Social Network Analysis
Lesson 7 CD-1

Community

- Community: It is formed by individuals such that those within a group interact with each other more frequently than with those outside the group
  - a.k.a. group, cluster, cohesive subgroup, module in different contexts
- Community detection: discovering groups in a network where individuals' group memberships are not explicitly given.

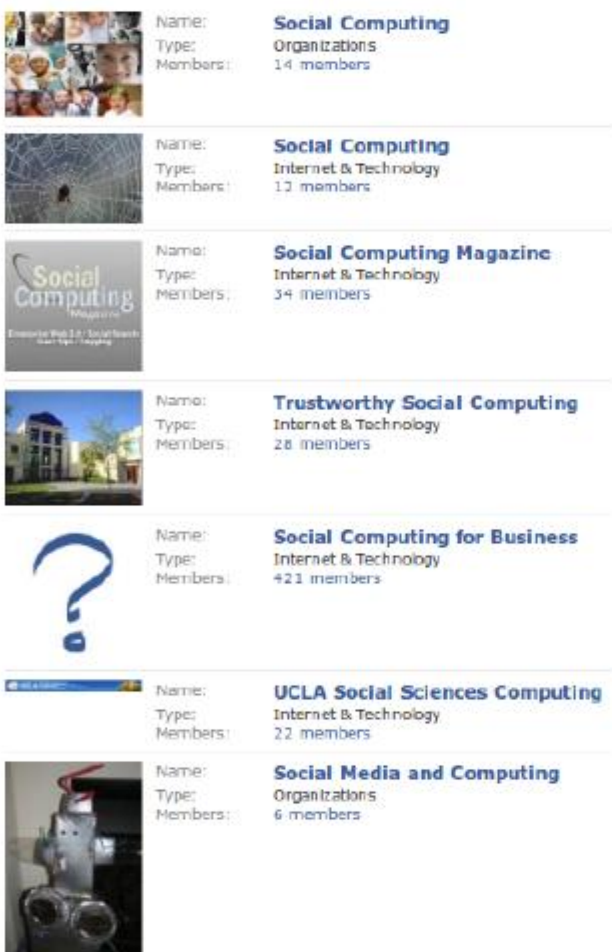
- Why communities in social media?
  - Human beings are social
  - Easy-to-use social media allows people to extend their social life in unprecedented ways
  - Difficult to meet friends in the physical world, but much easier to find friend online with similar interests
  - Interactions between nodes can help determine communities

Real-World Communities

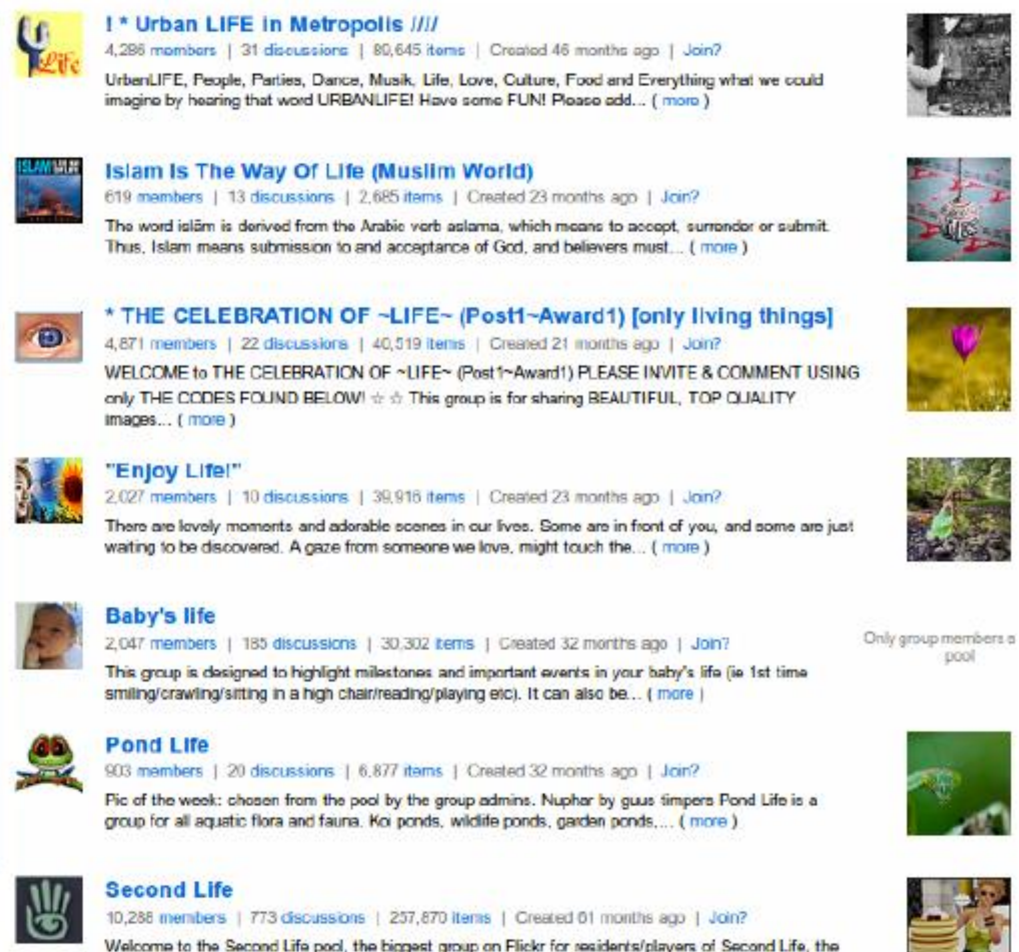
Egypt Protest, 2011

Real-World Communities

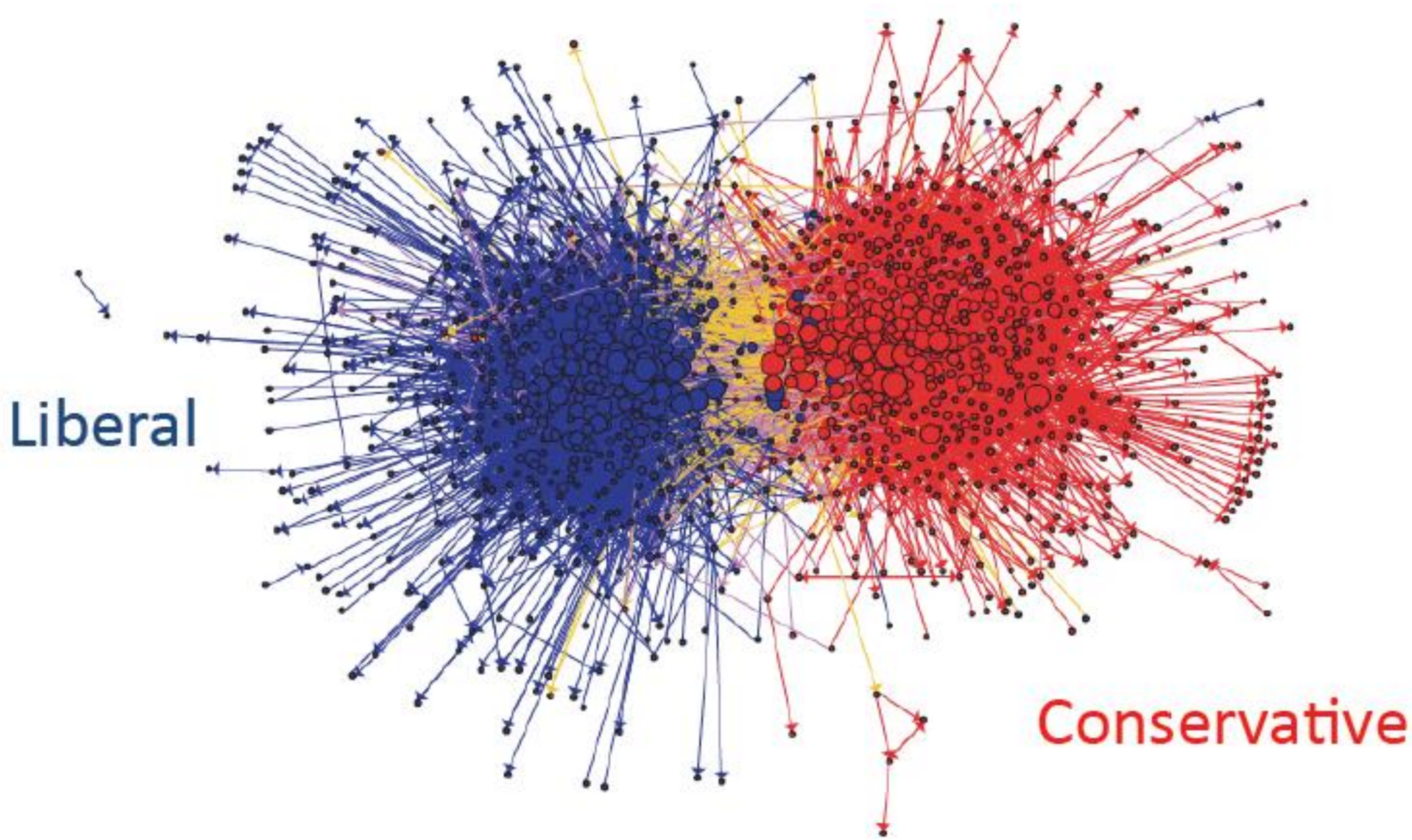
Communities from Facebook



Communities from Flickr

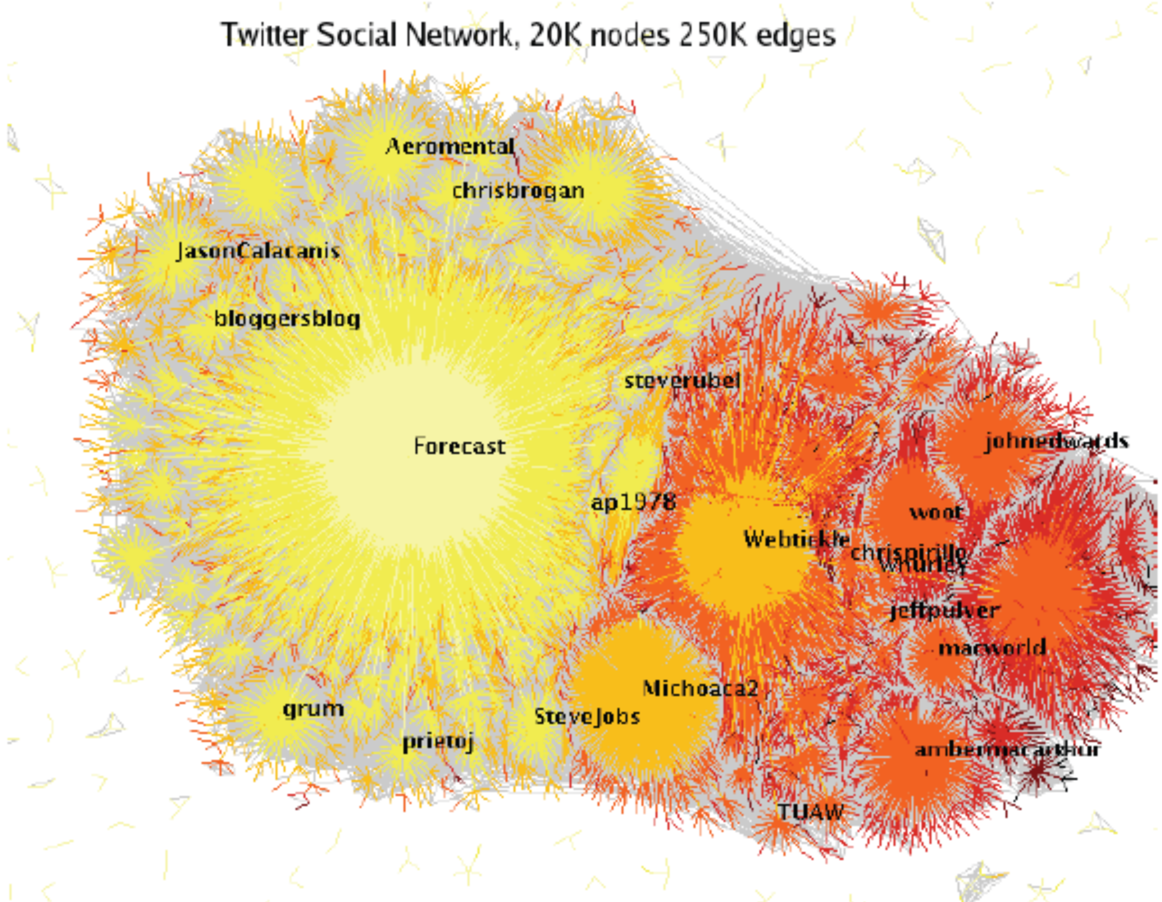


Real-World Communities



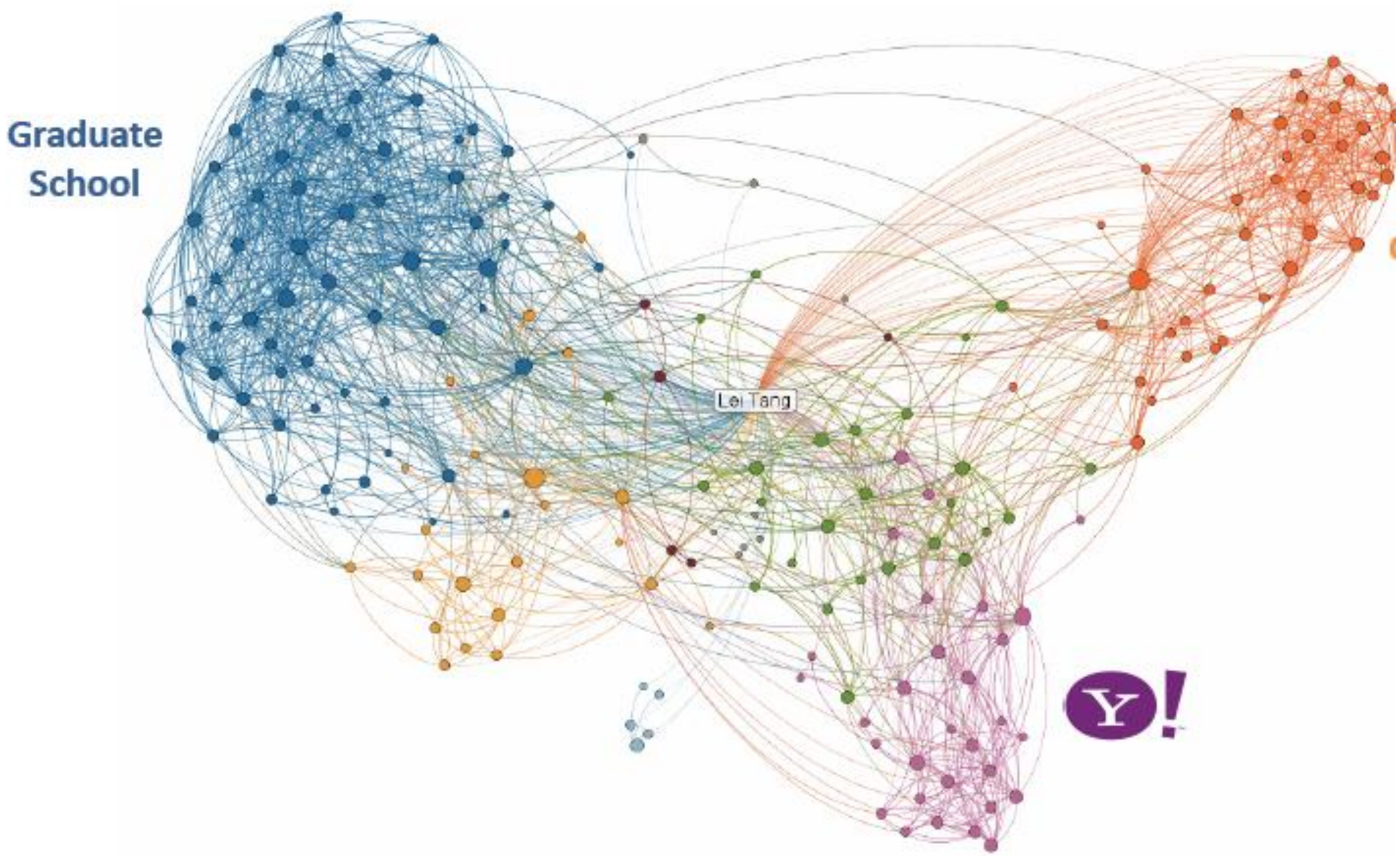
Lada Admic And Natalie Glance, *The Political Blogosphere and the 2004 U.S. Election: Divided They Blog,* 2005.

Communities in Twitter



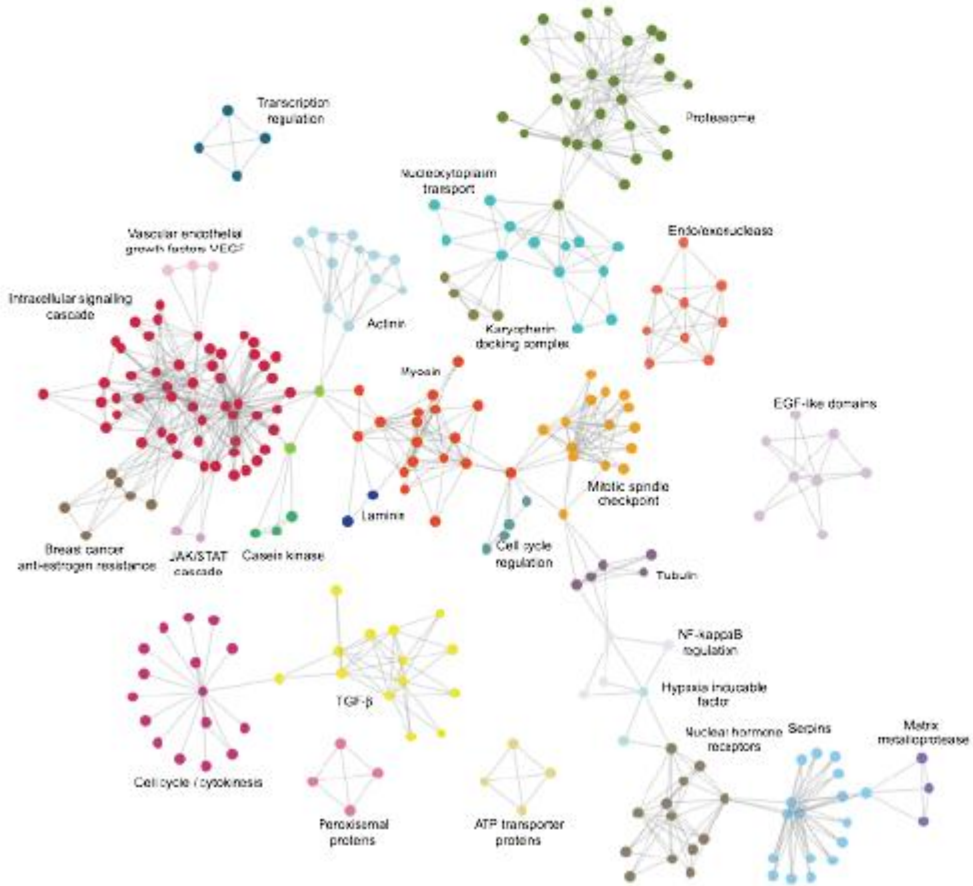
http://ebiquity.umbc.edu/blogger/2007/04/19/twitter-social-network-analysis/

Communities of Personal Social Network



Generated based on Lei Tang's LinkedIn connections on 2011/09/15

Communities in Protein-Protein Interaction Networks



Communities in Social Media

- Two types of groups in social media
  - Explicit Groups: formed by user subscriptions
  - Implicit Groups: implicitly formed by social interactions

- Some social media sites allow people to join groups, is it necessary to extract groups based on network topology?
  - Not all sites provide community platform
  - Not all people want to make effort to join groups
  - Groups can change dynamically
- Network interaction provides rich information about the relationship between users
  - Can complement other kinds of information, e.g. user profile
  - Help network visualization and navigation
  - Provide basic information for other tasks, e.g. recommendation

Note that each of the above three points can be a research topic.

Applications of Community Detection

- Visualization & network navigation
- Data compression
- Structural position and role analysis
- Topic detection in collaborative tagging systems
- Tag disambiguation
- Identifying #unique users in networks
- User profiling based on neighborhood smoothing
- Recommendation and targeting
- Event detection

Community Detection = Clustering?

- To some degree, community detection is essentially clustering.
- But why so many works on Community Detection? (in physical review, KDD, WWW)
- The network data pose challenges to classical clustering method.

Difference

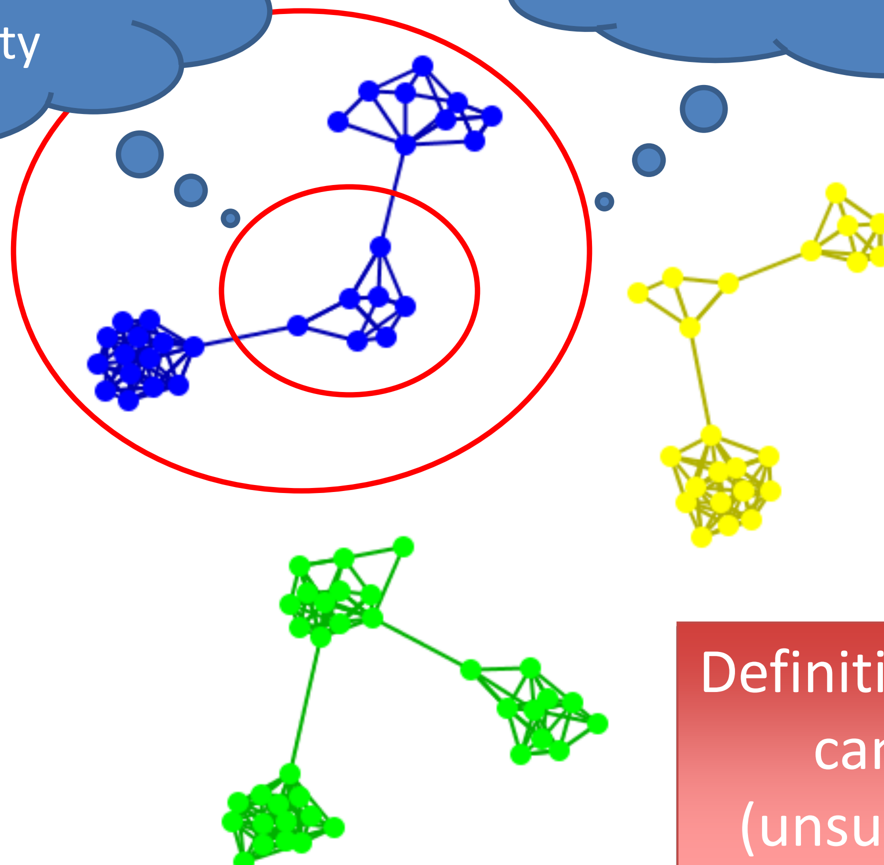
- Clustering works on the distance or similarity matrix (k-means, hierarchical clustering, spectral clustering)
- Network data tends to be "discrete", leading to algorithms using the graph property directly (k-clique, quasi-clique, vertex-betweenness, edge-betweenessetc.)
- Real-world network is large scale! Sometimes, even n^2 in unbearable for efficiency or space (local/distributed clustering, network approximation, sampling method)

- Overview of Community Detection Methods
  - Node-Centric
  - Group-Centric
  - Network-Centric
  - Hierarchy-Centric
- Communities in Social Media
  - Statistical Properties
  - Community Evolution
  - Heterogeneous Networks
  - Community Evaluation
  - Scaling Community Detection
- Application of Community Detection for Social Media Mining

Subjectivity of Community Definition

Each component is a community

A densely-knit community



Definition of a community can be subjective.
(unsupervised learning)

Taxonomy of Community Criteria

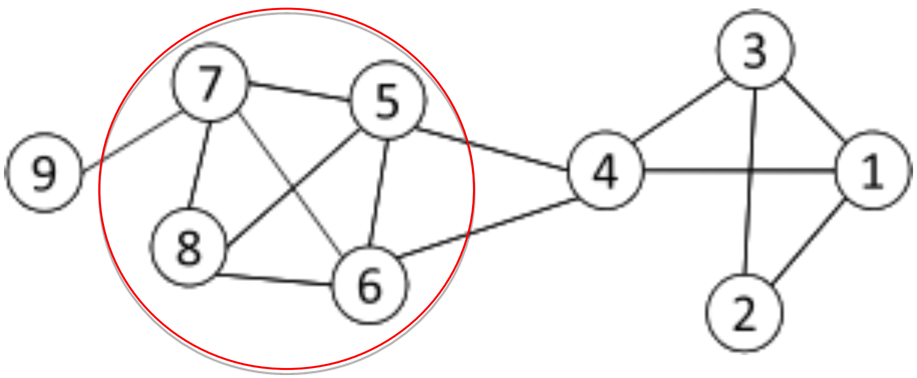
- Criteria vary depending on the tasks
- Roughly, community detection methods can be divided into 4 categories (not exclusive):
- Node-Centric Community
  - Each node in a group satisfies certain properties
- Group-Centric Community
  - Consider the connections within a group as a whole. The group has to satisfy certain properties without zooming into node-level
- **Network-Centric Community**
  - Partition the whole network into several disjoint sets
- Hierarchy-Centric Community
  - Construct a hierarchical structure of communities

Node-Centric Community Detection

- Nodes satisfy different properties
  - Complete Mutuality
    - cliques
  - Reachability of members
    - k-clique, k-club
  - Nodal degrees
    - k-plex, k-core
  - Relative frequency of Within-Outside Ties
    - LS sets, Lambda sets
- Commonly used in traditional social network analysis
- Here, we discuss some representative ones

Complete Mutuality: Cliques

- Clique: a maximum complete subgraph in which all nodes are adjacent to each other



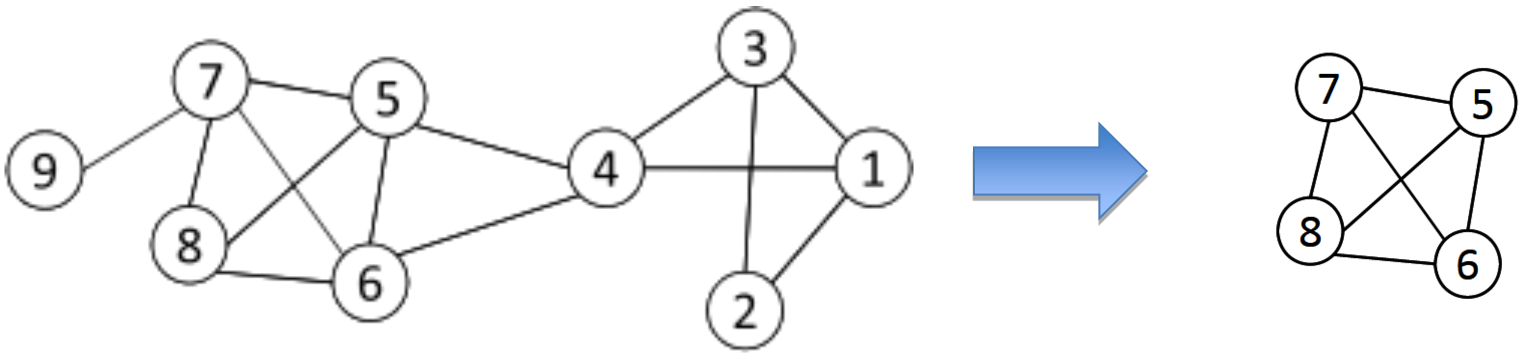
Nodes 5, 6, 7 and 8 form a clique

- NP-hard to find the maximum clique in a network
- Straightforward implementation to find cliques is very expensive in time complexity

Finding the Maximum Clique

- In a clique of size k, each node maintains degree >= k-1
  - Nodes with degree < k-1 will not be included in the maximum clique
- Recursively apply the following pruning procedure
  - Sample a sub-network from the given network, and find a clique in the sub-network, say, by a greedy approach
  - Suppose the clique above is size k, in order to find out a *larger* clique, all nodes with degree <= k-1 should be removed.
- Repeat until the network is small enough
- Many nodes will be pruned as social media networks follow a <u>power law distribution</u> for node degrees

Maximum Clique Example



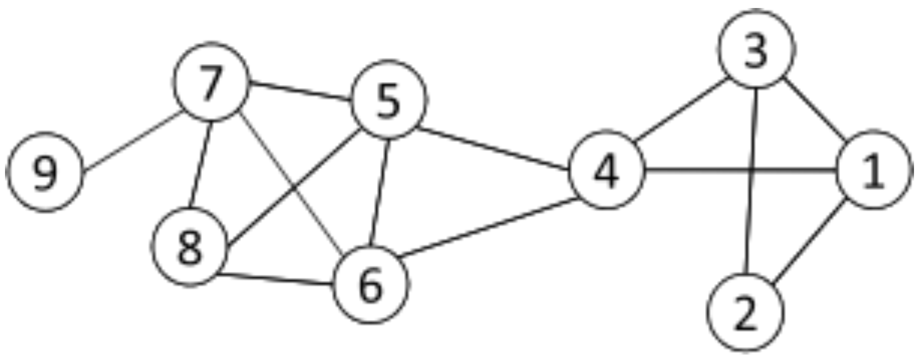
- Suppose we sample a sub-network with nodes {1-9} and find a clique {1, 2, 3} of size 3
- In order to find a clique >3, remove all nodes with degree <=3-1=2
  - Remove nodes 2 and 9
  - Remove nodes 1 and 3
  - Remove node 4

Clique Percolation Method (CPM)

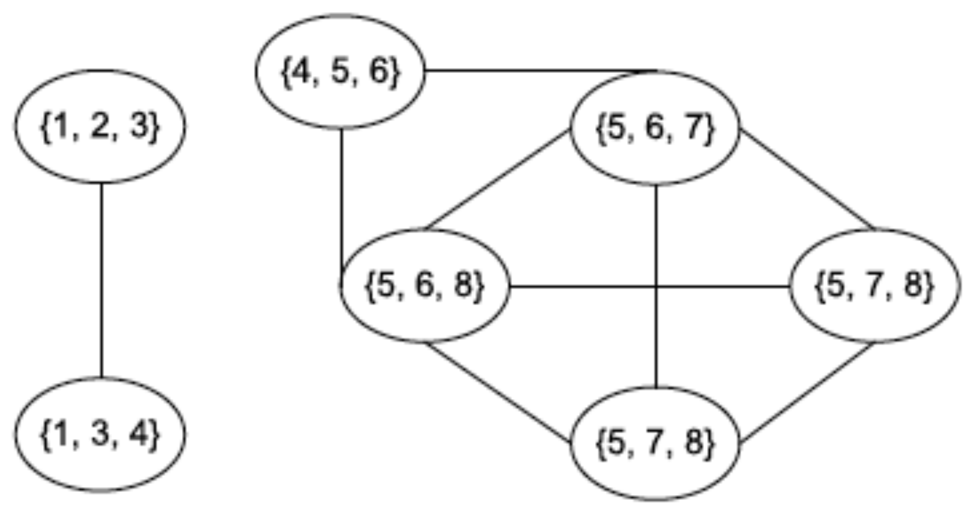
- Clique is a very strict definition, unstable
- Normally use cliques as a core or a seed to find larger communities

- CPM is such a method to find overlapping communities
  - **Input**
    - A parameter k, and a network
  - **Procedure**
    - Find out all cliques of size k in a given network
    - Construct a clique graph. Two cliques are adjacent if they share k-1 nodes
    - Each connected components in the clique graph form a community

CPM Example



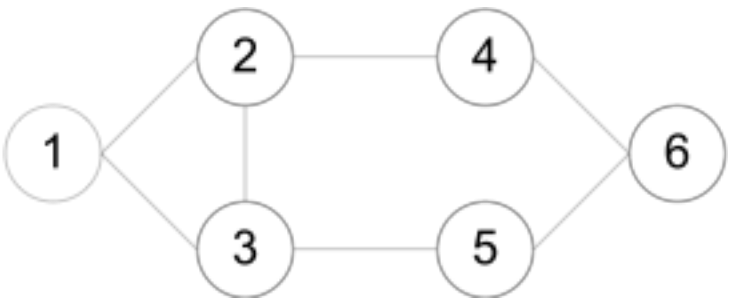
Cliques of size 3:
{1, 2, 3}, {1, 3, 4}, {4, 5, 6}, {5, 6, 7}, {5, 6, 8}, {5, 7, 8}, {6, 7, 8}



Communities:
{1, 2, 3, 4}
{4, 5, 6, 7, 8}

Reachability : k-clique, k-club

- Any node in a group should be reachable in k hops
- k-clique: a maximal subgraph in which the largest geodesic distance between any two nodes <= k
- k-club: a substructure of diameter <= k



Cliques: {1, 2, 3}
2-cliques: {1, 2, 3, 4, 5}, {2, 3, 4, 5, 6}
2-clubs: {1,2,3,4}, {1, 2, 3, 5}, {2, 3, 4, 5, 6}

- A k-clique might have diameter larger than k in the subgraph
  - E.g. {1, 2, 3, 4, 5}
- Commonly used in traditional SNA
- Often involves combinatorial optimization

Group-Centric Community Detection: Density-Based Groups

- The group-centric criterion requires the whole group to satisfy a certain condition
  - E.g., the group density >= a given threshold
- A subgraph $G_s(V_s, E_s)$ is a $\gamma - dense$ quasi-clique if

$$\frac{2|E_s|}{|V_s|(|V_s|-1)} \geq \gamma$$

  where the denominator is the maximum number of degrees.
- A similar strategy to that of cliques can be used
  - Sample a subgraph, and find a maximal $\gamma - dense$ quasi-clique (say, of size $|V_s|$)
  - Remove nodes with degree <u>less than</u> the average degree

    < $|V_s|\gamma \leq \frac{2|E_s|}{|V_s|-1}$

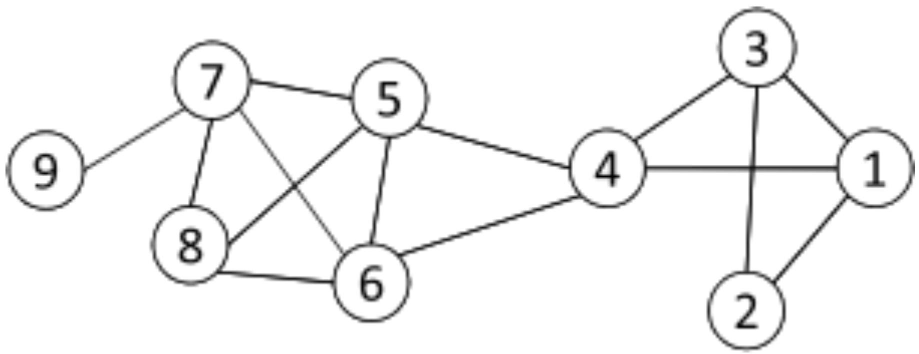
Network-Centric Community Detection

- Network-centric criterion needs to consider the connections within a network <u>globally</u>
- Goal: partition nodes of a network into <u>disjoint</u> sets
- Approaches:
  - (1) Clustering based on vertex similarity
  - **(2) Latent space models (multi-dimensional scaling)**
  - (3) Block model approximation
  - **(4) Spectral clustering**
  - **(5) Modularity maximization**

Clustering based on Vertex Similarity

- Apply k-means or similarity-based clustering to nodes
- Vertex similarity is defined in terms of the similarity of their neighborhood
- Structural equivalence: two nodes are structurally equivalent iff they are connecting to the same set of actors

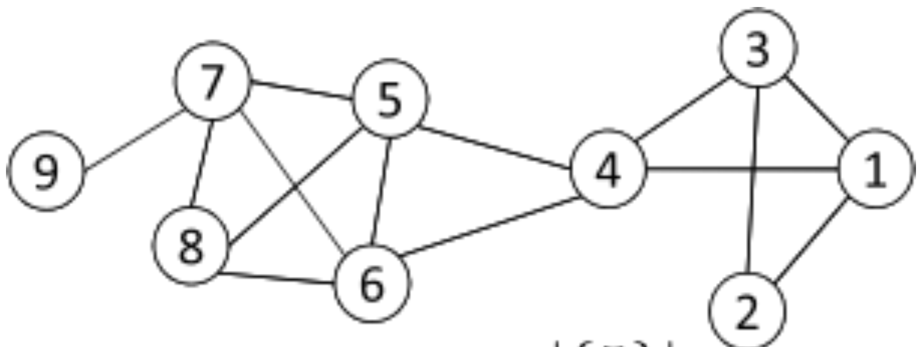
Nodes 1 and 3 are structurally equivalent; So are nodes 5 and 6.



- Structural equivalence is too restrict for practical use.

Vertex Similarity

- Jaccard Similarity $$Jaccard(v_i, v_j) = \frac{|N_i \cap N_j|}{|N_i \cup N_j|}$$
- Cosine similarity $$Cosine(v_i, v_j) = \frac{|N_i \cap N_j|}{\sqrt{|N_i| \cdot |N_j|}}$$



$$Jaccard(4,6) = \frac{|\{5\}|}{|\{1,3,4,5,6,7,8\}|} = \frac{1}{7}$$

$$cosine(4,6) = \frac{1}{\sqrt{4 \cdot 4}} = \frac{1}{4}$$

Latent Space Models

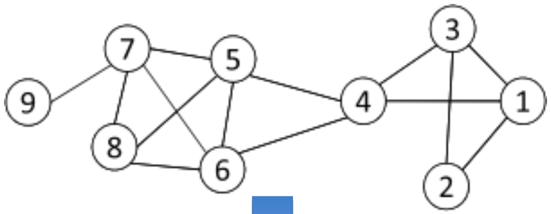
- Map nodes into a low-dimensional space such that the proximity between nodes based on network connectivity is preserved in the new space, then apply k-means clustering
- Multi-dimensional scaling (MDS)
  - Given a network, construct a proximity matrix P representing the pairwise distance between nodes (e.g., geodesic distance)
  - Let $S \in R^{n \times l}$ denote the coordinates of nodes in the low-dimensional space $$SS^T \approx -\frac{1}{2}(I - \frac{1}{n}\mathbf{1}\mathbf{1}^T)(P \circ P)(I - \frac{1}{n}\mathbf{1}\mathbf{1}^T) = \widetilde{P}$$

Centered matrix

  - Objective function: $\min \|SS^T - \widetilde{P}\|_F^2$
  - Solution: $S = V\Lambda^{\frac{1}{2}}$
  - V is the top ℓ eigenvectors of $\widetilde{P}$, and Λ is a diagonal matrix of top eigenvalues $\Lambda = diag(\lambda_1, \lambda_2, \cdots, \lambda_\ell)$

Reference: http://www.cse.ust.hk/~weikep/notes/MDS.pdf

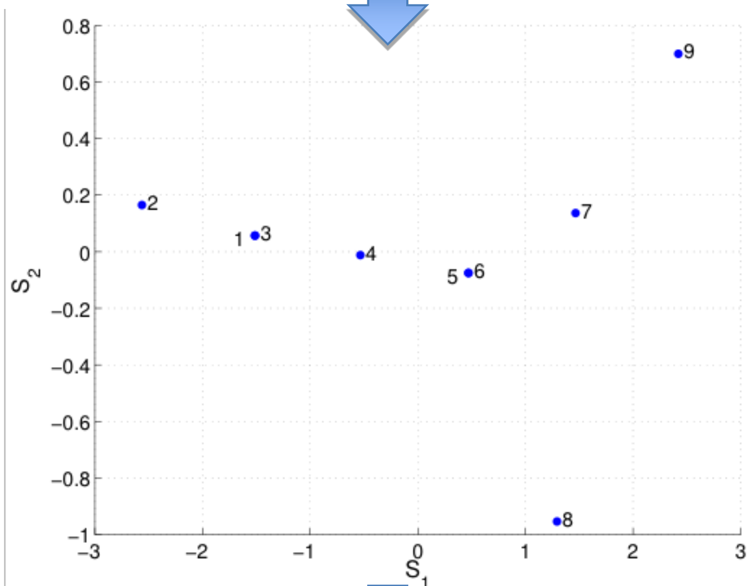
MDS Example



geodesic distance

$$P = \begin{bmatrix} 0 & 1 & 1 & 1 & 2 & 2 & 3 & 3 & 4 \\ 1 & 0 & 1 & 2 & 3 & 3 & 4 & 4 & 5 \\ 1 & 1 & 0 & 1 & 2 & 2 & 3 & 3 & 4 \\ 1 & 2 & 1 & 0 & 1 & 1 & 2 & 2 & 3 \\ 2 & 3 & 2 & 1 & 0 & 1 & 1 & 1 & 2 \\ 2 & 3 & 2 & 1 & 1 & 0 & 1 & 1 & 2 \\ 3 & 4 & 3 & 2 & 1 & 1 & 0 & 1 & 1 \\ 3 & 4 & 3 & 2 & 1 & 1 & 1 & 0 & 2 \\ 4 & 5 & 4 & 3 & 2 & 2 & 1 & 2 & 0 \end{bmatrix}$$

$$\widetilde{P} = \begin{bmatrix} 2.46 & 3.96 & 1.96 & 0.85 & -0.65 & -0.65 & -2.21 & -2.04 & -3.65 \\ 3.96 & 6.46 & 3.96 & 1.35 & -1.15 & -1.15 & -3.71 & -3.54 & -6.15 \\ 1.96 & 3.96 & 2.46 & 0.85 & -0.65 & -0.65 & -2.21 & -2.04 & -3.65 \\ 0.85 & 1.35 & 0.85 & 0.23 & -0.27 & -0.27 & -0.82 & -0.65 & -1.27 \\ -0.65 & -1.15 & -0.65 & -0.27 & 0.23 & -0.27 & 0.68 & 0.85 & 1.23 \\ -0.65 & -1.15 & -0.65 & -0.27 & -0.27 & 0.23 & 0.68 & 0.85 & 1.23 \\ -2.21 & -3.71 & -2.21 & -0.82 & 0.68 & 0.68 & 2.12 & 1.79 & 3.68 \\ -2.04 & -3.54 & -2.04 & -0.65 & 0.85 & 0.85 & 1.79 & 2.46 & 2.35 \\ -3.65 & -6.15 & -3.65 & -1.27 & 1.23 & 1.23 & 3.68 & 2.35 & 6.23 \end{bmatrix}$$



$$V = \begin{bmatrix} -0.33 & 0.05 \\ -0.55 & 0.14 \\ -0.33 & 0.05 \\ -0.11 & -0.01 \\ 0.10 & -0.06 \\ 0.10 & -0.06 \\ 0.32 & 0.11 \\ 0.28 & -0.79 \\ 0.52 & 0.58 \end{bmatrix}, \quad \Lambda = \begin{bmatrix} 21.56 & 0 \\ 0 & 1.46 \end{bmatrix}, \quad S = V\Lambda^{1/2} = \begin{bmatrix} -1.51 & 0.06 \\ -2.56 & 0.17 \\ -1.51 & 0.06 \\ -0.53 & -0.01 \\ 0.47 & -0.08 \\ 0.47 & -0.08 \\ 1.47 & 0.14 \\ 1.29 & -0.95 \\ 2.42 & 0.70 \end{bmatrix}$$

Two communities:
{1, 2, 3, 4} and {5, 6, 7, 8, 9}

Block Models

Table 3.1: Adjacency Matrix

-	1	1	1	0	0	0	0	0
1	-	1	0	0	0	0	0	0
1	1	-	1	0	0	0	0	0
1	0	1	-	1	1	0	0	0
0	0	0	1	-	1	1	1	0
0	0	0	1	1	-	1	1	0
0	0	0	0	1	1	-	1	1
0	0	0	0	1	1	1	-	0
0	0	0	0	0	0	1	0	-

$$\min ||A - S\Sigma S^T||_F^2$$

Table 3.2: Ideal Block Structure

1	1	1	1	0	0	0	0	0
1	1	1	1	0	0	0	0	0
1	1	1	1	0	0	0	0	0
1	1	1	1	0	0	0	0	0
0	0	0	0	1	1	1	1	1
0	0	0	0	1	1	1	1	1
0	0	0	0	1	1	1	1	1
0	0	0	0	1	1	1	1	1
0	0	0	0	1	1	1	1	1

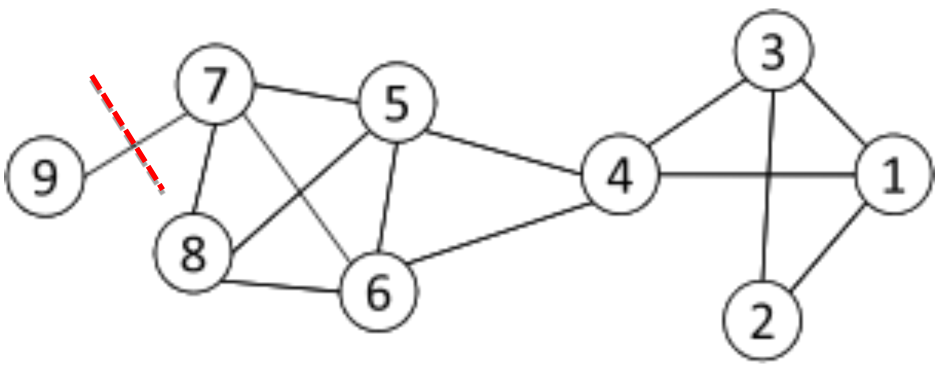
- S is the community indicator matrix (group memberships)
- Relax S to be numerical values, then the optimal solution corresponds to the top eigenvectors of A

$$S = \begin{bmatrix} 0.20 & -0.52 \\ 0.11 & -0.43 \\ 0.20 & -0.52 \\ 0.38 & -0.30 \\ 0.47 & 0.15 \\ 0.47 & 0.15 \\ 0.41 & 0.28 \\ 0.38 & 0.24 \\ 0.12 & 0.11 \end{bmatrix}, \Sigma = \begin{bmatrix} 3.5 & 0 \\ 0 & 2.4 \end{bmatrix}.$$

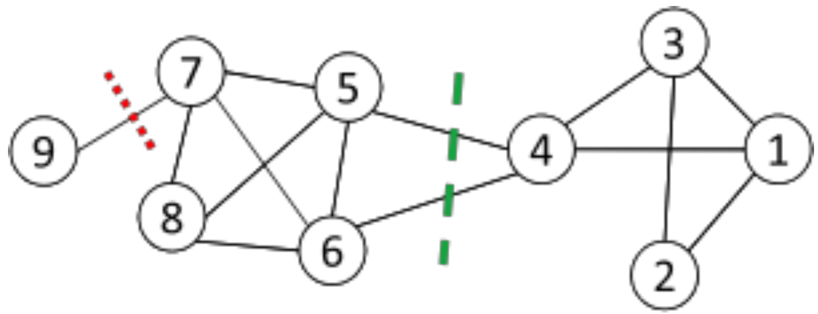
Two communities:
{1, 2, 3, 4} and {5, 6, 7, 8, 9}

Cut

- Most interactions are within group whereas interactions between groups are few
- community detection → minimum cut problem
- Cut: A partition of vertices of a graph into two disjoint sets
- Minimum cut problem: find a graph partition such that the number of edges between the two sets is minimized



Ratio Cut & Normalized Cut



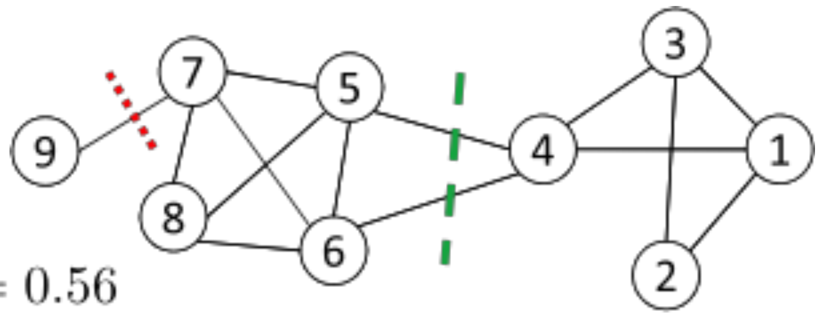
- Minimum cut often returns an <u>imbalanced</u> partition, with one set being a singleton, e.g. node 9
- Change the objective function to consider community size

$$\text{Ratio Cut}(\pi) = \frac{1}{k}\sum_{i=1}^{k}\frac{cut(C_i, \bar{C}_i)}{|C_i|},$$

$$\text{Normalized Cut}(\pi) = \frac{1}{k}\sum_{i=1}^{k}\frac{cut(C_i, \bar{C}_i)}{vol(C_i)}$$

$C_{i,}$: a community
$|C_i|$: number of nodes in C_i
$vol(C_i)$: sum of degrees in C_i

Ratio Cut & Normalized Cut Example



For partition in red: π_1

$$\text{Ratio Cut}(\pi_1) = \frac{1}{2}\left(\frac{1}{1} + \frac{1}{8}\right) = 9/16 = 0.56$$

$$\text{Normalized Cut}(\pi_1) = \frac{1}{2}\left(\frac{1}{1} + \frac{1}{27}\right) = 14/27 = 0.52$$

For partition in green: π_2

$$\text{Ratio Cut}(\pi_2) = \frac{1}{2}\left(\frac{2}{4} + \frac{2}{5}\right) = 9/20 = 0.45 < \text{Ratio Cut}(\pi_1)$$

$$\text{Normalized Cut}(\pi_2) = \frac{1}{2}\left(\frac{2}{12} + \frac{2}{16}\right) = 7/48 = 0.15 < \text{Normalized Cut}(\pi_1)$$

Both ratio cut and normalized cut prefer a balanced partition

Spectral Clustering

- Both ratio cut and normalized cut can be reformulated as

$$\min_{S \in \{0,1\}^{n \times k}} Tr(S^T \widetilde{L} S)$$

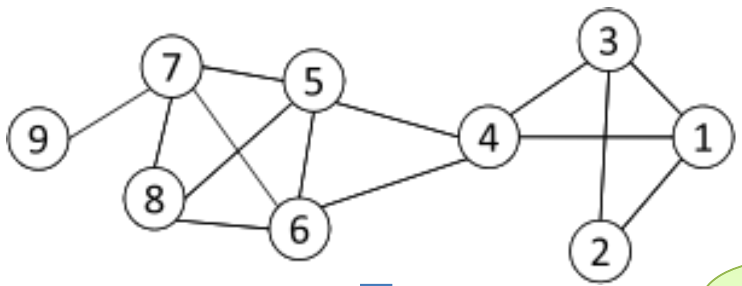
- Where $\widetilde{L} = \begin{cases} D - A & \text{graph Laplacian for ratio cut} \\ I - D^{-1/2} A D^{-1/2} & \text{normalized graph Laplacian} \end{cases}$

$D = diag(d_1, d_2, \cdots, d_n)$ A diagonal matrix of degrees

- Spectral relaxation: $\min_{S} Tr(S^T \widetilde{L} S) \quad s.t. \ S^T S = I_k$
- Optimal solution: top eigenvectors with the smallest eigenvalues

Reference: http://www.cse.ust.hk/~weikep/notes/clustering.pdf

Spectral Clustering Example



Two communities:
{1, 2, 3, 4} and {5, 6, 7, 8, 9}

The 1st eigenvector means all nodes belong to the same cluster, no use

k-means

$$D = diag(3,2,3,4,4,4,4,3,1)$$

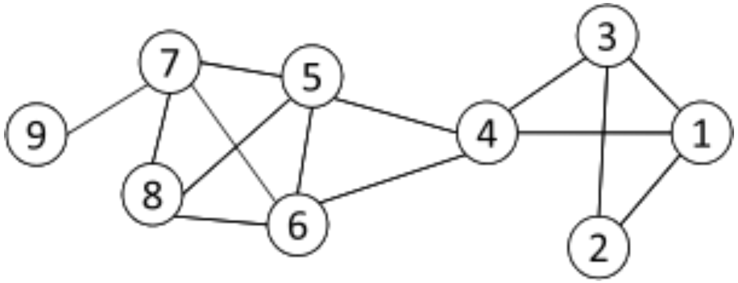
$$\widetilde{L} = D - A = \begin{bmatrix} 3 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & 3 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & 4 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 4 & -1 & -1 & -1 & 0 \\ 0 & 0 & 0 & -1 & -1 & 4 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & 4 & -1 & -1 \\ 0 & 0 & 0 & 0 & -1 & -1 & -1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} 0.33 & -0.38 \\ 0.33 & -0.48 \\ 0.33 & -0.38 \\ 0.33 & -0.12 \\ 0.33 & 0.16 \\ 0.33 & 0.16 \\ 0.33 & 0.30 \\ 0.33 & 0.24 \\ 0.33 & 0.51 \end{bmatrix}$$

Centered matrix

Modularity Maximization

- Modularity measures the strength of a community partition by taking into account the degree distribution
- Given a network with *m* edges, the expected number of edges between two nodes with degrees d_i and d_j is $d_i d_j / 2m$



The expected number of edges between nodes 1 and 2 is 3*2/ (2*14) = 3/14

- Strength of a community: $\sum_{i \in C, j \in C} A_{ij} - d_i d_j / 2m$

Given the degree distribution

- Modularity: $Q = \frac{1}{2m} \sum_{\ell=1}^{k} \sum_{i \in C_\ell, j \in C_\ell} (A_{ij} - d_i d_j / 2m)$
- A larger value indicates a good community structure

Modularity Matrix

Centered matrix

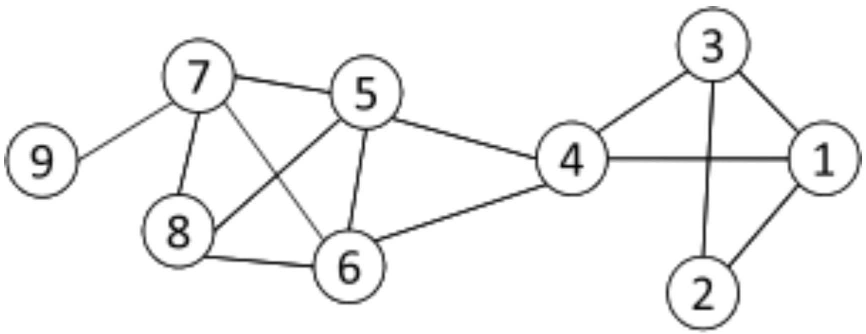
- Modularity matrix: $B = A - \mathbf{d}\mathbf{d}^T/2m \quad (B_{ij} = A_{ij} - d_i d_j/2m)$

- Similar to spectral clustering, Modularity maximization can be reformulated as

$$\max Q = \frac{1}{2m} Tr(S^T B S) \quad s.t.\ S^T S = I_k$$

- Optimal solution: top eigenvectors of the modularity matrix
- Apply k-means to S as a post-processing step to obtain community partition

Modularity Maximization Example



Two Communities:
{1, 2, 3, 4} and {5, 6, 7, 8, 9}

k-means

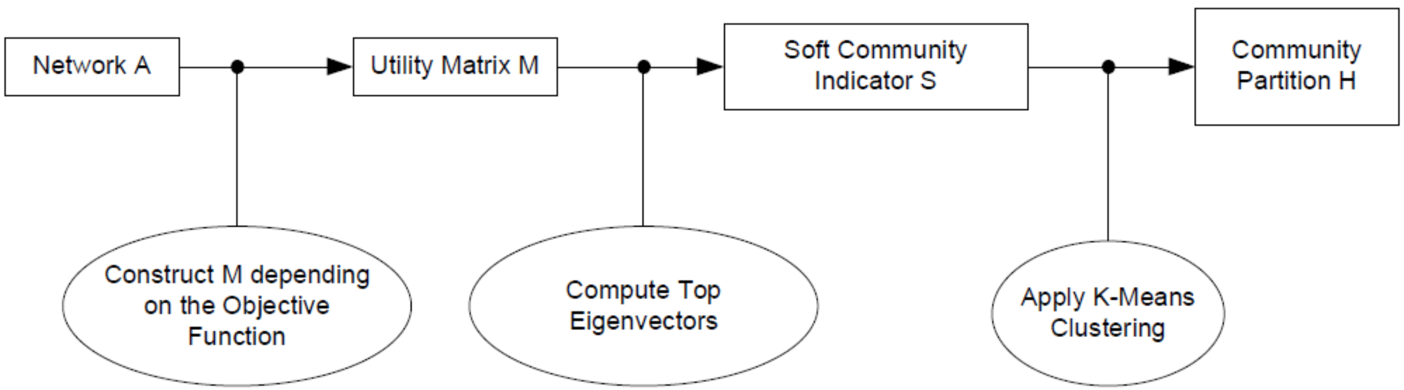
$$B = \begin{bmatrix} -0.32 & 0.79 & 0.68 & 0.57 & -0.43 & -0.43 & -0.43 & -0.32 & -0.11 \\ 0.79 & -0.14 & 0.79 & -0.29 & -0.29 & -0.29 & -0.29 & -0.21 & -0.07 \\ 0.68 & 0.79 & -0.32 & 0.57 & -0.43 & -0.43 & -0.43 & -0.32 & -0.11 \\ 0.57 & -0.29 & 0.57 & -0.57 & 0.43 & 0.43 & -0.57 & -0.43 & -0.14 \\ -0.43 & -0.29 & -0.43 & 0.43 & -0.57 & 0.43 & 0.43 & 0.57 & -0.14 \\ -0.43 & -0.29 & -0.43 & 0.43 & 0.43 & -0.57 & 0.43 & 0.57 & -0.14 \\ -0.43 & -0.29 & -0.43 & -0.57 & 0.43 & 0.43 & -0.57 & 0.57 & 0.86 \\ -0.32 & -0.21 & -0.32 & -0.43 & 0.57 & 0.57 & 0.57 & -0.32 & -0.11 \\ -0.11 & -0.07 & -0.11 & -0.14 & -0.14 & -0.14 & 0.86 & -0.11 & -0.04 \end{bmatrix}$$

Modularity Matrix

0.4384	-0.2709
0.3809	0.2671
0.4384	-0.2709
0.1716	0.6063
-0.2861	-0.3487
-0.2861	-0.3487
-0.3754	0.3355
-0.3421	0.1855
-0.1396	-0.1552

A Unified View for Community Partition

- Latent space models, block models, spectral clustering, and modularity maximization can be unified as



$$\text{Utility Matrix } M = \begin{cases} \text{modified proximity matrix } \widetilde{P} & \textit{if} \text{ latent space models} \\ \text{adjacency matrix } A & \textit{if} \text{ block models} \\ \text{graph Laplacian } \widetilde{L} & \textit{if} \text{ spectral clustering} \\ \text{modularity maximization } \; B & \textit{if} \text{ modularity maximization} \end{cases}$$

Reference: http://www.cse.ust.hk/~weikep/notes/Script_community_detection.m

Hierarchy-Centric Community Detection

- Goal: build a <u>hierarchical structure</u> of communities based on network topology

- Allow the analysis of a network <u>at different resolutions</u>

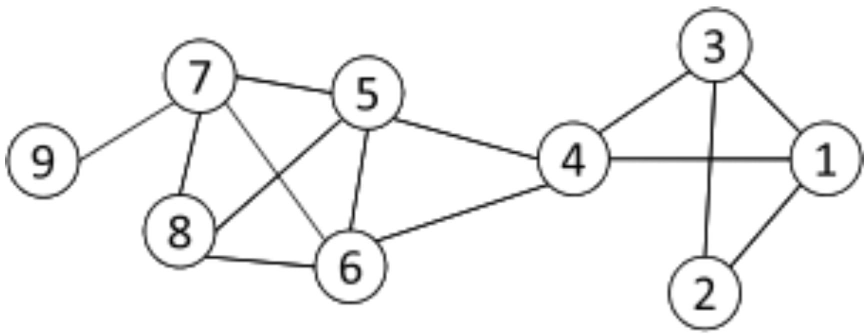
- Representative approaches:
  - Divisive Hierarchical Clustering (top-down)
  - Agglomerative Hierarchical clustering (bottom-up)

Divisive Hierarchical Clustering

- Divisive clustering
  - Partition nodes into several sets
  - Each set is further divided into smaller ones
  - Network-centric partition can be applied for the partition
- One particular example: recursively remove the "weakest" tie
  - Find the edge with the least strength
  - Remove the edge and update the corresponding strength of each edge
- Recursively apply the above two steps until a network is decomposed into desired number of connected components.
- Each component forms a community

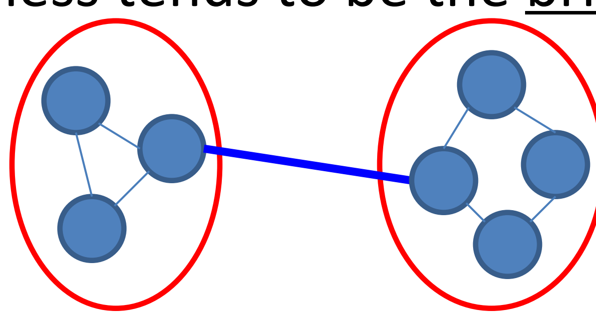
Edge Betweenness

- The strength of a tie can be measured by edge betweenness
- Edge betweenness: the number of shortest paths that pass along with the edge

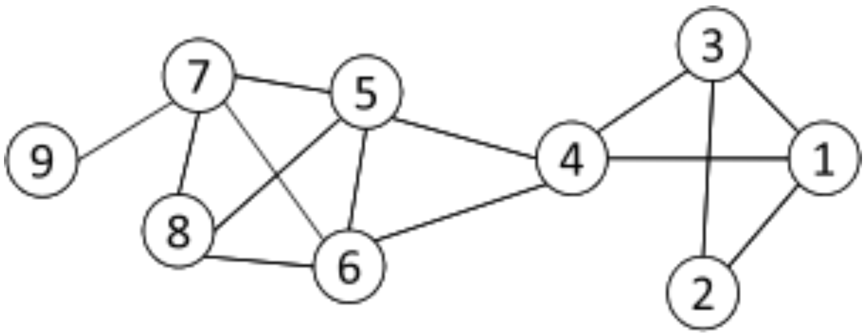


The edge betweenness of e(1, 2) is 4 (=6/2 + 1), as all the shortest paths from 2 to {4, 5, 6, 7, 8, 9} have to either pass e(1, 2) or e(2, 3), and e(1,2) is the shortest path between 1 and 2

- The edge with higher betweenness tends to be the <u>bridge</u> between two communities.



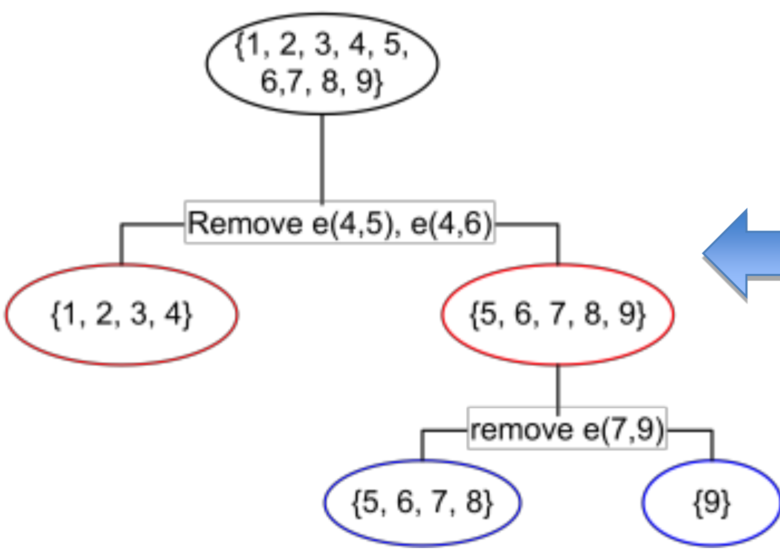
Divisive clustering based on edge betweenness



Initial betweenness value

Table 3.3: Edge Betweenness

	1	2	3	4	5	6	7	8	9
1	0	4	1	9	0	0	0	0	0
2	4	0	4	0	0	0	0	0	0
3	1	4	0	9	0	0	0	0	0
4	9	0	9	0	10	10	0	0	0
5	0	0	0	10	0	1	6	3	0
6	0	0	0	10	1	0	6	3	0
7	0	0	0	0	6	6	0	2	8
8	0	0	0	0	3	3	2	0	0
9	0	0	0	0	0	0	8	0	0



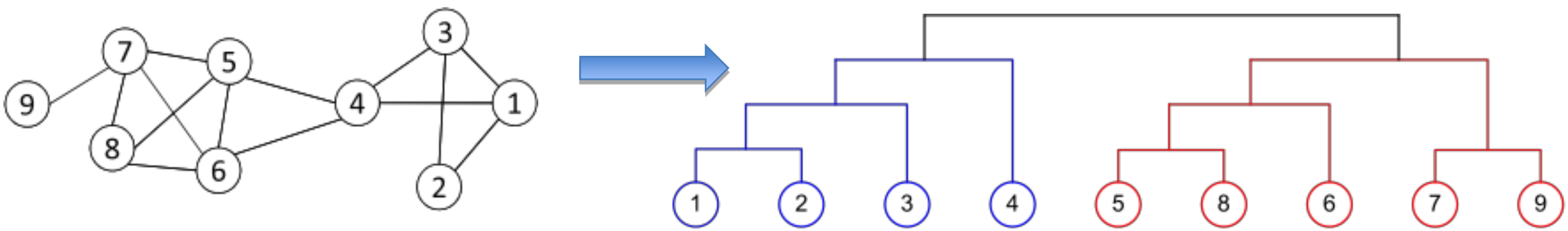
After remove e(4,5), the betweenness of e(4, 6) becomes 20, which is the highest;

After remove e(4,6), the edge e(7,9) has the highest betweenness value 4, and should be removed.

Idea: progressively removing edges with the highest betweenness

Agglomerative Hierarchical Clustering

- Initialize each node as a community
- Merge communities successively into larger communities following a certain criterion
  - E.g., based on modularity increase



Dendrogram according to Agglomerative Clustering based on Modularity

Summary of Community Detection

- Node-Centric Community Detection
  - *cliques, k-cliques, k-clubs*
- Group-Centric Community Detection
  - *quasi-cliques*
- Network-Centric Community Detection
  - *Clustering based on vertex similarity*
  - *Latent space models, block models, spectral clustering, modularity maximization*
- Hierarchy-Centric Community Detection
  - *Divisive clustering*
  - *Agglomerative clustering*

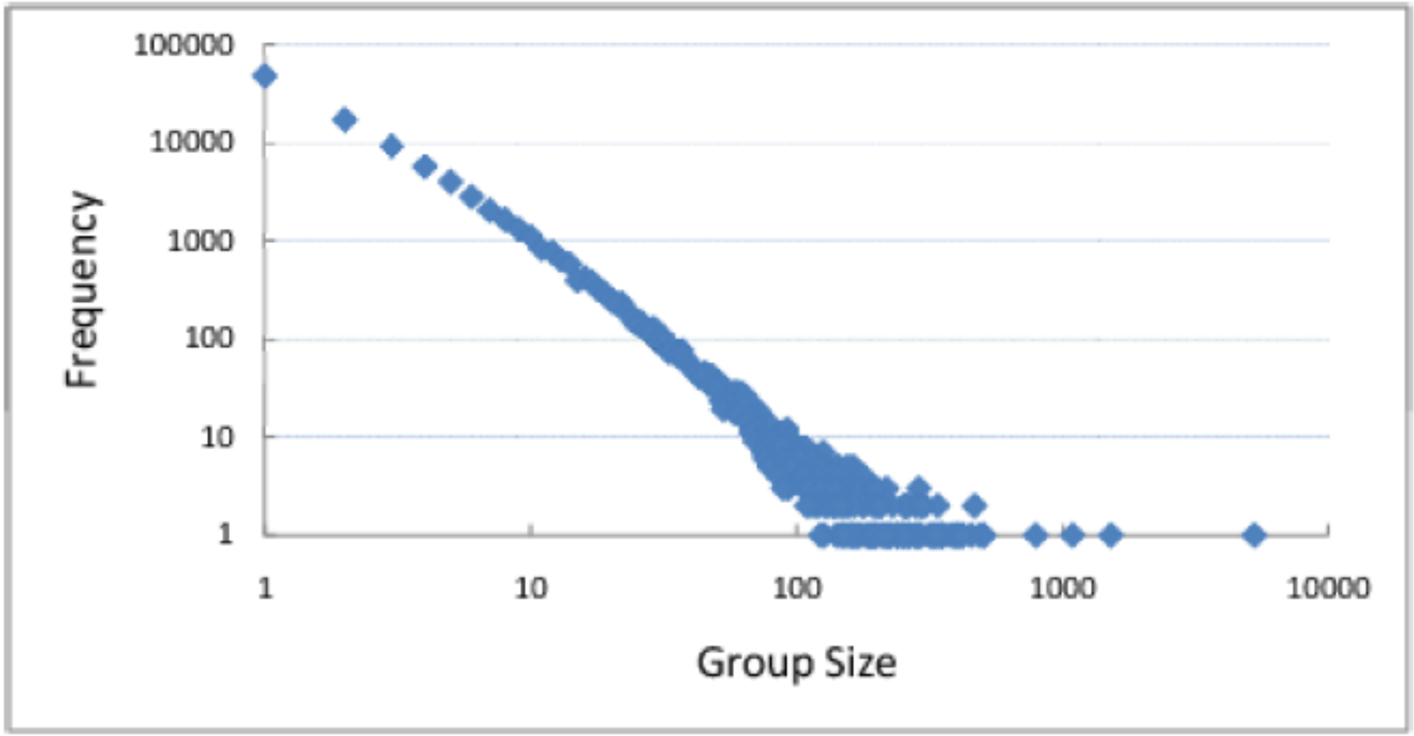
COMMUNITIES IN SOCIAL MEDIA

Questions & Challenges

- Statistical Properties of Communities
  - any structural patterns for community size, density and growth?
- Evolution
  - Timeliness is emphasized in social media
  - Interactions are highly dynamic
- Heterogeneity
  - Various types of entities and interactions are involved
- Evaluation
  - Lack of ground truth, and complete information due to privacy
- Scalability
  - Social networks are often in a scale of millions of nodes and connections
  - Traditional Network Analysis often deals with very limited number of of subjects

Statistical property

Size Distribution of Explicit Communities (LiveJournal)

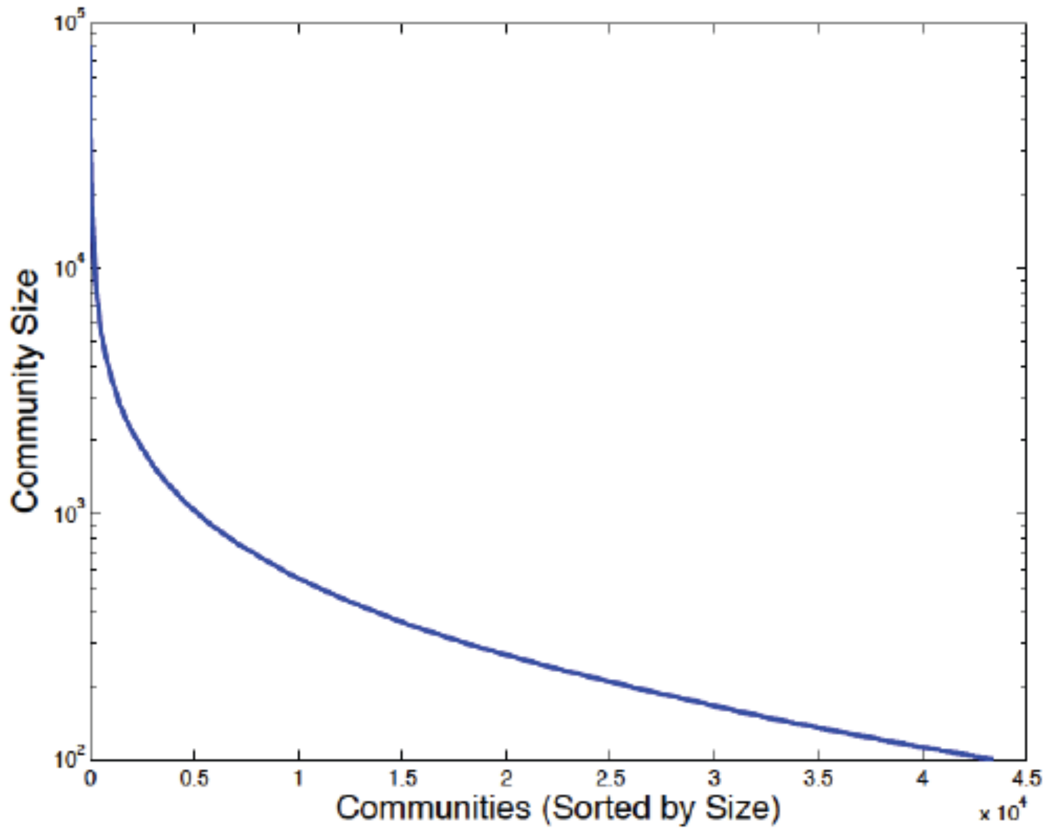


16K bloggers, 132k links, 100k groups

Lei Tang, Xufei Wang and Huan Liu, *Group Profiling for Understanding Social Structures*, TIST, 2011

Statistical property

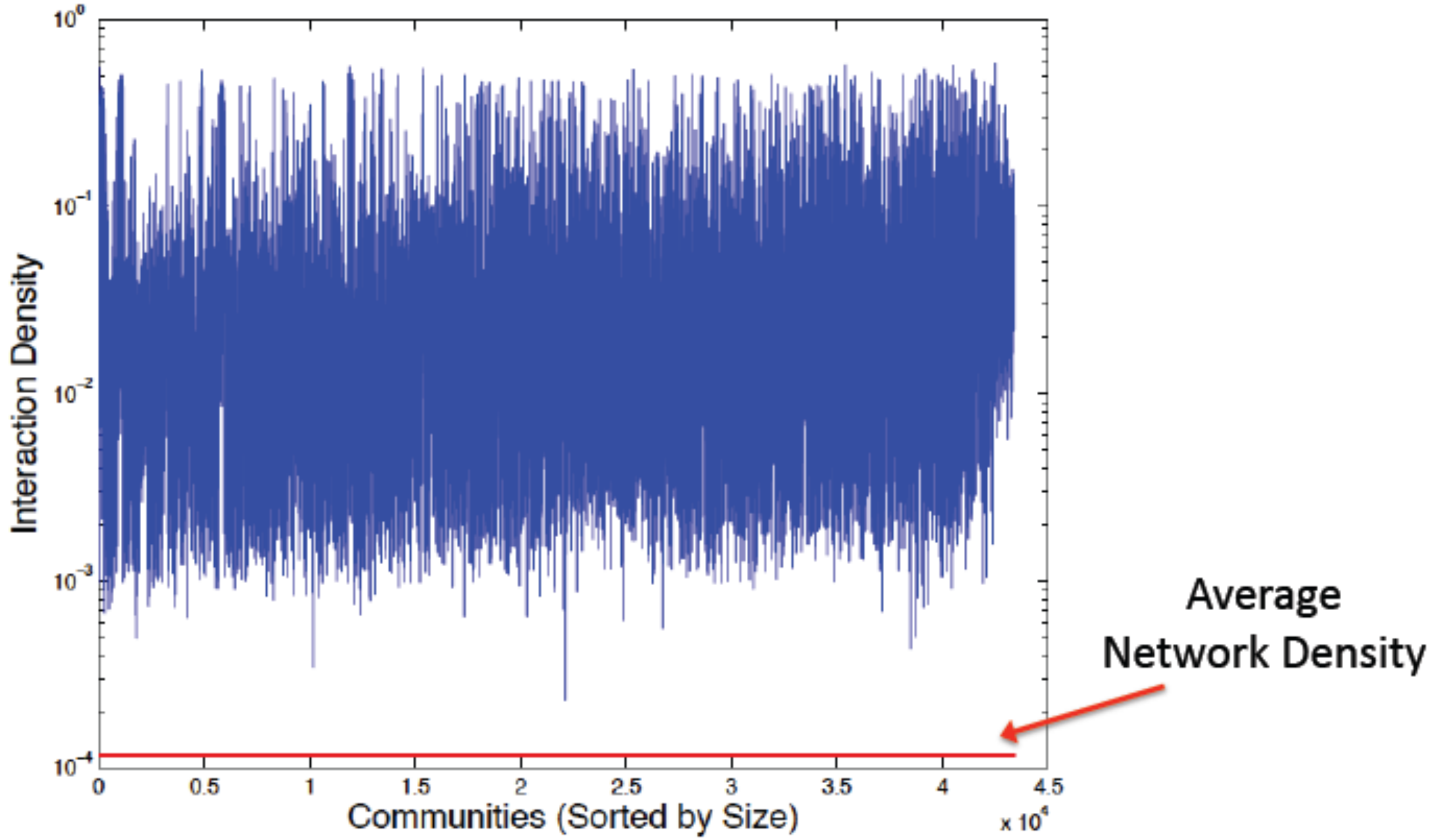
Size Distribution of Explicit Communities (Flickr)



907K users, 43k groups

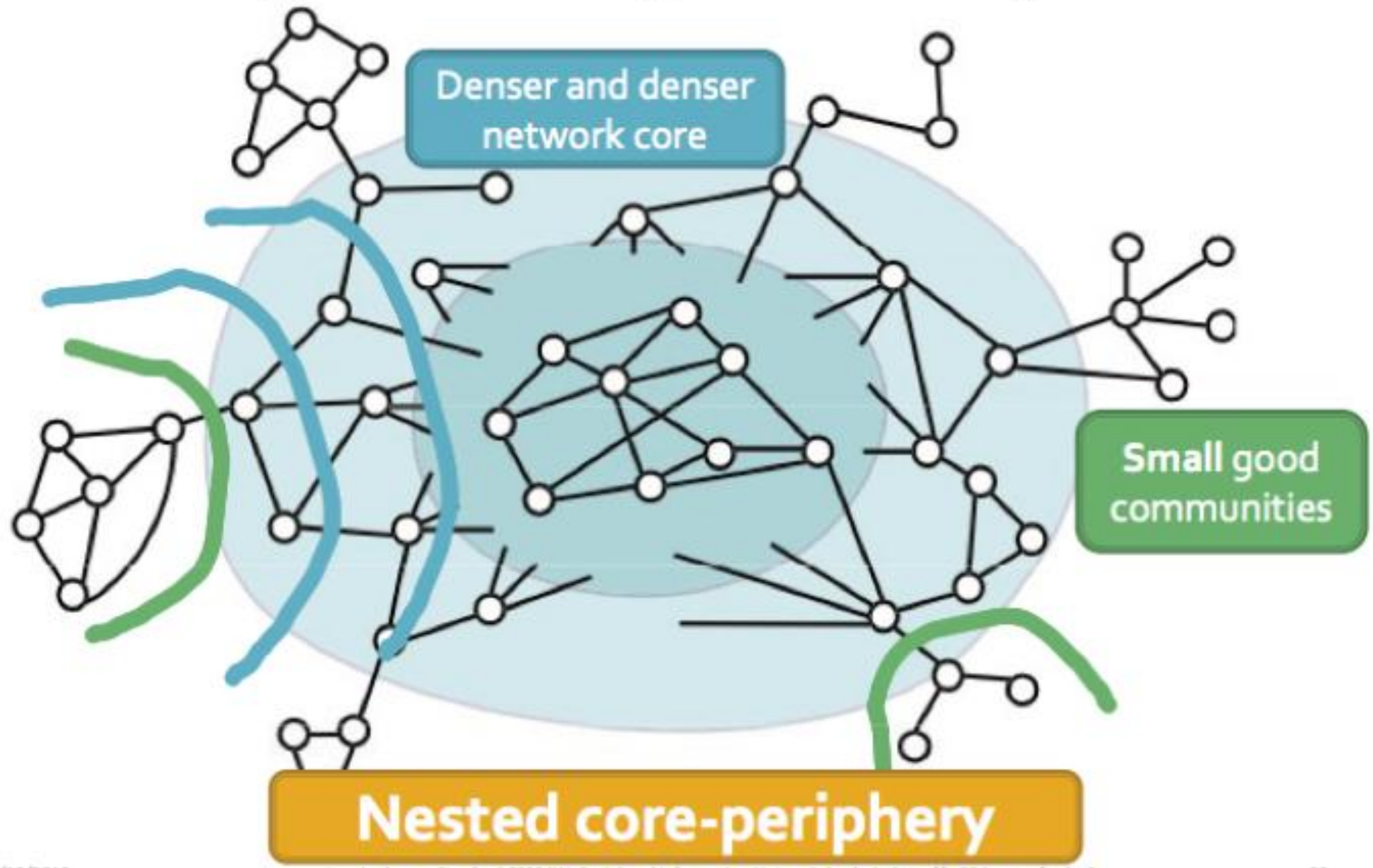
Statistical property

Densities of Flickr Groups



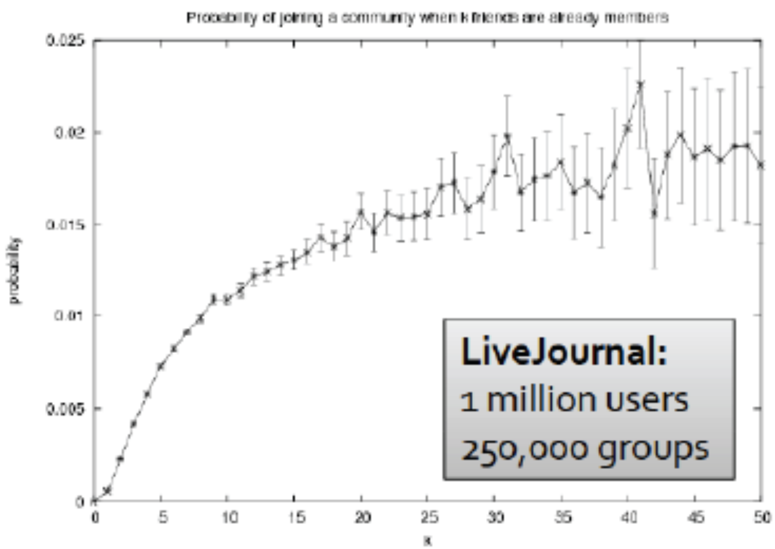
Statistical property

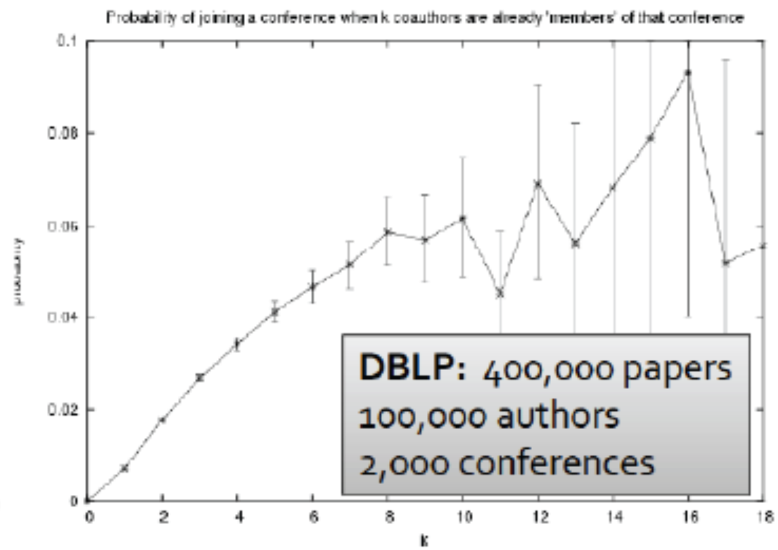
How does a network look like?
Zooming into the giant component



J. Leskovec, K. Lang, A. Dasgupta, M. Mahoney. *Statistical Properties of Community Structure in Large Social and Information Networks*, WWW 2008

Community Growth wrt. #friends





- Diminishing returns
  - Probability of joining increases with the number friends in the group
  - But increases get smaller and smaller

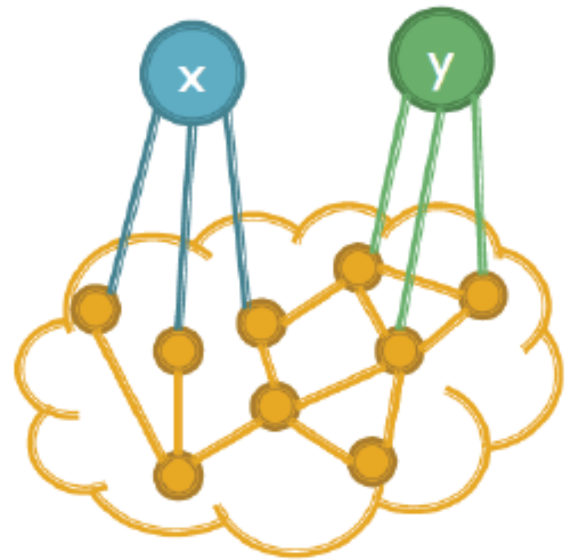
Group Formation in Large Social Networks: Membership, Growth, and Evolution, KDD 2006

Statistical property

Community Growth: More subtle features

- Connectedness of friends:
  - x and y have three friends in the group
  - x's friends are independent
  - y's friends are all connected

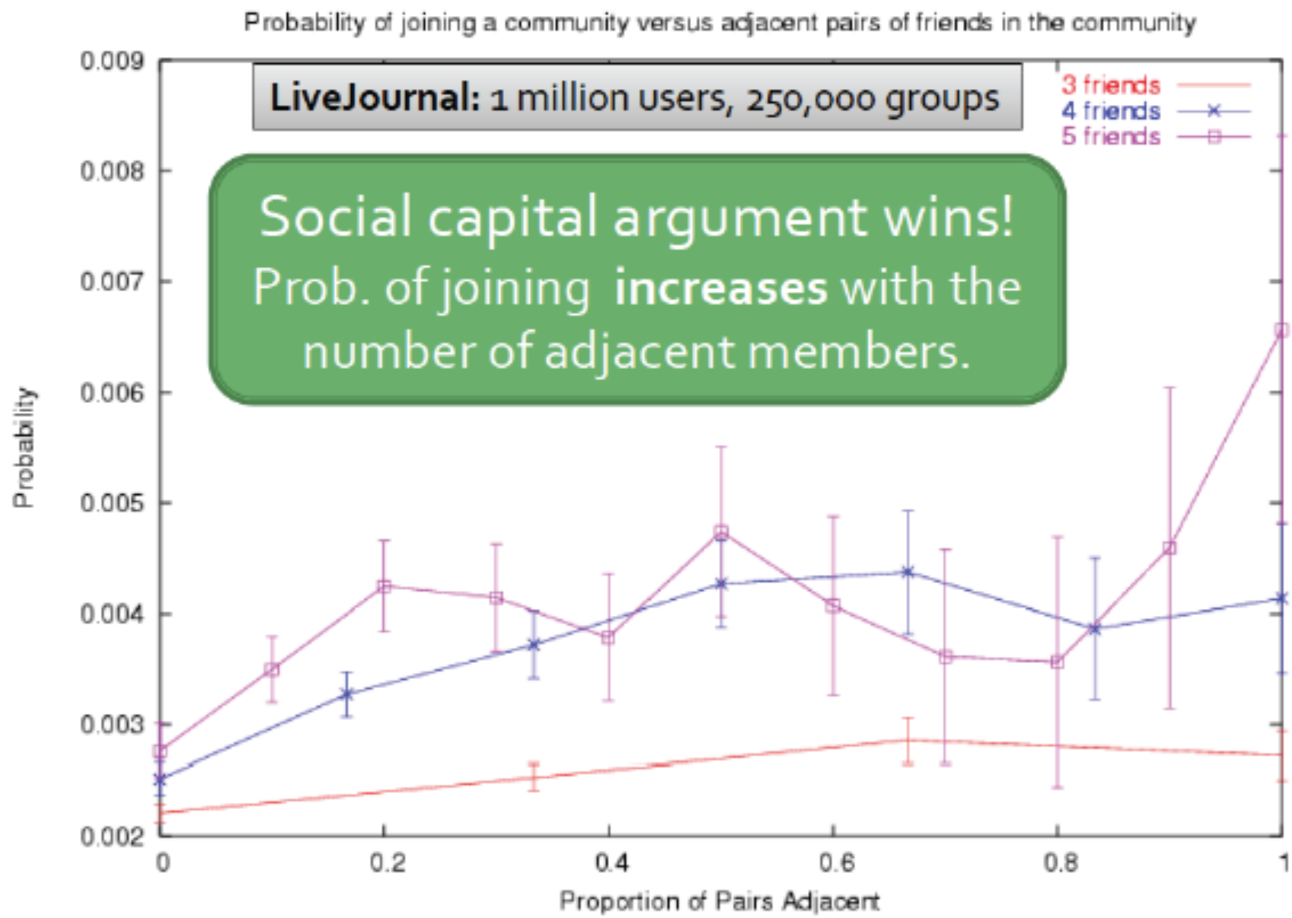
- Who is more likely to join?



Adapted from Jure Leskovec's lecture slides @Stanford

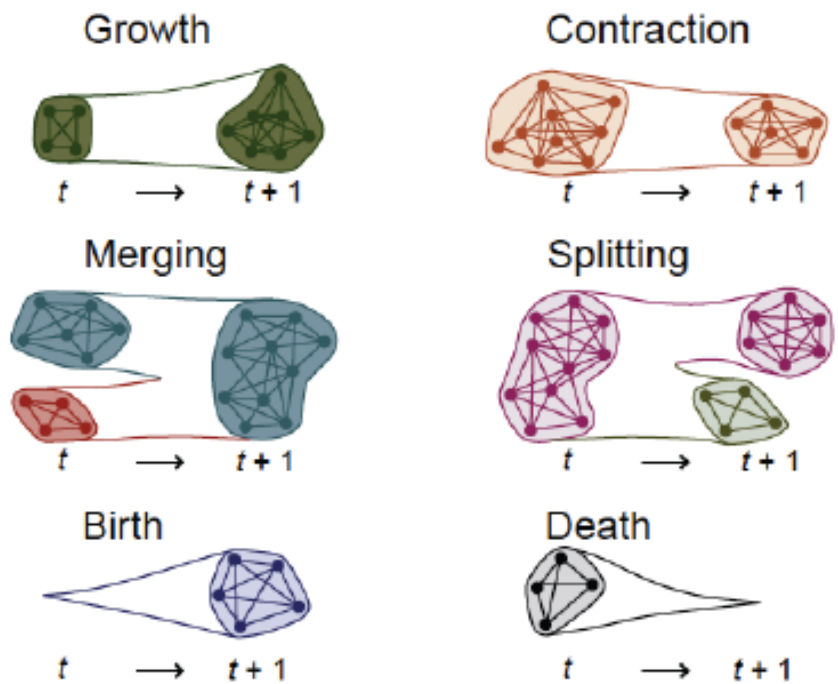
Statistical property

Community Growth wrt. Connectedness of Friends



Community Evolution

- Communities also expand, shrink , or dissolve in dynamic networks

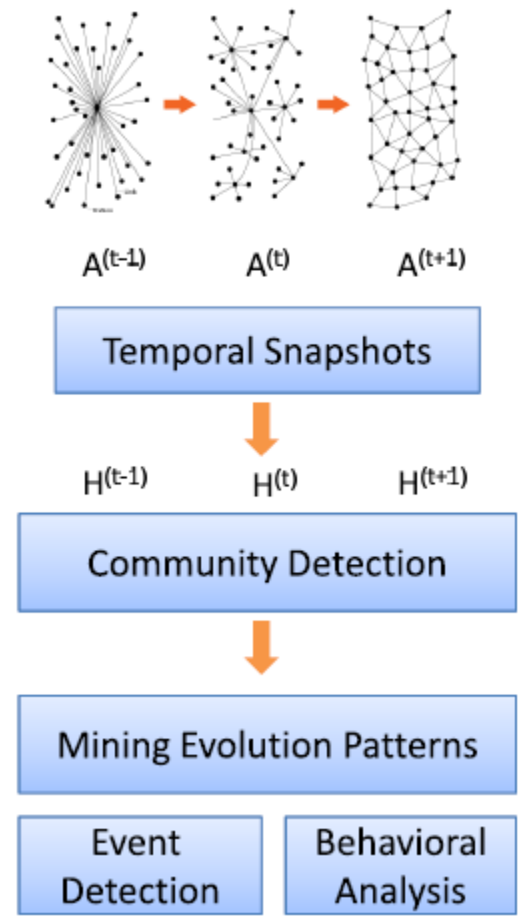


- How to uncover latent community change behind dynamic network interactions?

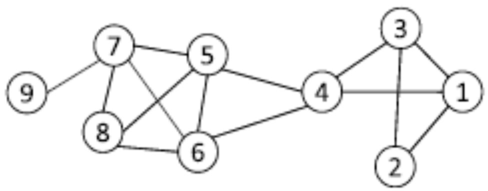
Naive Approach to Studying Community Evolution

- Take snapshots of a network
- find communities at each snapshot
- Clustering independently at each snapshot

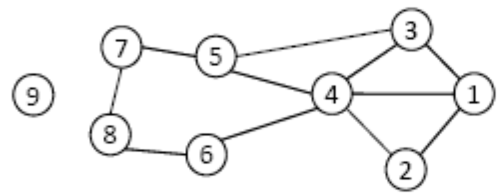
- Cons:
  – Most community detection methods produce local optimal solutions
  – Hard to determine if the evolution is due to the evolution or algorithm randomness



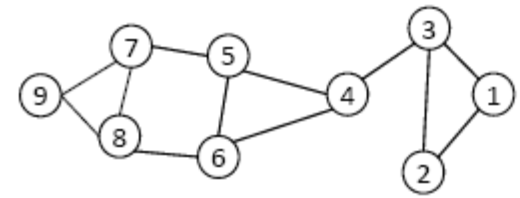
Naïve Approach Example



$A^{(1)}$ at T_1



$A^{(2)}$ at T_2



$A^{(3)}$ at T_3

$$H^{(1)} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \quad H^{(2)} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \quad H^{(3)} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$

- There is a sharp change at T_2
- This approach may report spurious structural changes

Evolutionary Clustering in Smoothly Evolving Networks

- Evolutionary Clustering: find a *smooth* sequence of communities given a series of network snapshots
- Objective function: snapshot cost (CS) + temporal cost (CT)

$$Cost = \alpha \cdot CS + (1-\alpha) \cdot CT$$

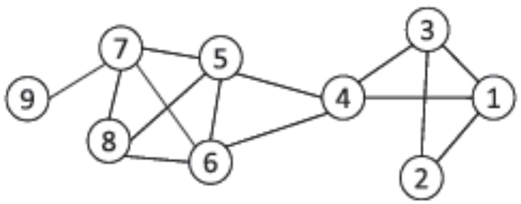
- Take spectral clustering as an example
  - Snapshot cost : $CS_t = Tr(S_t^T L_t S_t), \quad s.t. \quad S_t^T S_t = I_k$
  - Temporal cost: $CT_t = \cancel{\|S_t - S_{t-1}\|^2}$

$$CT_t = \frac{1}{2}\|S_t S_t^T - S_{t-1} S_{t-1}^T\|^2$$

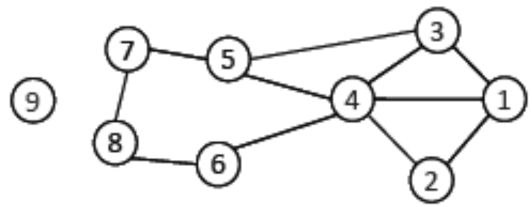
- Community Evolution: $Cost_t = Tr\left[S_t^T \widetilde{L}_t S_t\right]$

  where $\widetilde{L}_t = I - \alpha \cdot D_t^{-1/2} A^{(t)} D_t^{-1/2} - (1-\alpha) \cdot S_{t-1} S_{t-1}^T$

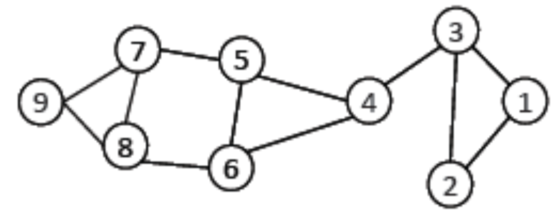
Evolutionary Clustering Example



$A^{(1)}$ at T_1



$A^{(2)}$ at T_2



$A^{(3)}$ at T_3

For T_1

$$S_1 = \begin{bmatrix} 0.33 & -0.44 \\ 0.27 & -0.43 \\ 0.33 & -0.44 \\ 0.38 & -0.16 \\ 0.38 & 0.24 \\ 0.38 & 0.24 \\ 0.38 & 0.38 \\ 0.33 & 0.30 \\ 0.19 & 0.23 \end{bmatrix}$$

For T_2

$$\tilde{L}_2 = \begin{bmatrix} 0.91 & -0.42 & -0.33 & -0.21 & -0.01 & -0.01 & 0.01 & 0.01 & 0.01 \\ -0.42 & 0.92 & -0.08 & -0.27 & 0.00 & 0.00 & 0.02 & 0.01 & 0.01 \\ -0.33 & -0.08 & 0.91 & -0.22 & -0.25 & -0.01 & 0.01 & 0.01 & 0.01 \\ -0.21 & -0.27 & -0.22 & 0.95 & -0.18 & -0.24 & -0.02 & -0.02 & -0.01 \\ -0.01 & 0.00 & -0.25 & -0.18 & 0.94 & -0.06 & -0.37 & -0.06 & -0.04 \\ -0.01 & 0.00 & -0.01 & -0.24 & -0.06 & 0.94 & -0.07 & -0.45 & -0.04 \\ 0.01 & 0.02 & 0.01 & -0.02 & -0.37 & -0.07 & 0.91 & -0.44 & -0.05 \\ 0.01 & 0.01 & 0.01 & -0.02 & -0.06 & -0.45 & -0.44 & 0.94 & -0.04 \\ 0.01 & 0.01 & 0.01 & -0.01 & -0.04 & -0.04 & -0.05 & -0.04 & 0.97 \end{bmatrix}$$

We obtain two communities based on spectral clustering with this modified graph Laplacian: {1, 2, 3, 4} and {5, 6, 7, 8, 9}

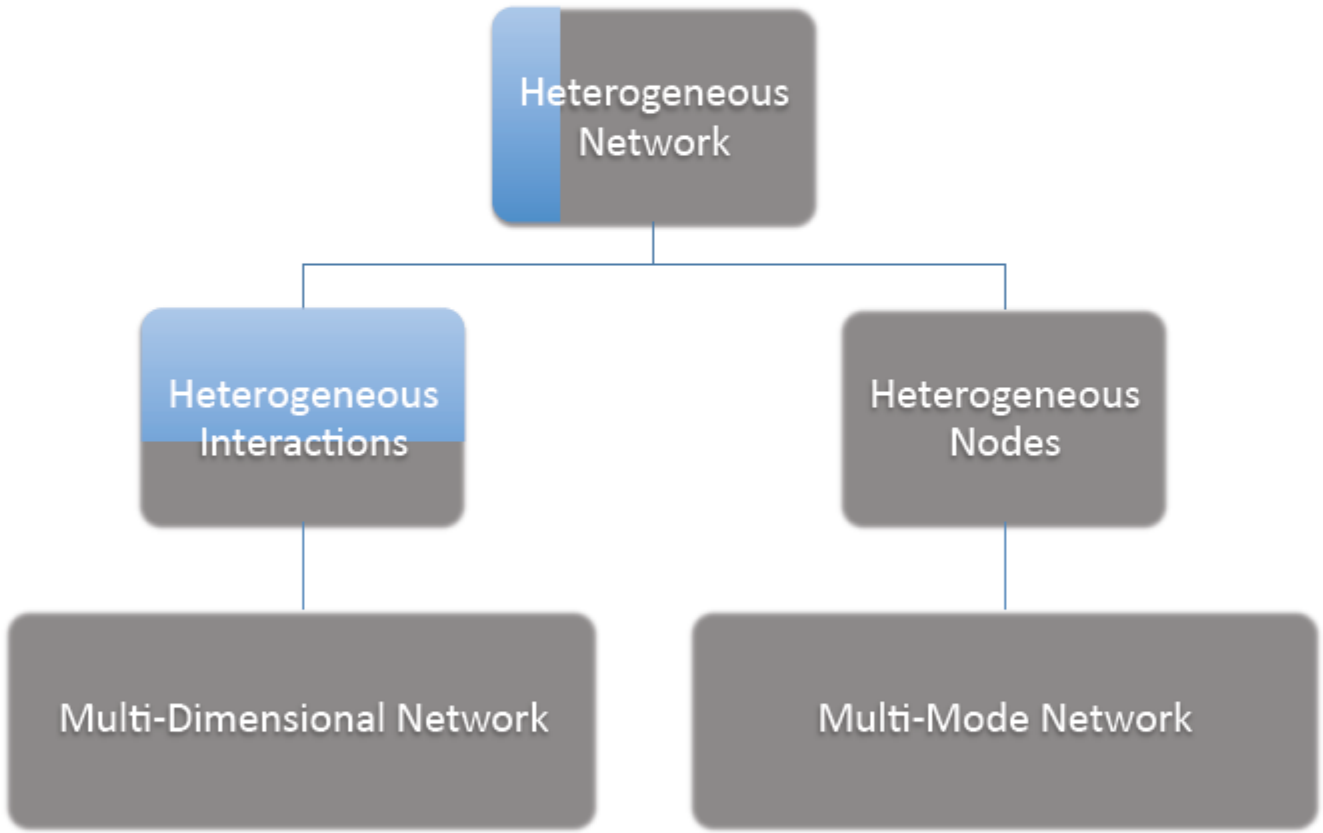
Segment-based Clustering with Evolving Networks

- Independent clustering at each snapshot
  - do not consider temporal information
  - Likely to output specious evaluation patterns
- Evolutionary clustering enforces smoothness
  - may fail to capture drastic change
- How to strike balance between *gradual changes* under normal circumstances and *drastic changes* caused by major events?
- Segment-based clustering:
  - Community structure remains unchanged in a segment of time
  - A change between consecutive segments
- Fundamental question: how to detect the change points?

Segment-based Clustering

- Segment-based Clustering assumes community structure remains unchanged in a segment of time
- GraphScope is one segment-based clustering method
  - If network connections do not change much over time, consecutive network snapshots should be grouped into one segment
  - If a new network snapshot does not fit into an existing segment (when current community structure induces a high cost on a new network snapshot), then introduce a change point and start a new segment

Heterogeneous Networks



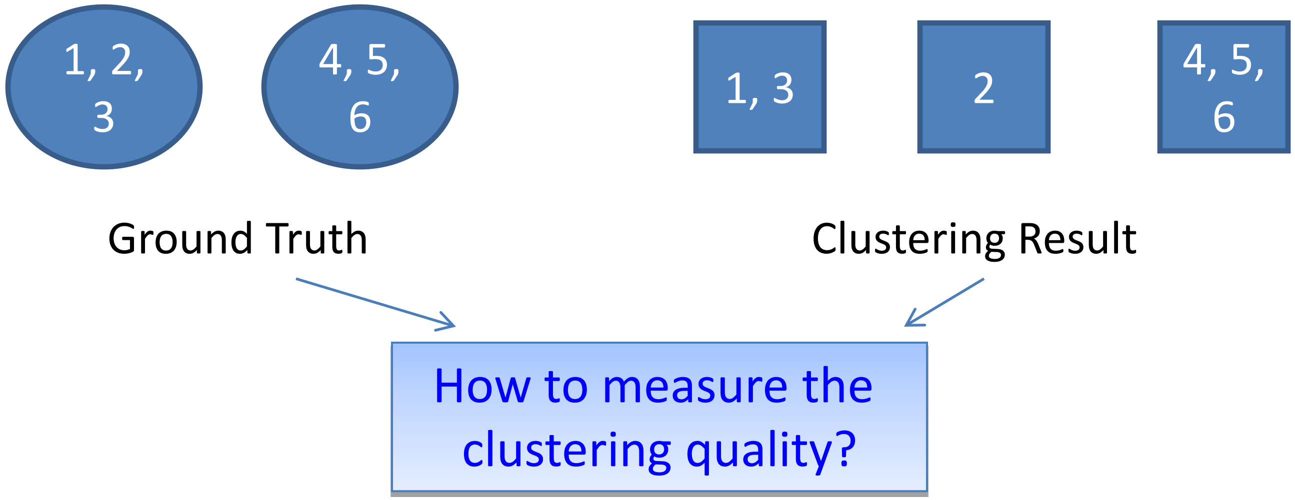
Why Does Heterogeneity Matter

- Social media introduces heterogeneity
- It calls for solutions to community detection in heterogeneous networks
  - Interactions in social media are noisy
  - Interactions in one mode or one dimension might be too noisy to detect meaningful communities
  - Not all users are active in all dimensions or with different modes
- Need integration of interactions at multiple dimensions or modes
- Details skipped due to time limit (check out chapter 4 of the lecture book)

Evaluating Community Detection (1)

- For groups with clear definitions
  - E.g., Cliques, k-cliques, k-clubs, quasi-cliques
  - Verify whether extracted communities satisfy the definition
- For networks with ground truth information
  - Normalized mutual information
  - Accuracy of pairwise community memberships

Measuring a Clustering Result



- The number of communities after grouping can be different from the ground truth
- No clear community <u>correspondence</u> between clustering result and the ground truth
- Normalized Mutual Information can be used

Normalized Mutual Information

- Entropy: the information contained in a distribution
$$H(X) = \sum_{x \in X} p(x) \log p(x)$$
- Mutual Information: the shared information between two distributions
$$I(X;Y) = \sum_{y \in Y} \sum_{x \in X} p(x,y) \log\left(\frac{p(x,y)}{p_1(x)p_2(y)}\right)$$
- Normalized Mutual Information (between 0 and 1)

$NMI(X;Y) = \frac{I(X;Y)}{\sqrt{H(X)H(Y)}}$ JMLR03, Strehl or $NMI(X;Y) = \frac{2I(X;Y)}{H(X)+H(Y)}$ KDD04, Dhilon

- Consider a partition as a distribution (probability of one node falling into one community), we can compute the matching between the clustering result and the ground truth

NMI

$$H(X) = \sum_{x \in X} p(x) \log p(x)$$

$$H(\pi^a) = \sum_{h}^{k^{(a)}} \frac{n_h^a}{n} \log(\frac{n_h^a}{n})$$

$$H(\pi^b) = \sum_{\ell}^{k^{(b)}} \frac{n_\ell^b}{n} \log(\frac{n_\ell^b}{n})$$

$$I(X;Y) = \sum_{y \in Y} \sum_{x \in X} p(x,y) \log \left(\frac{p(x,y)}{p_1(x) p_2(y)} \right)$$

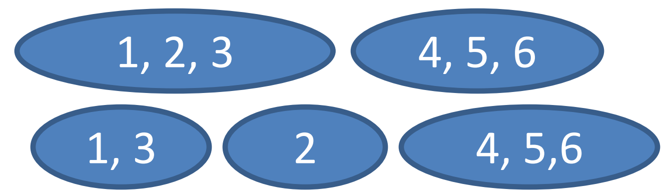
$$I(\pi^a, \pi^b) = \sum_{h} \sum_{\ell} \frac{n_{h,\ell}}{n} \log \left(\frac{\frac{n_{h,\ell}}{n}}{\frac{n_h^a}{n} \frac{n_\ell^b}{n}} \right)$$

$$NMI(X;Y) = \frac{I(X;Y)}{\sqrt{H(X)H(Y)}}$$

$$NMI(\pi^a, \pi^b) = \frac{\sum_{h=1}^{k^{(a)}} \sum_{\ell=1}^{k^{(b)}} n_{h,\ell} \log \left(\frac{n \cdot n_{h,l}}{n_h^{(a)} \cdot n_\ell^{(b)}} \right)}{\sqrt{\left(\sum_{h=1}^{k^{(a)}} n_h^{(a)} \log \frac{n_h^a}{n} \right) \left(\sum_{\ell=1}^{k^{(b)}} n_\ell^{(b)} \log \frac{n_\ell^b}{n} \right)}}$$

NMI-Example

- Partition a: [1, 1, 1, 2, 2, 2]
- Partition b: [1, 2, 1, 3, 3, 3]



$$n = 6$$
$$k^{(a)} = 2$$
$$k^{(b)} = 3$$

	n_h^a
h=1	3
h=2	3

	n_l^b
l=1	2
l=2	1
l=3	3

$n_{h,l}$	l=1	l=2	l=3
h=1	2	1	0
h=2	0	0	3

contingency table or confusion matrix

$$NMI(\pi^a, \pi^b) = \frac{\sum_{h=1}^{k^{(a)}} \sum_{\ell=1}^{k^{(b)}} n_{h,\ell} \log\left(\frac{n \cdot n_{h,l}}{n_h^{(a)} \cdot n_\ell^{(b)}}\right)}{\sqrt{\left(\sum_{h=1}^{k^{(a)}} n_h^{(a)} \log \frac{n_h^a}{n}\right)\left(\sum_{\ell=1}^{k^{(b)}} n_\ell^{(b)} \log \frac{n_\ell^b}{n}\right)}} = 0.8278$$

Reference: http://www.cse.ust.hk/~weikep/notes/NormalizedMI.m

Accuracy of Pairwise Community Memberships

- Consider all the possible pairs of nodes and check whether they reside in the same community
- An error occurs *if*
  - Two nodes belonging to the same community are assigned to different communities after clustering
  - Two nodes belonging to different communities are assigned to the same community
- Construct a contingency table or confusion matrix

		Ground Truth	
		$C(v_i) = C(v_j)$	$C(v_i) \neq C(v_j)$
Clustering Result	$C(v_i) = C(v_j)$	a	b
	$C(v_i) \neq C(v_j)$	c	d

$$accuracy = \frac{a+d}{a+b+c+d} = \frac{a+d}{n(n-1)/2}$$

Accuracy Example

Ground Truth

Clustering Result

		Ground Truth	
		$C(v_i) = C(v_j)$	$C(v_i) \mathrel{!=} C(v_j)$
Clustering Result	$C(v_i) = C(v_j)$	4	0
	$C(v_i) \mathrel{!=} C(v_j)$	2	9

Accuracy = (4+9)/ (4+2+9+0) = 13/15

Evaluation using Semantics

- For networks with semantics
  - Networks come with semantic or attribute information of nodes or connections
  - Human subjects can verify whether the extracted communities are coherent
- Evaluation is qualitative
- It is also intuitive and helps understand a community

An *animal* community

A *health* community

Evaluation without Ground Truth

- For networks without ground truth or semantic information
- This is the most common situation
- An option is to resort to cross-validation
  - Extract communities from a (training) network
  - Evaluate the quality of the community structure on a network constructed from a different date or based on a related type of interaction
- Quantitative evaluation functions
  - Modularity (M.Newman. Modularity and community structure in networks. PNAS 06.)
  - Link prediction (the predicted network is compared with the true network)

Scaling Community Detection

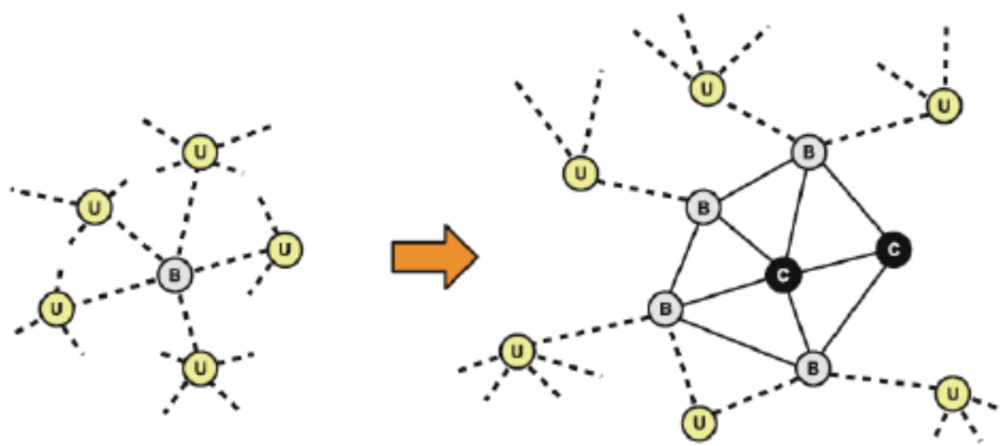
- Social media networks are huge. How to scale community detection methods?
  - Approximation
    - Sampling
    - Local graph processing
    - Multi-level methods
  - Exact
    - Streaming/Iterative schemes
    - Distributed and Parallel processing

Sampling

- Downsample the network such that classical community detection methods can be applied
  - Sample nodes (or edges)
- Which nodes should we sample?
  - A simple heuristic: keep those high-degree nodes
    - Node degrees follows a power-law distribution
    - Communities might form around those popular ones
  - Uncover the community membership for remaining nodes
    - Check if any of his friends has been assigned to any community
  - This simple heuristic works reasonably well
- Optimization may achieve better approximation

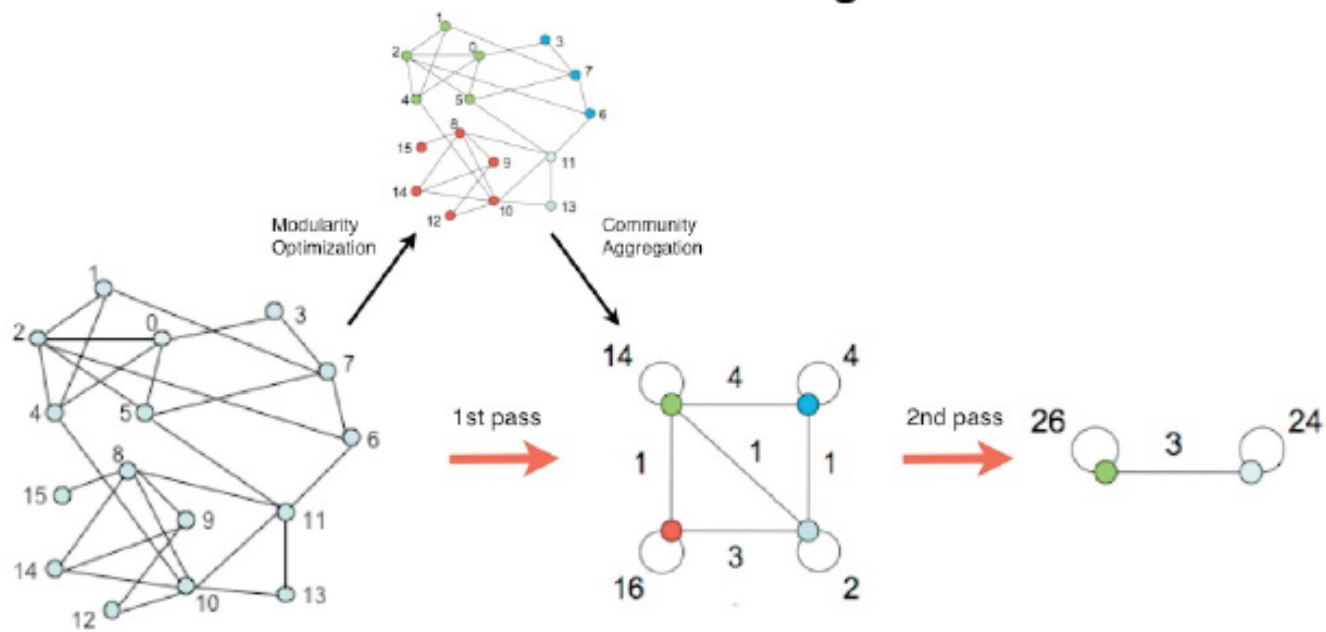
Local graph processing

- Circumvent the memory bottleneck
  - Rather than computing global quality, check local neighborhood (say, k-hop) for computation
    - Compute edge-betweenness
  - Construct communities from seeded nodes



Multi-Level Approaches

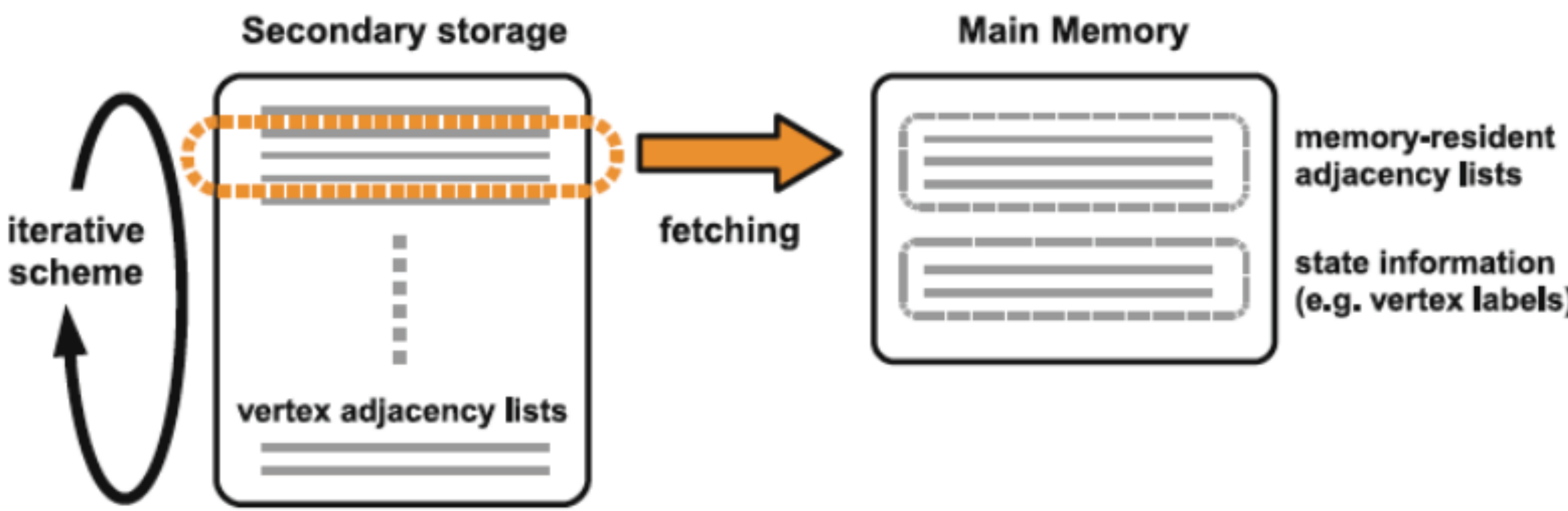
- Find a rough partition of the network into communities by use of a fast process (may at the expense of accuracy)
- Construct a meta network with nodes being communities and edges the connection between communities
- Meta-network is much smaller than the original network



http://sites.google.com/site/findcommunities/

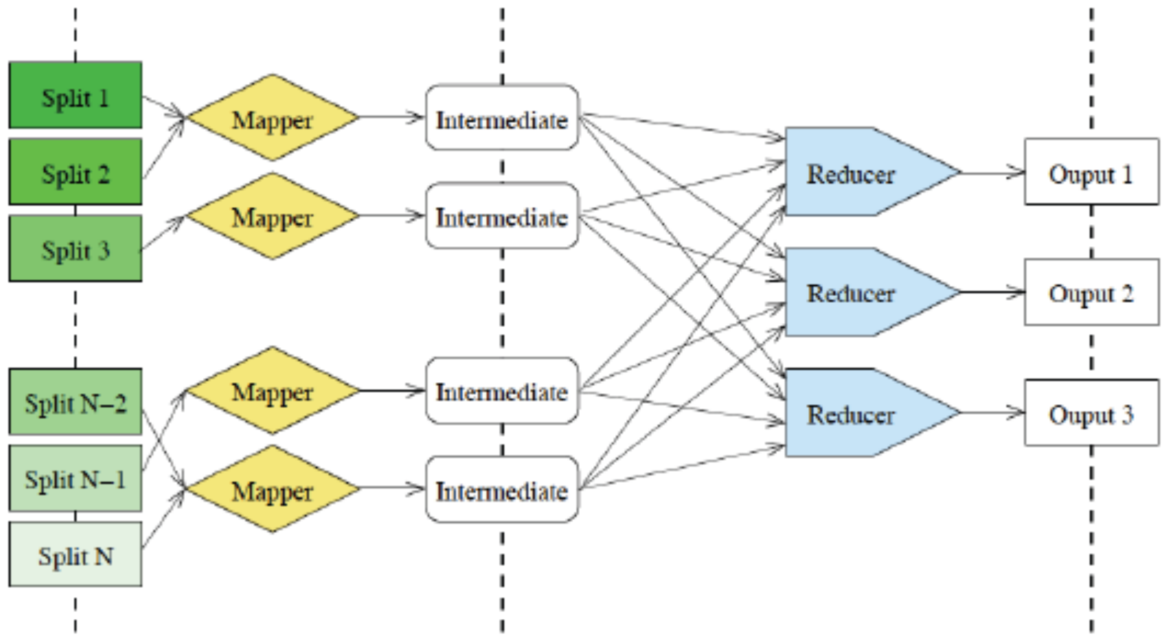
Streaming/Iterative schemes

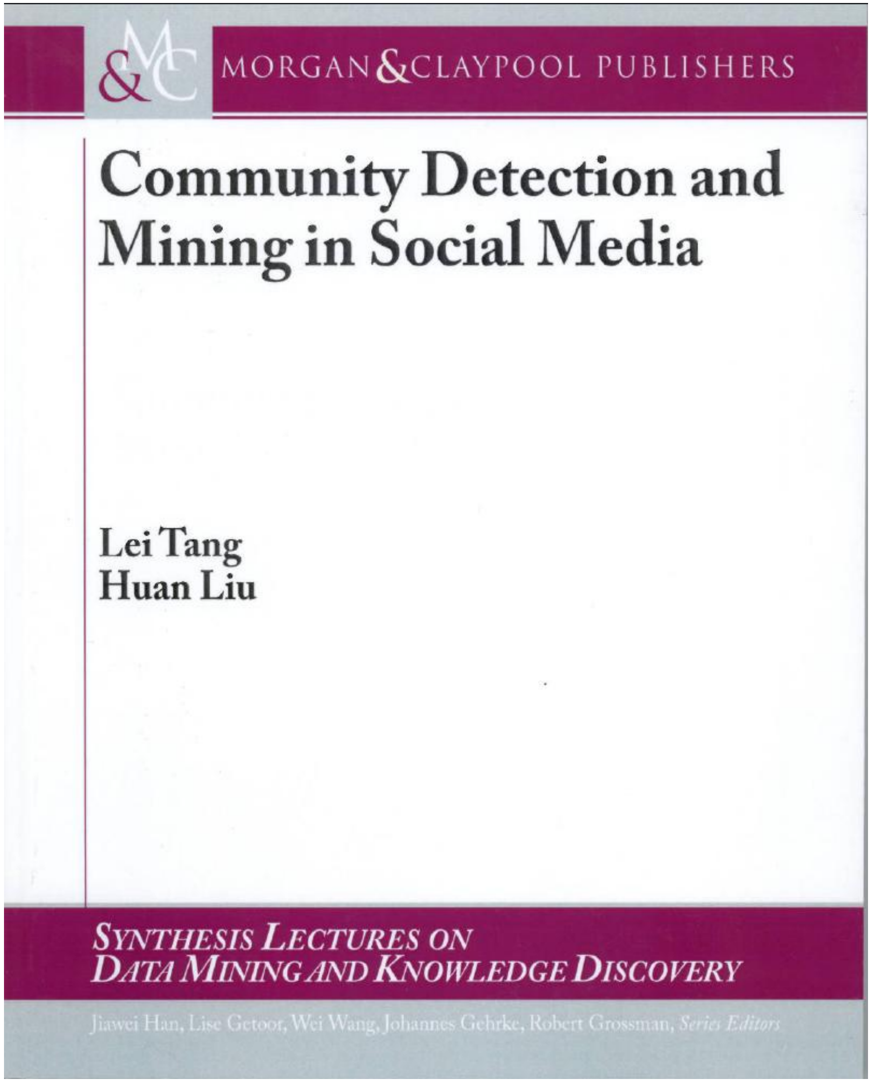
- An iterative process examines each vertex along with its neighbor in a given order and perform computation
- Repeat vertex iteration until convergence



Distributed/Parallel Computing

- Exploit the power of parallel computing
  - Extend community detection methods to Hadoop MapReduce
  - Typically requires multiple iterations as well
  - Move computation to data (split into N chunks)





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