☐ The phrase propositional logic is composed of two words:

proposition Logic

☐ Logic:

→ Logic is the science of reasoning. It helps us to understand and reason about different mathematical statement and finally we would be able to prove or disprove those mathematical statement precisely.

☐ Purpose of logic is to construct valid arguments (or proofs).
Once we prove a mathematical is true then we call it a theorem.
And this is the basis of whole mathematics.

Propositional Logic:

→ Proposition is a declarative sentence (a sentence that is declaring a fact or stating an argument) which can be either TRUE or FALSE but cannot be both.

Example:

1. Delhi is the capital of india.
2. Water Froze this morning
3. 1+1 = 2

Example of not propositions:

1. What time is it?
2. x+1 = 2 (can be both TRUE or FALSE)
3. Send us your resume before 11 A.M
4. I request you to please allow me a day off.
5. Fetch my umbrella!

⊛ Propositional logic →
Area of logic that studies ways of joining and/or modifying propositions to form more complicated propositions and it also studies the logical relationships and properties derived from these combined/altered propositions.

Ex: statment-1: "Adam is good in playing football"

Statement-2: "Adam is good in playing football and this time he is representing his college at National level."

"Adam is good in playing football and this time he is representing his college at National level"

proposition-1

joining two propositions with logical connective

proposition-2

* statment-3 → "I enjoy watching Television."

statment-4 → "$\underbrace{\text{It is not the case}}_{\text{Modifying the statement using negation}}$ that I enjoy watching television.

* Propositional logic sometimes called as "sentential logic" or "statement logic"

Propositional Variables:

→ Variables that are used to represent propositions are called propositionals variables.

Ex: "Adam is good in playing football and the time he is representing his college at National level."

if, p = Adam is good in playing football,

q = this time he is representing his college at National level.

⊛ $p \wedge q$ = (p and q) → $\wedge$ → and

⊞ There are 6 logical operators that we will focus on :-

1. Negation
2. Conjunction
3. Disjunction
4. Exclusive OR
5. Implication
6. Biconditional

Negation:

Let p be a proposition. $(\neg p)$ is called negation of p which simply states that

"It is not the case that p"

⊛ If p is true ¬p is false. If p is false the ¬p is true.

Example:

"Adam and Eve lived together for many years"

Negation: It is not the case that Adam and Eve lived together for many years.

OR, Adam and Eve haven't lived together for many years

⊛ Negation can be done differents ways by the meaning.

Truth table:

p	¬p
T	F
F	T

Conjunction: ($\wedge$) → and

conjunction of p and q is denoted by $p \wedge q$ (p and ($\wedge$) q). When both p and q are true then only the compound proposition $p \wedge q$ is true.

Example:

12 is divisible by 3 and 3 is a prime number.
('T') ('T') = 'T'

Truth table:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Note: Sometimes we use 'but' instead of 'and'.

propositional Logic :→ but, and → same

Disjunction: ($\vee$) $\to$ or

Disjunction of p and q is denoted by $p \vee q$.
When both p and q are false then only the compound proposition $p \vee q$ is false.

Examples "16 − 4 = 10 or 4 is an even number"

F T = T

($\vee \to$ OR)

Truth Table:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

In order to get a job in this multinational company, experience with c++ (q) $\boxed{\text{or}}$ Java (p) is mandatory."

or $\to$ Inclusive OR (Disjunction)

$\boxed{\text{p or q or both.}}$

Exclusive OR :

When you buy a car from XYZ company you get $2500 cashback or accessories worth $2500.

p = $2500 cashback

q = accessories worth $2500

or → Exclusive OR (X-OR)

$\boxed{\text{* p or q but not both}}$

definition: The exclusive OR of p and q (denoted by $p \oplus q$) is a proposition that simply means exactly one of p and q will be true but both can't be true.

Truth table:

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Example:

1. $\underbrace{\text{Coffee}}_{p}$ or $\underbrace{\text{Tea}}_{q}$ comes with dinner.

   p or q but not both

   so, statement is Exclusive OR

2. You can pay using $\underbrace{\text{US dollars}}_{p}$ or $\underbrace{\text{euros}}_{q}$.

   p or q but not both

   so, statement is Exclusive OR

3. Dinner for two includes $\underbrace{\text{two items}}_{p}$ from column A or $\underbrace{\text{three items}}_{q}$ from column B.

   p or q not both,

   so, statement is Exclusive OR

4. A password must have at least three digits or be at least eight characters long.

$\underbrace{\text{digits}}_{p}$ ... $\underbrace{\text{characters}}_{q}$

* p or q or both

so, statement is inclusive OR

5. To take discrete mathematics, you must have taken calculus or a course in computer science.

$\underbrace{\text{calculus}}_{p}$ ... $\underbrace{\text{computer science}}_{q}$

* p or q or both

so, statement is inclusive OR

Implication:

The proposition "if p then q" denoted by $p \to q$ ($\to$ implies) is called implication or conditional statement.

Truth table:

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

* if p is false then $p \to q$ will be true.
* p is called hypothesis (or premise) and q is called conclusion (or consequence)

Example:

"If you try hard for your exam then you will succeed"

p = you tried hard for your exam.

q = you succeed.

case-1: "You tried hard for your exam" $\boxed{P=T}$ and "you succeed" $\boxed{q=T}$

Compound proposition, $p \to q$ is True.

case-2: "You tried hard for your exam" $\boxed{P=T}$ but "you failed" $\boxed{q=F}$

Compound proposition $p \to q$ is False.

Case-3: "You haven't tried hard for your exam" $\boxed{p = F}$ and You succeeded" $\boxed{q = T}$.

compound proposition $p \to q$ is True.

why?

Because you can make the compound proposition false when you staisfy the first condition itself that is p. If that itself not satisfied then we can not make compound proposition False.

(Not False means True)

Case-4: "You haven't tried hard for your exam" ($p = F$) and "You failed" ($q = F$)

compound proposition $p \to q$ is True

(same reason above)

Example:

"If you have connection with seniors, then you will get promoted"

p = have connection with seniors; q = get promoted

p	q	
T	T	= T
T	F	= F
F	T or F	= T

Example:

If you get 100% marks on the final exam, then you will be awarded a Trophy

p = 100% marks; q = be awarded

p	q	
T	T	= T
T	F	= F
F	T or F	= T

CamScanner

If p is false then it doesn't matter what will be the truth value of q, p → q is always TRUE.

Example :

1. If 1+1=3, then dogs can fly.
   F   F

   so, statement is true

2. If 1+1=2, then dogs can fly.
   T   F

   so, statement is ~~true~~ false

3. If monkeys can fly, then 1+1=3
   F   F

   so, statement is true

4. If 1+1=2, then 2+2=5
   T   F

   so, statement is false

Implication & Representations

Different ways to represent conditional statements:

1. "if p then q"
2. "p implies q"
3. "q when p"
4. "q whenever p"
5. "q follows from p"

1. "p only if q"
2. "q is necessary for p"
3. "p is sufficient for q"
4. "q unless ¬p"

↓

very important (cause → confusing)

⊛ "p only if q"

How "if p then q" and "p only if q" can be same?

Example: I will stay at home only if I'm sick

Let p = 'I will stay at home'

and q = I'm sick

⊛ Above statement is of the form $\boxed{\text{p only if q}}$

⊛ Above statement, becoming sick is the necessary condition that will make you stay at home.

⊛ This means, "if you're not sick then, you can not stay at home at any cost."

⊛ In order to falsify the above statement, q must be FALSE and p must be TRUE.

That is, you are not sick and you still stay at home.

proof idea: truth value of p and q must be same, in order to falsify the statement.

if p then q:

"If I will stay at home then I'm sick"

The only way to falsify the above statement is by making p TRUE and q FALSE.

Therefore, p only if q is equivalent to if p then q (proved)

☐ Why "p only if q" is not equivalent to "if q then p"?

Examples:

p only if q: I'll stay at home only if I'm sick.

p = TRUE, q = FALSE = FALSE

If p then q:

~~p only if q is f:~~

If I'm sick than I'll stay at home

FALSE, TRUE = TRUE

So, p only if q and if q then p is not same. (proved) "q is necessary for p"

Example: "Good food is nececessary to keep us alive"

* According to the statement, if we won't have good food then, we'll soon die.

* But it is not the only factor to keep us alive.

* Therefore, when we say A is necessary for B then falsity of A guarantees the falsity of B but we cannot guarantee the truth of B from the truth of A.

* similarly, when we say q is necessary for p then we can only guarantee that when q is false then, p is definately false That's why, we say "q is necessary for p"

"p is sufficient for q"

Example: "It is ~~of~~ sufficient for you to travel by car in order to reach your destination on time."

Definitely, if you travel by car, you'll reach your destination on time. No doubt. but if you won't travel by car, does it mean you'll never reach your destination on time. May be flight or other means of transport, you'll reach your destination much earlier.

Therefore, when we say that when A is sufficient for B then, truth of A guarantees the truth of B but we cannot guarantee the falsity of B from the falsity of A.

Similarly, when we say "p is sufficient for q", then we can only guarantee that when p is true then, q is definitely true.

Why p is not necessary for q?

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Proof by contradiction:

Lets say p is necessary for q. if p is really necessary for q then if p is false then it is mandatory for q to be false in order to make the whole compound proposition $p \to q$ true. but it is not the case. According to the truth table, it is not mandatory for q to be false when p is false. Hence "p is not necessary for q."

CamScanner

Why q is not sufficient for p ?

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

Proof by contradiction:

Let say q is sufficient for p. if q is really sufficient for p then if q is true then it is mandatory for p to be true in order to make the whole compound proposition p→q true. but it is not the case.

According to the truth table it is not mandatory for p to be true when q is true

Hence "q is not sufficient for p"

D) "q unless $\neg p$"

↳ means 'except if'

Evaluation: "if p then q" (if p ~~d~~ is true then q must be true)

↓

"q is true when p is true"

↓

"q is true except when p is false"

↓

"q is true unless p is false"

↓

"q unless $\neg p$"

Implication, Converse, Contrapositive and Inverse

Implication or conditional statement : $p \rightarrow q$

Converse $\longrightarrow$ $q \rightarrow p$

Contrapositive $\longrightarrow$ $\neg q \rightarrow \neg p$

Inverse $\longrightarrow$ $\neg p \rightarrow \neg q$

Example: "If it rains today then, I will stay at home"

Converse: "If I will stay at home, then it rains today".

Contrapositive: "If I will not stay at home, then it does not rain today."

Inverse: "If it does not rain today then, I will not stay at home".

Facts:

→ Implication and contrapositive both are equivalent

→ converse and inverse both are equivalent

→ Neither converse nor inverse is equivalent to Implication.

Truth Table

p	q	$\neg p$	$\neg q$	$p \to q$ (implies)	$q \to p$ (converse)	$\neg q \to \neg p$ (contrapositive)	$\neg p \to \neg q$ (inverse)
T	T	F	F	T	T	T	T
T	F	F	T	F	T	F	T
F	T	T	F	T	F	T	F
F	F	T	T	T	T	T	T

* If you receive A grade in discrete mathematics then, you will be awarded a scholarship.

Converse: If you will be awarded a scholarship then you receive A grade in discrete mathematics.

contrapositive: If you will not be awarded a scholarship then you don't receive A grade in discrete mathematics.

Inverse: If you don't receive A grade in discrete mathematics then you will not be awarded a scholarship.

Biconditional Operator

let p and q be two propositions. The biconditional ~~statefor~~ statement of the form $p \leftrightarrow q$ is the proposition "p if and only if q".

$p \leftrightarrow q$ is true whenever the truth values of p and q are same.

Truth Table

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

How "p if and only if q" make sense?

"p if and only if q" composed of two statements—

"p if q" and "p only if q"

"p only if q" = if p then q and "p if q" = if q then p

$$(p \to q) \wedge (q \to p)$$

$$\equiv p \leftrightarrow q$$

Biconditional representations:

1. p is necessary and sufficient for q and vice-versa
2. if p then q, and conversely
3. p iff q.

Example: Let p a proposition "You get promoted" and let q be a proposition "You have connections"

then $p \leftrightarrow q$ is the statement:
"You get promoted if and only if you have connections."

* If you read newspapers everyday, you will be informed and conversely.

p if and only if q : ~~You will be informed~~

Everyday you read newspapers if and only if you will be informed

* It rains if it is a weekend day, and it is a weekend day if it rains.

p if and only if q : It is a weekend if and only if it is a rains.

Precedence of Logical Operators

* Precedence of operators helps us to decide which operator will get evaluated first in a complicated looking proposition.

Operators	Names	Precedence
$\neg$	Negation	1
$\wedge$	Conjunction	2
$\vee$	Disjunction	3
$\rightarrow$	Implication	4
$\leftrightarrow$	Biconditional	5

Examples: $p \rightarrow q \wedge \neg p$

Assuming, $p \rightarrow$ True, $q =$ false

$$p \rightarrow q \wedge \neg p$$

$$F \wedge F$$

$$\downarrow$$

$$T \rightarrow F = F$$

CamScanner

Construct the truth table for the compound proposition below:

$p \to \neg q \wedge \neg p \leftrightarrow \neg q$

$(\neg q \wedge \neg p) = r$

s

p	q	¬p	¬q	r	p→r	s⟺¬q
T	T	F	F	F	F	T
T	F	F	T	F	F	F
F	T	T	F	F	T	F
F	F	T	T	T	T	T

Translating English Sentences into Logical Expressions

Reasons:

1. Removes ambiguity
2. Easy manipulation
3. Able to solve puzzles

Example: "You are not allowed to watch adult movies if your age is less than 18 years or you have no age proof."

Step-1: Find the connectives which are connecting the two propositions together.

Step-2: Rename the propositions

Let, q = "You are allowed to watch adult movies"

r = "Your age is less than 18 years"

s = "You have age proof"

* logical expressions $\neg q$ $(r \vee \neg s) \rightarrow \neg q$

problems. Are these system specifications consistent?

"The system is in multiuser state if and only if it is operating normally."

"If the system is operating normally, then the kernel is functioning."

"The kernel is not functioning or the system is in interrupt mode."

"If the system is not in multiuser state, then it is in interrupt mode."

"The system is not in interrupt mode.

Solution:

Let p = "The system is in multiuser state"

q = "The system is operating normally."

r = "The kernel is functioning."

s = "The system is in interrupt mode."

F ← 1. $\overset{T}{p} \leftrightarrow \overset{F}{q}$

T ← 2. $\overset{F}{q} \to \overset{F}{r}$ (q=F)

T ← 3. $\overset{T}{\neg r} \vee \overset{F}{s}$ (r=F)

T ← 4. $\overset{F}{\neg p} \to \overset{F}{s}$ (p=T)

T ← 5. $\neg s \to T$ when s=F

cosistent means assigning truth values to propositional variables in such a way that finally we would be able to make all specification "True."

Tips: Always start from the statement which involves least numbers of propositions.

$\therefore$ these system specifications are not consistent.

Puzzle 1

In an island there are two kinds of inhabitants, knights, who always tell the truth, and their opposites, knaves, who always lie. You encounter two people A and B. Determine, if possible what A and B are if they address you in the ways described.

(a) A says "B is knight" and B says "The two of us are opposite types."

Solution: Let, p = A is knight and q = B is knight

$\neg p$ = A is knaves, $\neg q$ = B is knaves

Case-1: A is knight (assume).

$p = T$, $q = T$

The two of us are opposite types

$(p \wedge \neg q) \vee (\neg p \wedge q)$

$(T \wedge F) \vee (F \wedge T)$

$= F$

So, case-1 → False

Case-2: B is knight (assume)

q = T

$(p \wedge \neg q) \vee (\neg p \wedge q)$ → has to be true

→ $\neg p$ must be T

∴ p = F

F   T T

F   T

∴ A is knave

so what even A says it's lie. so,

B is knaves

∴ assumption and conclusion is not same so case-2 → F

Case-3: A is knave (assume)

∴ p = F, B is knave → q = F

has to be false

$(p \wedge \neg q) \vee (\neg q \vee q)$

$(F \wedge T) \vee (T \vee F)$

$F \vee F = F$

∴ case-3 → correct

Case-4:

B is knave $\therefore q = F$

$(p \wedge \neg q) \vee (\neg p \wedge q)$ → Has to be false

$(p \wedge T) \vee (T \wedge F)$

$= F$

$\therefore \boxed{p = F}$ → A is knave

$\therefore$ case-4 → ~~is~~ correct

$\therefore$ A is knave and B is also knave.

(b) A says "At least one of us is a knave" and B says nothing.

Solution:

(i) A is knight, $p = T$

$(p \wedge \neg q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$ → has to be true

$(T \wedge T) \vee (\quad) \vee (\quad)$

$= T$

$\therefore \neg q = T$

$q = F$ → B is knave

A is knight

B is knave

Tautology

Compound proposition which is always TRUE.

Ex:

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

Contradiction

Compound proposition which is always FALSE

Ex:

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

Contingency

Compound proposition which sometimes TRUE and sometimes FALSE.

Example:

p	q	p^q
T	T	T
F	T	F
T	F	F
F	F	F

Satisfiability

A compound proposition is satisfiable if there is at least one TRUE result in its truth table.

unsatisfiability: not even a single TRUE result in its truth table.

Tautology → satisfiable

Contradiction → unsatisfiable

Valid: A compound proposition is valid when it is a tautology.

Invalid: A compound proposition is invalid when it is either a contradiction or contingency.

Tautology	contradiction	Contingency
always true	always false.	Sometimes True or false
satisfiable	unsatisfiable	satisfiable
Valid	Invalid	Invalid

Logical Equivalences

The compound propositions p and q ~~are equal to~~ are said to be logically equivalent if $p \Leftrightarrow q$ is a Tautology. Logical Equivalence is denoted by $\equiv$ or $\Leftrightarrow$

Ex: $p \wedge T \equiv p$

p	T	$p \wedge T$
T	T	T
F	T	F

1. Identity Laws: a) $p \wedge T \equiv p$ b) $p \vee F \equiv p$

2. Domination Laws: a) $p \vee T \equiv T$ b) $p \wedge F \equiv F$

3. Idempotent Laws: a) $p \vee p \equiv p$ b) $p \wedge p \equiv p$

4. Double Negation Law: $\neg(\neg p) \equiv p$

5. Commutative Laws: a) $p \vee q \equiv q \vee p$ b) $p \wedge q \equiv q \wedge p$

6. Associative Laws: a) $(p \vee q) \vee r \equiv p \vee (q \vee r)$

b) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

7. Distributive Laws: a) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

b) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

8. De Morgan's Laws: a) $\neg(p \wedge q) \equiv \neg p \vee \neg q$

b) $\neg(p \vee q) \equiv \neg p \wedge \neg q$

9. Absorption Laws: a) $p \vee (p \wedge q) \equiv p$

<u>proof:</u> $p \vee (p \wedge q)$

$= (p \vee p) \wedge (p \vee q)$

$= p \wedge (p \vee q)$

$= (p \wedge p) \vee (p \wedge q)$

$= p \vee (p \wedge q)$

$= p \vee$ T বা F

$= p$

(b) $p \wedge (p \vee q) \equiv p$

proof:

$$\cancel{p \wedge (p \vee q)}$$

$$\cancel{= (p \wedge p) \vee (p \wedge q)}$$

$$\cancel{= p \vee (p \wedge q)}$$

$$p \wedge (p \vee q)$$

$$= (p \wedge p) \vee (p \wedge q)$$

$$= p \vee \underset{T \cap F}{\underset{\downarrow}{(p \wedge q)}}$$

$$= p$$

10. Negation Laws: (a) $p \vee \neg p \equiv T$

(b) $p \wedge \neg p \equiv F$

Logical equivalences involving conditional statements:

1. $p \to q \equiv \neg p \vee q$

$\boxed{\text{proof:}}$

$p \to q \equiv \neg p \vee q$

T T F T

T T

Let, $\neg p = F$

$p = T$

and $q = T$

$p \to q \equiv \neg p \vee q$

F $\square$ T $\square$

T T

Let $\neg p = T$

$p = F$

2. $p \to q \equiv \neg q \to \neg p$ (Implication and contrapositive are equivalent)

3. $p \vee q \equiv \neg p \to q$

proof: $\neg p \to q = \neg(\neg p) \vee q$ (1 থেকে)

$\equiv p \vee q$

4. $\cancel{p \wedge q = \neg(p \to \neg}$

4. $p \wedge q \equiv \neg(q \to \neg p)$

<u>proof:</u> $\neg(q \to \neg p) \equiv \neg(\neg q \vee \neg p)$ [ডি মরগান]

$\equiv q \wedge p$

$= p \wedge q$

5. $\neg(p \to q) \equiv p \wedge \neg q$

<u>proof:</u> $\neg(p \to q) \equiv \neg((\neg p) \vee q)$ [ডি মরগান]

$\equiv p \wedge \neg q$

6. $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

<u>proof:</u> $(p \to q) \wedge (p \to r)$

$= (\neg p \vee q) \wedge (\neg p \vee r)$ [Distributive Laws]

$= \neg p \vee \underbrace{(q \wedge r)}_{s}$

$= \neg p \vee s$

$= p \to s = p \to (q \wedge r)$

7. $(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$

8. $(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$

9. $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

6 এর similar logic এ proof করা যাবে

☐ Logical Equivalences involving biconditionals:

1. $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

2. $p \leftrightarrow q = \neg p \leftrightarrow \neg q$

<u>proof:</u> $p \leftrightarrow q = (p \rightarrow q) \wedge (q \rightarrow p)$

$= (\neg q \rightarrow \neg p) \wedge (\neg p \rightarrow \neg q)$ [Implication and contrapositive are equivalent]

$= (\underbrace{\neg p}_{s} \rightarrow \underbrace{\neg q}_{r}) \wedge (\neg q \rightarrow \neg p)$

$= (s \rightarrow r) \wedge (r \rightarrow s)$

$= (s \leftrightarrow r)$

$= (\neg p \leftrightarrow \neg q)$

CamScanner

3. $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

<u>proof:</u>

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p) \quad [p \rightarrow q = \neg p \vee q]$$

$$\equiv (\neg p \vee q) \wedge (\neg q \vee p)$$

$$\equiv (\neg p \wedge \neg q) \vee \underbrace{(\neg p \wedge p)}_{F} \vee \underbrace{(q \wedge \neg q)}_{F} \vee (q \wedge p)$$

$$\equiv (\neg p \wedge \neg q) \vee F \vee F \vee (q \wedge p)$$

$$\equiv (\neg p \wedge \neg q) \vee (q \wedge p)$$

$$= (\neg p \wedge \neg q) \vee (p \wedge q)$$

$$= (p \wedge q) \vee (\neg p \wedge \neg q)$$

4. $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

<u>proof:</u>

$$\neg(p \leftrightarrow q) \equiv \neg\big((p \rightarrow q) \wedge (q \rightarrow p)\big)$$

$$\equiv \neg\big((\neg p \vee q) \wedge (\neg q \vee p)\big)$$

$$\equiv \neg(\neg p \vee q) \vee \neg(\neg q \vee p)$$

$$\equiv (\underbrace{p}_{s} \wedge \underbrace{\neg q}_{r}) \vee (\underbrace{q}_{\neg r} \wedge \underbrace{\neg p}_{\neg s})$$

$$= (s \wedge r) \vee (\neg r \wedge \neg s)$$

[3 No. দেখ]

$$\equiv (s \leftrightarrow r)$$

$$= (p \leftrightarrow \neg q)$$

Problem:

In a room there are only two types of people, namely Type 1 and Type 2. Type 1 people always tell the truth and Type 2 people always lie. You give a fair coin to a person in that room, without knowing which type ~~hi~~ he is from and tell him to toss it and hide the result from you till you ask for it. Upon asking, the person replies the following.

"The Result of the toss is head if and only if I am saying the truth."

which of the following options is correct?

a) The result is head.

b) The result is tail.

c) If the person is of Type 2, then the result is tail.

d) If the person is of Type 1, then ~~th~~ the result is til.

Sol^n:

Let, p = the result of toss is head

q = I am saying the truth.

$\therefore$ statements, $p \Leftrightarrow q$ (if and only if $\leftrightarrow$)

Type 1:

$p \Leftrightarrow q \to T \to$ Type-I always say true

$q \to T$

$p \to T$

$\therefore p \Leftrightarrow q$ is true

$\therefore$ Result is head

Type-2:

$p \Leftrightarrow q \to F \hookrightarrow$ Type-2 always say lie.

$q \to F$

So, for ($p \Leftrightarrow q \to F$) p must be true.

$\therefore p \to T$

$\therefore$ Result is head

$\therefore$ Answer: a

[4] P and Q are two propositions. Which of the following logical expressions are equivalent?

1. $p \vee \sim Q$
2. $\sim(\sim p \wedge Q)$
3. $(p \wedge Q) \vee (p \wedge \sim Q) \vee (\sim p \wedge \sim Q)$
4. $(p \wedge Q) \vee (p \wedge \sim Q) \vee (\sim p \wedge Q)$

Soln:

(2) $\sim(\sim p \wedge Q) = p \vee \sim Q = (1)$

(3) $(p \wedge Q) \vee (p \wedge \sim Q) \vee (\sim p \wedge \sim Q)$

$= p \wedge (Q \vee \sim Q) \vee (\sim p \wedge \sim Q)$

$= p \wedge (T) \vee (\sim p \wedge \sim Q)$

$= p \vee (\sim p \wedge \sim Q)$

$= p \vee \sim p \wedge p \vee \sim Q$

$= T \wedge p \vee \sim Q$

$= p \vee \sim Q = 1$

4. $(p\wedge q)\vee(p\wedge\sim q)\vee(\sim p\wedge q)$

$= p(q\vee\sim q)\vee(\sim p\wedge q)$

$= p\vee(\sim p\wedge q)$

$=(p\vee\sim p)\wedge(p\vee q)$

$= T\wedge(p\vee q)$

$= p\vee q$

$\therefore$ 1, 2, 3 are equivalent.

Problem: which one of the following boolean expression is NOT a tautology?

a) $((a \to b) \wedge (b \to c)) \to (a \to c)$

b) $(a \leftrightarrow c) \to (\sim b \to (a \wedge c))$

c) $(a \wedge b \wedge c) \to (c \vee a)$

d) $a \to (b \to a)$

Solution:

If we make those expression false ~~atleo~~ at least one combination of ~~the~~ truth value of a, b, c then it's not a tautology.

a) $(\underbrace{(a \to b) \wedge (b \to c)}_{S}) \to (a \to c)$

$T \to F \quad F \; F \quad T \; F$

$F \wedge T \quad F$

$F \to F = T$

$\therefore$ a is ~~not~~ a tautology,

For making it's false $a \to c$ has to be False. For this reason $c \to F$, $a \to T$

$S \to$ has to be true

(b) $(a \leftrightarrow c) \rightarrow (\sim b \rightarrow (a \wedge c))$ → For making it's false first part has to be true and 2nd part has to be false

$F \leftrightarrow F \qquad T \rightarrow \sim F$

$= T \rightarrow F$

$= F$

$\therefore$ it's not a tautology.

so let $\sim b = T$

Problem:

Let P, Q and R be three atomic propositional assertions. Let X denote $(P \vee Q) \to R$ and Y denote $(P \to R) \vee (Q \to R)$. Which one of the following is a tautology?

a) $X \equiv Y$

b) $X \to Y$

c) $Y \to X$

d) $\sim Y \to X$

Soln:

$$(P \vee Q) \to R \equiv \neg (P \vee Q) \vee R$$
$$= (\neg P \wedge \neg Q) \vee R$$
$$= (\neg P \vee R) \wedge (\neg Q \vee R)$$
$$= (P \to R) \wedge (Q \to R) = X$$

$\therefore Y = (P \to R) \vee (Q \to R)$

$\therefore X \neq Y$ $\therefore$ a is incorrect.

(b) $X \to X$

$$\left[\begin{array}{l} X = \underset{A}{(P\to R)} \wedge \underset{B}{(Q \to R)} \\ Y = \underset{A}{(P\to R)} \vee \underset{B}{(Q\to R)} \end{array}\right.$$

$\therefore A \wedge B \to A \vee B$

$T \wedge T \quad T \vee T$

$T \to T$

$= T$

it's true in every cases.

so it's a tautology.

it's a tautology. So all possible composition it's true. If we make it false for at least one condition then it's not be a tautology, For making it's false 1st part must be T and 2nd $\to$ F

(c) $Y \to X$

$A \vee B \to A \wedge B$

$T \vee F \quad T \wedge F$

$T \to F$

$= F$

so, it's not a tautology

(d) $\sim Y \to X$

$\neg(A \vee B) \to (A \wedge B)$

$\Rightarrow \neg A \wedge \neg B \to A \wedge B$

$F \wedge F \quad F \wedge F$

$= T \to F$

$= F$

$\therefore$ so it's not a tautology

□ The binary operation □ is defined as follows;

p	q	$p \square q$
T	T	T
T	F	T
F	T	F
F	F	T

which one of the following is equivalent to $p \vee q$?

a) $\sim q \square \sim p$ b) $p \square \sim q$

c) $\sim p \square q$ d) $\sim p \square \sim q$

Solution:

$$p \square q = p \wedge q + p \wedge q' + p' \wedge q'$$

$$= p(q \vee q') + p' \wedge q'$$

$$= p \wedge T + p' \wedge q'$$

$$= p + p' \wedge q'$$

$$= p \vee p' \wedge q'$$

$$= (p \vee p') \wedge (p \vee q')$$

$$= T \wedge (p \vee q')$$

$$= p \vee q'$$

$$= p \vee \sim q$$

$p \wedge q$
T T $= T$

$p \wedge q'$
$T \wedge T = T$ $(q = F)$

$p' \wedge q'$
$T \wedge T = T$ $(p = F, q = F)$

$$\therefore \; p \square q = p \vee \sim q$$

(a) $\sim q \,\square\, \sim p$ [previous page proof]

$= \sim q \vee \sim(\sim p)$

$= \sim q \vee p$ it's not correct

(b) $p \,\square\, \sim q$

$= p \vee \sim(\sim q)$

$= p \vee q$ → it's correct

(c) $\sim p \,\square\, q$

$= \sim p \vee \sim q$ → incorrect

(d) $\sim p \,\square\, \sim q$

$= \sim p \vee \sim(\sim q)$

$= \sim p \vee q$ → incorrect

Problem:

Consider the following expressions:

1. False 2. Q 3. True 4. $P \vee Q$ 5. $\neg Q \vee P$

The number of expressions given above that are logically implied by $p \wedge (p \rightarrow Q)$ is ?

Solution:

"logically implied means it's a tautology"

Theory: If $P \Leftrightarrow Q$, $P \rightarrow Q$ is a tautology or valid
$P \leftrightarrow Q$ is a tautology or valid

Solve this problem:

$$p \wedge (p \rightarrow Q) \equiv p \wedge (\neg p \vee Q)$$
$$\equiv (p \wedge \neg p) \vee (p \wedge Q)$$
$$= F \vee p \wedge Q$$
$$= p \wedge Q$$

(i) $p \wedge Q \Rightarrow F$ [$\Rightarrow$ logically implied]

$T \wedge T \Rightarrow F \rightarrow F$

so, it's not a tautology. & That's why, it's incorrect

(ii) $p \wedge q \Rightarrow q$

$T \quad T \qquad T$

$T \rightarrow T$

$= T$

it's a tautology, it's correct

(iii) $p \wedge q \Rightarrow T$

$T \wedge T \Rightarrow T$

it's a tautology ; it's correct

(iv) $p \wedge q \Rightarrow p \vee q$

$T \wedge T \qquad T \vee T$

$T \Rightarrow T$

it's a tautology, it's correct

(v) $p \wedge q \Rightarrow \neg q \vee p$

$T \wedge T \Rightarrow F \vee T$

$T \Rightarrow T$

it's a tautogy. it's correct

Rules of Inference in Propositional Logic

Premise: is a proposition on the basis of which we would able to draw a conclusion. You can think of premise as an evidence or assumption.

Therefore, initially we assume something is true and on the ~~bas~~ basis of that assumption, we draw some conclusion.

Conclusion: is a proposition that is reached from the given set of premises. ~~Youa~~ You can think of it as the result of the assumptions that we made in an argument.

$\boxed{\text{if premise then conclusion}}$

Argument: Sequence of statements that ends ~~at~~ with a conclusion.

OR

it is a set of one or more premises and a conclusion.

Valid Argument: An Argument is said to be valid if and only if it is not possible to make all premises true and a conclusion false.

Example of an argument:

P_1: "If I love cat then I love dog." → $p \to q$

P_2: "I love cat." → p

C: Therefore, "I love dog." → $\therefore q$

∴ if premises then conclusion

$$((p \to q) \wedge p) \to q$$

Lets: Assume premises are true

∴ $p \to q \to T$ and $p \to T$

q → T has to be true

So, we Assume premises true and get conclusion True all possible cases.

hence, it's a valid argument

Ex: "If I love ~~cant~~ cat then I love dog." → $p \to q$

"I love dog."

Therefore, "I love cat."

$$\begin{array}{l} p \to q \\ q \\ \hline \therefore p \end{array}$$

$$\therefore ((p \to q) \wedge q) \to p$$

Lets Assume, $p \to q \to T$ and $q \to T$

$\swarrow$ T

p can be T or F

so, there is a case when $p \to F$ and conclusion is false.

hence it's a invalid argument.

Rules of Inference: are the templates for constructing valid arguments.

deriving conclusions from evidences

Types of Inference Rules:

1. Modus Ponens:

$$\begin{array}{ll} p \to q & T \\ p & T \\ \hline \therefore q & T \end{array}$$

or $[(p \to q) \wedge p] \to q$

2. Modus Tollens:

$$\begin{array}{ll} p \to q & T \\ \sim q & T \\ \hline \sim p & T \end{array}$$

or $[(p \to q) \wedge \sim q] \to \sim p$

3. Hypothetical Syllogism:

$$\begin{array}{ll} p \to q & T \\ q \to r & T \\ \hline \therefore p \to r & \end{array}$$

$T \quad T = T$

or $[(p \to q) \wedge (q \to r)] \to (p \to r)$

4. Disjunctive Syllogism:

$$\begin{array}{ll} p \vee q & T \\ \sim p & T \\ \hline \therefore q & T \end{array}$$

or $[(p \vee q) \wedge \sim p] \to q$

5. Addition: $\dfrac{p}{\therefore p \vee q}$ or $p \rightarrow (p \vee q)$

$\;\;\;\;\;\;\;\;\;\;\;\;$ T $\boxed{}$

$\;\;\;\;\;\;\;\;\;\;\;\;$ = T

6. Simplification: $\dfrac{\overset{T}{p} \wedge \overset{T}{q}\ T}{\therefore p\ T}$ or $\dfrac{\overset{T}{p} \wedge \overset{T}{q}\ T}{\therefore q\ T}$ or $(p \wedge q) \rightarrow p$ or $(p \wedge q) \rightarrow q$

7. Conjunction: $\dfrac{\begin{array}{c} p\ T \\ q\ T \end{array}}{p \wedge q\ T}$ or $(p \wedge q) \rightarrow (p \wedge q)$

8. Resolution:

$$\dfrac{\begin{array}{c} \overset{T}{p} \vee \overset{F}{q}\ T \\ \sim \overset{F}{p} \vee \overset{T}{r}\ T \end{array}}{\therefore \underset{F}{q} \vee \underset{T}{r}}$$

$= T$

or $[(p \vee q) \wedge (\sim p \vee r)] \rightarrow (q \vee r)$

How to build arguments using rules of inference

Premises: "Randy works hard," "If Randy works hard, then he is a dull boy." and "If Randy is a dull boy, then he will not get the job."

Conclusion: "Randy will not get the job."

Let, H = Randy works hard
D = Randy is a dull boy
J = Randy will get the job

$\therefore$

$$\boxed{\begin{array}{l} H \\ H \to D \\ D \to \sim J \\ \hline \sim J \end{array}}$$

case-1: choose two premises

$$\begin{array}{ll} H & \\ H \to D & \text{[Modus Ponens]} \\ \hline \therefore D & \\ D \to \sim J & \text{[Modus Ponens]} \\ \hline \therefore \sim J & \end{array}$$

case-2:

$$\begin{array}{ll} H \to D & \\ D \to \sim J & \text{[Hypothetical syllogism]} \\ \hline H \to \sim J & \\ H & \text{[Modus ponens]} \\ \hline \sim J & \end{array}$$

(*) It's not mandatory that we have to choose ~~pre~~ given premises. we can choose ~~conclusion (for~~ conclusion of one argument as a premises for other argument

<u>Premises</u>: "If it does not rain or if it is not foggy, then the sailing race will be held and lifesaving demonstration will go on".
"If the sailing race is held, then the trophy will be awarded" and "The trophy was not awarded".

<u>Conclusion</u>: "It rained"

R = it rains, F = it is foggy, S = the sailing race will be held

D = the lifesaving demonstration will go on and

T = trophy will be awarded

$$(\sim R \vee \sim F) \rightarrow S \wedge D$$

$$S \rightarrow T$$

$$\frac{\sim T}{\therefore R}$$

$$S \rightarrow T$$

$$\frac{\sim T}{\sim S} \quad \text{[Modus Tollens]}$$

$$\sim S \vee \sim D = \overbrace{\sim (S \wedge D)}^{\sim q}$$

$$\underbrace{(\sim R \vee \sim F)}_{p} \rightarrow \underbrace{S \wedge D}_{q}$$

$$p \rightarrow q$$

$$\frac{\sim q}{\sim p} \quad \text{[Modus Tollens]}$$

$$\sim p = \sim (\sim R \vee \sim F)$$

$$= \frac{R \wedge F}{R} \quad \text{[Simplification rule]}$$

Checking the validity of the argument

Problem: Show that the following argument is valid. If today is Tuesday, then I have a test in Mathematics or Economics. If my Economics Professor is sick, then I will not have a test in Economics. Today is Tuesday and my Economics Professor is sick. Therefore I have a test in Mathematics.

Solution:

T = Today is Tuesday

M = I have a test in Mathematics

E = I have a test in Economics

S = My Economics Professor is sick

$$
\begin{array}{l|c}
{}^{T}T \rightarrow M^{F} \vee E^{F} & T \; \times \\
T\,S \rightarrow \sim E\,T & T \\
{}^{T}T \wedge S\,T & T \\
\hline
M\;F & F \\
\end{array}
$$

Shortcut Method:

we have to make all premises T and conclusion F. If we don't make it then our argument is valid.

∴ we are unsuccessful to make it. So, our argument is valid.

Problem: Consider the following logical inferences

I1: If it rains then the cricket match will not be played.
The cricket match was played.

Inferences: There was no rain.

I2: If it rains the cricket match will not be played.
It did not rain.

Inferences: The cricket match was played.

Which of the following is TRUE?

a) Both I1 and I2 are correct inferences

b) I1 is correct but I2 is not a correct inference

c) I1 is not correct but I2 is correct inference

d) Both I1 and I2 are not correct inferences

Let R = it rains

$C \Rightarrow$ the cricket match was played

I1 : (shortcut method)

$$\begin{array}{ll} {}^{F}R \rightarrow \sim C^{F} & \boxed{T} \\ {}^{T}C & T \\ \hline \therefore \sim R_{T} & F \end{array}$$

$\therefore$ argument is valid

I2 :

$$\begin{array}{ll} {}^{F}R \rightarrow \sim C^{T} & T \\ \sim R_{T} & T \\ \hline \therefore C_{F} & F \end{array}$$

$\therefore$ argument is invalid.

$\therefore$ Answer: (b)

Problem: Considers the following two statements

S1: If a candidate is known to be corrupt, then he will not be elected.

S2: If a candidate is kind, he will be elected.

Which one of the following statements from S1 and S2 as per sound inference rules of logic?

a) If a person is known to be corrupt, he is kind.

b) If a person is not known to be corrupt, he is not kind.

c) If a person is kind, he is not known to be corrupt.

d) If a person is not kind, he is not known to be corrupt.

C → ~~kind~~ corrupt
E → elected
K → kind

$C \to \sim E$

$K \to E$

(a) T T

$C \to \sim E$

F $K \to E$ F

$\therefore C \to K$

T F

T
T
F

argument is invalid

(b) F F

$C \to \sim E$

T $K \to E$ T

$\sim C \to \sim K$

T F

T
T
F

argument is invalid.

(c) T $C \to \sim E$ F

T $K \to E$ T

$K \to \sim C$

T F

T×
T
F

argument is valid.

(d) T T

$C \to \sim E$

F $K \to E$ F

$\sim K \to \sim C$

T F

T
T
F

argument is invalid

Limitation of Propositional Logic

The argument which propositional logic can handle

If it snows today, then we will go skiing. $p \to q$ T

It is snowing today. p T

we will go skiing $\therefore q$ T

This is true by the rule of Modus ponens

The argument which propositional logic could not handle :

p Every one enrolled in the university has lived a dormitory

q Mira has never lived in a dormitory

$\therefore r$ ∴ Mira is not enrolled in the university

↳ It can't be proved by propositional logic