Nyquist Theorem and Shannon Theorem

1. Nyquist Theorem (Nyquist Rate)

Definition:

The Nyquist theorem, also known as the Nyquist sampling theorem, is a fundamental principle in signal processing. It states that:

> **"To reconstruct a continuous-time signal perfectly from its samples, it must be sampled at least at twice the highest frequency present in the signal."**

Mathematically, if the maximum frequency component of a signal is $f_{\max}$, then the sampling frequency f_s must satisfy:

$$f_s \geq 2f_{\max}$$

This minimum rate $2f_{\max}$ is known as the **Nyquist Rate**.

Example:

Suppose we have an analog signal with maximum frequency component of $f_{\max} = 5$ kHz. According to Nyquist theorem:

$$f_s \geq 2 \times 5\,\text{kHz} = 10\,\text{kHz}$$

Therefore, we must sample the signal at least at 10 kHz to reconstruct it without aliasing.

Importance:

Sampling below the Nyquist rate can lead to **aliasing**, where different signals become indistinguishable from each other in the sampled data.

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2. Shannon Theorem (Shannon-Hartley Theorem)

Definition:

Shannon's theorem defines the maximum rate at which data can be transmitted over a communication channel with a specific bandwidth and noise level. It is given by the formula:

$$C = B \log_2 \left(1 + \frac{S}{N}\right) \quad \text{(bits per second)}$$

Where:

- C = Channel capacity (bps)
- B = Bandwidth of the channel in Hz
- $\frac{S}{N}$ = Signal-to-noise ratio (unitless)

Example:

If a communication channel has a bandwidth of $B = 3\,\text{MHz}$ and a signal-to-noise ratio $S/N = 15$, then the maximum data rate is:

$$C = 3 \times 10^6 \log_2(1 + 15) = 3 \times 10^6 \log_2(16)$$
$$C = 3 \times 10^6 \times 4 = 12\,\text{Mbps}$$

Importance:

Shannon's theorem sets the theoretical upper bound for data transmission over a channel, helping engineers design efficient communication systems.

Conclusion

- **Nyquist Theorem** helps in determining the minimum sampling rate to avoid aliasing in signal processing.
- **Shannon Theorem** defines the maximum error-free data transmission rate over a noisy communication channel.