

## Pattern Printing:-

for  $r = 4$  to 1 do  
    {  
        for  $c = r$  to 4 do \*  
        }  
    }  
}

so, if ( $r = 0$ ) return;

if ( $c < r$ ) {  
    print (\*)  
    triangle( $r, c+1$ )  
}

else,  
    {  
        new line  
        triangle( $r-1, 0$ )  
    }

■ Bubble sort: → each pass (swap if previous > next)

$$\text{arr} = \{4, 3, 6, 2, 8\}$$

bubble (arr, arr.length-1, 0)      ↳ index

if (~~n~~ n == 0) return;

if (c < r) {

    if (arr[c] > arr[c+1]) {

        swap (arr[c], arr[c+1])

}.

    bubble (arr, c, c+1);

}.

else, bubble (arr, r-1, 0);

田 Selection Sort: each pass  $\rightarrow$  max element

sort (arr, length, 0, 0)      right pos.  
                ↑      ↓      ↓  
                r      c      max

if ( $r == 0$ )      return

if ( $c < r$ ) {

    if ( $arr(c) \geq arr(max)$ ) {

        sort (arr, r, c+1, c)

}

    else {

        sort (arr, r, c+1, max)

}

}

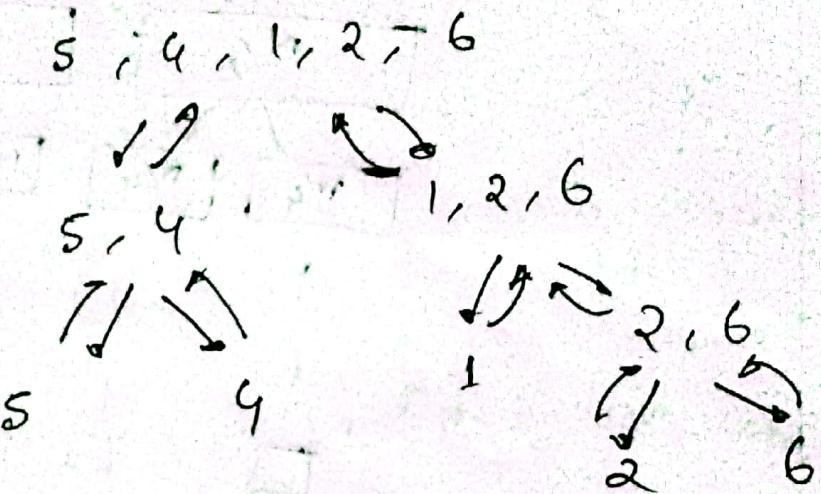
    else {

        swap (arr (max), arr (r-1))

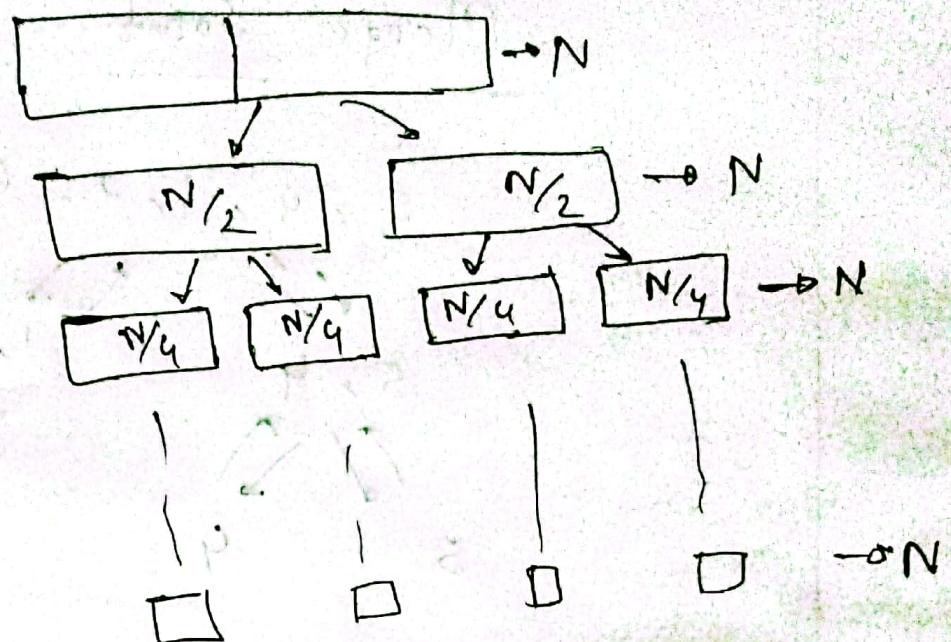
        sort (arr, r-1, 0, 0)

}

## Merge Sort



- ④ First divide. 2 parts.
- ④ sort each parts.
- ④ merge the 2 parts



At every level,  $N$  elements are being merged.

Levels

$$0 \rightarrow N \quad N$$

$$1 \rightarrow N/2 \quad N/2$$

$$\square \quad N/4$$

1st  
1

$$\therefore 1 = \frac{N}{2^k} \Rightarrow k = \log_2 N$$

∴ Total Time Complexity :  $O(N \times \log N)$

Space Complexity :

Auxiliary Space :  $O(N)$



$$\begin{aligned} T(N) &= T(N/2) + T(N/2) + (N-1) \\ &= 2T(N/2) + (N-1) \end{aligned}$$

from Akna-Bazzi.

$$a = 2, b = \frac{1}{2}$$

$$2 \times \frac{1}{2^p} = 1 \Rightarrow p = 1$$

$$\begin{aligned} T(x) &= x + x \int_1^x \left( \frac{u+1}{u^{1+1}} \right) du \\ &= x + x \int_1^x \left( \frac{1}{u} - \frac{1}{u^2} \right) du \\ &= x + x \left[ \log u + \frac{1}{u} \right]_1^x \\ &= x + x \left[ \log x + \frac{1}{x} - 1 \right] \\ &= x + x \log x + 1 - x \\ &= x \log x \end{aligned}$$

$\therefore T.C = O(N \times \log N)$

## Quick Sort

### ④ Pick Pivot

In left side of pivot bring all elements that are  $<$  pivot and In right side of pivot bring all elements that are  $>$  pivot

⑤ 5, 4, 1, 3, 2

Let's pivot = 4

1, 3, 2, 4, 5

so, each pass pivot element will come at its right position

④ Put pivot at its correct position

$s, p$   
5, 4, 3, 2, 1  
 $e$

$s, p$   
1, 4, 3, 2, 5  
 $e$

$s, e$   
1, 2, 3, 4, 5

low                          High

pivot comes at  
correct position

while  $\leftarrow \text{arr}[s] < p$

$s++;$

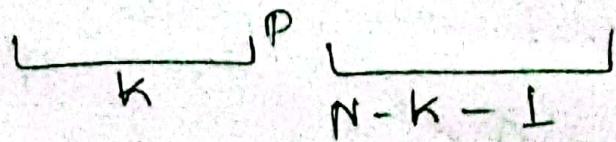
while  $\rightarrow \text{arr}[e] > p$

$e--;$

⑤ pivot

1. random
2. connect
3. mid

⊕



$$T(N) = T(k) + T(n-k-1) + O(n)$$

worst case: pivot  $\rightarrow$  largest/smallest

$$k = 0$$

$$\begin{aligned} T(N) &= T(n-1) + O(n) \\ &= O(n^v) \end{aligned}$$

Best case: pivot  $\rightarrow$  mid

$$\begin{aligned} T(N) &= T(N/2) + T(N/2) + O(n) \\ &= 2T(N/2) + O(n) \\ &= O(N * \log N) \end{aligned}$$

→ Quick sort

④ Not stable

⑤ In-place (preferred over merge sort  
because merge sort taking  
 $O(N)$  extra space)

• ⑥ Merge sort is better in linked lists  
due to memory allocation.

☒ Hybrid Sorting Algorithms (Tim sort)

Merge Sort + Insertion Sort

↓  
works better for partially  
sorted arrays