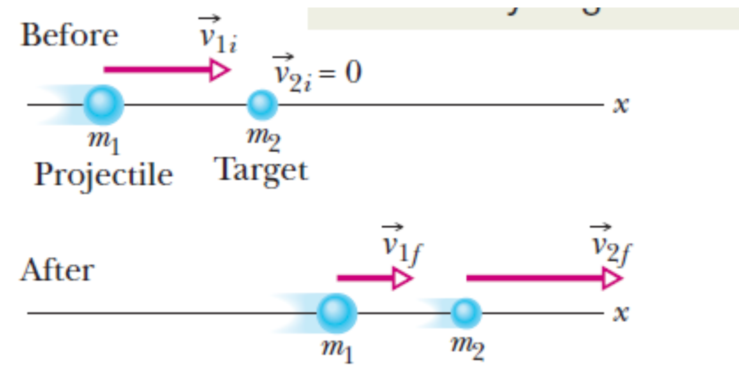
Summer 2021
UB CMS Group Research Intro

Undergrad Physics —> Groundbreaking Research

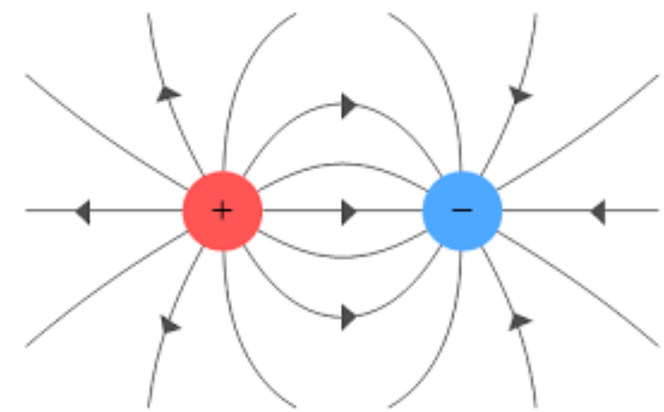
- You know basic mechanics and E+M, like:
  - Kinematics:

$$K = \frac{p^2}{2m}$$



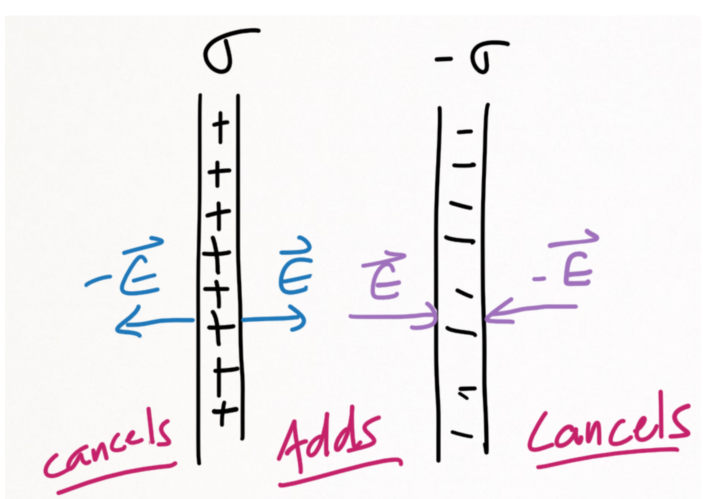
  - Point charges, dipoles:

$$U = -\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$$



  - Gauss's law, Electric fields in capacitors, parallel plate capacitor:

$$E = \frac{\sigma}{\epsilon_0}$$



(Sorry, this "E" is electric field and other "E's" may specify energy)

Undergrad Physics : Work and Energy

- Remember that work is defined as:

Work = Force x distance

– In calculus: $$W = \int \vec{F} \cdot \vec{dx}$$

- Also that (conservative) force is the gradient of the potential energy:

$$\vec{F} = -\vec{\nabla} U$$

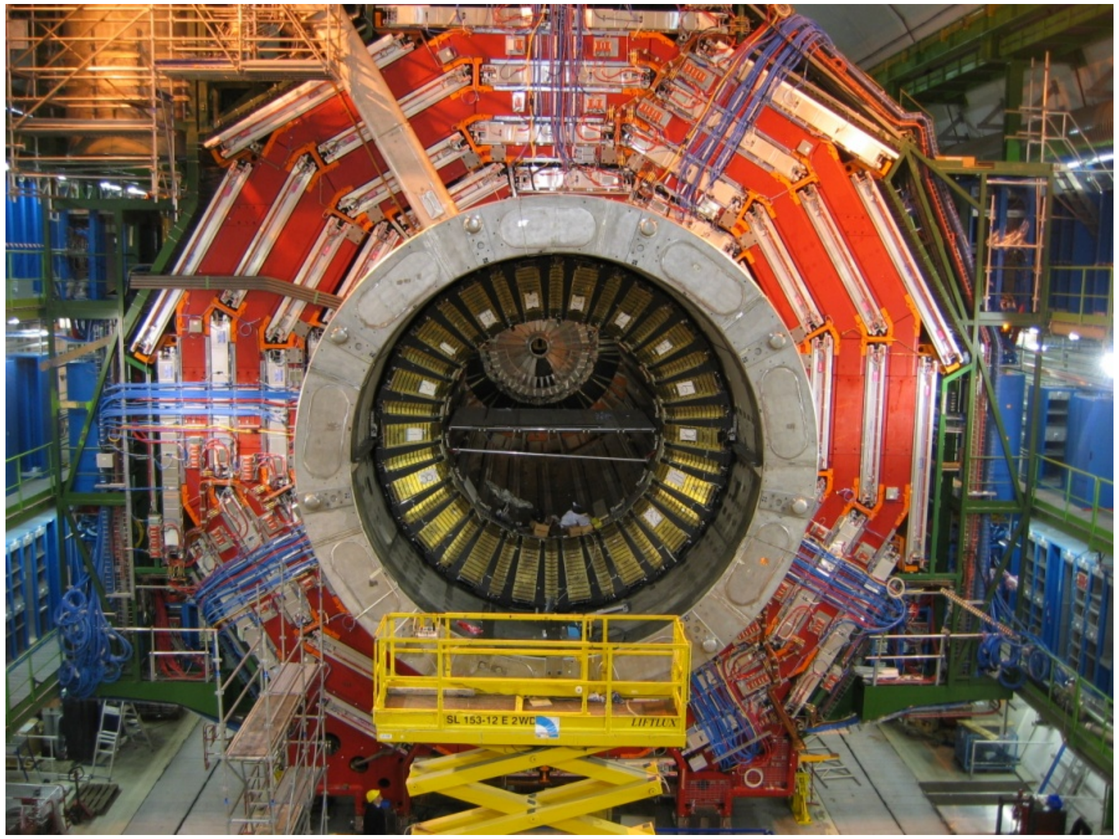
– Only differences in potential energy matter!

Undergrad Physics —> Groundbreaking Research

- How do you get to…

$$
\begin{gathered}
\mathcal{L}_{SM} = -\tfrac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \tfrac{1}{4}g^2_s f^{abc}f^{ade}g^b_\mu g^c_\nu g^d_\mu g^e_\nu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - \\
M^2 W^+_\mu W^-_\mu - \tfrac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \tfrac{1}{2c^2_w}M^2 Z^0_\mu Z^0_\mu - \tfrac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - igc_w(\partial_\nu Z^0_\mu (W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - Z^0_\nu (W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu (W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)) - \\
igs_w(\partial_\nu A_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu (W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + A_\mu (W^+_\nu \partial_\nu W^-_\mu - \\
W^-_\nu \partial_\nu W^+_\mu)) - \tfrac{1}{2}g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \tfrac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c^2_w (Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - \\
Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s^2_w (A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w (A_\mu Z^0_\nu (W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu) - \tfrac{1}{2}\partial_\mu H \partial_\mu H - 2M^2 \alpha_h H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - \tfrac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \\
\beta_h \left(\tfrac{2M^2}{g^2} + \tfrac{2M}{g}H + \tfrac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-)\right) + \tfrac{2M^4}{g^2}\alpha_h - \\
g\alpha_h M\left(H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-\right) - \\
\tfrac{1}{8}g^2\alpha_h\left(H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2H^2\right) - \\
gMW^+_\mu W^-_\mu H - \tfrac{1}{2}g\tfrac{M}{c^2_w}Z^0_\mu Z^0_\mu H - \\
\tfrac{1}{2}ig\left(W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)\right) + \\
\tfrac{1}{2}g\left(W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) + W^-_\mu(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)\right) + \tfrac{1}{2}g\tfrac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) + \\
M\left(\tfrac{1}{c_w}Z^0_\mu\partial_\mu\phi^0 + W^+_\mu\partial_\mu\phi^- + W^-_\mu\partial_\mu\phi^+\right) - ig\tfrac{s^2_w}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + igs_w MA_\mu(W^+_\mu\phi^- - \\
W^-_\mu\phi^+) - ig\tfrac{1-2c^2_w}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + igs_w A_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \\
\tfrac{1}{4}g^2 W^+_\mu W^-_\mu\left(H^2 + (\phi^0)^2 + 2\phi^+\phi^-\right) - \tfrac{1}{8}g^2\tfrac{1}{c^2_w}Z^0_\mu Z^0_\mu\left(H^2 + (\phi^0)^2 + 2(2s^2_w - 1)^2\phi^+\phi^-\right) - \\
\tfrac{1}{2}g^2\tfrac{s^2_w}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) - \tfrac{1}{2}ig^2\tfrac{s^2_w}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \tfrac{1}{2}g^2 s_w A_\mu\phi^0(W^+_\mu\phi^- + \\
W^-_\mu\phi^+) + \tfrac{1}{2}ig^2 s_w A_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\tfrac{s_w}{c_w}(2c^2_w - 1)Z^0_\mu A_\mu\phi^+\phi^- - \\
g^2 s^2_w A_\mu A_\mu\phi^+\phi^- + \tfrac{1}{2}ig_s\,\lambda^a_{ij}(\bar{q}^\sigma_i\gamma^\mu q^\sigma_j)g^a_\mu - \bar{e}^\lambda(\gamma\partial + m^\lambda_e)e^\lambda - \bar{\nu}^\lambda(\gamma\partial + m^\lambda_\nu)\nu^\lambda - \bar{u}^\lambda_j(\gamma\partial + \\
m^\lambda_u)u^\lambda_j - \bar{d}^\lambda_j(\gamma\partial + m^\lambda_d)d^\lambda_j + igs_w A_\mu\left(-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \tfrac{2}{3}(\bar{u}^\lambda_j\gamma^\mu u^\lambda_j) - \tfrac{1}{3}(\bar{d}^\lambda_j\gamma^\mu d^\lambda_j)\right) + \\
\tfrac{ig}{4c_w}Z^0_\mu\{(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s^2_w - 1 - \gamma^5)e^\lambda) + (\bar{d}^\lambda_j\gamma^\mu(\tfrac{4}{3}s^2_w - 1 - \gamma^5)d^\lambda_j) + \\
(\bar{u}^\lambda_j\gamma^\mu(1 - \tfrac{8}{3}s^2_w + \gamma^5)u^\lambda_j)\} + \tfrac{ig}{2\sqrt{2}}W^+_\mu\left((\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)U^{lep}{}_{\lambda\kappa}e^\kappa) + (\bar{u}^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)\right) + \\
\tfrac{ig}{2\sqrt{2}}W^-_\mu\left((\bar{e}^\kappa U^{lep\dagger}{}_{\kappa\lambda}\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\kappa\lambda}\gamma^\mu(1+\gamma^5)u^\lambda_j)\right) + \\
\tfrac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_e(\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa}(1-\gamma^5)e^\kappa) + m^\lambda_\nu(\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa}(1+\gamma^5)e^\kappa\right) + \\
\tfrac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_e(\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa}(1+\gamma^5)\nu^\kappa) - m^\kappa_\nu(\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa}(1-\gamma^5)\nu^\kappa\right) - \tfrac{g}{2}\tfrac{m^\lambda_\nu}{M}H(\bar{\nu}^\lambda\nu^\lambda) - \\
\tfrac{g}{2}\tfrac{m^\lambda_e}{M}H(\bar{e}^\lambda e^\lambda) + \tfrac{ig}{2}\tfrac{m^\lambda_\nu}{M}\phi^0(\bar{\nu}^\lambda\gamma^5\nu^\lambda) - \tfrac{ig}{2}\tfrac{m^\lambda_e}{M}\phi^0(\bar{e}^\lambda\gamma^5 e^\lambda) - \tfrac{1}{4}\,\bar{\nu}_\lambda\,M^R_{\lambda\kappa}\,(1-\gamma_5)\hat{\nu}_\kappa - \\
\tfrac{1}{4}\,\bar{\nu}_\lambda\,M^R_{\lambda\kappa}\,(1-\gamma_5)\hat{\nu}_\kappa + \tfrac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_d(\bar{u}^\lambda_j C_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) + m^\lambda_u(\bar{u}^\lambda_j C_{\lambda\kappa}(1+\gamma^5)d^\kappa_j\right) + \\
\tfrac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_d(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m^\kappa_u(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1-\gamma^5)u^\kappa_j\right) - \tfrac{g}{2}\tfrac{m^\lambda_u}{M}H(\bar{u}^\lambda_j u^\lambda_j) - \\
\tfrac{g}{2}\tfrac{m^\lambda_d}{M}H(\bar{d}^\lambda_j d^\lambda_j) + \tfrac{ig}{2}\tfrac{m^\lambda_u}{M}\phi^0(\bar{u}^\lambda_j\gamma^5 u^\lambda_j) - \tfrac{ig}{2}\tfrac{m^\lambda_d}{M}\phi^0(\bar{d}^\lambda_j\gamma^5 d^\lambda_j) + \bar{G}^a\partial^2 G^a + g_s f^{abc}\partial_\mu\bar{G}^a G^b g^c_\mu + \\
\bar{X}^+(\partial^2 - M^2)X^+ + \bar{X}^-(\partial^2 - M^2)X^- + \bar{X}^0(\partial^2 - \tfrac{M^2}{c^2_w})X^0 + \bar{Y}\partial^2 Y + igc_w W^+_\mu(\partial_\mu\bar{X}^0 X^- - \\
\partial_\mu\bar{X}^+ X^0) + igs_w W^+_\mu(\partial_\mu\bar{Y}X^- - \partial_\mu\bar{X}^+ Y) + igc_w W^-_\mu(\partial_\mu\bar{X}^- X^0 - \\
\partial_\mu\bar{X}^0 X^+) + igs_w W^-_\mu(\partial_\mu\bar{X}^- Y - \partial_\mu\bar{Y}X^+) + igc_w Z^0_\mu(\partial_\mu\bar{X}^+ X^+ - \\
\partial_\mu\bar{X}^- X^-) + igs_w A_\mu(\partial_\mu\bar{X}^+ X^+ - \\
\partial_\mu\bar{X}^- X^-) - \tfrac{1}{2}gM\left(\bar{X}^+ X^+ H + \bar{X}^- X^- H + \tfrac{1}{c^2_w}\bar{X}^0 X^0 H\right) + \tfrac{1-2c^2_w}{2c_w}igM\left(\bar{X}^+ X^0\phi^+ - \bar{X}^- X^0\phi^-\right) + \\
\tfrac{1}{2c_w}igM\left(\bar{X}^0 X^-\phi^+ - \bar{X}^0 X^+\phi^-\right) + igMs_w\left(\bar{X}^0 X^-\phi^+ - \bar{X}^0 X^+\phi^-\right) + \\
\tfrac{1}{2}igM\left(\bar{X}^+ X^+\phi^0 - \bar{X}^- X^-\phi^0\right).
\end{gathered}
$$

List of all known particle interactions

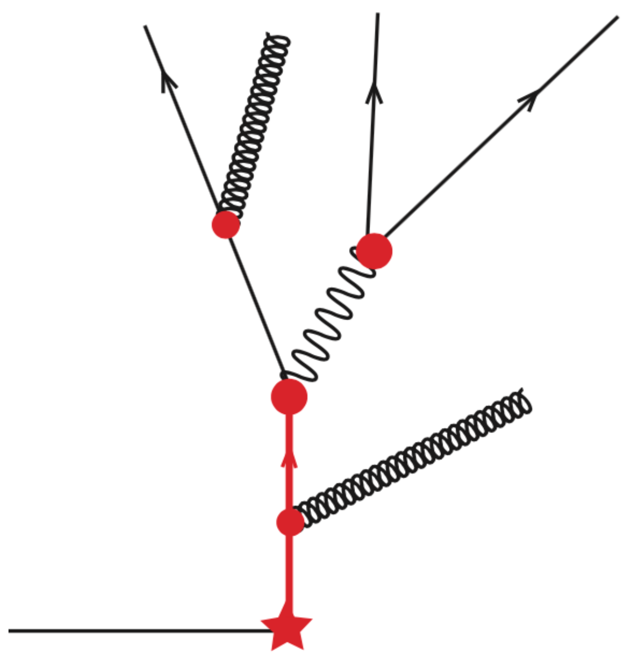


The detector you'll work on
(Compact Muon Solenoid)

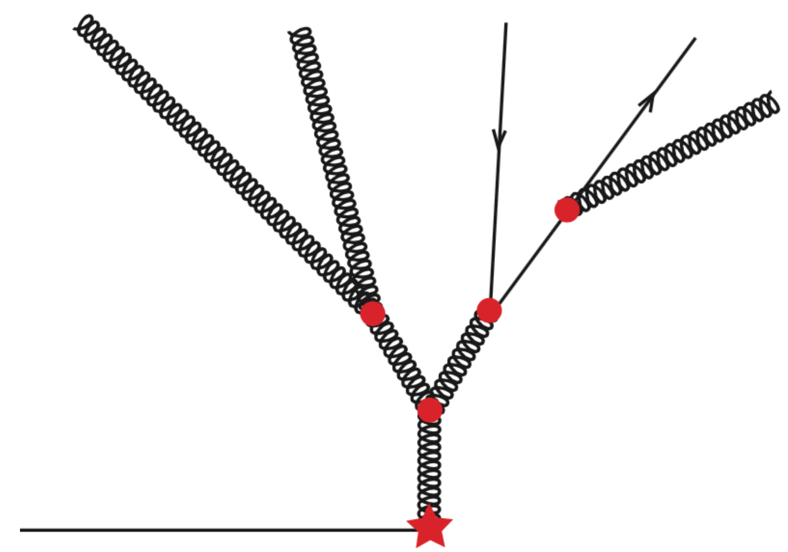
…????

Undergrad Physics —> Groundbreaking Research

- Goal: Classify sprays of strongly interacting particles ("jets") :



Category A: Heavy particles decaying into jets



Category B: Light particles decaying into jets

How do we get there?

Undergrad Physics : Quantum Mechanics

- May not yet know quantum mechanics (that's okay)

- Fast overview:
  - Classical energy conservation:

$$E = KE + PE$$
$$E = \frac{p^2}{2m} + U$$

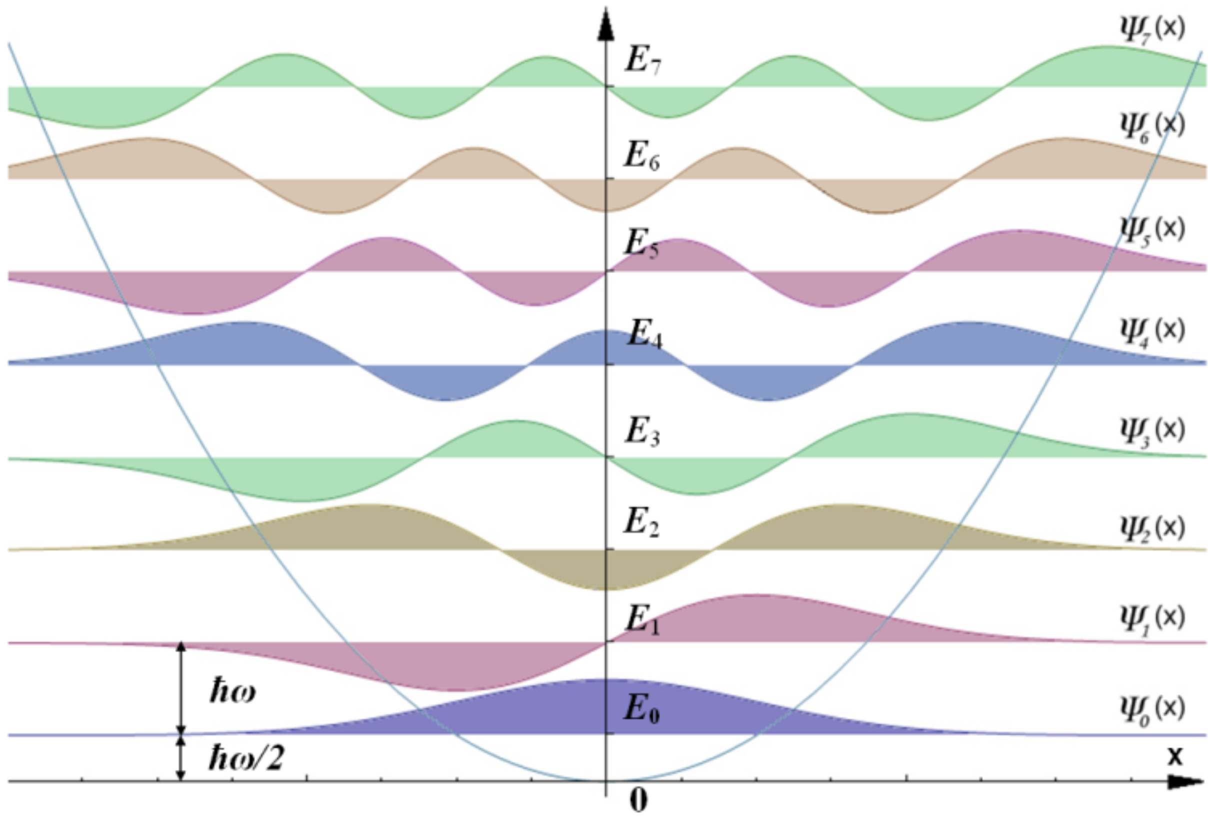
  - Quantum energy conservation (1d)
    - Add the uncertainty principle: $E \rightarrow i\hbar\frac{\partial}{\partial t} \quad p \rightarrow -i\hbar\frac{\partial}{\partial x}$
    - Get the Schroedinger equation:

Now operates on wave functions

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + U(x,t)\psi(x,t)$$

Undergrad Physics : Quantum Mechanics

- Non-relativistic quantum mechanics: Describes motion of small things moving slowly compared to speed of light
  - Hydrogen atom
  - Quantum harmonic oscillator



  - Particles should now be thought of as little wave packets

Undergrad Physics : Special Relativity

- Problem 1: This is incomplete:

$$E = \frac{p^2}{2m} + U$$

Assumes infinite speed of light (false)

  - Nonrelativistic! We know the universe is relativistic.

$$E^2 = p^2c^2 + m^2c^4$$

Rest energy

Relativistic kinetic energy

  - Or:

$$E = \pm\sqrt{p^2c^2 + m^2c^4}$$

$$E = mc^2\sqrt{1 + \frac{p^2c^2}{m^2c^4}}$$

  - If p << m:

$$E = mc^2\left(1 + \frac{1}{2}\frac{p^2}{m^2c^2}\right) \Rightarrow E = mc^2 + \frac{p^2}{2m}$$

Rest energy

Nonrelativistic kinetic energy

~~Under~~grad Physics : Relativistic Quantum Mechanics

- Move toward relativistic quantum mechanics:
  - Probability density:

$$\rho = \psi^* \psi = |\psi|^2$$

  - Evolves according to the probability “current” vector:

$$J = -\frac{i\hbar}{2m}\left(\psi^* \nabla \psi - \psi \nabla \psi^*\right)$$

  - Conservation equation:

$$\nabla \cdot J + \frac{\partial \rho}{\partial t} = 0$$

~~Under~~ grad Physics : Relativistic Quantum Mechanics

- Problem 2: Try Schroedinger's trick:

$$\vec{p} \rightarrow -i\hbar\vec{\nabla} \qquad E \rightarrow i\hbar\frac{\partial}{\partial t}$$

- We get the Klein-Gordon equation

$$\left(-\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \nabla^2\right)\phi = \frac{m^2c^2}{\hbar^2}\phi$$

- Problem: Cannot interpret phi as a simple probability density anymore
  - Hyperbolic equation can arbitrarily specify ϕ and $\partial\phi/\partial t$
- This describes motion of spin-zero fields

~~Under~~ grad Physics : Relativistic Quantum Mechanics

- What about probability density?

$$\rho = \phi^* \phi = |\phi|^2$$

- Doesn't work, because can now arbitrarily specify ϕ and $\partial\phi/\partial t$
- Need a density symmetric in space and time
  - Makes sense! Special relativity is!
- Adjust (purposely changing phi to psi):

$$\rho = \frac{i\hbar}{2mc^2}\left(\psi^* \partial_t \psi - \psi \partial_t \psi^*\right)$$

- Make that zeroth component of 4-vector:

$$J^\mu = \frac{i\hbar}{2m}\left(\psi^* \partial^\mu \psi - \psi \partial^\mu \psi^*\right)$$

~~Under~~grad Physics : Relativistic Quantum Mechanics

- Does it work?

- Still no. If we specify the time derivative arbitrarily, can get negative values (i.e. unphysical for a probability density)

- This means : cannot simply generalize Schroedinger equation assuming SCALAR fields

~~Under~~grad Physics : Relativistic Quantum Mechanics

- Dirac came up with solution:
  - Expand:

$$\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} = \left(A\partial_x + B\partial_y + C\partial_z + \frac{i}{c}D\partial_t\right)^2$$

  - Cross terms like $\partial_x\partial_y$ cancel if $\{A, B\} = 0$
    but $A^2 = B^2 = \ldots = 1$

$AB + BA = 0$

- This works if A, B, C, and D are MATRICES, not numbers!
- Arrive at Dirac equation:

$$i\hbar\gamma^\mu\partial_\mu\psi - mc\psi = 0$$

where γ^μ are special matrices:

$$\gamma^0 = \begin{pmatrix} I_2 & 0 \\ 0 & -I_2 \end{pmatrix}, \gamma^1 = \begin{pmatrix} 0 & \sigma_x \\ -\sigma_x & 0 \end{pmatrix}, \gamma^2 = \begin{pmatrix} 0 & \sigma_y \\ -\sigma_y & 0 \end{pmatrix}, \gamma^3 = \begin{pmatrix} 0 & \sigma_z \\ -\sigma_z & 0 \end{pmatrix}$$

~~Under~~grad Physics : Relativistic Quantum Mechanics

- These are 4x4 matrices:

$$\gamma^0 = \begin{pmatrix} I_2 & 0 \\ 0 & -I_2 \end{pmatrix}, \gamma^1 = \begin{pmatrix} 0 & \sigma_x \\ -\sigma_x & 0 \end{pmatrix}, \gamma^2 = \begin{pmatrix} 0 & \sigma_y \\ -\sigma_y & 0 \end{pmatrix}, \gamma^3 = \begin{pmatrix} 0 & \sigma_z \\ -\sigma_z & 0 \end{pmatrix}$$

- Where I is the identity matrix and the sigmas are the Pauli matrices:

$$\sigma_1 = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_2 = \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_3 = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

~~Under~~ grad Physics : Relativistic Quantum Mechanics

- What does this MEAN, though?
- There are 4 components of ψ!
  - Relativistic spin-1/2 field
  - Includes antimatter!

(Geek out about that for a second)

- Define "slash" notation as a sum over these matrices times the components :

$$\not{v} = \sum_i \gamma_i v_i$$

- Set c = 1, and we get the Dirac equation:

$$(i\not{\partial} - m)\,\psi = 0$$

~~Under~~grad Physics : Relativistic Quantum Mechanics

- Now define the adjoint spinor:

$$\bar{\psi} = \psi^\dagger \gamma^0$$

- And Now the current density equation becomes:

$$\partial_\mu \left(\bar{\psi} \gamma^\mu \psi \right) = 0$$

- The probability density is:

$$J^0 = \bar{\psi} \gamma^0 \psi = \psi^\dagger \psi$$

~~Under~~grad Physics : Quantum Field Theory

- If you remember E+M, it is the relationship between CURRENTS and FIELDS
  - We have the currents
  - What about the fields?
- Recall Maxwell's equations, but now in 4-vector notation:
  - Electric and magnetic potentials put to 4-vector:

$$A^\mu = \left(\phi, \vec{A}\right)$$

  - Then the field strength tensor is:

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

  - Maxwell's equations become:

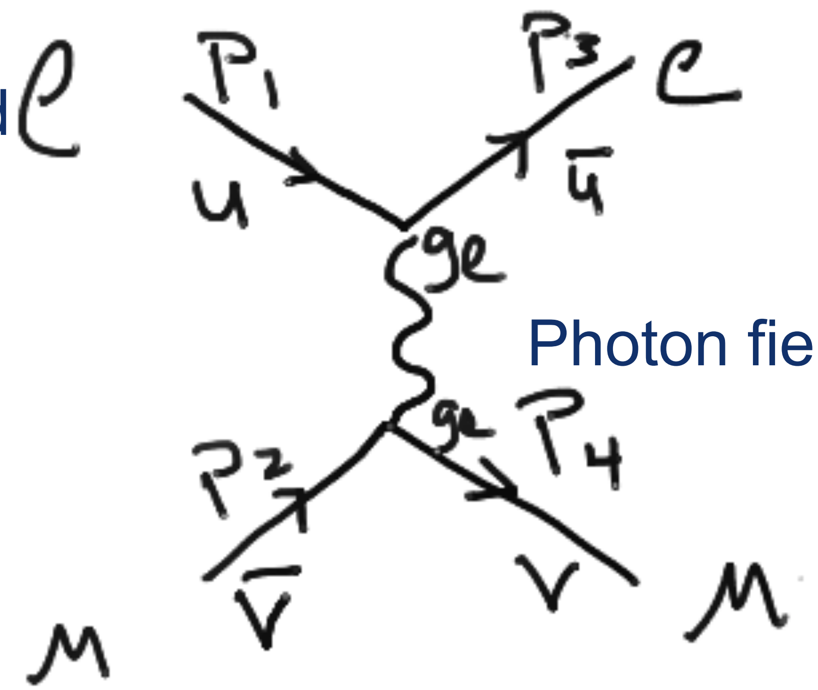
$$\partial_\mu F^{\mu\nu} = 4\pi J^\mu$$

This is quantum electrodynamics!

~~Under~~grad Physics : Quantum Field Theory

- Similar currents exist for matter (electrons, etc)
  – Quantum field theory!
- Interactions are then specified in terms of interactions of the currents and the fields

Electron field

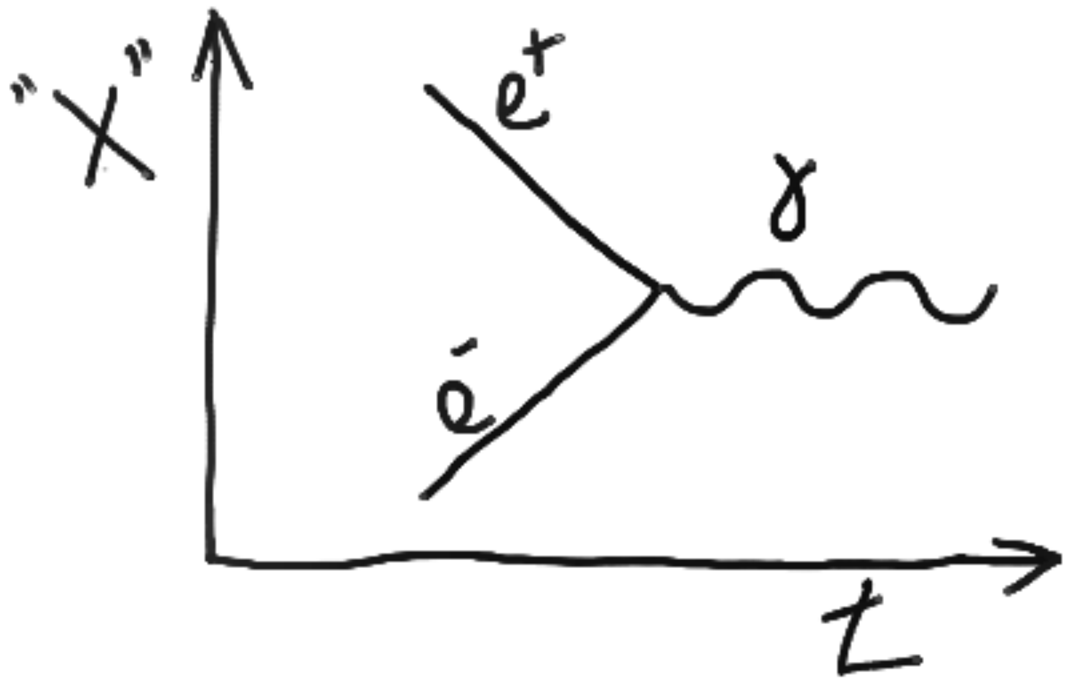


Photon field : interaction!

Muon field

~~Under~~grad Physics : Quantum Field Theory

- Graphical way to represent **interaction**
- Time goes left-to-right
- “Space” is up-and-down
  - This is a very vague notion of “space”, it’s merely symbolic



~~Under~~grad Physics : Quantum Field Theory

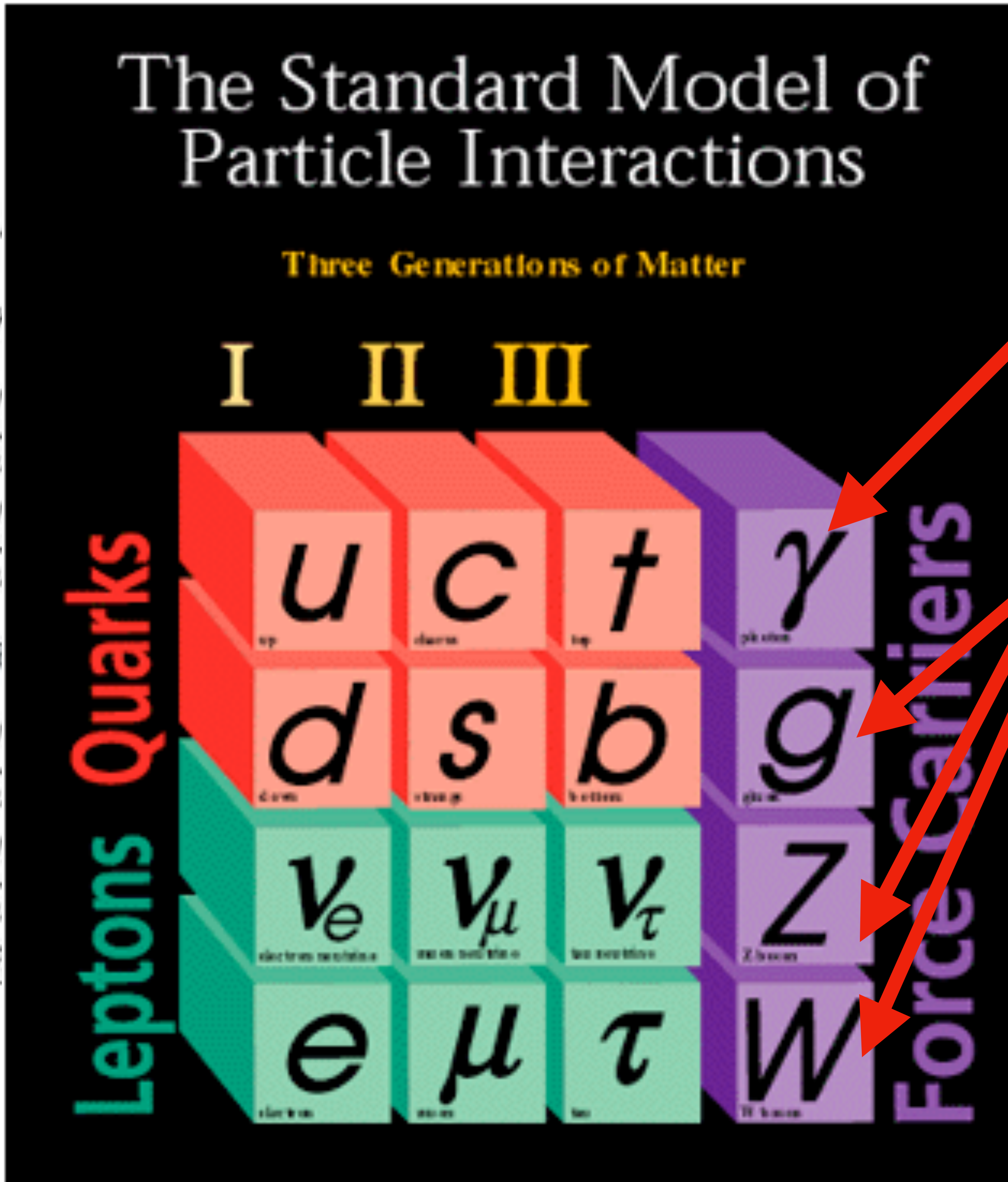
$$
\begin{gathered}
\mathcal{L}_{SM} = -\tfrac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \tfrac{1}{4}g_s^2 f^{abc}f^{ade}g^b_\mu g^c_\nu g^d_\mu g^e_\nu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - \\
M^2 W^+_\mu W^-_\mu - \tfrac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \tfrac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu - \tfrac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - igc_w(\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)) - \\
igs_w(\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + A_\mu(W^+_\nu \partial_\nu W^-_\mu - \\
W^-_\nu \partial_\nu W^+_\mu)) - \tfrac{1}{2}g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \tfrac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - \\
Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w(A_\mu Z^0_\nu(W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu) - \tfrac{1}{2}\partial_\mu H \partial_\mu H - 2M^2\alpha_h H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - \tfrac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \\
\beta_h\left(\tfrac{2M^2}{g^2} + \tfrac{2M}{g}H + \tfrac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-)\right) + \tfrac{2M^4}{g^2}\alpha_h - \\
g\alpha_h M\left(H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-\right) - \\
\tfrac{1}{8}g^2\alpha_h\left(H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2H^2\right) - \\
gMW^+_\mu W^-_\mu H - \tfrac{1}{2}g\tfrac{M}{c_w^2}Z^0_\mu Z^0_\mu H - \\
\tfrac{1}{2}ig\left(W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)\right) + \\
\tfrac{1}{2}g\left(W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) + W^-_\mu(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)\right) + \tfrac{1}{2}g\tfrac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) + \\
M\left(\tfrac{1}{c_w}Z^0_\mu\partial_\mu\phi^0 + W^+_\mu\partial_\mu\phi^- + W^-_\mu\partial_\mu\phi^+\right) - ig\tfrac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + igs_w MA_\mu(W^+_\mu\phi^- - \\
W^-_\mu\phi^+) - ig\tfrac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + igs_w A_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \\
\tfrac{1}{4}g^2 W^+_\mu W^-_\mu\left(H^2 + (\phi^0)^2 + 2\phi^+\phi^-\right) - \tfrac{1}{8}g^2\tfrac{1}{c_w^2}Z^0_\mu Z^0_\mu\left(H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2\phi^+\phi^-\right) - \\
\tfrac{1}{2}g^2\tfrac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) - \tfrac{1}{2}ig^2\tfrac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \tfrac{1}{2}g^2 s_w A_\mu\phi^0(W^+_\mu\phi^- + \\
W^-_\mu\phi^+) + \tfrac{1}{2}ig^2 s_w A_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\tfrac{s_w}{c_w}(2c_w^2 - 1)Z^0_\mu A_\mu\phi^+\phi^- - \\
g^2 s_w^2 A_\mu A_\mu\phi^+\phi^- + \tfrac{1}{2}ig_s\lambda^a_{ij}(\bar{q}^\sigma_i\gamma^\mu q^\sigma_j)g^a_\mu - \bar{e}^\lambda(\gamma\partial + m^\lambda_e)e^\lambda - \bar{\nu}^\lambda(\gamma\partial + m^\lambda_\nu)\nu^\lambda - \bar{u}^\lambda_j(\gamma\partial + \\
m^\lambda_u)u^\lambda_j - \bar{d}^\lambda_j(\gamma\partial + m^\lambda_d)d^\lambda_j + igs_w A_\mu\left(-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \tfrac{2}{3}(\bar{u}^\lambda_j\gamma^\mu u^\lambda_j) - \tfrac{1}{3}(\bar{d}^\lambda_j\gamma^\mu d^\lambda_j)\right) + \\
\tfrac{ig}{4c_w}Z^0_\mu\{(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda) + (\bar{d}^\lambda_j\gamma^\mu(\tfrac{4}{3}s_w^2 - 1 - \gamma^5)d^\lambda_j) + \\
(\bar{u}^\lambda_j\gamma^\mu(1 - \tfrac{8}{3}s_w^2 + \gamma^5)u^\lambda_j)\} + \tfrac{ig}{2\sqrt{2}}W^+_\mu\left((\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)U^{lep}{}_{\lambda\kappa}e^\kappa) + (\bar{u}^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)\right) + \\
\tfrac{ig}{2\sqrt{2}}W^-_\mu\left((\bar{e}^\kappa U^{lep\dagger}{}_{\kappa\lambda}\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\kappa\lambda}\gamma^\mu(1+\gamma^5)u^\lambda_j)\right) + \\
\tfrac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_e(\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa}(1-\gamma^5)e^\kappa) + m^\lambda_\nu(\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa}(1+\gamma^5)e^\kappa\right) + \\
\tfrac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_e(\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa}(1+\gamma^5)\nu^\kappa) - m^\kappa_\nu(\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa}(1-\gamma^5)\nu^\kappa\right) - \tfrac{g}{2}\tfrac{m^\lambda_\nu}{M}H(\bar{\nu}^\lambda\nu^\lambda) - \\
\tfrac{g}{2}\tfrac{m^\lambda_e}{M}H(\bar{e}^\lambda e^\lambda) + \tfrac{ig}{2}\tfrac{m^\lambda_\nu}{M}\phi^0(\bar{\nu}^\lambda\gamma^5\nu^\lambda) - \tfrac{ig}{2}\tfrac{m^\lambda_e}{M}\phi^0(\bar{e}^\lambda\gamma^5 e^\lambda) - \tfrac{1}{4}\bar{\nu}_\lambda M^R_{\lambda\kappa}(1-\gamma_5)\hat{\nu}_\kappa - \\
\tfrac{1}{4}\overline{\bar{\nu}_\lambda M^R_{\lambda\kappa}(1-\gamma_5)\hat{\nu}_\kappa} + \tfrac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_d(\bar{u}^\lambda_j C_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) + m^\lambda_u(\bar{u}^\lambda_j C_{\lambda\kappa}(1+\gamma^5)d^\kappa_j\right) + \\
\tfrac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_d(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m^\kappa_u(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1-\gamma^5)u^\kappa_j\right) - \tfrac{g}{2}\tfrac{m^\lambda_u}{M}H(\bar{u}^\lambda_j u^\lambda_j) - \\
\tfrac{g}{2}\tfrac{m^\lambda_d}{M}H(\bar{d}^\lambda_j d^\lambda_j) + \tfrac{ig}{2}\tfrac{m^\lambda_u}{M}\phi^0(\bar{u}^\lambda_j\gamma^5 u^\lambda_j) - \tfrac{ig}{2}\tfrac{m^\lambda_d}{M}\phi^0(\bar{d}^\lambda_j\gamma^5 d^\lambda_j) + \bar{G}^a\partial^2 G^a + g_s f^{abc}\partial_\mu\bar{G}^a G^b g^c_\mu + \\
\bar{X}^+(\partial^2 - M^2)X^+ + \bar{X}^-(\partial^2 - M^2)X^- + \bar{X}^0(\partial^2 - \tfrac{M^2}{c_w^2})X^0 + \bar{Y}\partial^2 Y + igc_w W^+_\mu(\partial_\mu\bar{X}^0 X^- - \\
\partial_\mu\bar{X}^+ X^0) + igs_w W^+_\mu(\partial_\mu\bar{Y}X^- - \partial_\mu\bar{X}^+ Y) + igc_w W^-_\mu(\partial_\mu\bar{X}^- X^0 - \\
\partial_\mu\bar{X}^0 X^+) + igs_w W^-_\mu(\partial_\mu\bar{X}^- Y - \partial_\mu\bar{Y}X^+) + igc_w Z^0_\mu(\partial_\mu\bar{X}^+ X^+ - \\
\partial_\mu\bar{X}^- X^-) + igs_w A_\mu(\partial_\mu\bar{X}^+ X^+ - \\
\partial_\mu\bar{X}^- X^-) - \tfrac{1}{2}gM\left(\bar{X}^+ X^+ H + \bar{X}^- X^- H + \tfrac{1}{c_w^2}\bar{X}^0 X^0 H\right) + \tfrac{1-2c_w^2}{2c_w}igM\left(\bar{X}^+ X^0\phi^+ - \bar{X}^- X^0\phi^-\right) + \\
\tfrac{1}{2c_w}igM\left(\bar{X}^0 X^-\phi^+ - \bar{X}^0 X^+\phi^-\right) + igMs_w\left(\bar{X}^0 X^-\phi^+ - \bar{X}^0 X^+\phi^-\right) + \\
\tfrac{1}{2}igM\left(\bar{X}^+ X^+\phi^0 - \bar{X}^- X^-\phi^0\right) .
\end{gathered}
$$

- This is the just the Lagrangian of the Standard Model of particle physics:
  - SU(3) : quantum chromodynamics
  - SU(2): weak interaction
  - U(1): quantum electrodynamics

~~Under~~ grad Physics : Quantum Field Theory

$$
\begin{aligned}
\mathcal{L} = & -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{8}tr(\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}) - \frac{1}{2}tr(\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu}) && \text{(U(1), SU(2) and SU(3) gauge terms)} \\
& +(\bar{\nu}_L, \bar{e}_L)\,\tilde{\sigma}^\mu iD_\mu \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} + \bar{e}_R\sigma^\mu iD_\mu e_R + \bar{\nu}_R\sigma^\mu iD_\mu \nu_R + \text{(h.c.)} && \text{(lepton dynamical term)} \\
& -\frac{\sqrt{2}}{v}\left[(\bar{\nu}_L, \bar{e}_L)\,\phi M^e e_R + \bar{e}_R \bar{M}^e \bar{\phi}\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}\right] && \text{(electron, muon, tauon mass term)} \\
& -\frac{\sqrt{2}}{v}\left[(-\bar{e}_L, \bar{\nu}_L)\,\phi^* M^\nu \nu_R + \bar{\nu}_R \bar{M}^\nu \phi^T \begin{pmatrix} -e_L \\ \nu_L \end{pmatrix}\right] && \text{(neutrino mass term)} \\
& +(\bar{u}_L, \bar{d}_L)\,\tilde{\sigma}^\mu iD_\mu \begin{pmatrix} u_L \\ d_L \end{pmatrix} + \bar{u}_R\sigma^\mu iD_\mu u_R + \bar{d}_R\sigma^\mu iD_\mu d_R + \text{(h.c.)} && \text{(quark dynamical term)} \\
& -\frac{\sqrt{2}}{v}\left[(\bar{u}_L, \bar{d}_L)\,\phi M^d d_R + \bar{d}_R \bar{M}^d \bar{\phi}\begin{pmatrix} u_L \\ d_L \end{pmatrix}\right] && \text{(down, strange, bottom mass term)} \\
& -\frac{\sqrt{2}}{v}\left[(-\bar{d}_L, \bar{u}_L)\,\phi^* M^u u_R + \bar{u}_R \bar{M}^u \phi^T \begin{pmatrix} -d_L \\ u_L \end{pmatrix}\right] && \text{(up, charmed, top mass term)} \\
& +\overline{(D_\mu\phi)}D^\mu\phi - m_h^2[\bar{\phi}\phi - v^2/2]^2/2v^2. && \text{(Higgs dynamical and mass term)} \quad (1)
\end{aligned}
$$

~~Under~~grad Physics : Quantum Field Theory



$\mathcal{L} = -\frac{1}{4}$ (U(1), SU(2) and SU(3) gauge terms)

$+(\bar{\nu}$ (lepton dynamical term)

$-\frac{\sqrt{\ }}{v}$ (electron, muon, tauon mass term)

$-\frac{\sqrt{\ }}{v}$ (neutrino mass term)

$+(\bar{u}$ (quark dynamical term)

$-\frac{\sqrt{\ }}{v}$ (down, strange, bottom mass term)

$-\frac{\sqrt{\ }}{v}$ (up, charmed, top mass term)

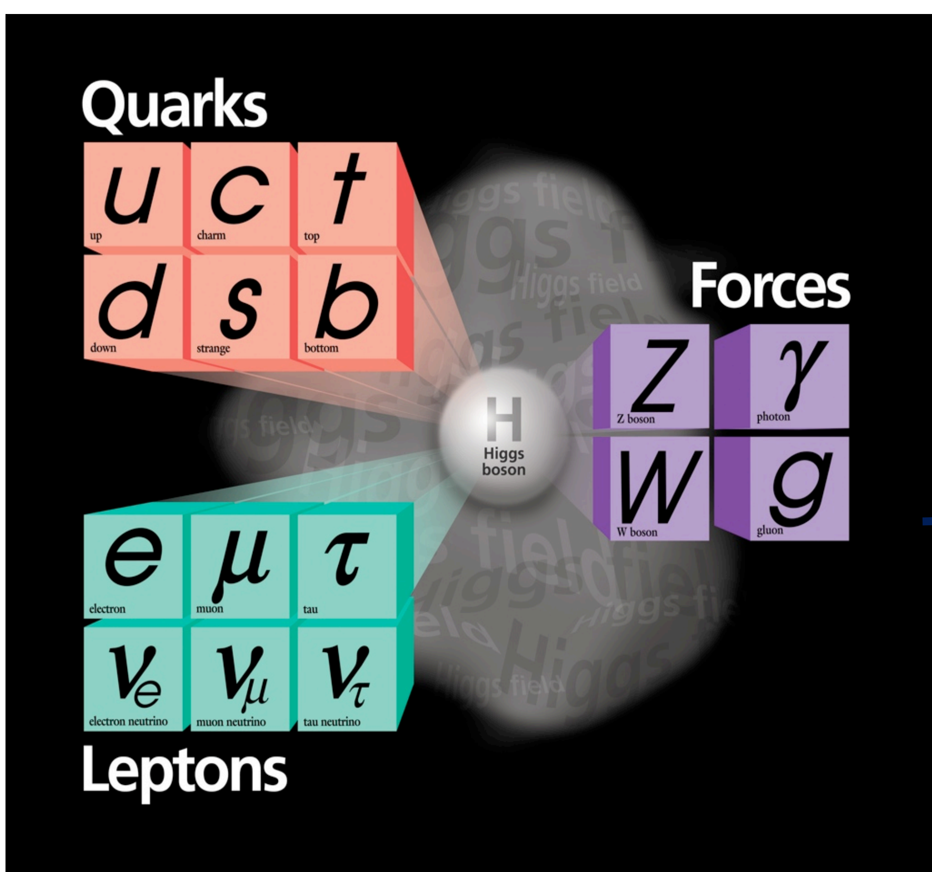
$+(\overline{D}$ (Higgs dynamical and mass term) (1)

~~Under~~grad Physics : Quantum Field Theory

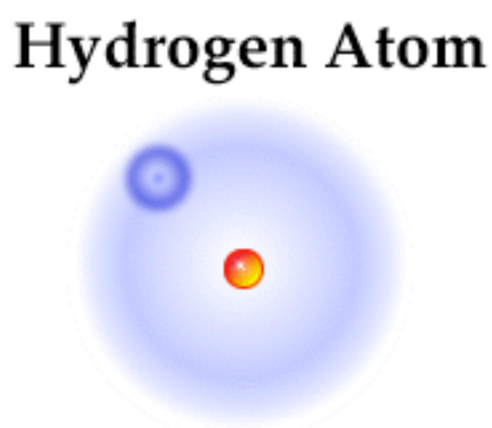
- Electromagnetic interaction and Weak interaction are UNIFIED
  - This is the "electroweak symmetry"
  - It is broken ("electroweak symmetry breaking") by the Higgs field
  - If there were no Higgs, the weak and EM interactions would be identical
    - Would live in a structureless universe

Phys.Lett. B716 (2012) 1-29

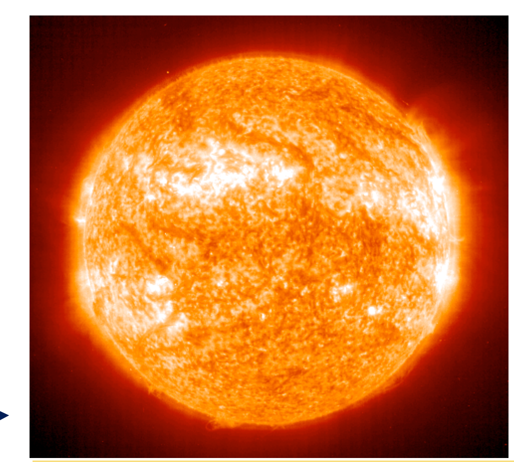
Phys.Lett. B716 (2012) 30-61

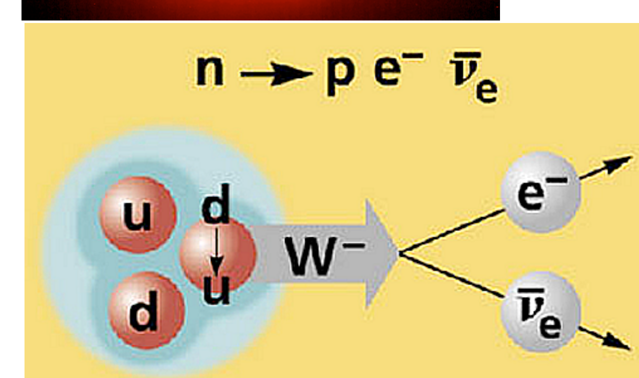


Need electron mass for stable atoms



Need W mass for fusion in stars





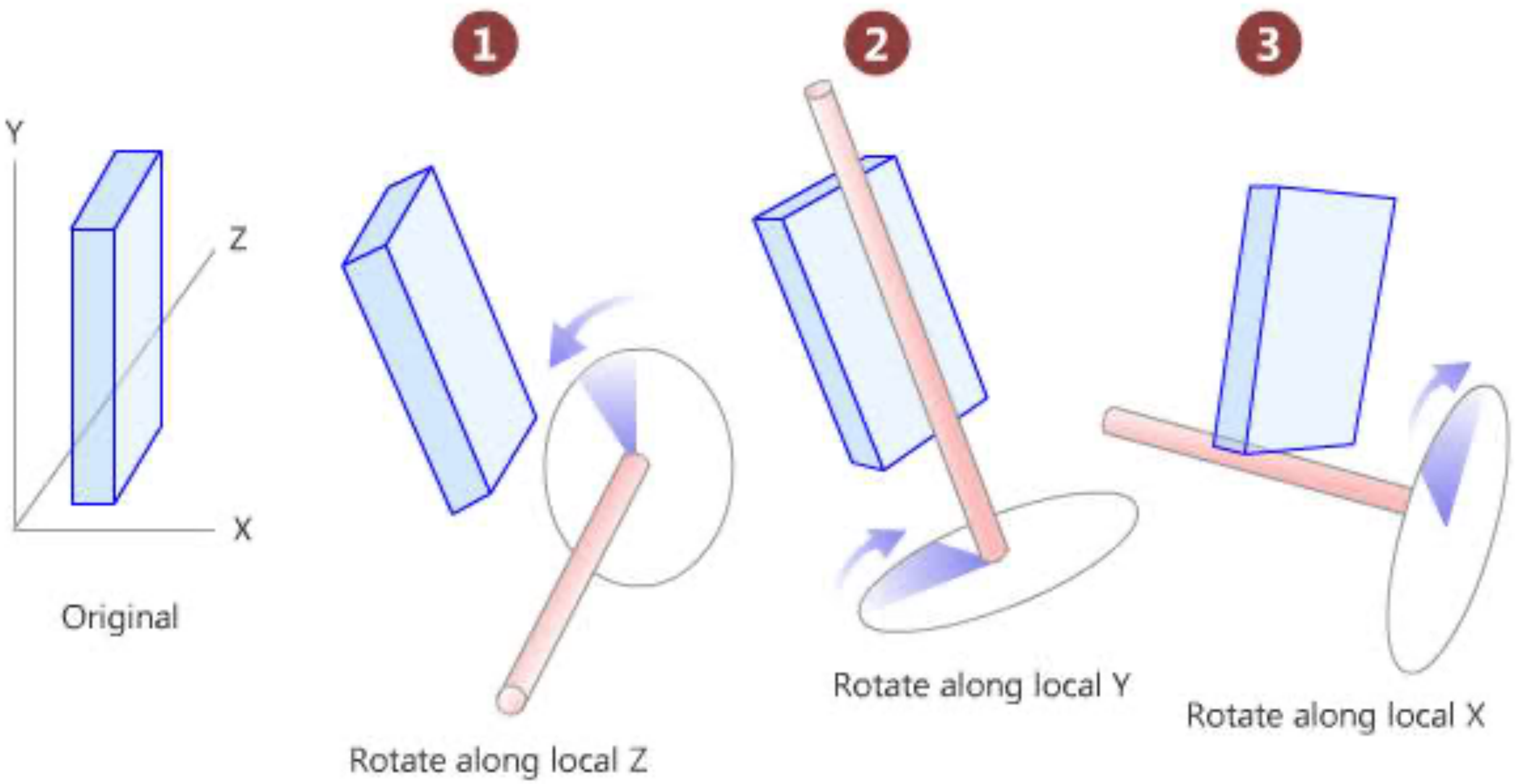
Undergrad Physics : Rotations

- What is a "unification"?
  - Symmetry groups are part of a larger symmetry

- Example: Rotations

"Special Orthogonal" == rotation

  - Three 2d rotations in space, all "SO(2)"
    - Rotations about X
    - Rotations about Y
    - Rotations about Z
  - All are contained within 3d rotations! "SO(3)"



Undergrad Physics : EM interaction symmetry

- Remember?

$$\vec{F} = -\vec{\nabla} U$$

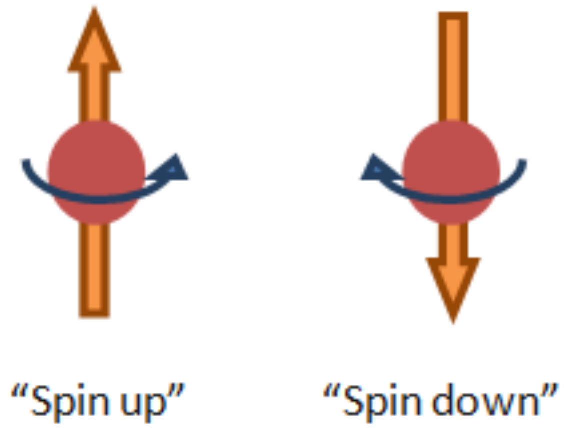
- Adding a constant does not matter

- This is a U(1) symmetry!
  - One force carrier: the photon

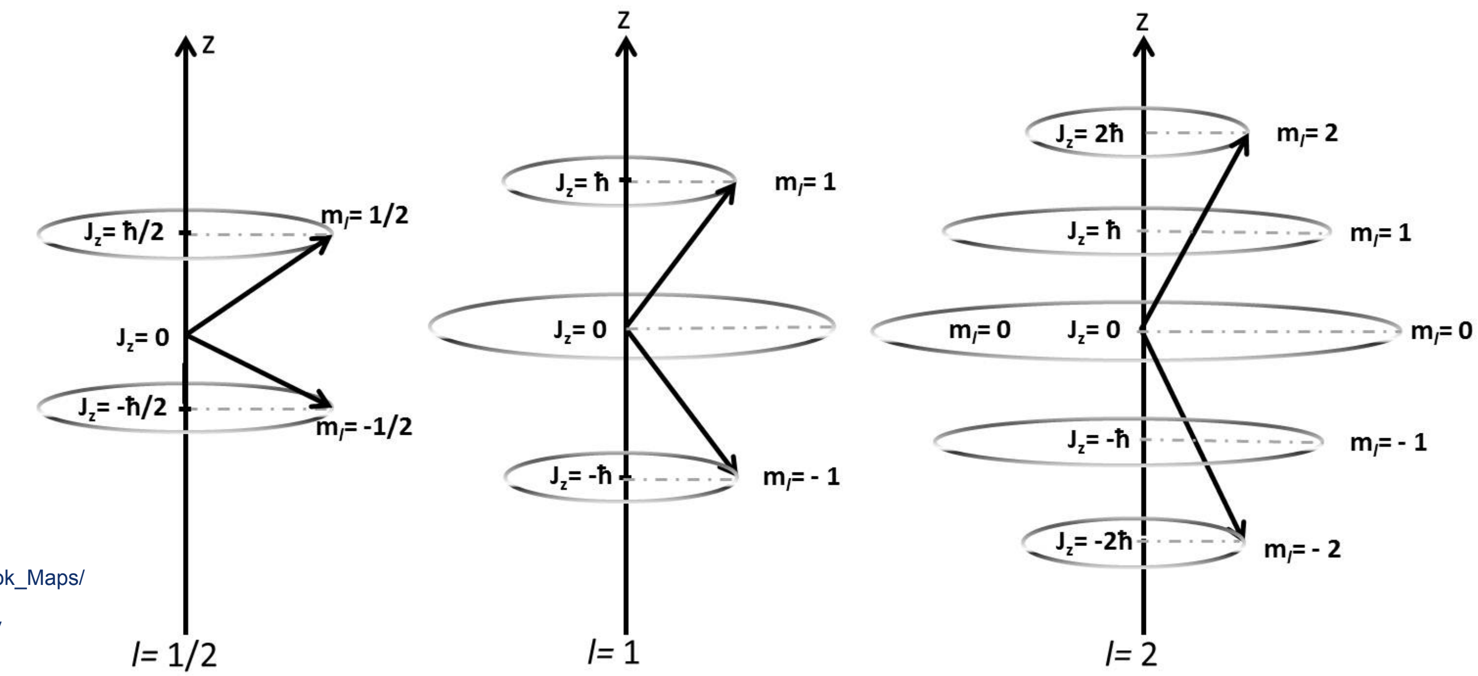
Undergrad Physics : Spin + SU(2)

- SU(2): Like quantum mechanical spin

$$L = \hbar\sqrt{\ell(\ell+1)}$$



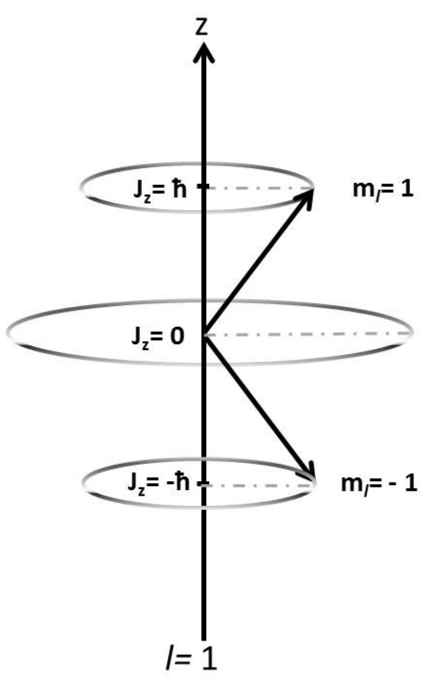
  - Electrons are fermions, have spin 1/2. In hydrogen atom can have "spin up" (+1/2) or "spin down" (-1/2)
  - W, Z, photon are bosons, have spin 1. Weak interaction has an SU(2) symmetry too, but there are now THREE states (+1, 0, -1), so there are THREE force carriers (W+, Z0, W-)



https://chem.libretexts.org/Bookshelves/Physical_and_Theoretical_Chemistry_Textbook_Maps/Supplemental_Modules_(Physical_and_Theoretical_Chemistry)/Spectroscopy/Magnetic_Resonance_Spectroscopies/Nuclear_Magnetic_Resonance/NMR_-_Theory

Electroweak Symmetry

- SU(2) x U(1) symmetry means there is a U(1) symmetry AND an SU(2) symmetry
  - Similar to as if you had an SO(3) symmetry AND an SO(2) symmetry, in 4-d space, but not SO(4) symmetry
  - There are thus 4 force carriers in SU(2) x U(1):



W^+

W^0

W^-

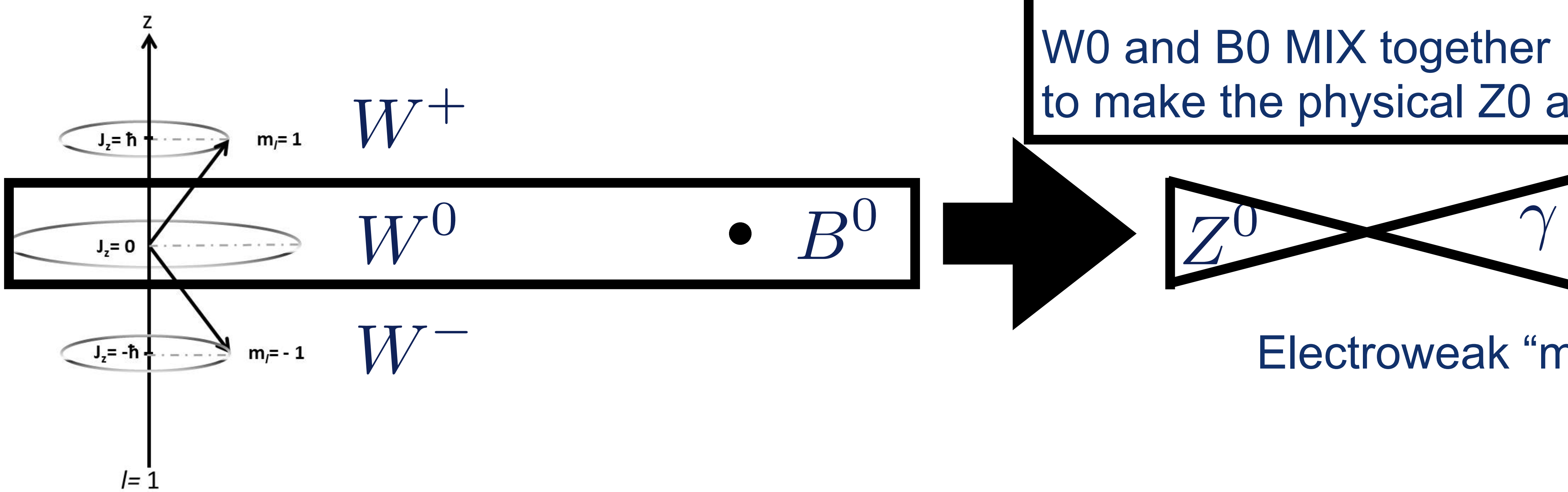
- B^0

Electroweak Symmetry

- SU(2) x U(1) symmetry means there is a U(1) symmetry AND an SU(2) symmetry
  - Similar to as if you had an SO(3) symmetry AND an SO(2) symmetry, in 4-d space, but not SO(4) symmetry
  - There are thus 4 force carriers in SU(2) x U(1):

QM Weirdness:

W0 and B0 MIX together
to make the physical Z0 and photon



Electroweak “mixing”

Electroweak Symmetry Breaking

- SU(2) x U(1) fields
  "massless" fields

$$W^+$$

$$W^0 \quad B^0$$

$$W^-$$

Electroweak Symmetry Breaking

- SU(2) x U(1) fields
  "massless" fields
- W0 and B0 "mix" to form Z0
  and photon

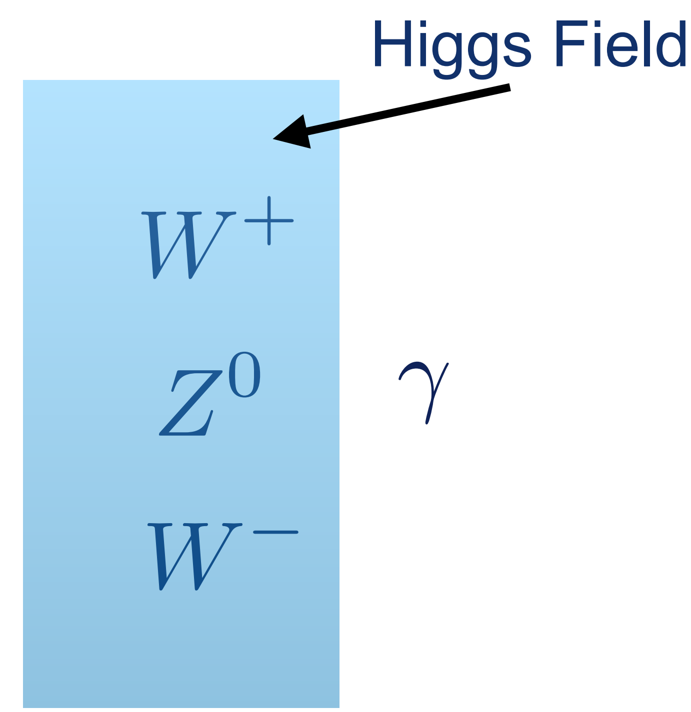
W^+

W^0 Z^0 $\quad$ B^0 γ

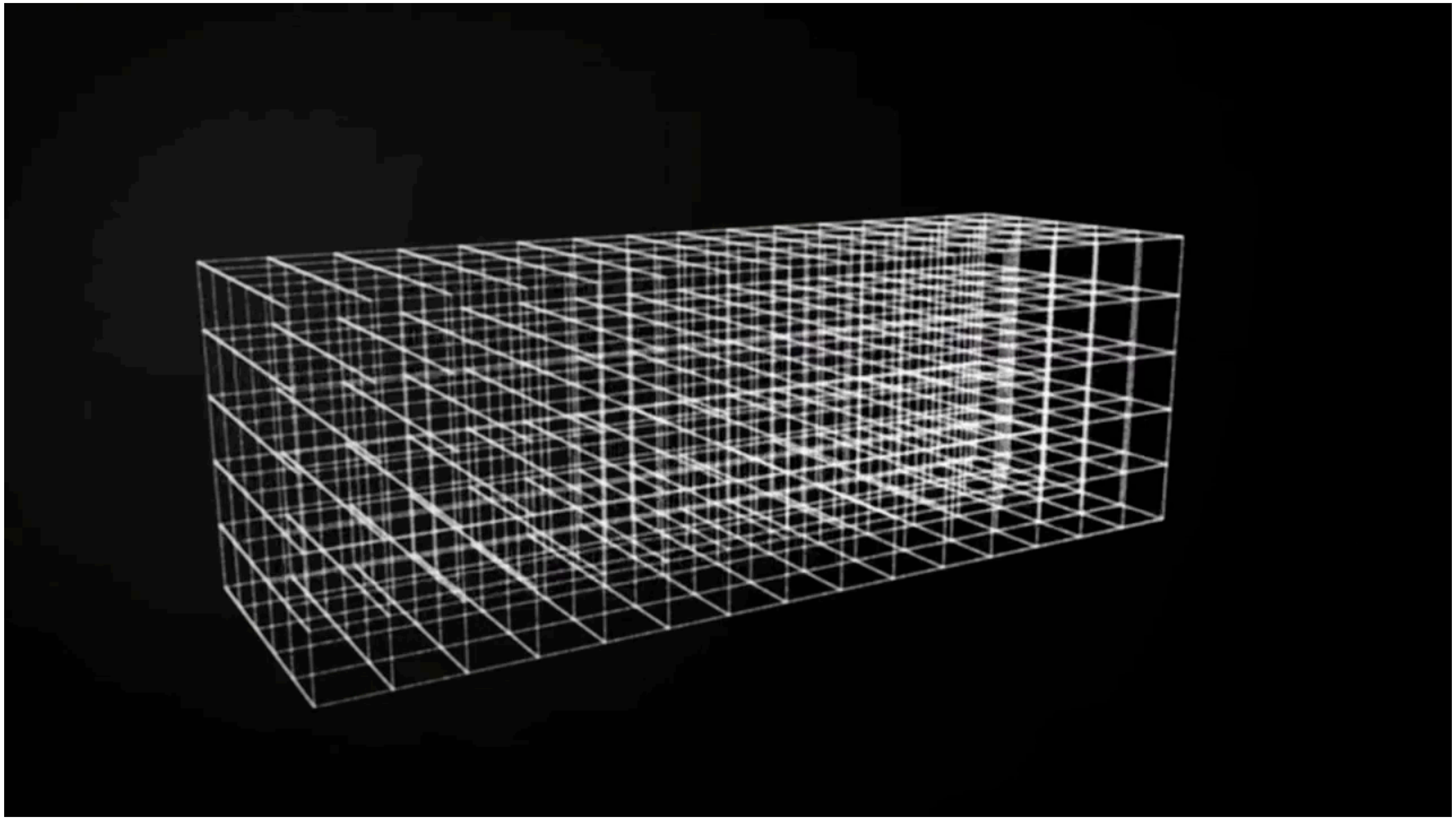
W^-

Electroweak Symmetry Breaking

- SU(2) x U(1) fields
  "massless" fields
- W0 and B0 "mix" to form Z0 and photon
- Interactions with Higgs fields gives mass to W/Z bosons and leaves photon massless



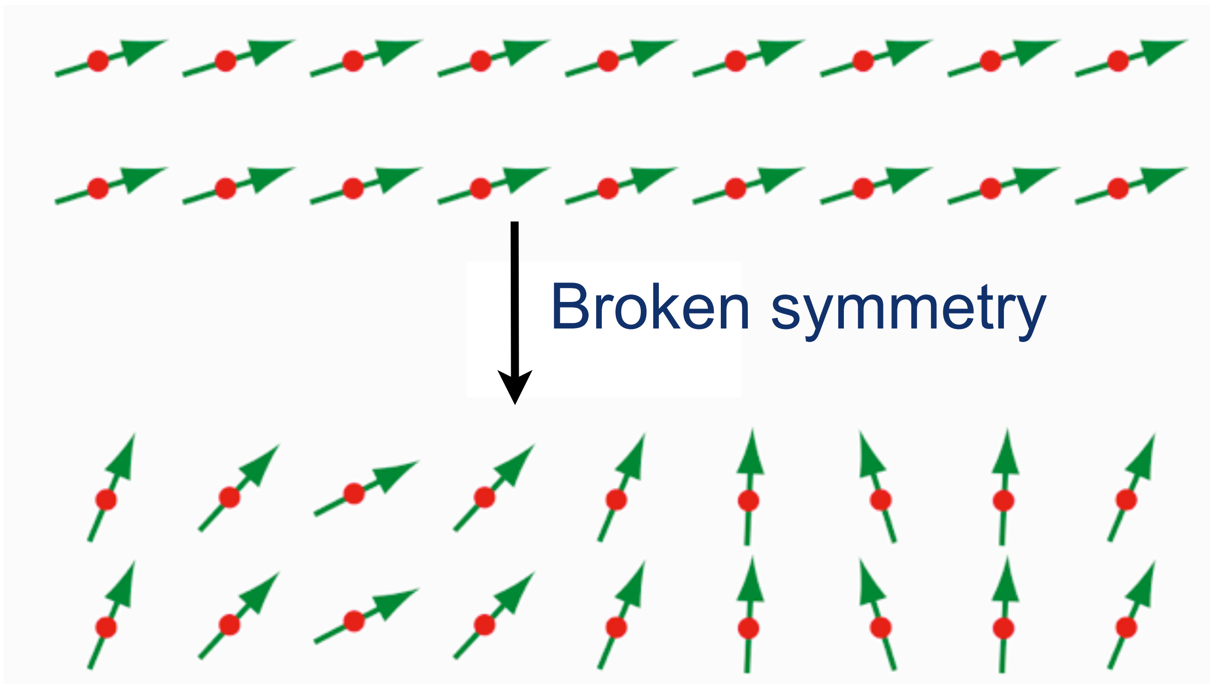
Higgs Mechanism



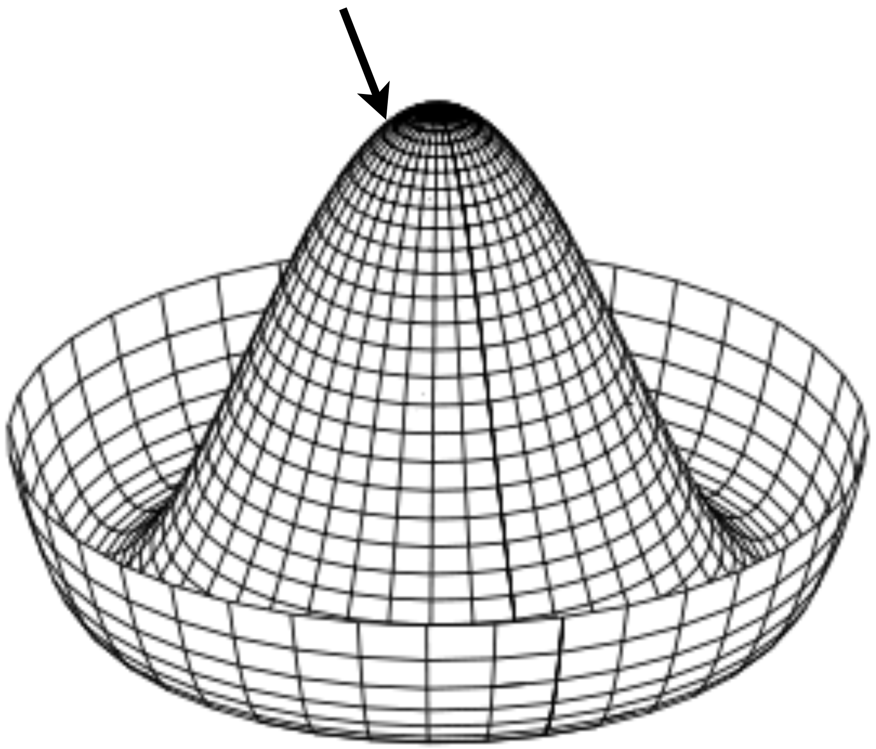
Higgs Mechanism

- Simplest explanation for EWSB is the "Higgs mechanism"
- Nambu-Goldstone boson for the spontaneously broken symmetry of SU(2)XU(1)
- Similar to superconductor or ferromagnet!
- Simplest field to do this is a "Sombrero" potential

(OK, it's really the Englert-Brout-Higgs-Guralnik-Hagen-Kibble Mechanism, but most just call it the Higgs Mechanism)



Nambu-Goldstone boson "generated" by the broken symmetry



Ground state breaks the EW symmetry!

Quantum Chromodynamics (QCD)

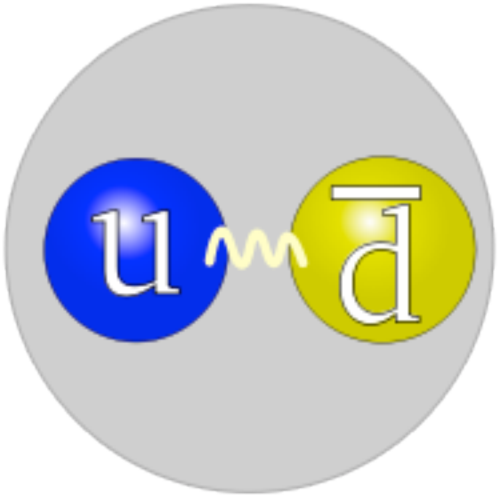
- The strong interaction (quantum chromodynamics — QCD) is left for last
  - This is the one we're going to work with this summer
  - U(1): 1 force carrier
  - SU(2): 3 force carriers
  - SU(3) : 8 force carriers (gluons)

- QCD has an SU(3) symmetry, and gluons are massless
  - Quantum interactions "look like" QED, but…
  - Additional complication: Asymptotic freedom

QCD

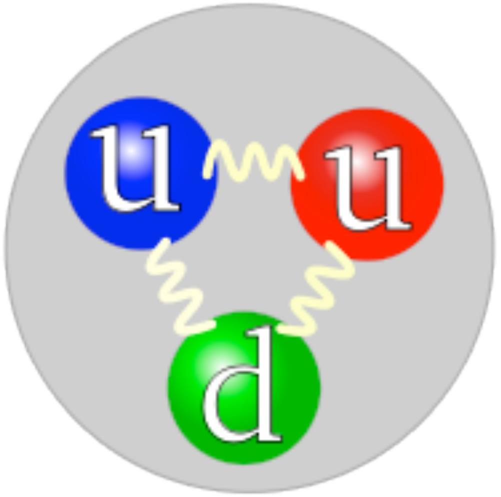
- Because observed particles are color-neutral, the colored particles must exist in **bound states**
  - Held together by gluons!

- Two quarks : meson
  - Example : pion



http://en.wikipedia.org/wiki/Pion

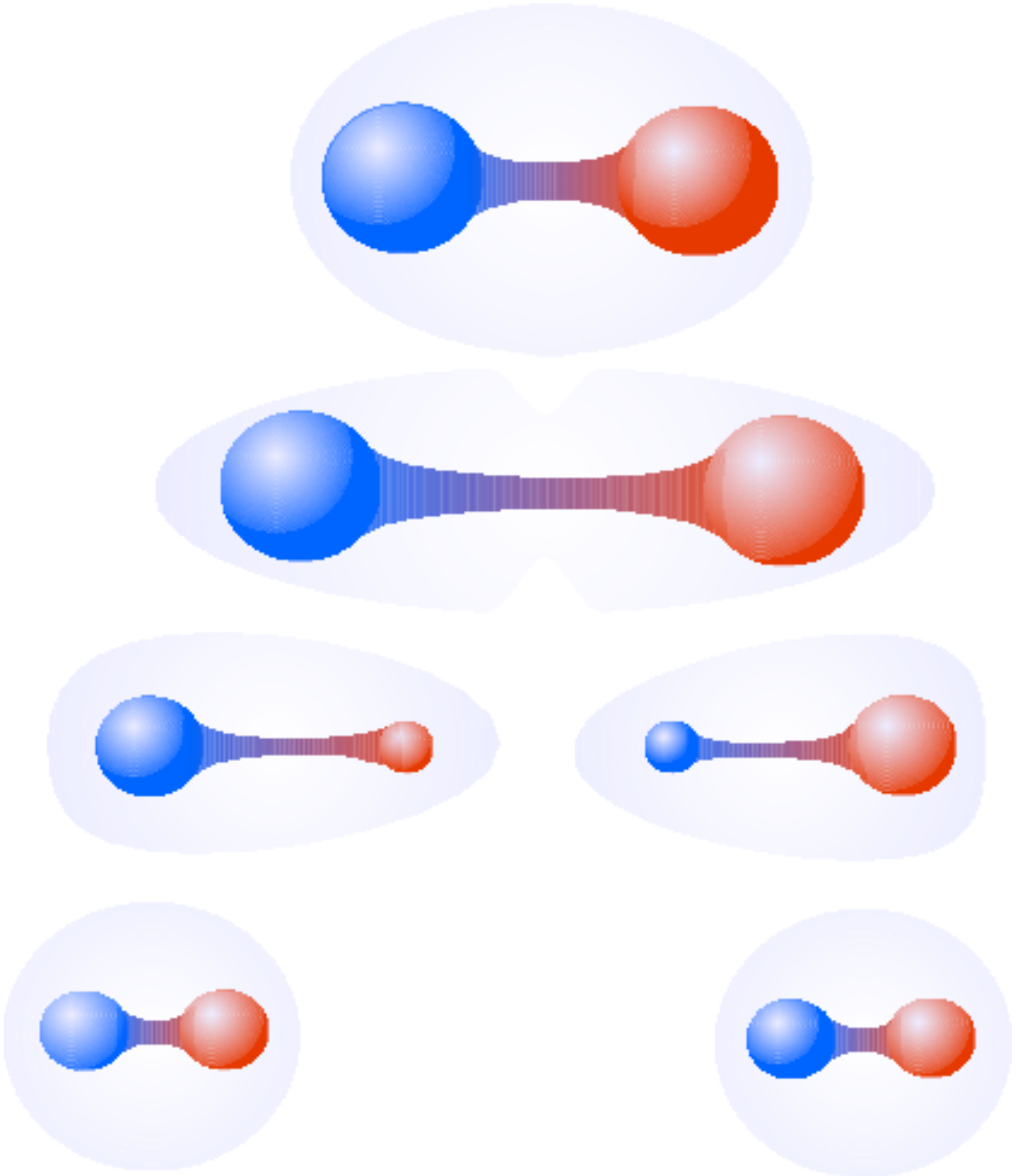
- Three quarks : baryons
  - Example : proton



http://en.wikipedia.org/wiki/Proton

QCD

- So what happens if you try to, say, pull apart a pion?



As you try to break the quarks apart, a quantum pair of quarks is pulled from the vacuum, and you get two hadrons instead of one!

Quarks are confined!

http://zam581.zam.kfa-juelich.de/nic/Publikationen/Broschuere/elementarteilchenphysik-e.html