$$\begin{cases} \tilde{W}_t = W_t + (1-\alpha)P_t S_{t-1} \\ B_{N+1,t} = P_t S_t \\ R_{N+1,t+1} = (1-\alpha)\frac{P_{t+1}}{P_t} \end{cases}$$

Then the intraperiod identity and the intertemporal budget constraint can be rewritten as :

$$\begin{cases} \sum_{i=0}^{N} B_{it} = \tilde{W}_t - Ct \\ \\ \tilde{W}_{t+1} = \sum_{i=0}^{N+1} B_{it} R_{i,t+1} \end{cases}$$

Define the portfolio shares $w_{it} = \frac{B_{it}}{(\tilde{W}_t - C_t)}$ for all i = 0,...,N + 1. Then the last two equations are equivalent to :

$$1 = \sum_{i=0}^{N+1} w_{it}$$

$$\tilde{W}_{t+1} = (\tilde{W}_t - C_t) \sum_{i=0}^{N+1} w_{it} R_{i,t+1}$$

We have that $P_t S_t = w_{N+1,t}(\tilde{W}_t - C_t)$, S_t can be substituted out of intraperiod utility as :

$$u(C_t, S_t) = C_t[1 - \gamma + \gamma(\frac{w_{N+1,t}(\tilde{W}_t / C_t - 1)}{P_t})^{1-\frac{1}{\epsilon}}]^{1/(1-1/\epsilon)} = C_t \tilde{v}_t(\frac{C_t}{\tilde{W}_t}, w_{N+1,t})$$

Finally, the agent's problem can be restated as follows. Given its current level of wealth W_t, which includes the stock of the durable good, the agent chooses its consumption and portfolio shares $(Ct, w_{0t}, ..., wN+1, t)$to maximize utility subject to the constraints detailed in the last two equations. The solution to this problem, that is the agent's optimal consumption and portfolio shares, will be denoted by $(C_t^*, {}^*_{0t}, ..., w^*_{N+1,t})$. The Bellman equation for the problem is given by :

$$J_t(\tilde{W}_t) = \max_{Ct, w_{0t}, ..., wN+1, t} [(1-\delta)[C_t \tilde{v}_t(\frac{C_t}{\tilde{W}_t}, w_{N+1,t}]^{1-1/\psi} + \delta(E_t[J_{t+1}(\tilde{W}_{t+1})^{1-\alpha}])^{1/k}]^{1/(1-1/\psi)}$$

By the homogeneity of the optimization problem, the value function is proportional to wealth $J_t(\widetilde{W}_t) = \phi_t \widetilde{W}_t$. Using arguments similar to Epstein and Zin (1991), we can show that :

$$\phi_t = [(1-\delta)(1-\gamma)\widetilde{v}(\frac{C_t^*}{\widetilde{W}_t}, w_{N+1,t}^*)^{1/\epsilon - 1/\psi}]^{1/(1-1/\psi)}(\frac{C_t^*}{\widetilde{W}_t})^{1/(1-\psi)}$$

Let $R_{W,t+1}^* = \sum_{i=0}^{N+1} w_{it}^* R_{i,t+1}$ be the return on wealth from the optimal portfolio.

We note u_c and u_s denote the marginal utility of C and S, respectively.

After tedious algebra along the lines of Epstein and Zin (1991), the FOC with respect to C_t is given by :

$$E_t[M_{t+1}^* R_{W,t+1}^*] = (1 - \frac{w_{N+1,t}^* u_{st}}{P_t u_{ct}})^k$$

Similarly, the FOC with respect to it is given by:

$$E_t[M_{t+1}^*(R_{i,t+1} - R_{0,t+1}] = 0$$

for all assets i = 1,...,N. The FOC with respect to $w_{N+1,t}$, that is the fraction of wealth held in the durable good, is given by:

$$E_t[M_{t+1}^*(R_{0,t+1} - R_{N+1,t+1}] = ((1 - \frac{w_{N+1,t}^* u_{dt}}{P_t u_{ct}})^{k-1})\frac{u_{st}}{P_t u_{ct}}$$

Straightforward algebra reveals that the last 3 equations imply :

$$E_t[M_{t+1}^* R_{i,t+1}] = (1 - \frac{w_{N+1,t}^* u_{st}}{P_t u_{ct}})^{k-1}$$

This Euler equation is the basis for consumption-based asset pricing. Marginal utility is the appropriate measure of risk for an investor who cares about consumption. Assets that deliver low returns when marginal utility is high must have high expected returns to reward the investor for bearing risk. On the other hand, assets that deliver high returns when marginal utility is high provide a good hedge and consequently must have low expected returns.

To derive the nominal bond and equity prices, they used an numeraire-adjusted stochastic factor :

$$\widetilde{M}_{t+1} = M_{t+1}\frac{\widetilde{\prod}_t}{\widetilde{\prod}_{t+1}}$$

When interested in asset prices, we need to be careful with the usual "macroeconomic" linearization. Such an approximation is certainty-equivalent, meaning that coefficients of linearized solution do not depend on size of shocks. Moreover, all variables in linearized

solution will fluctuate around their deterministic steady states. As a result, risk premia are zero, which kind of defies the point. Here they used The linearization of the relative share to shuts off the variation in the asset volatilities and risk premia due to the relative share movements because these fluctuations are not likely to be important for the unconditional levels of prices, which is the key focus of the paper

We note Z_t The relative share of non-durable consumption of the agent :

$$Z_t = \frac{C_t}{C_t + Q_t S_t}$$

Where Q_t is the user cost of durable goods given by the ratio of the marginal utilities of durable to non-durable consumption :

$$Q_t = \frac{u_{st}}{u_{ct}} = \frac{\gamma}{1-\gamma}(\frac{S_t}{C_t})^{-\frac{1}{\epsilon}}$$

We show that optimal consumption of the durable good requires an intratemporal FOC of the form :

$$\frac{u_{st}}{u_{ct}} = P_t - (1-\alpha)E_t[M_{t+1}P_{t+1}]$$

Since a unit of the durable consumption good costs P_t today and can be sold for $(1\alpha)P_{t+1}$ tomorrow, after depreciation, Q_t has a natural interpretation as the user cost of the service flow for the durable good. the equation found simply says that the marginal rate of substitution between the durable and nondurable consumption goods must equal the relative price of the durable good.

When $\alpha = 1$ and $\epsilon = 1$, this equation reduces to $\delta/(1\delta)$ = P S/C, so can be interpreted as the expenditure share of the "durable" good. When $\alpha < 1$, δ loses this economic interpretation since S is the stock, rather than the expenditure, of the durable good.

As we discussed in the previous section, what matters for the asset prices are the dynamics of non-durable consumption and relative expenditure share. The growth rate of the latter can be written in terms of the underlying states of the economy:

$$\Delta Z_{t+1} = (1-\frac{1}{\epsilon})(\Delta s_{t+1} - \Delta c_{t+1}) = (1-\frac{1}{\epsilon}) * (i_s - i_c)' g_{t+1}$$

where i_c and i_s are the vectors which pick non-durable and durable consumption growth from the vector Z_t.We approximate the dynamics of $log(1+e^{Z_t})$ in the following way :

$$\Delta log(1+e^{Z_t}) \approx X \Delta Z_t$$

The parameter $X \in (0,1)$ is an approximating constant equal to the average expenditure on durables in the economy.Hence, this parameter captures the importance of durable goods in the economy. For X = 0 (e.g., when preference weight to durables $\delta = 0$), the specification reduces to a one-good economy, while high X indicates relative importance of the durable channel.

They solved the equilibrium asset prices given the preferences and the stochastic discount factor, and the exogenous dynamics for non-durable and durable consumption growth rates, inflation, and dividend cash flows to show that they play an important role in interpreting the evidence in the asset markets.

To solve the model, conjecture that the equilibrium price-consumption ratio is a function of the economic states x_t :

$$pc_t = A_0 + A_x' x_t$$

Using the Euler equation for the consumption asset, we obtain that the price consumption loadings satisfy :

$$A_x = (1 - \frac{1}{\psi})(I - k_1 \prod^{'})^{-1}((1 - X)i_c + Xi_c)$$

where k1 is the log-linearization coefficient whose value is provided in the Appendix in the paper.

The intuition for the signs of the responses of asset valuations to economic state naturally extends that in one-good long-run risks model (see Bansal and Yaron (2004)). Investors are concerned about future long-run expectations of non-durable consumption ct and effective durable consumption atst , where the relative weight to the two is determined by the average expenditure parameter . When inter-temporal elasticity of substitution is above one, substitution effect dominates wealth effect. So, the states which increase expected non-durable or effective durable consumption positively affect asset valuations.

You can find in the paper the real stochastic discount factor, expressed in units of non-durable numeraire but to gain further intuition on the sources and compensation for risks in the economy, we can decompose discount factor loadings and market prices of risks to the components related to expected non-durable and durable consumption (see equation 24 in th paper).

Note: When X = 0 the specification reduces to a one-good non-durable model, and the loading captures the expected non-durable consumption divided by the IES. With durable goods, both the inter-temporal elasticity of substitution and elasticity of substitution between the two goods play a key role in determining the response of the discount factor to the underlying economic states. In a two-good economy, similar to a one-good one, the loading to expected non-durable consumption is negative. When $\epsilon < \psi$, the loading to expected durable consumption is positive.

In a similar way, we can decompose the market prices of risks in the economy (equation 19 in the paper).

In a one-good non-durable economy, X = 0, and price of short-run consumption news is γ, while the price of expected growth news is given by $(1)k1A_x$. With durables, the price of non-durable consumption risks changes to a weighted average between the risk aversion and the inverse of elasticity of substitution, where the weight is determined by the importance of durables in the economy. The prices of short-run durable risk depend on the relative magnitude of the risk-aversion coefficient and the inverse of elasticity of substitution, and are expected to be positive when intra-temporal elasticity ϵ is small enough.

It is important to note that with CRRA expected utility, $\delta = 1/\psi$, and market prices of expected durable and non-durable consumption risks are equal to zero. Then, investors are concerned only with short-run innovations in consumption.

1.1.3 Equilibrium Asset Prices

Using the solution for the stochastic discount factor, we can characterize equilibrium prices of bond and equity claims in the model (see Appendix B in the paper).

Notably, our economy is solved under the assumption that non-durable consumption is the numeraire. We denote by $p_{t,n}$ the price of a risk-free bond which delivers one unit of non-durable consumption n periods in the future. Using Euler equation seen before we can show that **the equilibrium price is linear in the states of the economy:**

$$p_{t,n} = -B_{0,n} - B_{x,n}^{\prime x_t}$$

where the recursive solutions for the bond loadings are provided in the Appendix B. In particular, the solution to a one-period bond yield satisfies :

$$r_t = B_{0,n} - m_x^{'} x_t$$

As we qee before, the one-period claim to unit nondurables positively responds to expected consumption news, and negatively to news to expected durables if $\epsilon < \psi$.

We denote $rx_{t+m,n}$ the excess log return on buying an n month bond at time t and selling it at time t + m as an n m period bond as :

$$rx_{t+m,n} = -p_{t,n} + p_{t+m,n-m} + p_{t,m}$$

The expected excess return for 1-period strategies is given by the covariance of the discount factor with the excess bond return, up to Jensen's term as you see in equation 23 in the paper.

The overall risk premium depends on market prices of long-run risks, bond loadings to those risks, as well as the variance-covariance matrix of expected growth shocks. When the real bond premium is positive, this leads to a positive slope of the term structure.

In the approach above the real bonds deliver one unit of nondurable consumption. Unlike a one-good economy, in multiple good economy this is not the only way tothink about the real risk-free asset. More generally, we can define a real bond as delivering a basket of goods in the future. Define P_t^* the price of the basked in unitsof nondurables. Then, the price of the bond which delivers this basked satisfies the following Euler equation:

$$P_{t,n}^* = E_t M_{t+1} \frac{P_t^*}{P_{t+1}^*} P_{t+1,n-1}^*$$

For instance, taking P_t^* we are back to the definition of real bond delivering unit of nondurables.Another particular case of the Euler condition above obtains when we use nominal price of non-durables in P_t^*. This allows us to compute equilibrium prices of nominal claims. Indeed, the nominal discount factor takes into account the dynamics of the inflation process, and can be solved in the following way:

$$m_{t+1}^{\$} = m_{t+1} - \pi_{t+1} = m_0^{\$} + m_x^{\$\prime} x_t + m_z z_t - (\lambda_g^{\$\prime} \Sigma_g + \lambda_x^{'} \Sigma_x)\eta_{t+1}$$

The nominal discount factor parameters are provided in the Appendix in the paper. For nominal bonds, the exposure of expected non-durable and durable consumption to expected inflation risks changes the exposure of nominal bonds to expected growth risks. This, the inflation premium can come through both the non-durable and durable channel: if both expected non-durables and durables respond negatively to expected inflation, then non-durable and durable based inflation premia are all positive.

Note: With expected CRRA utility, market prices of expected durable and nondurable risks are equal to zero. Hence, in this benchmark model, the real and nominal premia are equal to zero, which will lead to a flat real and nominal term structure.

1.1.4 Empirical facts:

As shown in the paper in the data, durable consumption is very persistent and highly and negatively predictable by nominal yields and inflation, and these effects are much more pronounced for durable consumption then the nondurable one. that's why they set up a two-good, long-run risks type nominal economy which features nonseparable utility over consumption of durable and nondurable goods, fluctuations in relative preference for durables, inflation, and recursive utility function with preference for early resolution of uncertainty. They show that the model is consistent with the above empirical facts. Further, the model can successfully, and effectively out-of-sample, explain unconditional moments and the conditional movements in the term structure.Model-implied equilibrium real yields are upward sloping, which cannot be obtained in a one-good economy, for a range of numeraire choices. Empirically, they find that most of the inflation premium in the model comes from a durable risk channel. Overall, there findings suggests that long-run durable risks and preference for early resolution of uncertainty play a key role to explain the bond prices in the data.

Note that : In the current model specification, all the risk premia are constant, so future bond returns are not predictable.Extensions of the model to generate timevariation in risk premia, such as through time-varying aggregate volatility can be find in Bansal and Yaron (2004), Bansal and Shaliastovich (2010) and Hasseltoft (2010).

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