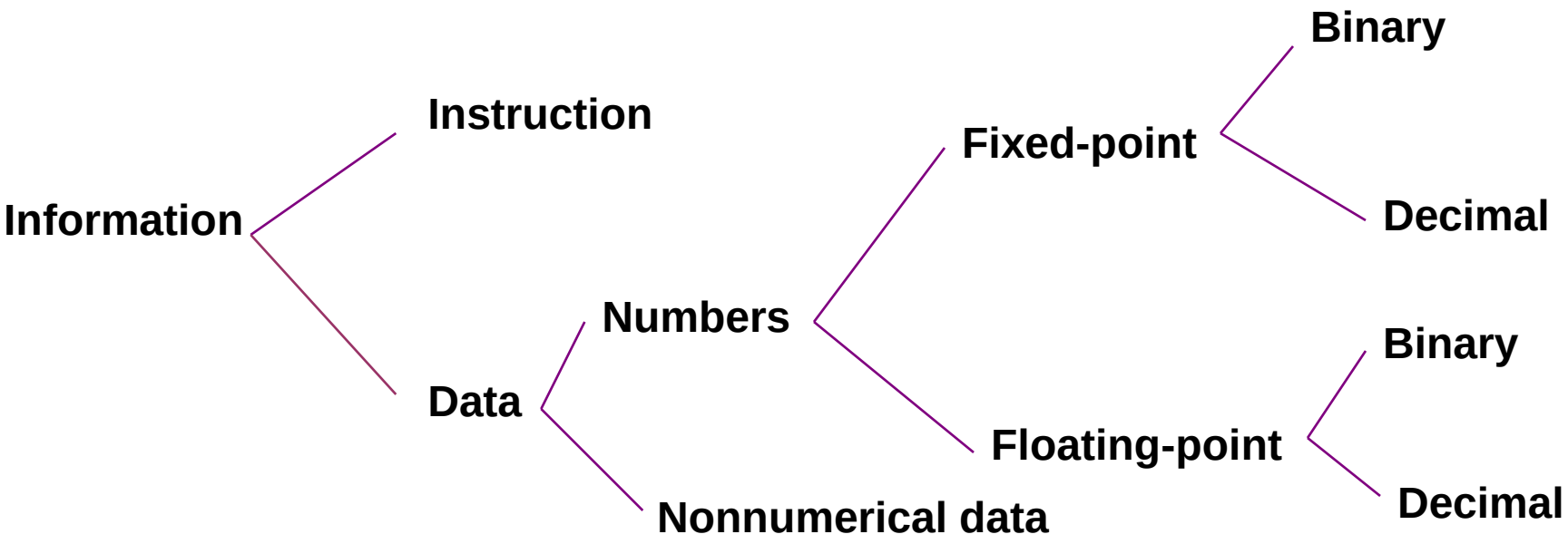
Lecture - 3

The Basic Information Types

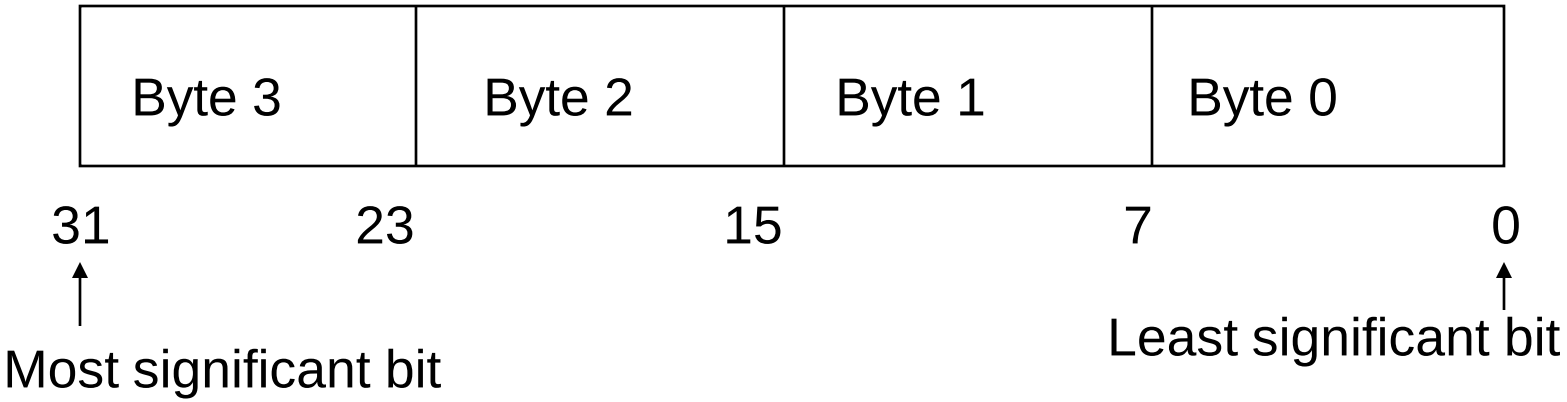


Word Length

Bits	Name	Description
1	Bit	Logic variable, flag.
8	Byte	Smallest addressable memory item, Binary-coded decimal digit pair.
16	Halfword	Short fixed-point number. Short address, Short instruction.
32	Word	Fixed or floating point number, Memory address, Instruction.
64	Double word	Long instruction, Double-precision floating point number.

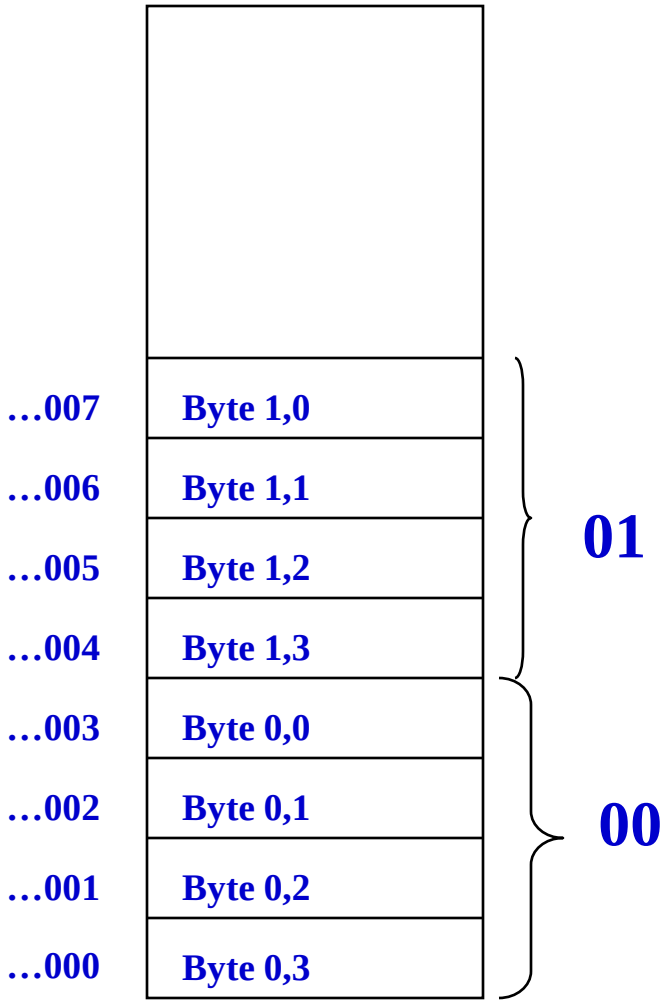
Some Instruction formats of the Motorola 680X0 microprocessor series

Byte Storage



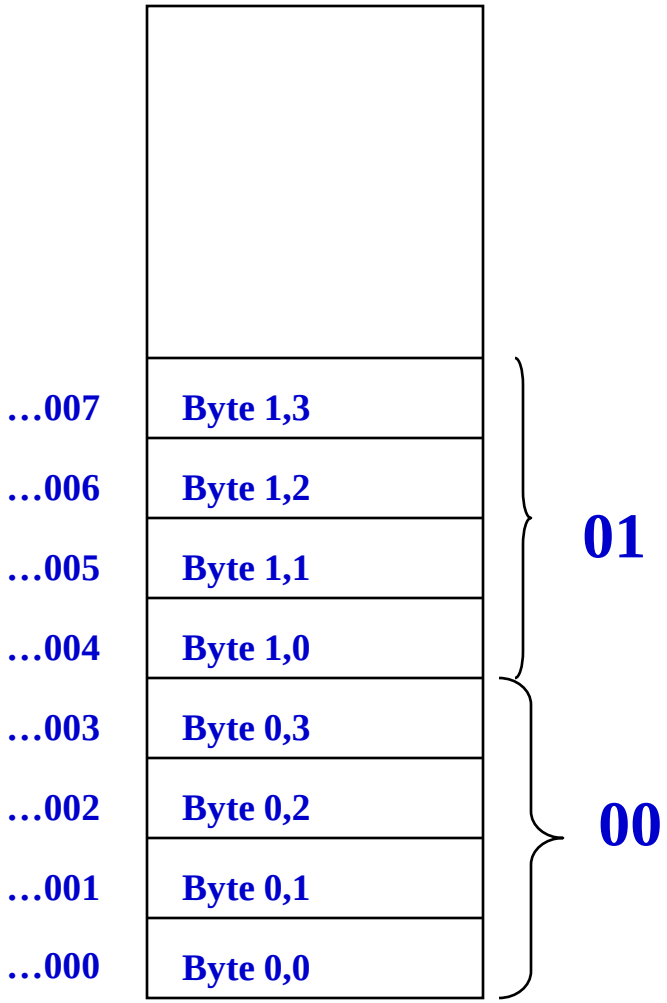
Indexing convention for the bits and bytes of a word

Big-Endian Storage Method



- Suppose a word W_i is represented as $B_{i,3}\,B_{i,2}\,B_{i,1}\,B_{i,0}$
- Most significant byte $B_{i,3}$ of W_i is assigned the lowest address and the least significant byte $B_{i,0}$ is assigned the highest address.
- It is so named because it assigns highest address to byte 0 of a word.

Little-Endian Storage Method



- Suppose a word W_i is represented as $B_{i,0}\ B_{i,1}\ B_{i,2}\ B_{i,3}$
- Most significant byte $B_{i,3}$ of W_i is assigned the highest address and the least significant byte $B_{i,0}$ is assigned the lowest address.
- It is so named because it assigns lowest address to byte 0 of a word.

Tags

- It is a technique of determining the type of a word.
- This is done by associating with each information word a group of bits, called a tag, that identifies the word's type.

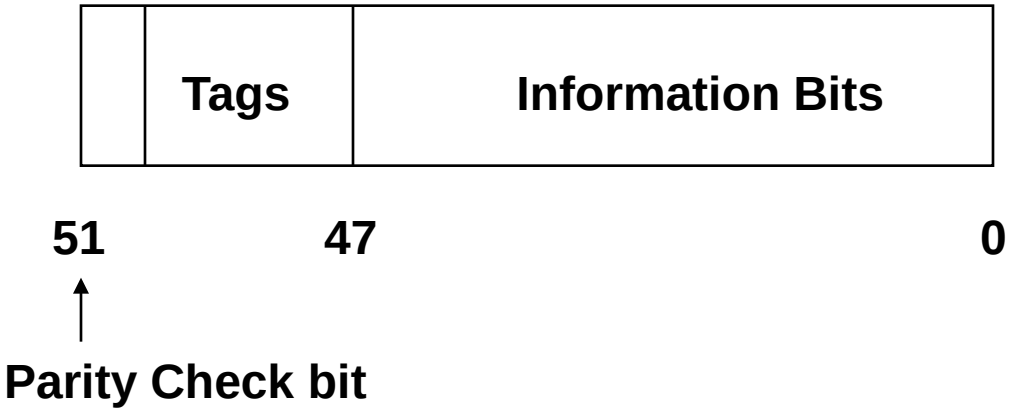
Advantages:

- It simplifies instruction specifications.
- Tag inspection permits the hardware to check for software errors.

Disadvantages:

- They increase memory size.
- Add system hardware costs without increasing computing performance.

Tags



Tag	Interpretation
000	Single-precision number
001	Indirect reference word
010	Double-precision number
011	Segment descriptor
100	Step-index control word
101	Data descriptor
110	Uninitialized operand
111	Instruction

Tagged-word format of the B6500/7500 Series

Error Detection and Correction

- The parity bit is appended to an n-bit word $X = (x_o, x_i, \ldots, x_{n-1})$ to form the $(n+1)$-bit word $X^* = (x_o, x_i, \ldots, x_{n-1}, c_0)$.
- Bit c_0 is assigned the value 0 or 1 that makes the number of ones in X^* even.

Even Parity:

- $c_0 = x_o \oplus x_i \oplus \ldots \oplus x_{n-1}$

Odd Parity:

- $c_0' = x_o \oplus x_i \oplus \ldots \oplus x_{n-1}$

Error Detection and Correction

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- Let, *B* receive the word $X' = (x'_o, x'_i, \ldots, x'_{n-1}, c'_0)$.
- Then, $c^*_0 = x'_o \oplus x'_i \oplus \ldots \oplus x'_{n-1}$
- If, $c'_0 \neq c^*_0$, the received information contains an error.
- If, $c'_0 = c^*_0$, there is no single-bit error.

Factors to select a number representation

- The number types to be represented; for example, integers or real numbers.
- The range of values (number magnitudes) likely to be encountered.
- The precision of numbers, which refers to the maximum accuracy of the representation.
- The cost of the hardware required to store and process the numbers.

Fixed Point Binary Numbers

- The unsigned binary fixed-point format takes the form $b_N \ldots b_1 b_0 . b_{-1} b_{-2} \ldots b_M$ where each b_i is 0 or 1, representing the number $\sum b_i 2^i$ where $M <= i <= N$.

- This is a positional notation in which each digit has a fixed-weight according to it's position relative to the binary point.

  The signed binary number is represented as

  $$x_{n-1} \underbrace{x_{n-2} x_{n-3} \ldots x_2 x_1 x_0}$$

  Sign — Magnitude

- Using n-bit number format we can represent all integers N with magnitude $|N|$ in the range $0 <= |N| <= 2^{n-1}$.

Signed Numbers

3 types of representations of a signed number:

- **Sign magnitude**
  +75=01001011
  -75=11001011

- **1’s complement**
  +75= 01001011
  -75 = 10110100

- **2’s complement**
  +75= 01001011
  -75 = 10110101

Signed Numbers

- Sign magnitude number employ the positional notation for magnitude and simply change the sign bit to represent + or -.
- In both complement code x_{n-1} retains the role as sign bit, but remaining bits no longer form a simple positional code when the number is negative.
- The advantage of the 2'scomplement code is that subtraction can be performed by logical complementation and addition only. 2's complement addition and subtraction can be implemented by a simple adder for unsigned numbers.
- Multiplication and division are more difficult to implement of 2's complement code.

Signed Numbers

Decimal Representation	Sign Magnitude	1's Complement	2's Complement
+7	0111	0111	0111
+6	0110	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
+0	0000	0000	0000
-0	1000	1111	0000
-1	1001	1110	1111
-2	1010	1101	1110
-3	1011	1100	1101
-4	1100	1011	1100
-5	1101	1010	1011
-6	1110	1001	1010
-7	1111	1000	1001

Decimal Codes

BCD (Binary coded decimal)

- In BCD format each digit d_i of a decimal number is denoted by a 4-bit equivalent $b_{i,3}b_{i,2}b_{i,1}b_{i,0}$.
- BCD is a weighted (positional) number code where each $b_{i,j}$ has the weight 10^i2^j.

ASCII

- The 8-bit ASCII code represents the 10 decimal digits by a 4-bit BCD field and the remaining 4-bits of the ASCII word have no numerical significance.

Decimal Codes

Excess-Three Code

- The excess-three code can be formed by adding 0011_2 to the corresponding BCD number.
- The advantage of the excess-three code is that it may be processed using the same logic used for binary codes.
- Some arithmetic operations are difficult to implement using excess-three code, mainly because it is a non-weighted code.

Decimal Codes

Excess-Three Code

- If two excess-three numbers are added like binary numbers, the required decimal carry is automatically generated from the high-order bits. The sum must be corrected by adding +3.

	1000 = 5	
	+ 1100 = 9	
Carry 1	0100	Binary sum
	+ 0011	Correction
	0111 = 4	Excess-three sum

Decimal Codes

Two-out-of-Five

- Each decimal digit is represented by a 5-bit sequence containing two 1s and three 0s. There are exactly 10 distinct sequence of this type.
- It is single-error detecting code, since changing any one bit results in a sequence that does not correspond to a valid code word.
- It uses 5 rather than 4 bits per decimal digit.

Decimal Codes

Decimal Digit	BCD	ASCII	Excess-three	Two-out-of-five
0	0000	00110000	0011	11000
1	0001	00110001	0100	00011
2	0010	00110010	0101	00101
3	0011	00110011	0110	00110
4	0100	00110101	0111	01001
5	0101	00110101	1000	01010
6	0110	00110110	1001	01100
7	0111	00110111	1010	10001
8	1000	00111000	1011	10010
9	1001	00111001	1100	10100

Decimal Codes

Hexadecimal Numbers:

- Uses 16 digits, consisting of decimal digits 0, 1, … , 9 augmented by the six digits A, B, C, D, E and F, which have the numerical values 10, 11, 12, 13, 14 and 15, respectively.

- Useful for representing long binary numbers.

Need for Floating-Point Number

- The range of number represented by a fixed-point number is insufficient for many applications, particularly, when very large and very small numbers are required.

- Scientific notation allows to represent such numbers using relatively few digits.

- For example, it is easier to write 1.0×10^{18} than

  1 000 000 000 000 000 000

Basic Format of Floating-Point Number

- Three numbers are associate with a floating-point number: a mantissa *M*, an exponent *E* and a base *B*. The mantissa *M* is also referred to as the significand or fraction in the literature.

- A real number is represented as $M \times B^E$.

- Example: 1.0×10^{18} where 1.0 = mantissa, 10 = base and 18 = exponent.

Representation of Floating-Point Number

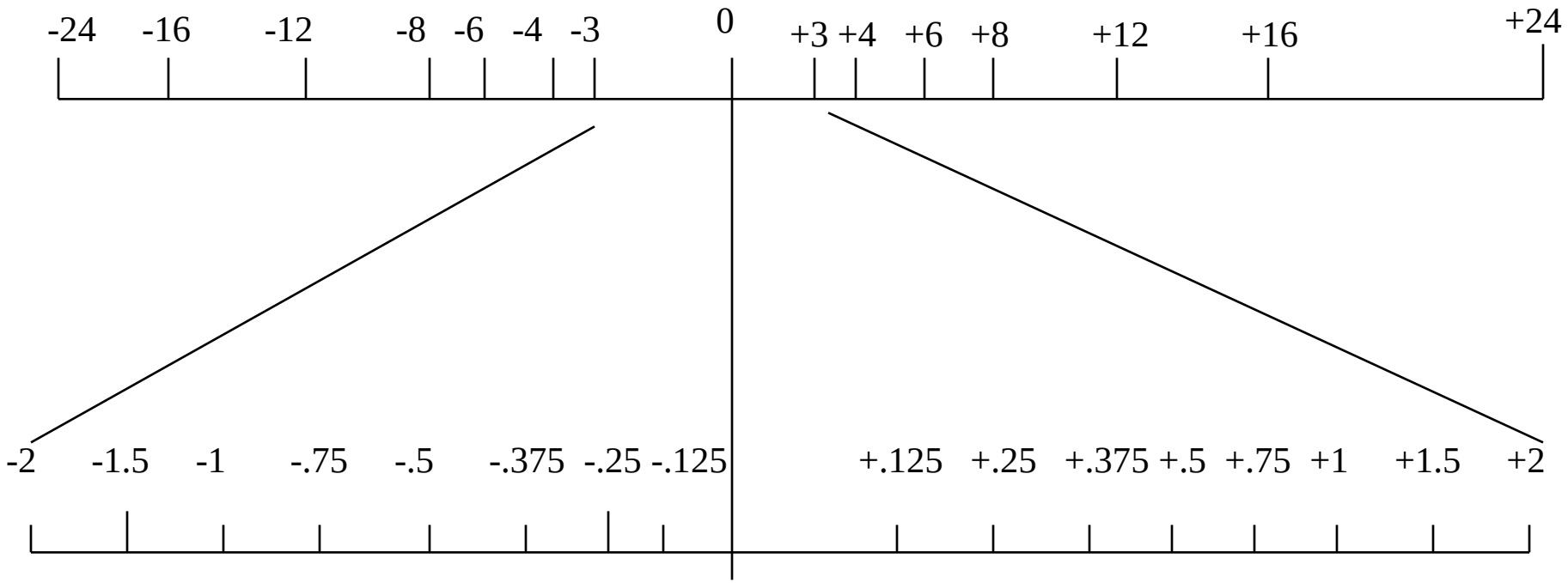
- A floating point number is represented as a word (M,E) consisting of a pair of signed fixed-point numbers: M, which is usually a fraction or integer and E, which is an integer.

- Since, B is constant, it is not stored but is simply built into the circuit that process the number.

- The number of digits in M determines the precision of (M,E); B and E determines the range.

Example of Floating-Point Number

- Let M and E are both 3-bit, sign magnitude numbers. Then M and E each can assume the values ±0, ±1, ±2, ±3. Let B=2.
- All binary words of the form (M, E) = (x00, xxx) represent 0, where x denotes either 0 or 1.



The real numbers representable by a hypothetical 6-bit, floating-point format

Floating-Point Number

- Smallest nonzero positive number is (001, 111), denoting $1 \times 2^{-3} = 0.125$
- (101, 111), denoting $-1 \times 2^{-3} = -0.125$
- Smallest nonzero negative number is (111, 111), denoting $-3 \times 2^{-3} = -0.375$
- Largest positive number is (011, 011), denoting $3 \times 2^{3} = 24$
- Largest negative number is (111, 011), denoting $-3 \times 2^{-3} = -24$

Floating-Point Number

- The floating point representation of most real numbers is only approximate. For example, 1.25 is approximated by (011,101) representing 1.5 or by either (001, 000) or (001, 100) representing 1.0.

- The result of most calculations with floating point arithmetic only approximate the correct result. For example, the exact result of the addition (011,001) + (011, 010) =18 which is not representable. The closest representative number of 18 is 16 (010, 011).

- Include a few extra mantissa digits termed *guard digits* to reduce approximation errors; the guard digits are removed automatically from the final results.

Normalization

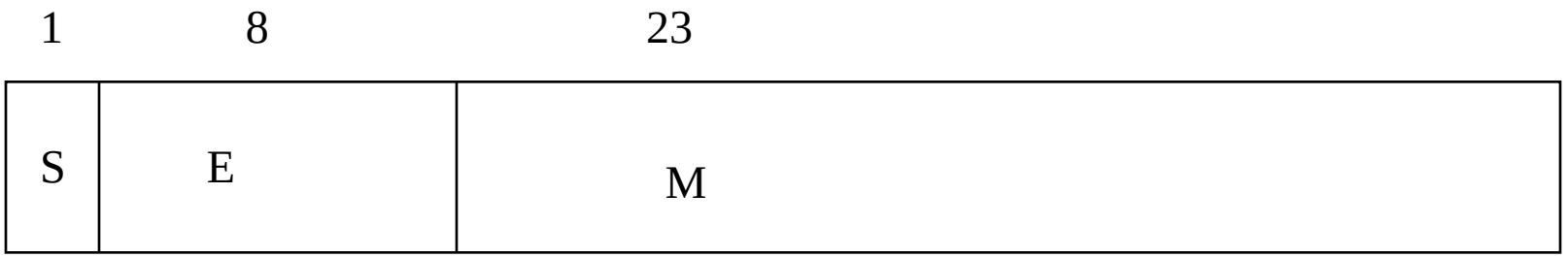
- The same number can be represented in many ways. For example: 1.0×10^{18}, 0.1×10^{19}, 1000000×10^{12} etc.
- It is desirable to have a unique or normal form for each representable number in a floating point system.
- A binary number is normalized when it has one non-zero digit to the left of the decimal point.
- Example: for IEEE 754 the normalized form 1.M.
- An un-normalized number is normalized by shifting the mantissa to the right or left and appropriately incrementing or decrementing the exponent.

Normalization

Advantages:

- It simplifies the exchange of data.
- It simplifies the floating point arithmetic algorithm
- It increases the accuracy of the numbers that can be stored in a word, since the unnecessary leading 0s are replaced by real digits to the right of the binary point.

IEEE 754 Floating Point Number



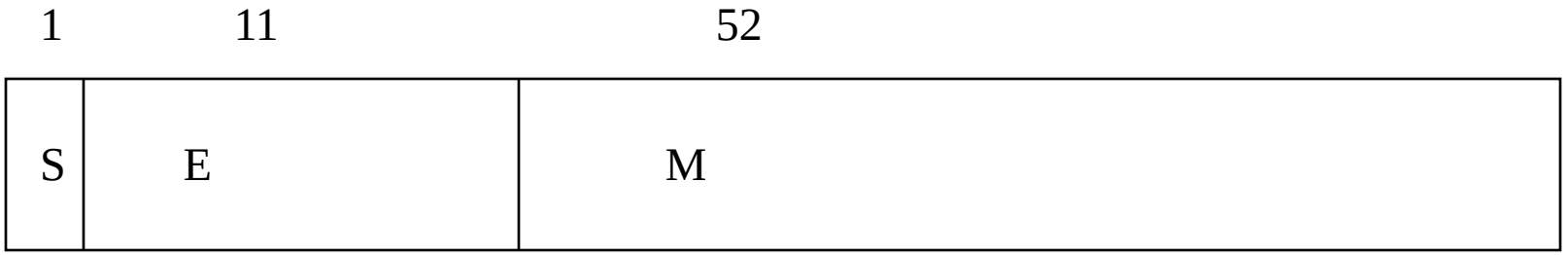
IEEE 754 standard 32-bit floating-point number format

- S= 1 bit sign representation
- E = 8 bit excess-127. The actual exponent is E-127.
- M=23 bit mantissa [fraction part of sign-magnitude binary significand with hidden bit]

IEEE 754 Floating Point Number

- A real number $N = (-1)^s 2^{E-127}(1.M)$ where, $0 < E < 255$
- N=-1.5 is represented as
  1 01111111 10000000000…..0

IEEE 754 Floating Point Number



IEEE 754 standard 64-bit floating-point number format

- A real number $N = (-1)^{s}2^{E-1023}(1.M)$ where, $0< E<2047$

Biasing

- If we use 2's complement or other notation in which negative exponents have 1 in MSB, a negative number will look like a big number. For example, 1.0×2^{-1} is represented as

  0 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 …….

  1.0×2^{1} is represented as,

  0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 ……

- It is desirable to represent most negative exponent as 00...000 and most positive as 11….111. This convention is called biased notation.

- The exponents are encoded in excess-*K* code where the exponent field *E* contains an integer that is the desired exponent value plus *K*. The quantity *K* is called **bias** and exponent encoded in this way is called a **biased exponent**.

- -1+127=126=01111110 and +1+127=128=10000000

Biasing

Exponent E	Unsigned value	Bias 127	Bias 128
111…11	255	+128	+127
111…10	254	+127	+126
……	……	……	……
100…01	129	+2	+1
100…00	128	+1	0
011…11	127	0	-1
011…10	126	-1	-2
……	……	……	……
000…01	1	-126	-127
000…00	0	-127	-128

8-bit biased exponent with bias=127 (excess-127) and bias = 128 (excess-128)

Converting from Decimal to Binary Floating Point

- What is the binary representation for the single-precision floating point number that corresponds to $X = -12.25_{10}$?

- What is the normalized binary representation for the number?

  $-12.25_{10} = -1100.01_2 = -1.10001_2 \times 2^3$

- What are the sign, stored exponent, and normalized mantissa?

  S = 1 (since the number is negative)

  $E = 3 + 127 = 130 = 128 + 2 = 10000010_2$

  $M = 10001000000000000000000_2$

  $X = 11000001010001000000000000000000_2$

Converting from Decimal to Binary Floating Point

Overflow:

A situation in which a positive exponent becomes too large to fit in the exponent field.

Underflow:

A situation in which a negative exponent becomes too large to fit in the exponent field.