DeepLearning.AI

Optimization in Neural Networks and Newton's Method

Gradient Descent and Backpropagation

Back Propagation Introduction

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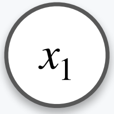

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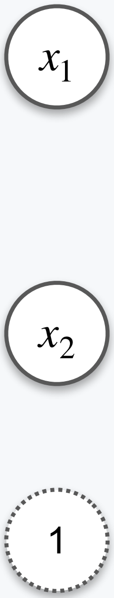

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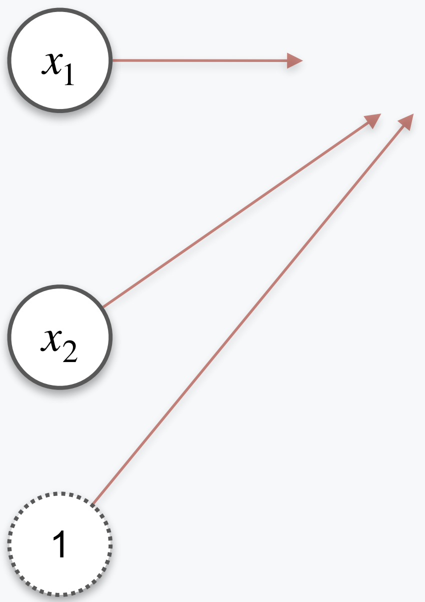

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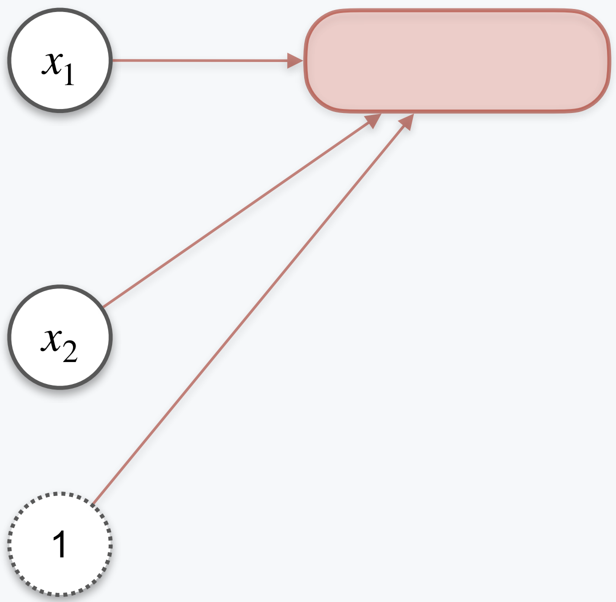

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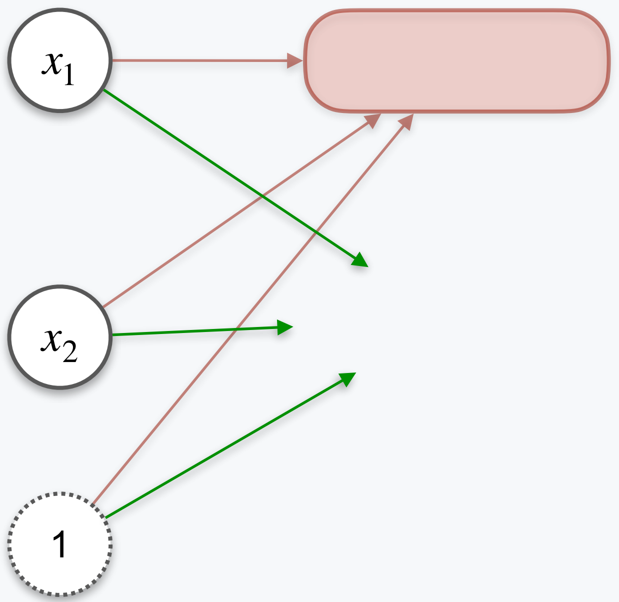

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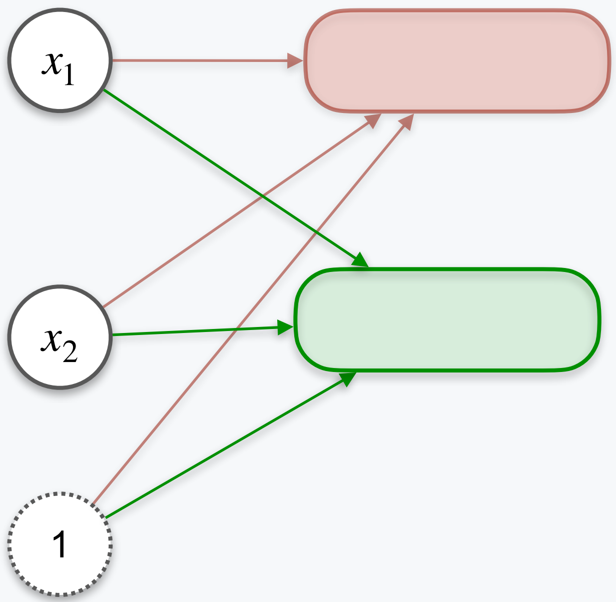

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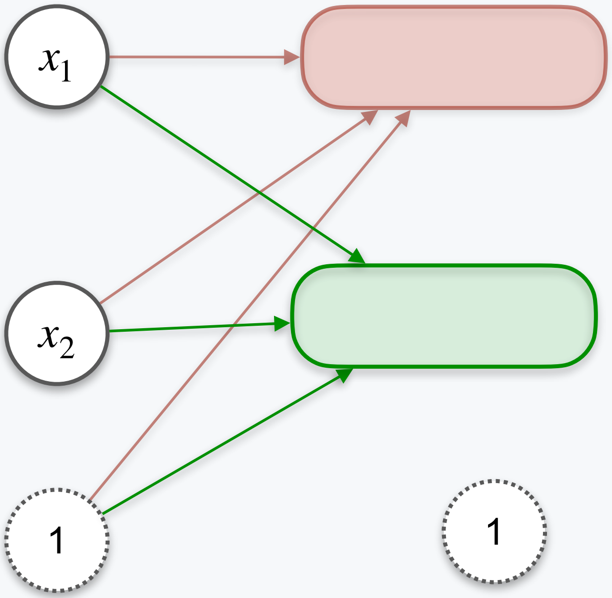

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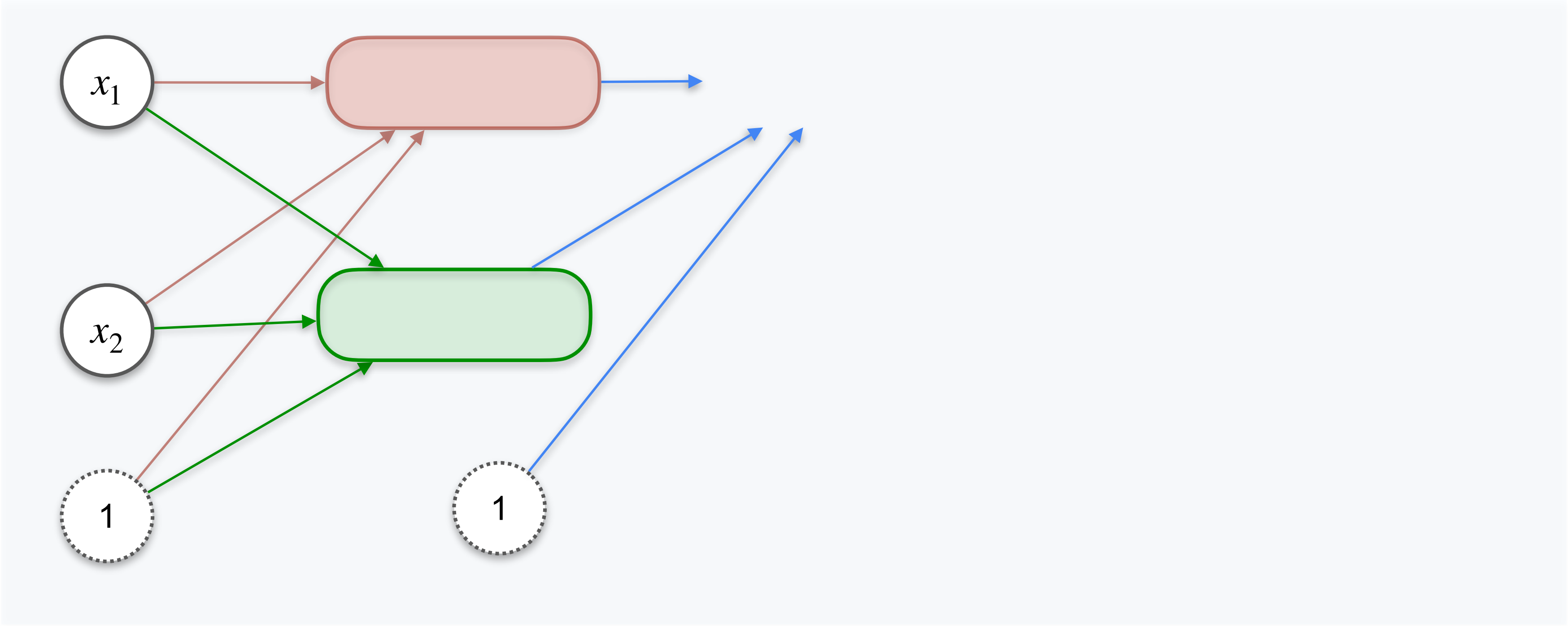

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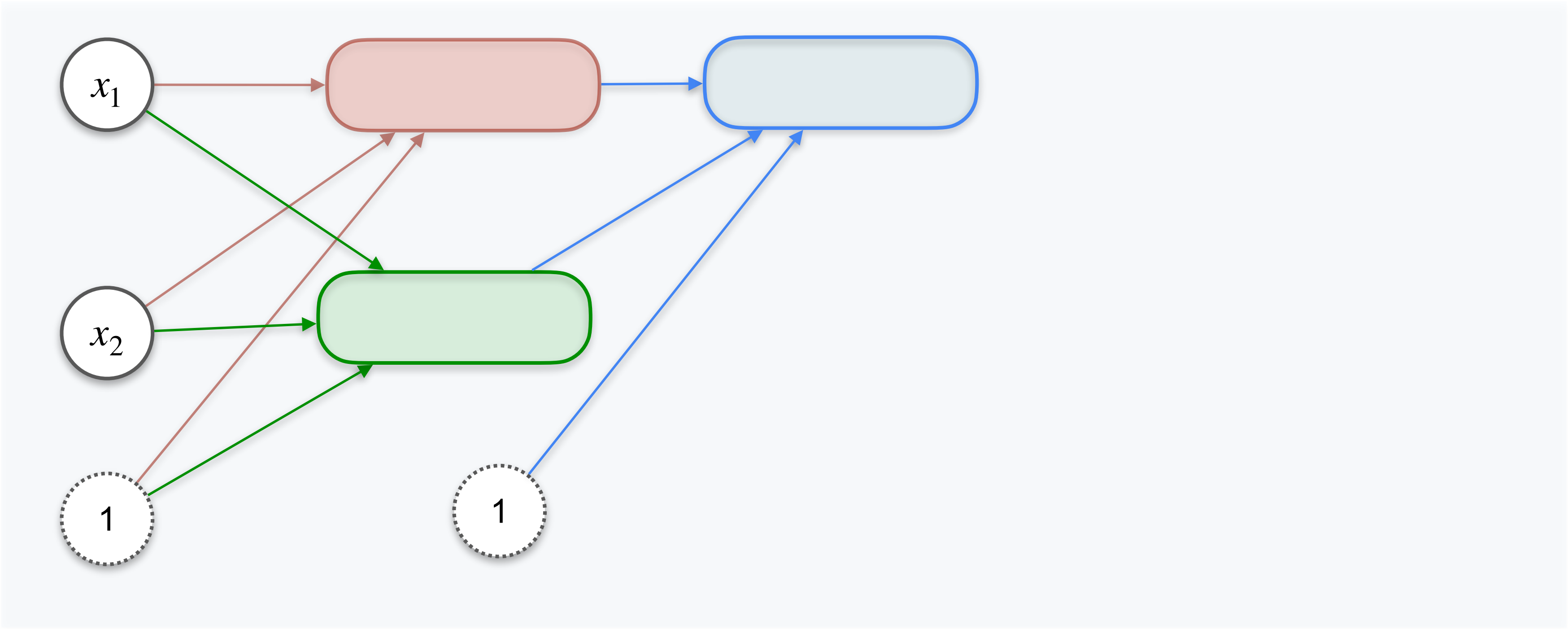

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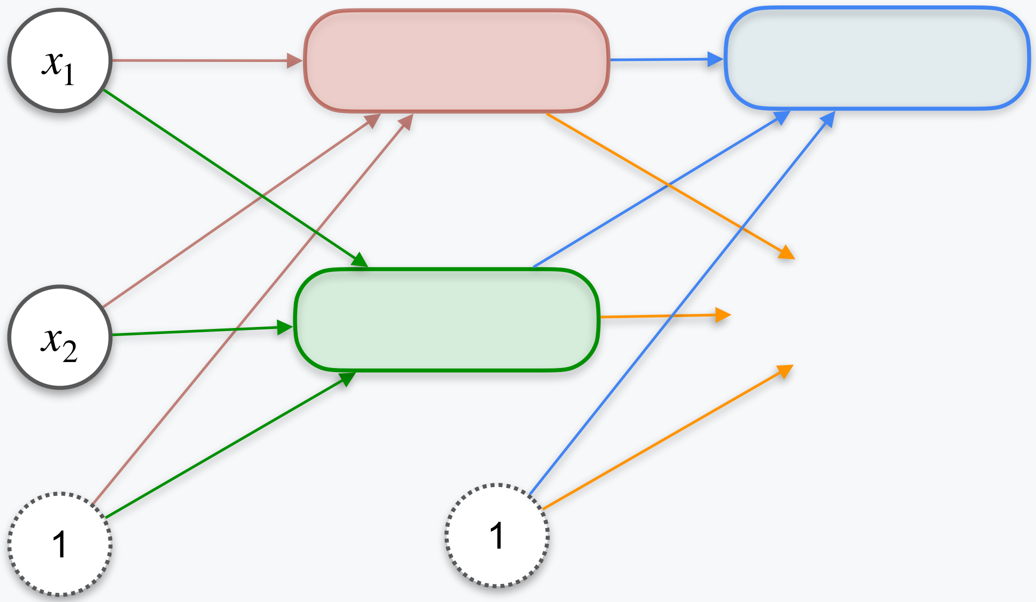

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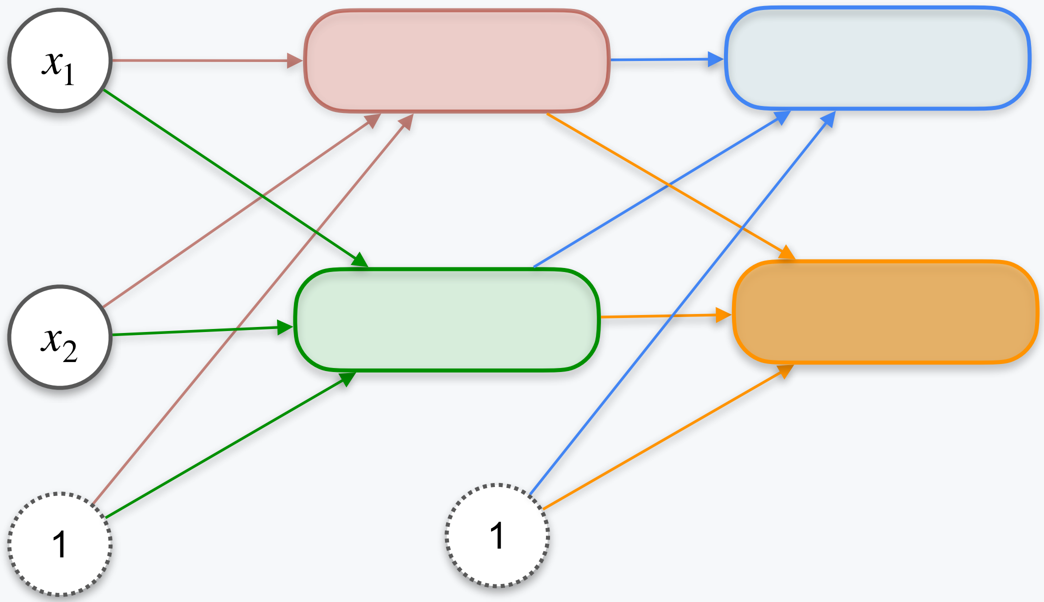

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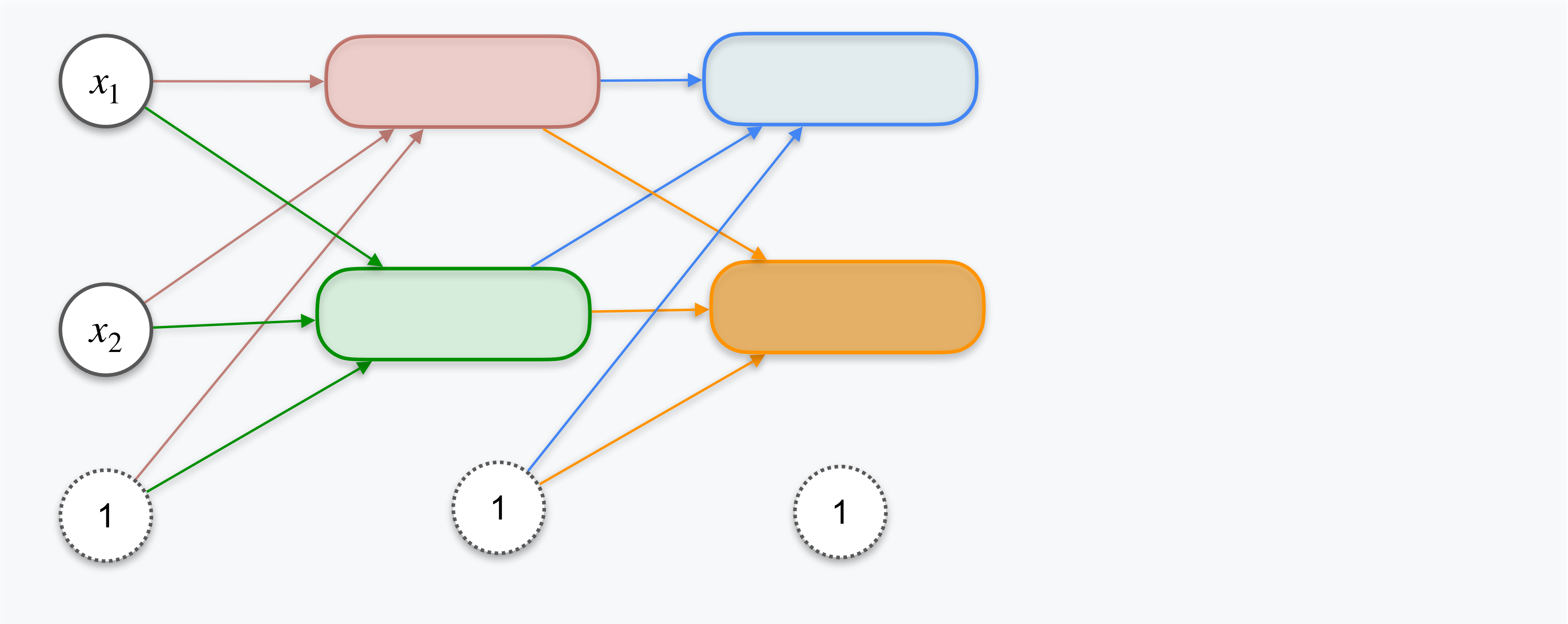

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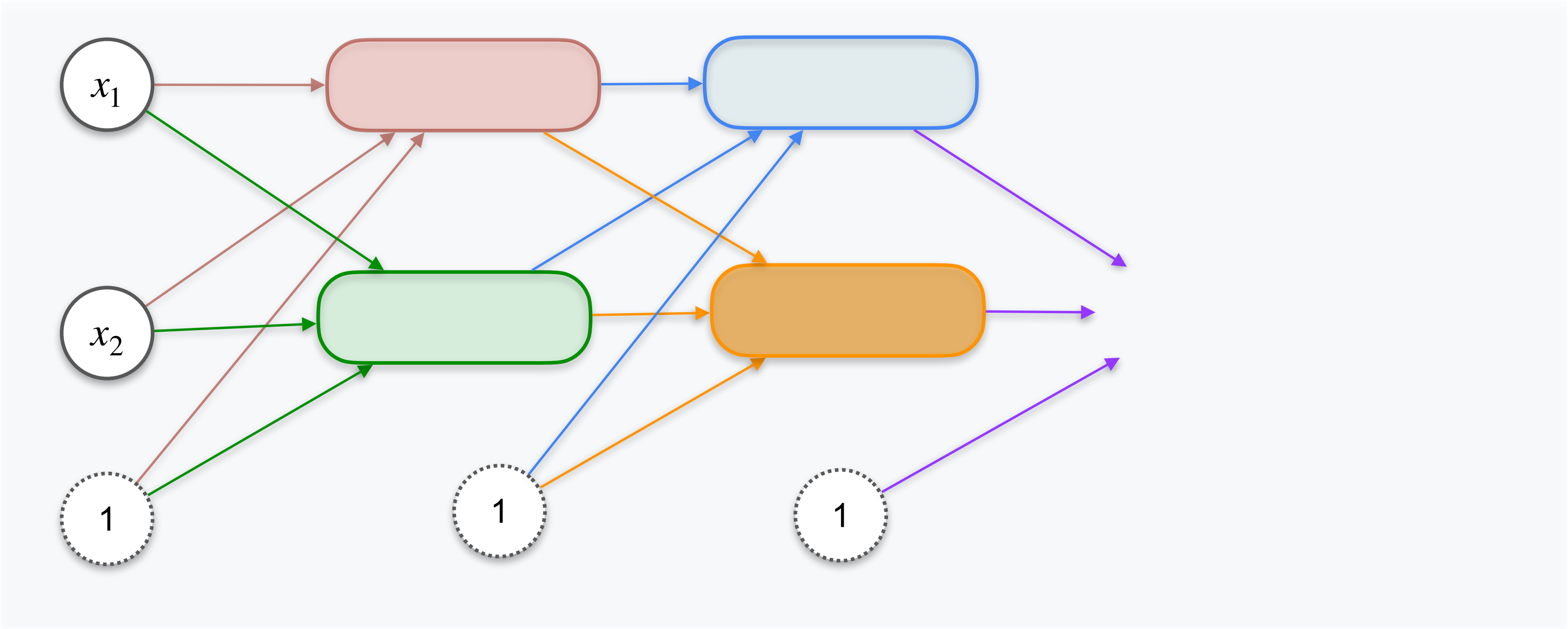

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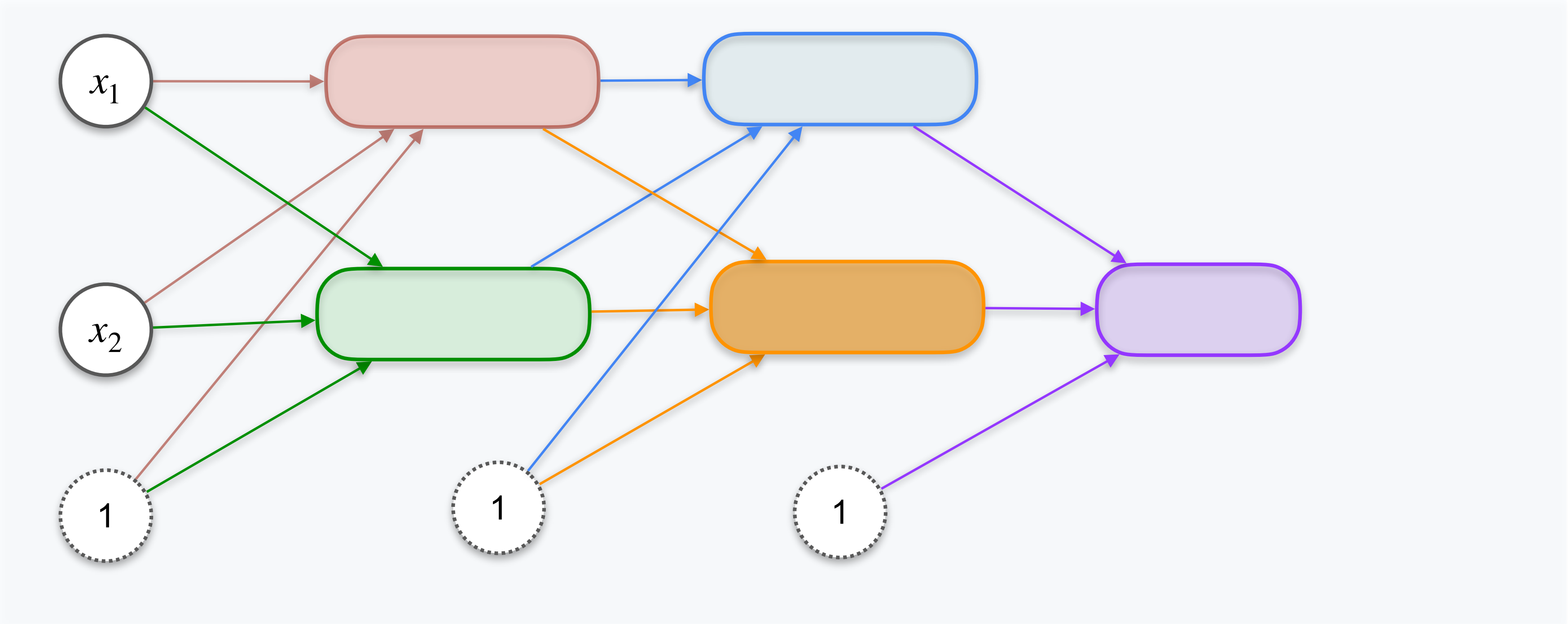

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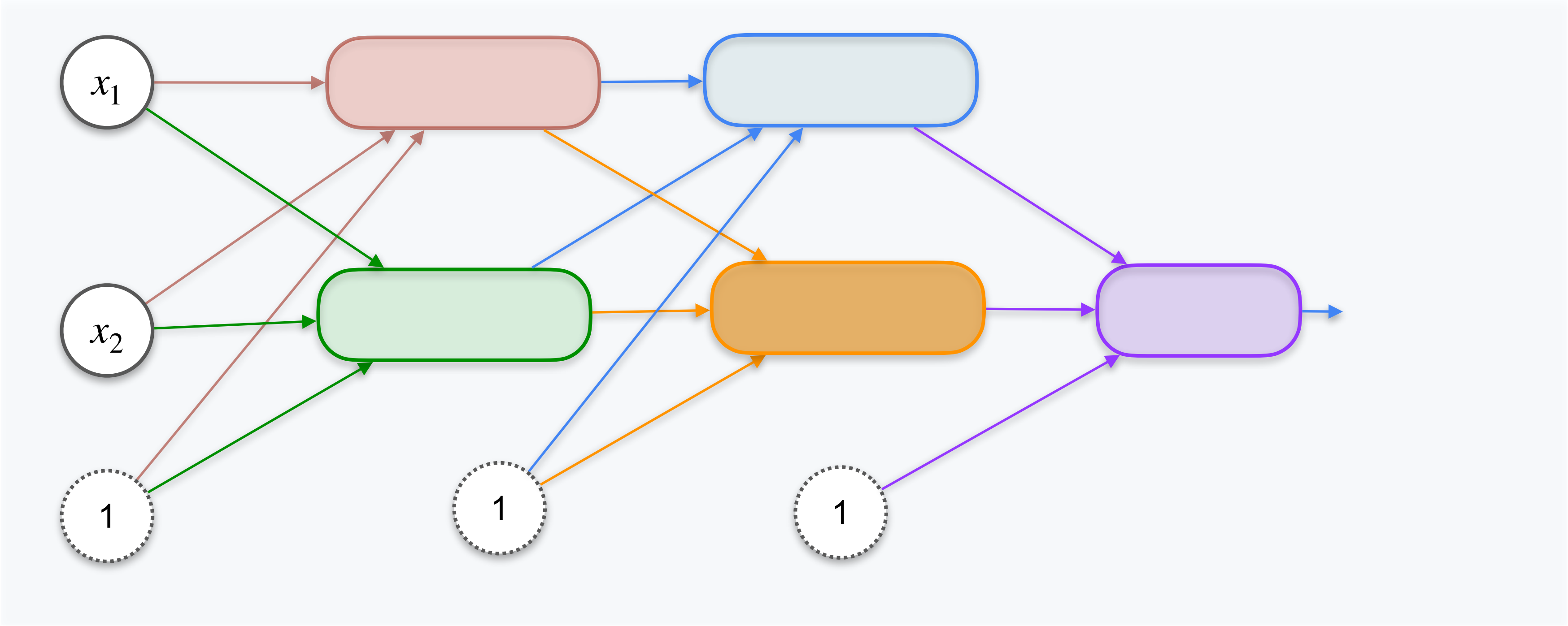

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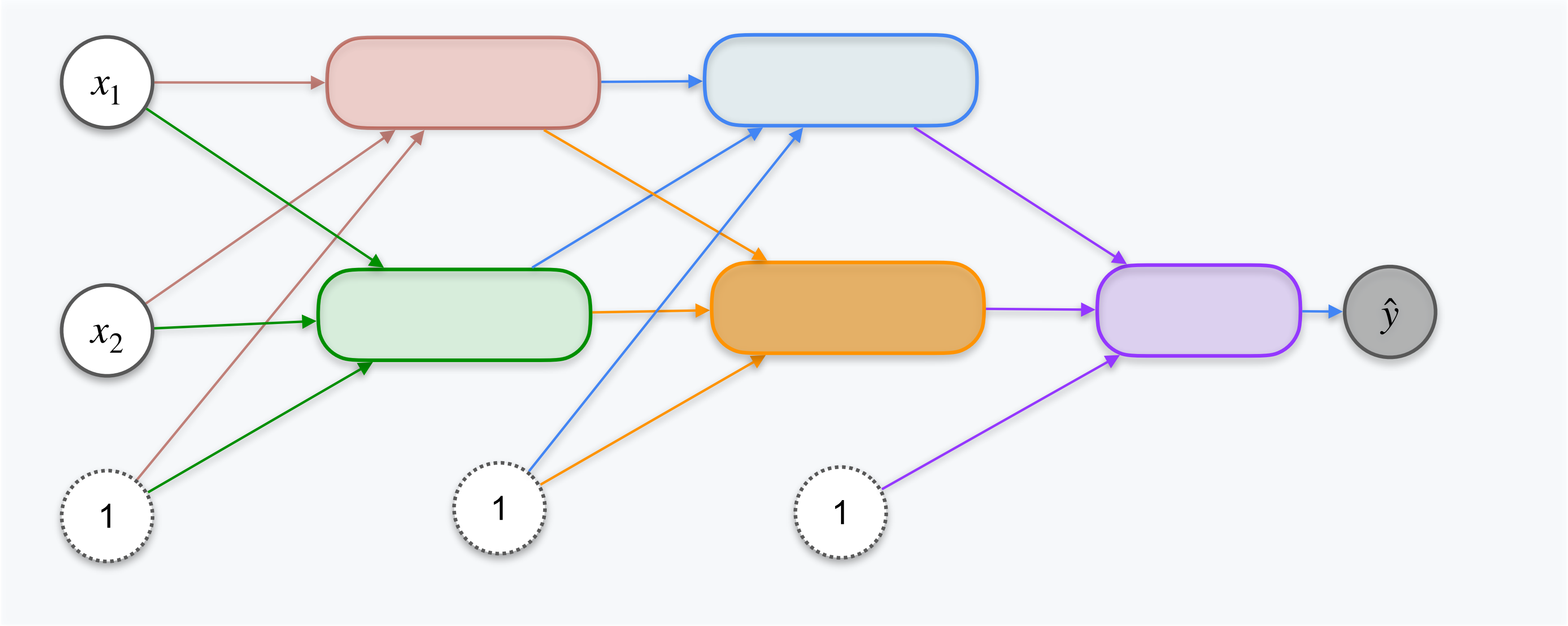

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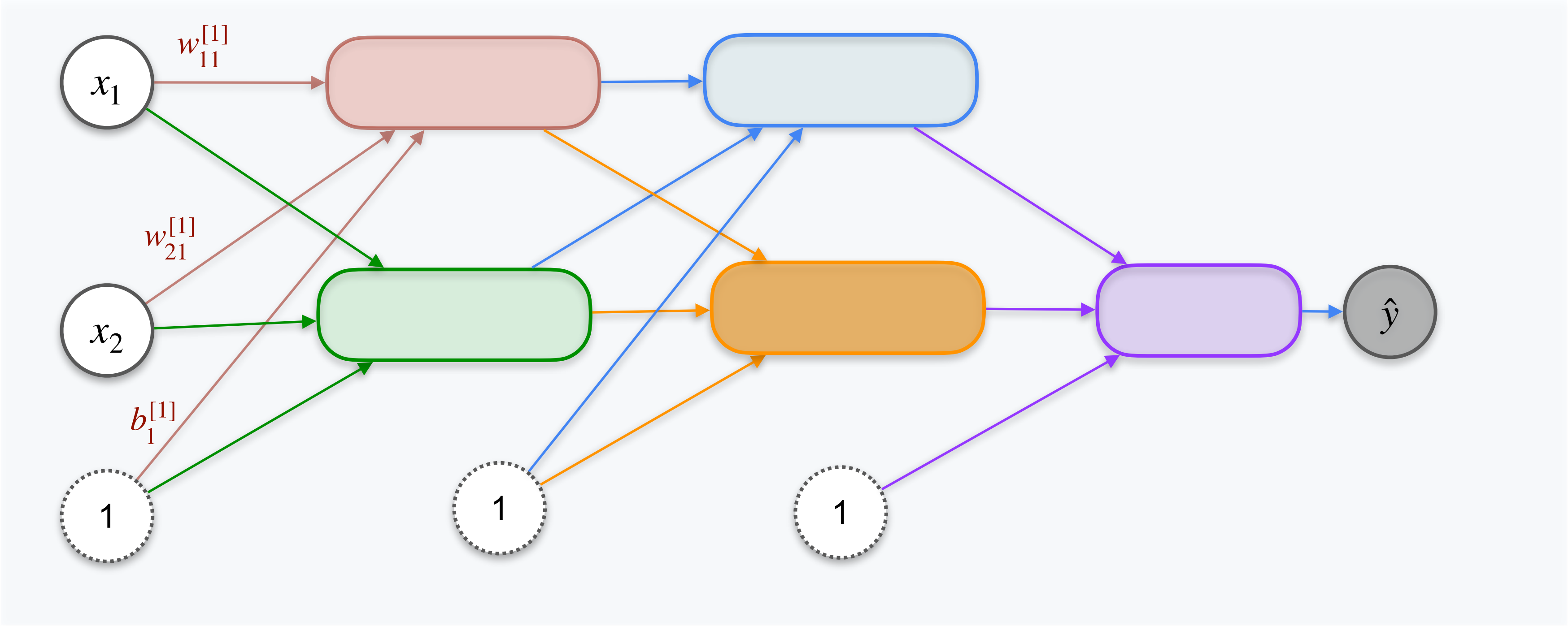

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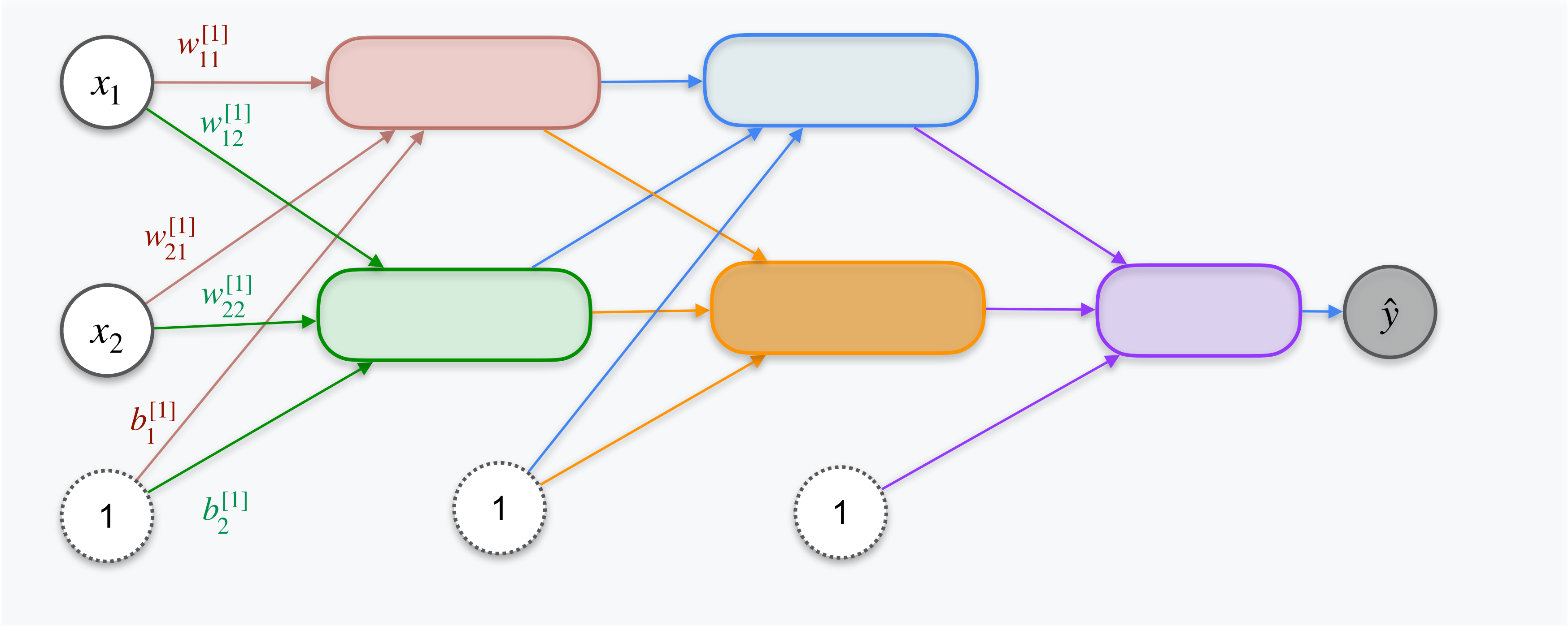

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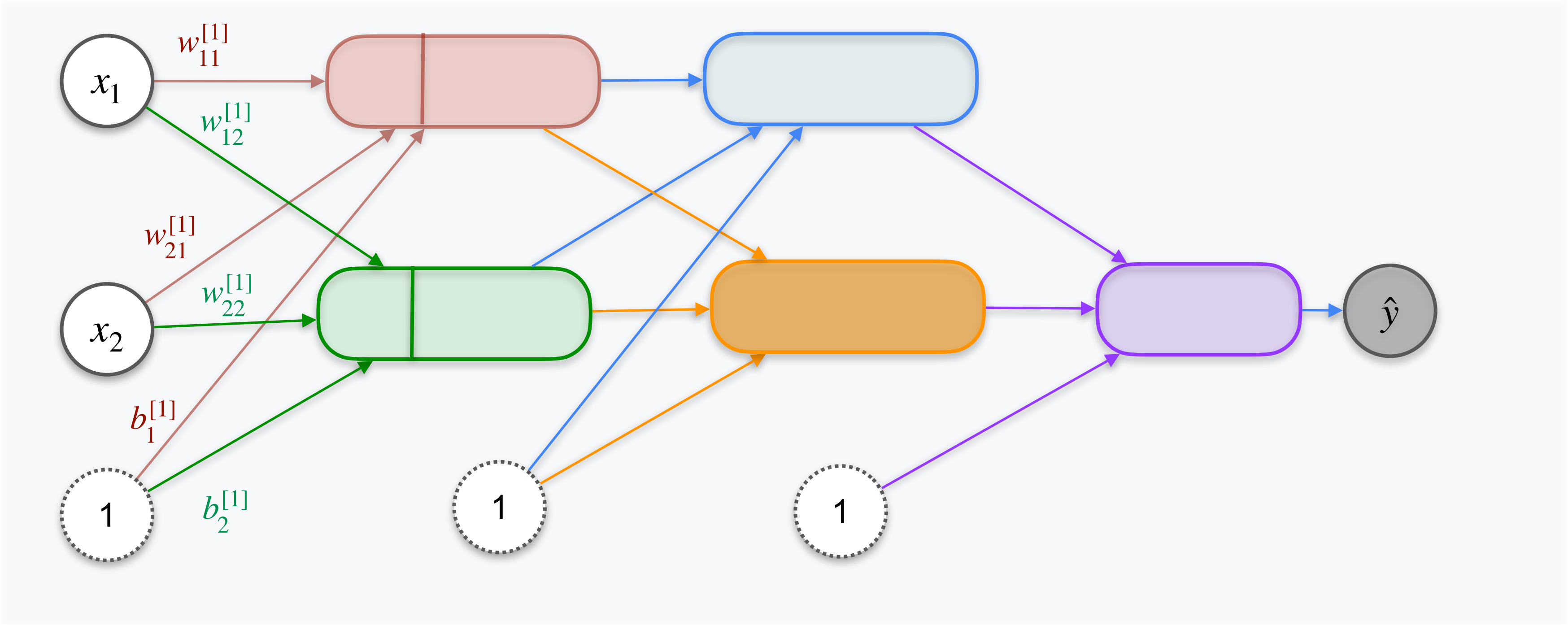

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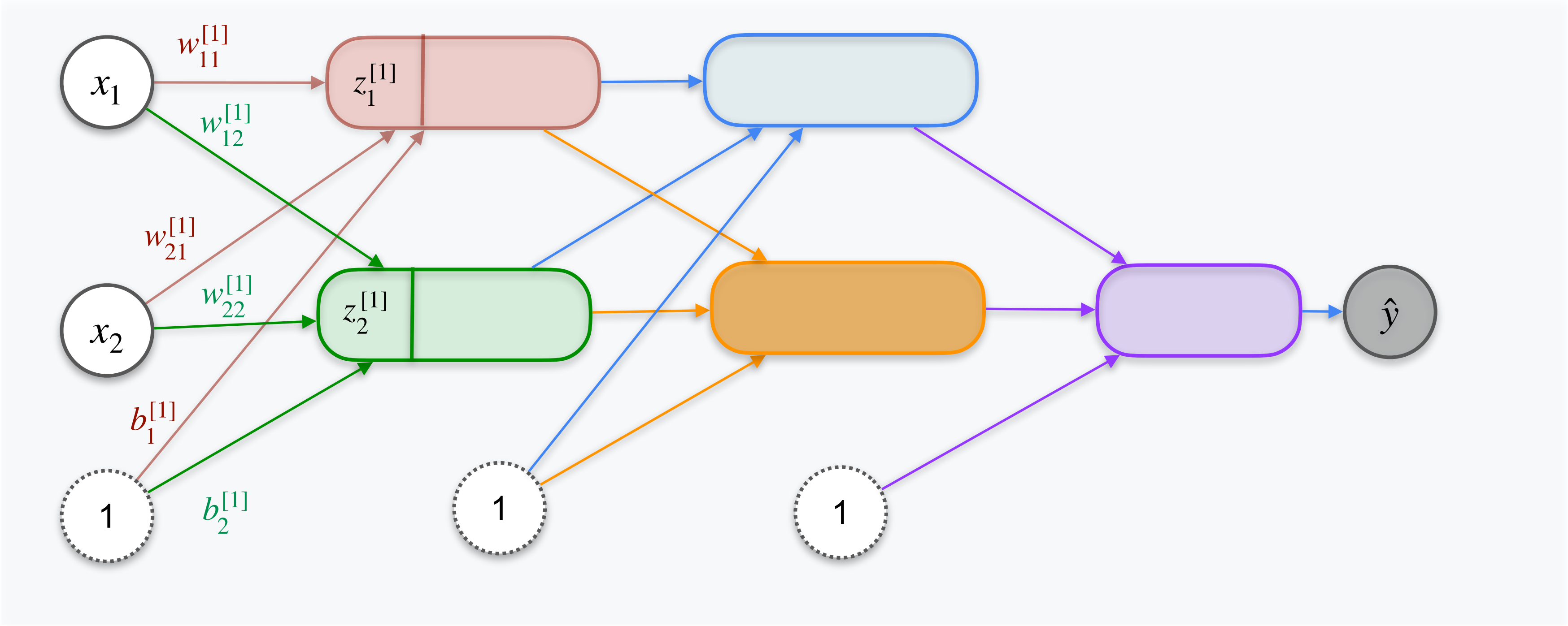

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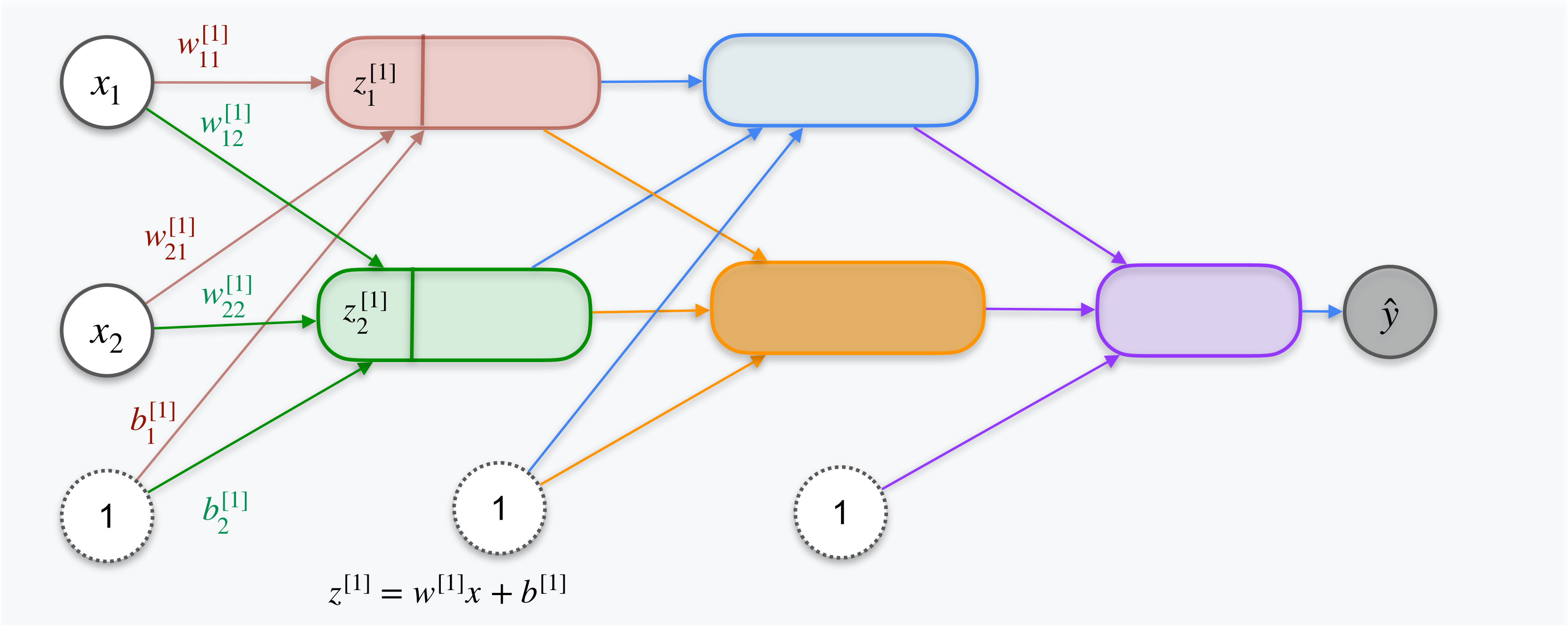

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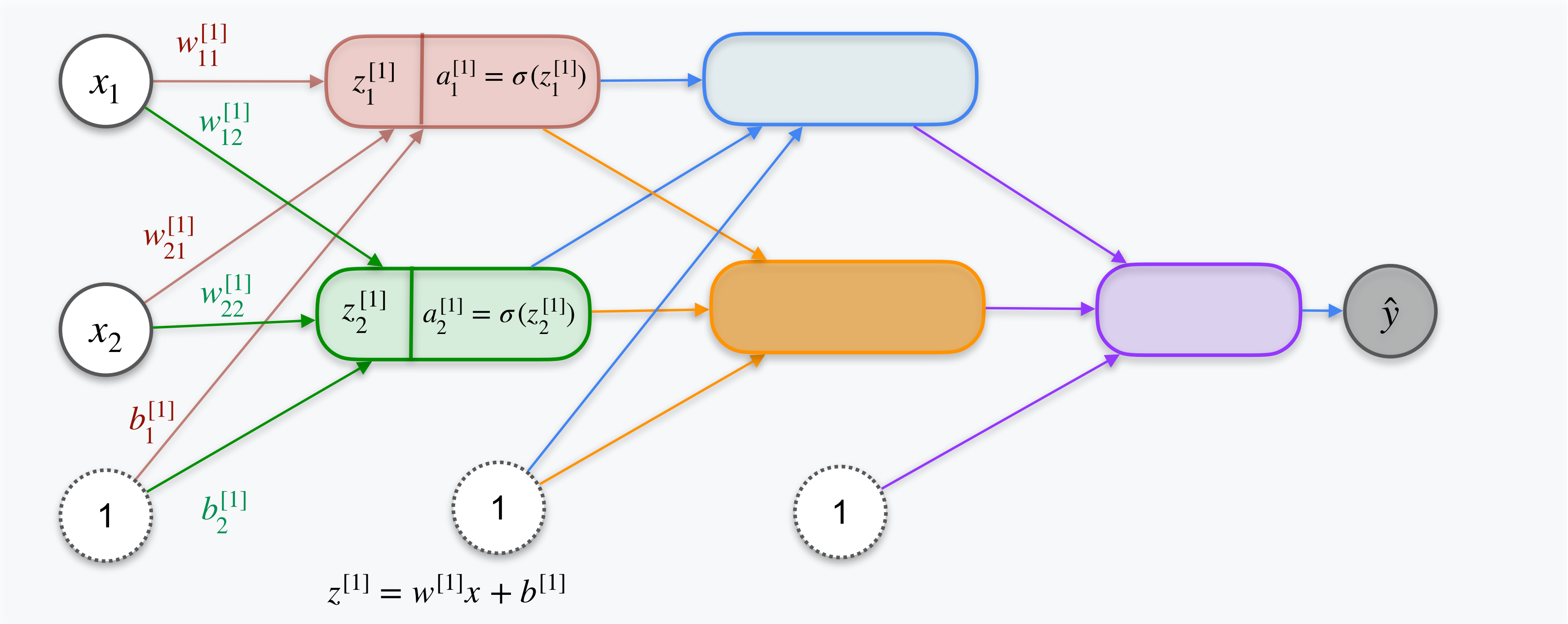

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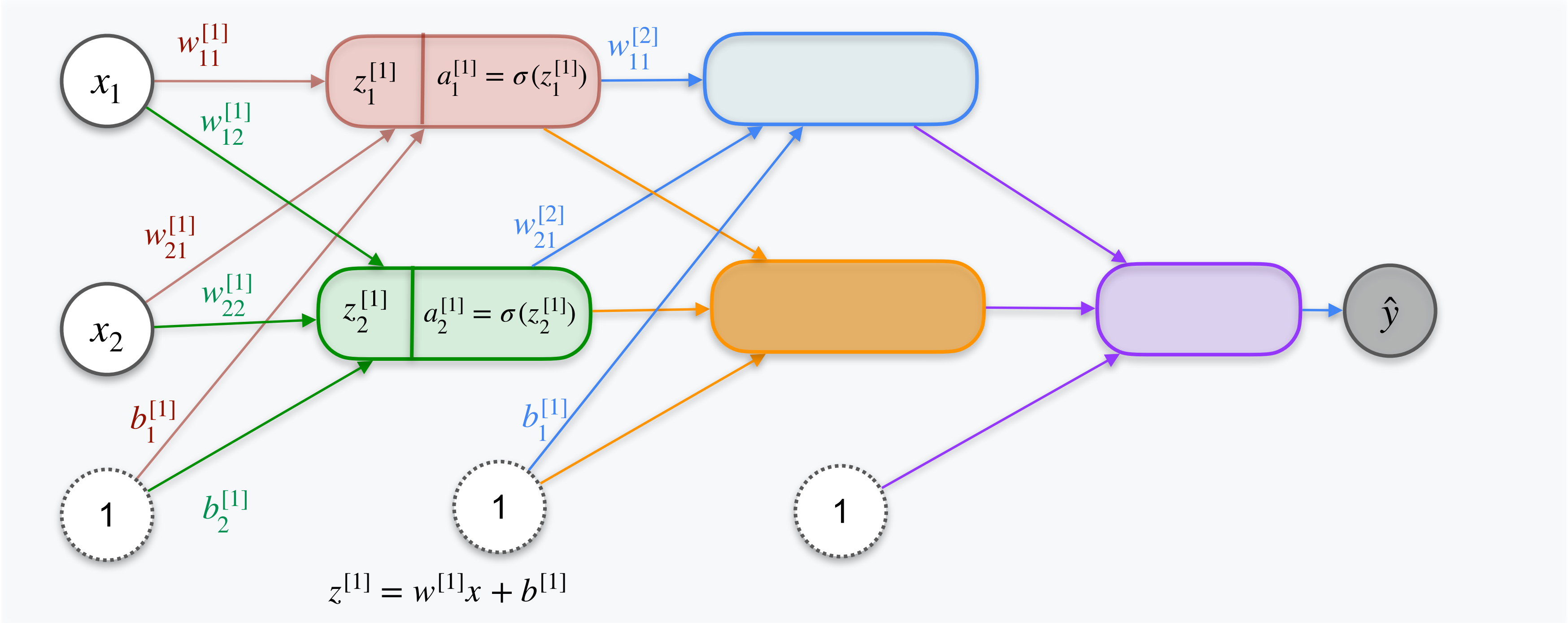

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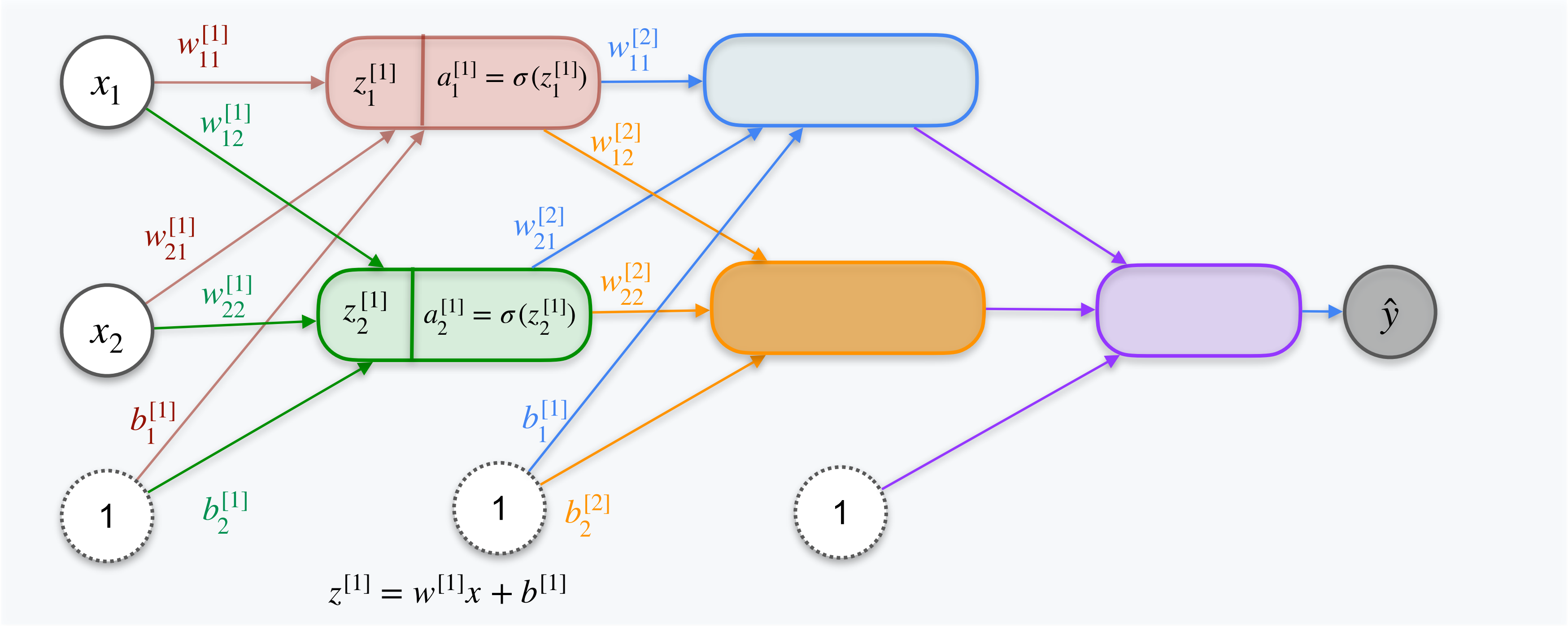

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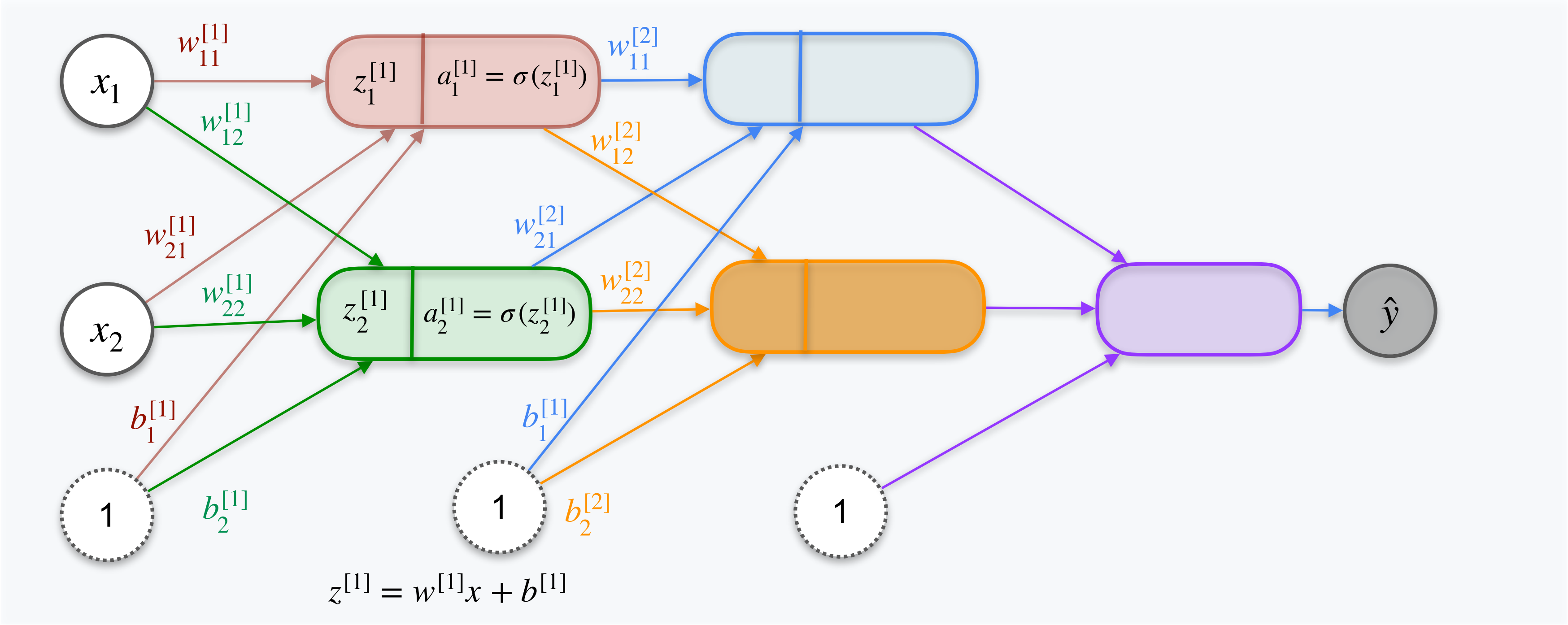

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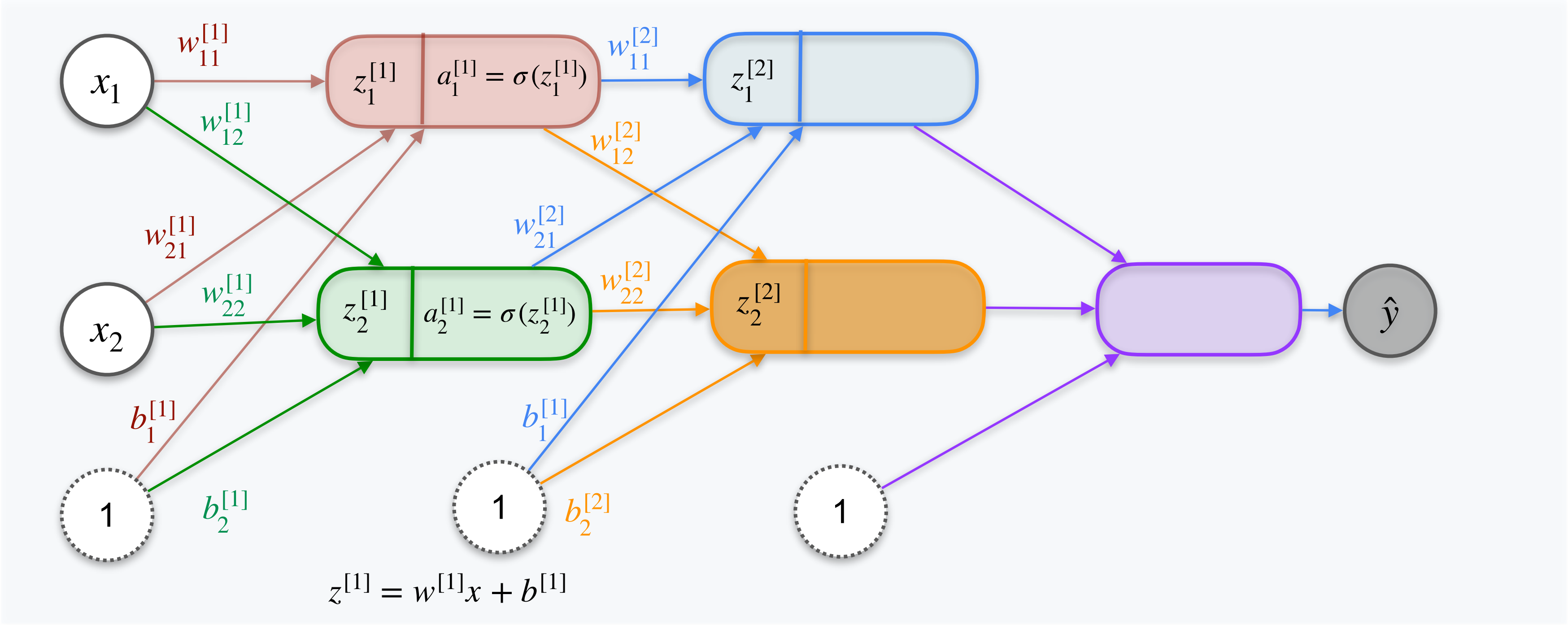

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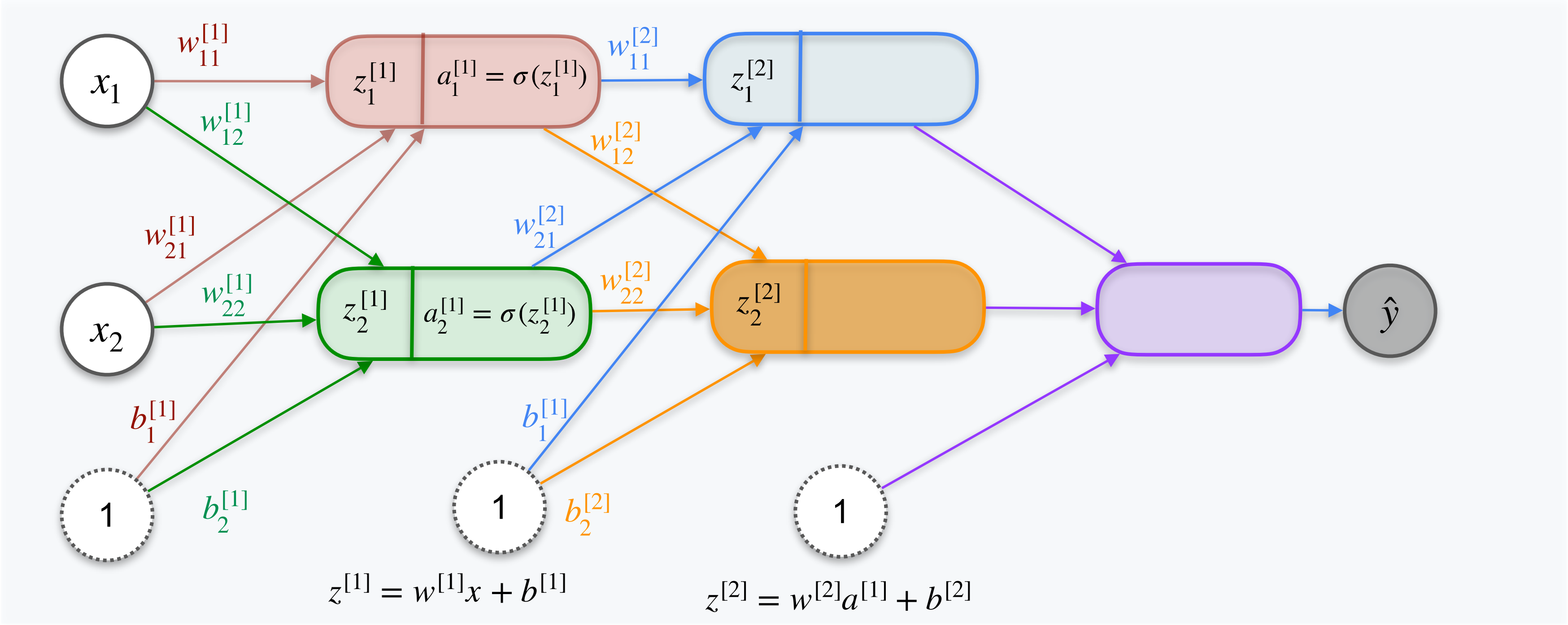

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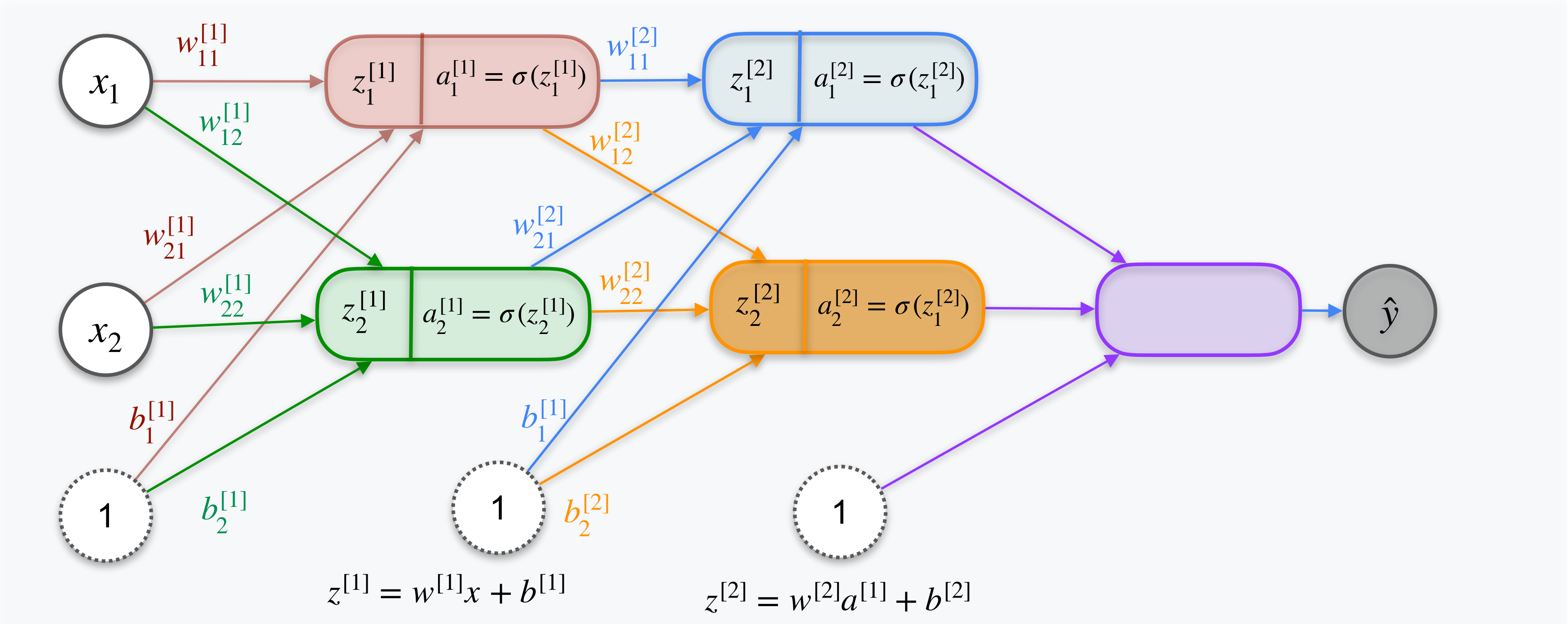

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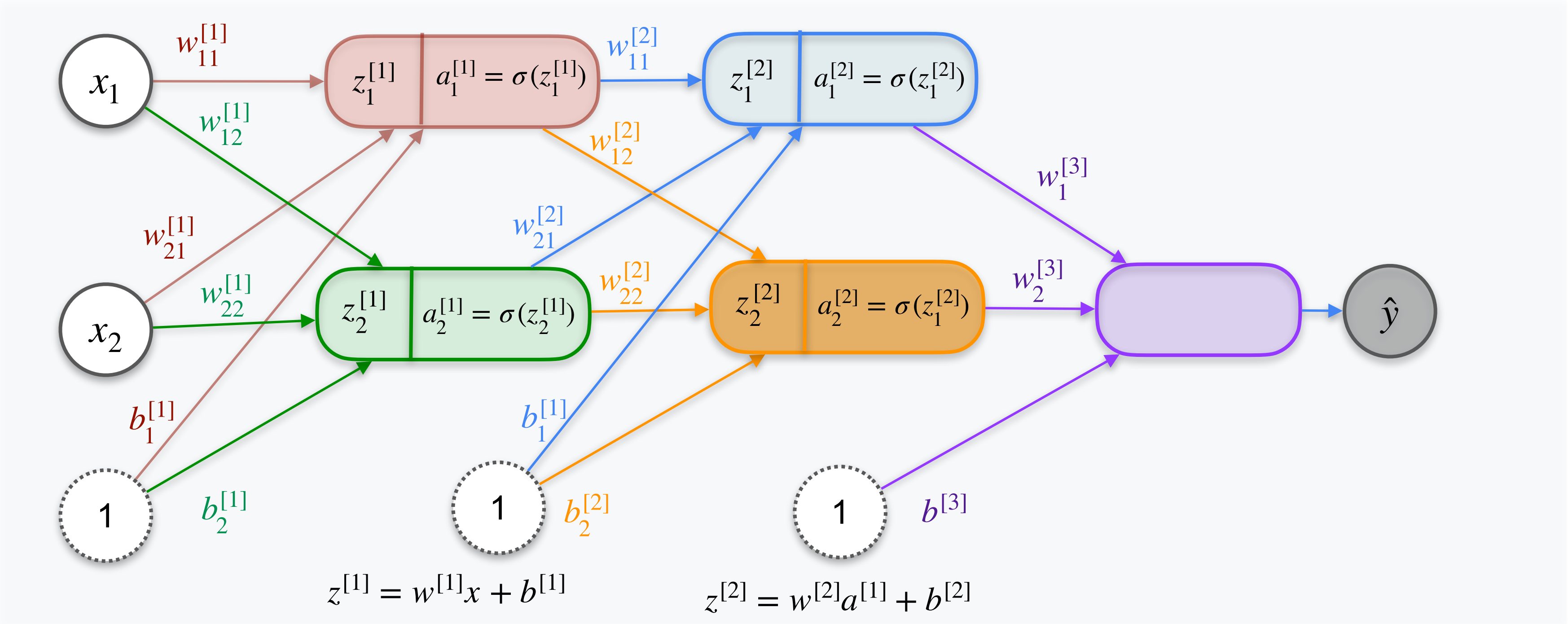

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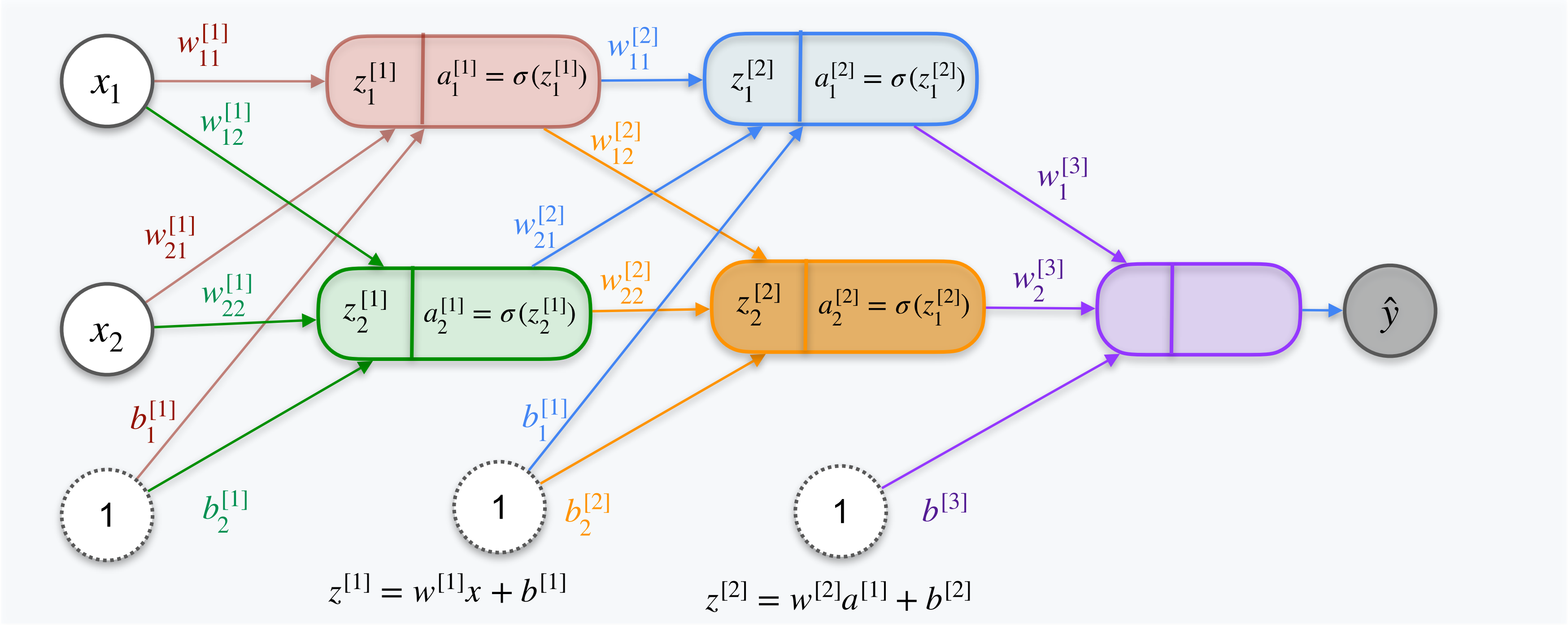

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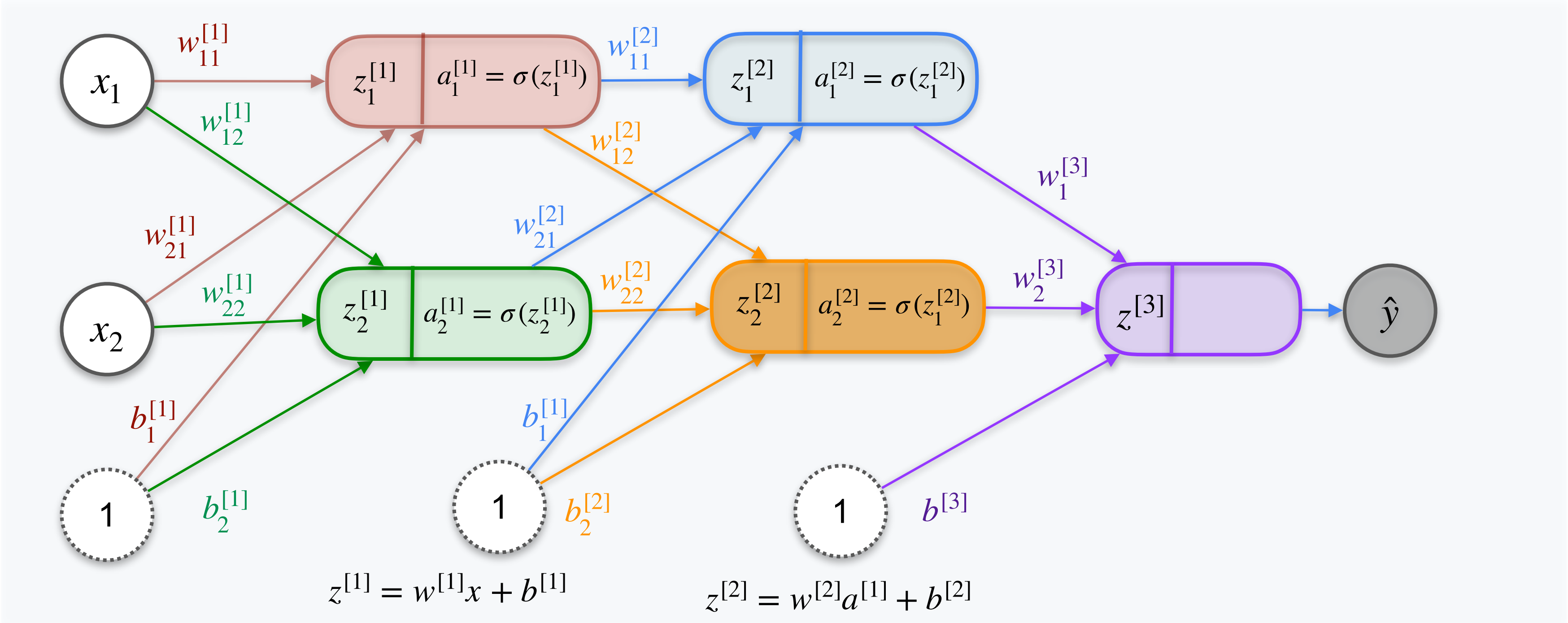

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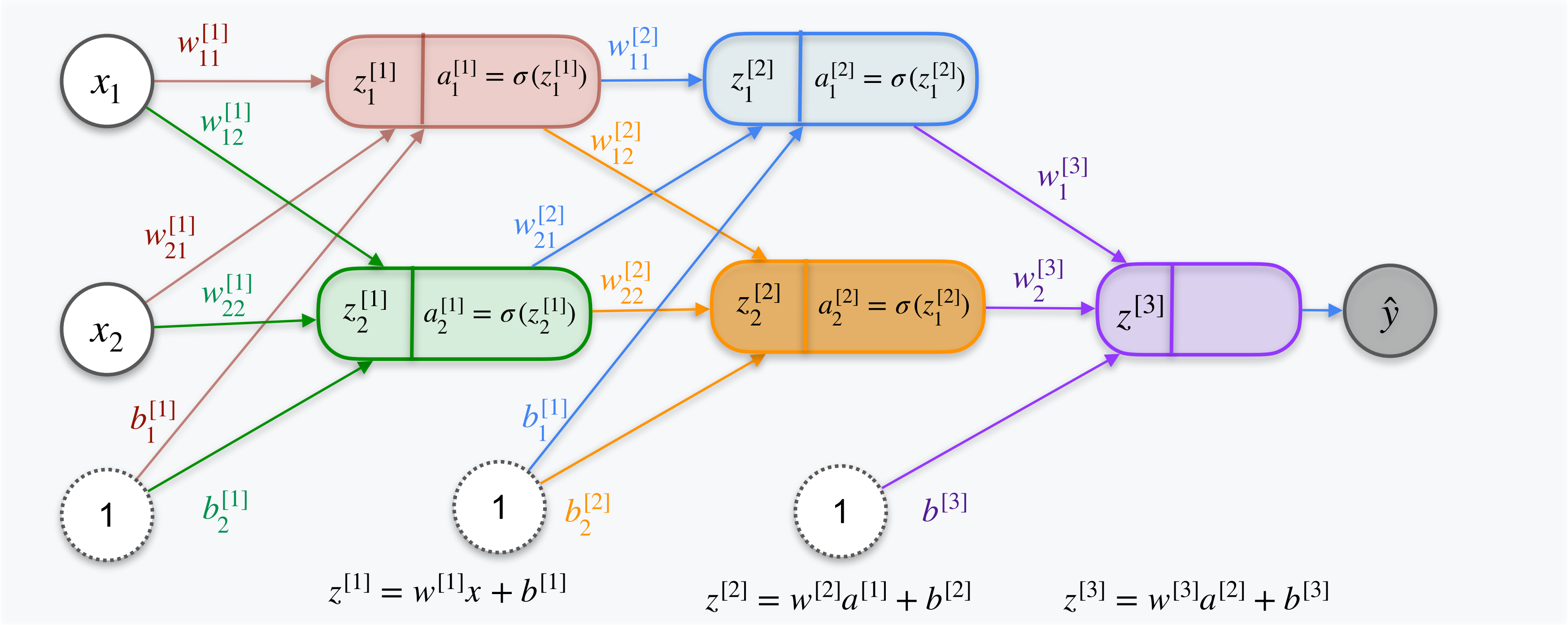

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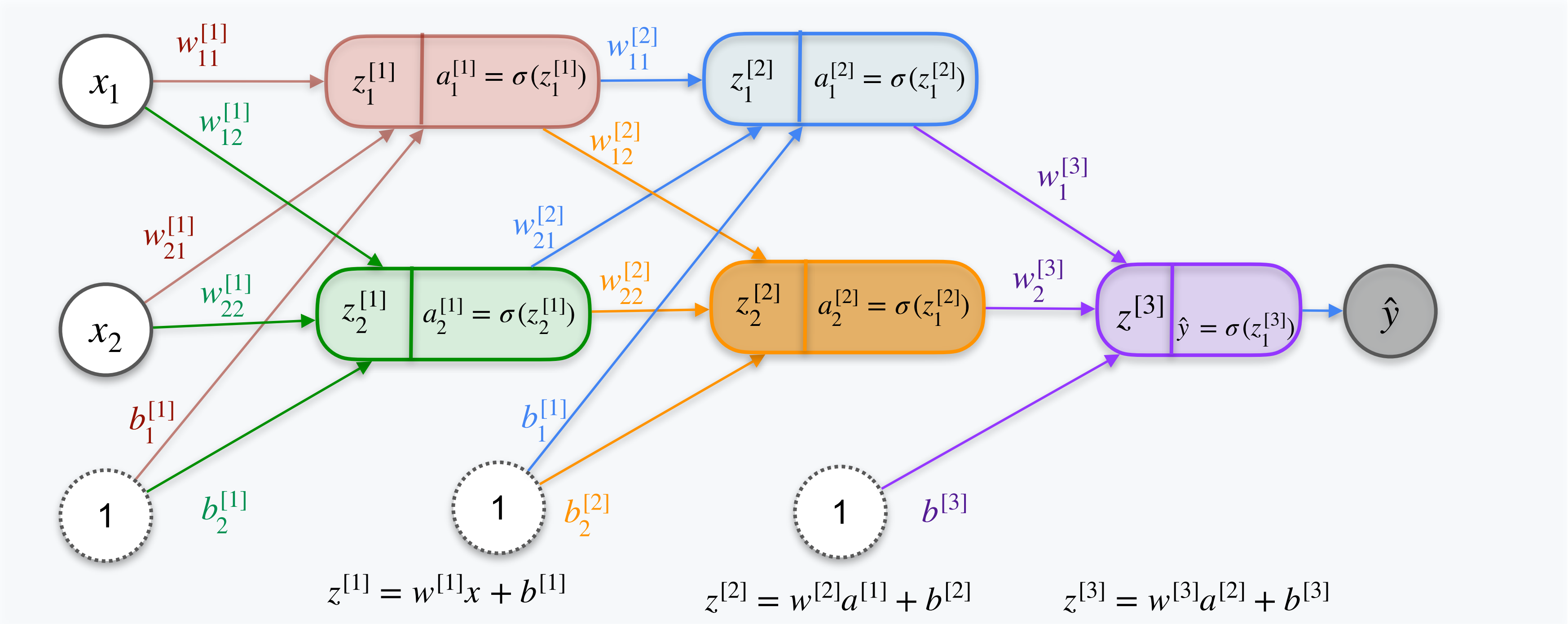

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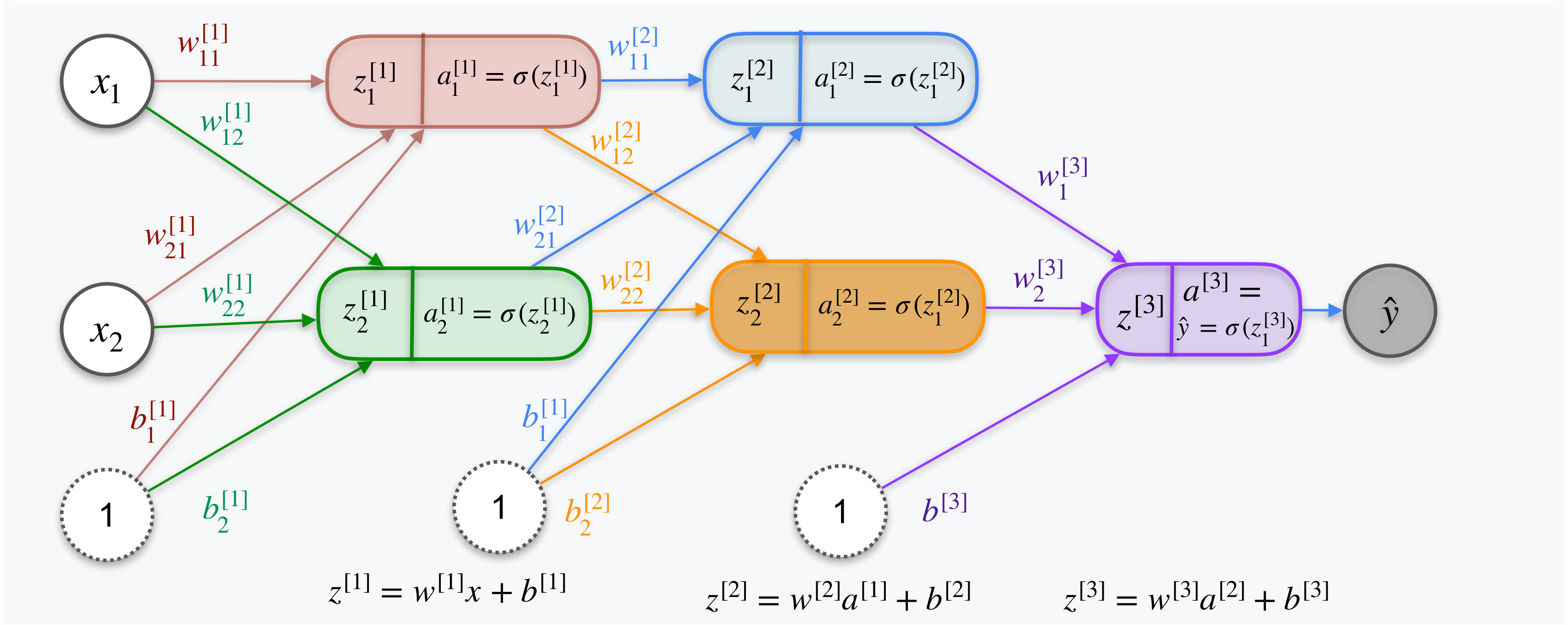

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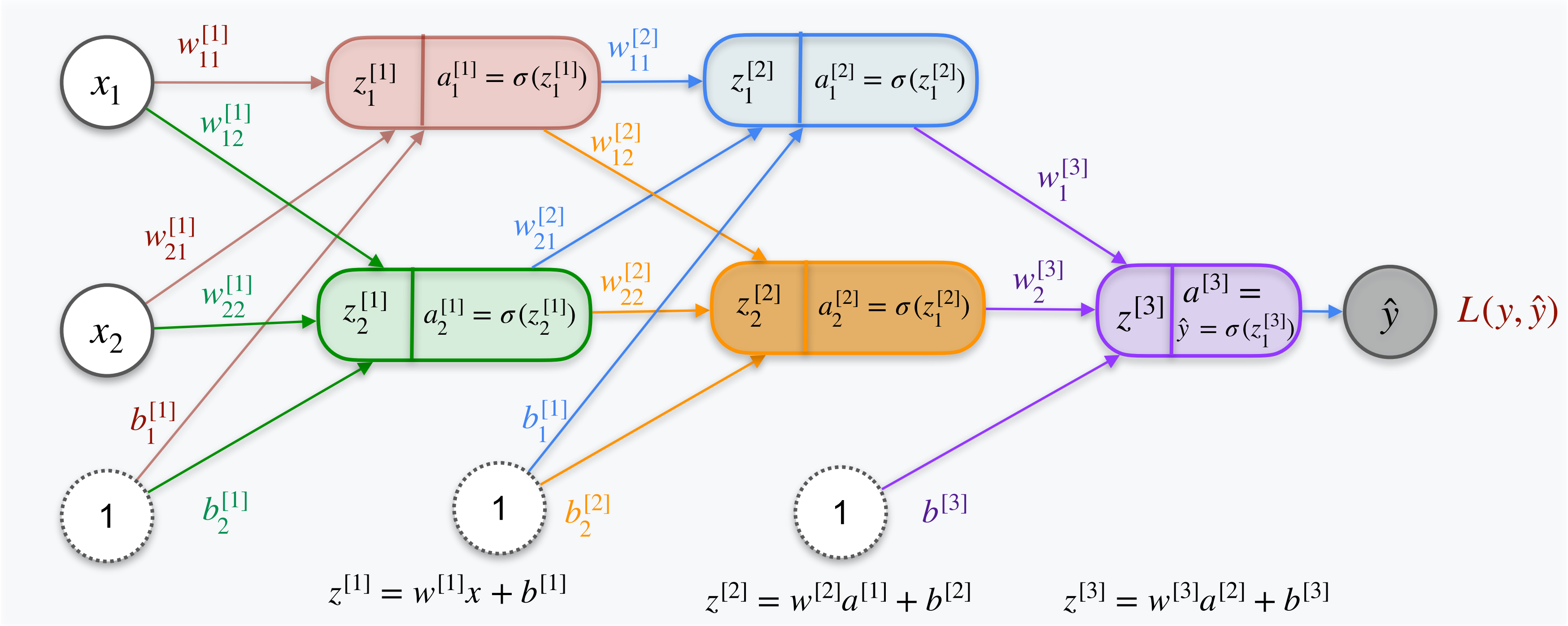

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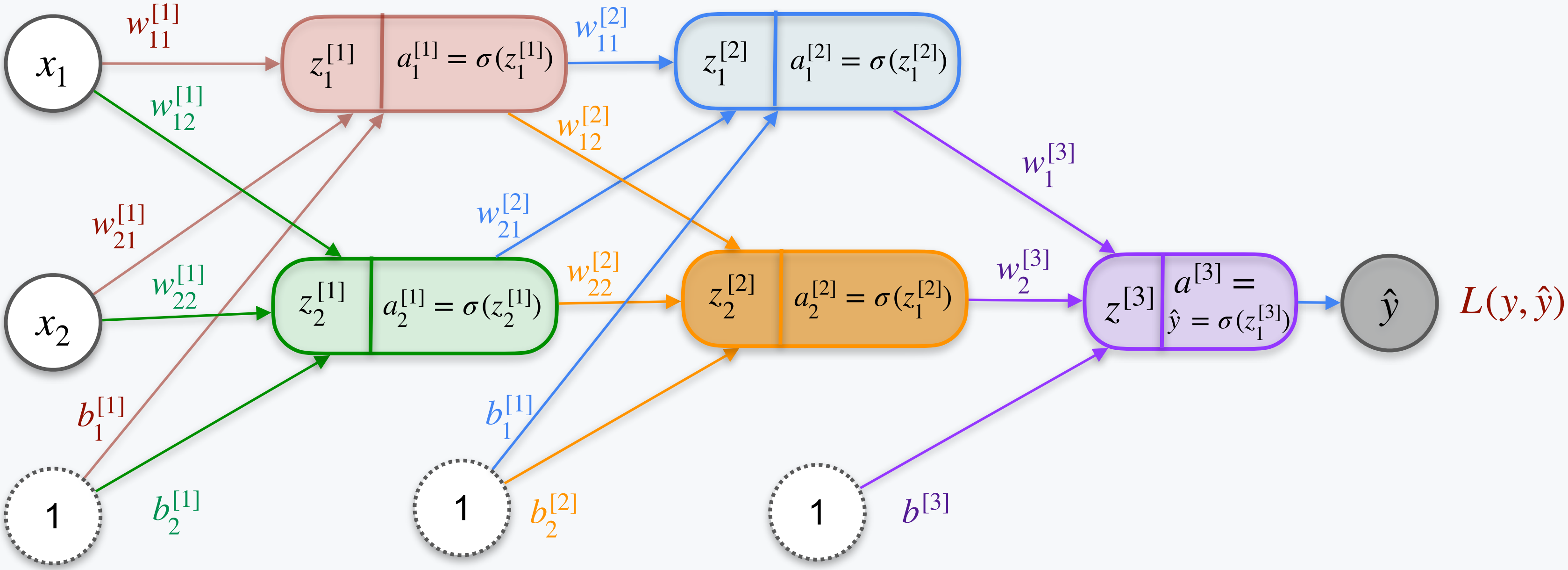

$z^{[1]} = w^{[1]}x + b^{[1]}$

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$z^{[3]} = w^{[3]}a^{[2]} + b^{[3]}$

$L(y, \hat{y})$

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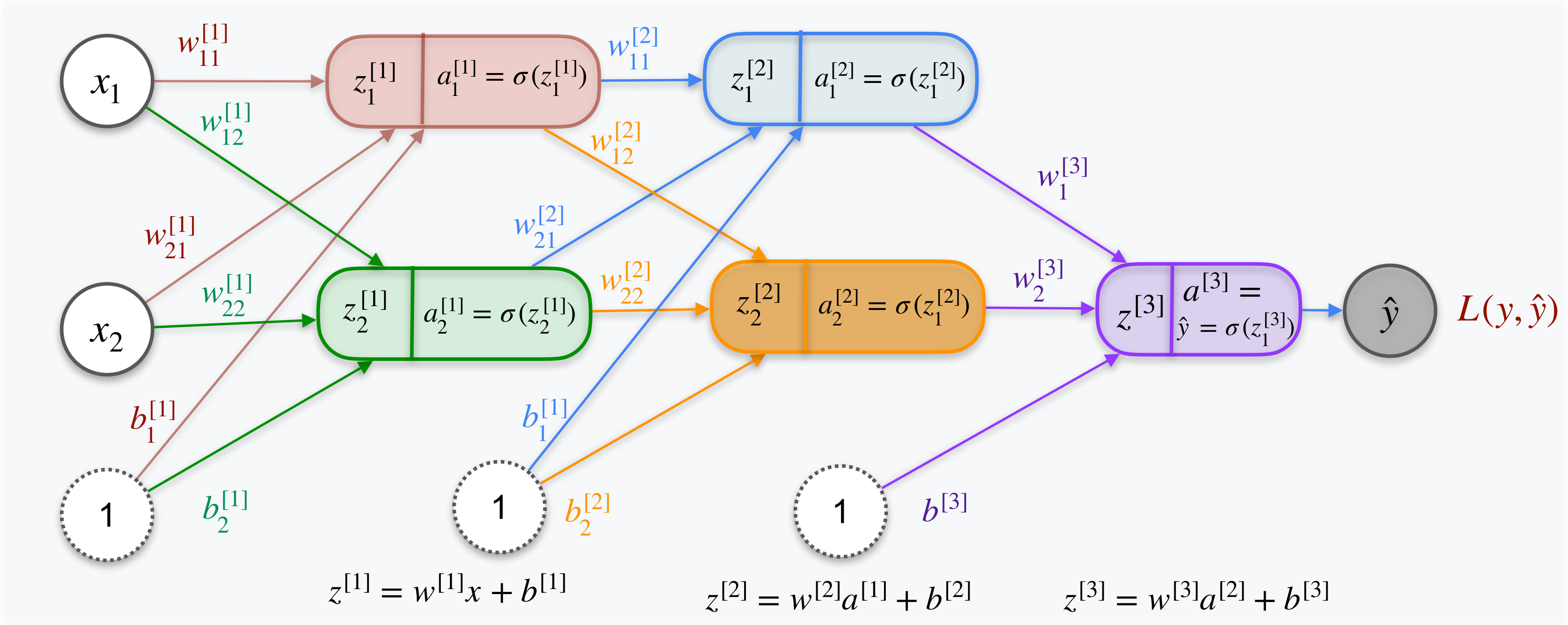

$$L(y, \hat{y}) = -ylog(\hat{y}) - (1 - y)log(1 - \hat{y})$$

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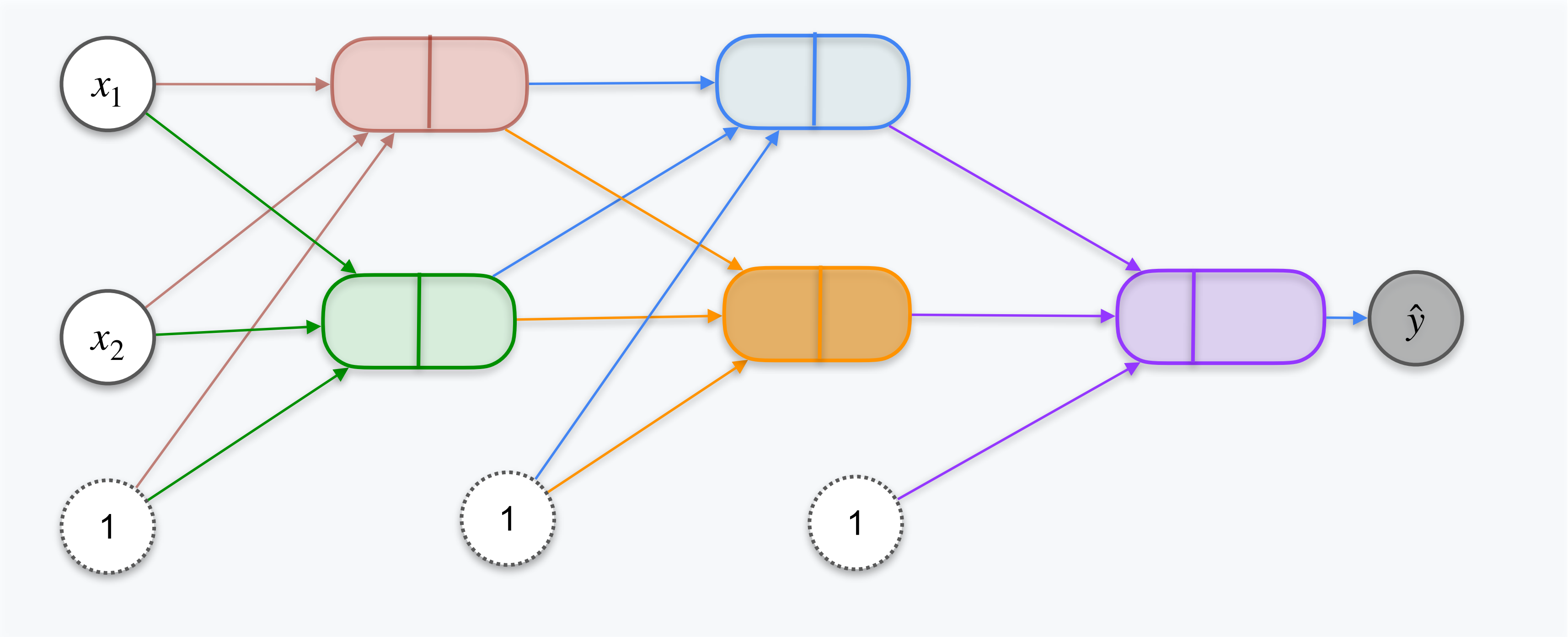

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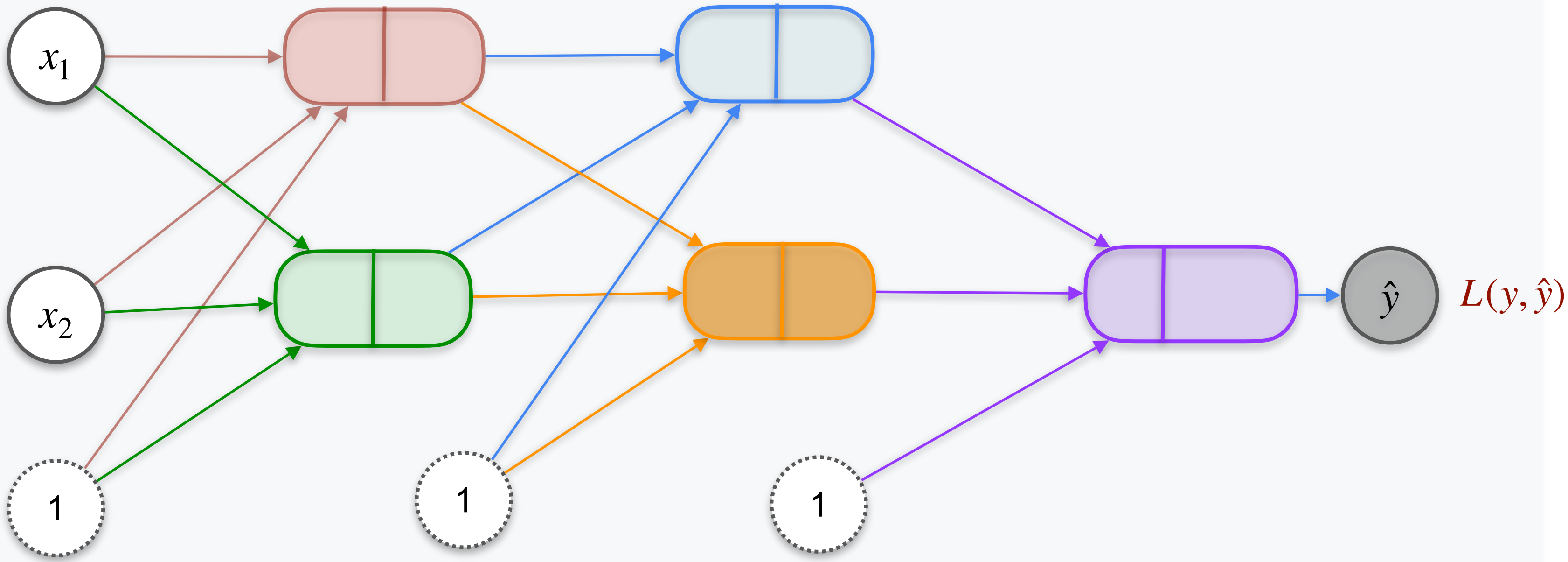

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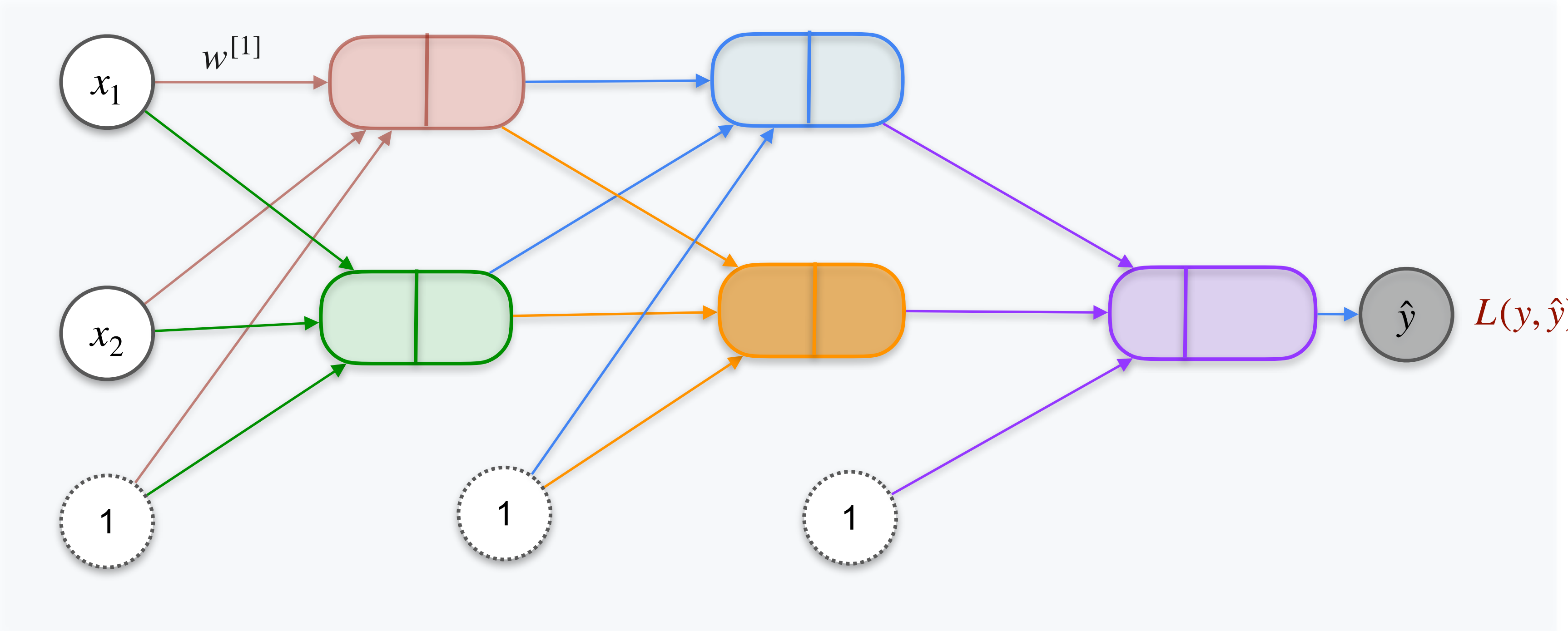

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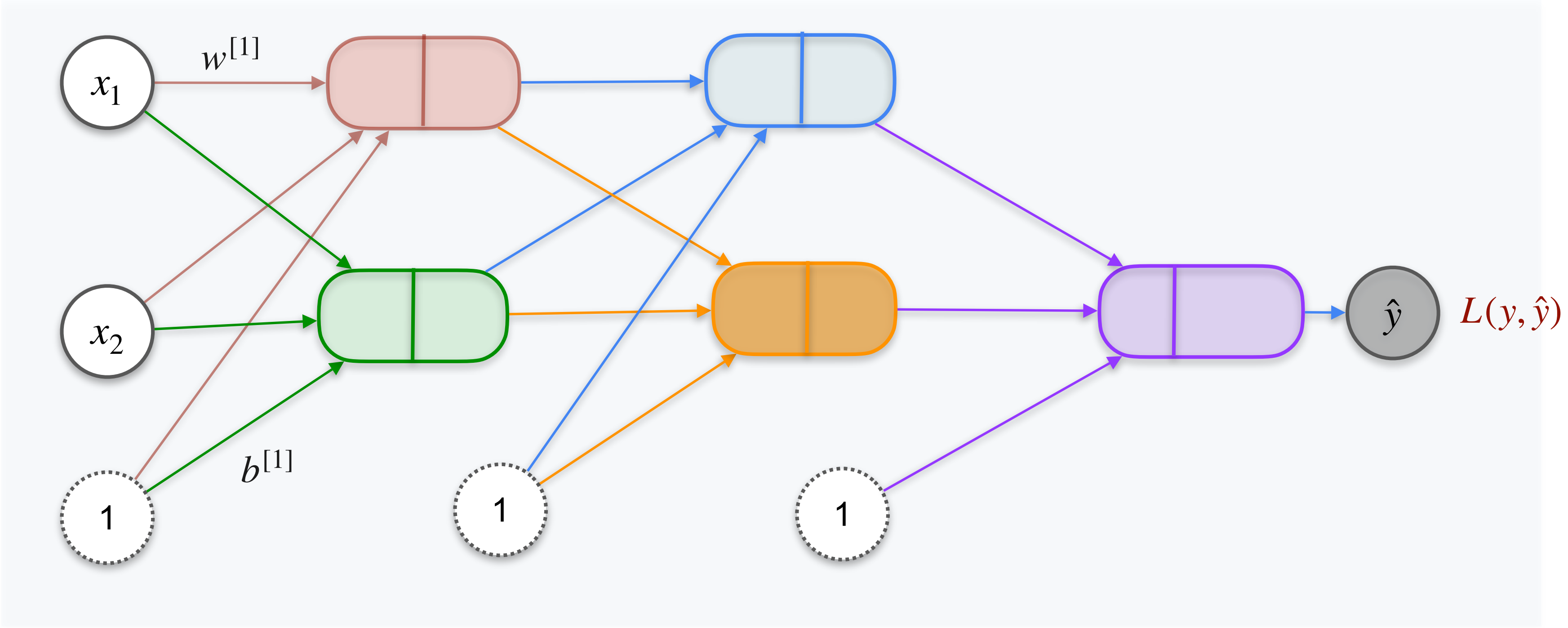

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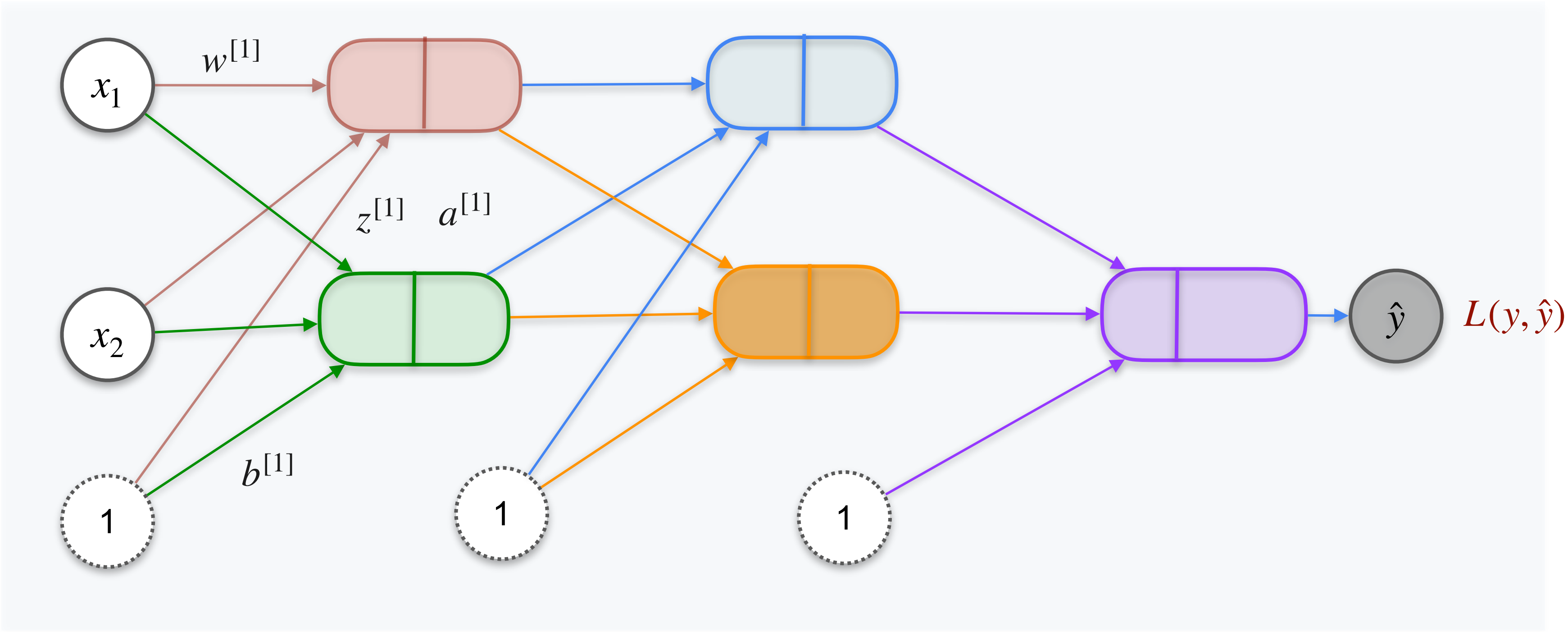

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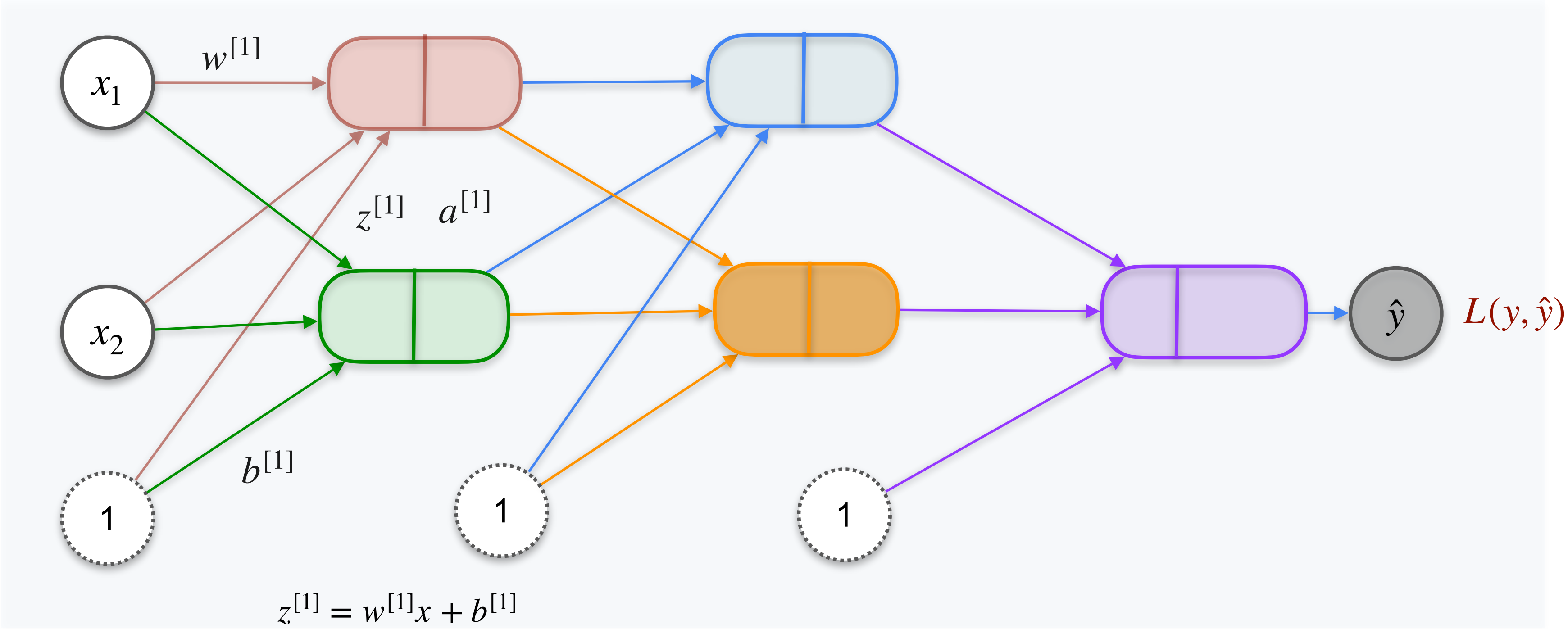

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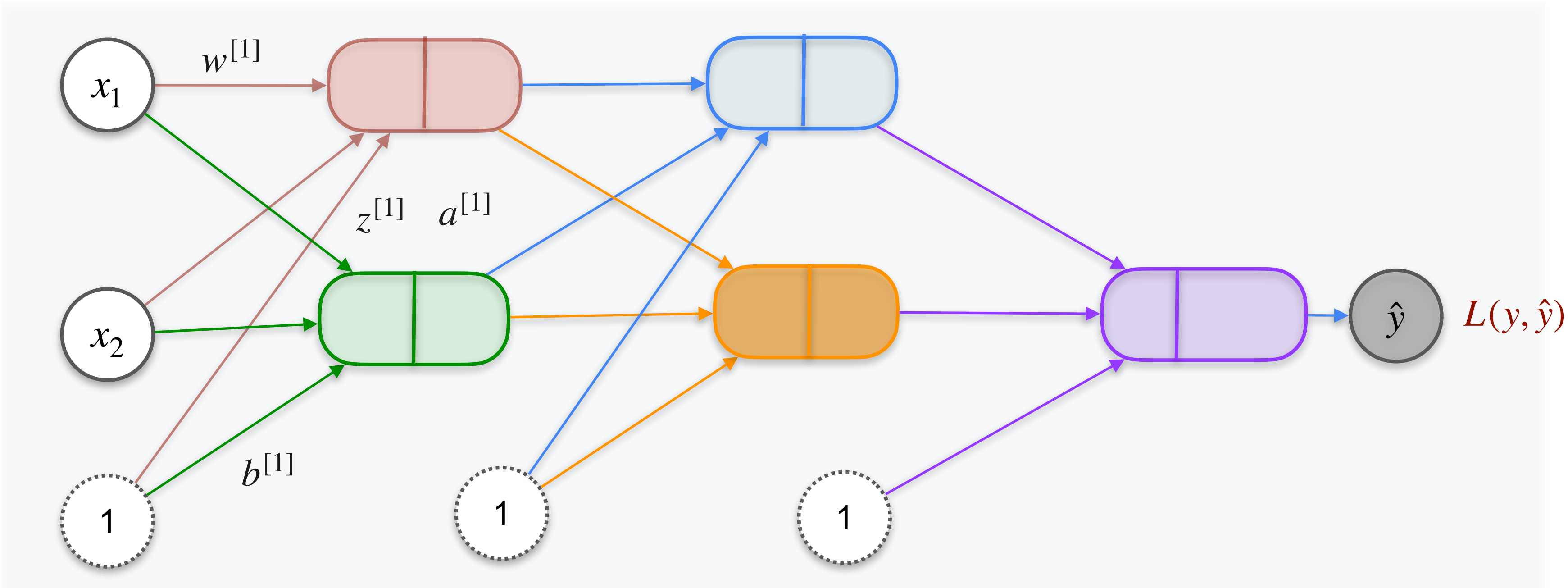

$$z^{[1]} = w^{[1]}x + b^{[1]}$$
$$a^{[1]} = \sigma(z^{[1]})$$

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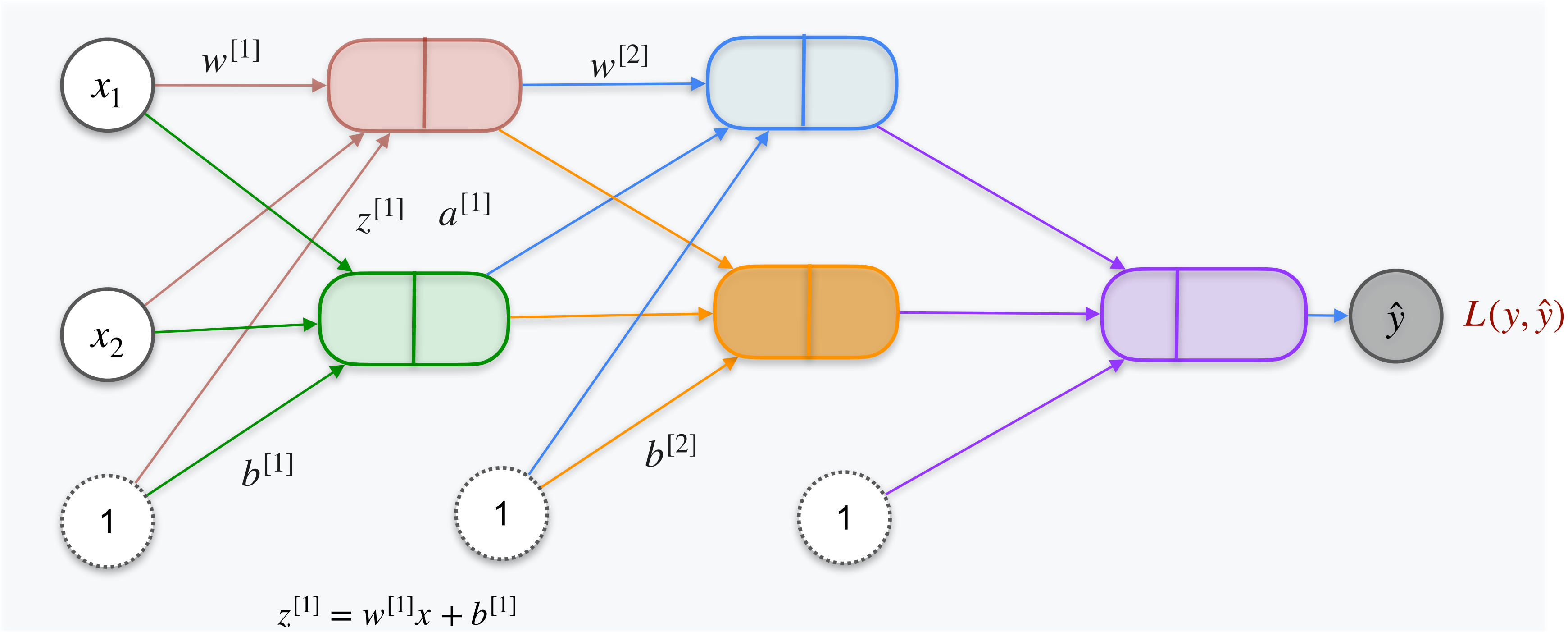

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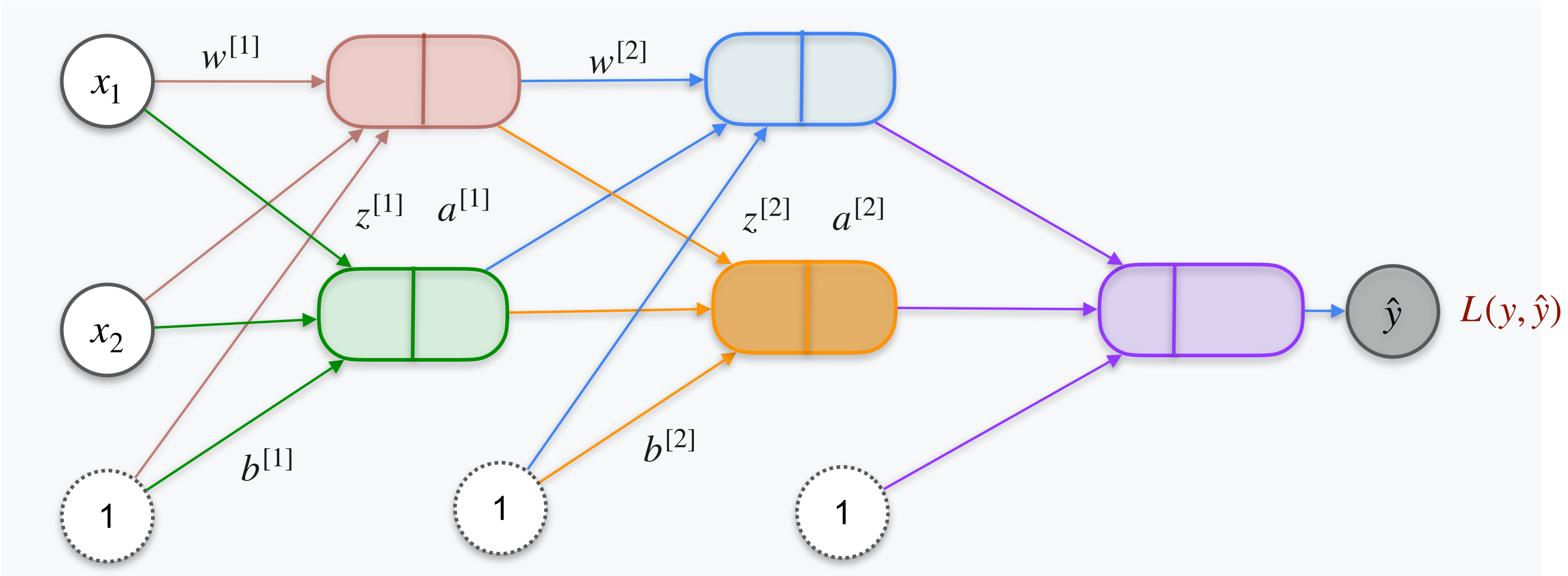

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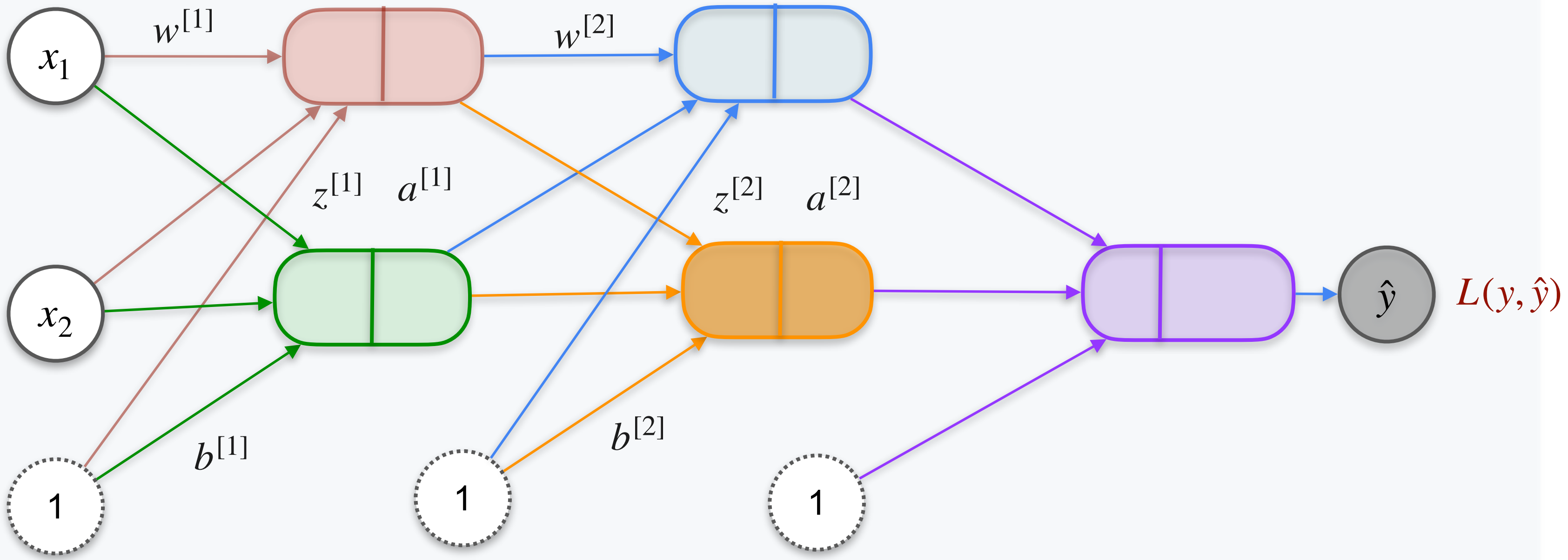

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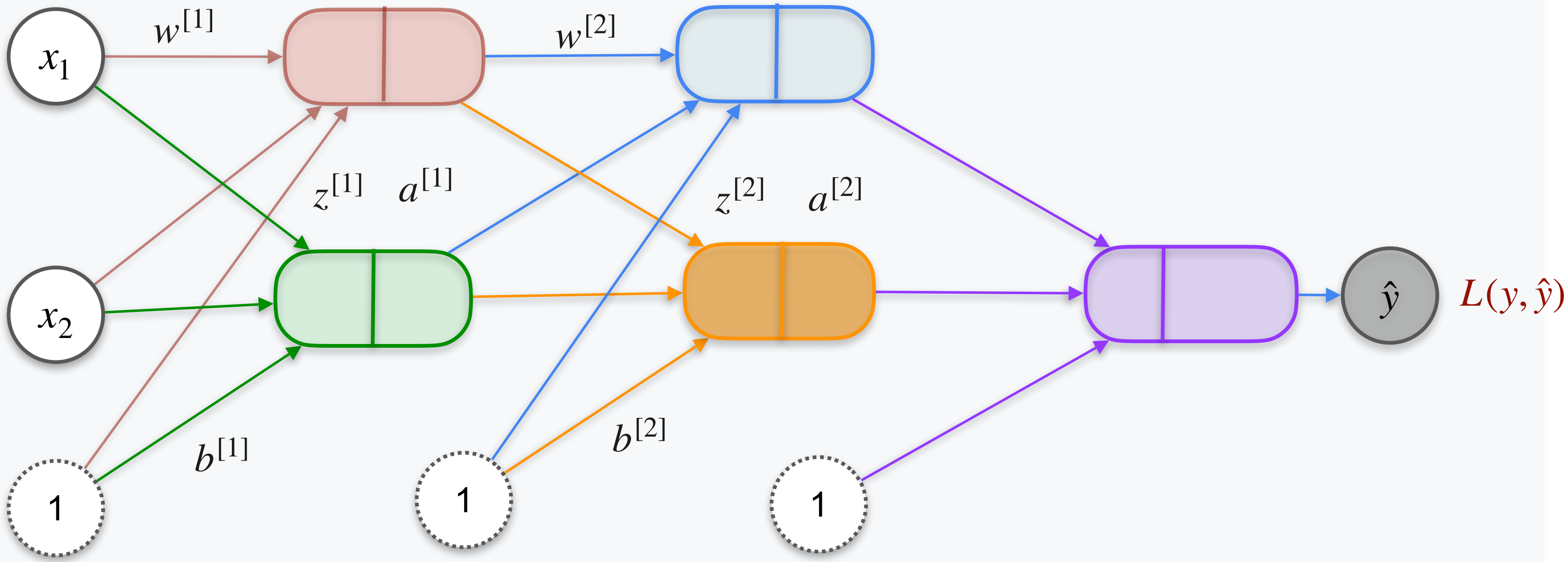

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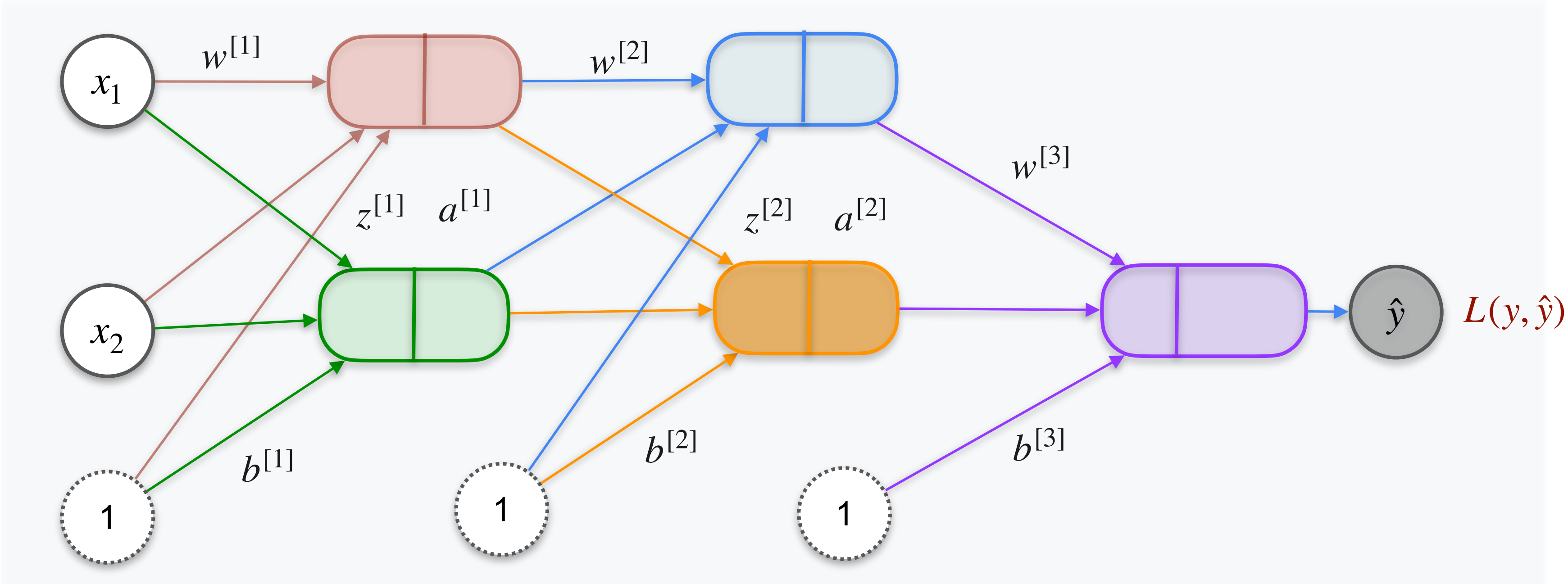

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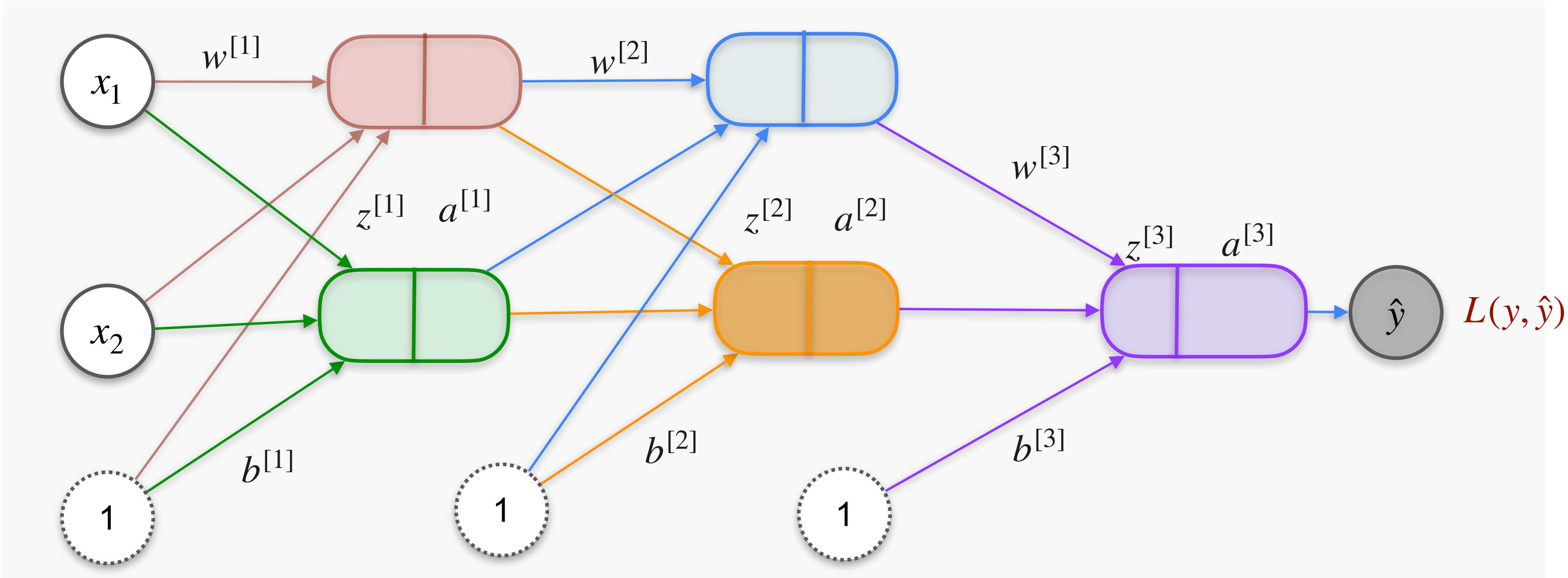

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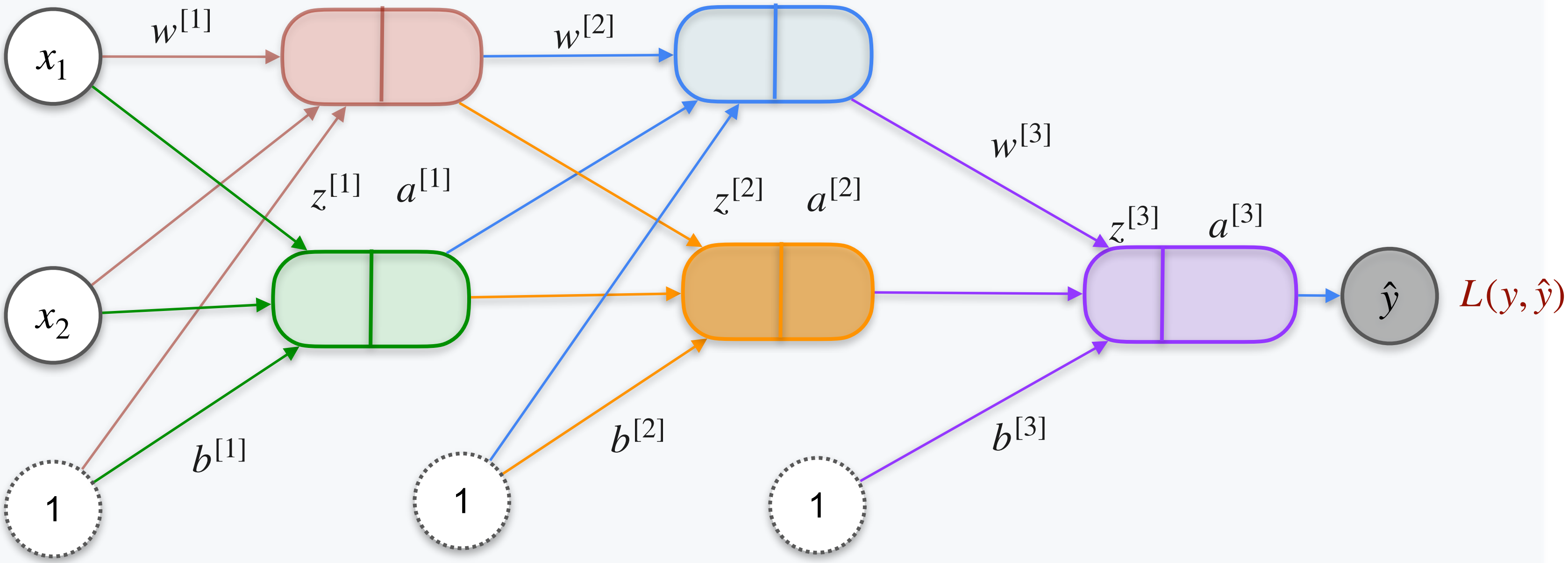

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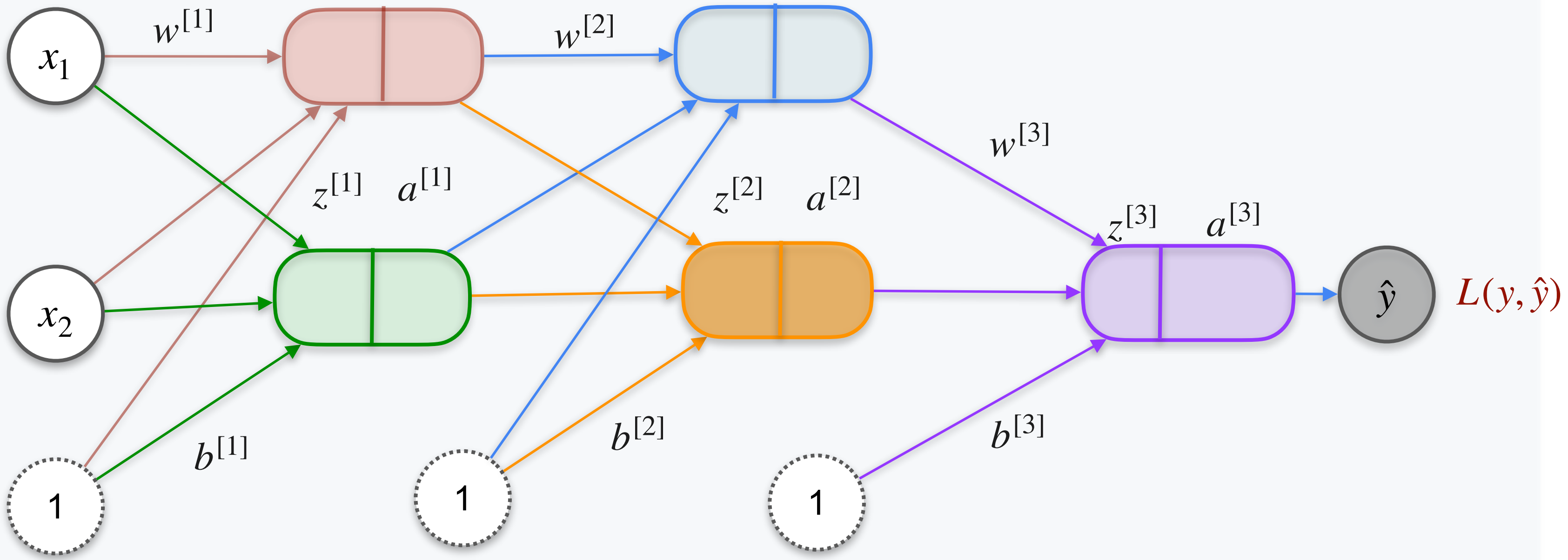

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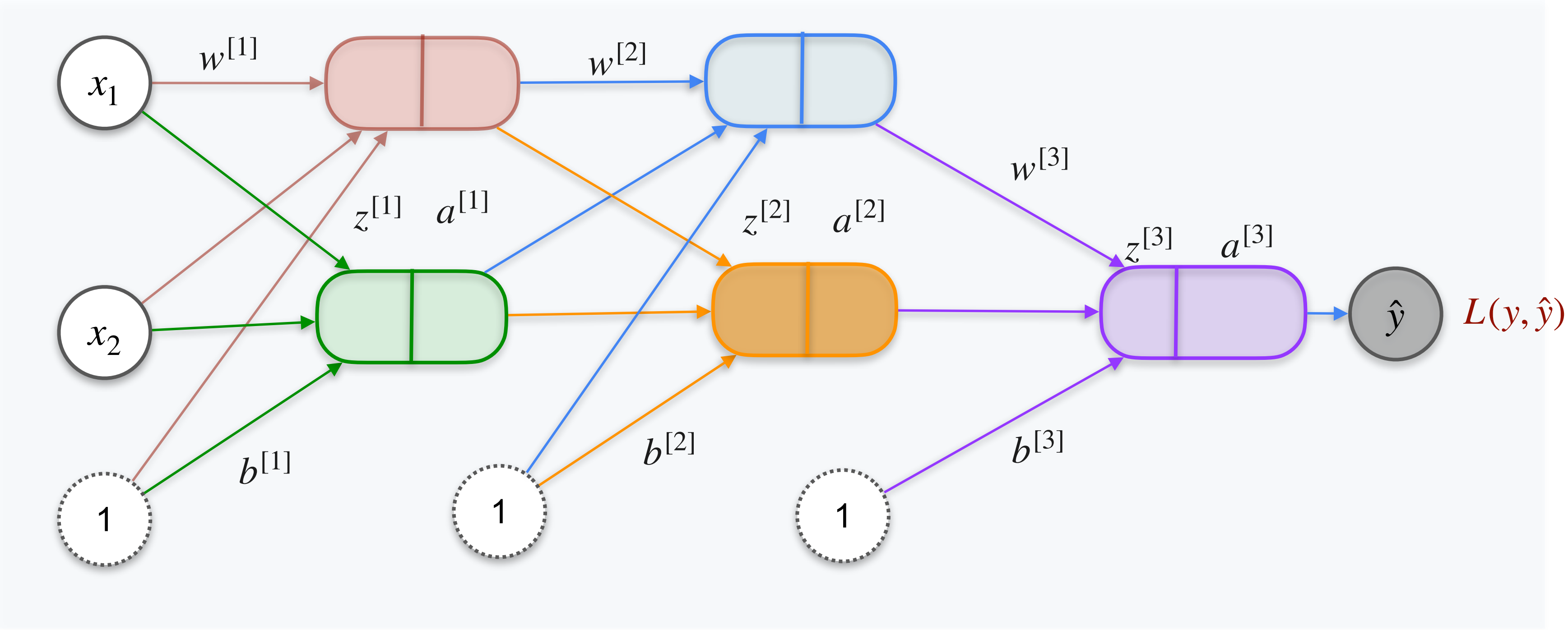

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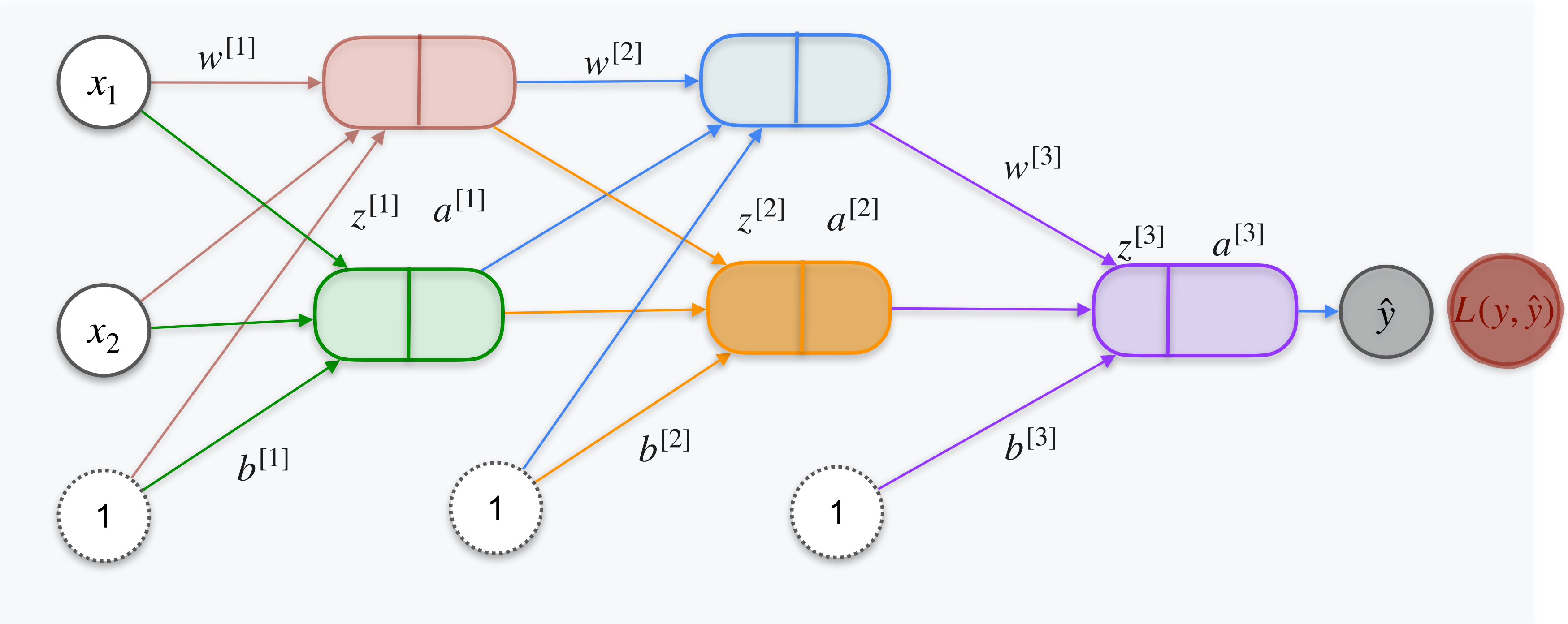

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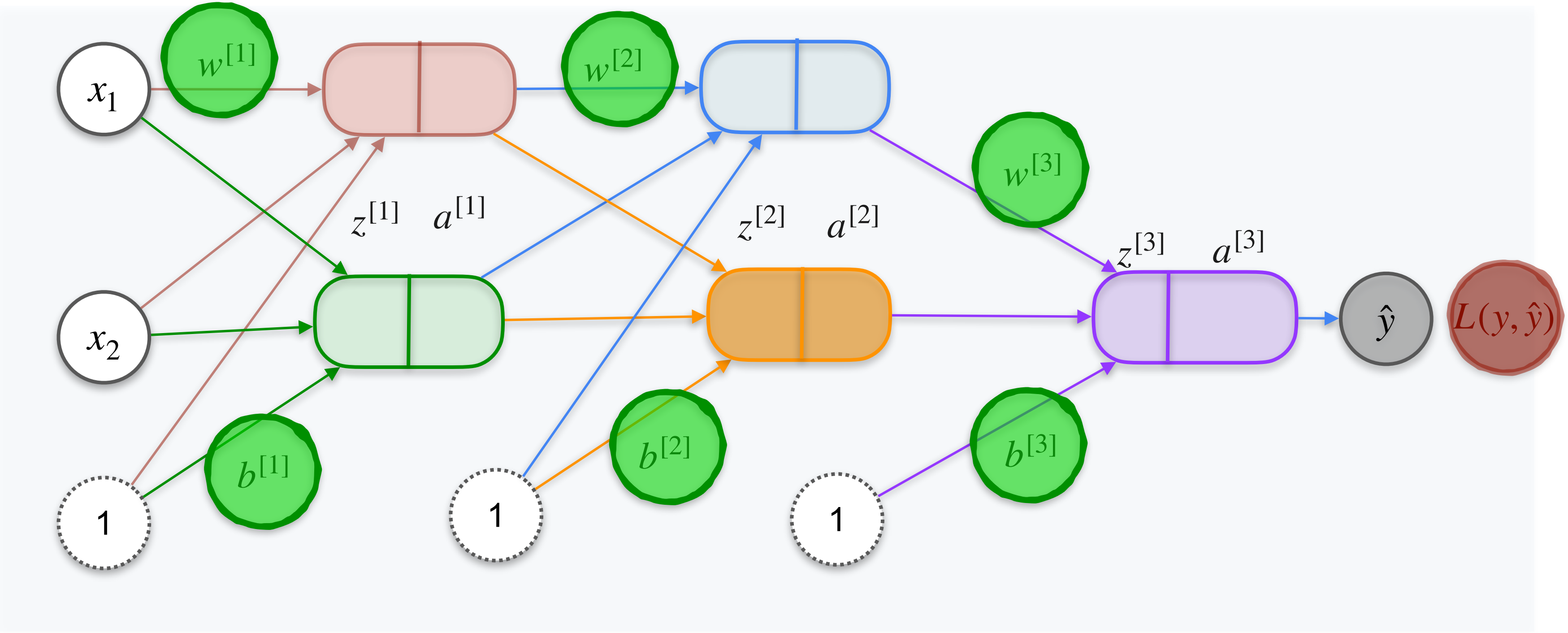

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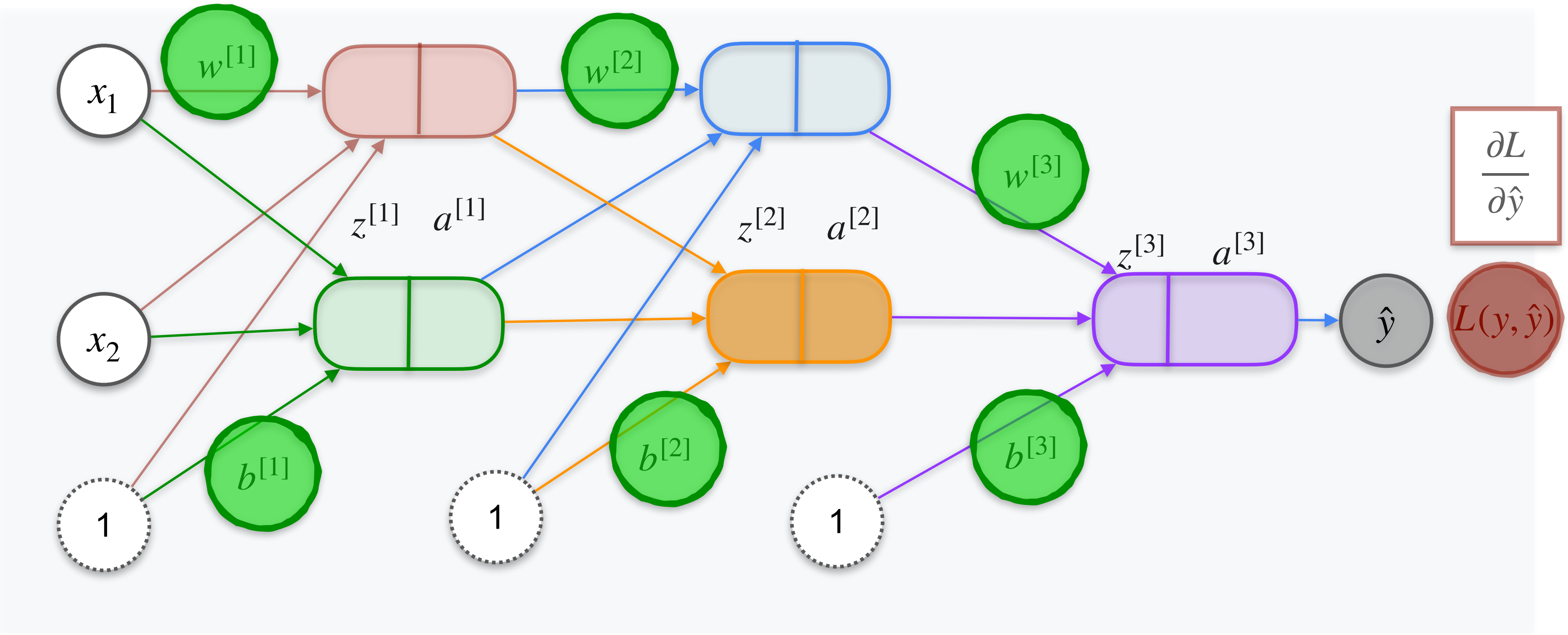

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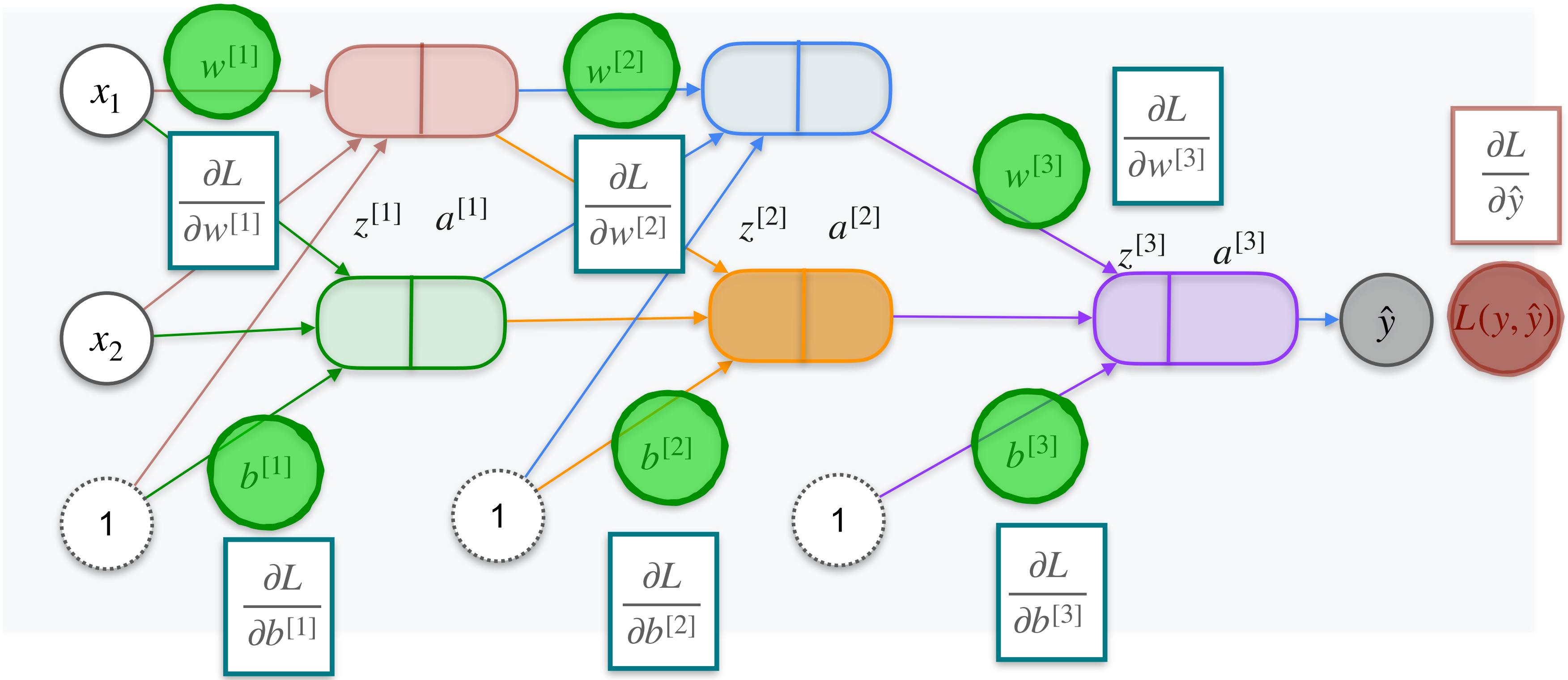

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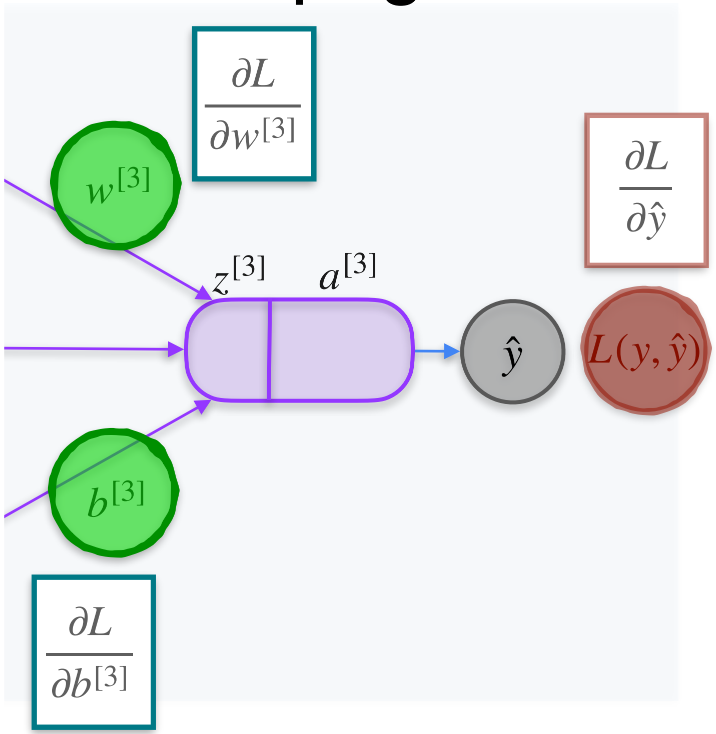

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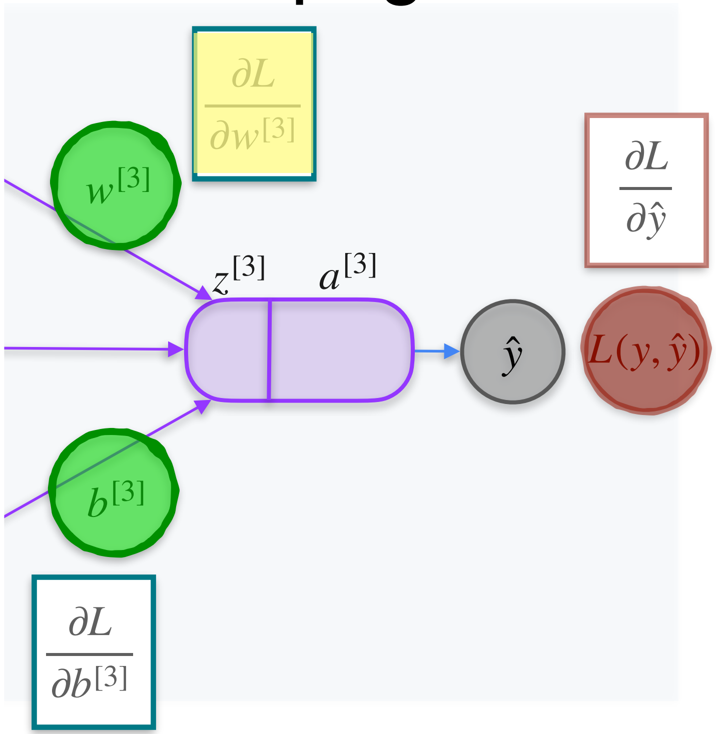

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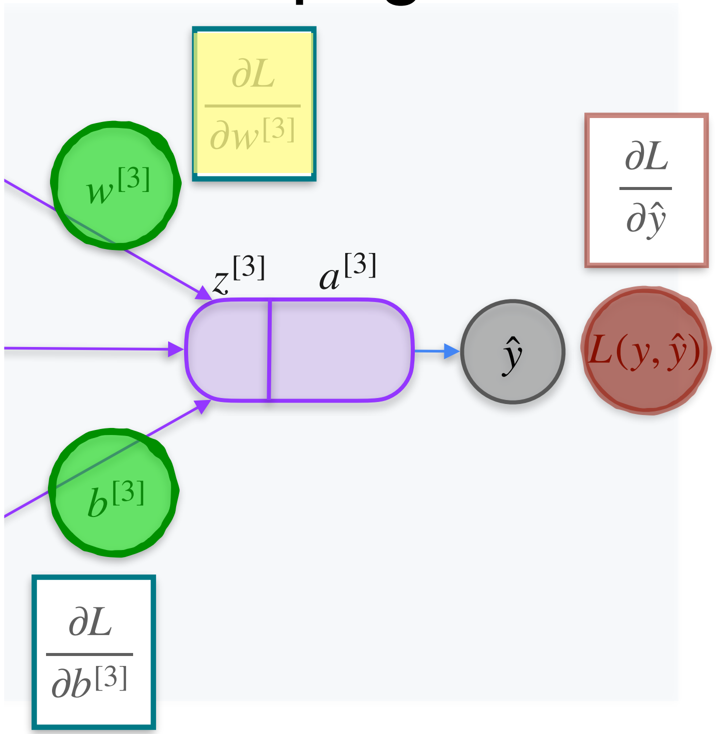

$$\boxed{\frac{\partial L}{\partial w^{[3]}}}$$

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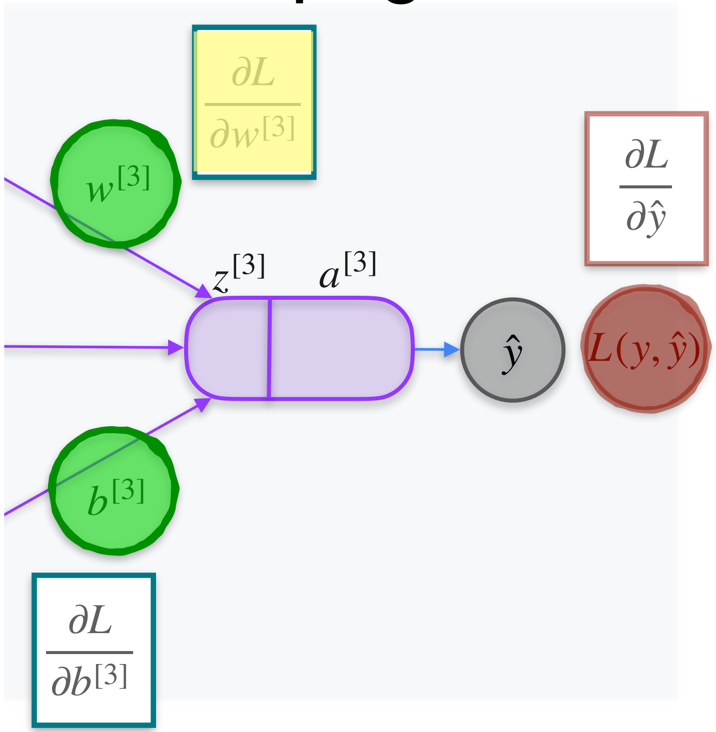

$$\frac{\partial L}{\partial w^{[3]}}$$

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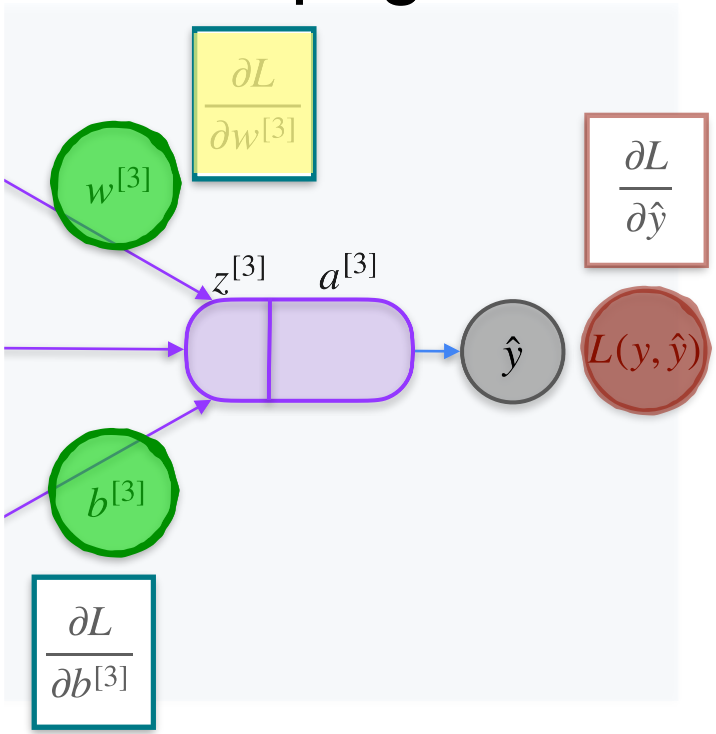

$$\frac{\partial L}{\partial w^{[3]}} \qquad \frac{\partial a^{[3]}}{\partial z^{[3]}} \qquad \frac{\partial L}{\partial a^{[3]}}$$

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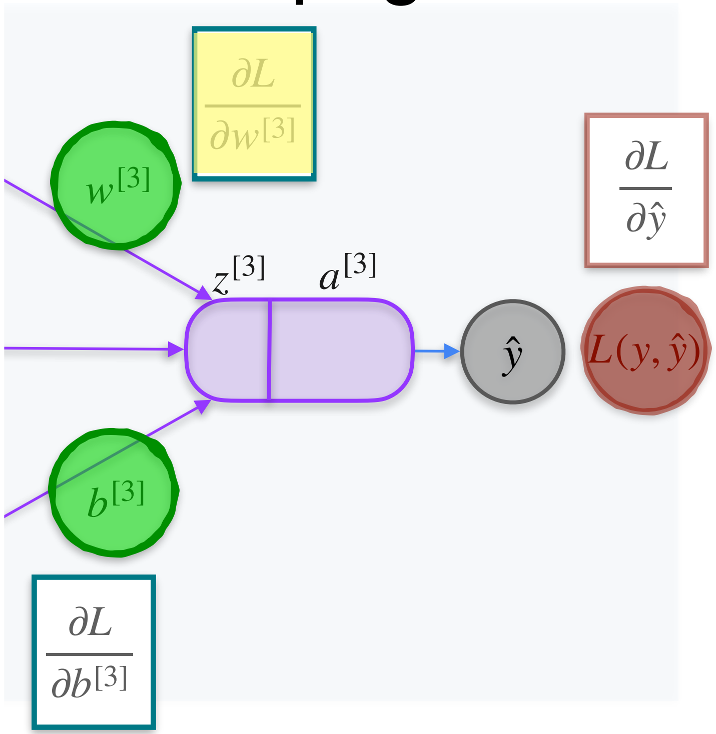

$\frac{\partial L}{\partial w^{[3]}}$ $\frac{\partial z^{[3]}}{\partial w^{[3]}}$ $\frac{\partial a^{[3]}}{\partial z^{[3]}}$ $\frac{\partial L}{\partial a^{[3]}}$

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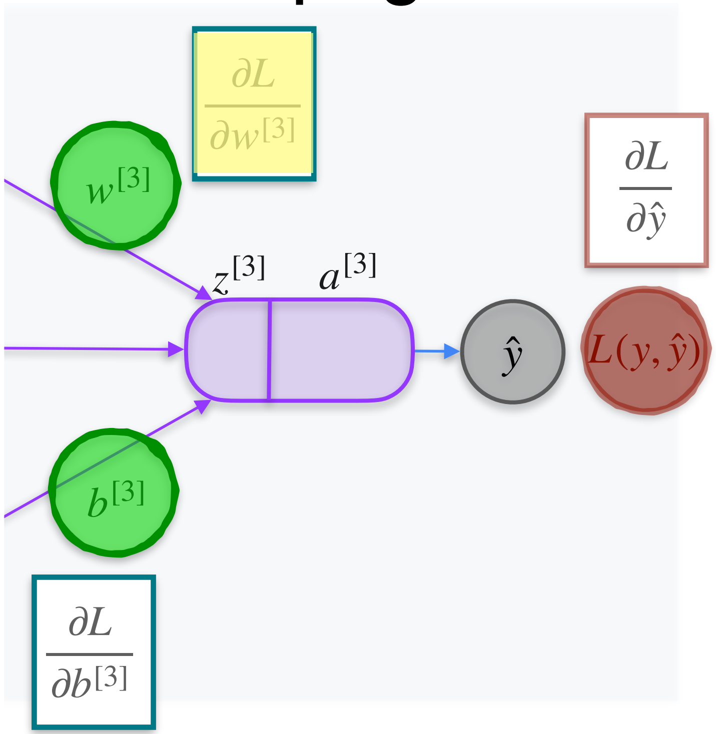
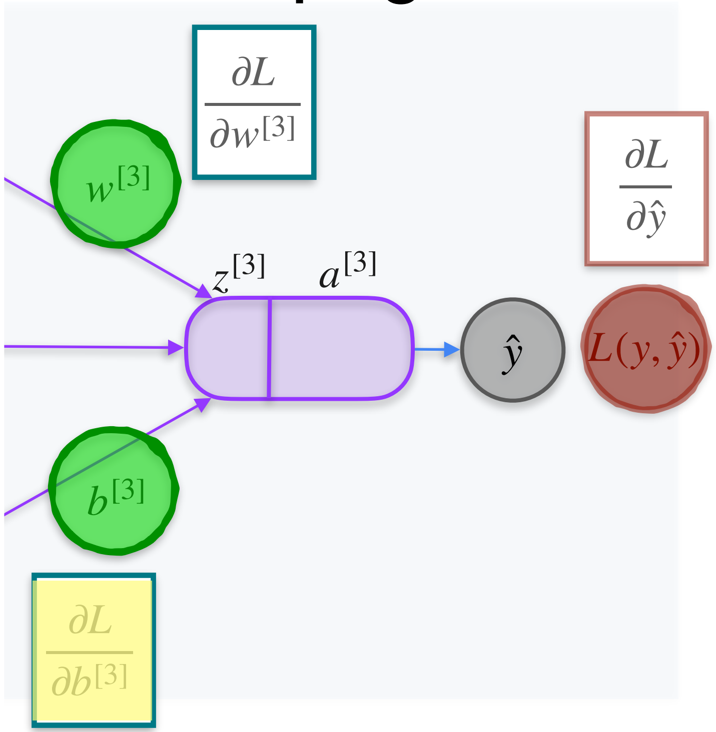


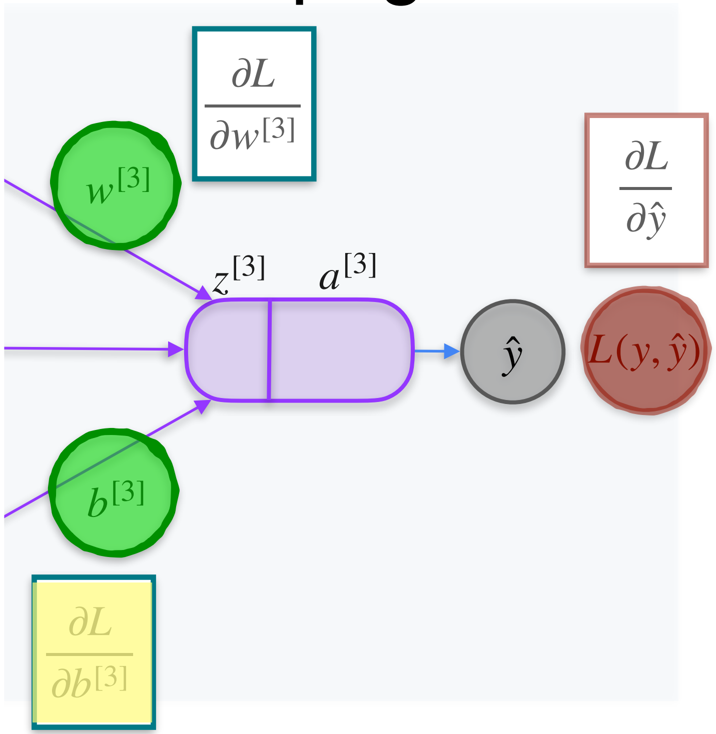
$$\boxed{\frac{\partial L}{\partial w^{[3]}}} = \boxed{\frac{\partial z^{[3]}}{\partial w^{[3]}}} \cdot \boxed{\frac{\partial a^{[3]}}{\partial z^{[3]}}} \cdot \boxed{\frac{\partial L}{\partial a^{[3]}}}$$

Back Propagation Introduction



$$\boxed{\frac{\partial L}{\partial w^{[3]}}} = \boxed{\frac{\partial z^{[3]}}{\partial w^{[3]}}} \cdot \boxed{\frac{\partial a^{[3]}}{\partial z^{[3]}}} \cdot \boxed{\frac{\partial L}{\partial a^{[3]}}}$$

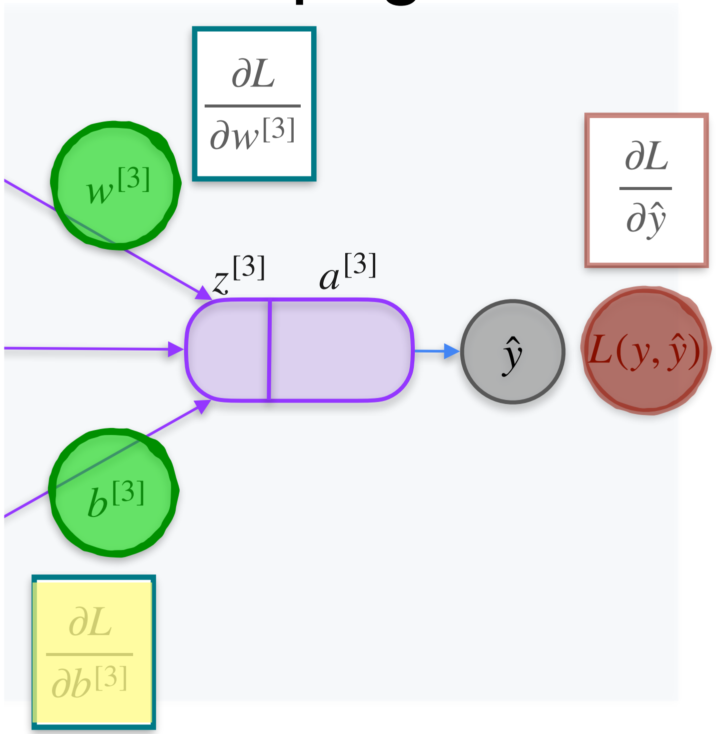
Back Propagation Introduction



$$\boxed{\frac{\partial L}{\partial w^{[3]}}} = \boxed{\frac{\partial z^{[3]}}{\partial w^{[3]}}} \cdot \boxed{\frac{\partial a^{[3]}}{\partial z^{[3]}}} \cdot \boxed{\frac{\partial L}{\partial a^{[3]}}}$$

$$\boxed{\frac{\partial L}{\partial b^{[3]}}}$$

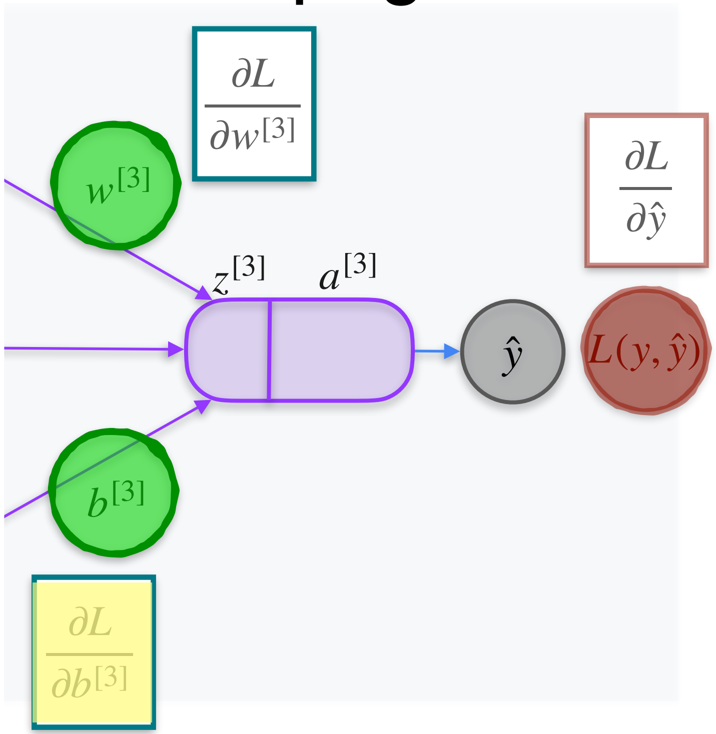
Back Propagation Introduction



$$\frac{\partial L}{\partial w^{[3]}} = \frac{\partial z^{[3]}}{\partial w^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial L}{\partial a^{[3]}}$$

$$\frac{\partial L}{\partial b^{[3]}} \quad \frac{\partial z^{[3]}}{\partial b^{[3]}} \quad \frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

Back Propagation Introduction



$$\boxed{\frac{\partial L}{\partial w^{[3]}}} = \boxed{\frac{\partial z^{[3]}}{\partial w^{[3]}}} \cdot \boxed{\frac{\partial a^{[3]}}{\partial z^{[3]}}} \cdot \boxed{\frac{\partial L}{\partial a^{[3]}}}$$

$$\boxed{\frac{\partial L}{\partial b^{[3]}}} = \boxed{\frac{\partial z^{[3]}}{\partial b^{[3]}}} \cdot \boxed{\frac{\partial a^{[3]}}{\partial z^{[3]}}} \cdot \boxed{\frac{\partial L}{\partial a^{[3]}}}$$

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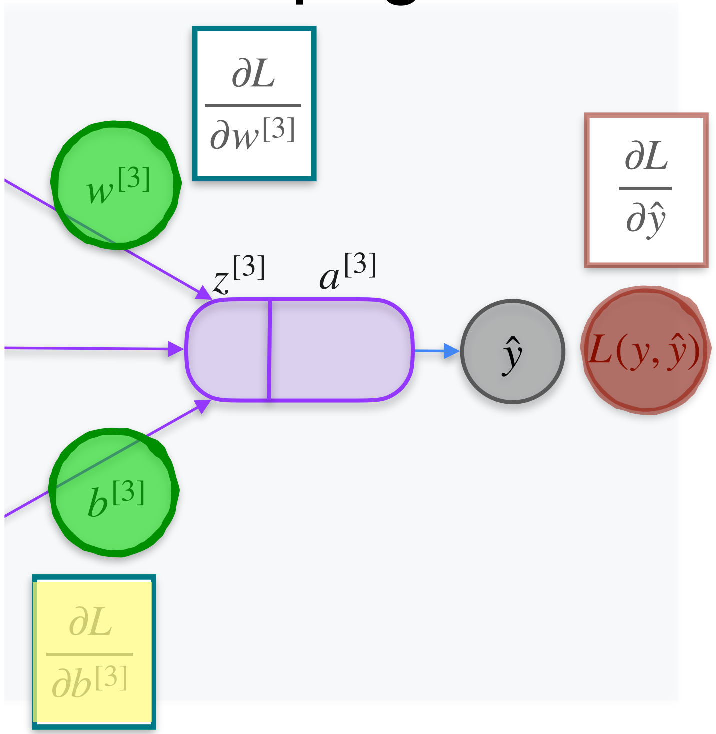

$$\frac{\partial L}{\partial w^{[3]}} = \frac{\partial z^{[3]}}{\partial w^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial L}{\partial a^{[3]}}$$

$$\frac{\partial L}{\partial b^{[3]}} = \frac{\partial z^{[3]}}{\partial b^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial L}{\partial a^{[3]}}$$

$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

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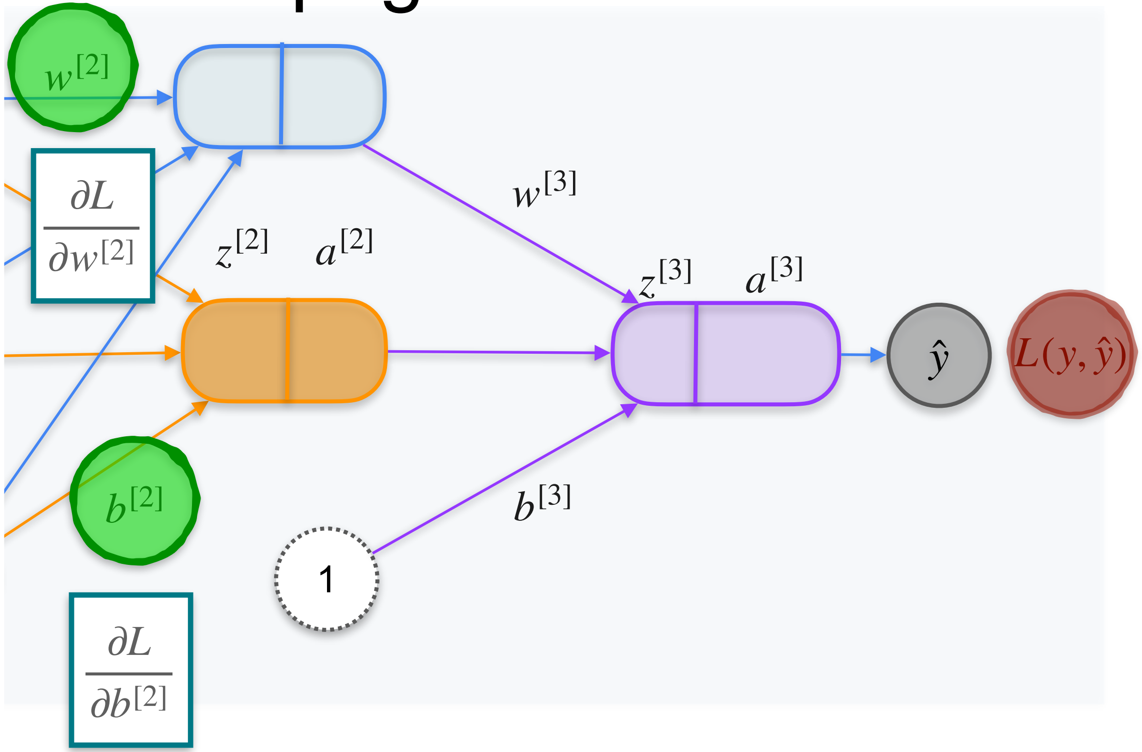

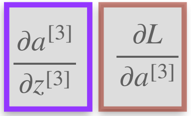

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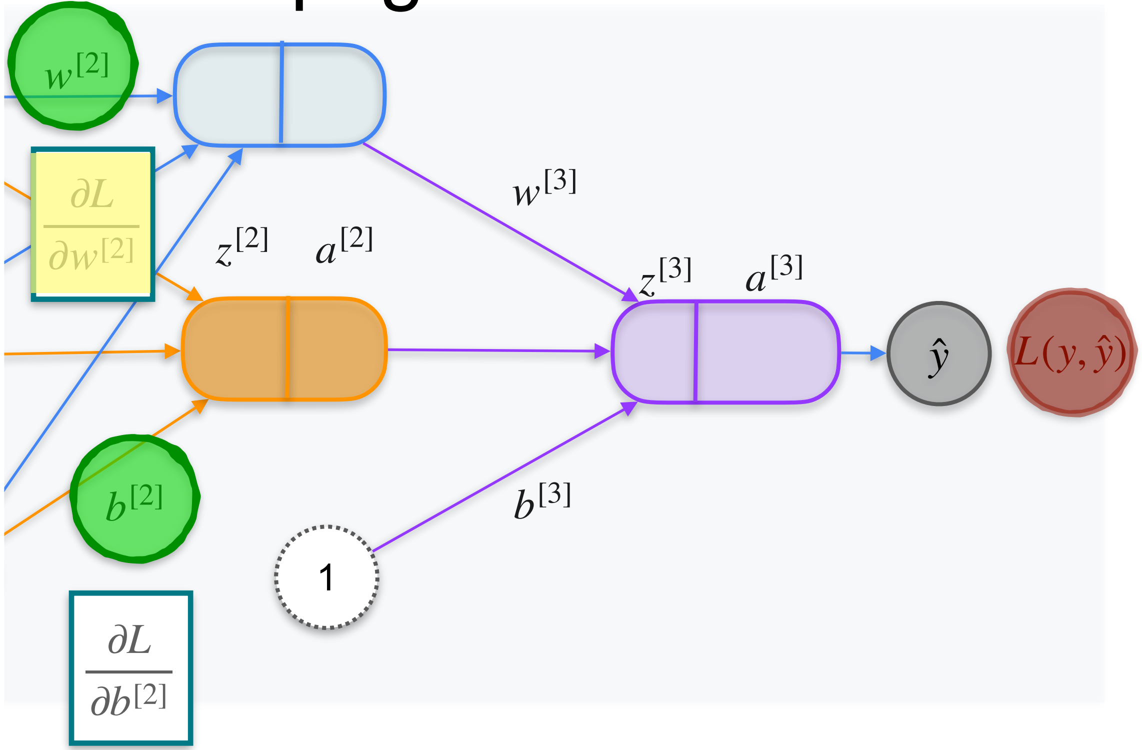

$\frac{\partial a^{[3]}}{\partial z^{[3]}}$ $\frac{\partial L}{\partial a^{[3]}}$

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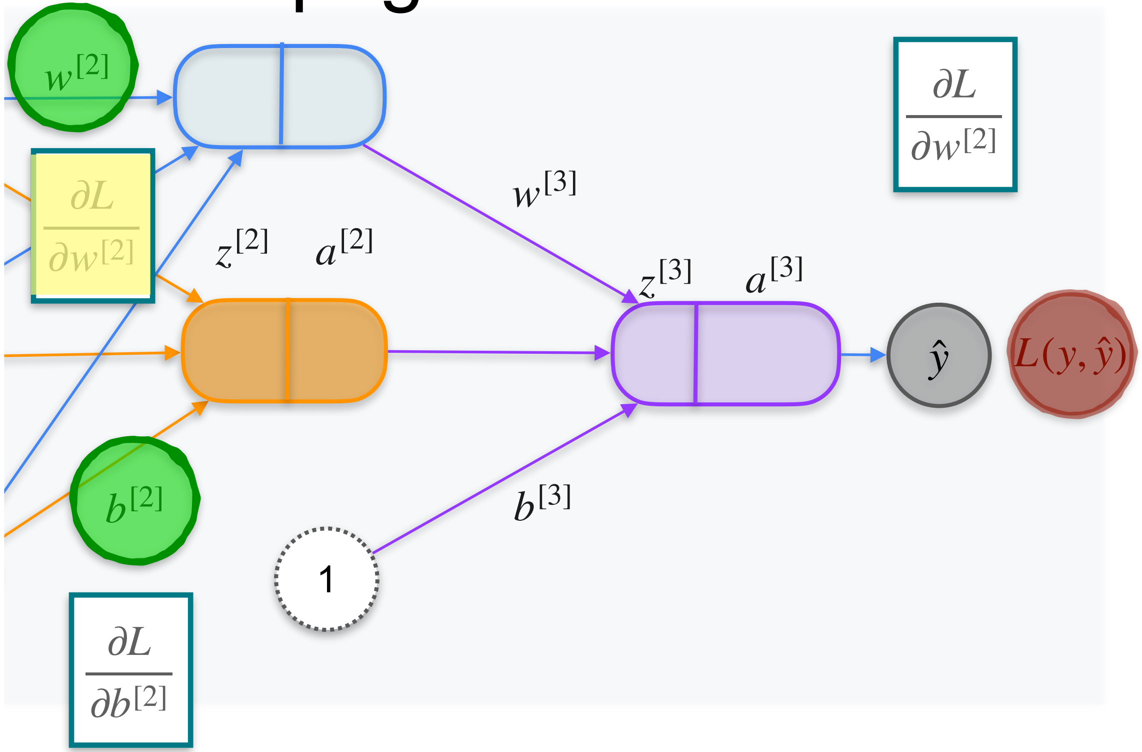

$\frac{\partial a^{[3]}}{\partial z^{[3]}}$ $\frac{\partial L}{\partial a^{[3]}}$

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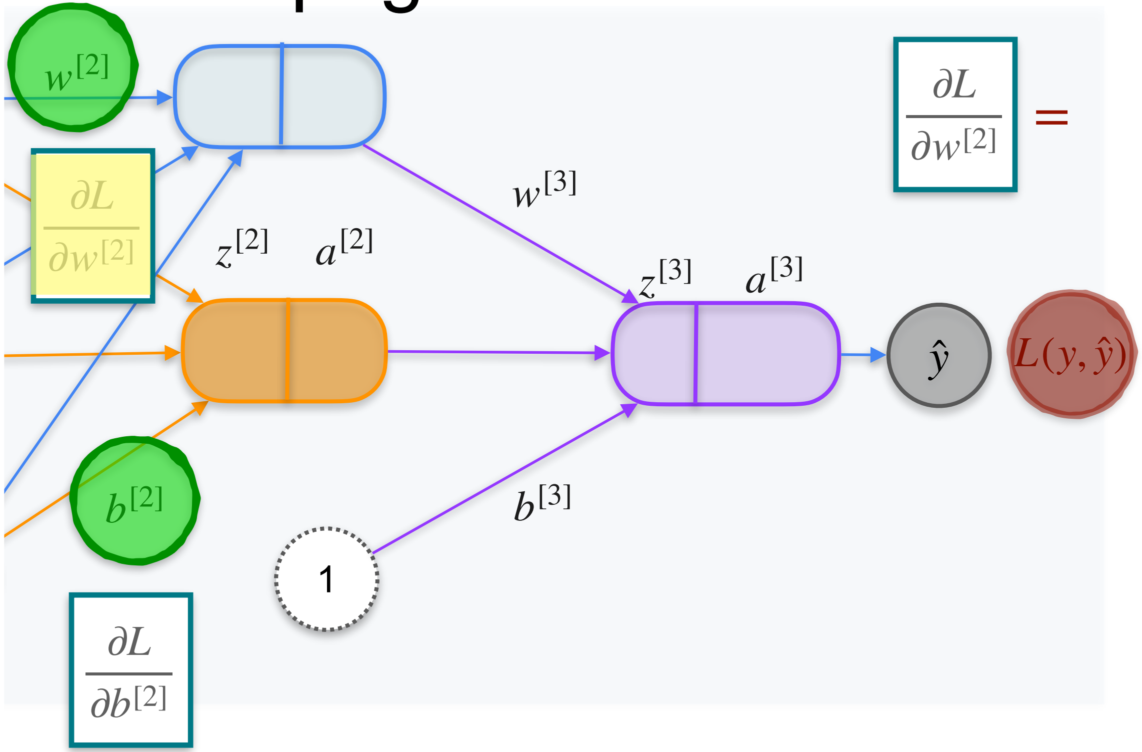
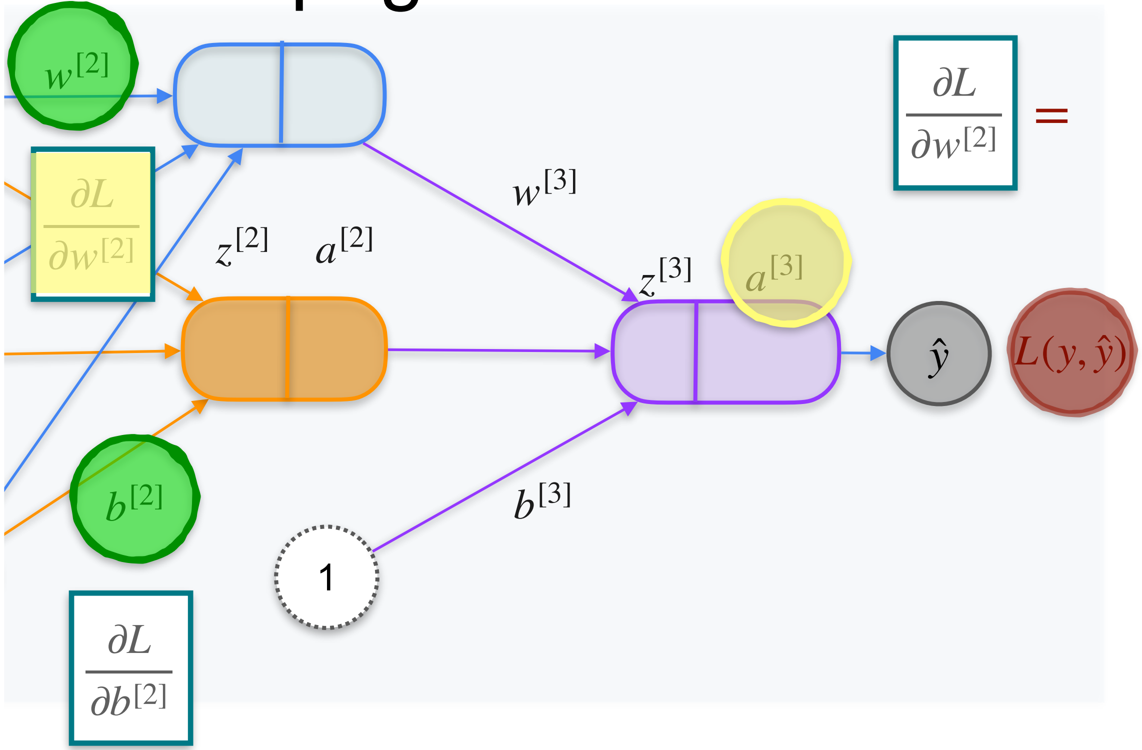


$\frac{\partial a^{[3]}}{\partial z^{[3]}}$ $\frac{\partial L}{\partial a^{[3]}}$

Back Propagation Introduction



$$\frac{\partial L}{\partial w^{[2]}} =$$

$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

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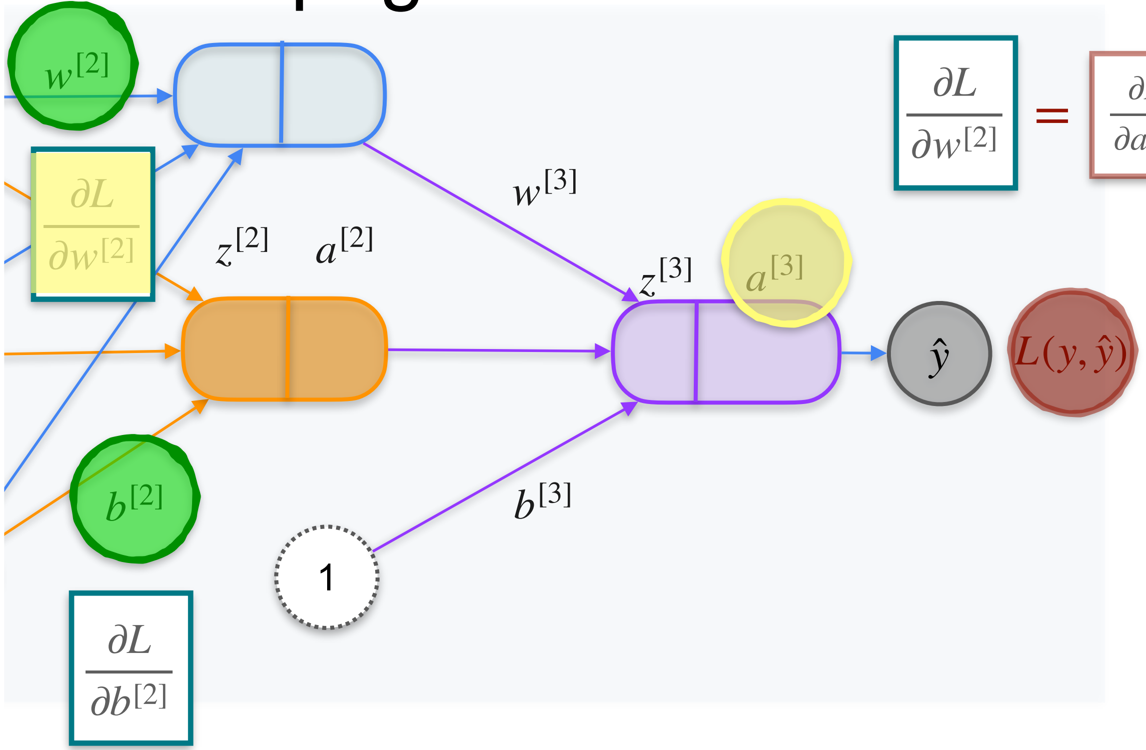

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}}$$

$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

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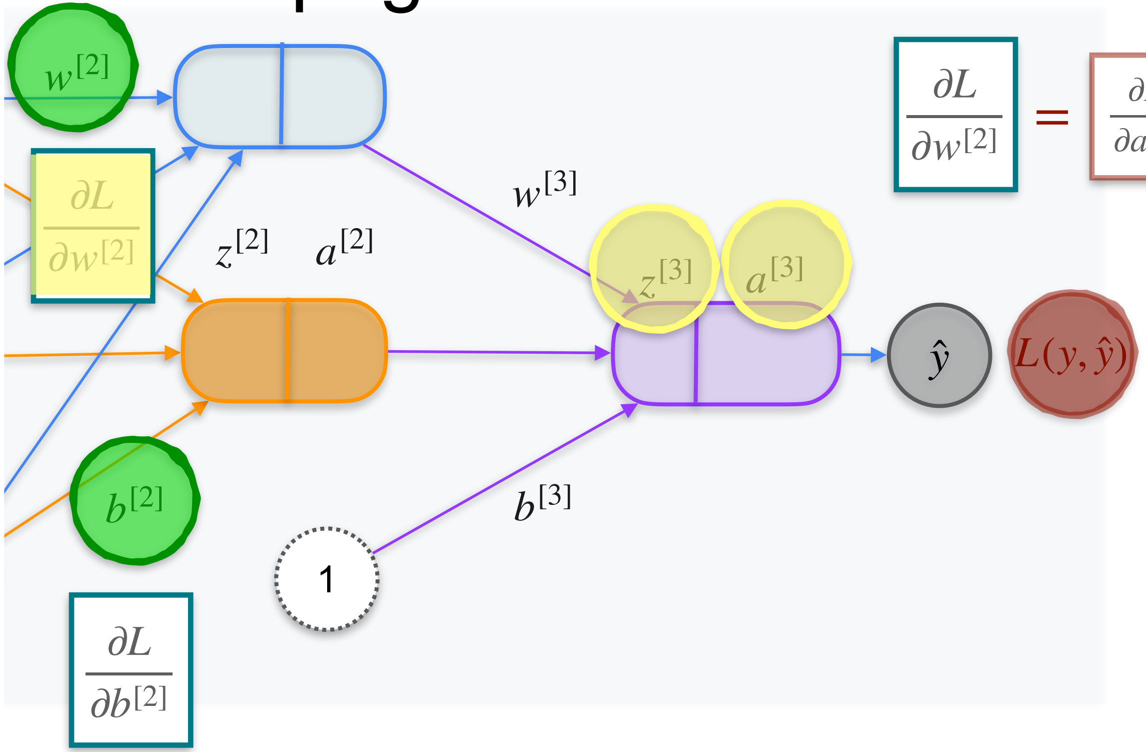

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}}$$

$\frac{\partial a^{[3]}}{\partial z^{[3]}}$ $\frac{\partial L}{\partial a^{[3]}}$

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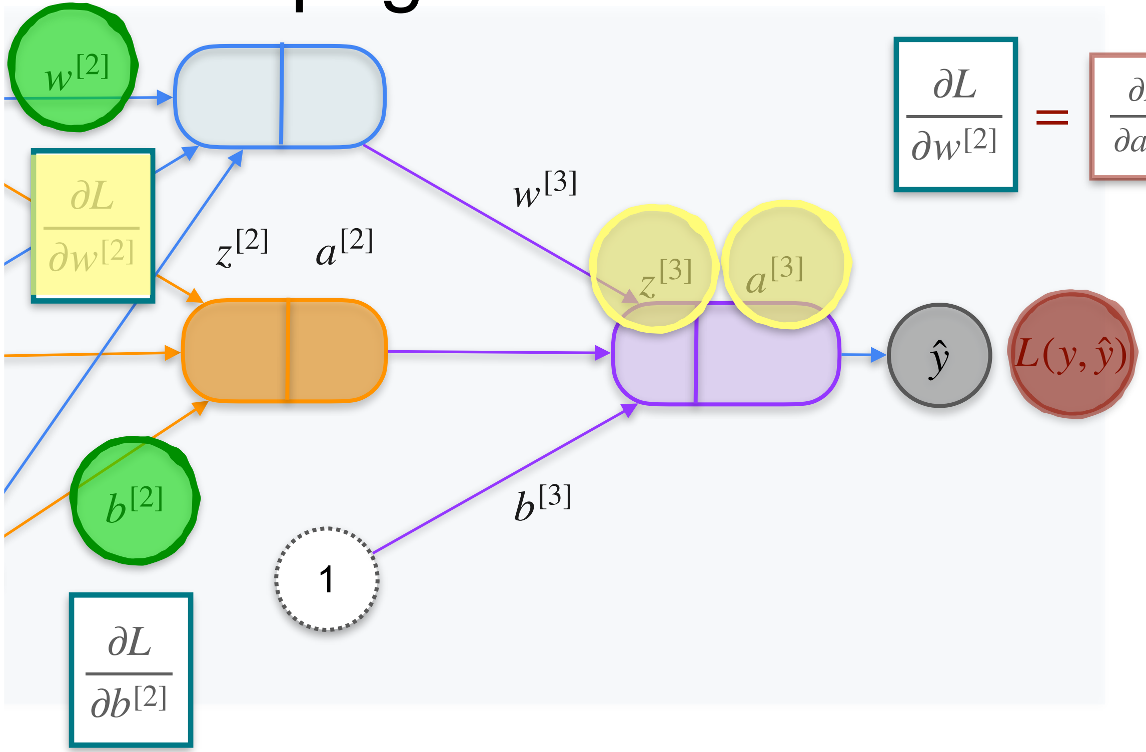

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \frac{\partial a^{[3]}}{\partial z^{[3]}}$$

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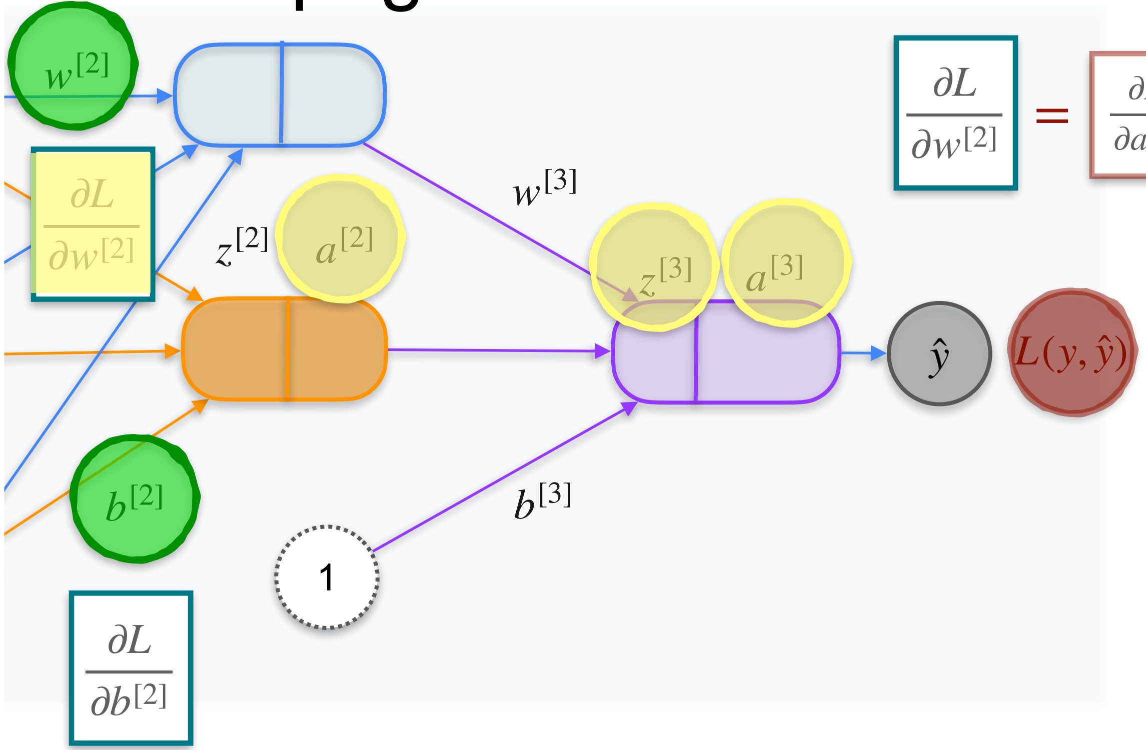

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \frac{\partial a^{[3]}}{\partial z^{[3]}}$$

$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

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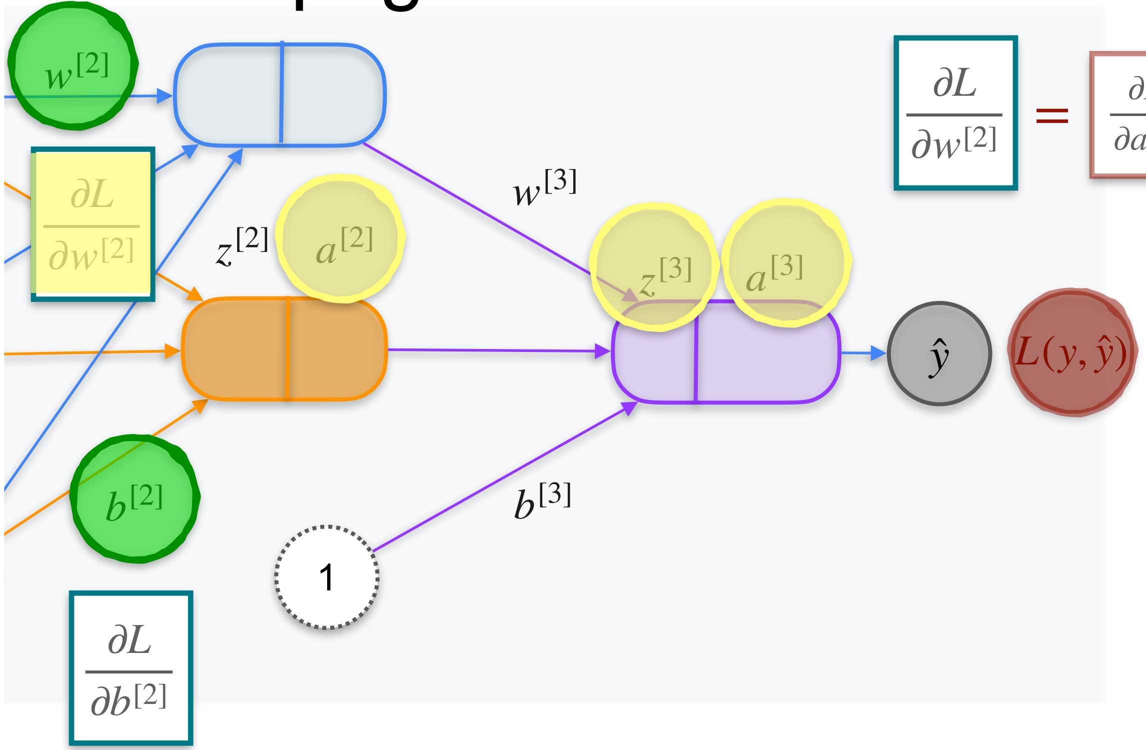

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \frac{\partial a^{[3]}}{\partial z^{[3]}} \frac{\partial z^{[3]}}{\partial a^{[2]}}$$

$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

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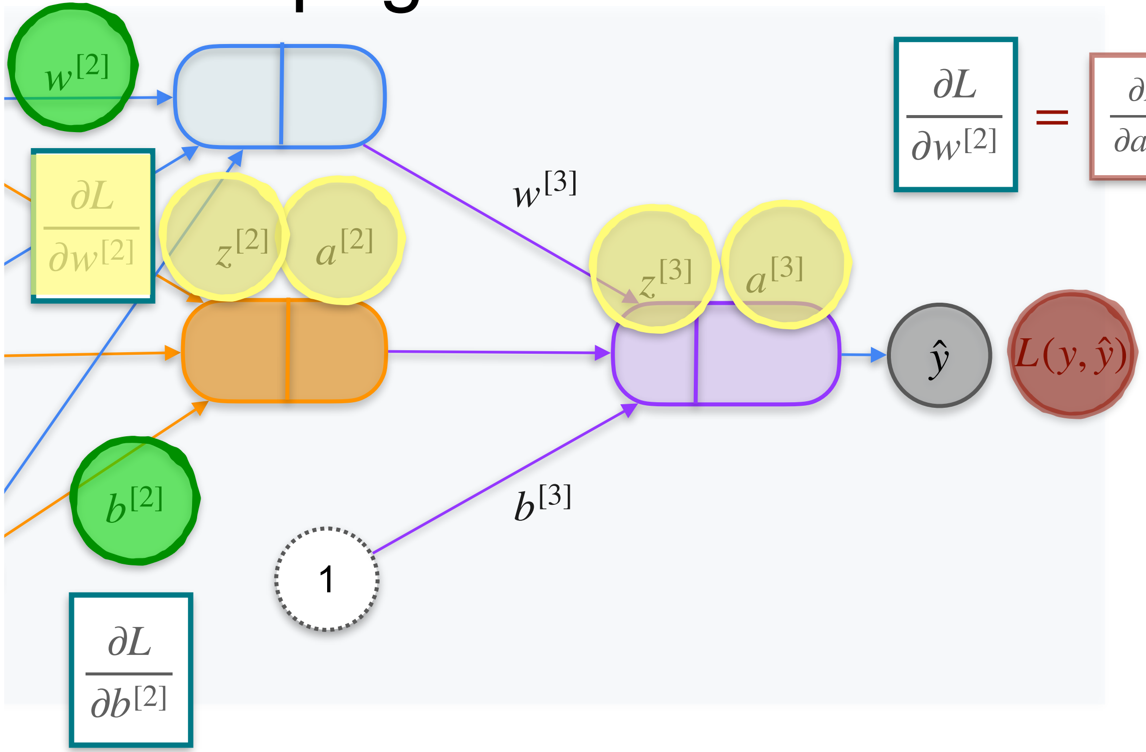

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \frac{\partial a^{[3]}}{\partial z^{[3]}} \frac{\partial z^{[3]}}{\partial a^{[2]}}$$

$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \frac{\partial L}{\partial a^{[3]}}$$

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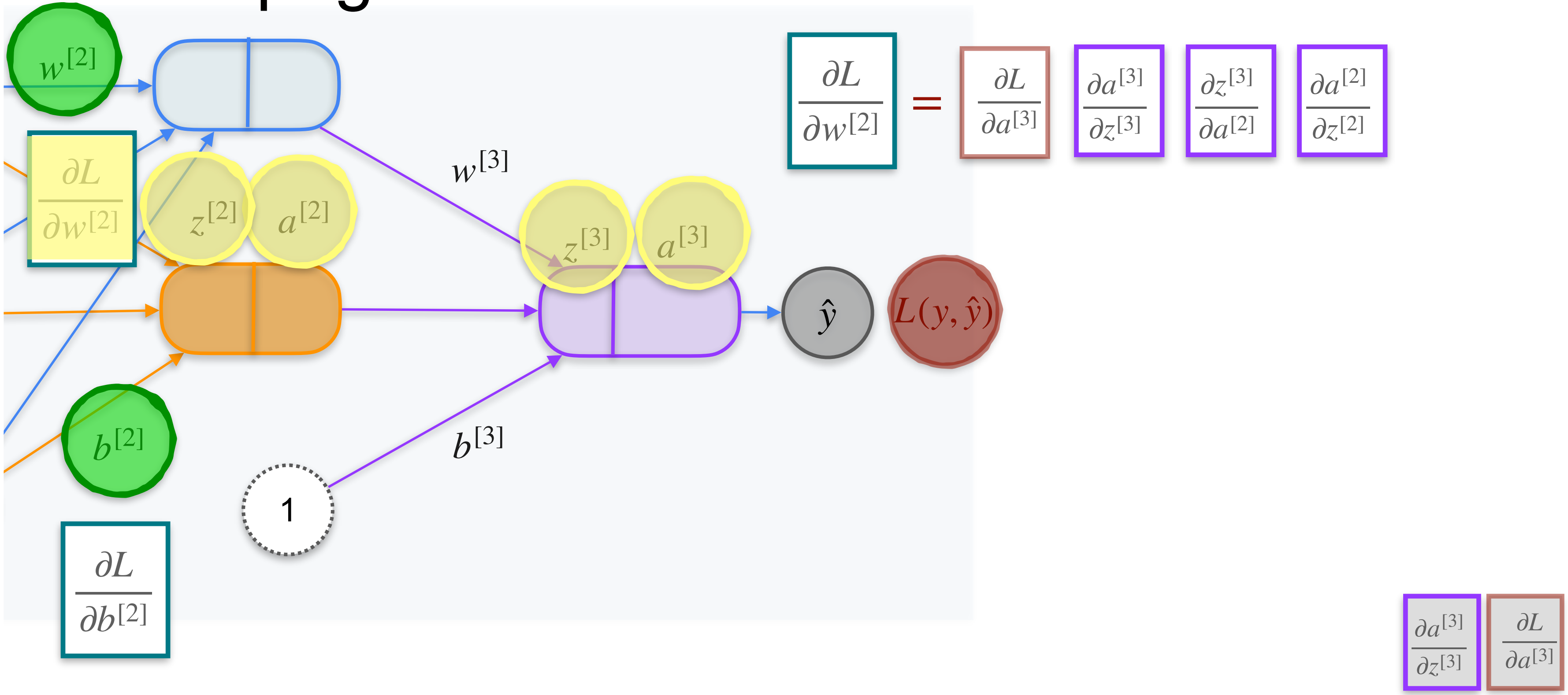

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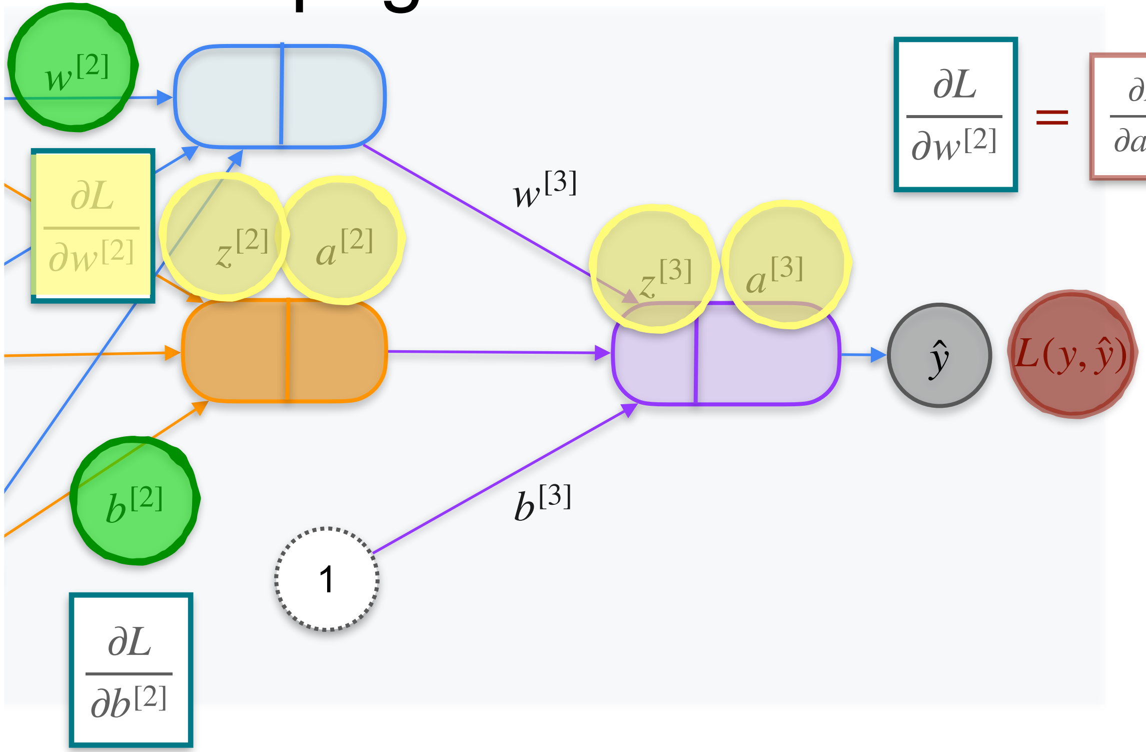

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \frac{\partial a^{[3]}}{\partial z^{[3]}} \frac{\partial z^{[3]}}{\partial a^{[2]}} \frac{\partial a^{[2]}}{\partial z^{[2]}} \frac{\partial z^{[2]}}{\partial w^{[2]}}$$

$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

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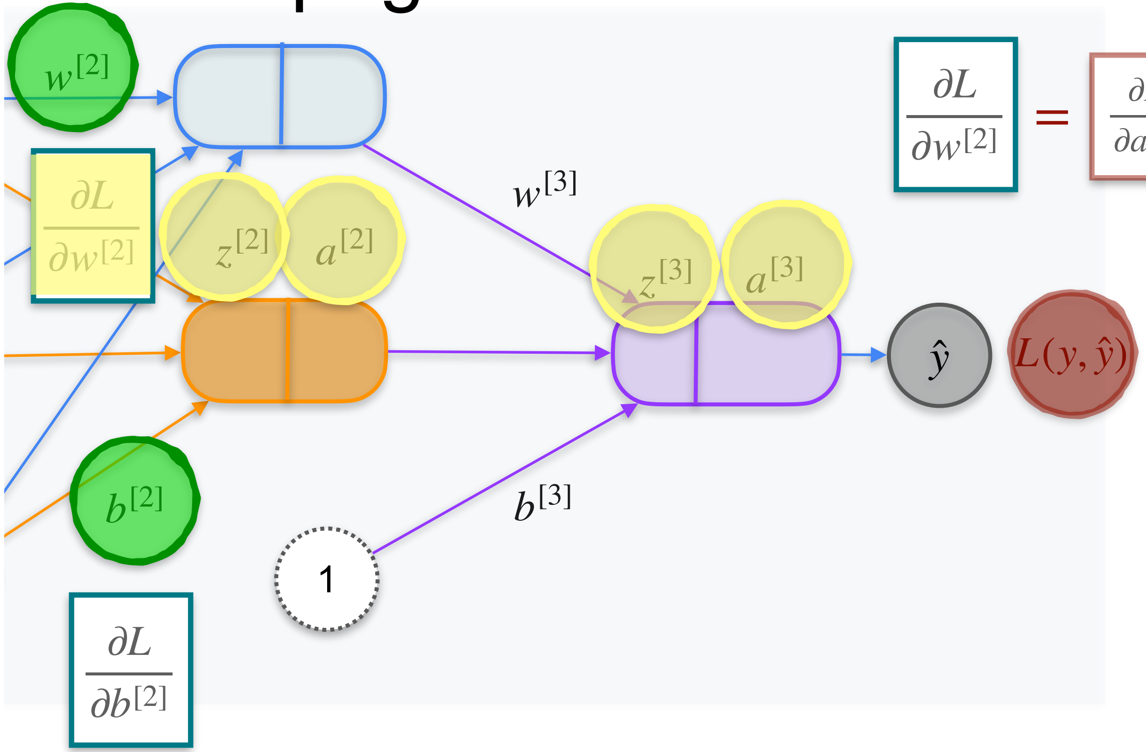

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial z^{[3]}}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial w^{[2]}}$$

$\frac{\partial a^{[3]}}{\partial z^{[3]}}$ $\frac{\partial L}{\partial a^{[3]}}$

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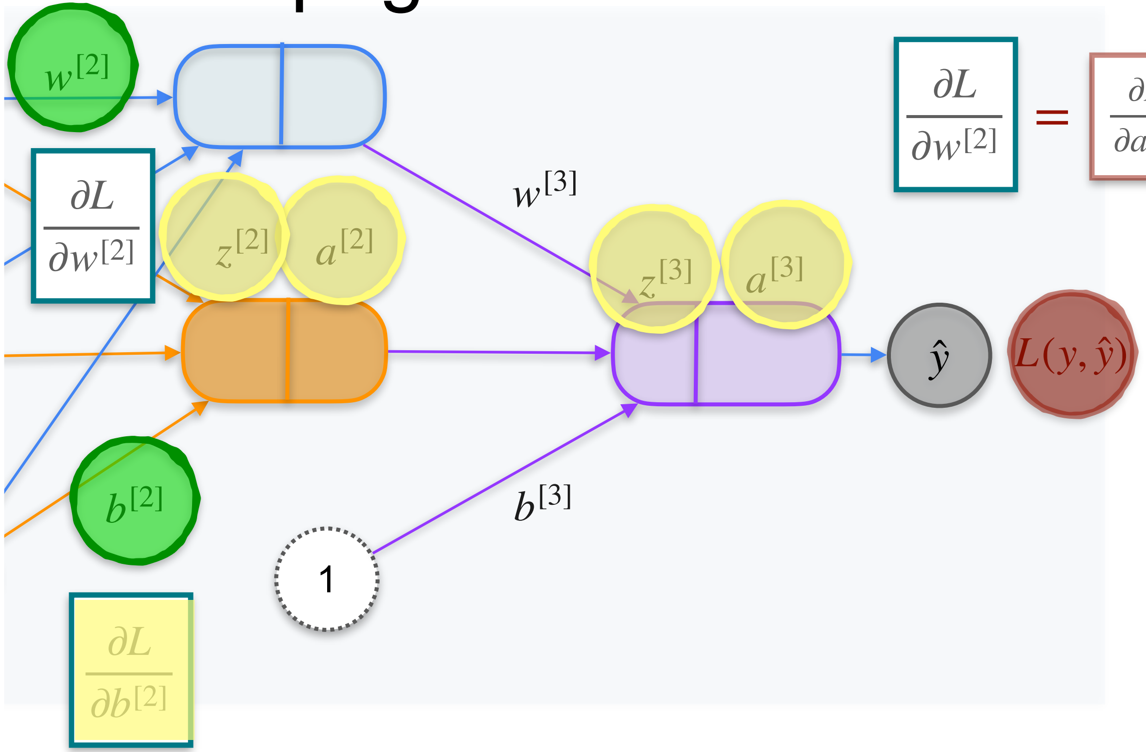

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial z^{[3]}}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial w^{[2]}}$$

$\frac{\partial a^{[3]}}{\partial z^{[3]}}$ $\frac{\partial L}{\partial a^{[3]}}$

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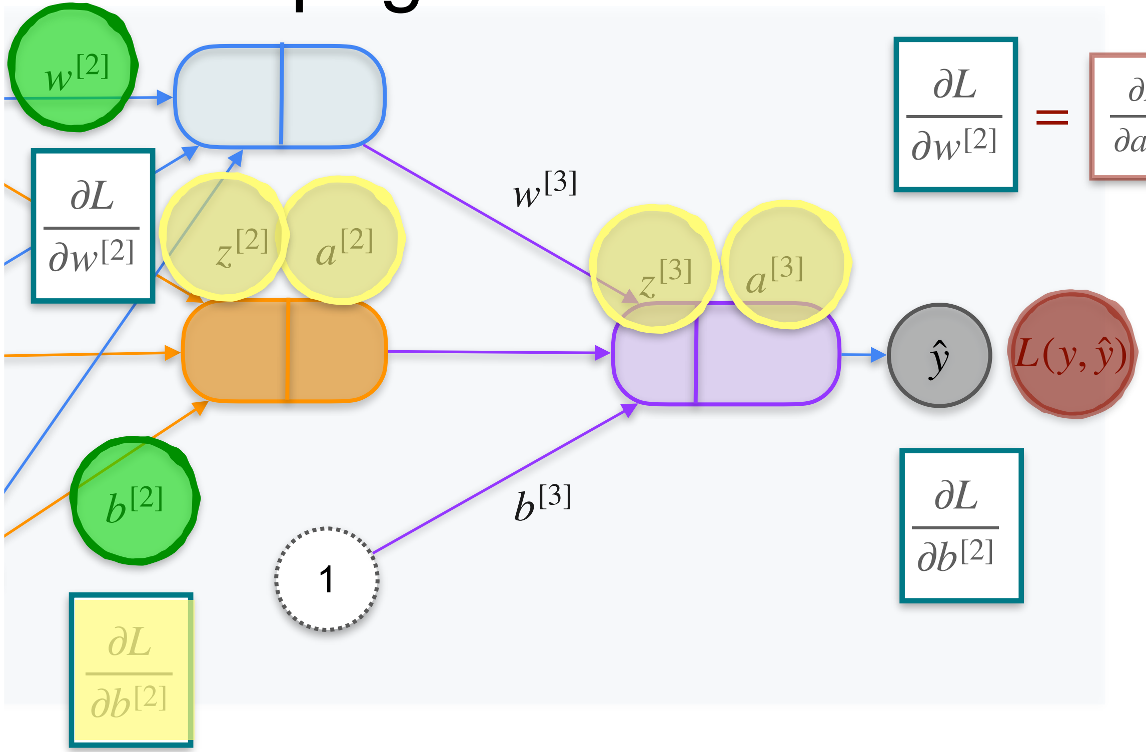

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial z^{[3]}}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial w^{[2]}}$$

$\frac{\partial a^{[3]}}{\partial z^{[3]}}$ $\frac{\partial L}{\partial a^{[3]}}$

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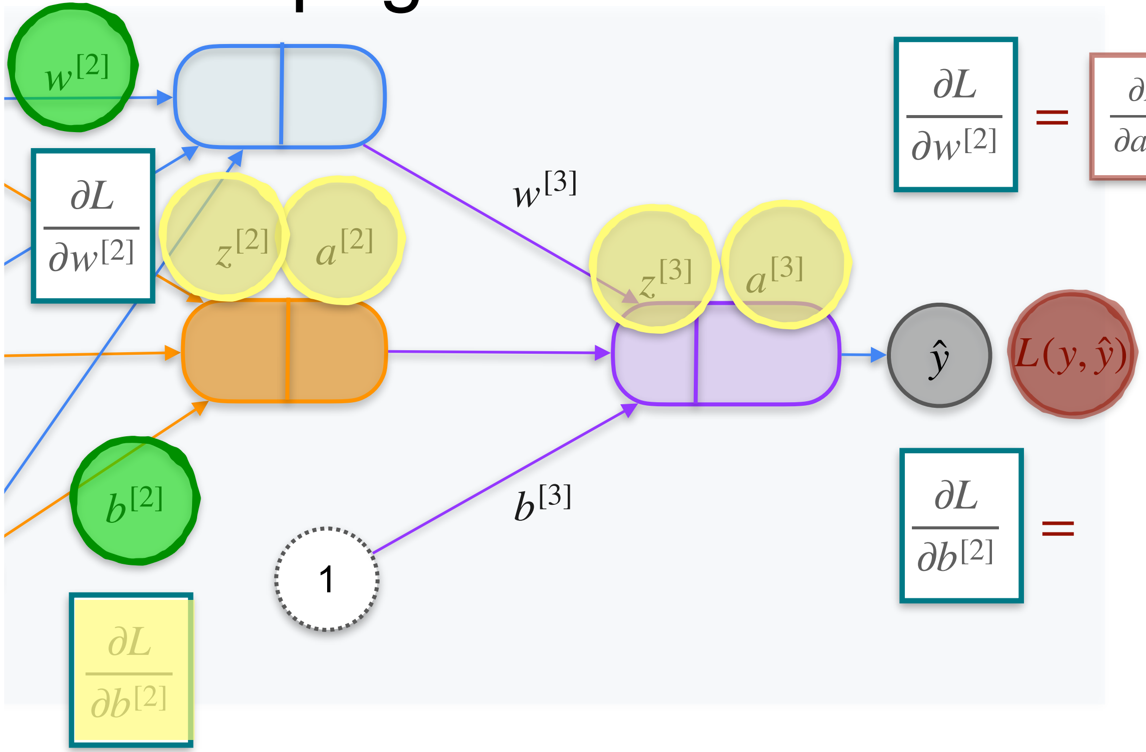

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial z^{[3]}}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial w^{[2]}}$$

$$\frac{\partial L}{\partial b^{[2]}} =$$

$\frac{\partial a^{[3]}}{\partial z^{[3]}}$ $\frac{\partial L}{\partial a^{[3]}}$

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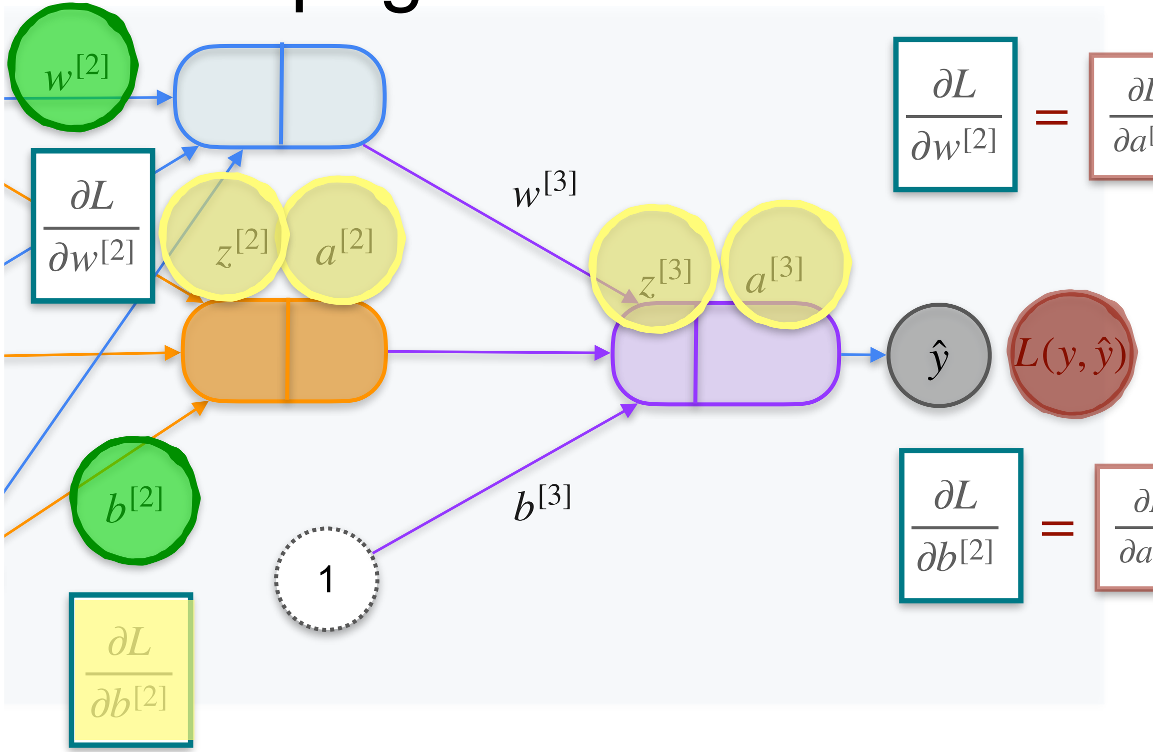

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial z^{[3]}}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial w^{[2]}}$$

$$\frac{\partial L}{\partial b^{[2]}} = \frac{\partial L}{\partial a^{[3]}}$$

$\frac{\partial a^{[3]}}{\partial z^{[3]}}$ $\frac{\partial L}{\partial a^{[3]}}$

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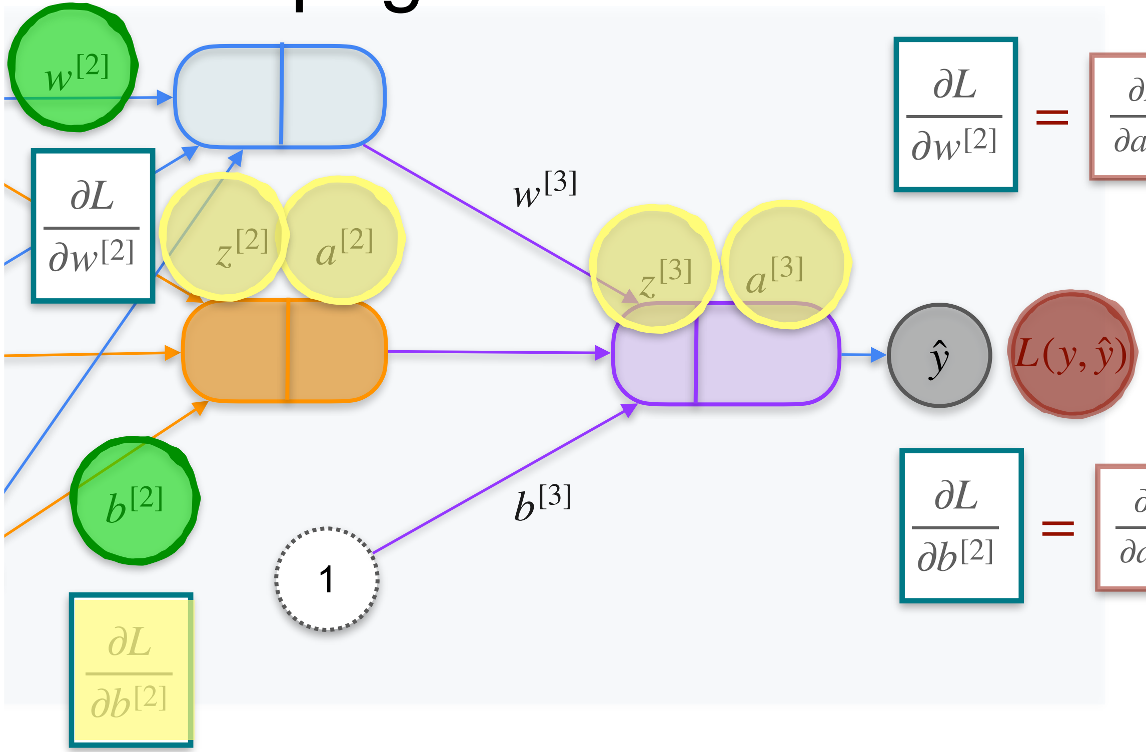

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial z^{[3]}}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial w^{[2]}}$$

$$\frac{\partial L}{\partial b^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \quad \frac{\partial a^{[3]}}{\partial z^{[3]}}$$

$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

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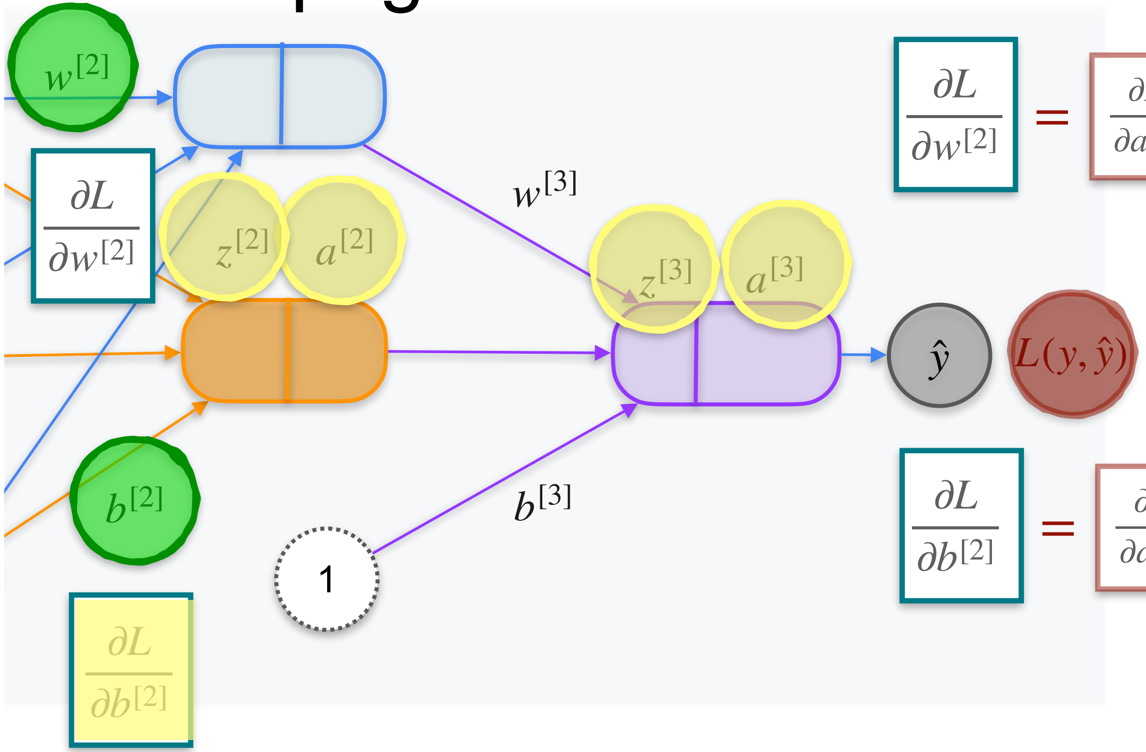

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial z^{[3]}}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial w^{[2]}}$$

$$\frac{\partial L}{\partial b^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \quad \frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial z^{[3]}}{\partial a^{[2]}}$$

$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

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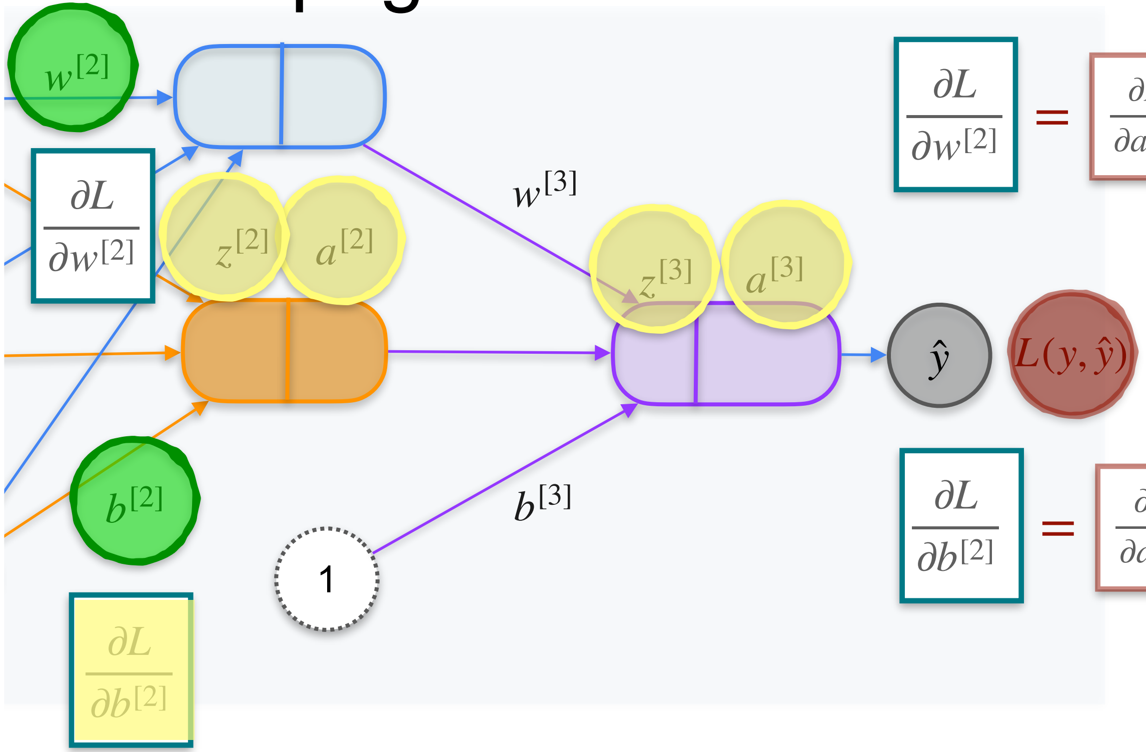

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial z^{[3]}}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial w^{[2]}}$$

$$\frac{\partial L}{\partial b^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \quad \frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial z^{[3]}}{\partial a^{[2]}} \quad \frac{\partial a^{[2]}}{\partial z^{[2]}}$$

$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

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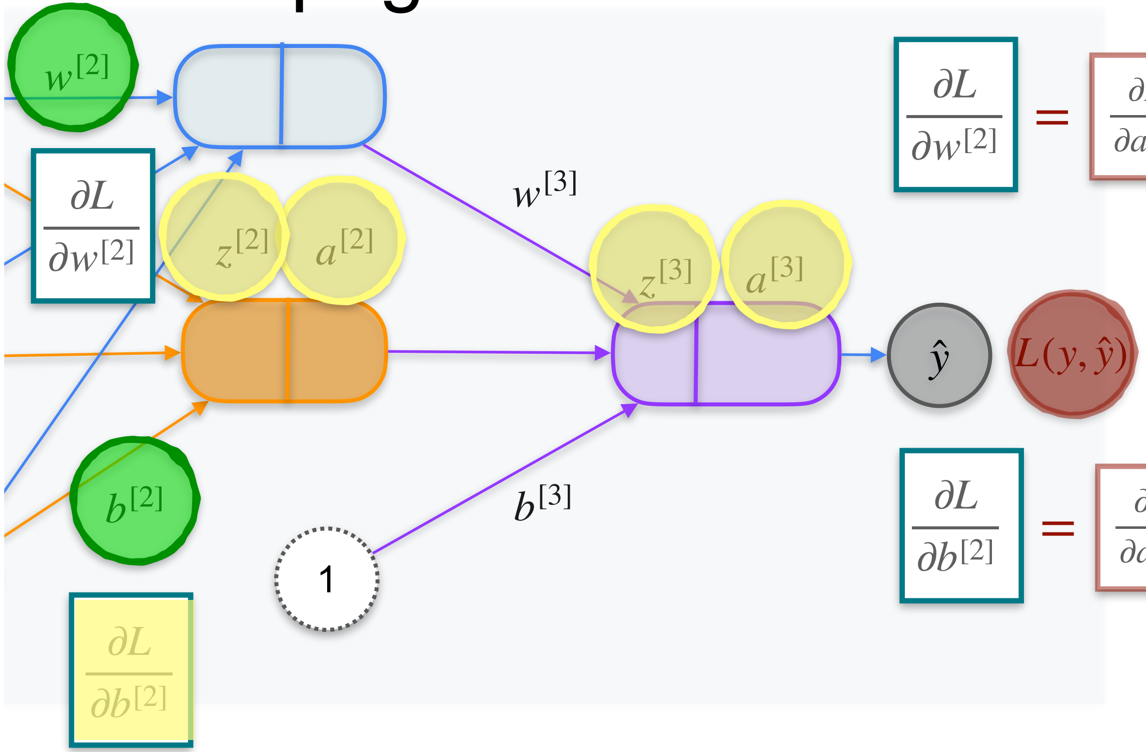

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial z^{[3]}}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial w^{[2]}}$$

$$\frac{\partial L}{\partial b^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \quad \frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial z^{[3]}}{\partial a^{[2]}} \quad \frac{\partial a^{[2]}}{\partial z^{[2]}} \quad \frac{\partial z^{[2]}}{\partial b^{[2]}}$$

$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

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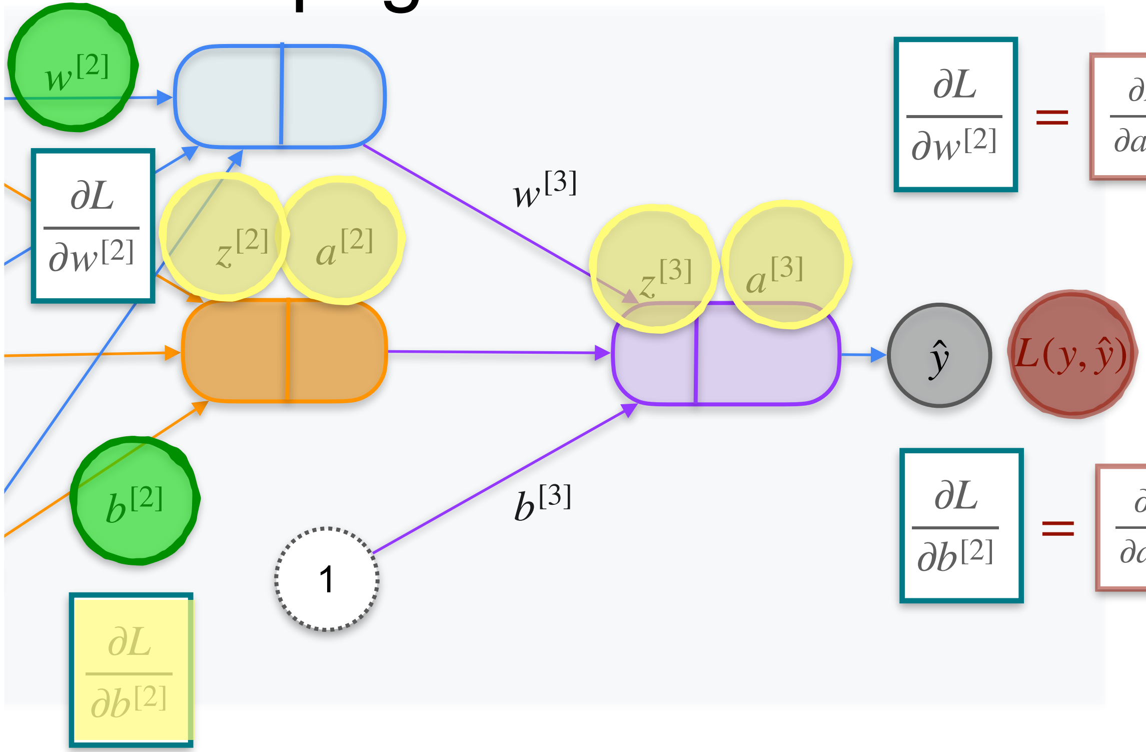

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial z^{[3]}}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial w^{[2]}}$$

$$\frac{\partial L}{\partial b^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial z^{[3]}}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial b^{[2]}}$$

$\frac{\partial a^{[3]}}{\partial z^{[3]}}$ $\frac{\partial L}{\partial a^{[3]}}$

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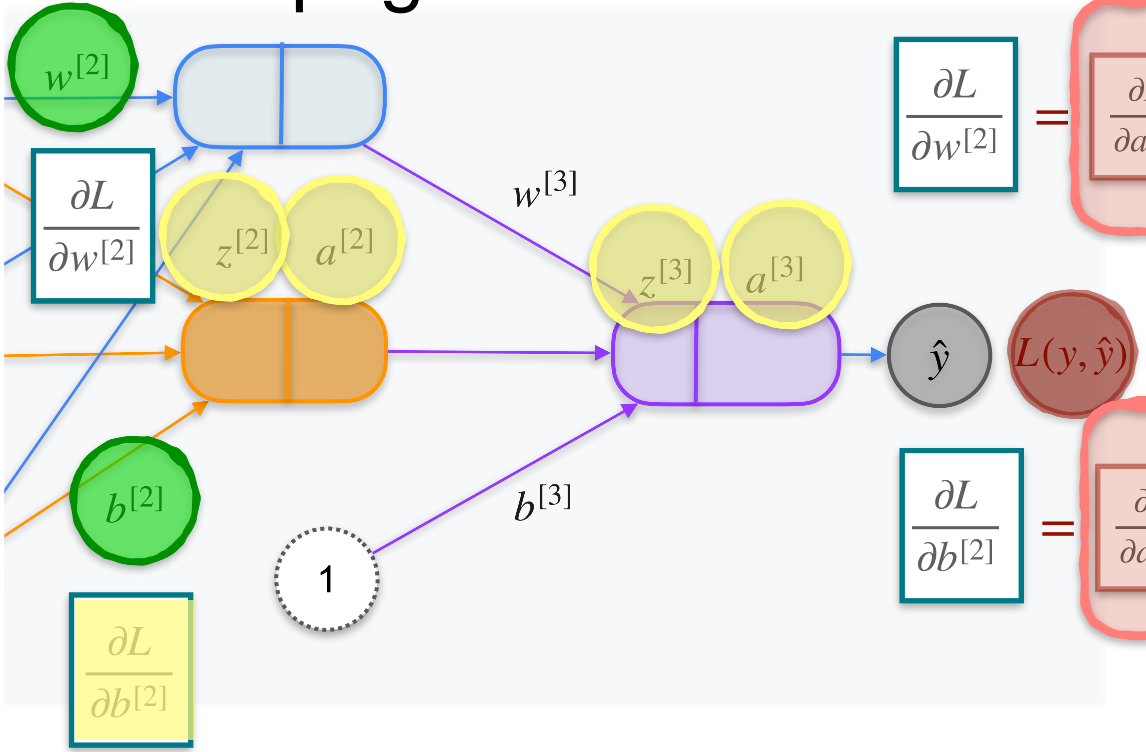

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial z^{[3]}}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial w^{[2]}}$$

$$\frac{\partial L}{\partial b^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial z^{[3]}}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial b^{[2]}}$$

$\frac{\partial a^{[3]}}{\partial z^{[3]}}$ $\frac{\partial L}{\partial a^{[3]}}$

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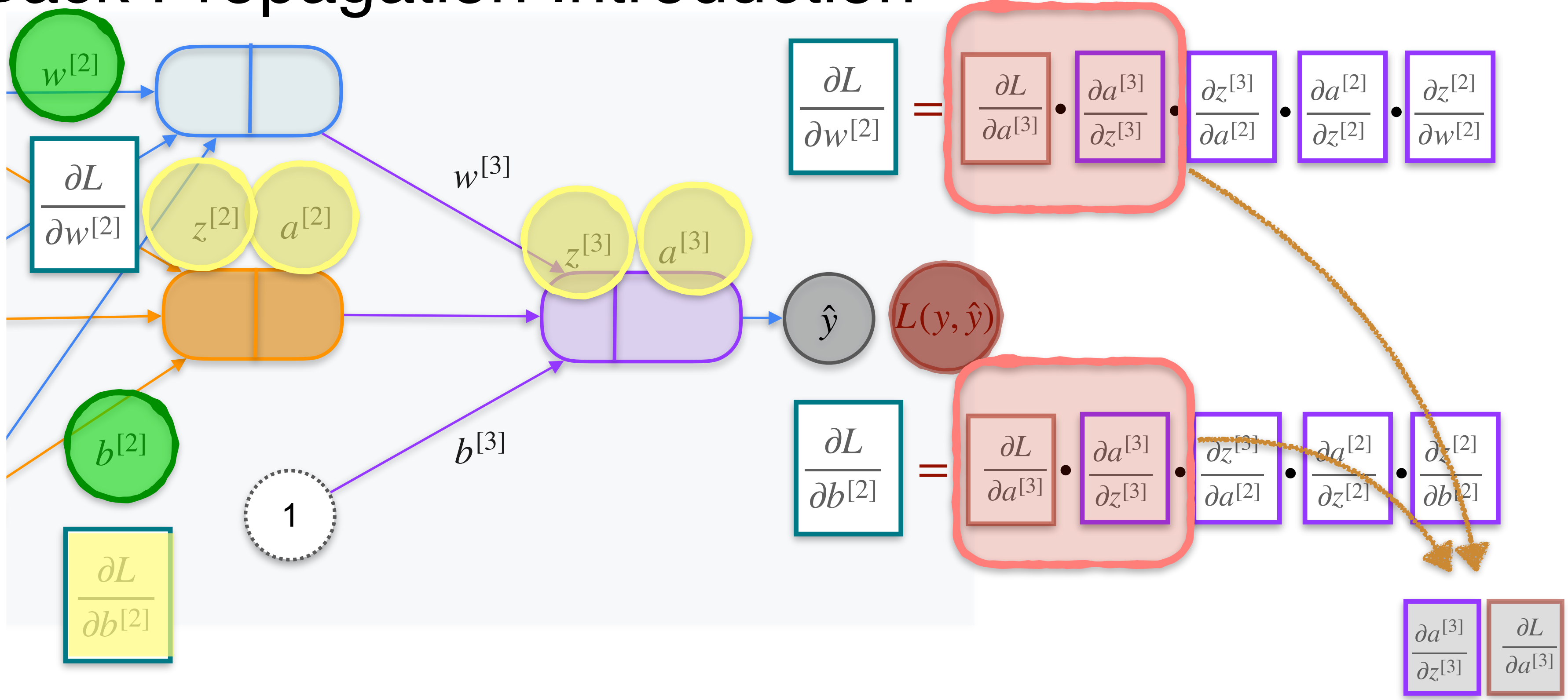

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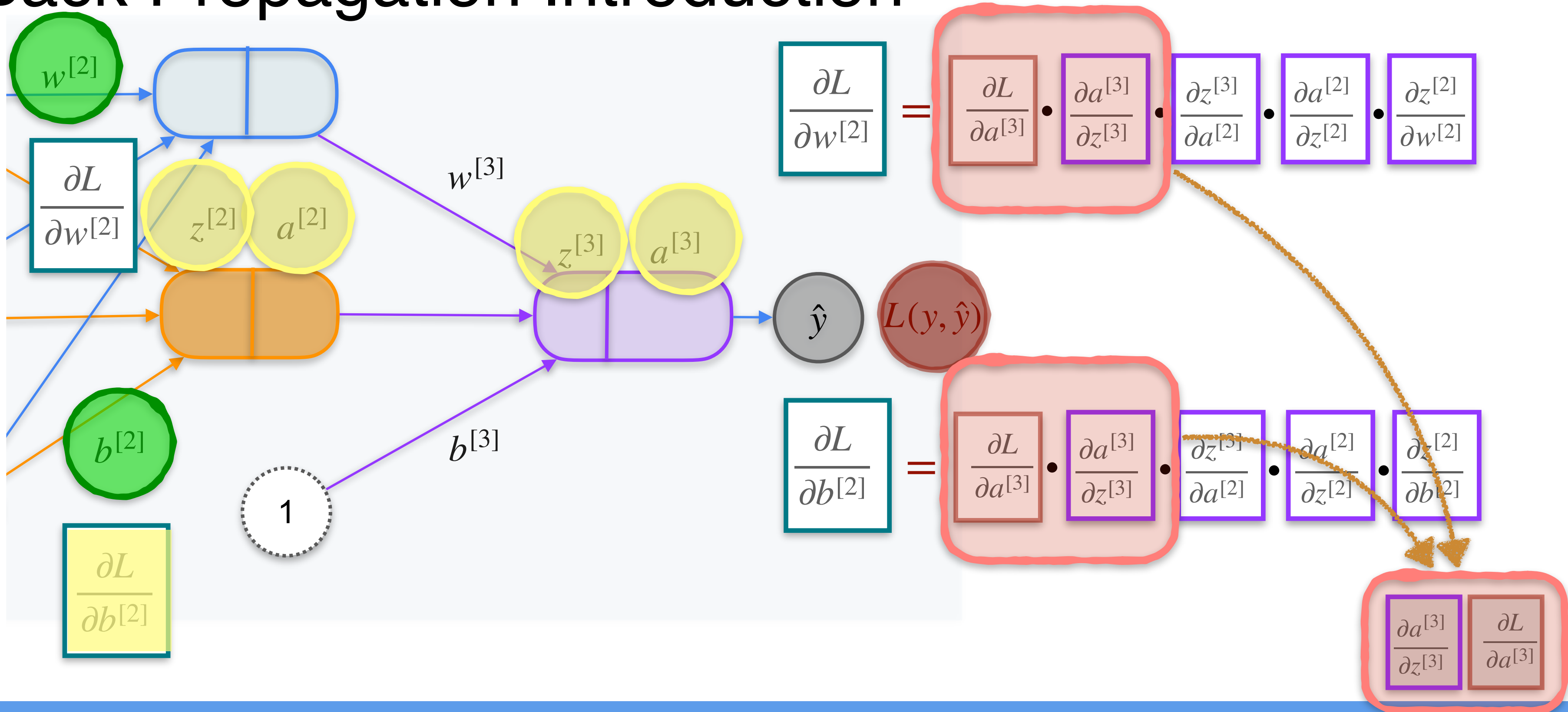

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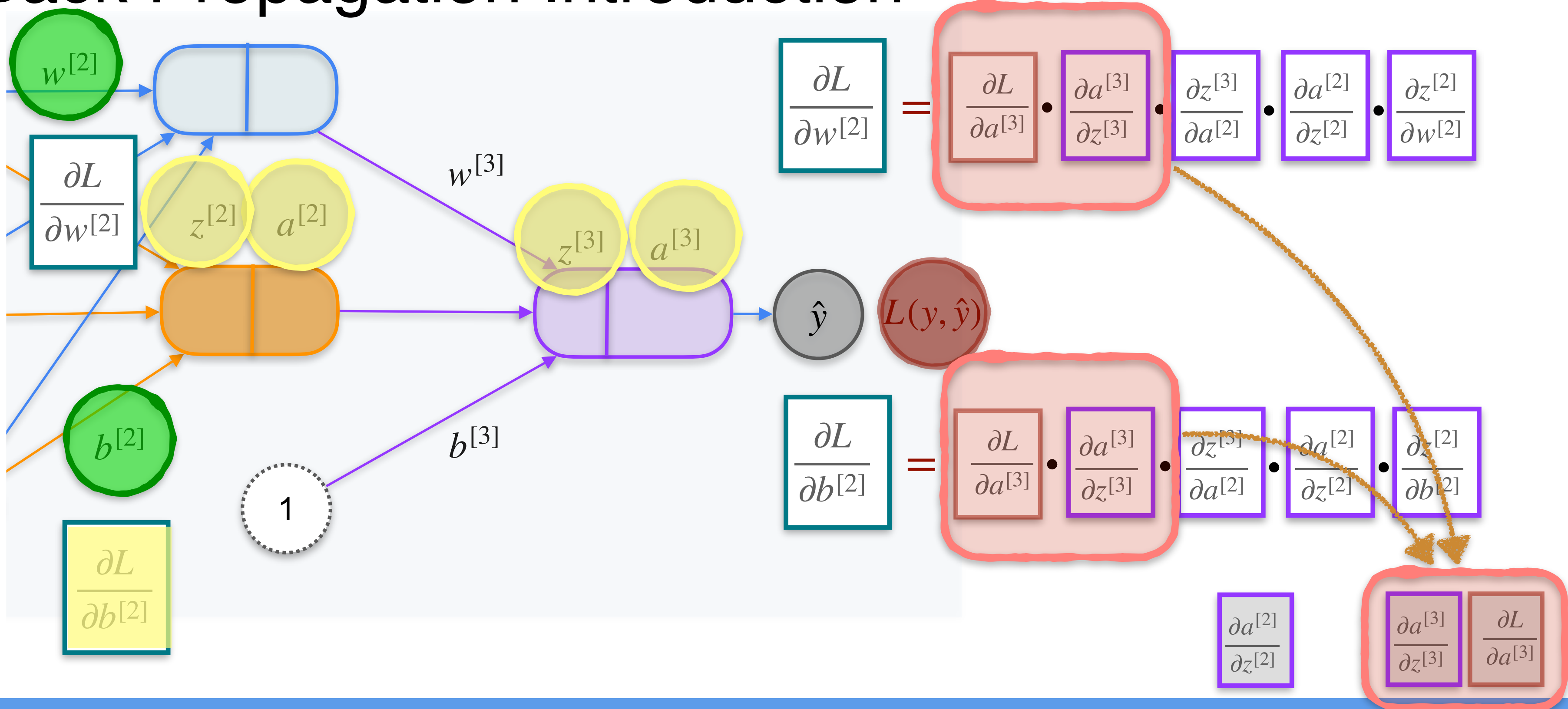

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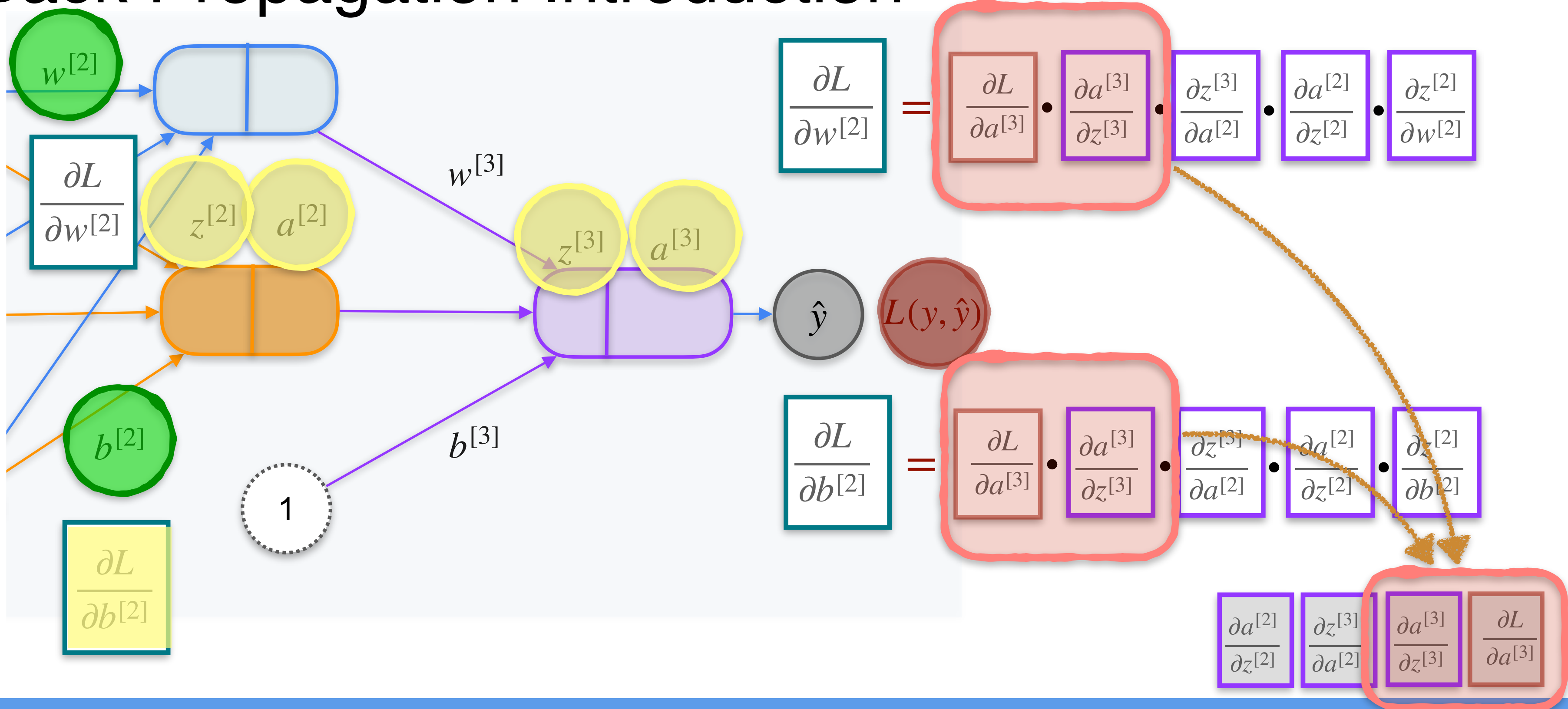

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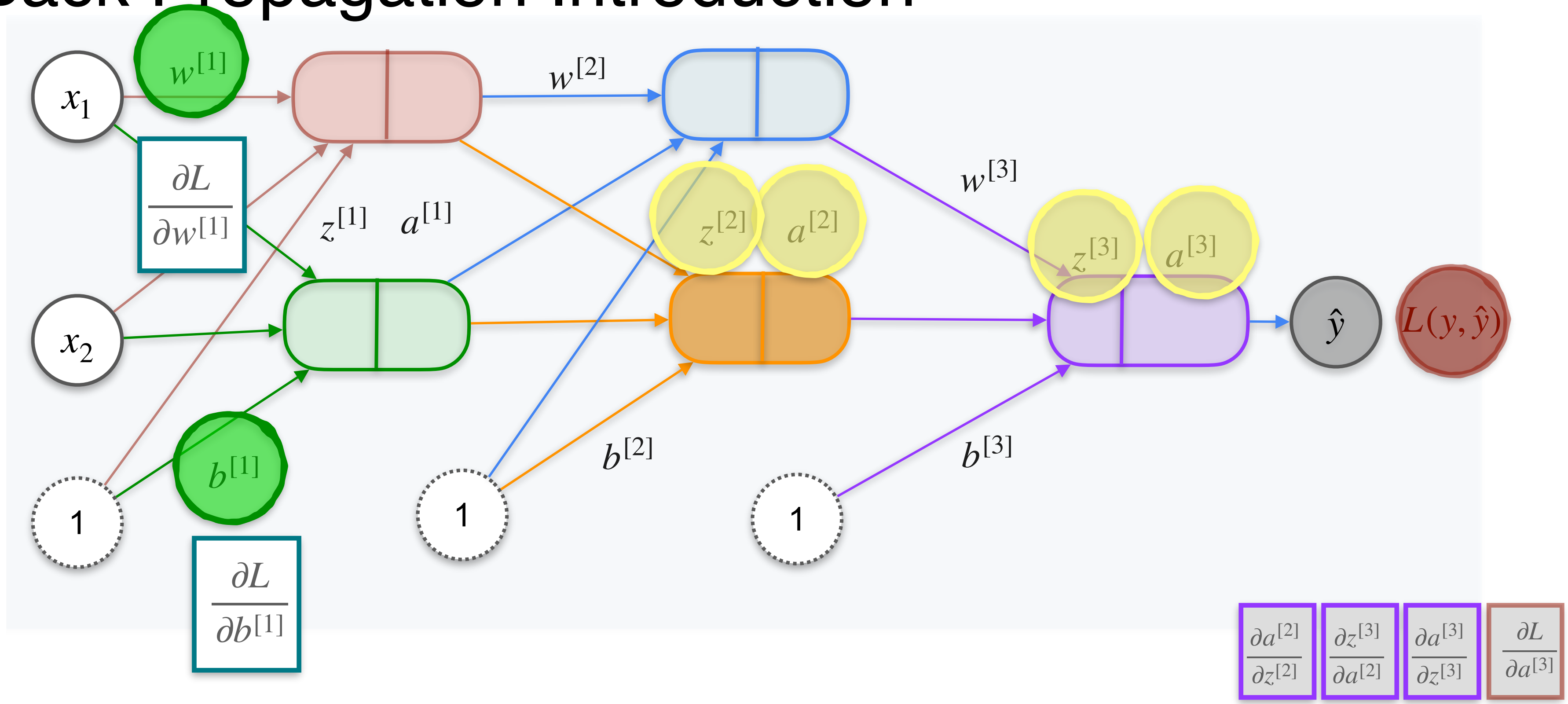

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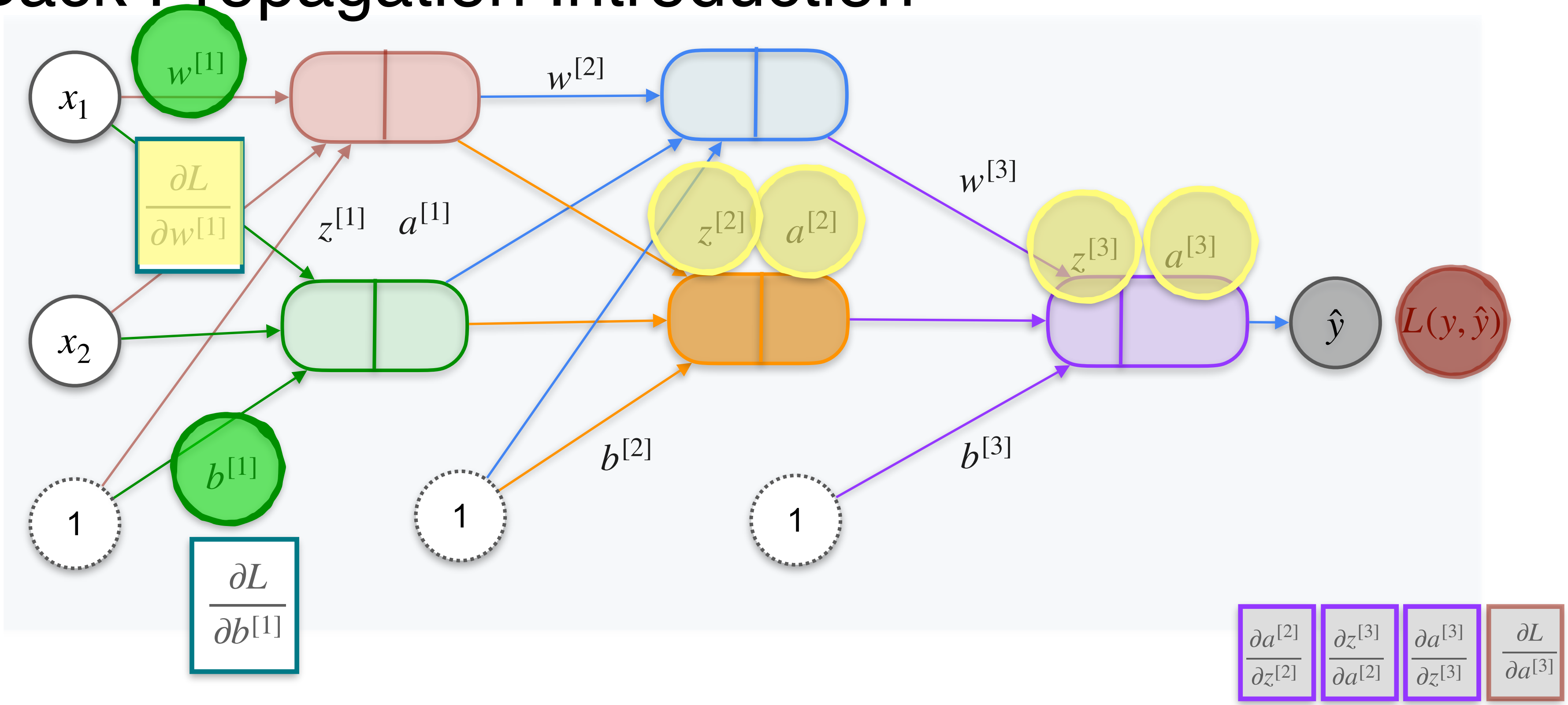

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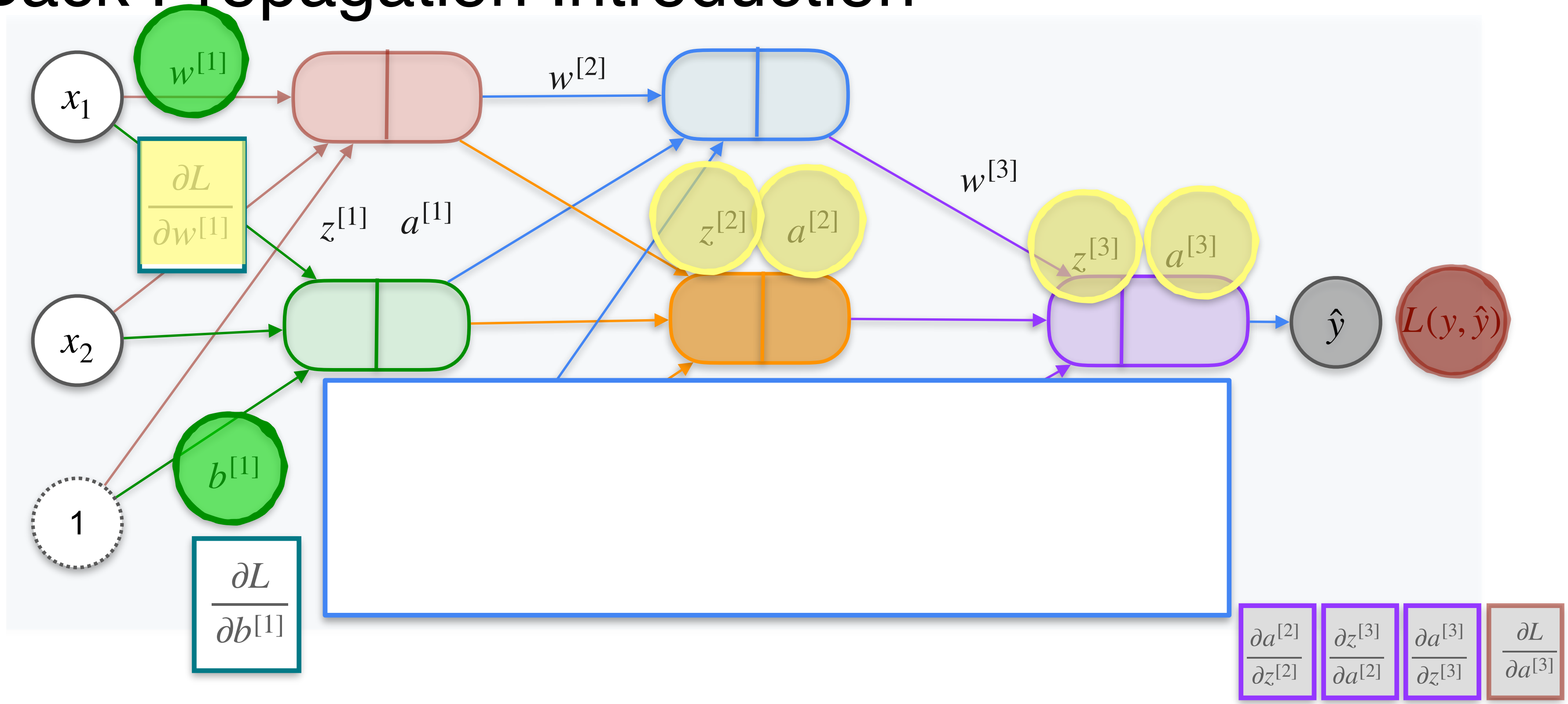

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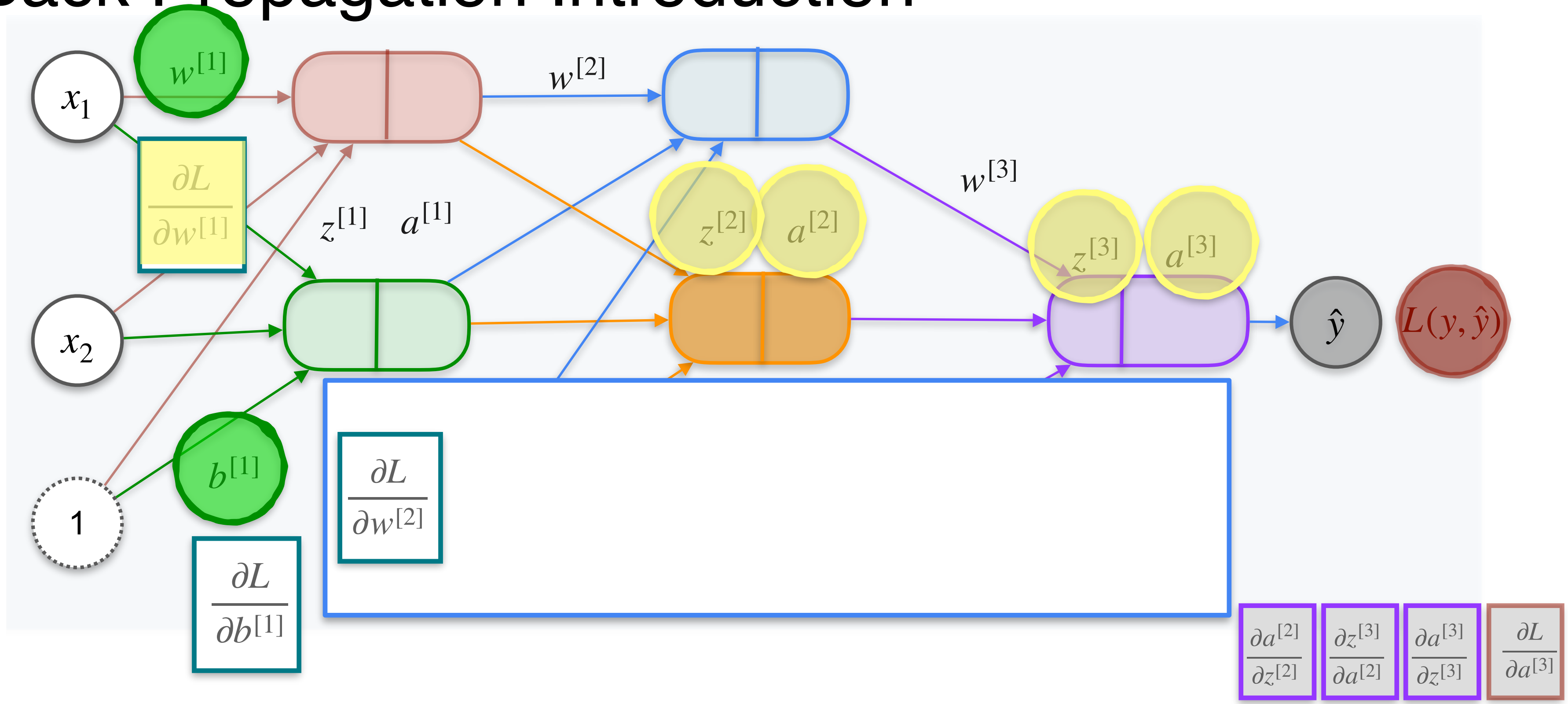

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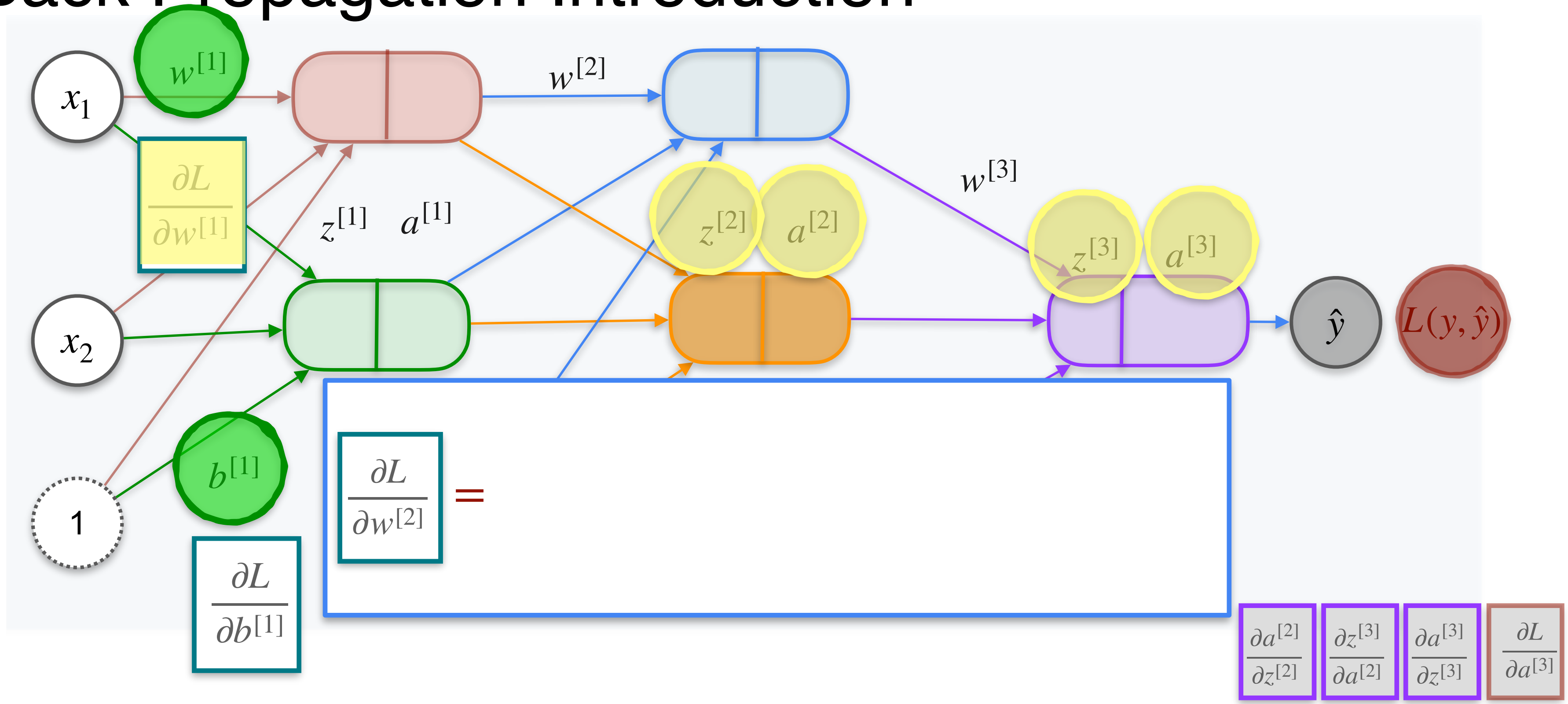

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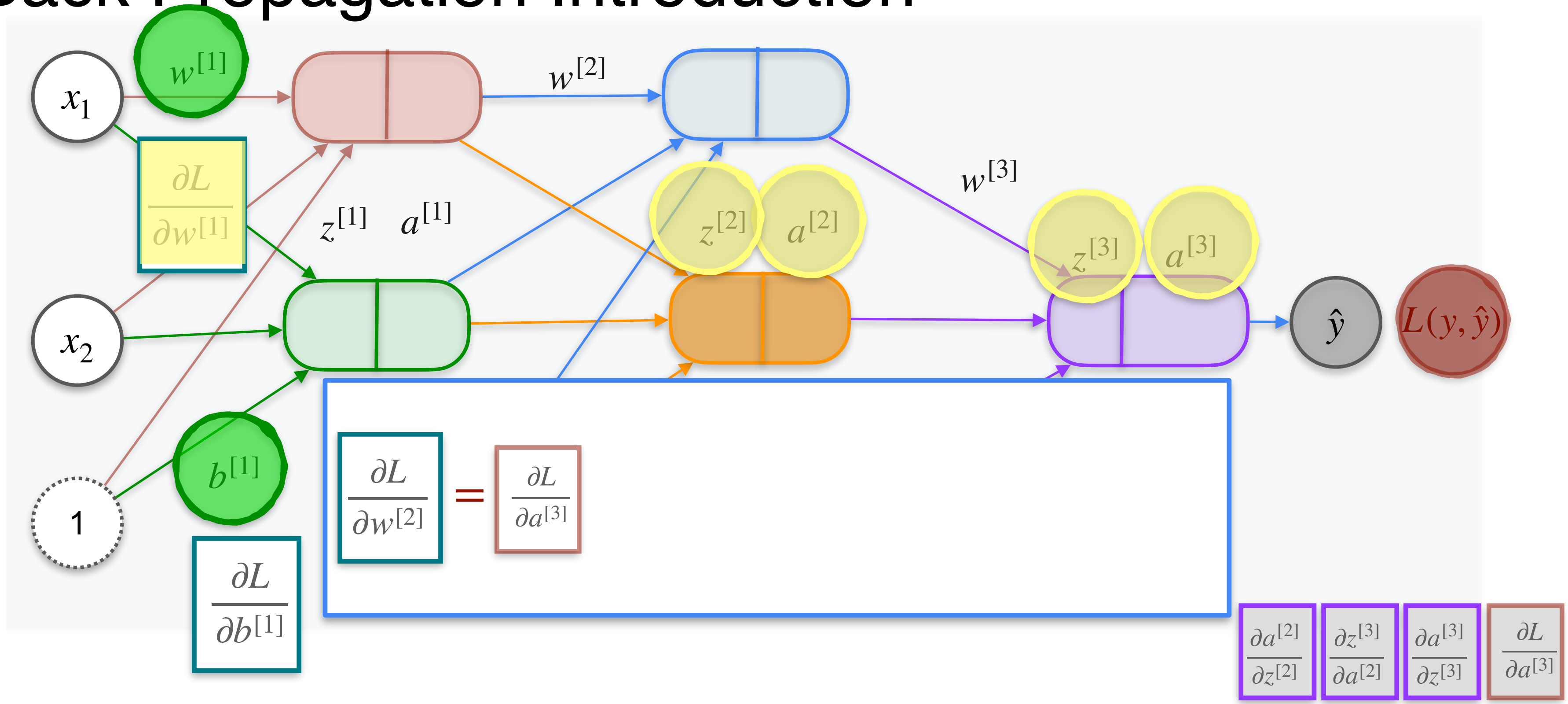

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}}$$

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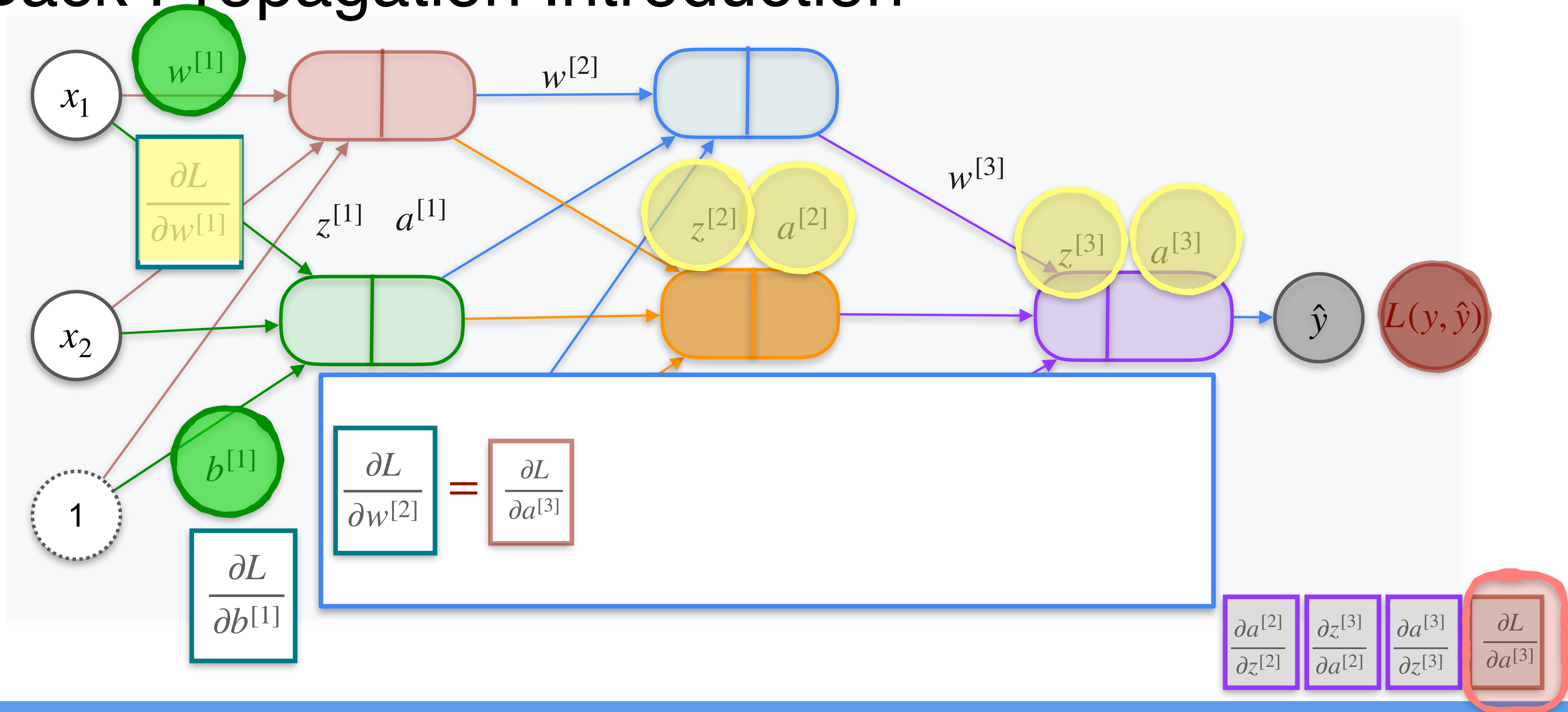

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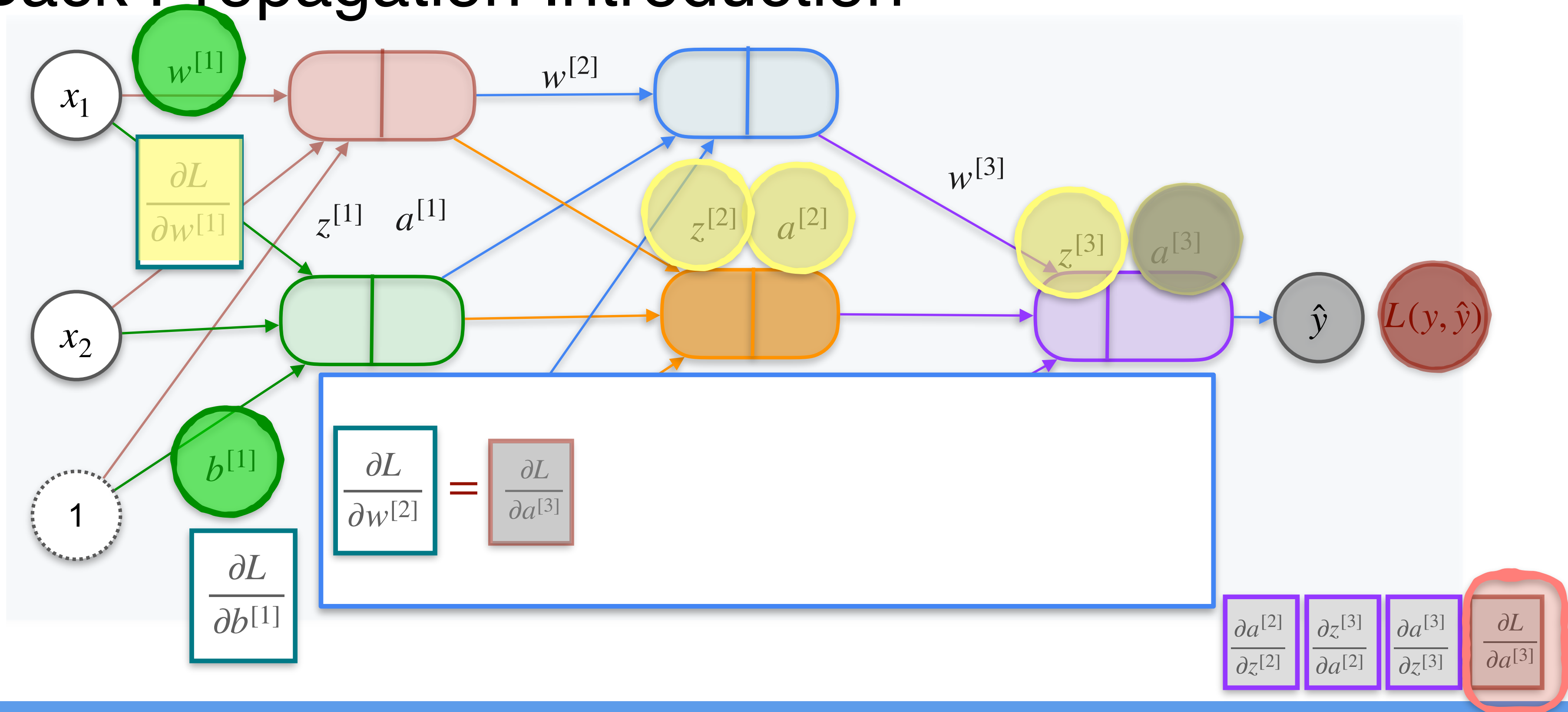

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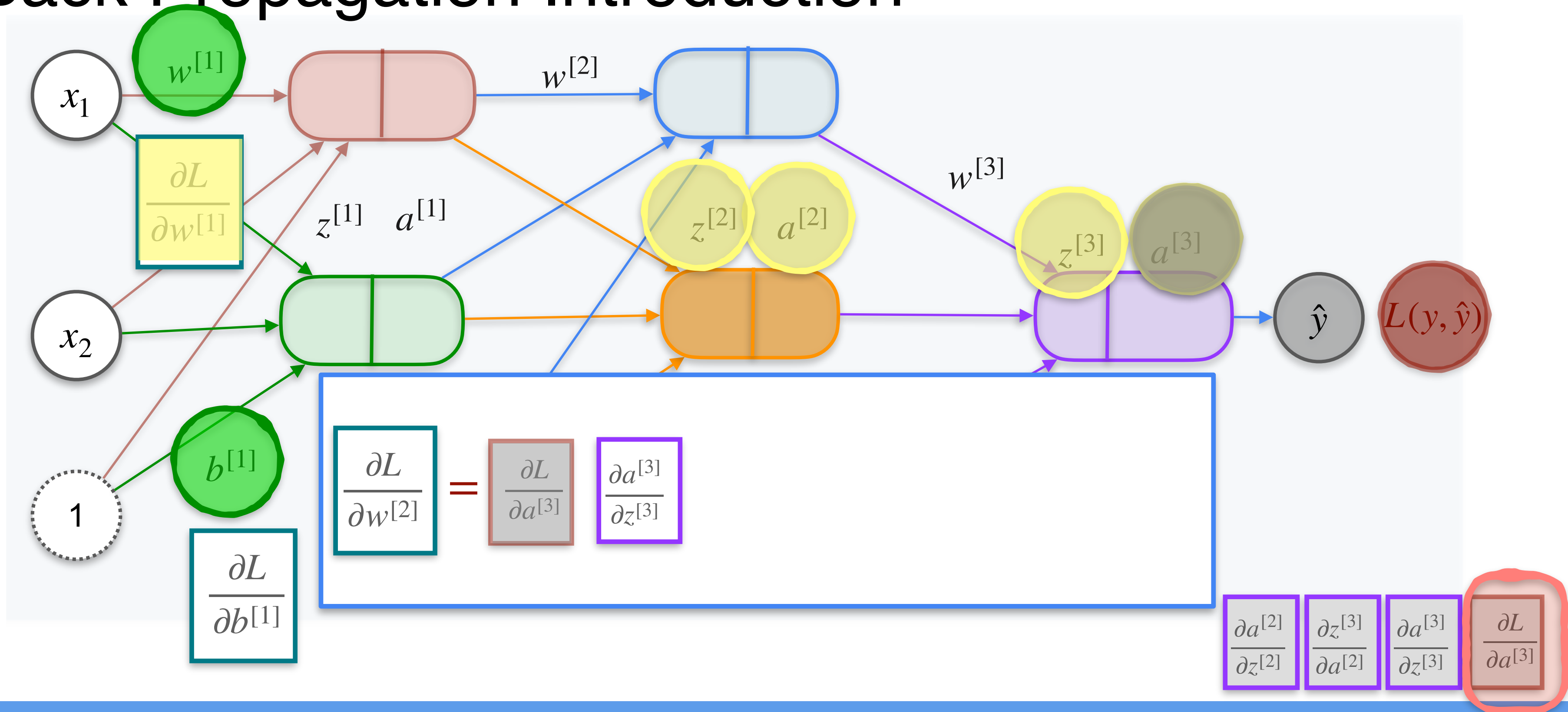

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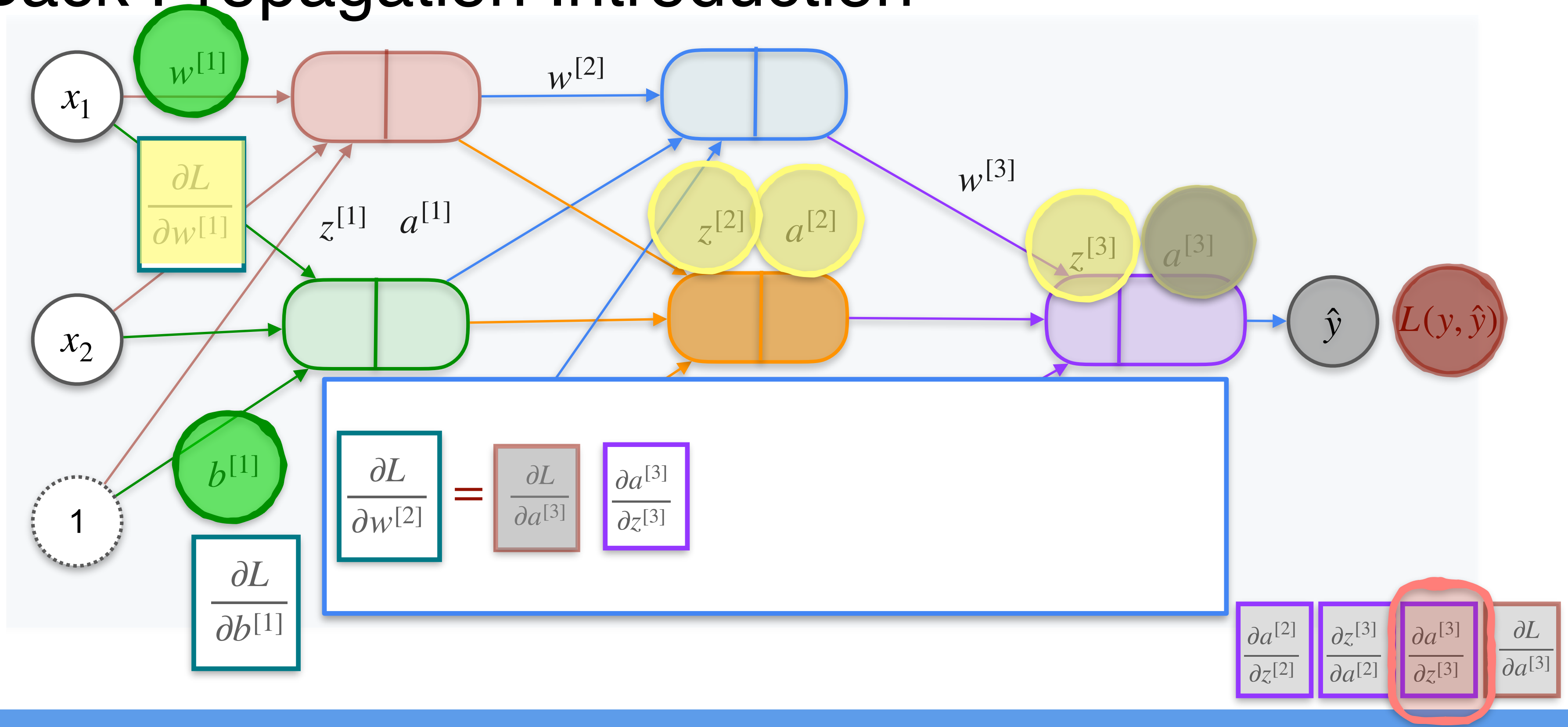

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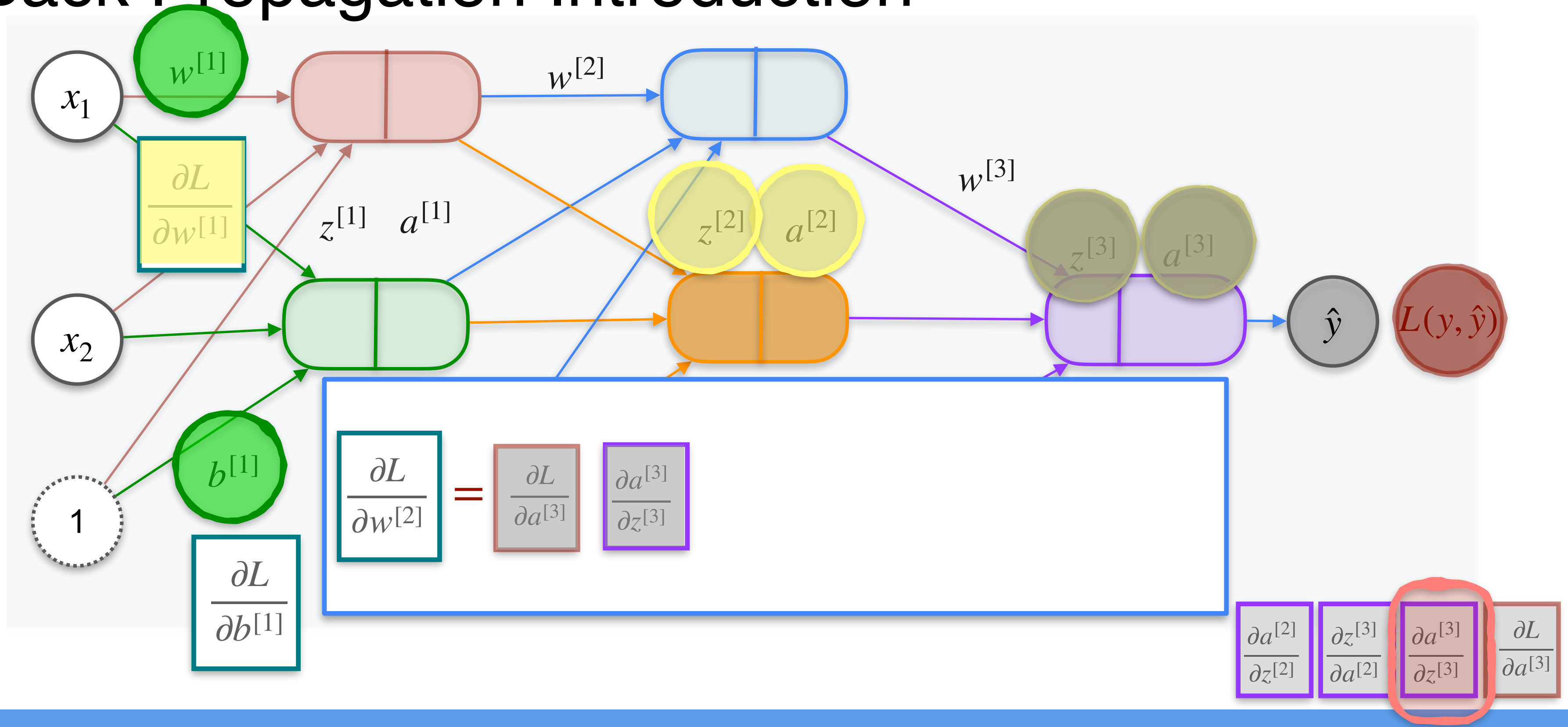
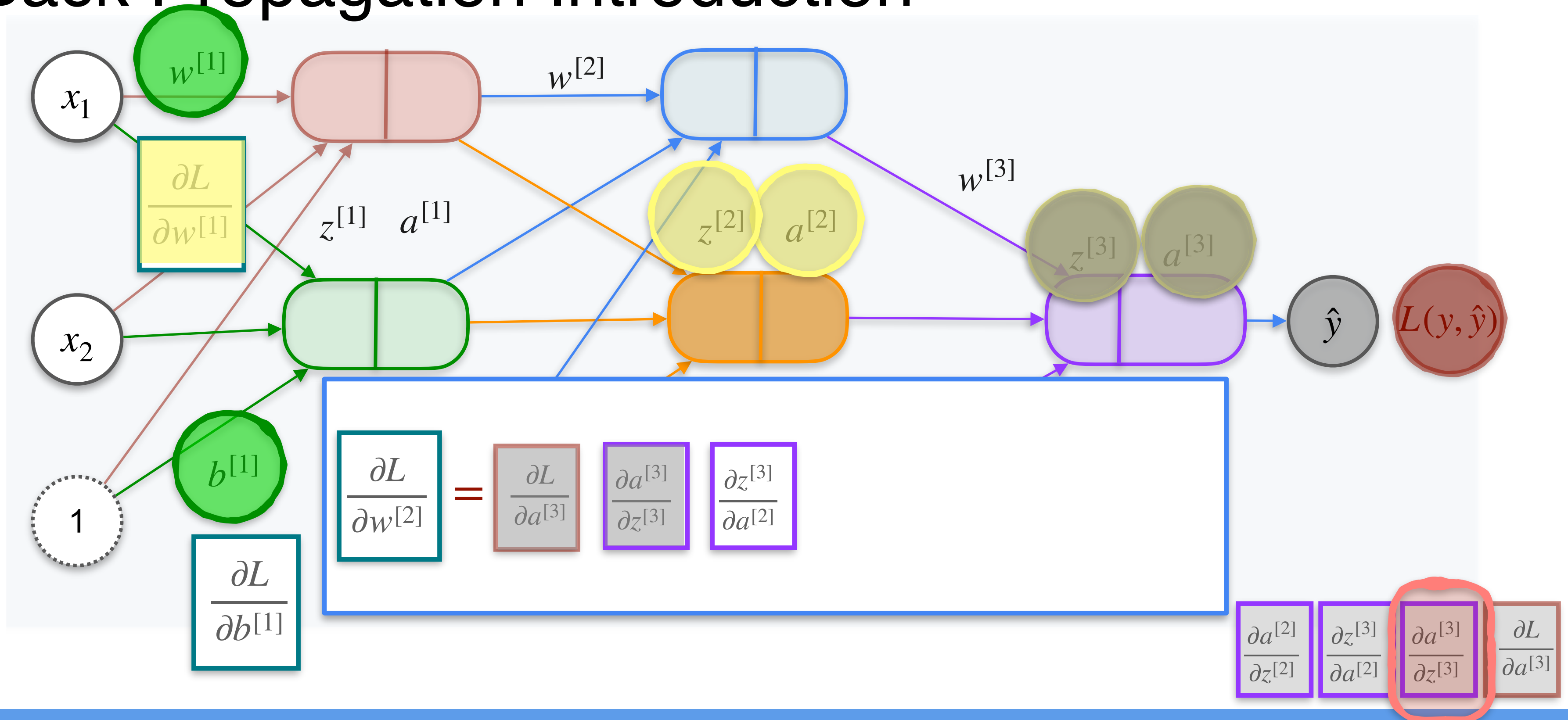
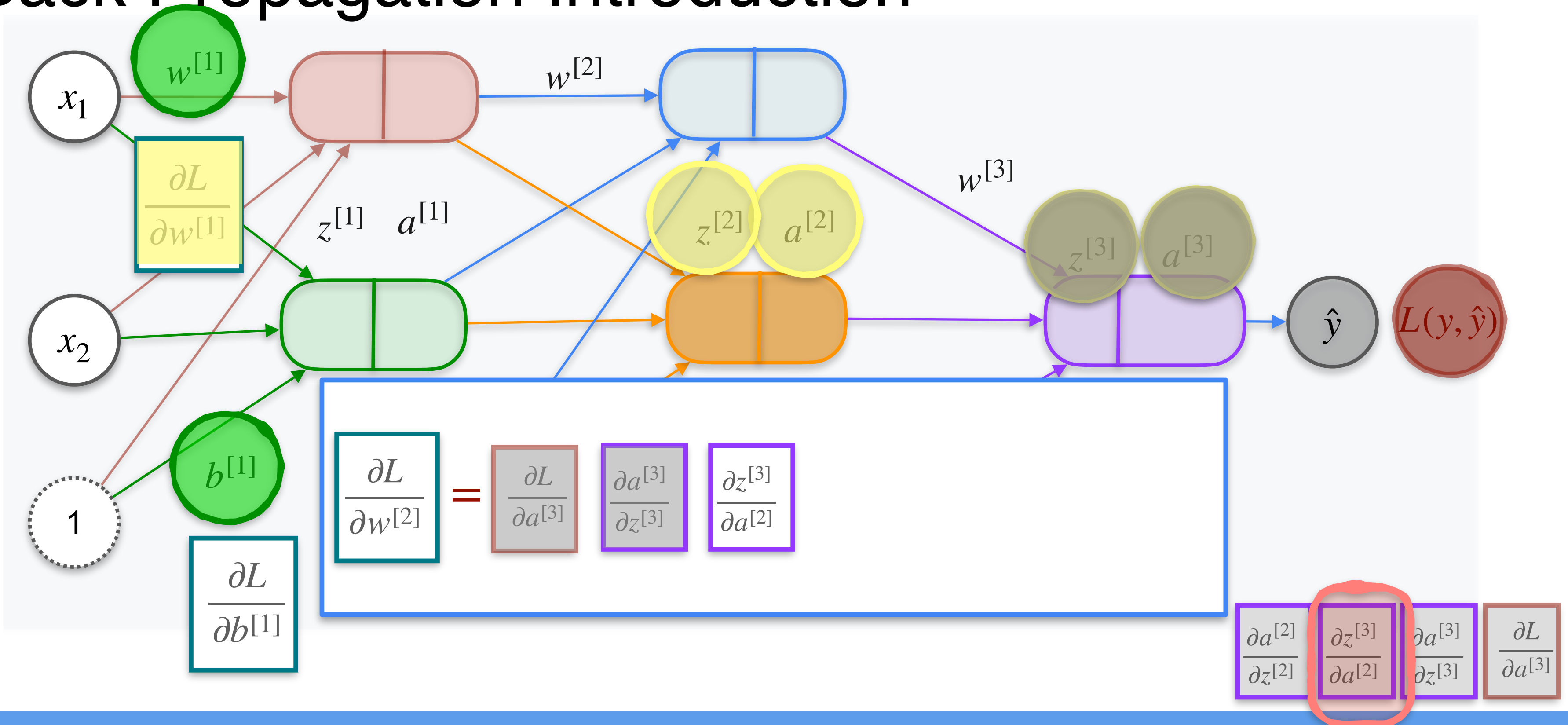


Back Propagation Introduction



Back Propagation Introduction



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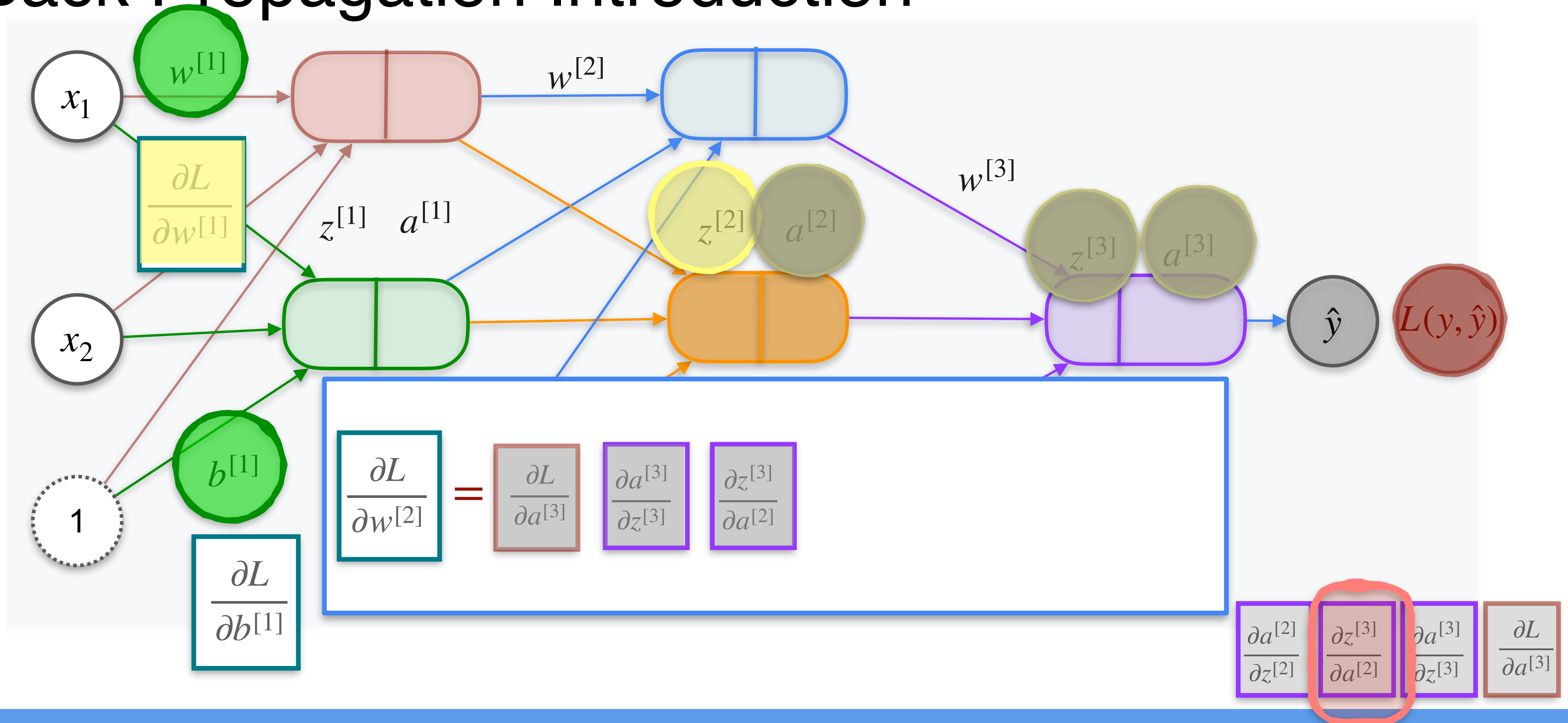

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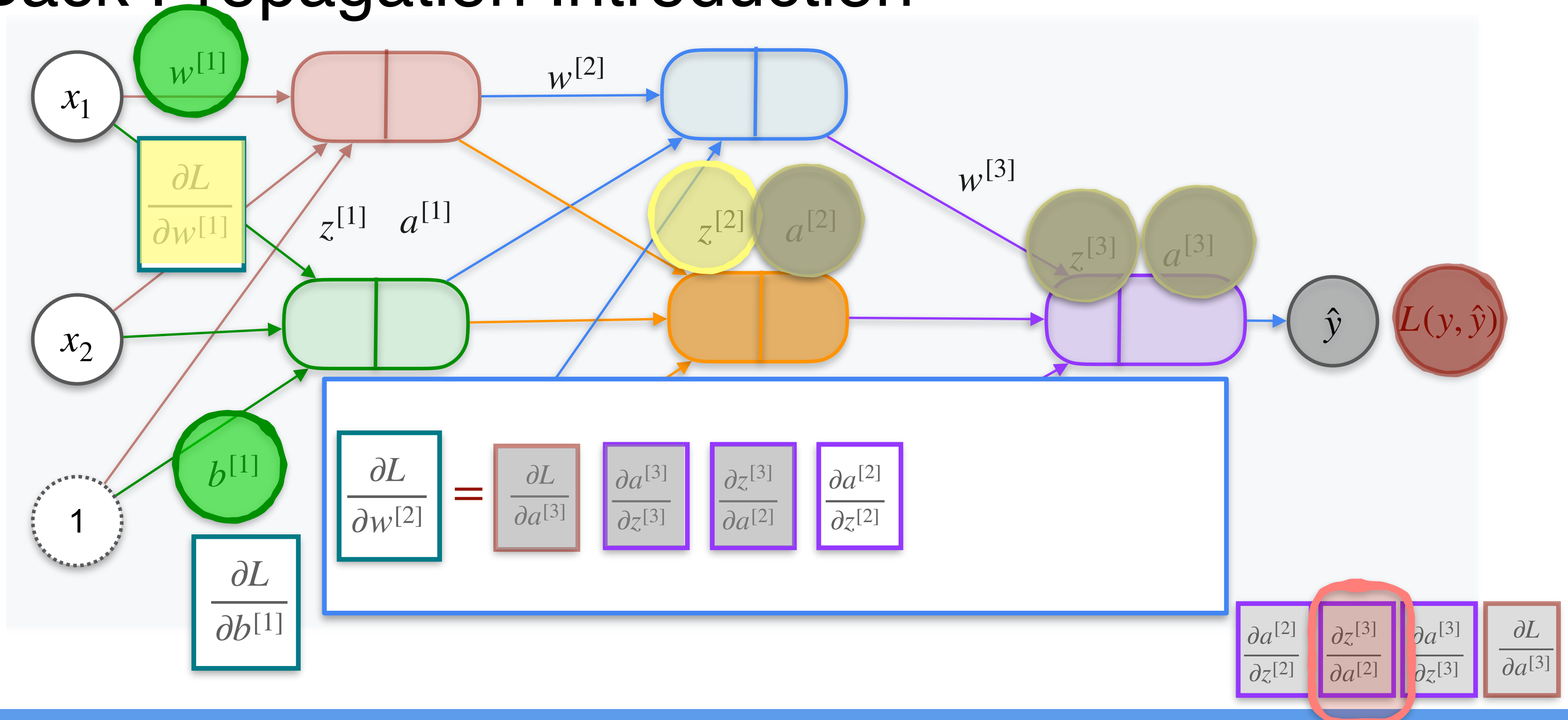

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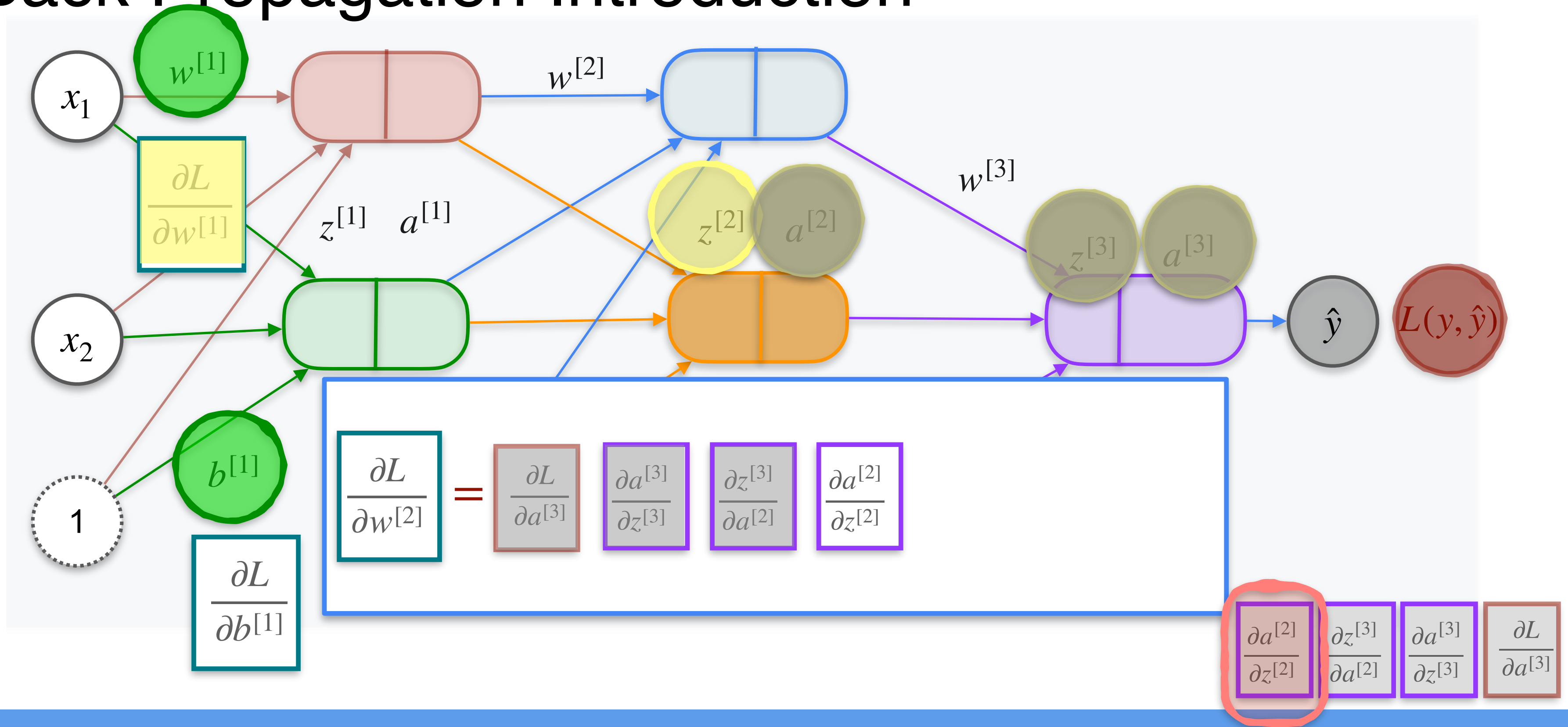

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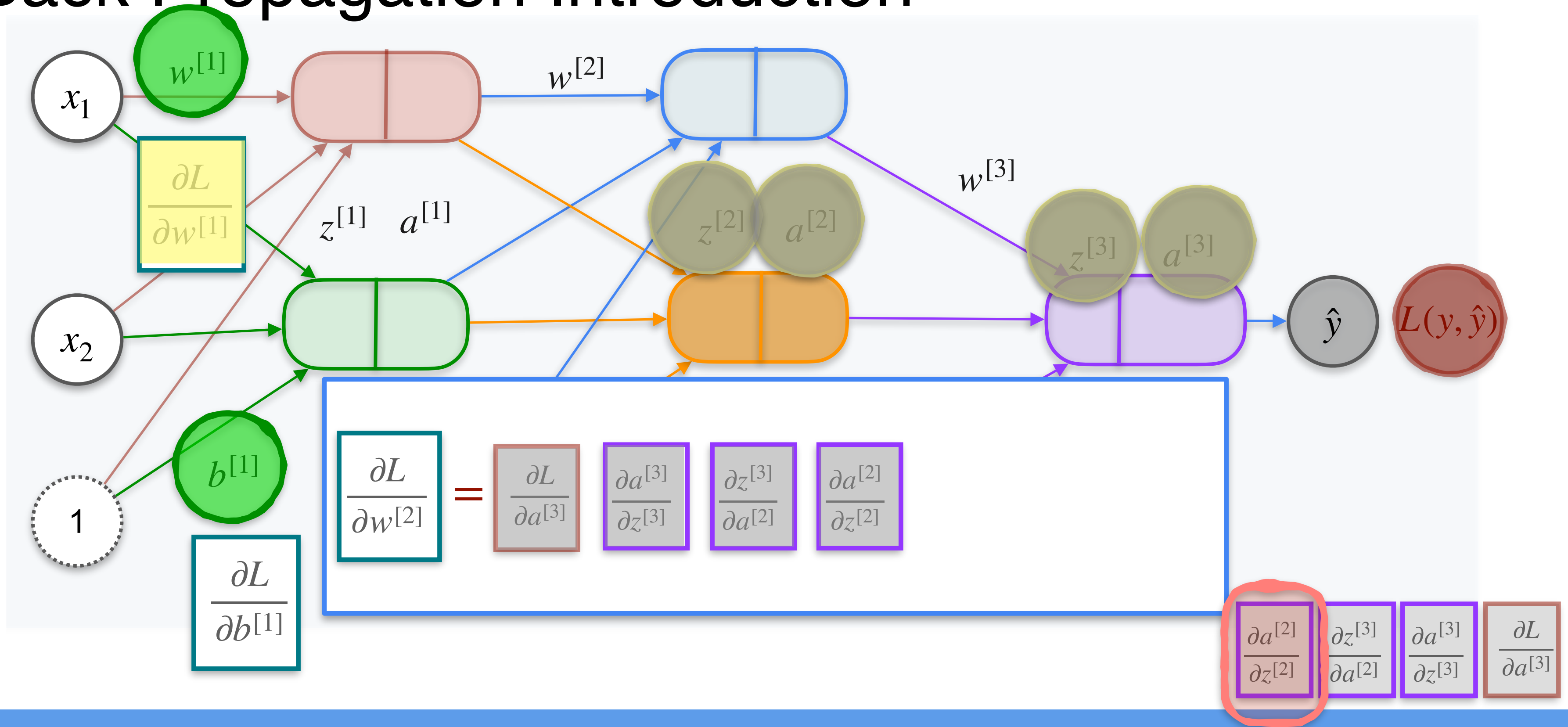

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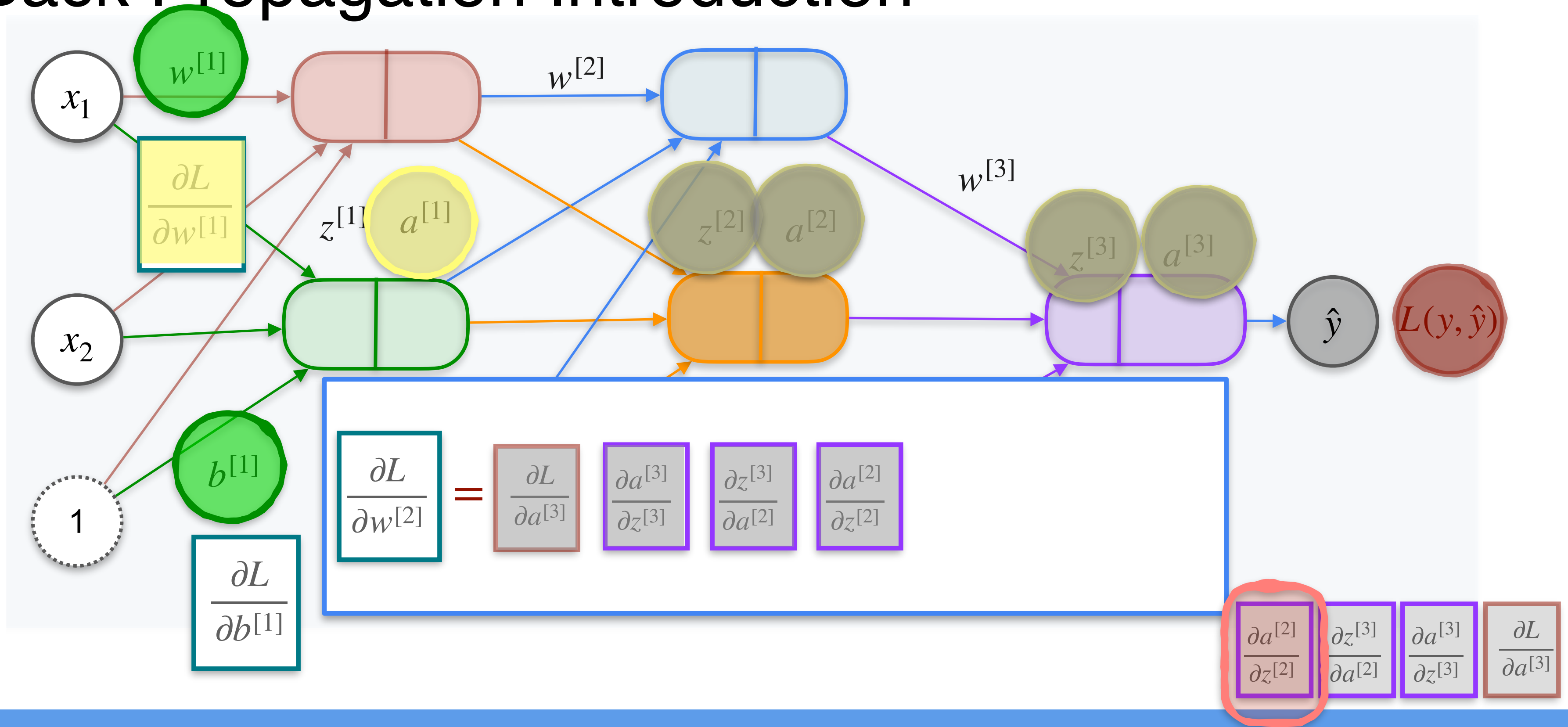

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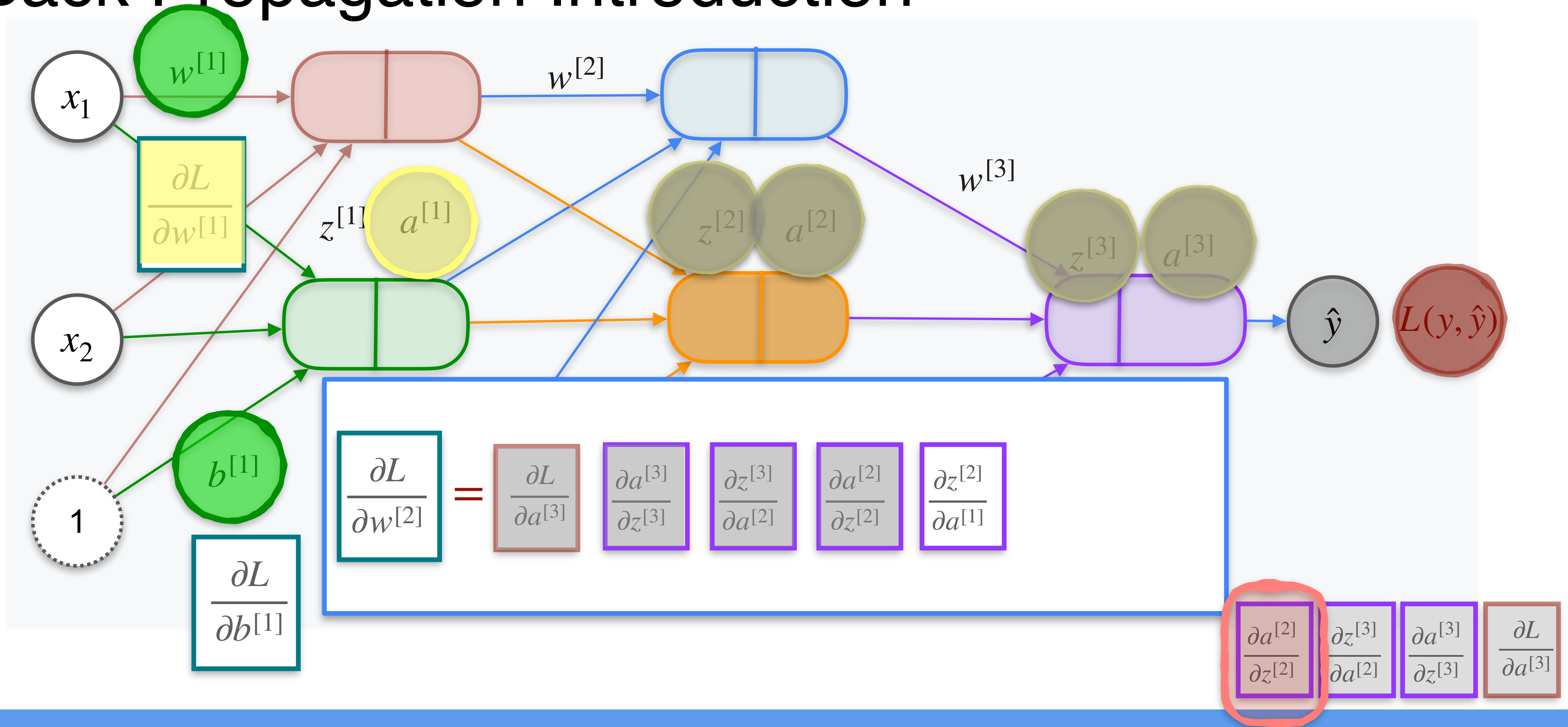

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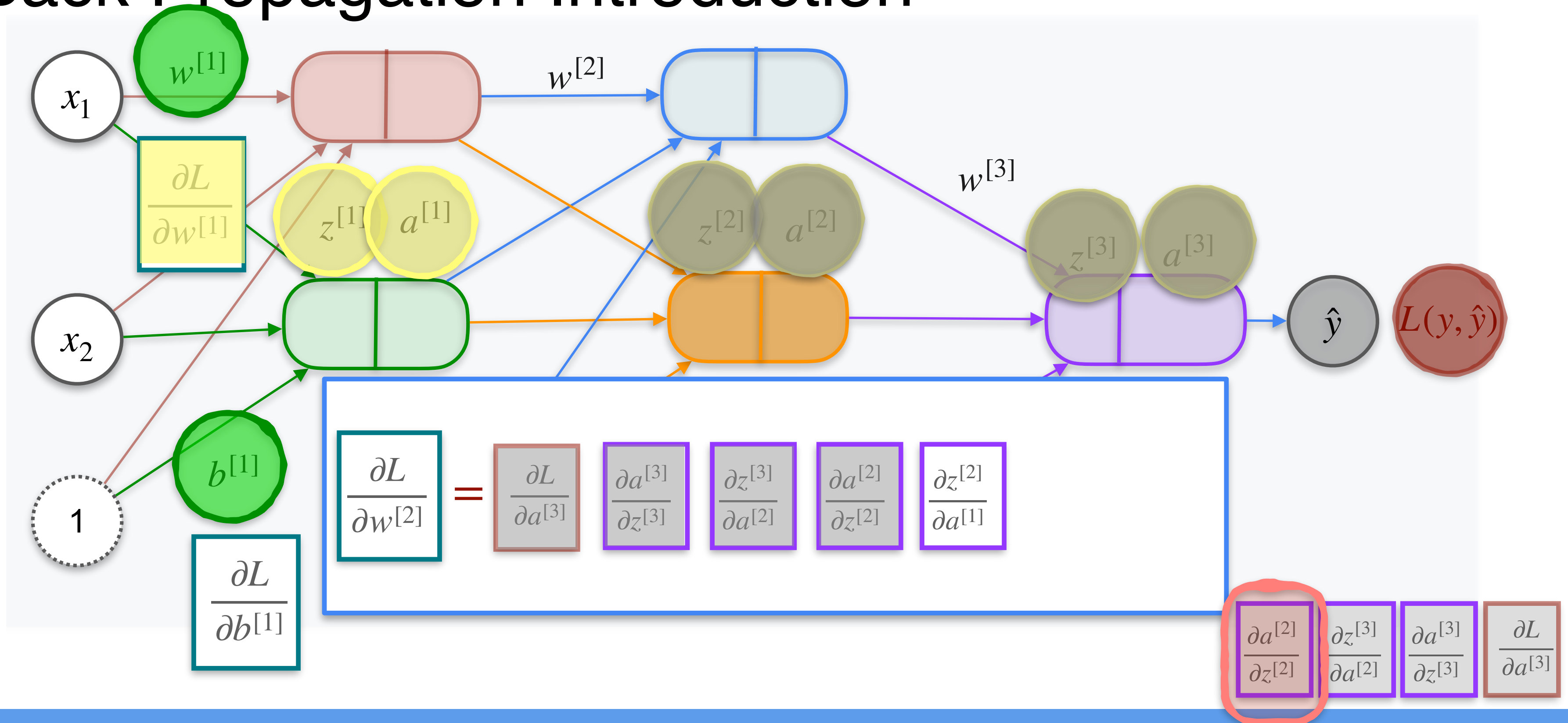
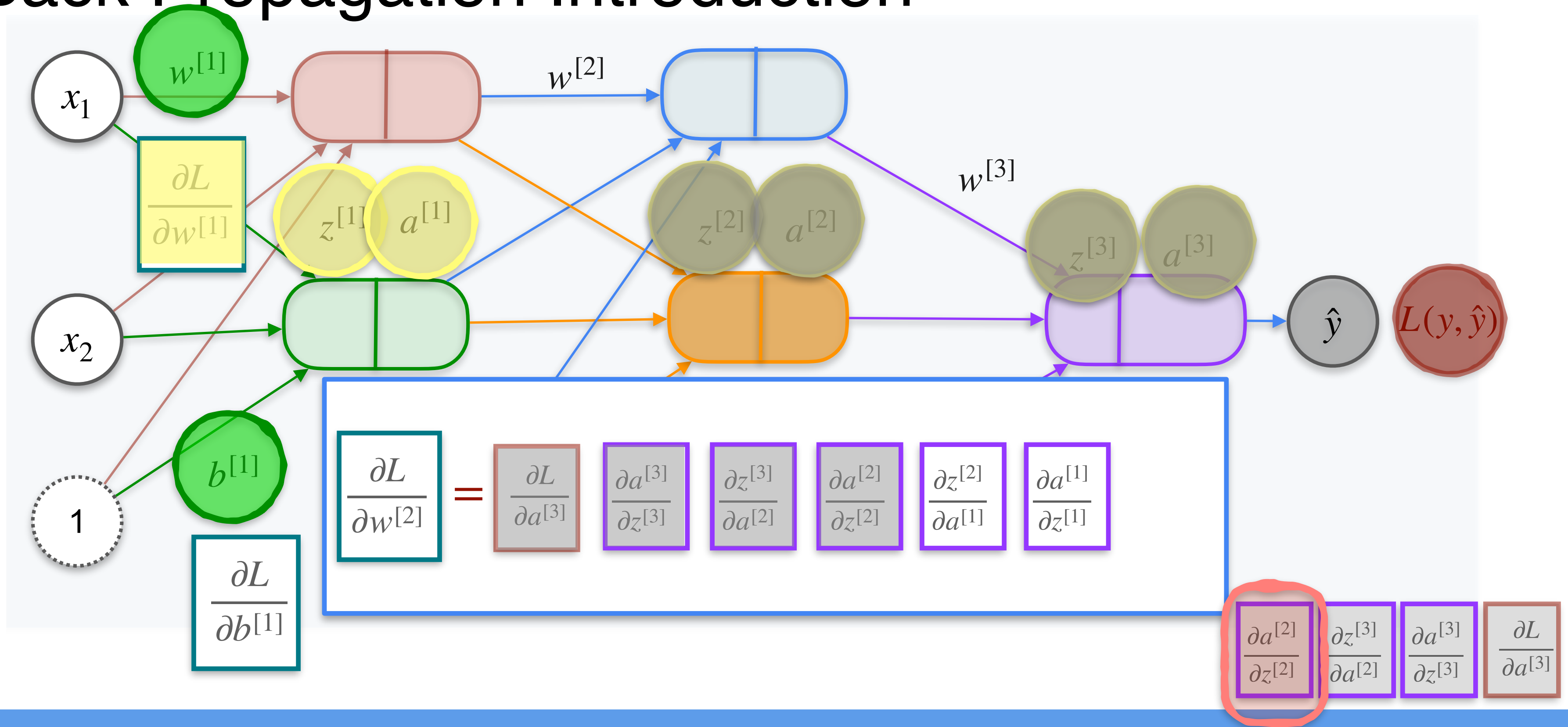
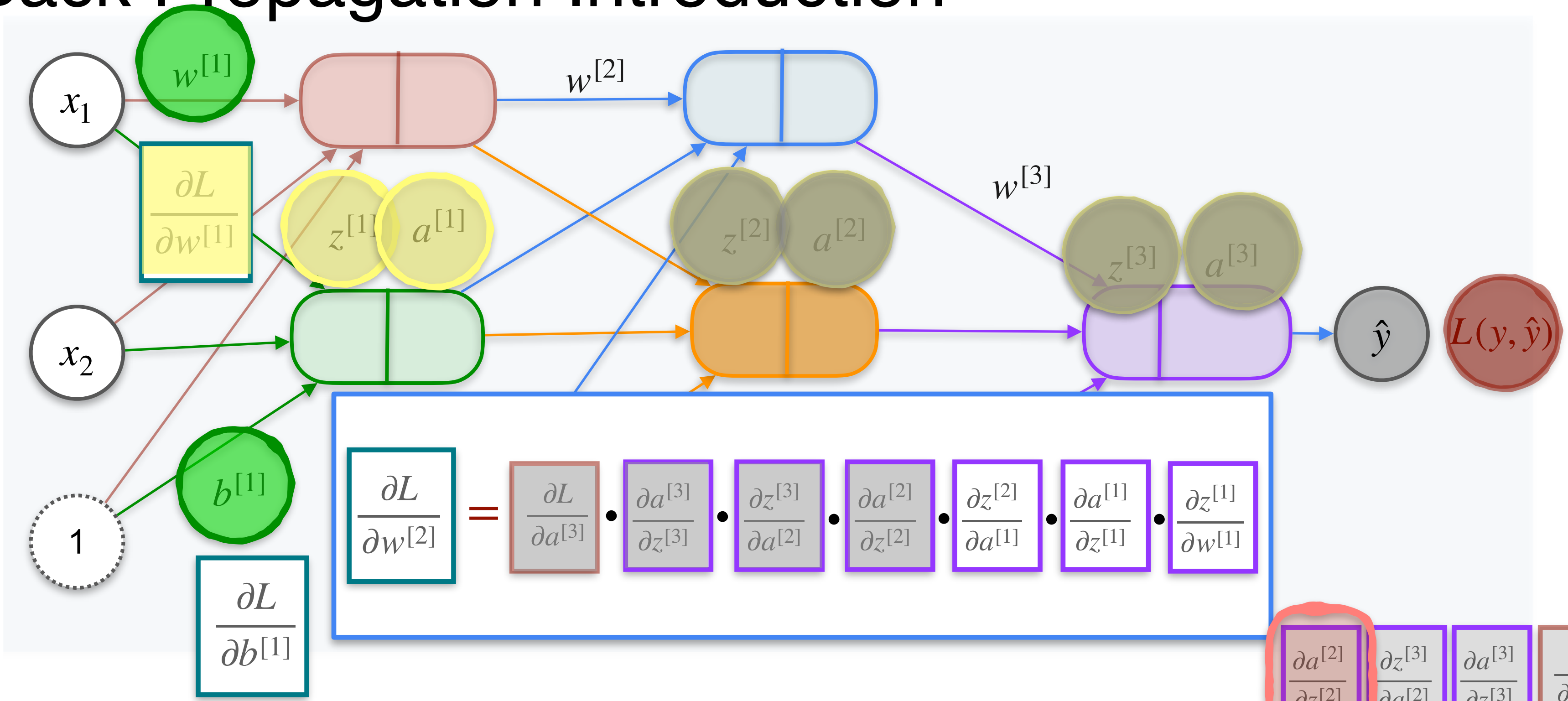


$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \frac{\partial a^{[3]}}{\partial z^{[3]}} \frac{\partial z^{[3]}}{\partial a^{[2]}} \frac{\partial a^{[2]}}{\partial z^{[2]}} \frac{\partial z^{[2]}}{\partial a^{[1]}}$$

Back Propagation Introduction



Back Propagation Introduction



$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial z^{[3]}}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial a^{[1]}} \cdot \frac{\partial a^{[1]}}{\partial z^{[1]}} \cdot \frac{\partial z^{[1]}}{\partial w^{[1]}}$$

$\frac{\partial a^{[2]}}{\partial z^{[2]}}$ $\frac{\partial z^{[3]}}{\partial a^{[2]}}$ $\frac{\partial a^{[3]}}{\partial z^{[3]}}$ $\frac{\partial L}{\partial a^{[3]}}$

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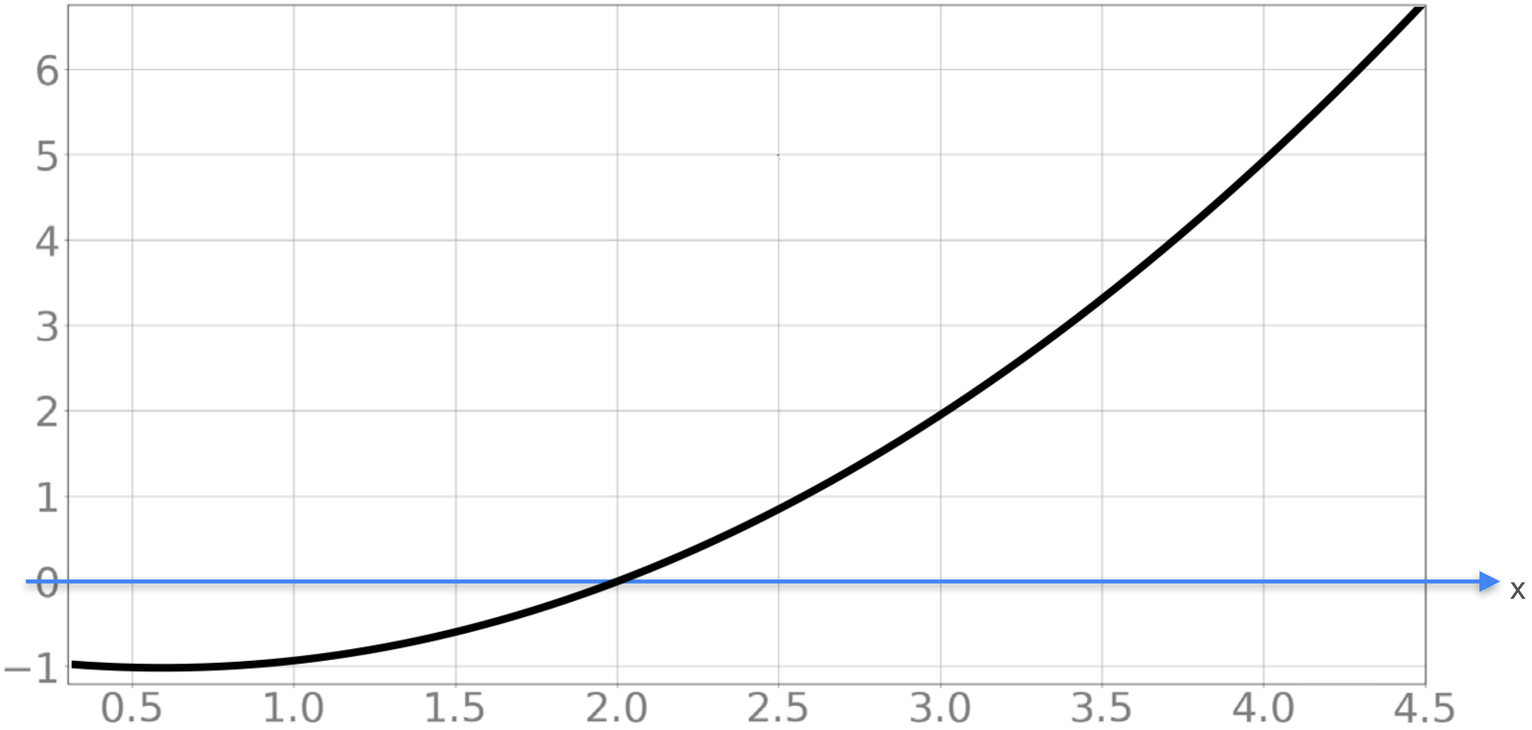
Optimization in Neural Networks and Newton's Method

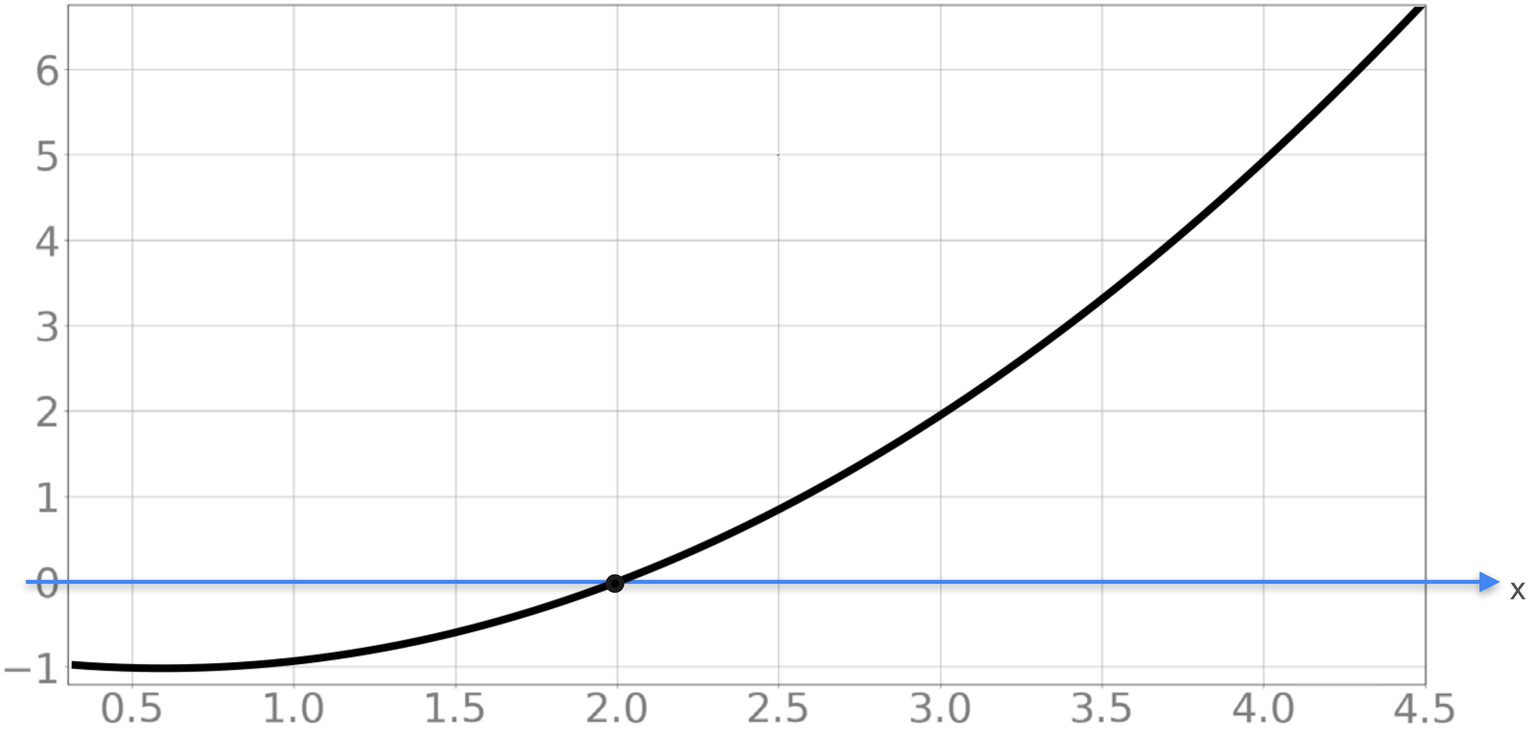
Newton's method

Newton's Method

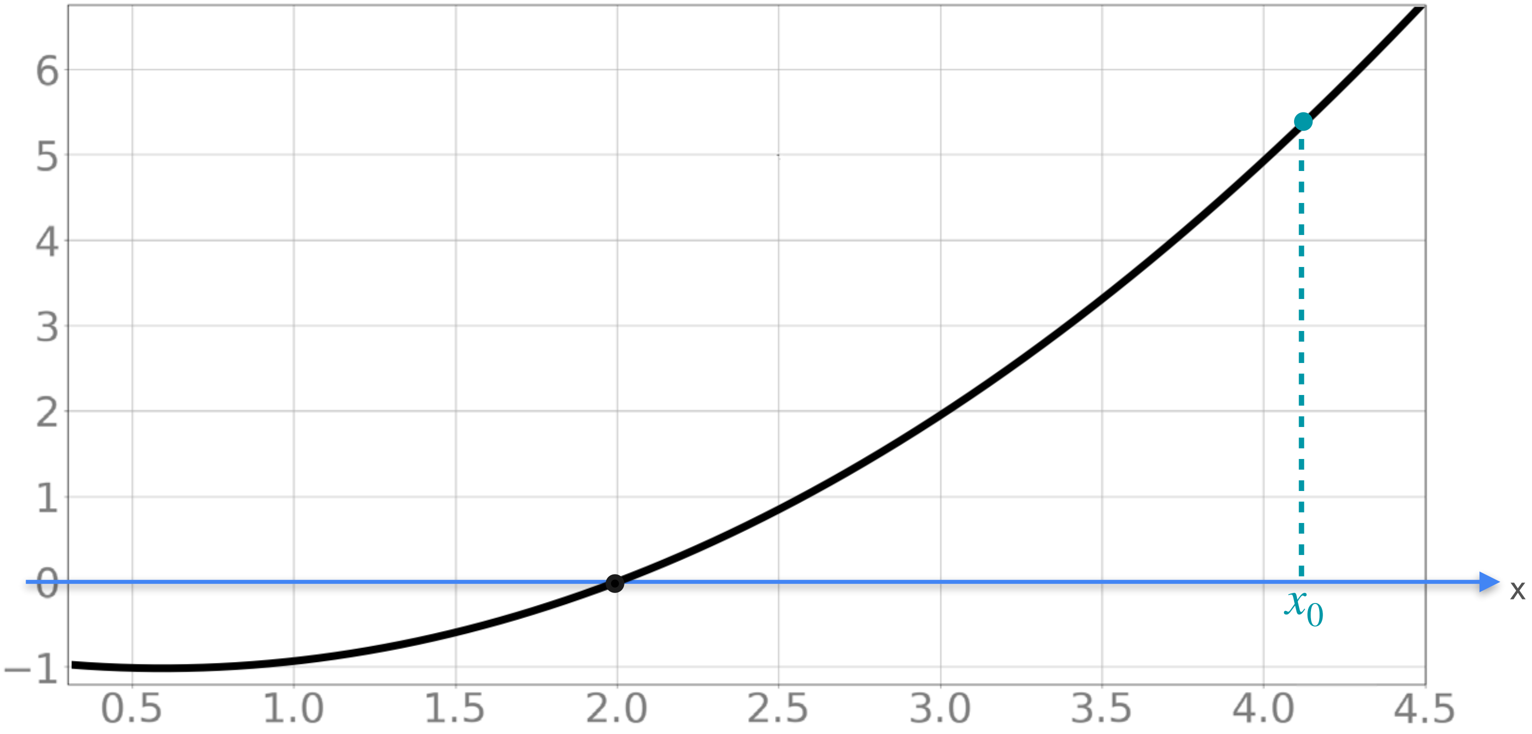
Newton's Method



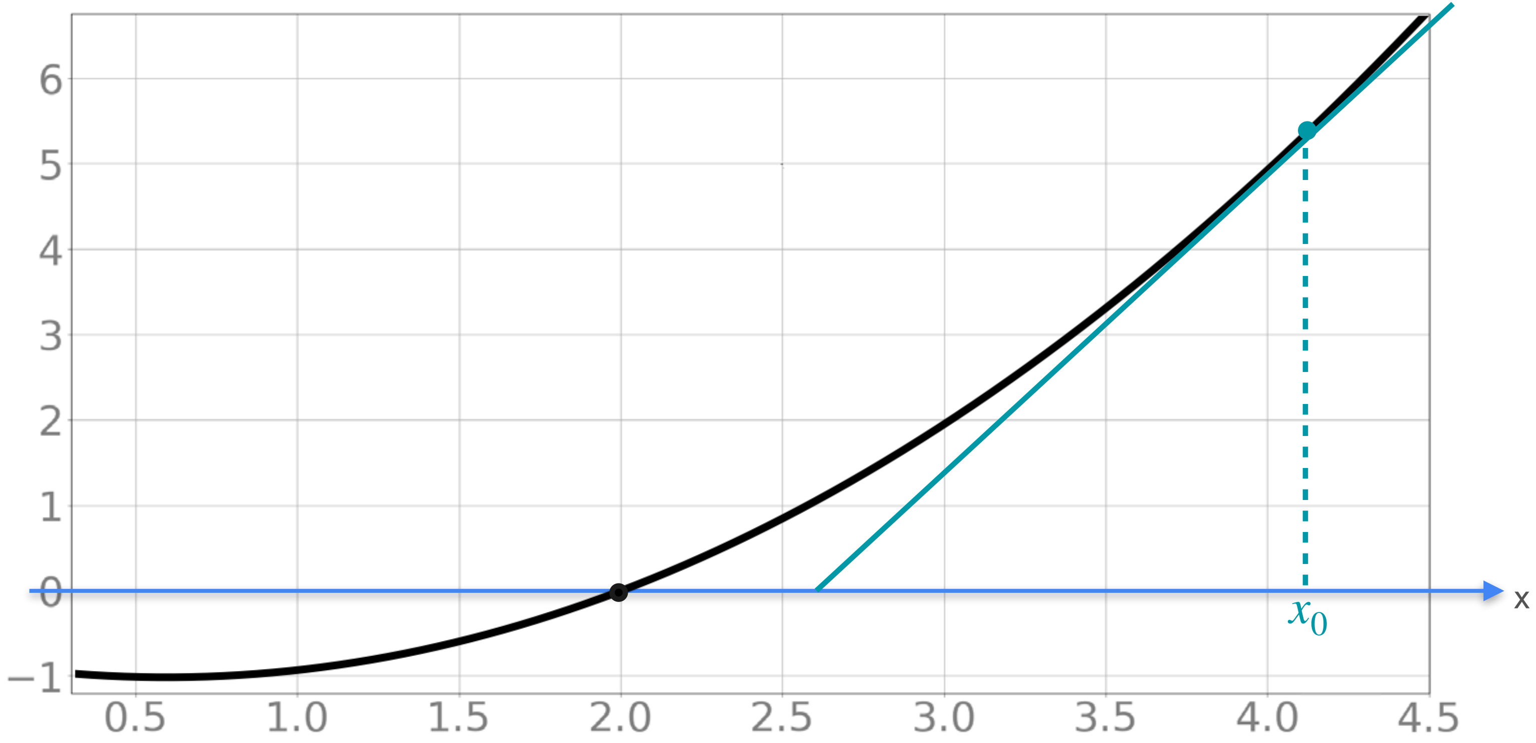
Newton's Method



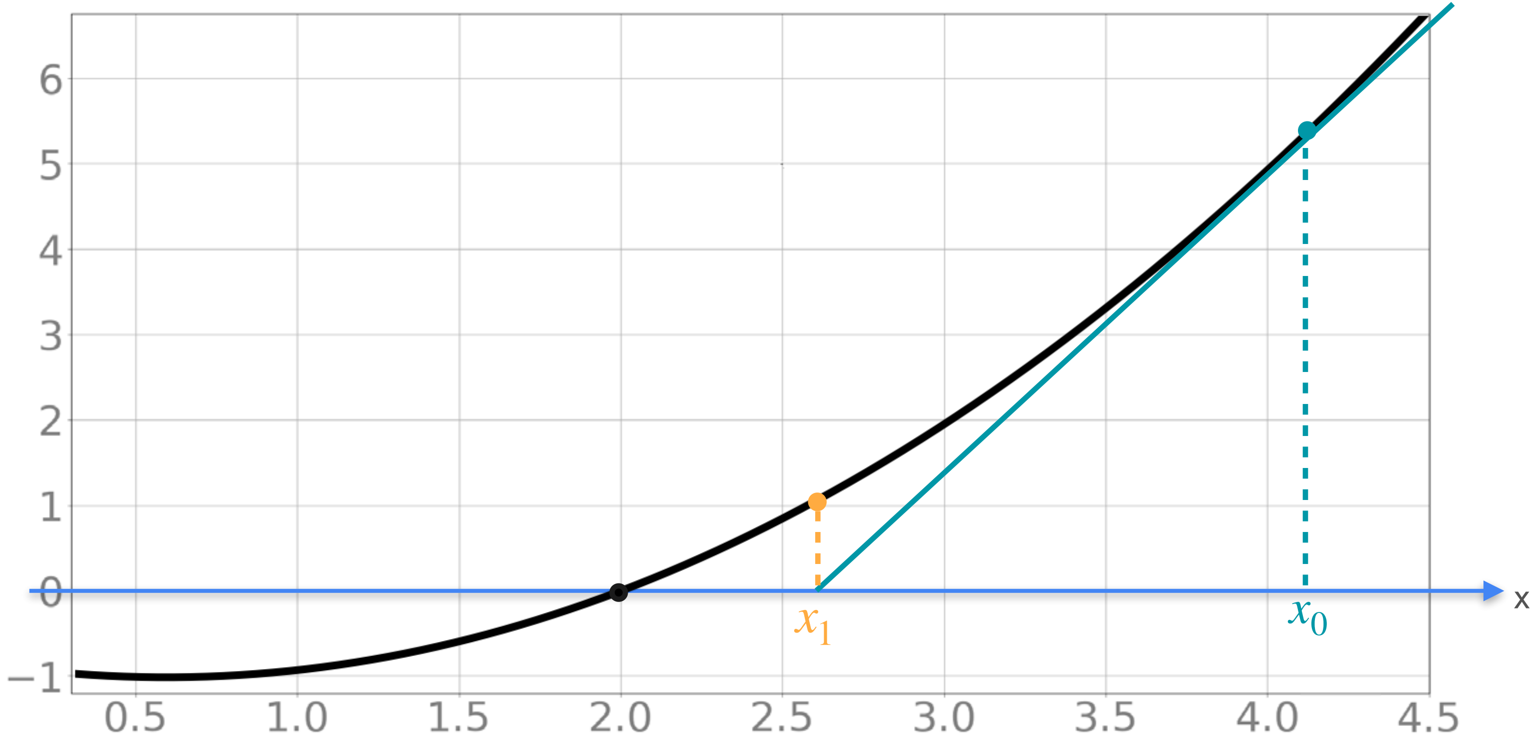
Newton's Method



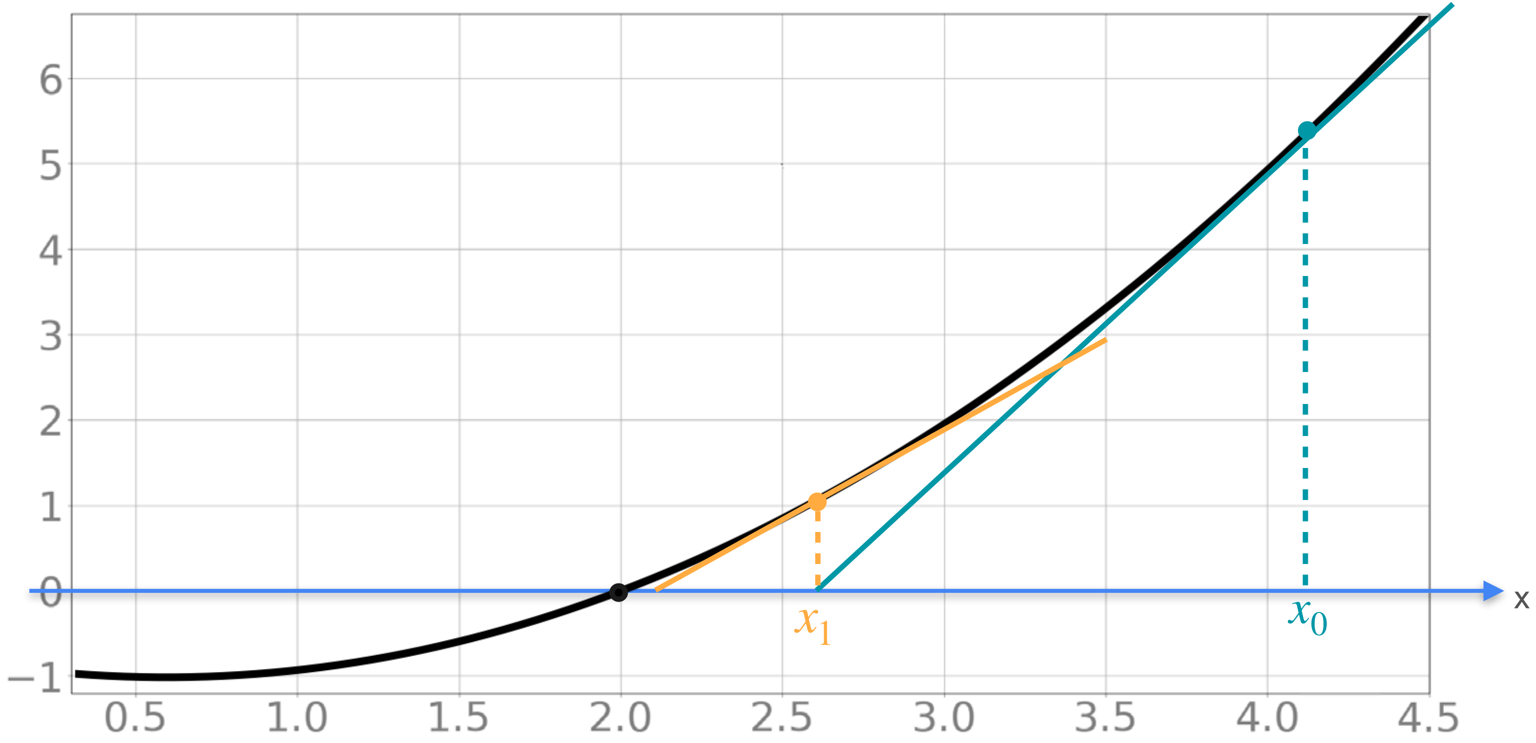
Newton's Method



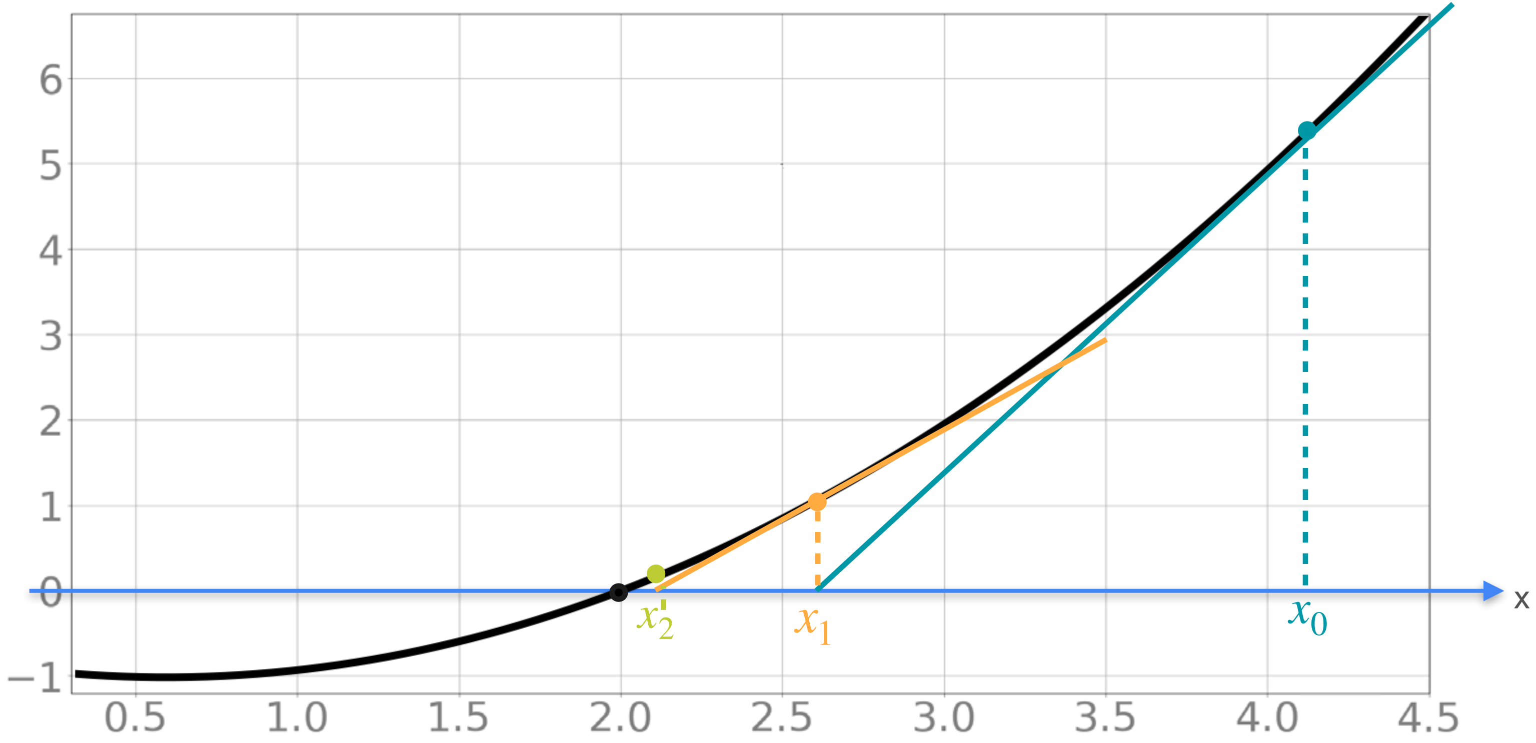
Newton's Method



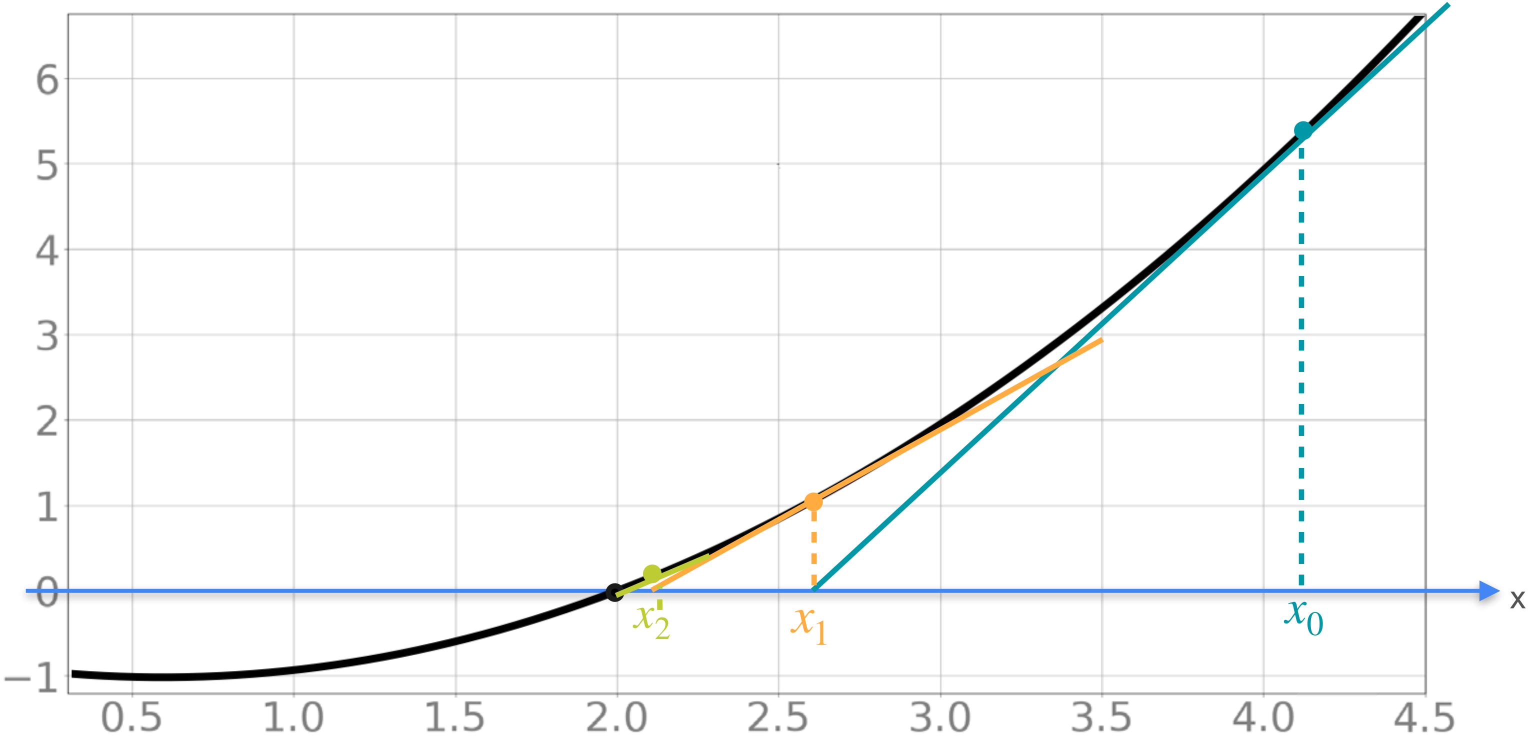
Newton's Method



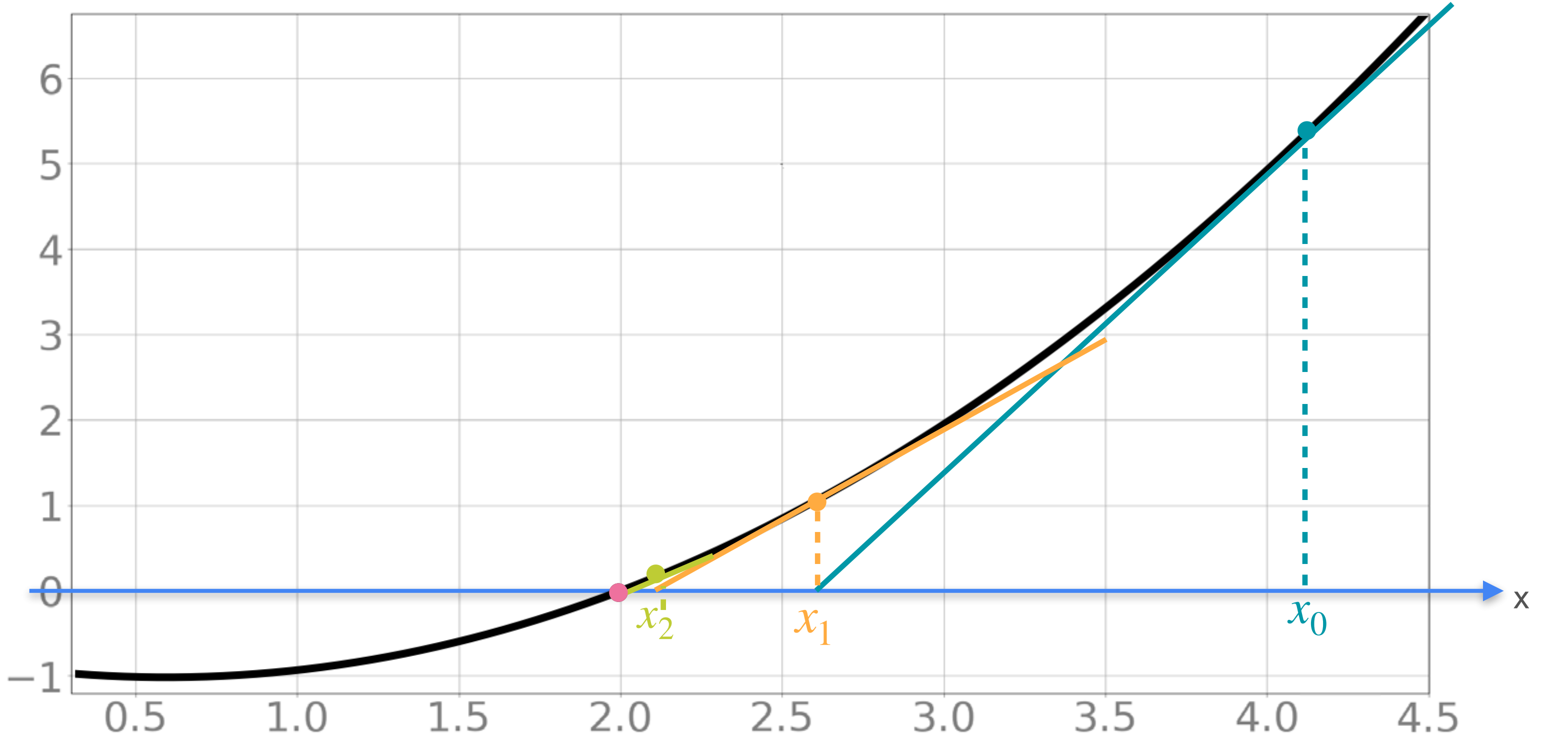
Newton's Method



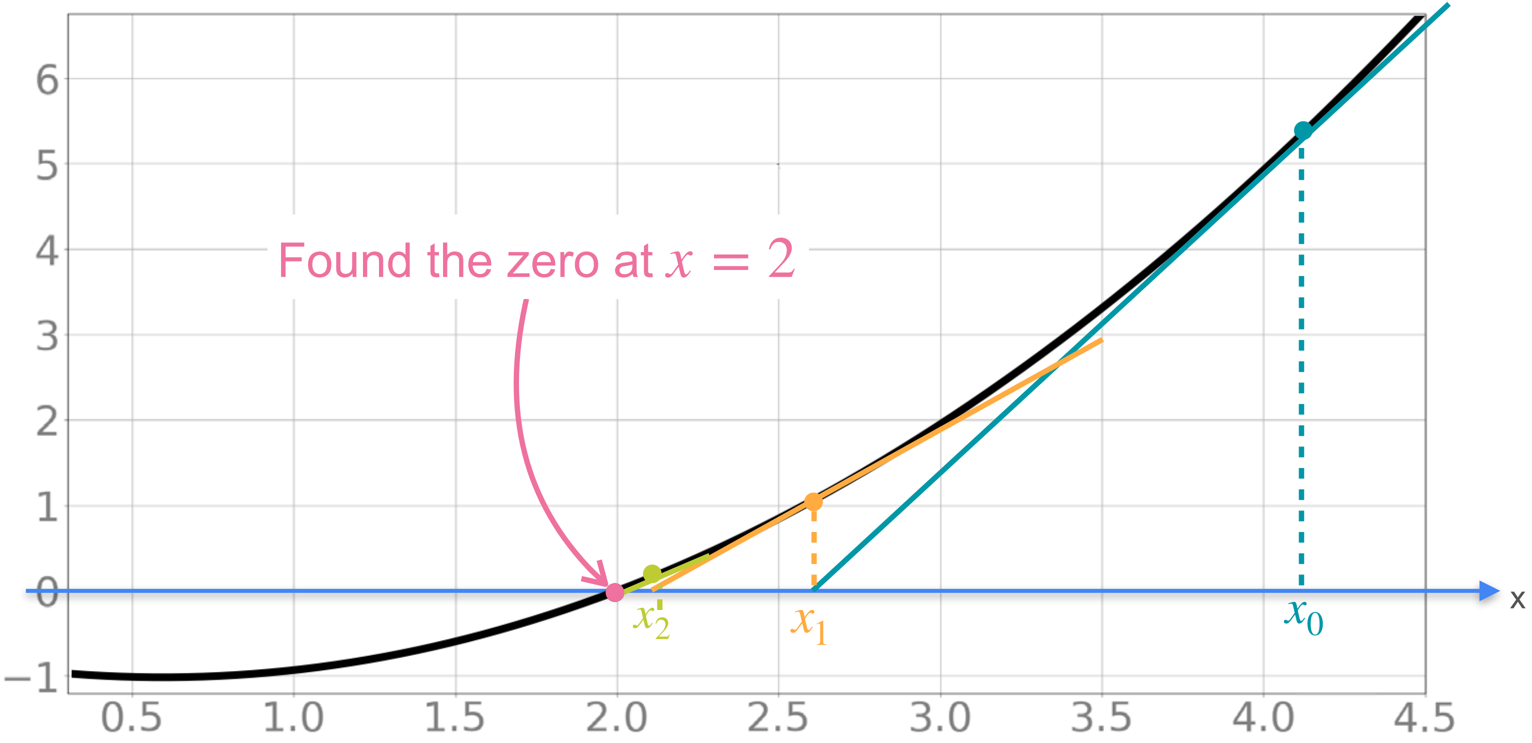
Newton's Method



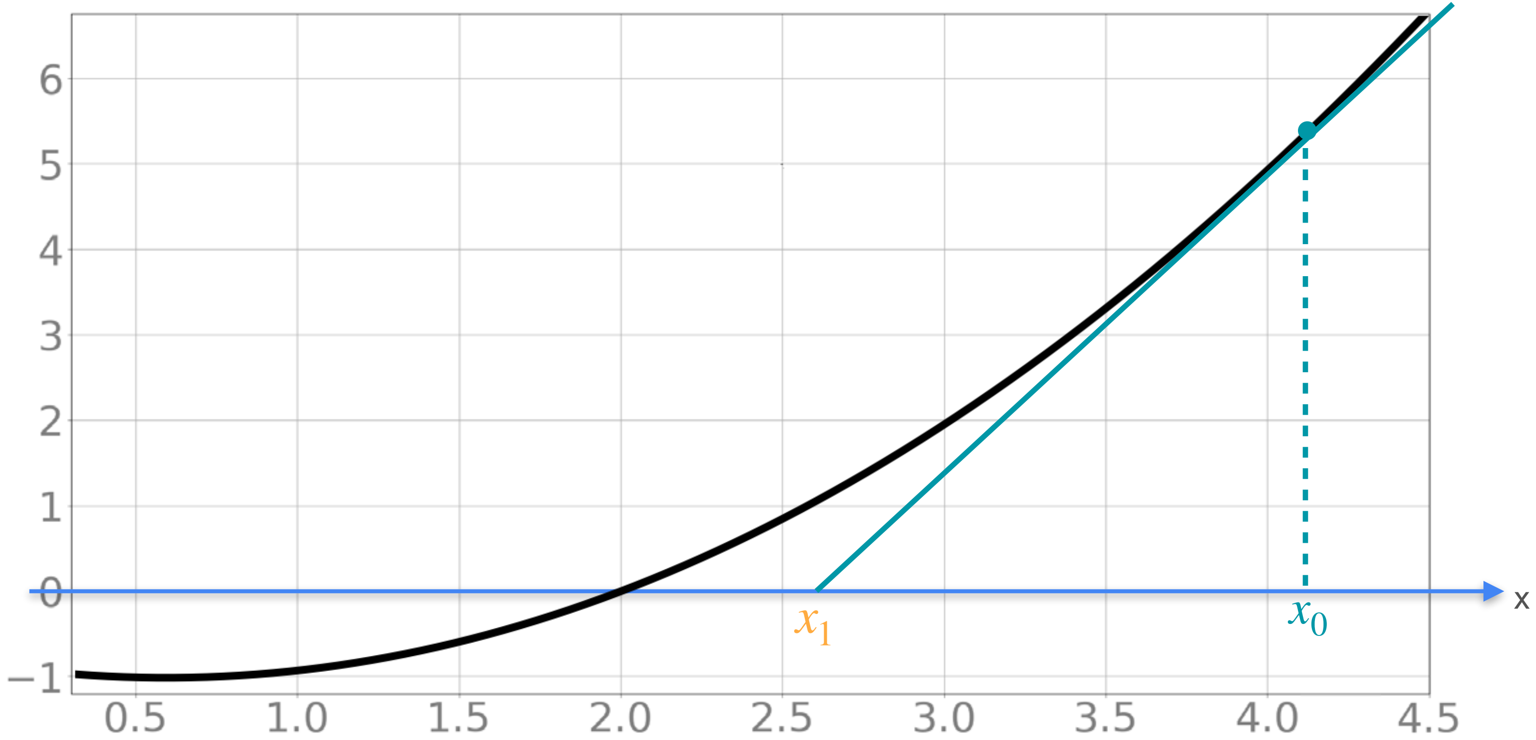
Newton's Method



Newton's Method



Update Approximation



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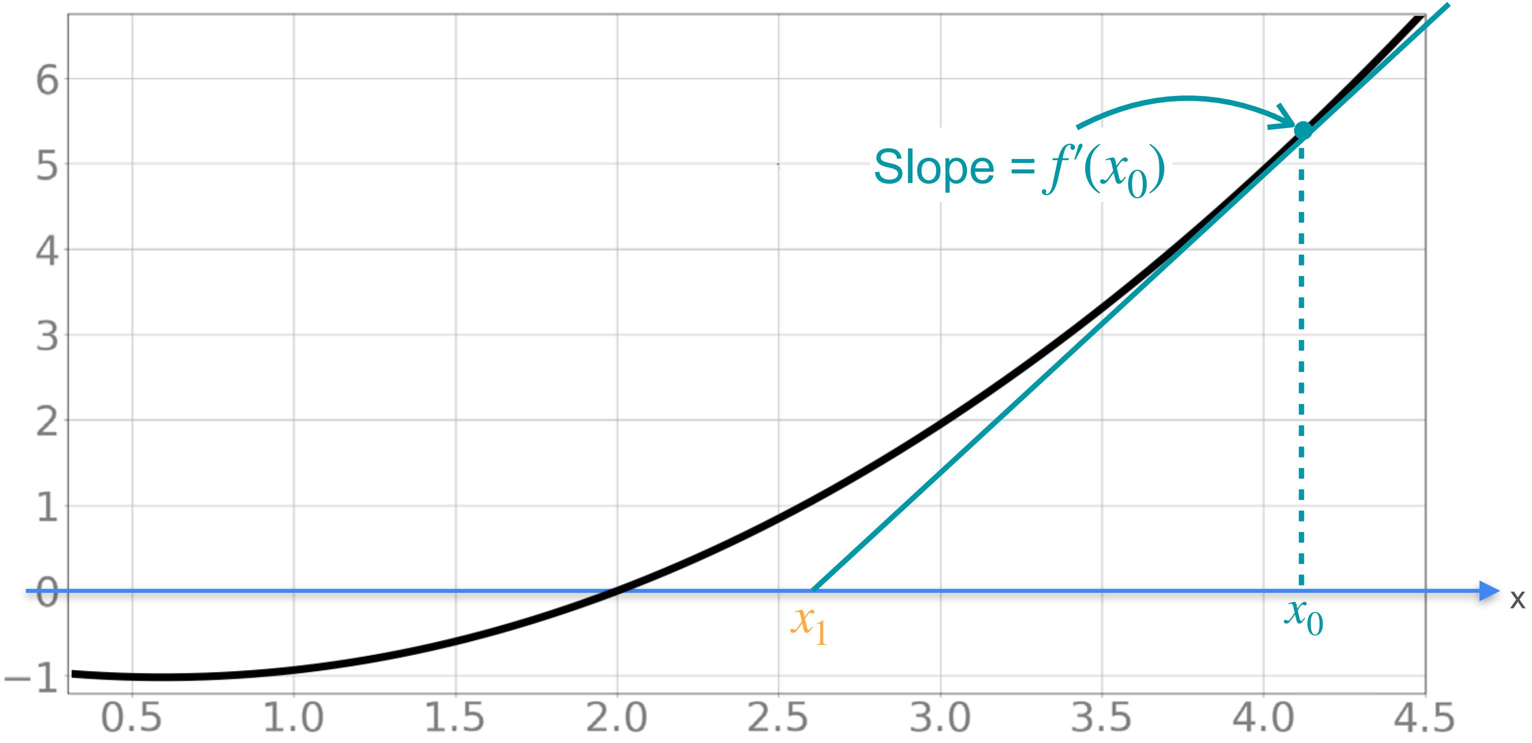

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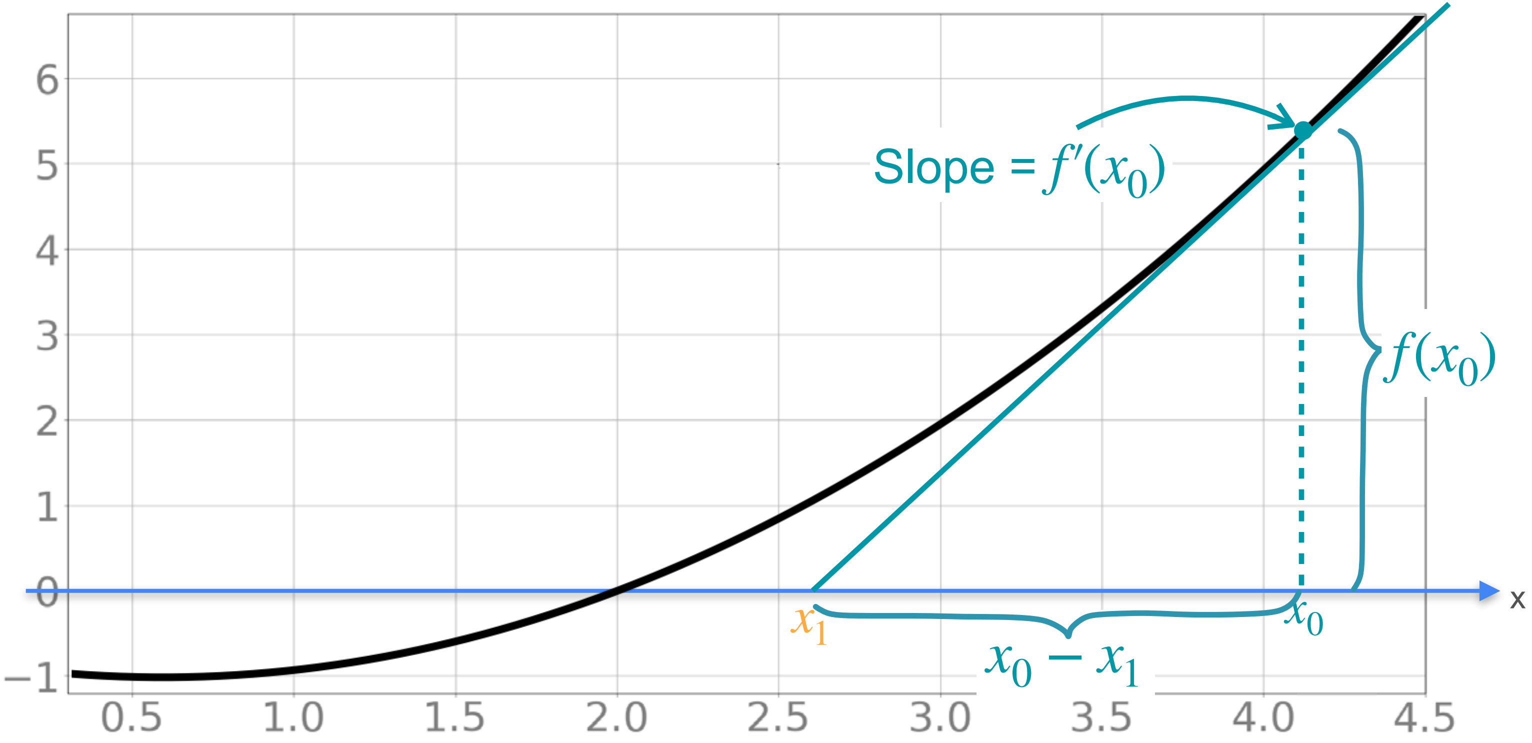

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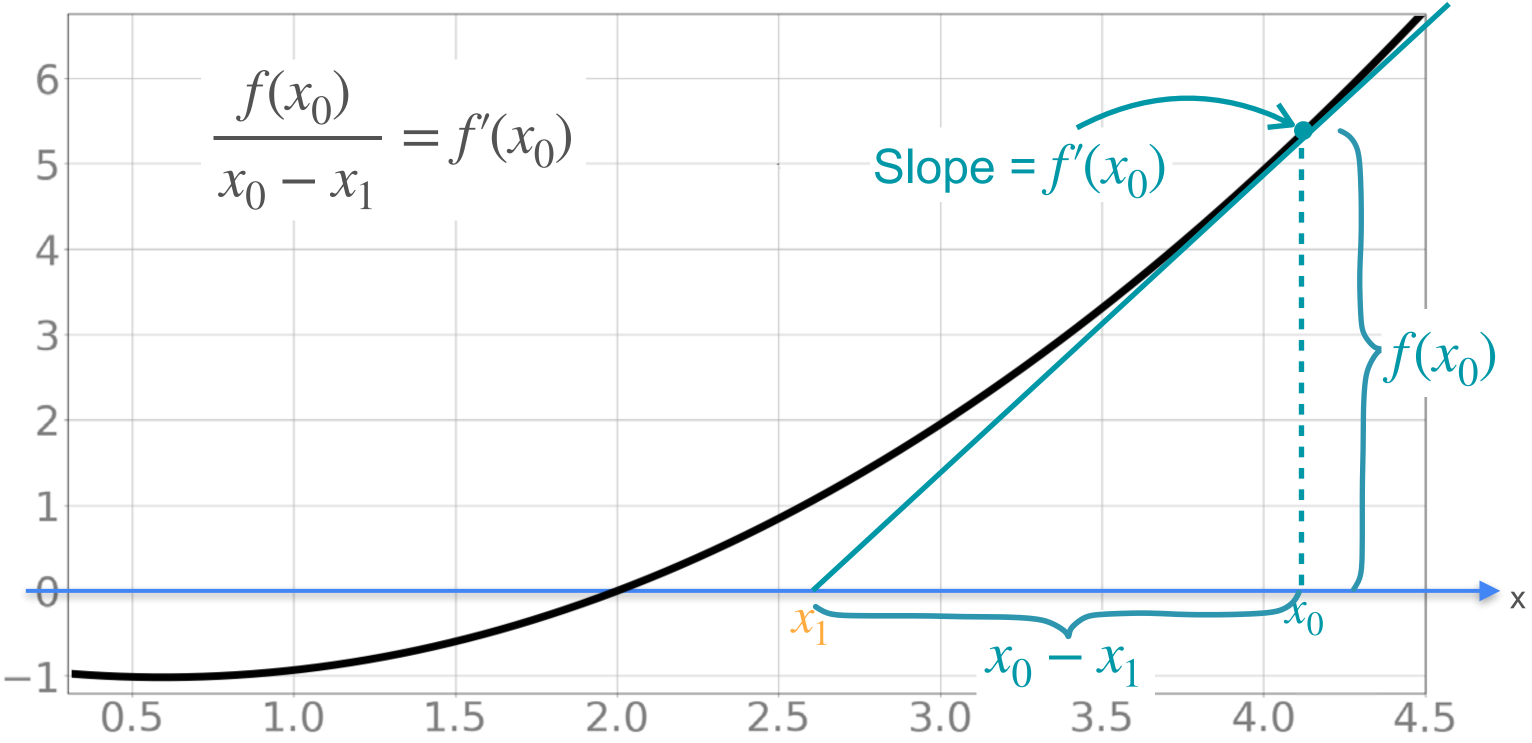

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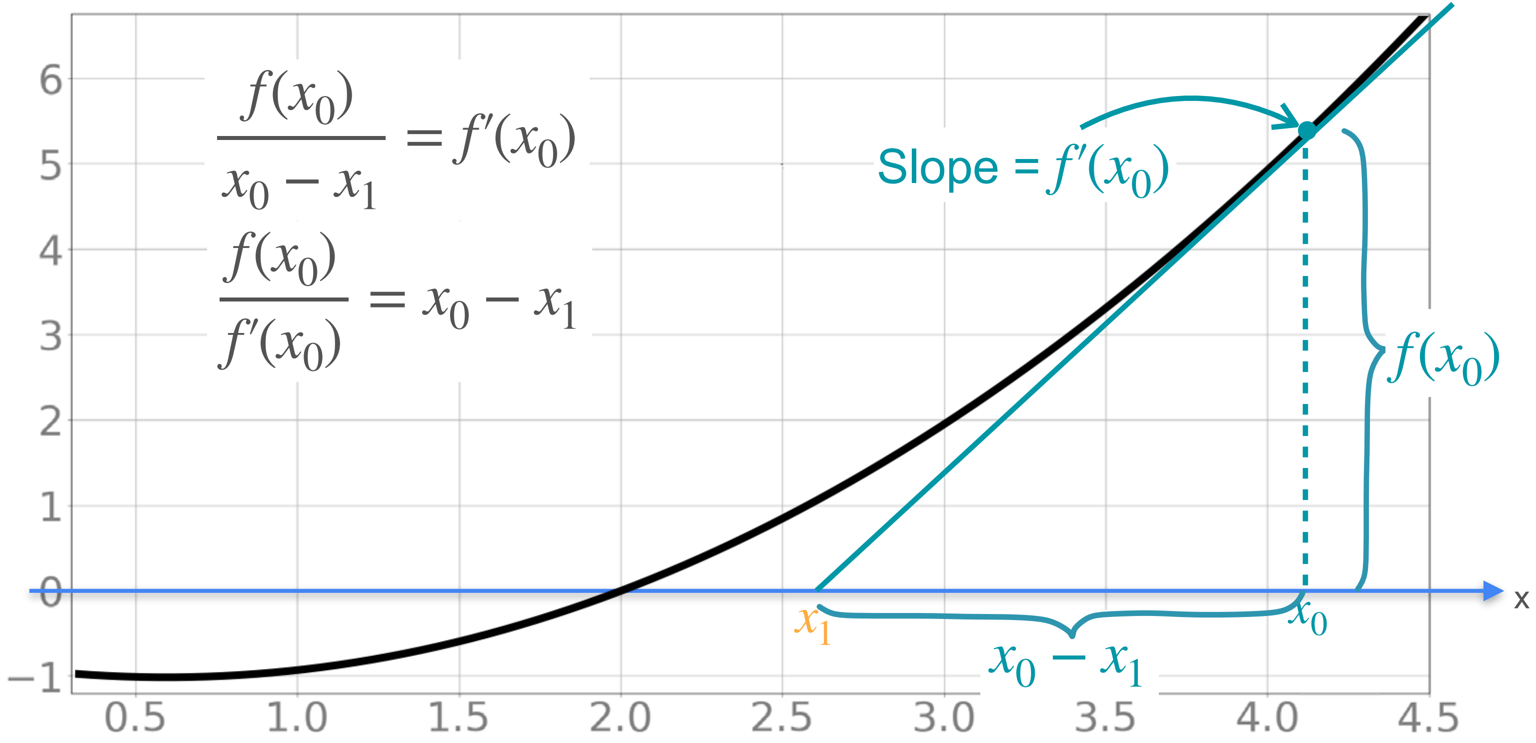

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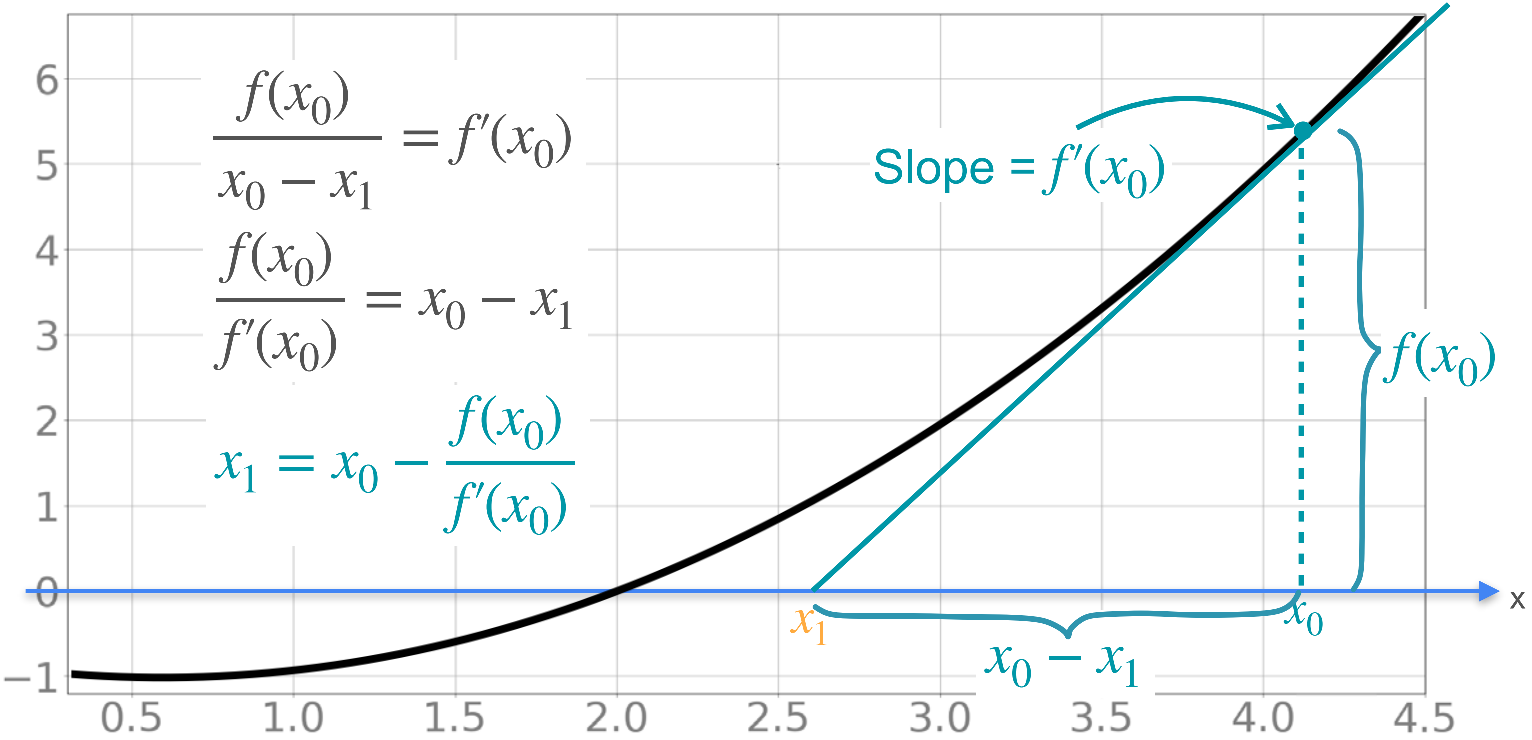

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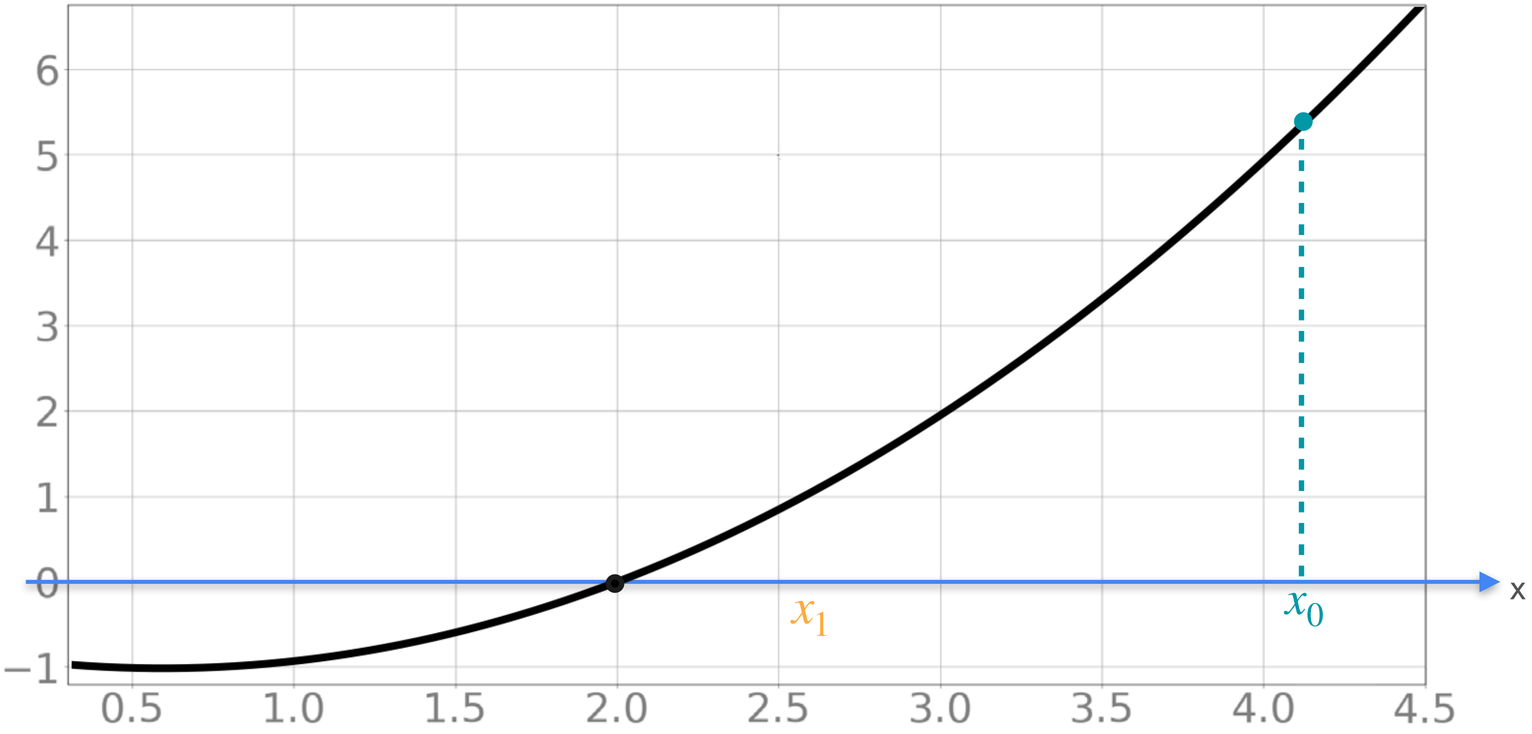

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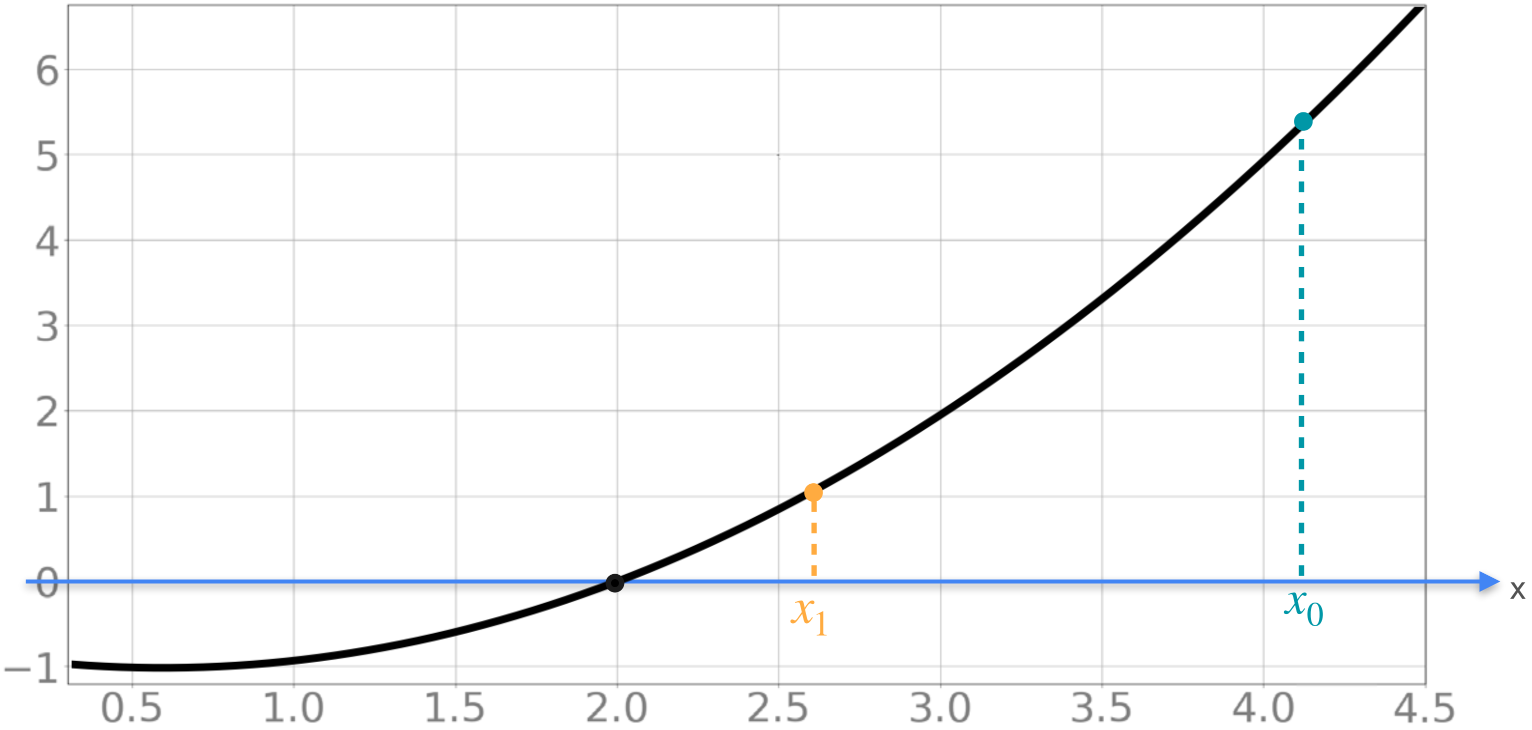
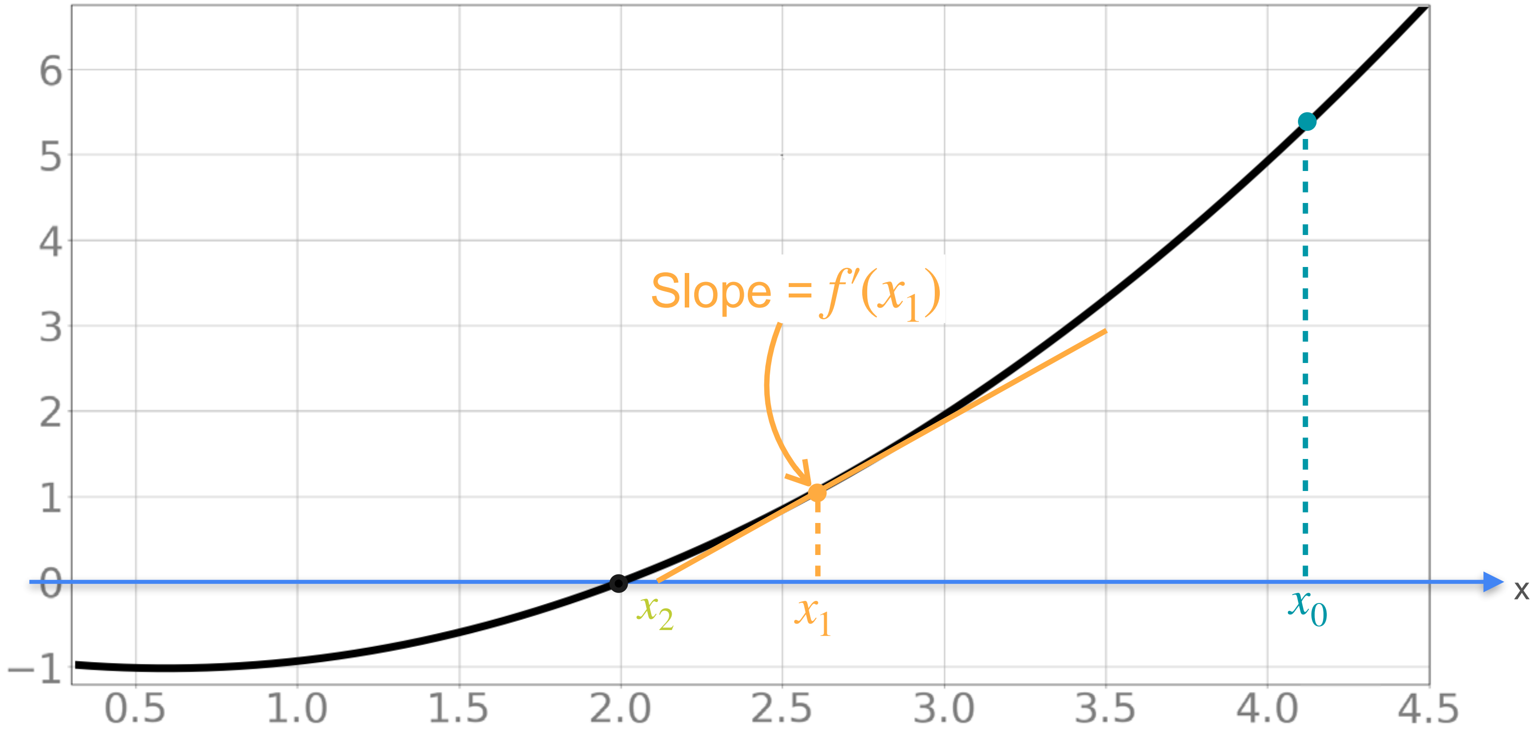
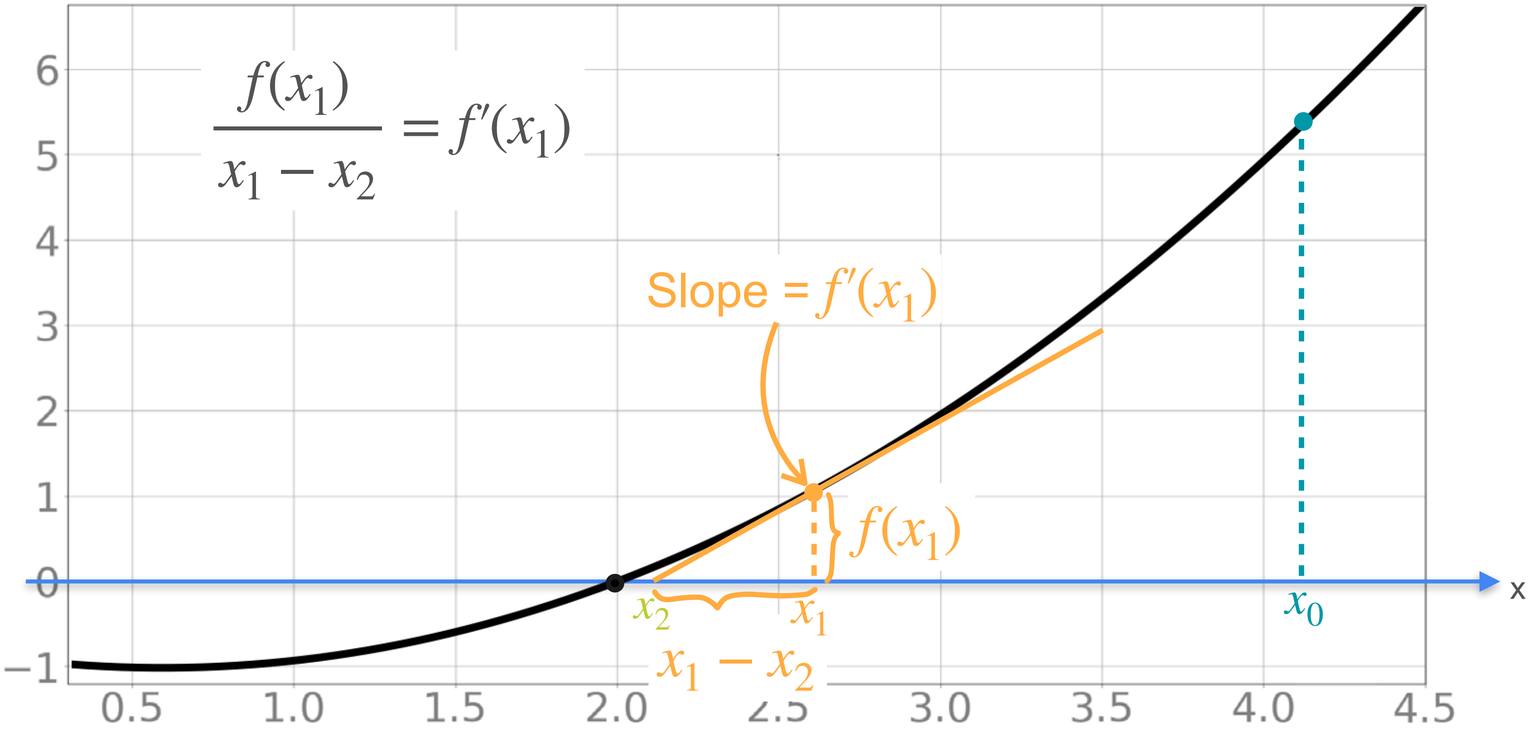
Update Approximation



Update Approximation



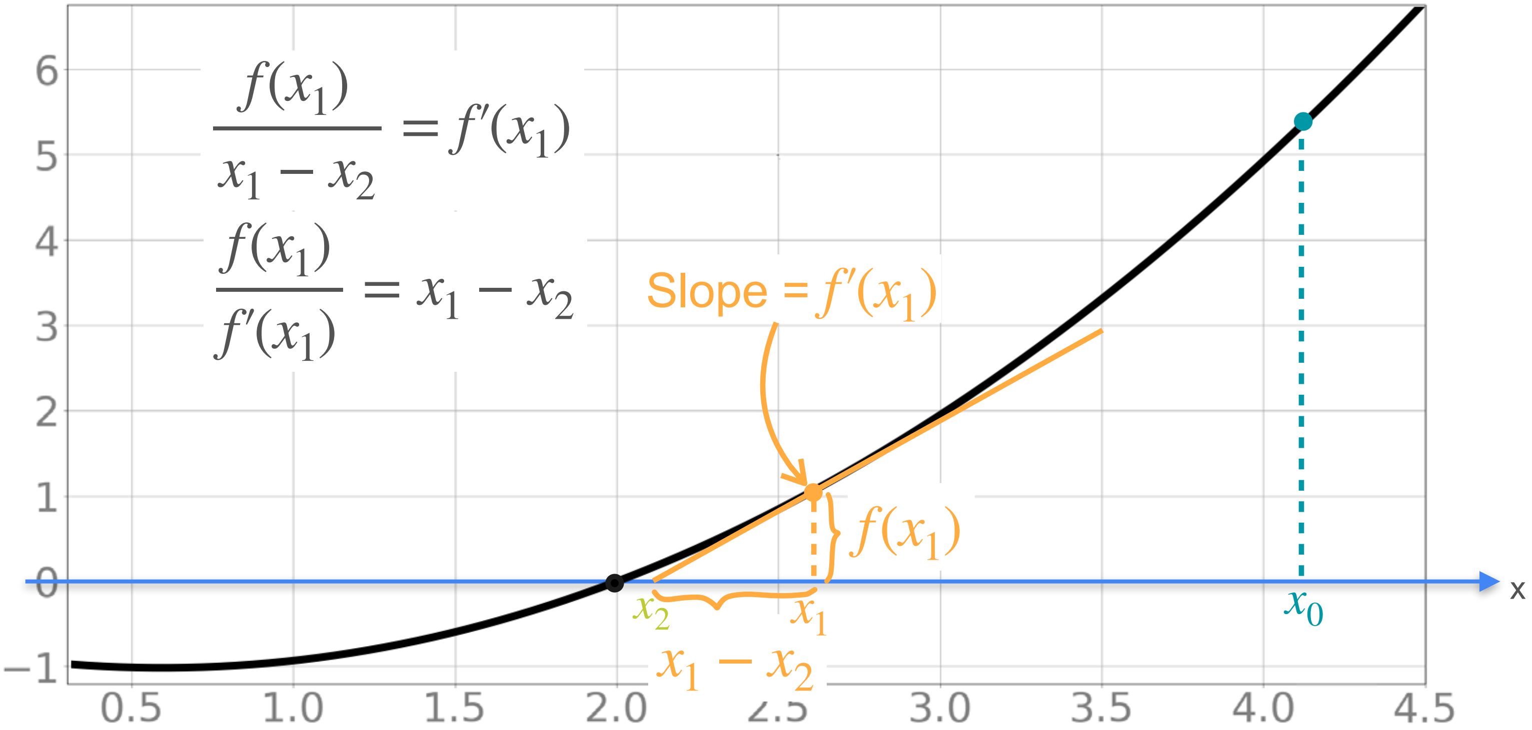
Update Approximation



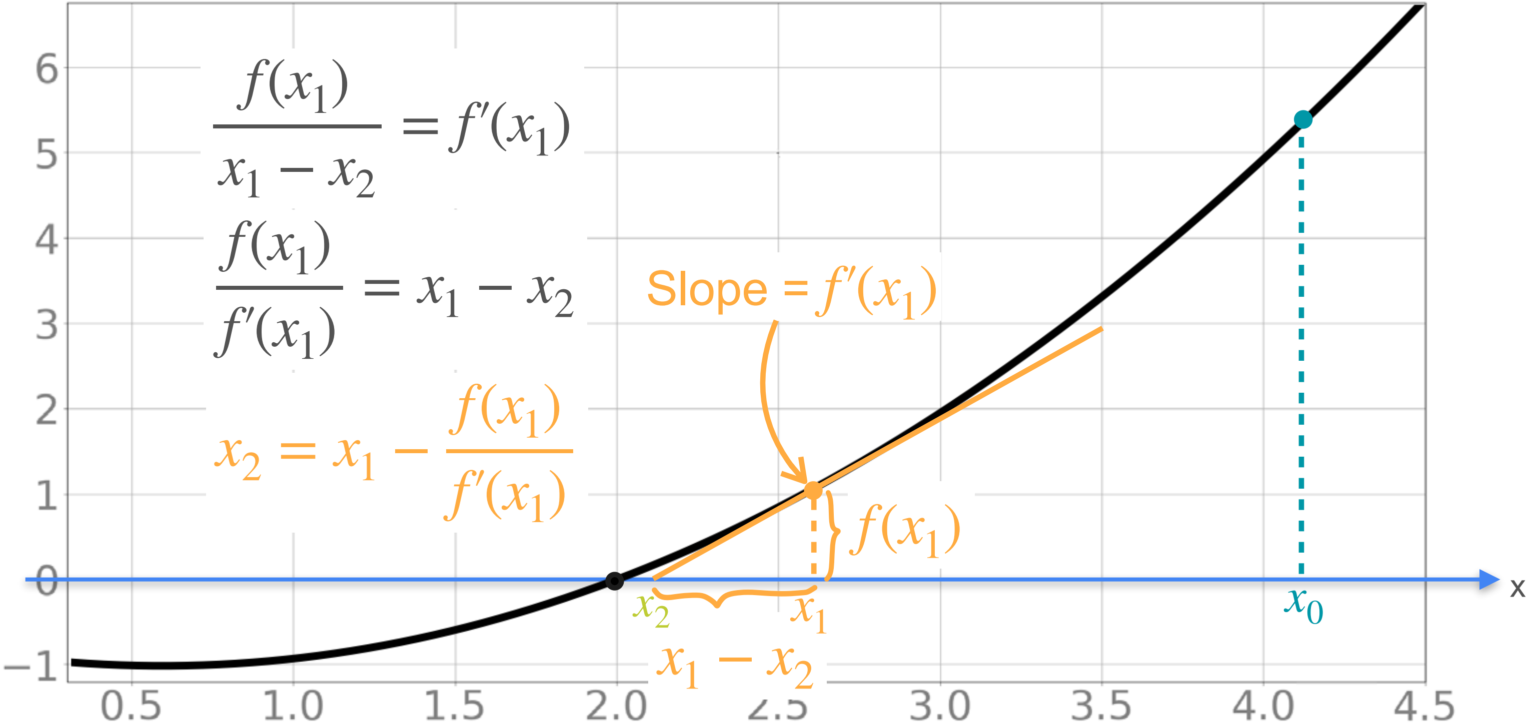
Update Approximation

$$\frac{f(x_1)}{x_1 - x_2} = f'(x_1)$$

$$\frac{f(x_1)}{f'(x_1)} = x_1 - x_2$$



Update Approximation



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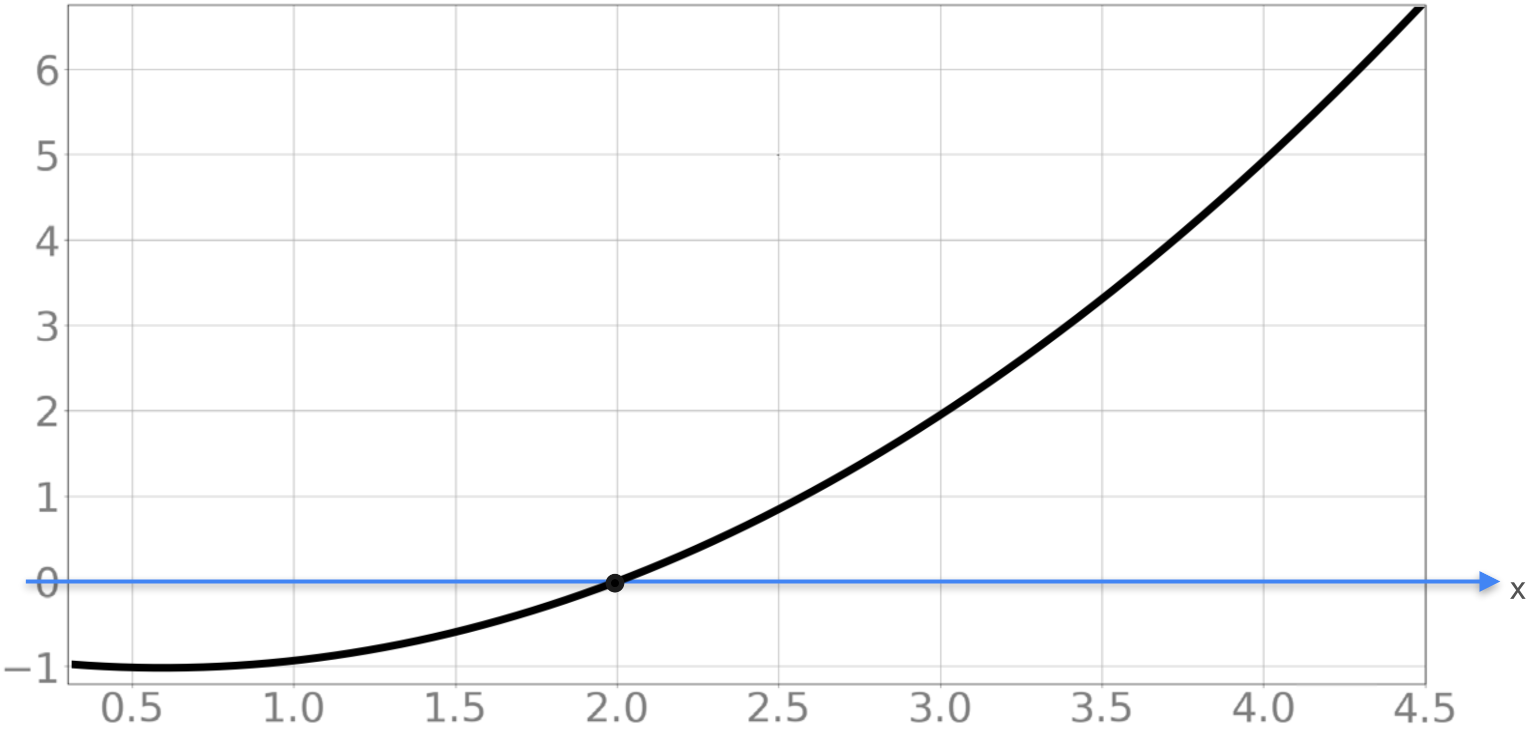

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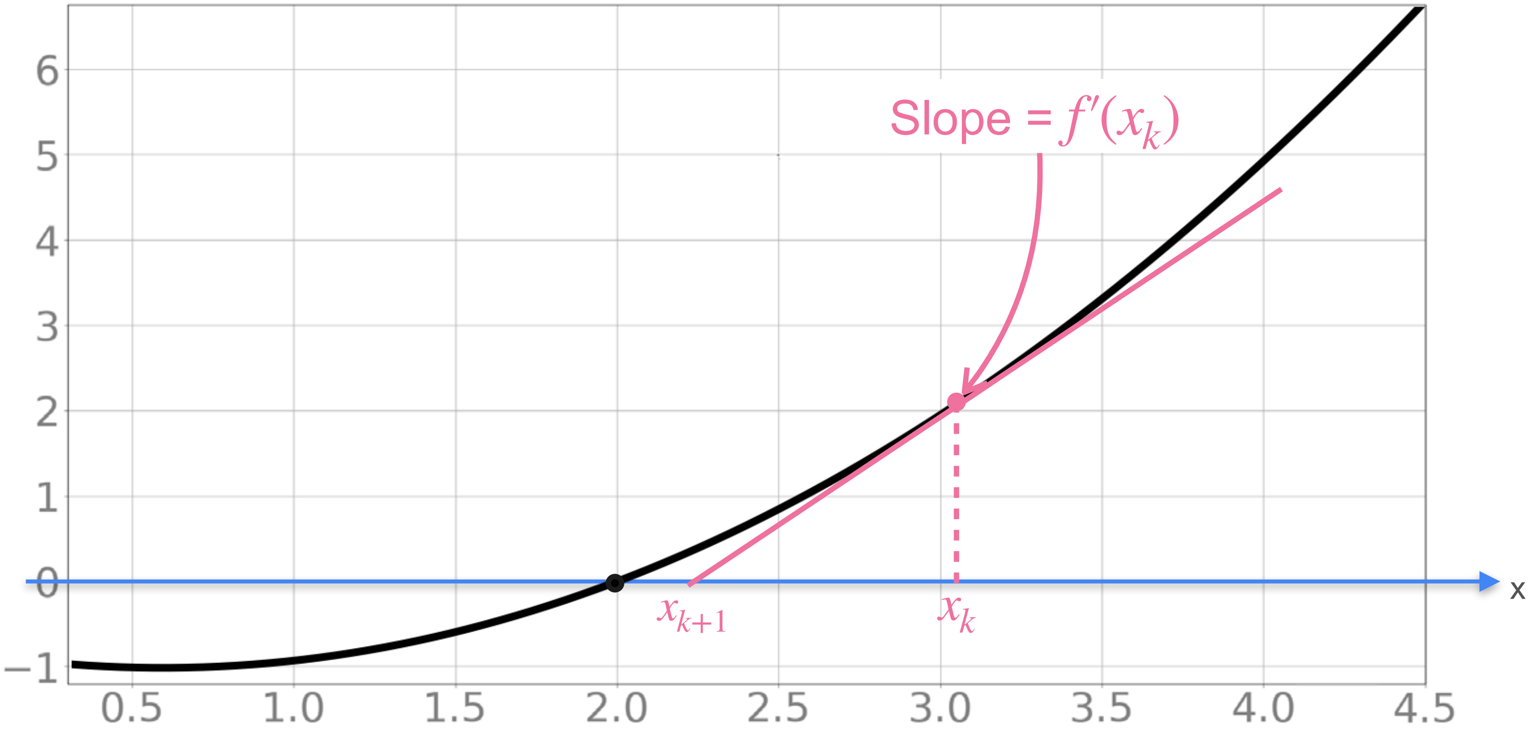

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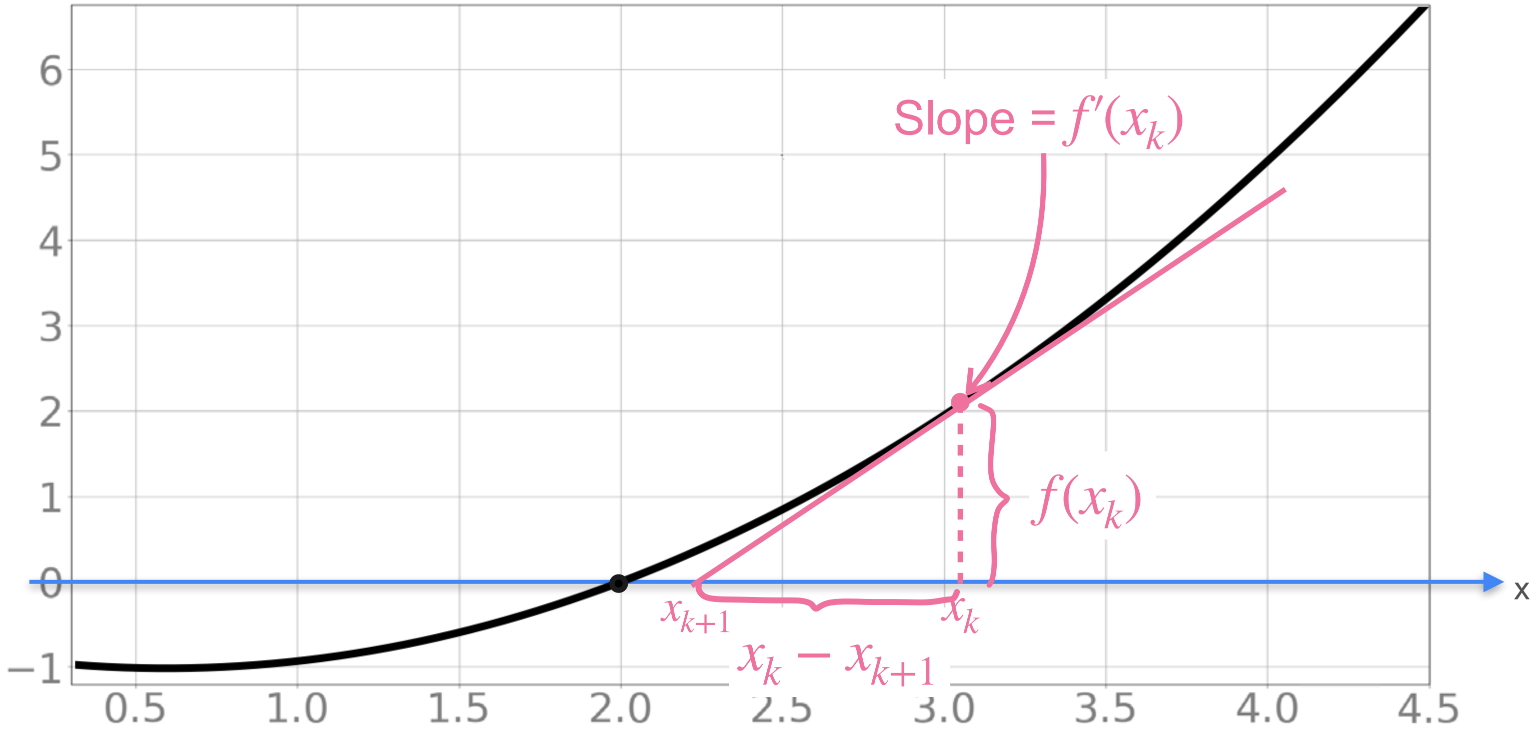

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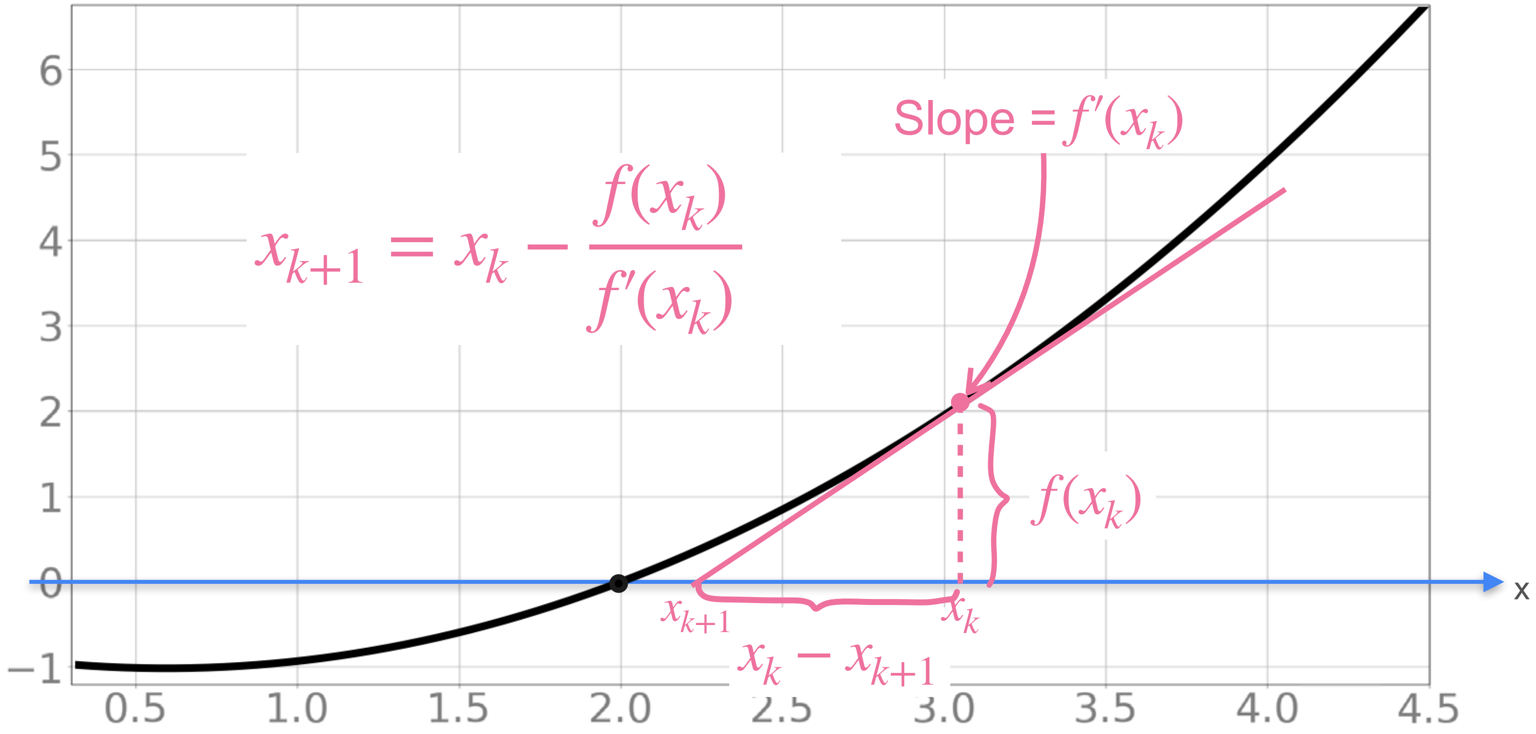

Newton's Method for Optimization

Newton’s Method for Optimization

Newton’s Method for Optimization

Newton’s method

Goal: find a zero of $f(x)$

Newton's Method for Optimization

Newton's method

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NM for Optimization

Goal: minimize $g(x) \Rightarrow$ find zeros of $g'(x)$

Newton's Method for Optimization

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NM for Optimization

Goal: minimize $g(x) \Rightarrow$ find zeros of $g'(x)$

$$f(x) \mapsto g'(x) \qquad f'(x) \mapsto (g'(x))'$$

Newton's Method for Optimization

Newton's method

Goal: find a zero of $f(x)$

1) Start with some x_0

NM for Optimization

Goal: minimize $g(x) \Rightarrow$ find zeros of $g'(x)$

$$f(x) \mapsto g'(x) \qquad f'(x) \mapsto (g'(x))'$$

1) Start with some x_0

Newton's Method for Optimization

Newton's method

Goal: find a zero of $f(x)$

1) Start with some x_0

2) Update:

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

NM for Optimization

Goal: minimize $g(x) \Rightarrow$ find zeros of $g'(x)$

$f(x) \mapsto g'(x) \qquad f'(x) \mapsto (g'(x))'$

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Newton's Method for Optimization

Newton's method

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Newton's Method for Optimization

Newton's method

Goal: find a zero of $f(x)$

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2) Update:

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

3) Repeat 2) until you find the root.

NM for Optimization

Goal: minimize $g(x) \Rightarrow$ find zeros of $g'(x)$

$f(x) \mapsto g'(x)$ $\quad$ $f'(x) \mapsto (g'(x))'$

1) Start with some x_0

2) Update:

$$x_{k+1} = x_k - \frac{g'(x_k)}{(g'(x_k))'}$$

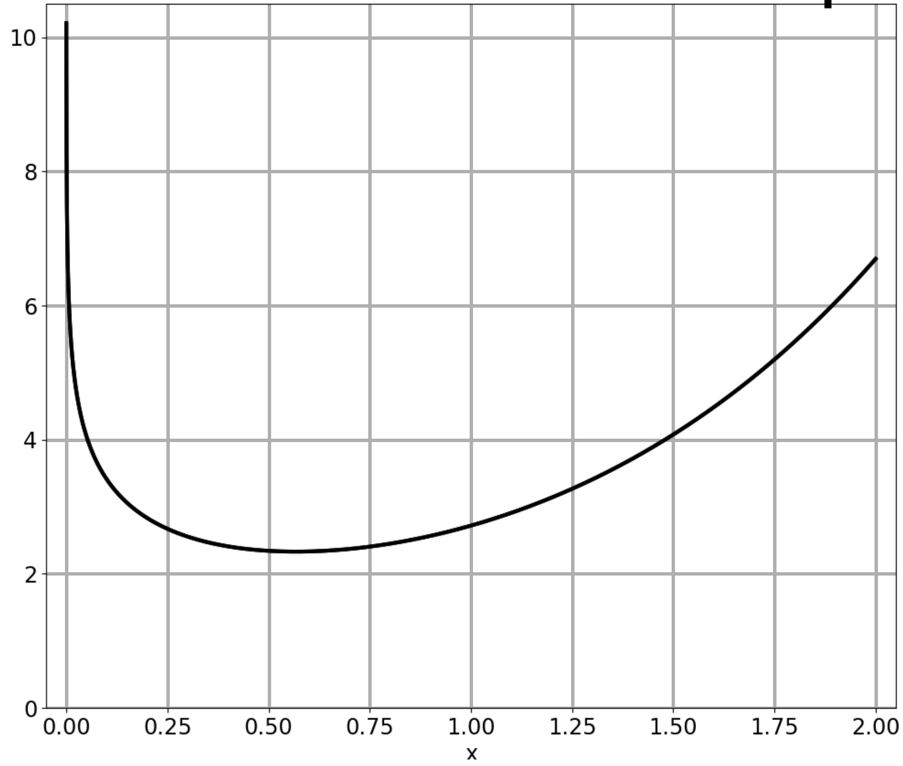
3) Repeat 2) until you find the root.

Optimization in Neural Networks and Newton's Method

Newton's method: An example

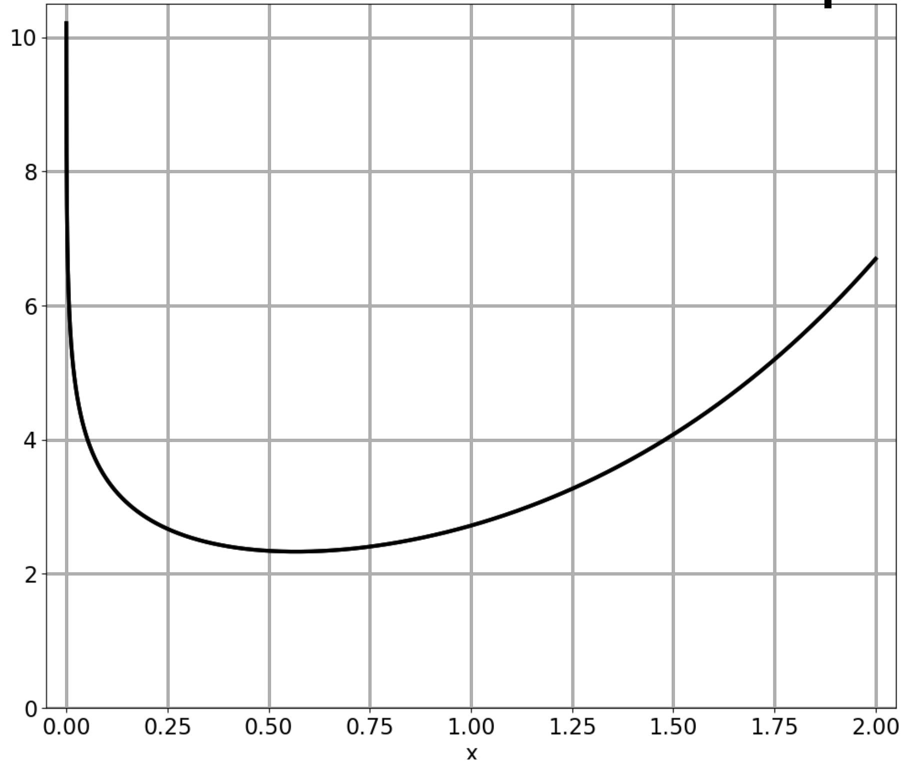
Newton's Method for Optimization

Newton's Method for Optimization



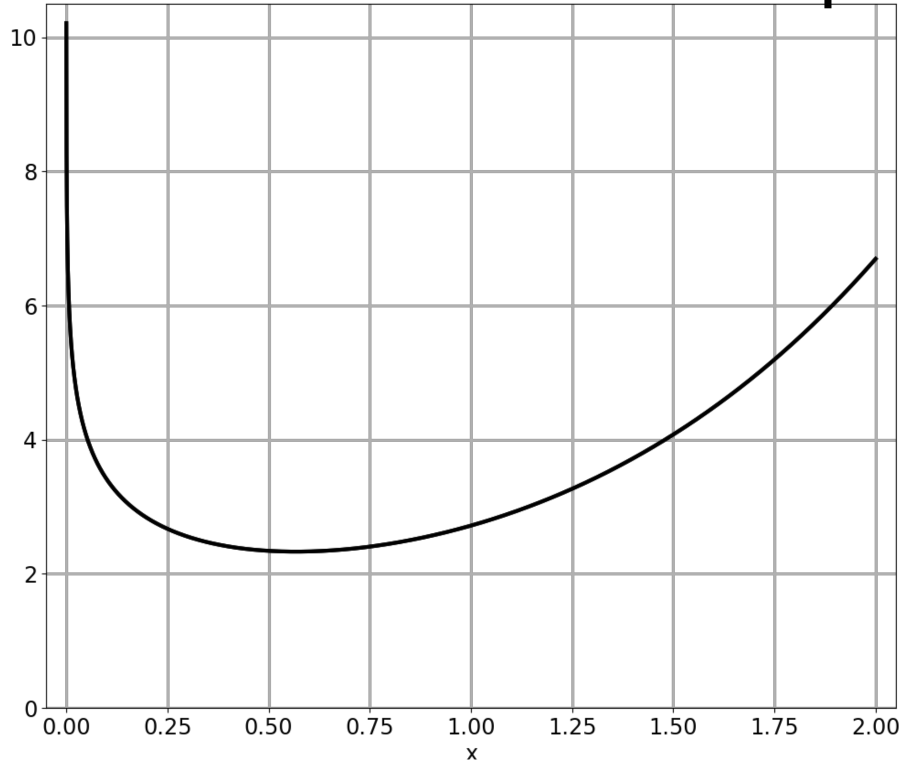
$$g(x) = e^x - \log(x)$$

Newton's Method for Optimization



$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

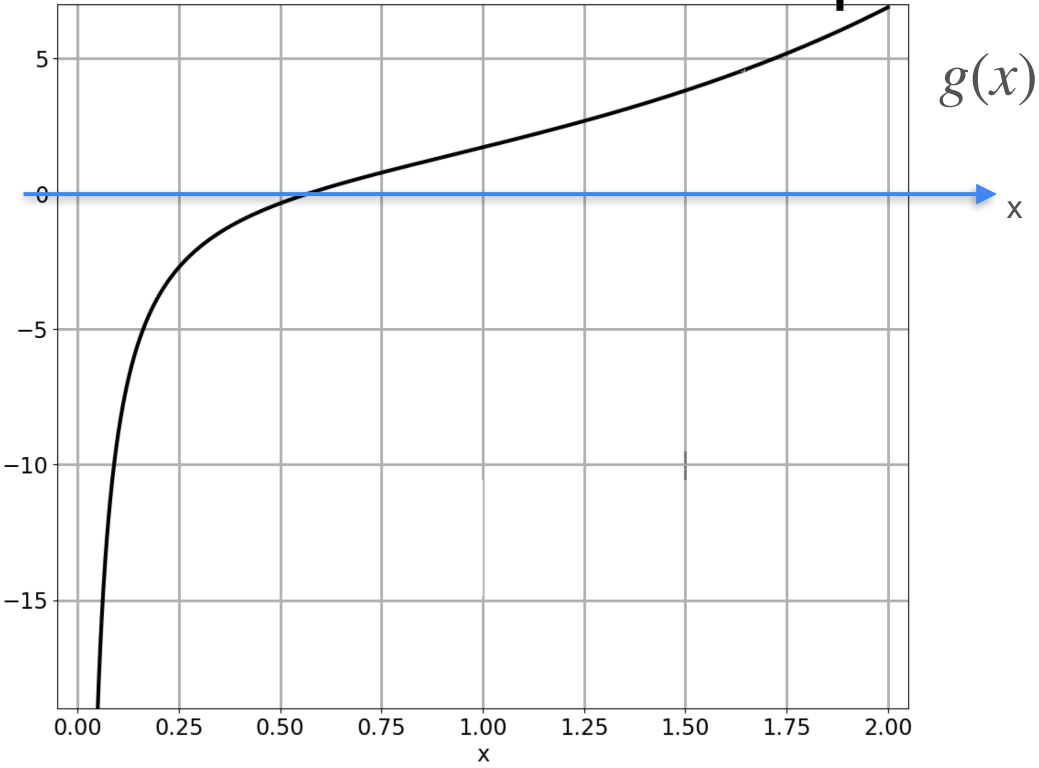
Newton's Method for Optimization



$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

Newton's Method for Optimization



$g(x) = e^x - \log(x)$ $\quad g'(x) = e^x - 1/x$

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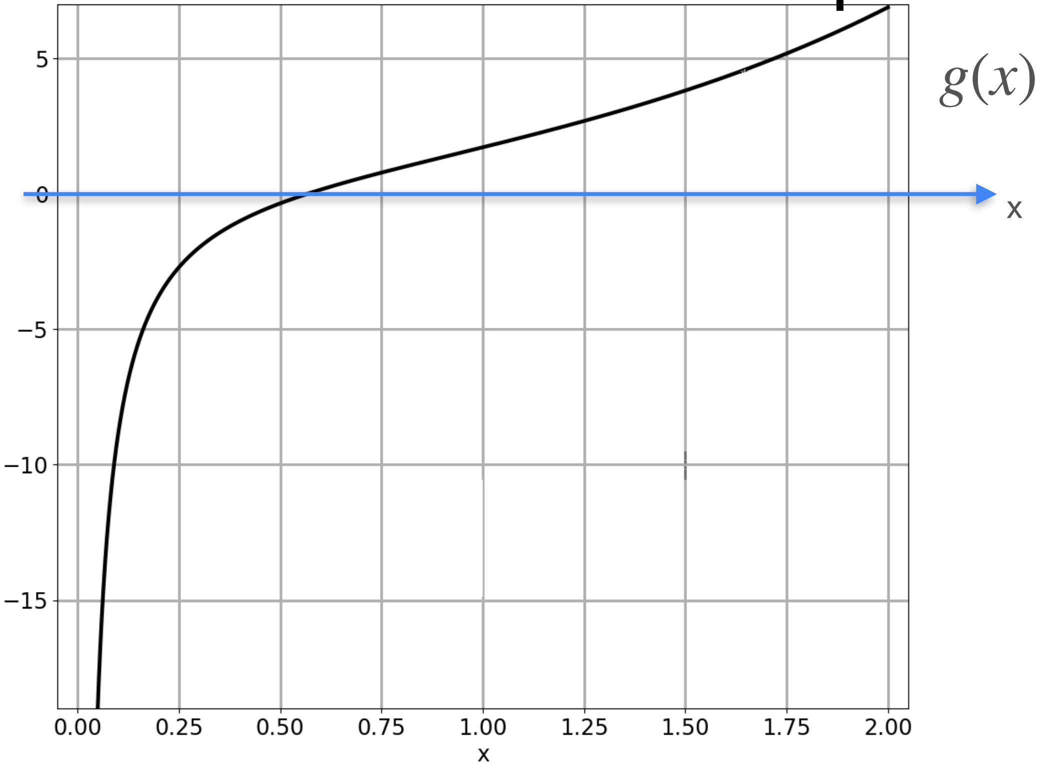
Newton's Method for Optimization

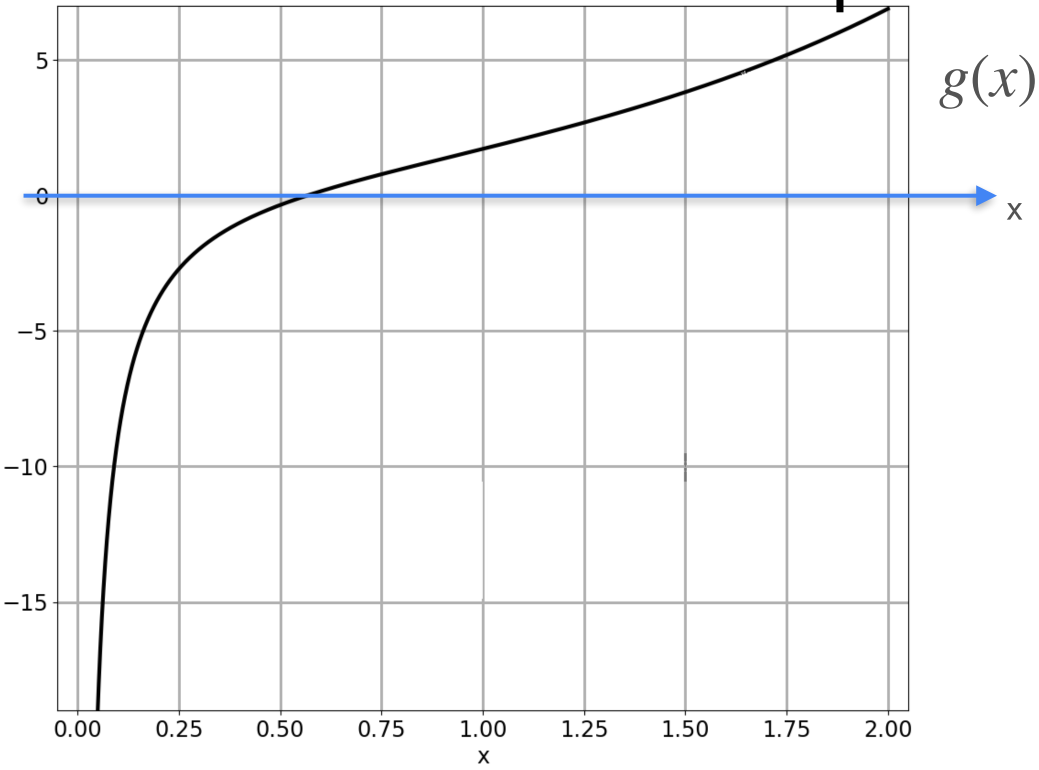


$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

Newton's Method for Optimization

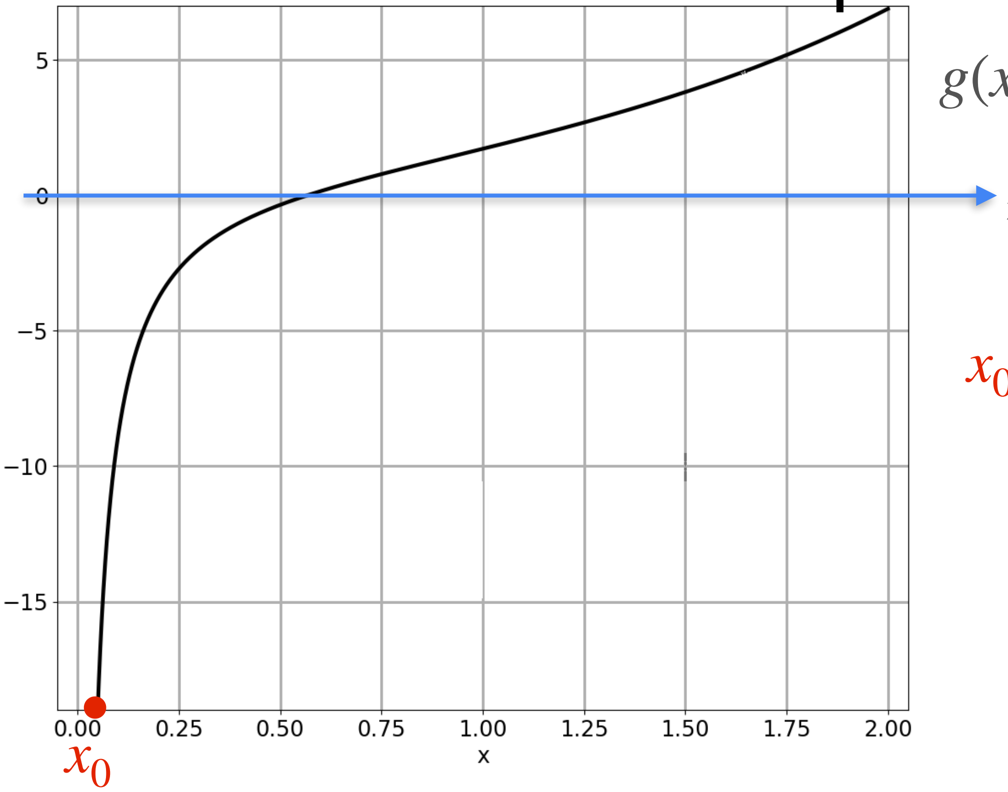


$$g(x) = e^x - \log(x) \qquad \overbrace{g'(x) = e^x - 1/x}^{f(x)}$$

Minimum: $x^* = 0.5671$

$$\underbrace{(g'(x))' = e^x + \frac{1}{x^2}}_{f'(x)}$$

Newton's Method for Optimization



$g(x) = e^x - \log(x)$ $\quad \overbrace{g'(x) = e^x - 1/x}^{f(x)}$

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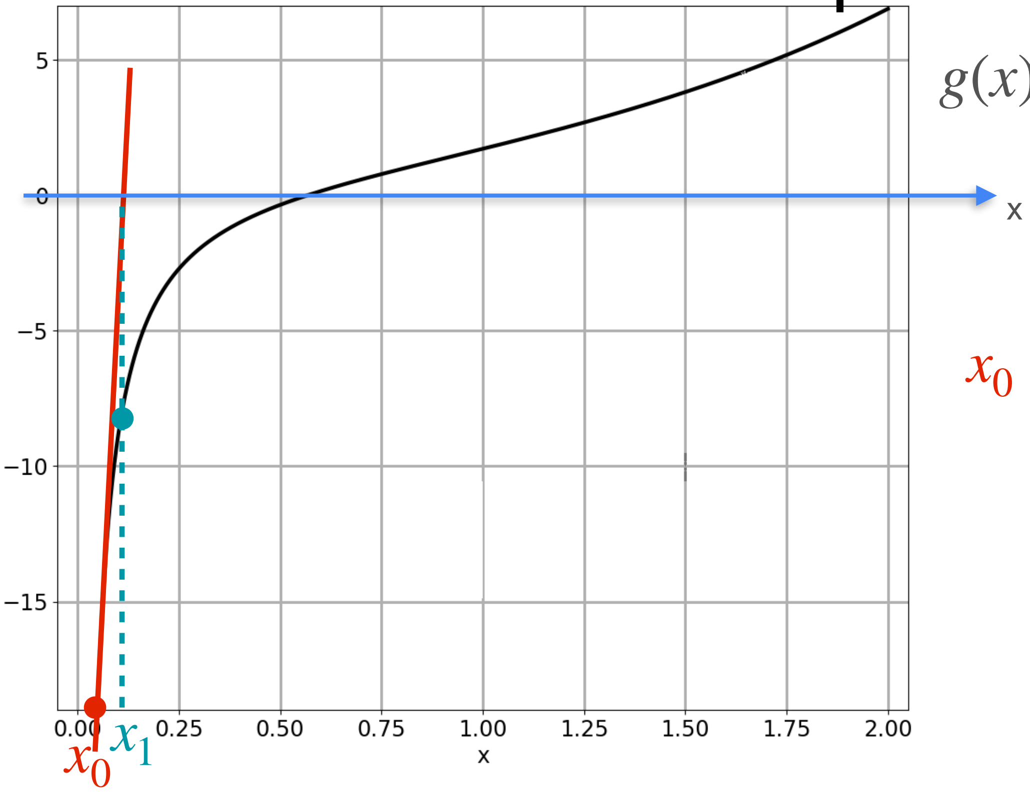
$x_0 = 0.05$

Newton's Method for Optimization



$$g(x) = e^x - \log(x) \qquad \overbrace{g'(x) = e^x - 1/x}^{f(x)}$$

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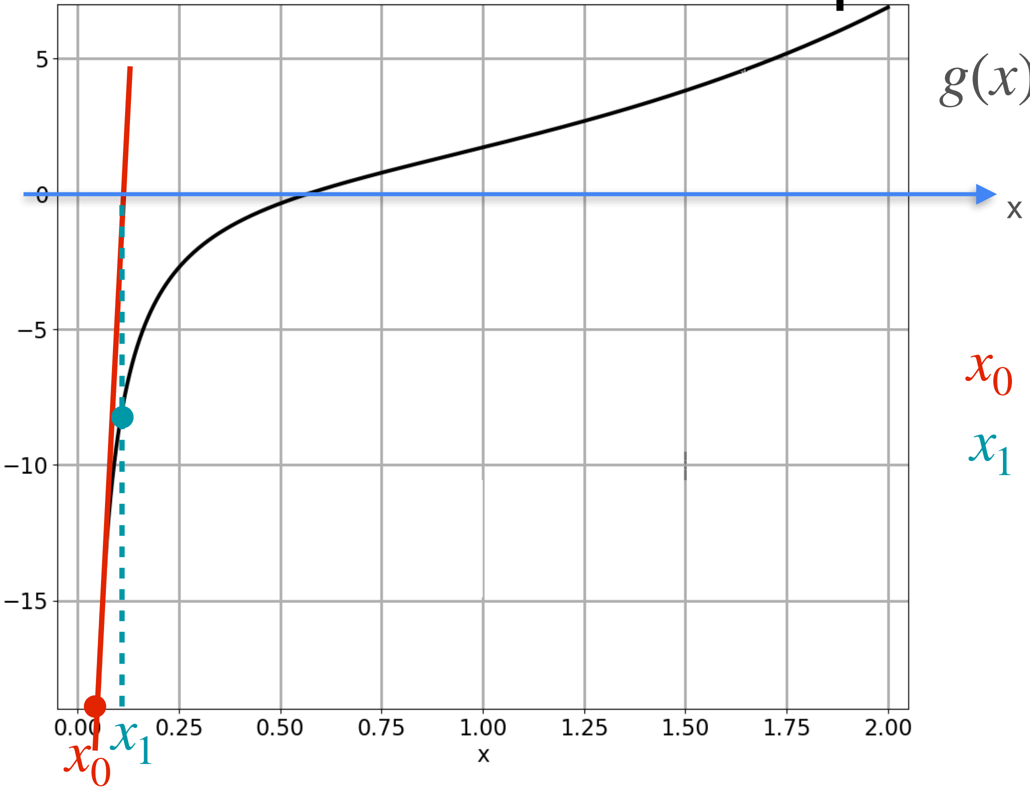
Newton's Method for Optimization



$$g(x) = e^x - \log(x) \qquad \overbrace{g'(x) = e^x - 1/x}^{f(x)}$$

Minimum: $x^* = 0.5671$

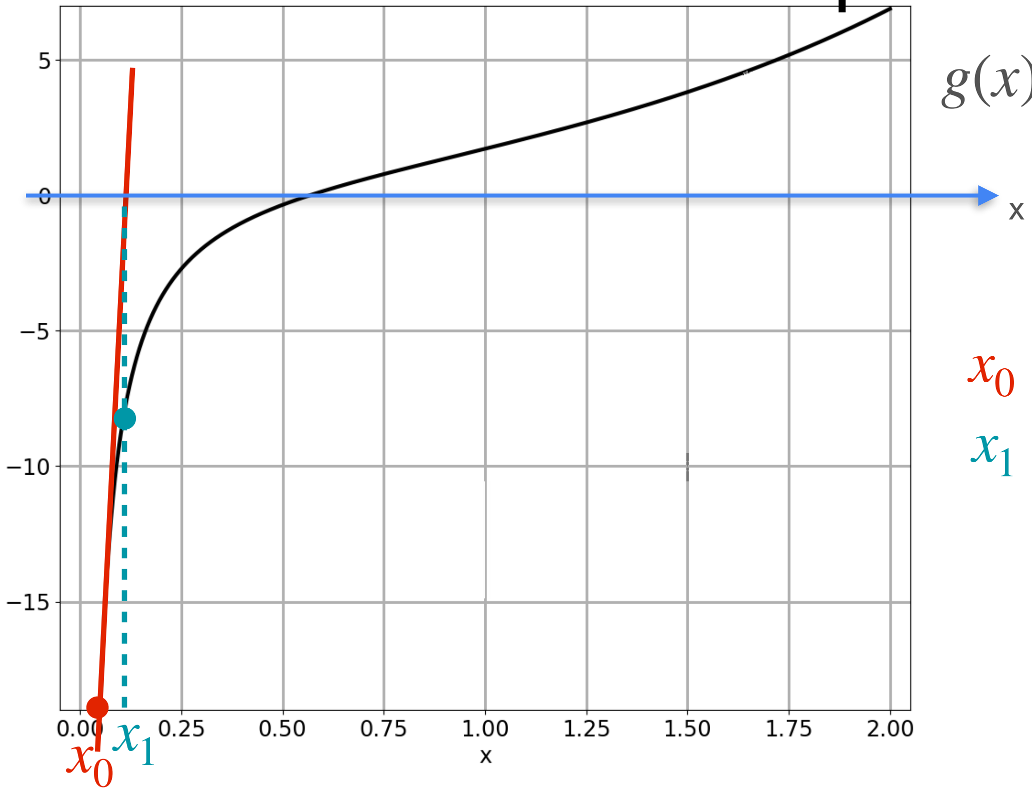
$$\underbrace{(g'(x))' = e^x + \frac{1}{x^2}}_{f'(x)}$$

$$x_0 = 0.05$$

$$x_1 = x_0 - \frac{g'(x_0)}{(g'(x_0))'}$$

$$= 0.05 - \frac{\left(e^{0.05} - \frac{1}{0.05}\right)}{\left(e^{0.05} + \frac{1}{0.05^2}\right)}$$

Newton's Method for Optimization



$$g(x) = e^x - \log(x) \qquad \overbrace{g'(x) = e^x - 1/x}^{f(x)}$$

Minimum: $x^* = 0.5671$

$$\underbrace{(g'(x))' = e^x + \frac{1}{x^2}}_{f'(x)}$$

$$x_0 = 0.05$$

$$x_1 = x_0 - \frac{g'(x_0)}{(g'(x_0))'}$$

$$= 0.05 - \frac{\left(e^{0.05} - \frac{1}{0.05}\right)}{\left(e^{0.05} + \frac{1}{0.05^2}\right)} = 0.097$$

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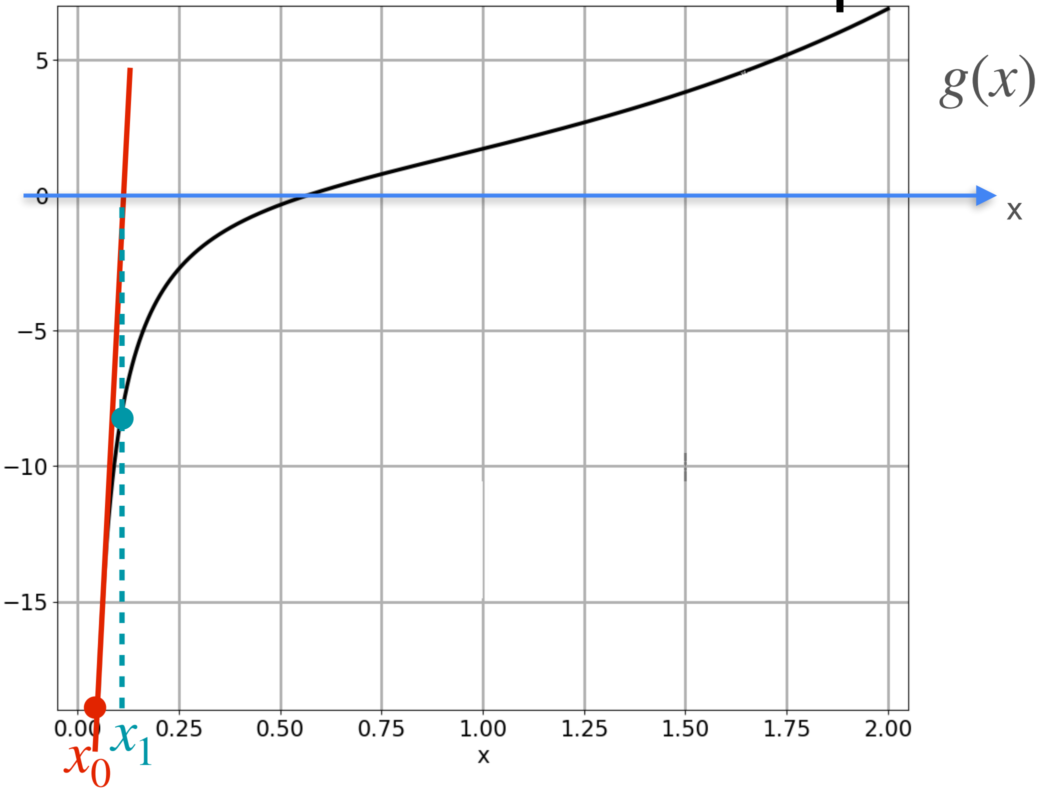
Newton's Method for Optimization

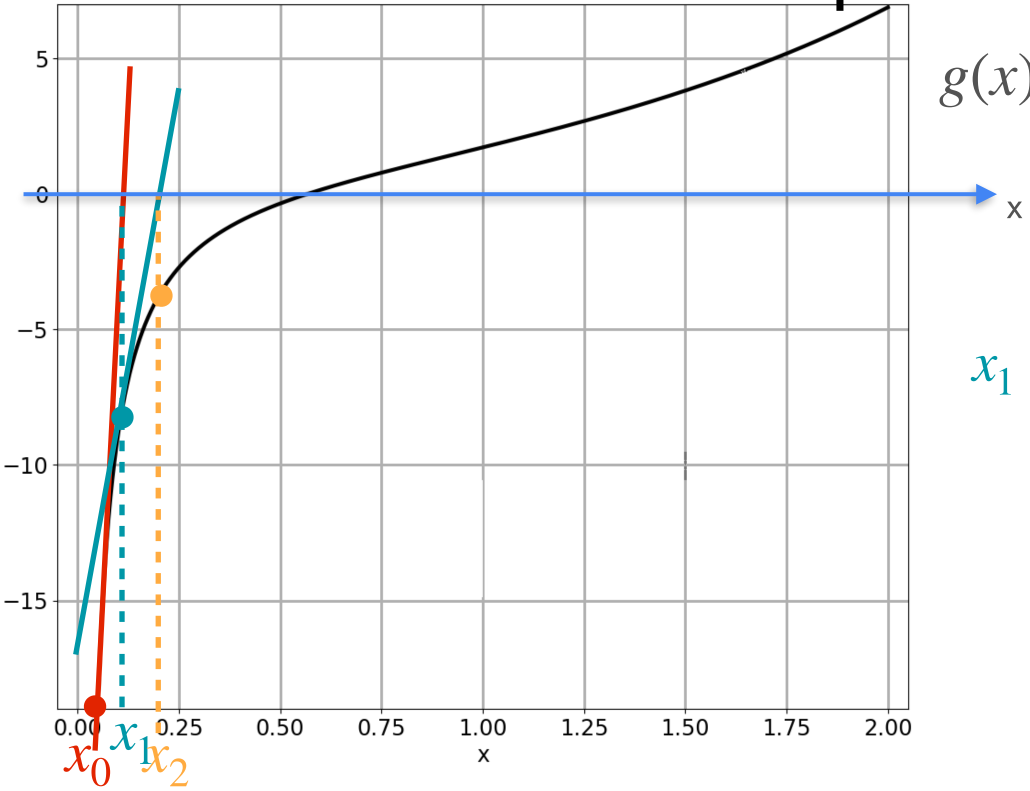


$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

Newton's Method for Optimization



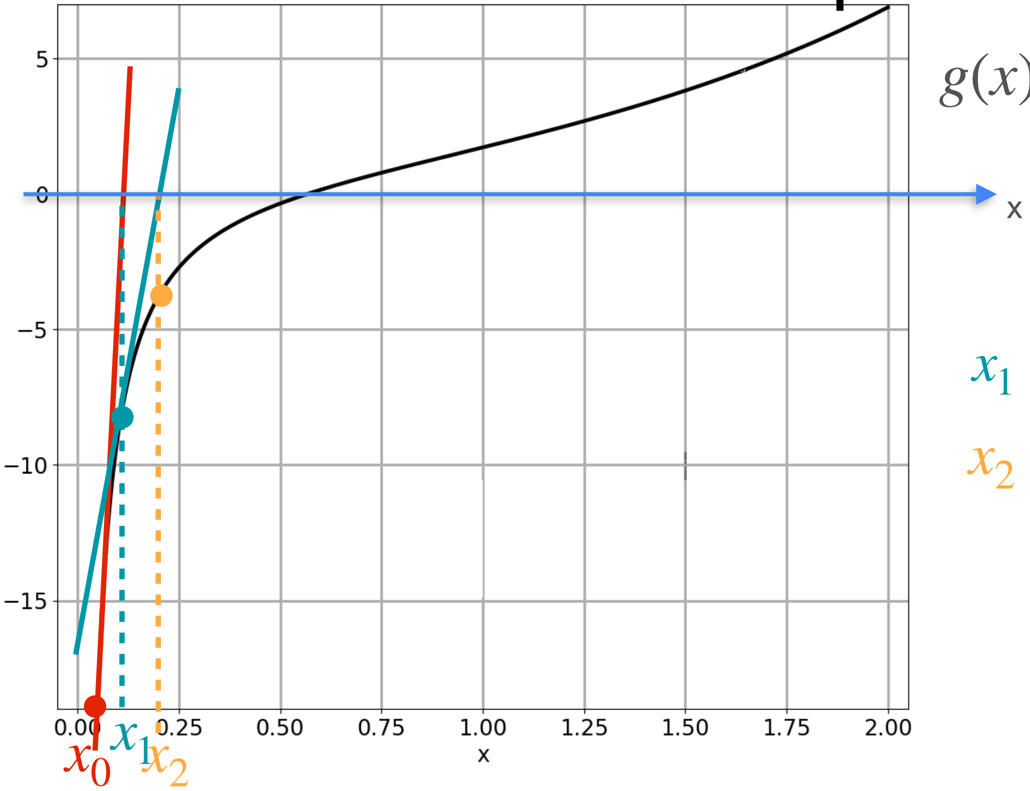
$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

$x_1 = 0.097$

Newton's Method for Optimization



$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

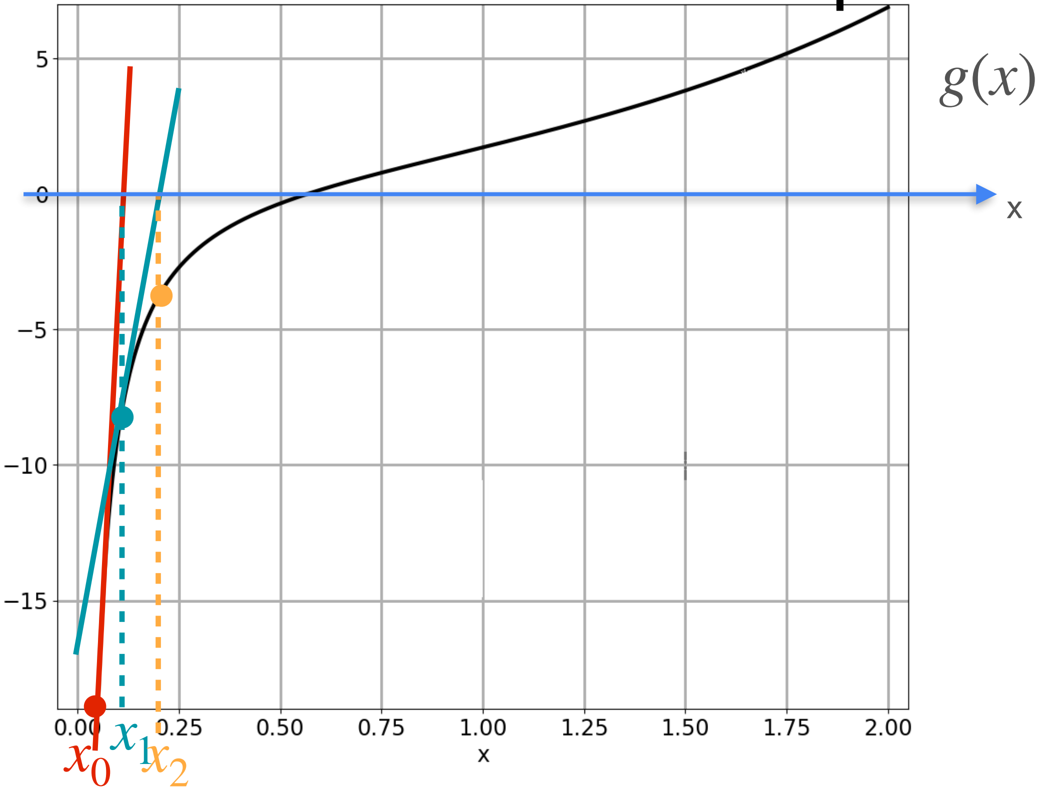
$$(g'(x))' = e^x + \frac{1}{x^2}$$

$x_1 = 0.097$

$$x_2 = x_1 - \frac{g'(x_1)}{(g'(x_1))'}$$

$$= 0.097 - \frac{\left(e^{0.097} - \frac{1}{0.097}\right)}{\left(e^{0.097} + \frac{1}{0.097^2}\right)} = 0.183$$

Newton's Method for Optimization



$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

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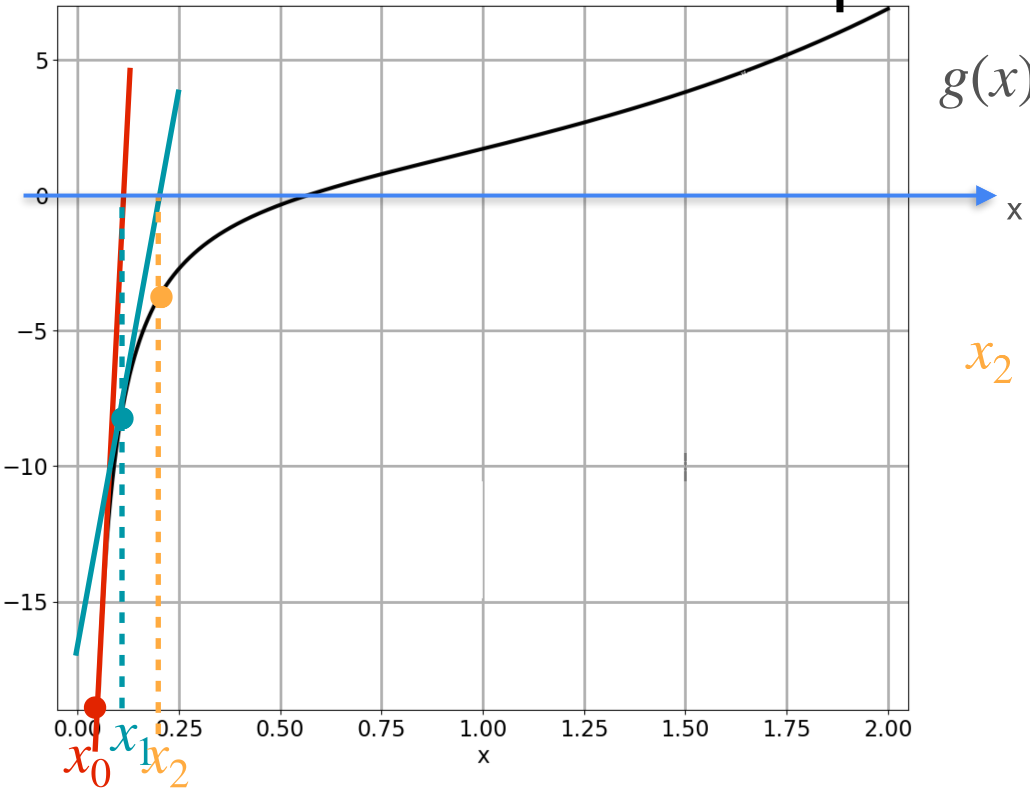
$$(g'(x))' = e^x + \frac{1}{x^2}$$

Newton's Method for Optimization



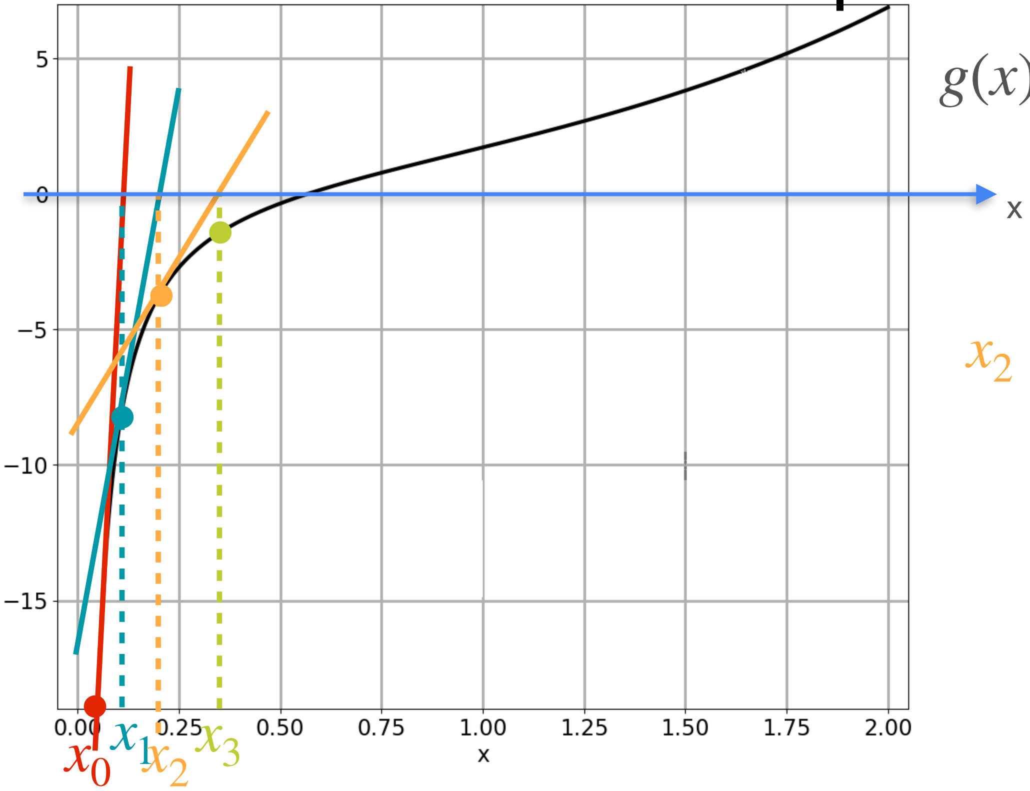
$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

$x_2 = 0.183$

Newton's Method for Optimization



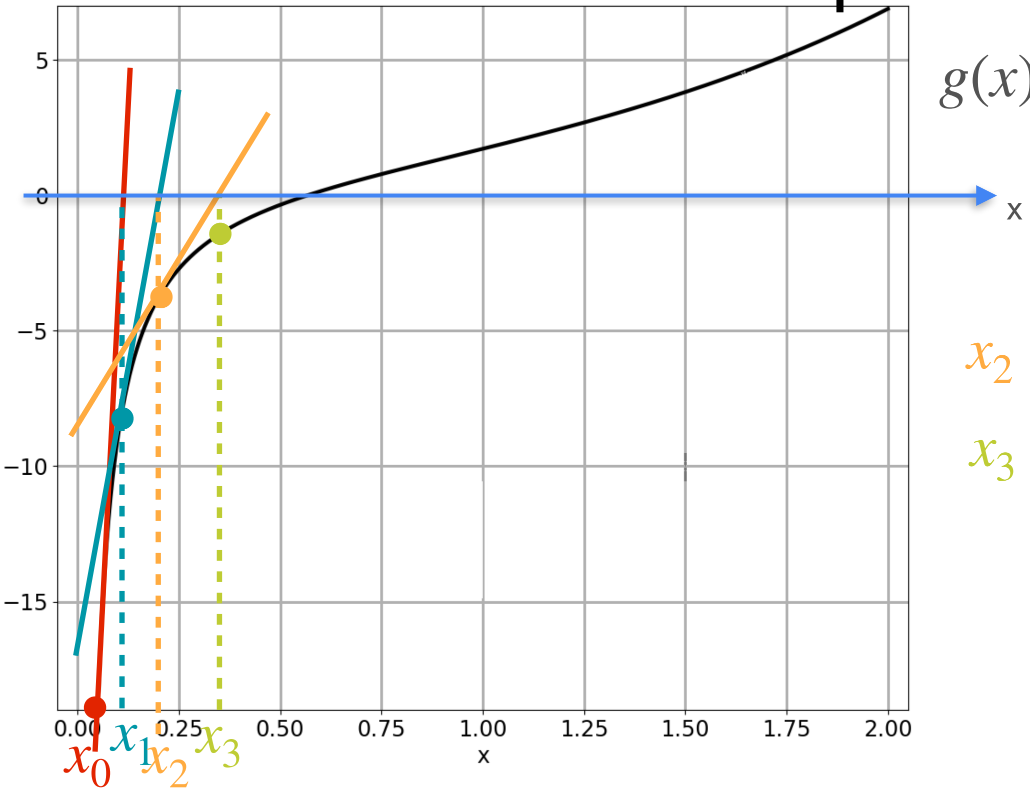
$g(x) = e^x - \log(x)$ $\quad g'(x) = e^x - 1/x$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

$x_2 = 0.183$

Newton’s Method for Optimization



$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

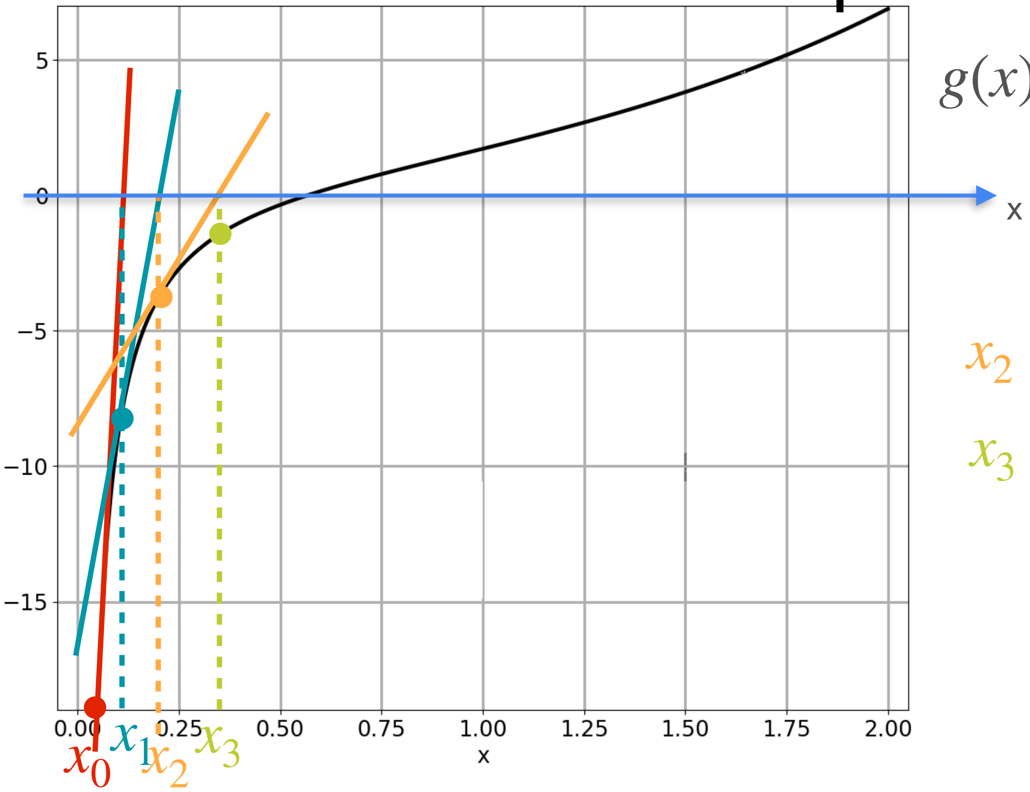
$$(g'(x))' = e^x + \frac{1}{x^2}$$

$$x_2 = 0.183$$

$$x_3 = x_2 - \frac{g'(x_2)}{(g'(x_2))'}$$

$$= 0.183 - \frac{\left(e^{0.183} - \frac{1}{0.183}\right)}{\left(e^{0.183} + \frac{1}{0.183^2}\right)}$$

Newton's Method for Optimization



$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

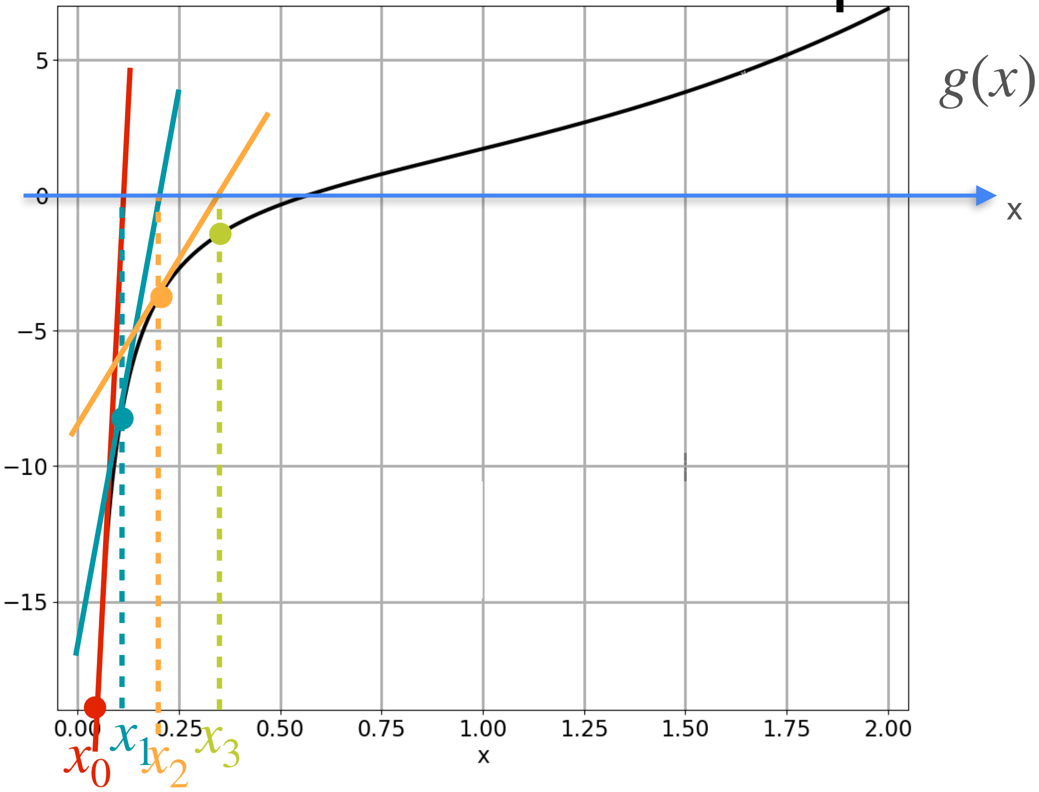
$$(g'(x))' = e^x + \frac{1}{x^2}$$

$x_2 = 0.183$

$$x_3 = x_2 - \frac{g'(x_2)}{(g'(x_2))'}$$

$$= 0.183 - \frac{\left(e^{0.183} - \frac{1}{0.183}\right)}{\left(e^{0.183} + \frac{1}{0.183^2}\right)} = 0.320$$

Newton's Method for Optimization

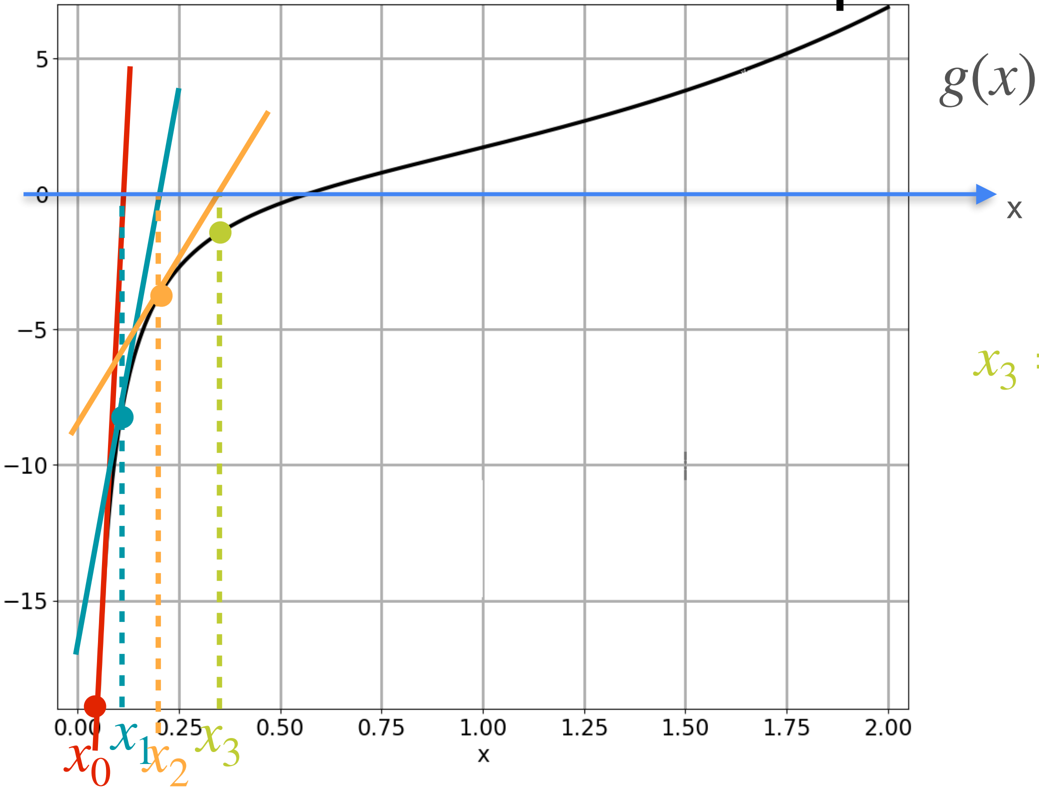


$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

Newton's Method for Optimization



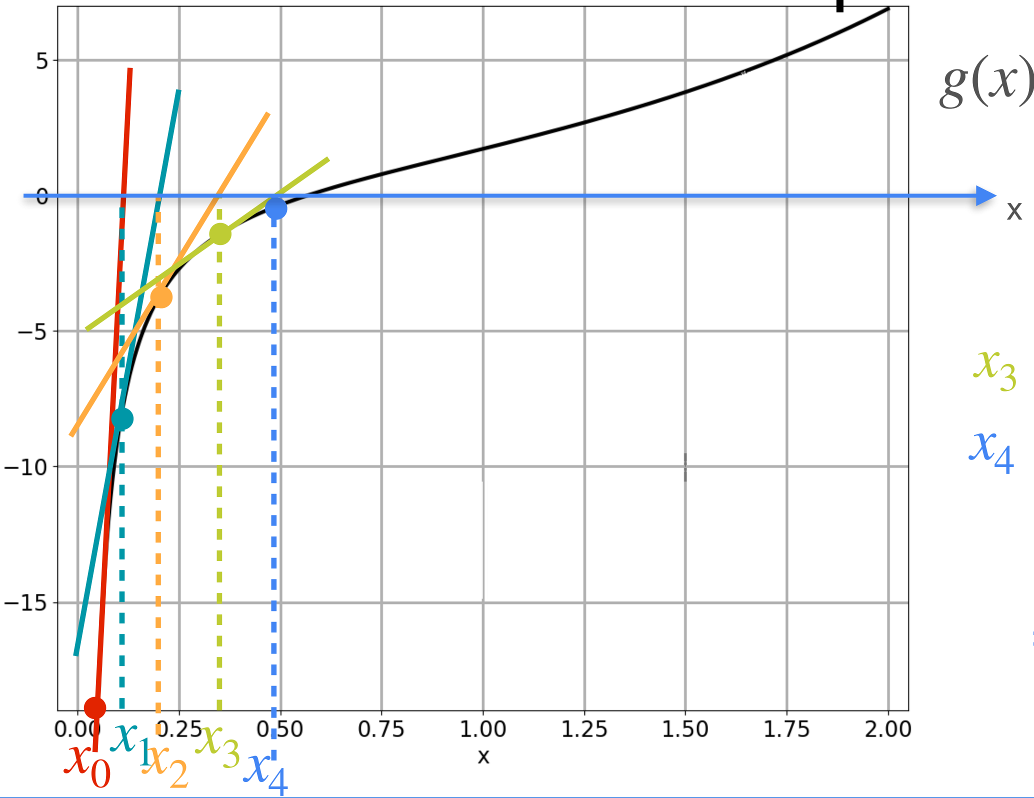
$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

$x_3 = 0.320$

Newton's Method for Optimization



$g(x) = e^x - \log(x)$ $\quad g'(x) = e^x - 1/x$

Minimum: $x^* = 0.5671$

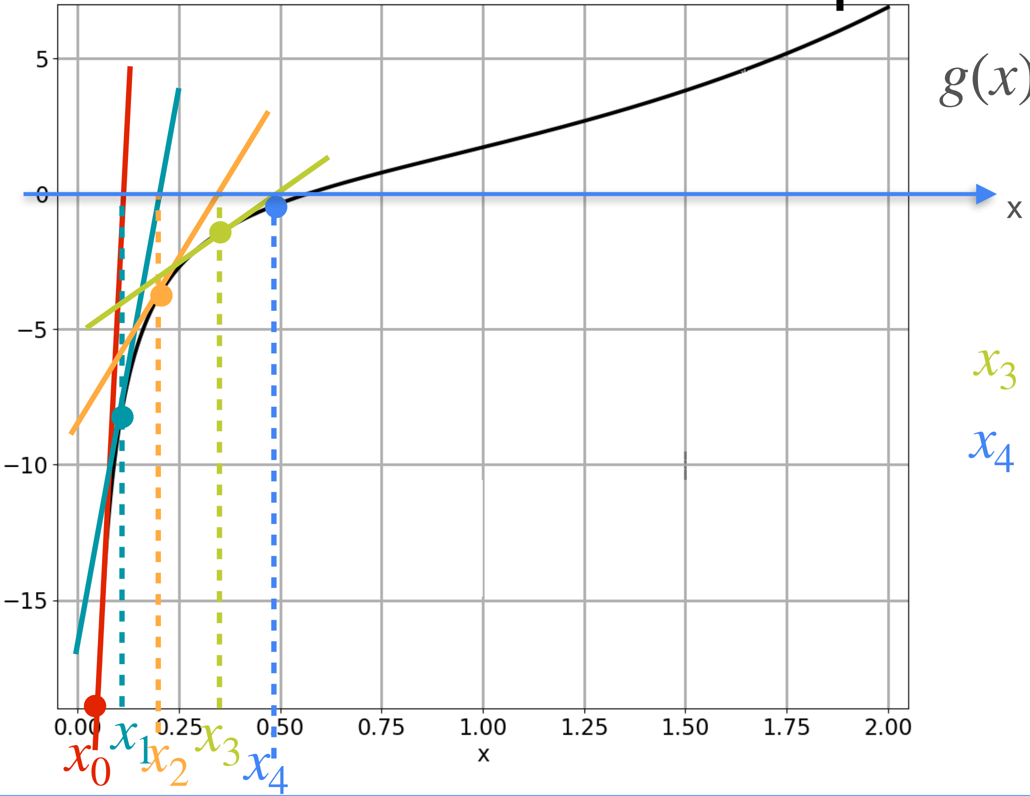
$$(g'(x))' = e^x + \frac{1}{x^2}$$

$x_3 = 0.320$

$$x_4 = x_3 - \frac{g'(x_3)}{(g'(x_3))'}$$

$$= 0.320 - \frac{\left(e^{0.320} - \frac{1}{0.320}\right)}{\left(e^{0.320} + \frac{1}{0.320^2}\right)}$$

Newton's Method for Optimization



$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

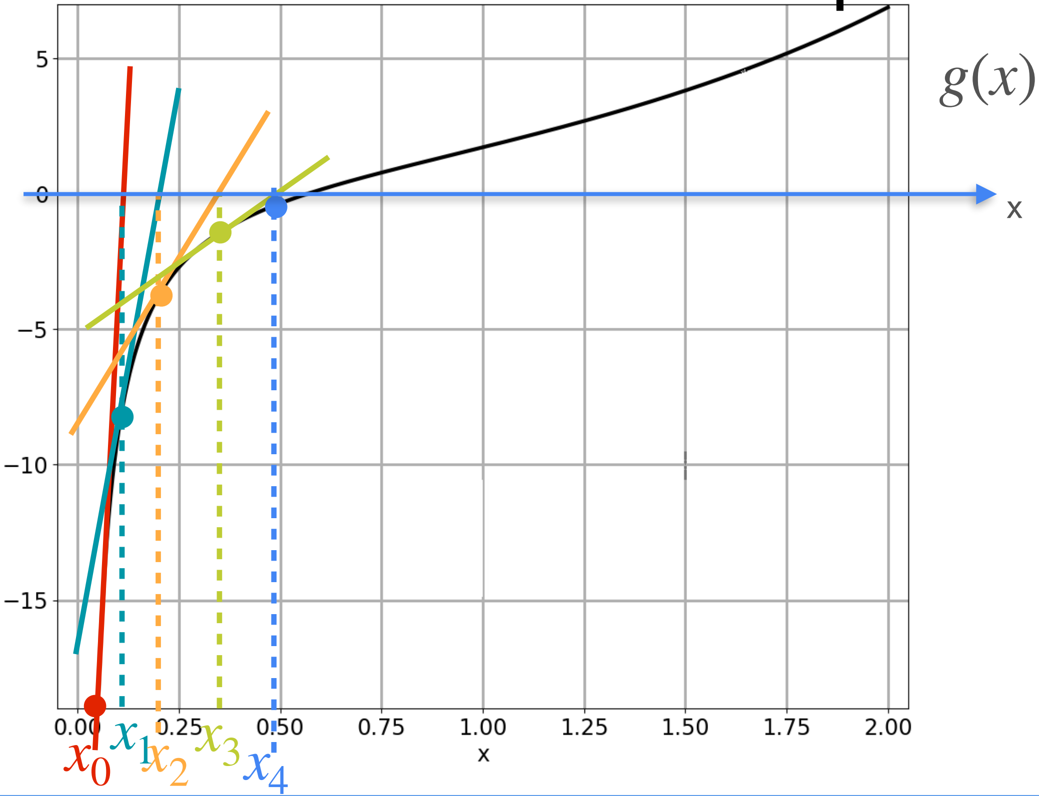
$$(g'(x))' = e^x + \frac{1}{x^2}$$

$$x_3 = 0.320$$

$$x_4 = x_3 - \frac{g'(x_3)}{(g'(x_3))'}$$

$$= 0.320 - \frac{\left(e^{0.320} - \frac{1}{0.320}\right)}{\left(e^{0.320} + \frac{1}{0.320^2}\right)} = 0.477$$

Newton's Method for Optimization

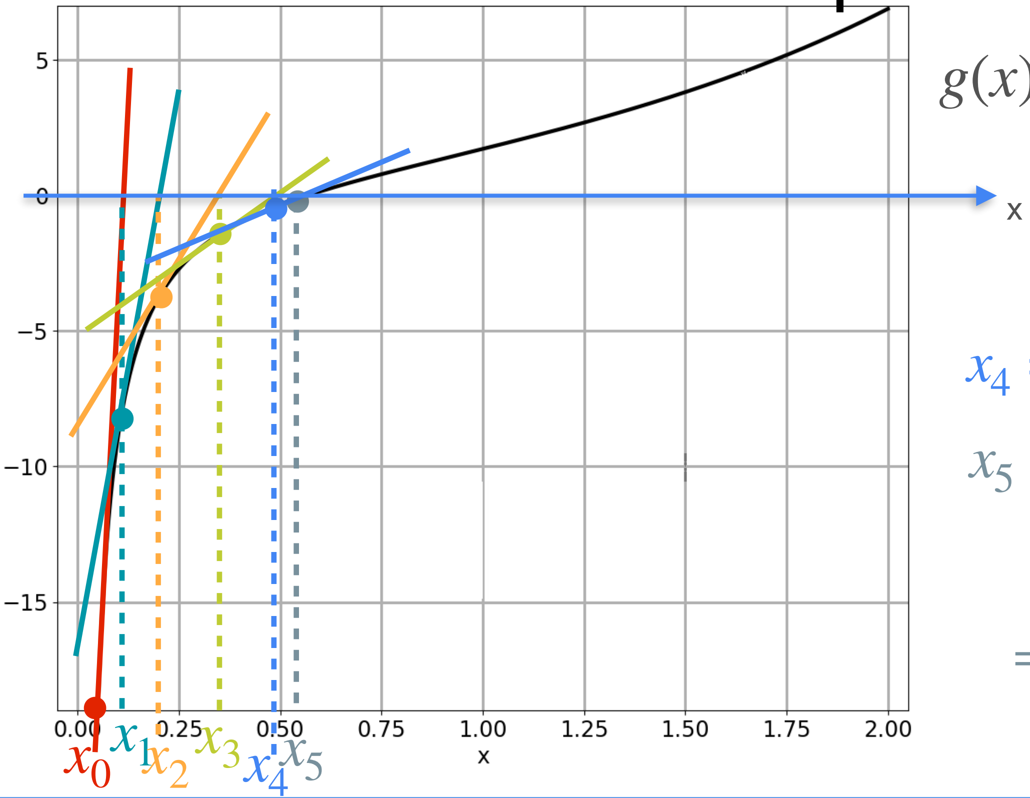


$g(x) = e^x - \log(x)$ $\quad$ $g'(x) = e^x - 1/x$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

Newton's Method for Optimization



$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

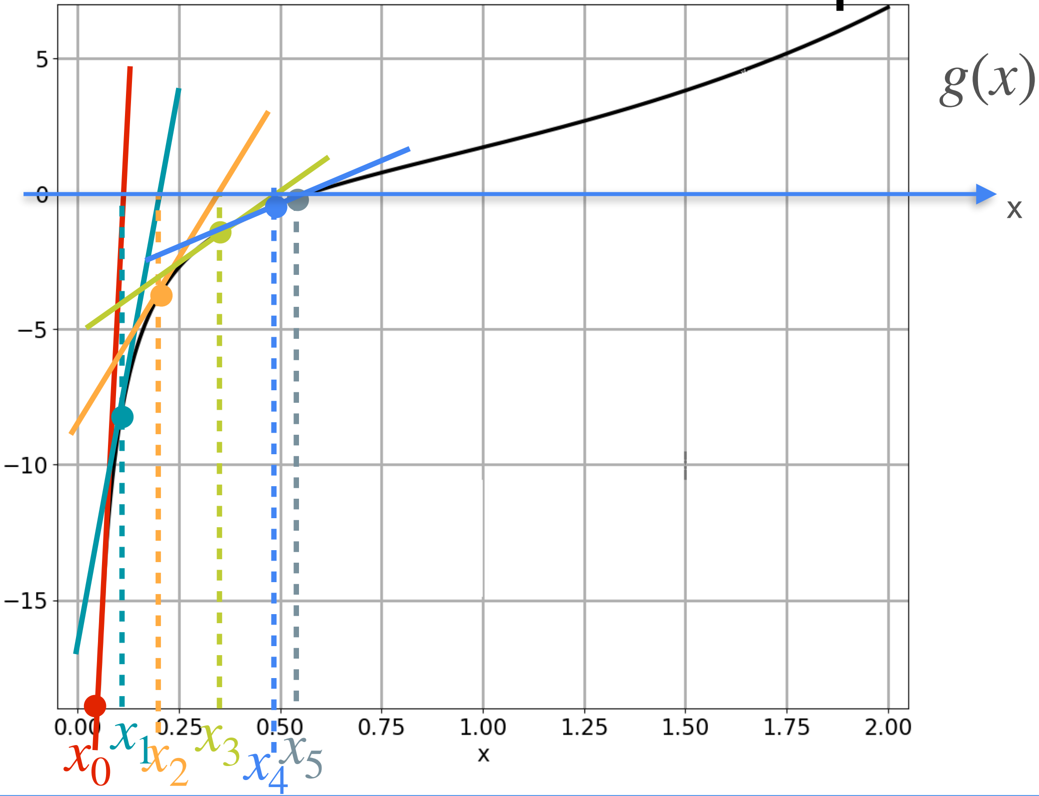
$$(g'(x))' = e^x + \frac{1}{x^2}$$

$$x_4 = 0.477$$

$$x_5 = x_4 - \frac{g'(x_4)}{(g'(x_4))'}$$

$$= 0.447 - \frac{\left(e^{0.447} - \frac{1}{0.447}\right)}{\left(e^{0.447} + \frac{1}{0.447^2}\right)} = 0.558$$

Newton's Method for Optimization

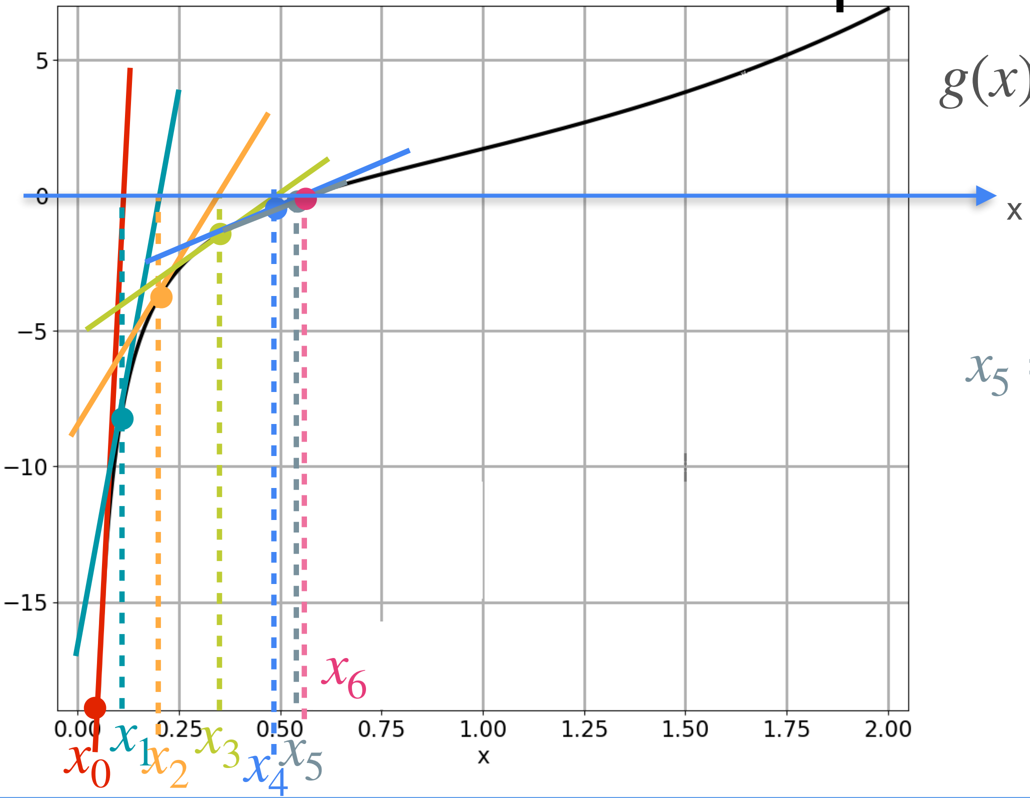


$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.567$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

Newton's Method for Optimization



$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

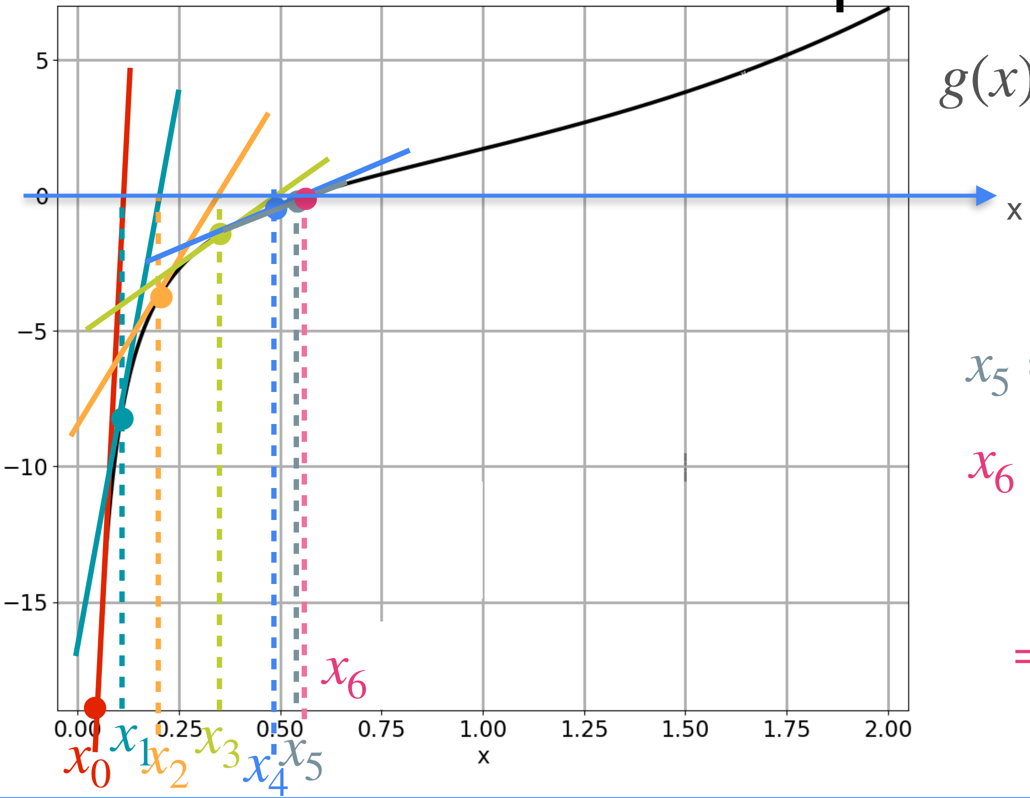
Minimum: $x^* = 0.567$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

$x_5 = 0.558$

DeepLearning.AI

Newton's Method for Optimization



$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.567$

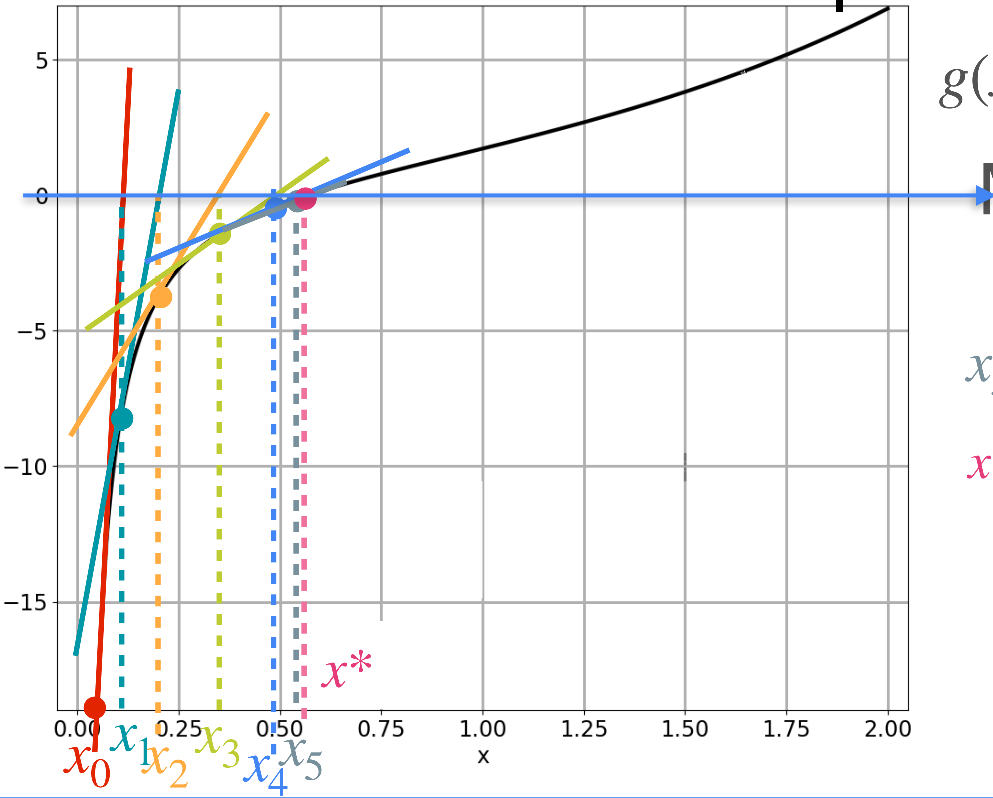
$$(g'(x))' = e^x + \frac{1}{x^2}$$

$$x_5 = 0.558$$

$$x_6 = x_5 - \frac{g'(x_5)}{(g'(x_5))'}$$

$$= 0.558 - \frac{\left(e^{0.558} - \frac{1}{0.558}\right)}{\left(e^{0.558} + \frac{1}{0.558^2}\right)}$$

Newton's Method for Optimization



$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.567$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

$$x_5 = 0.558$$

$$x^* = x_5 - \frac{g'(x_5)}{(g'(x_5))'}$$

$$= 0.558 - \frac{\left(e^{0.558} - \frac{1}{0.558}\right)}{\left(e^{0.558} + \frac{1}{0.558^2}\right)} = 0.567$$

Optimization in Neural Networks and Newton's Method

The second derivative

Second Derivative

Second Derivative

Newton's method:

Second Derivative

Newton's method: $$x_{k+1} = x_k - \frac{g'(x_k)}{(g'(x_k))'}$$

Second Derivative

Newton's method: $$x_{k+1} = x_k - \frac{g'(x_k)}{\boxed{(g'(x_k))'}}$$??

Second Derivative

Newton's method: $x_{k+1} = x_k - \dfrac{g'(x_k)}{\boxed{(g'(x_k))'}}$??

Second derivative

Second Derivative

Newton's method: $$x_{k+1} = x_k - \frac{g'(x_k)}{\boxed{(g'(x_k))'}}$$??

Second derivative

Leibniz notation: $$\frac{d^2 f(x)}{dx^2} = \frac{d}{dx}\left(\frac{df(x)}{dx}\right)$$

Second Derivative

Newton's method: $x_{k+1} = x_k - \dfrac{g'(x_k)}{\boxed{(g'(x_k))'}}$??

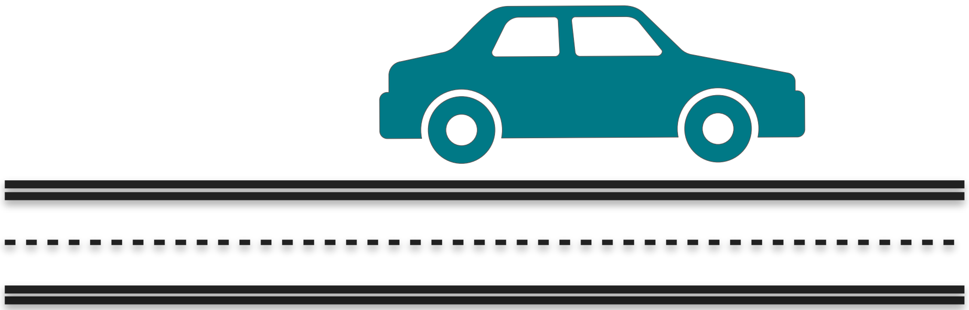
Second derivative

Leibniz notation: $\dfrac{d^2 f(x)}{dx^2} = \dfrac{d}{dx}\left(\dfrac{df(x)}{dx}\right)$

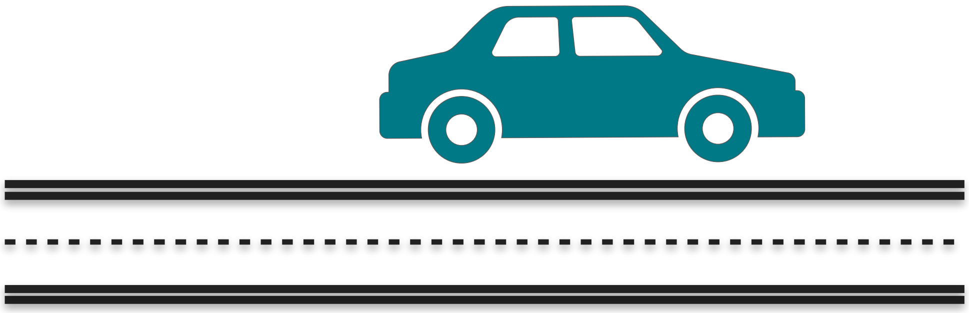
Lagrange notation: $f''(x)$

Understanding Second Derivative

Understanding Second Derivative

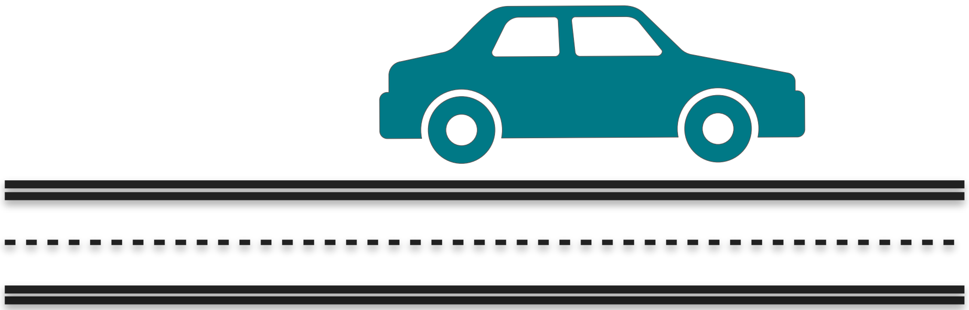


Understanding Second Derivative



x Distance

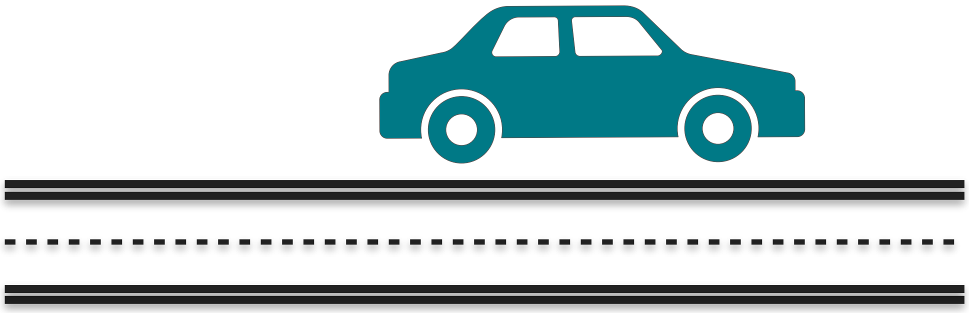
Understanding Second Derivative



x Distance

v Velocity $\frac{dx}{dt}$

Understanding Second Derivative



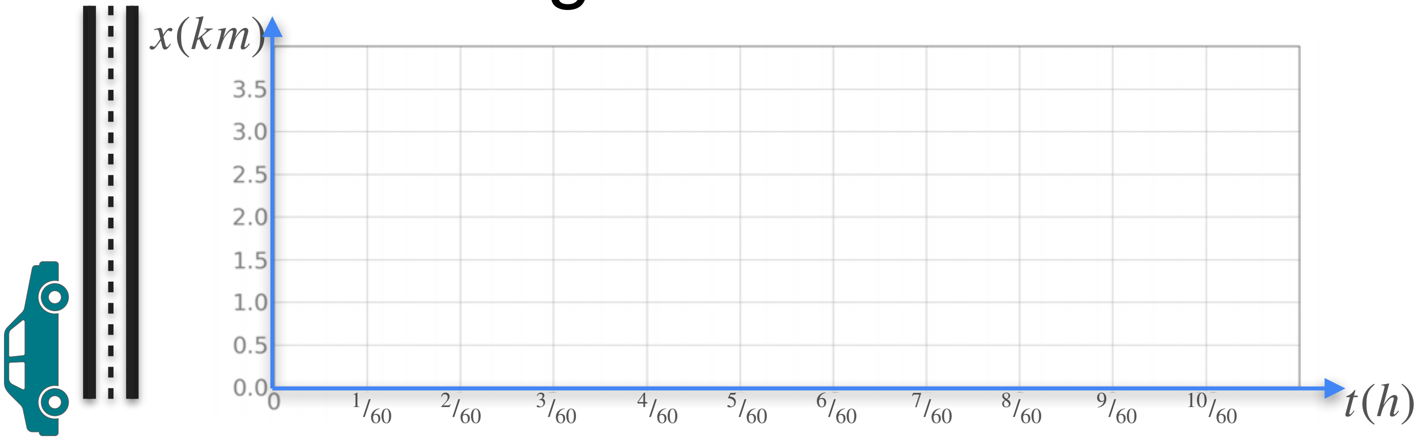
x Distance

v Velocity $\dfrac{dx}{dt}$

a Acceleration $\dfrac{dv}{dt} = \dfrac{d^2x}{dt^2}$

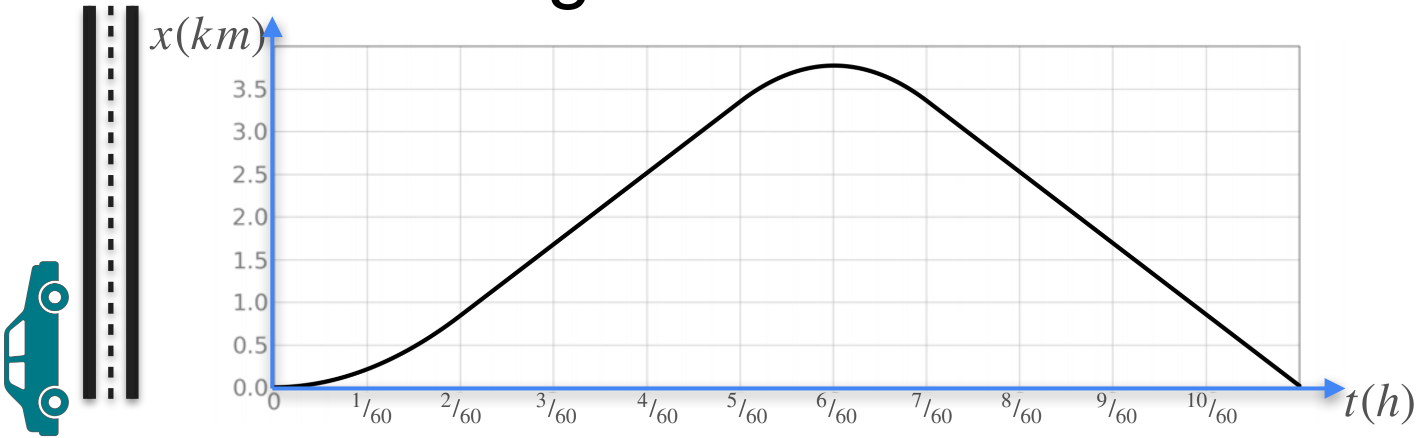
Understanding Second Derivative

Understanding Second Derivative



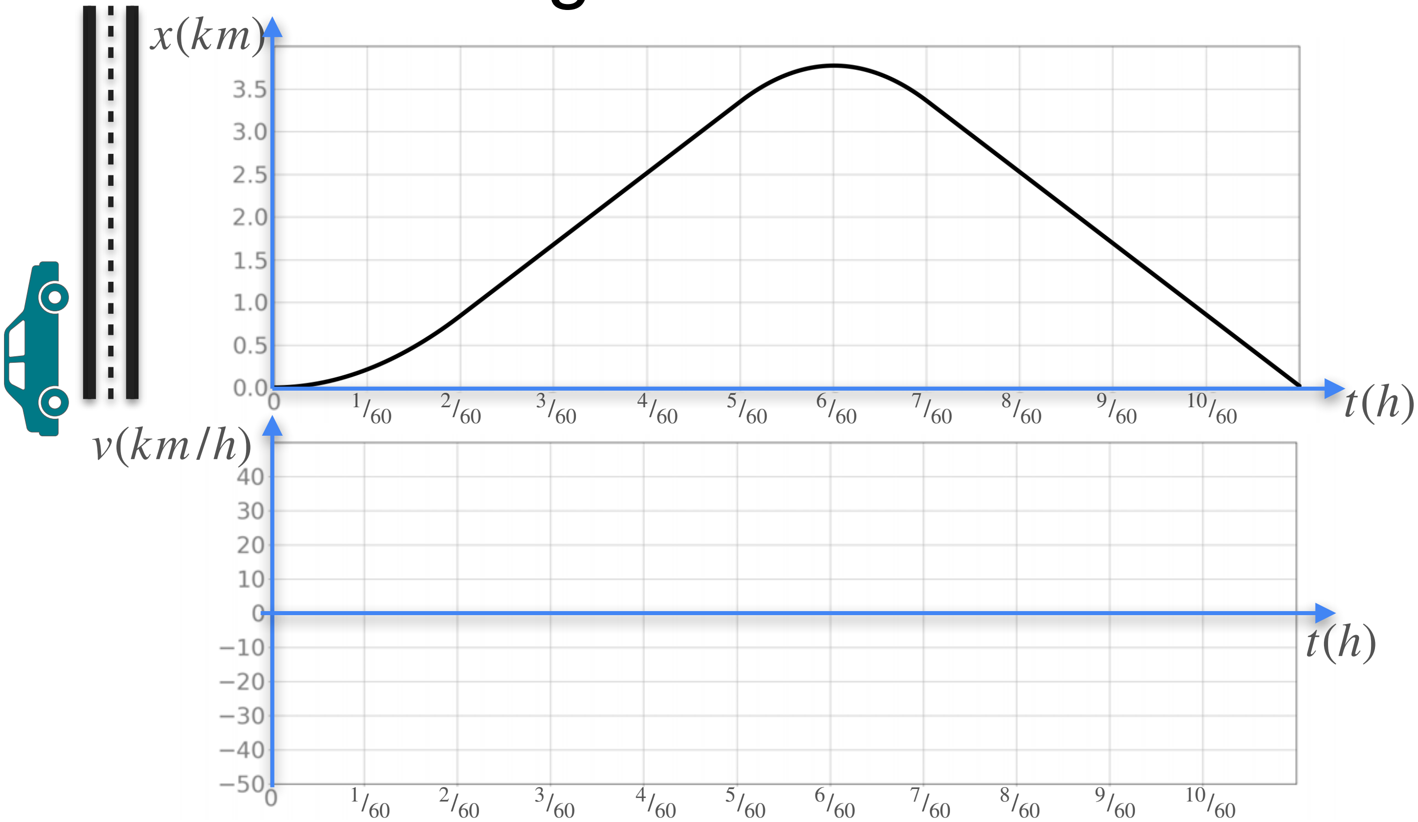
x Distance

Understanding Second Derivative



x Distance

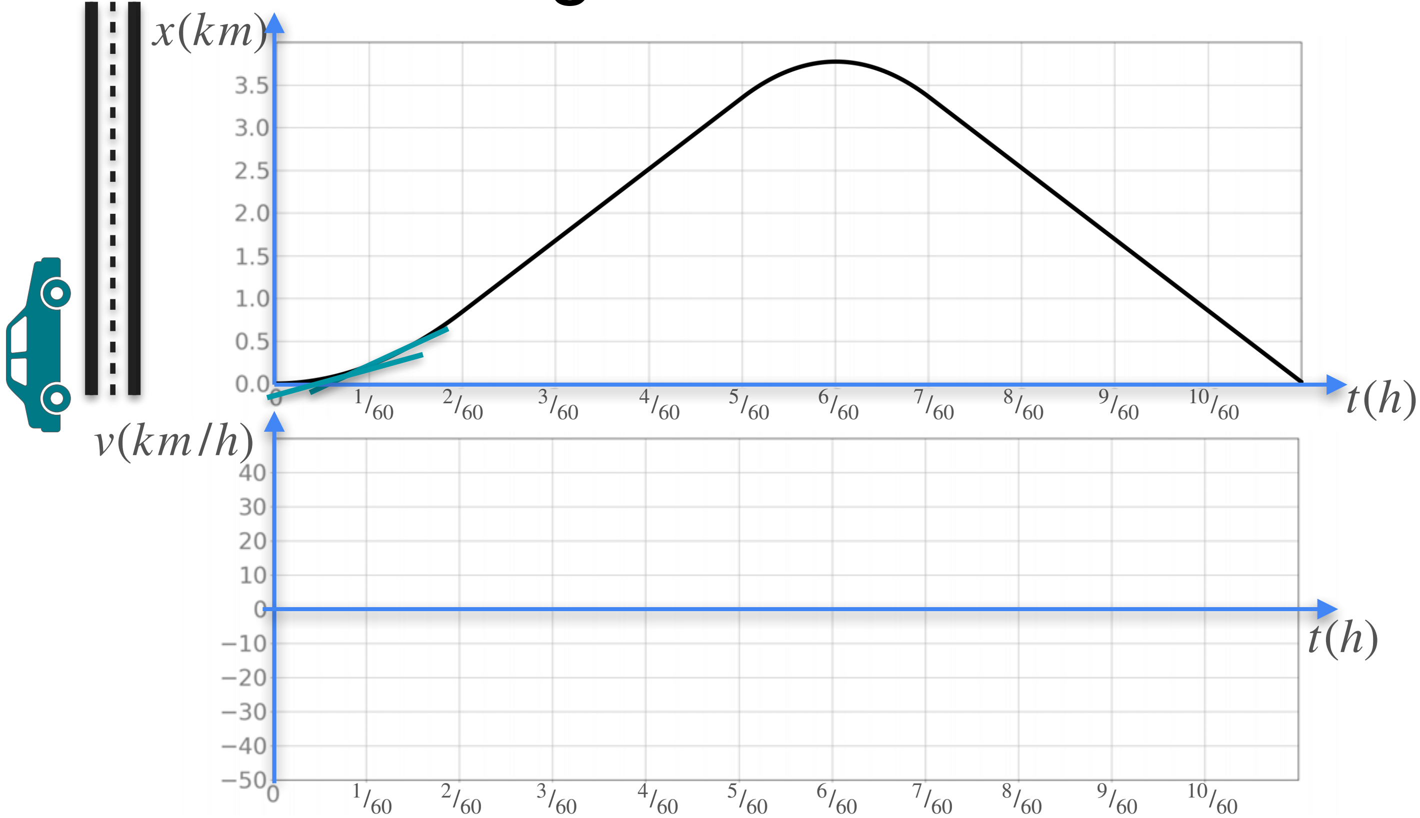
Understanding Second Derivative



x Distance

v Velocity $\dfrac{dx}{dt}$

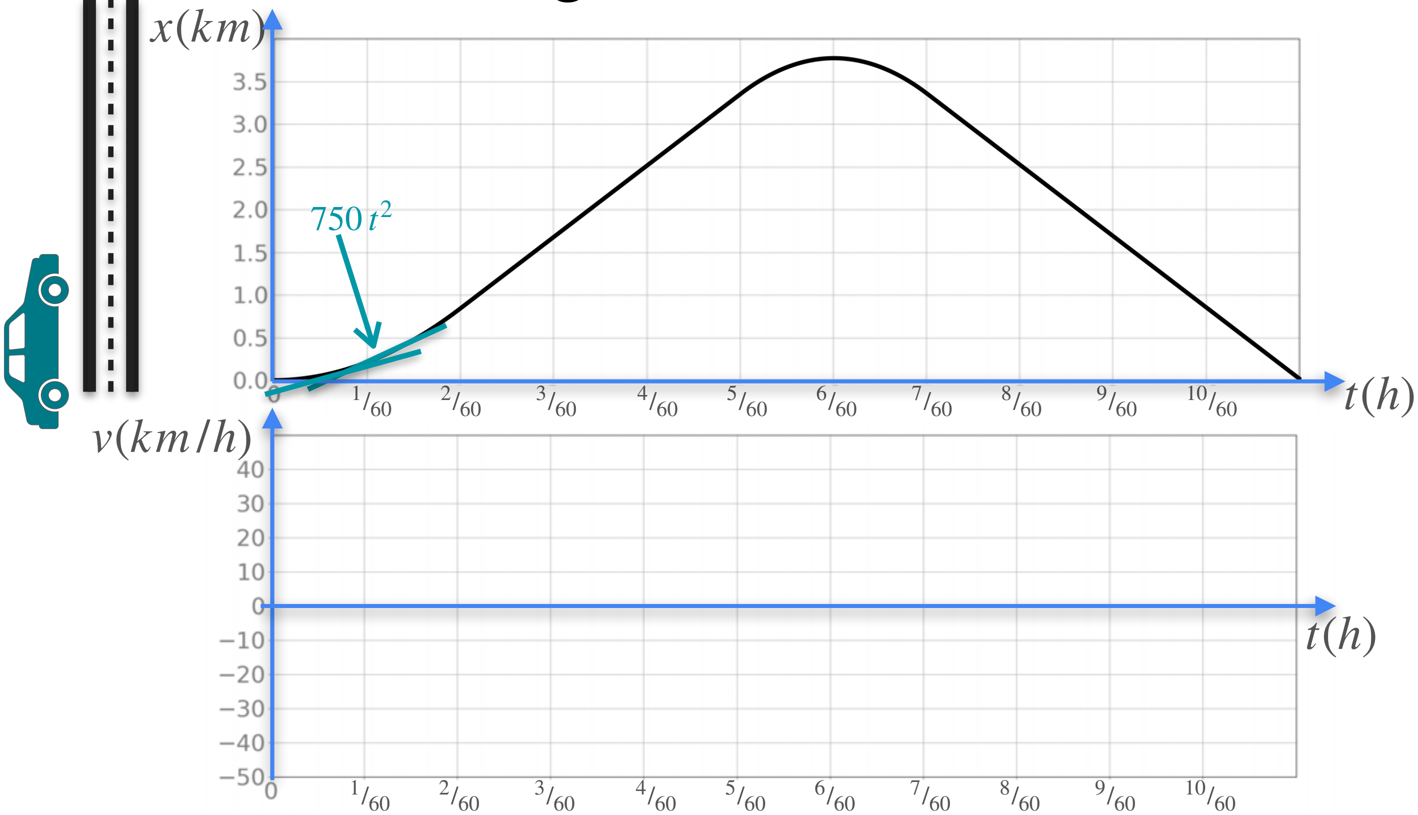
Understanding Second Derivative



x Distance

v Velocity $\frac{dx}{dt}$

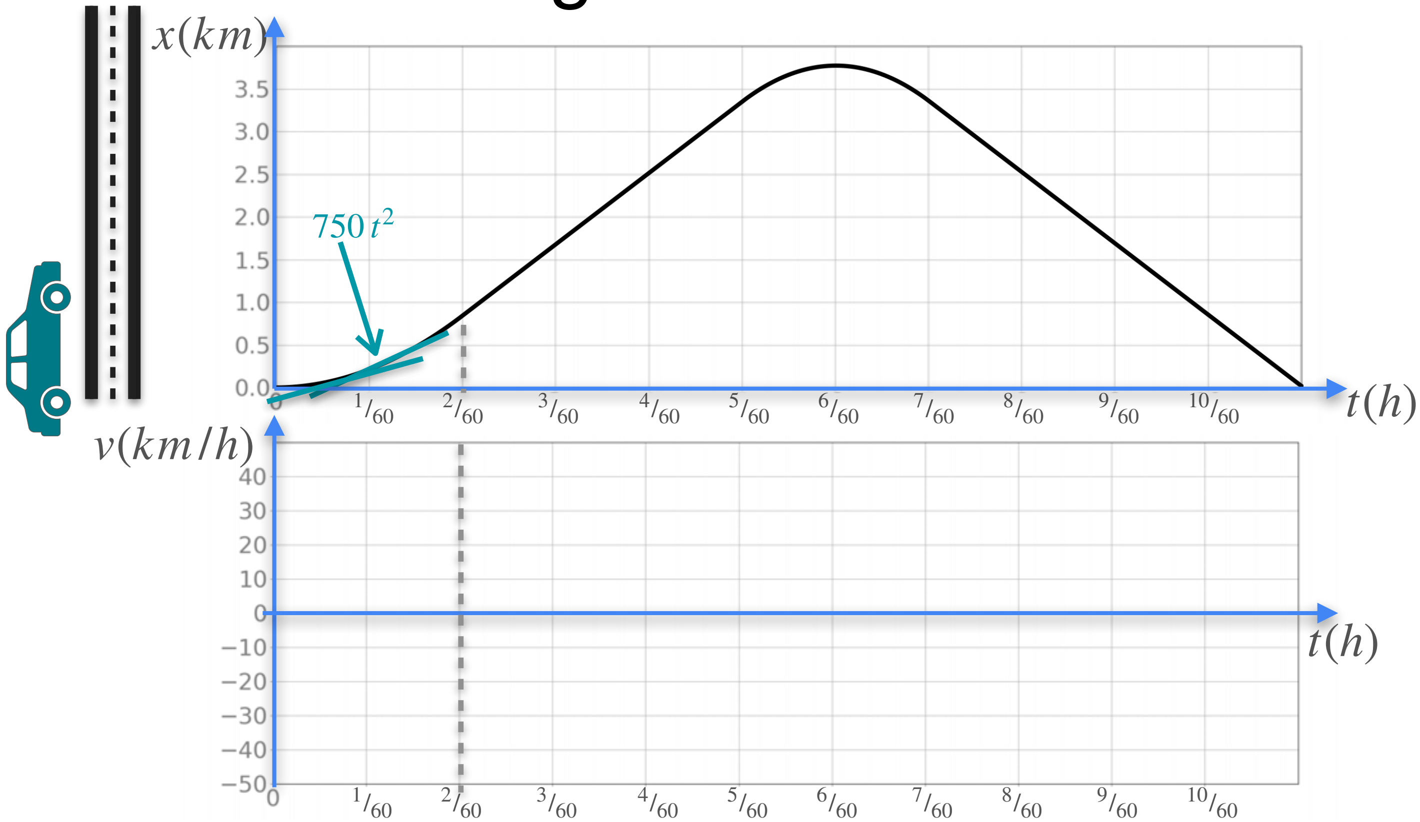
Understanding Second Derivative



x Distance

v Velocity $\dfrac{dx}{dt}$

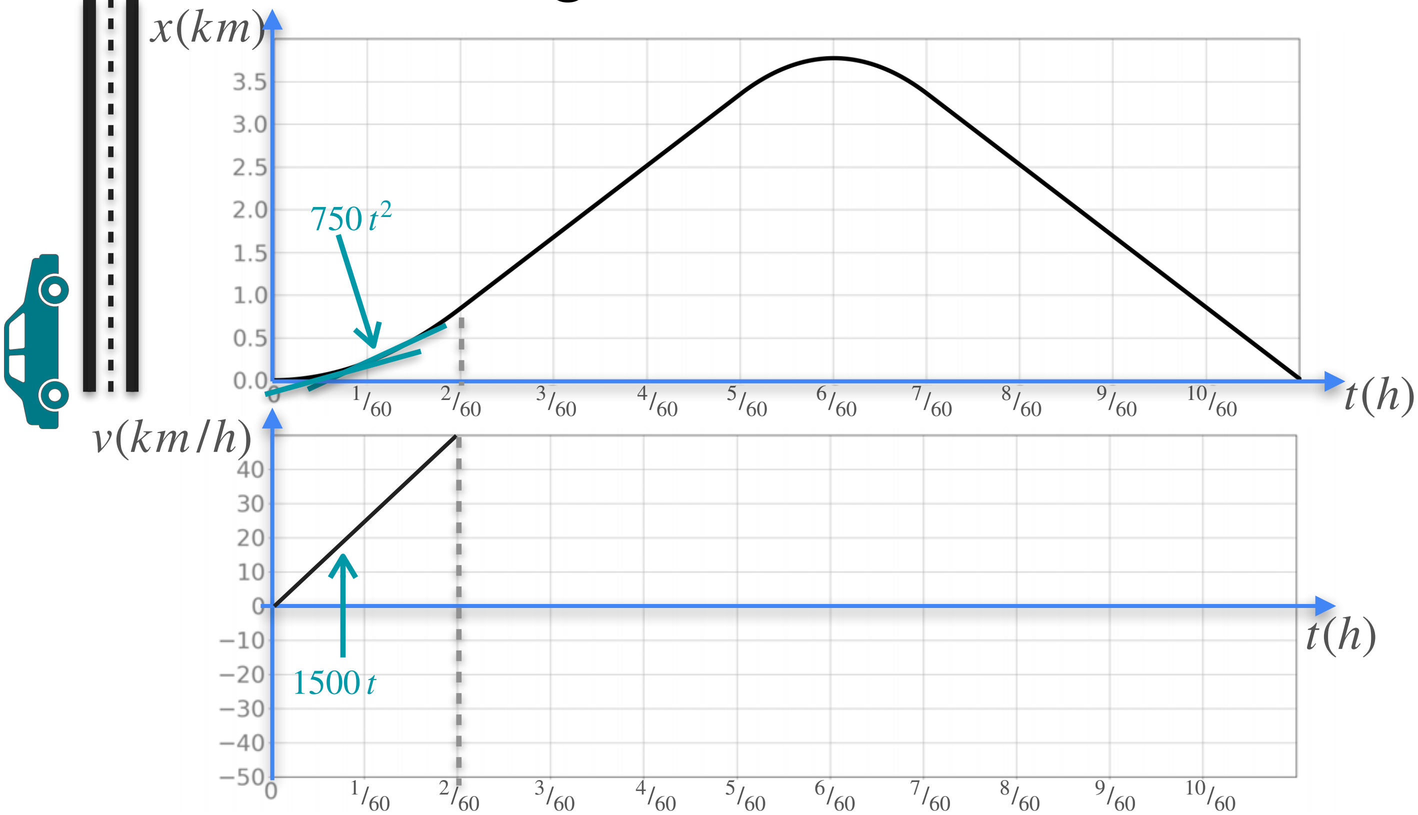
Understanding Second Derivative



x Distance

v Velocity $\dfrac{dx}{dt}$

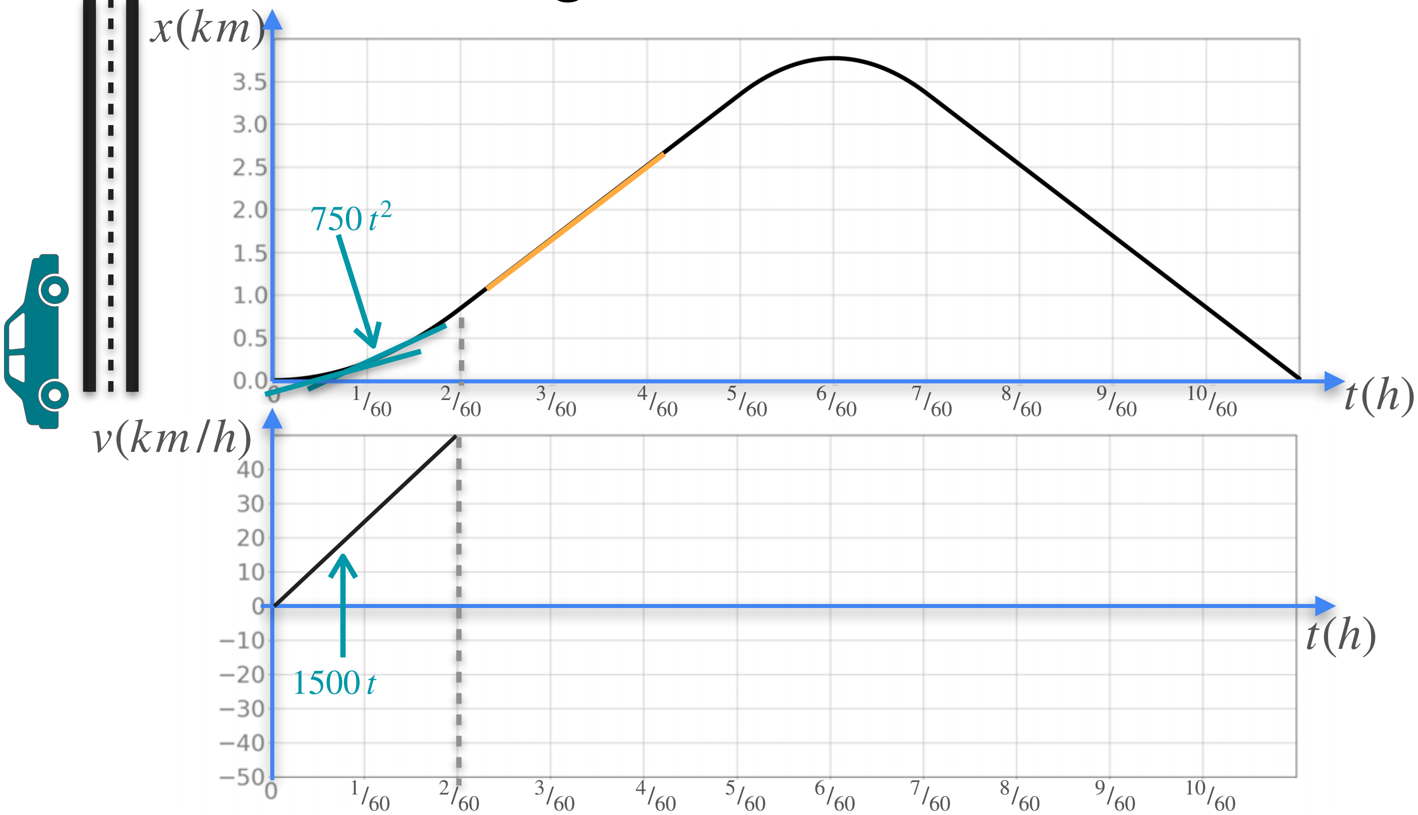
Understanding Second Derivative



x Distance

v Velocity $\dfrac{dx}{dt}$

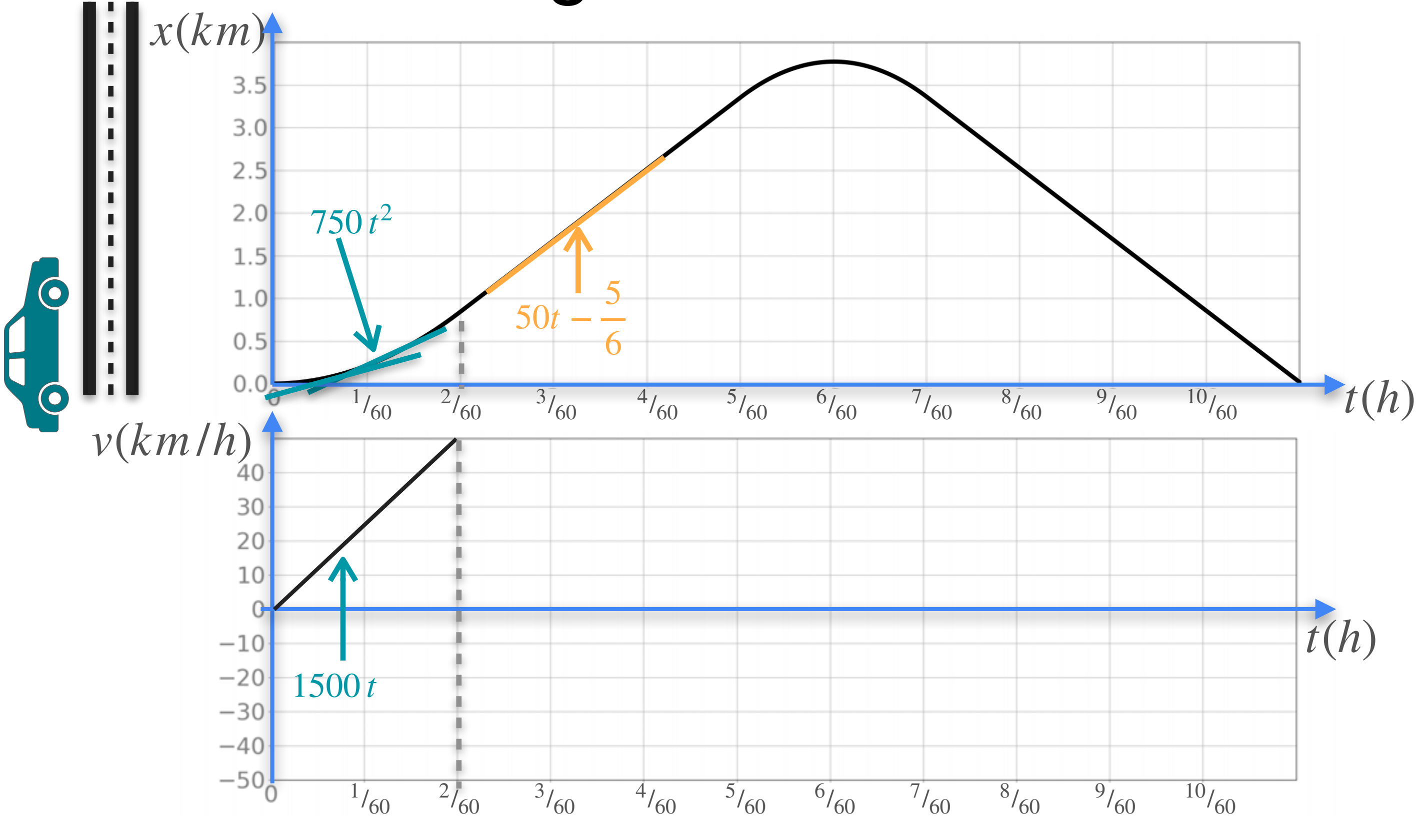
Understanding Second Derivative



x Distance

v Velocity $\frac{dx}{dt}$

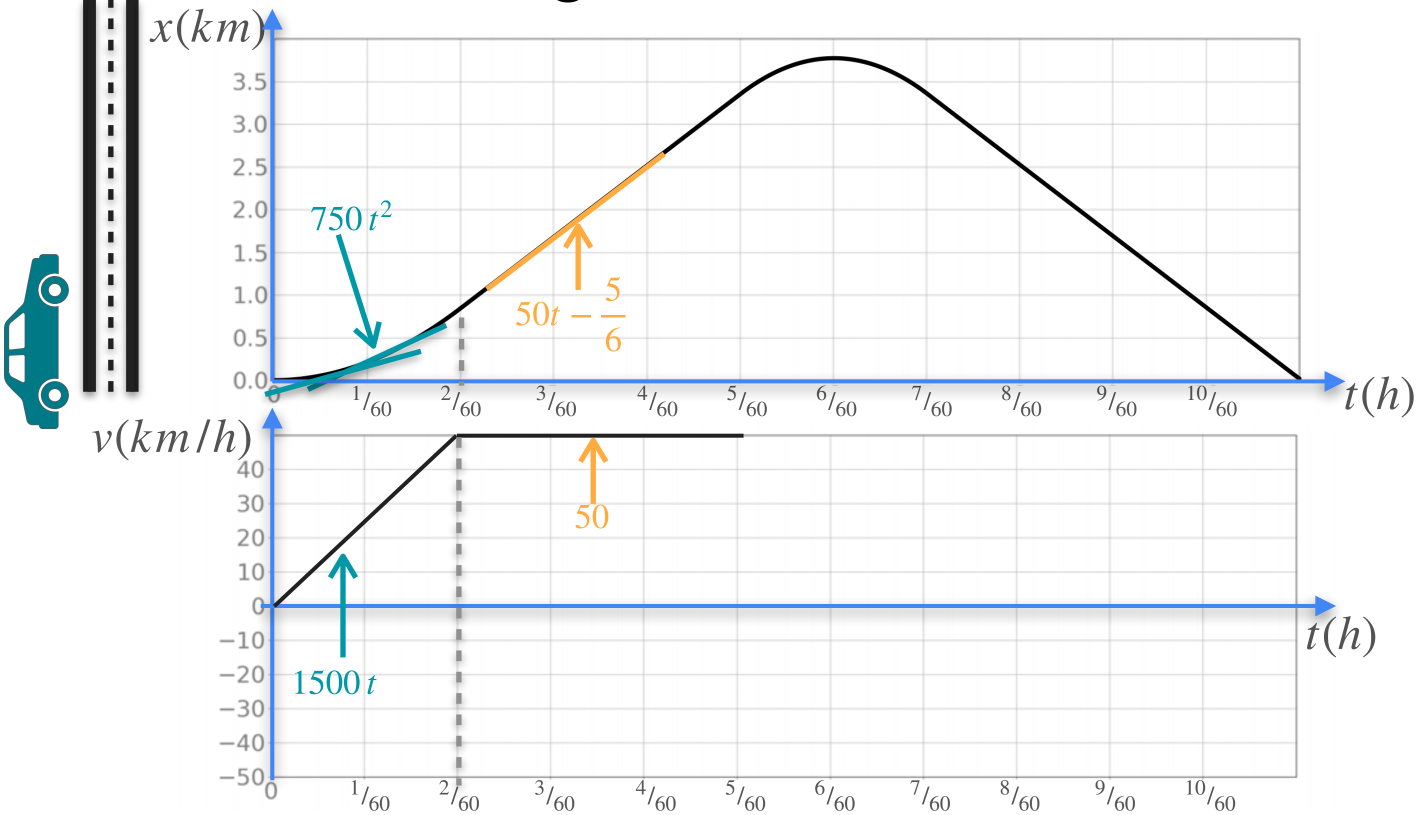
Understanding Second Derivative



x Distance

v Velocity $\frac{dx}{dt}$

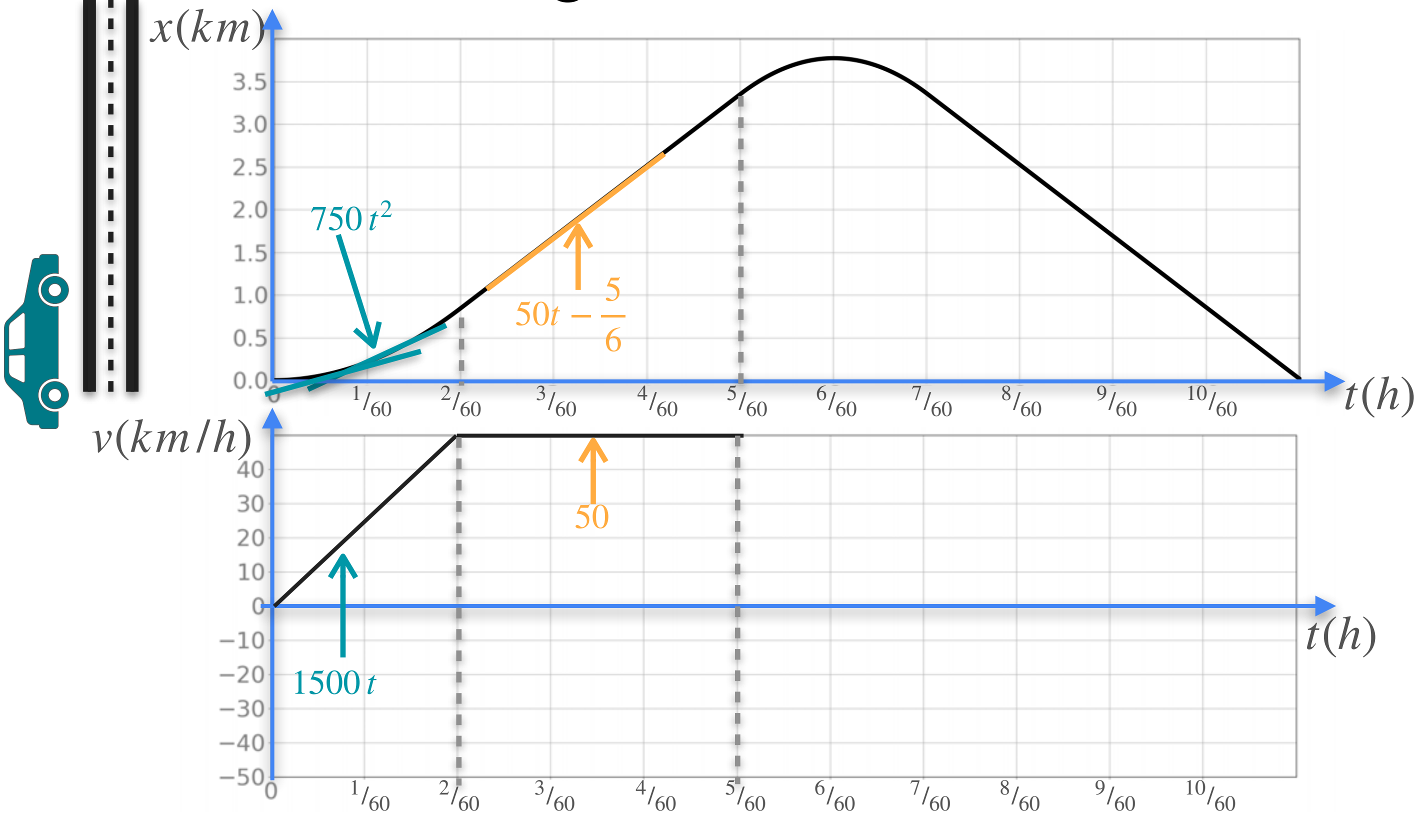
Understanding Second Derivative



x Distance

v Velocity $\frac{dx}{dt}$

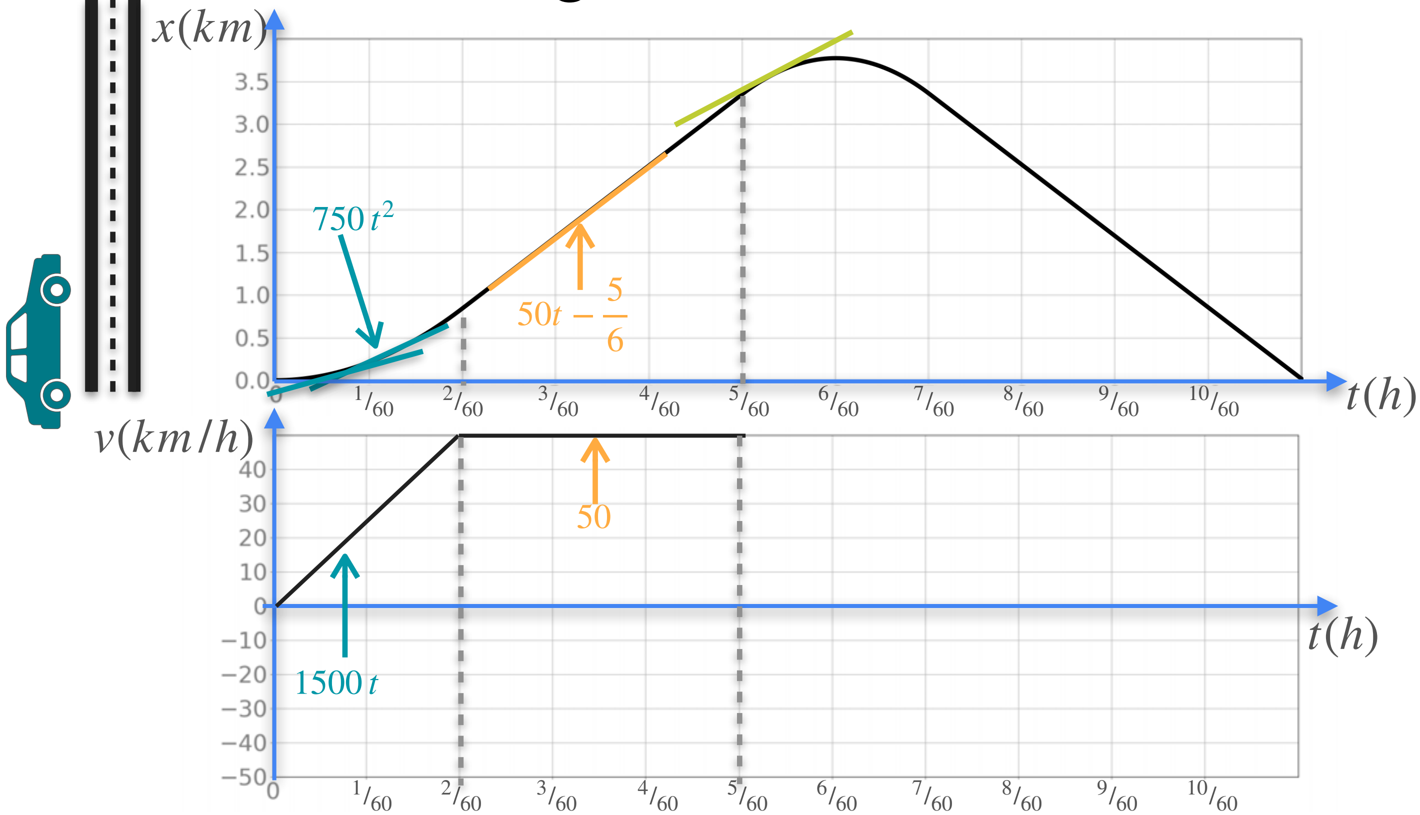
Understanding Second Derivative



x Distance

v Velocity $\frac{dx}{dt}$

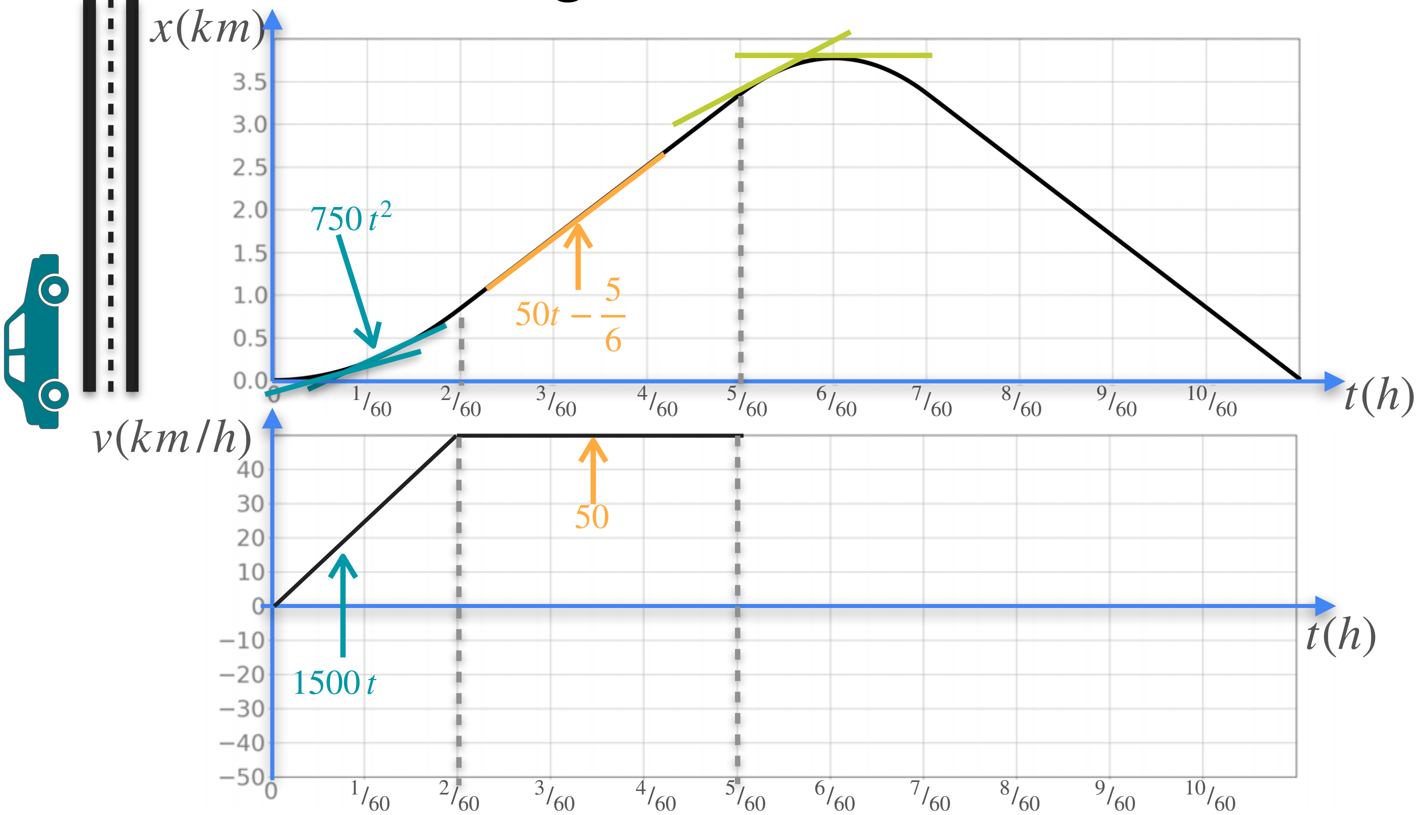
Understanding Second Derivative



x Distance

v Velocity $\frac{dx}{dt}$

Understanding Second Derivative



x Distance

v Velocity $\frac{dx}{dt}$

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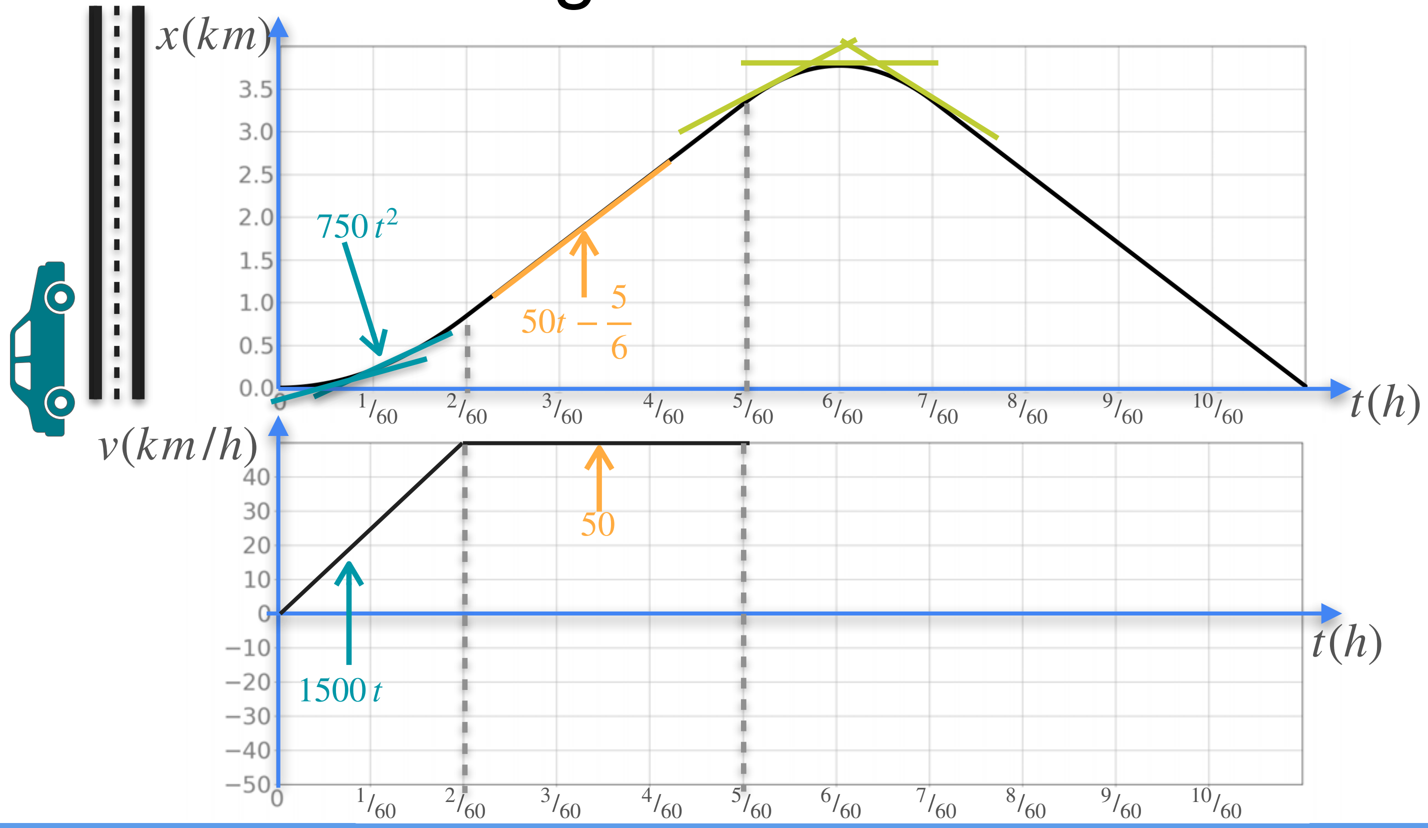
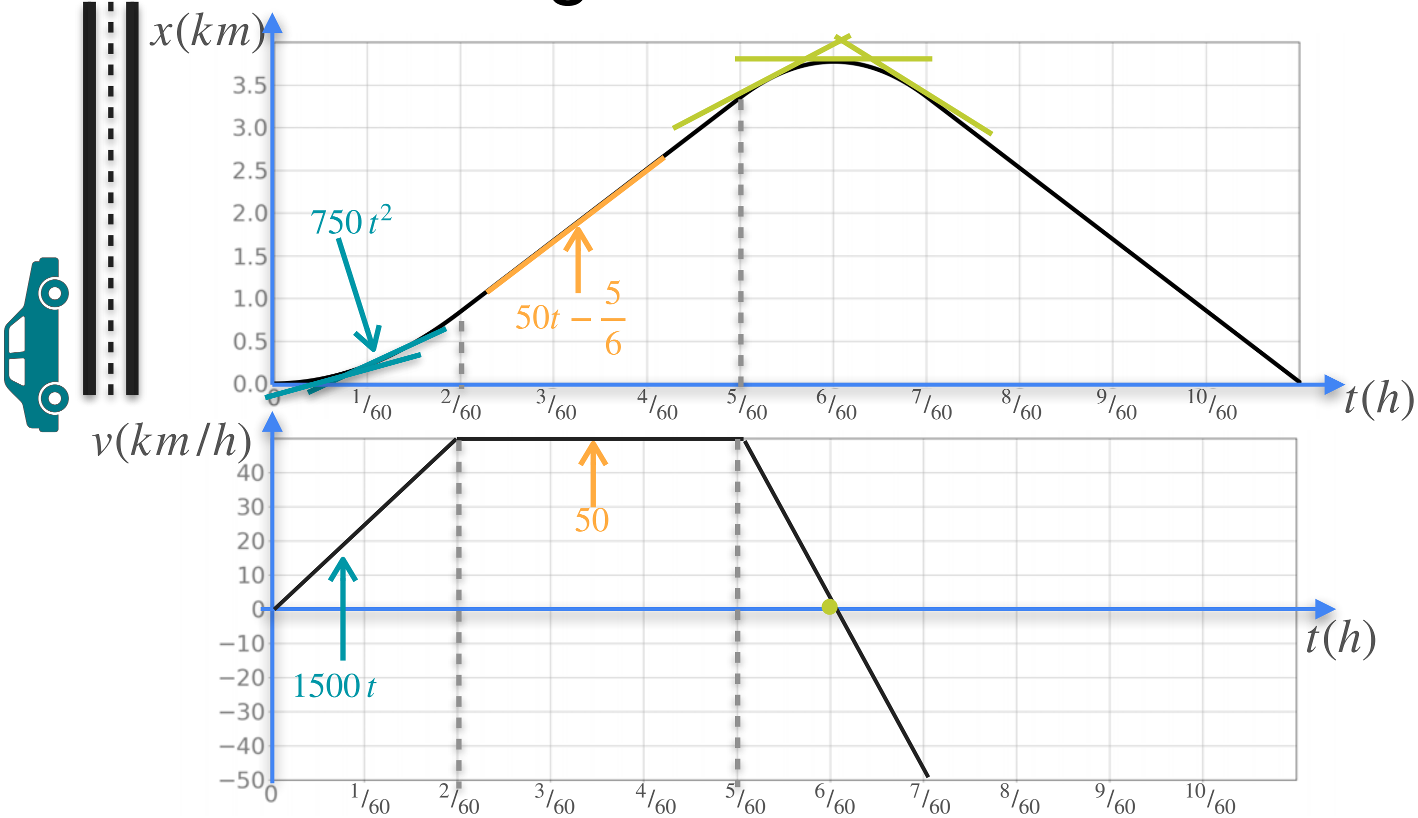
Understanding Second Derivative



x Distance

v Velocity $\frac{dx}{dt}$

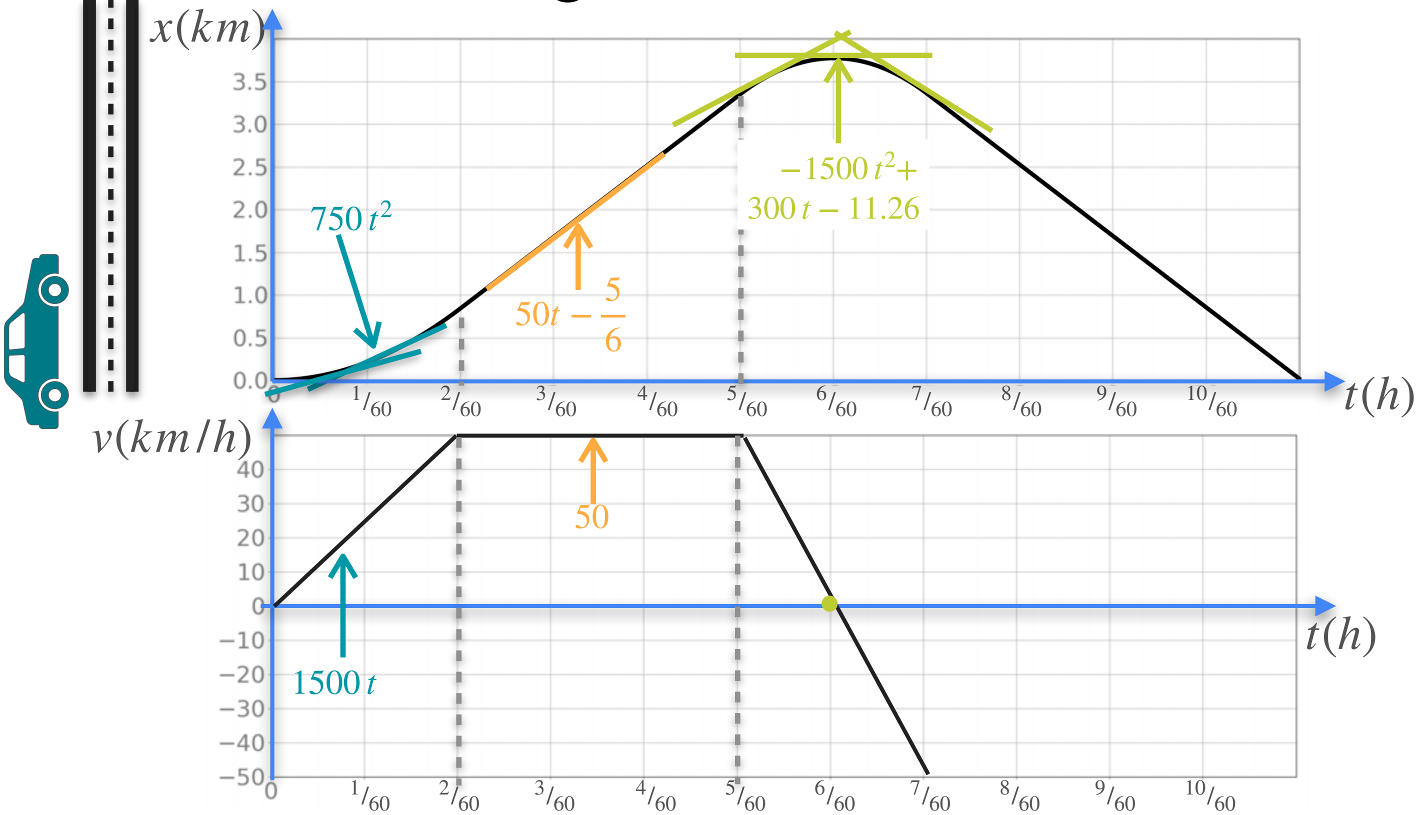
Understanding Second Derivative



x Distance

v Velocity $\frac{dx}{dt}$

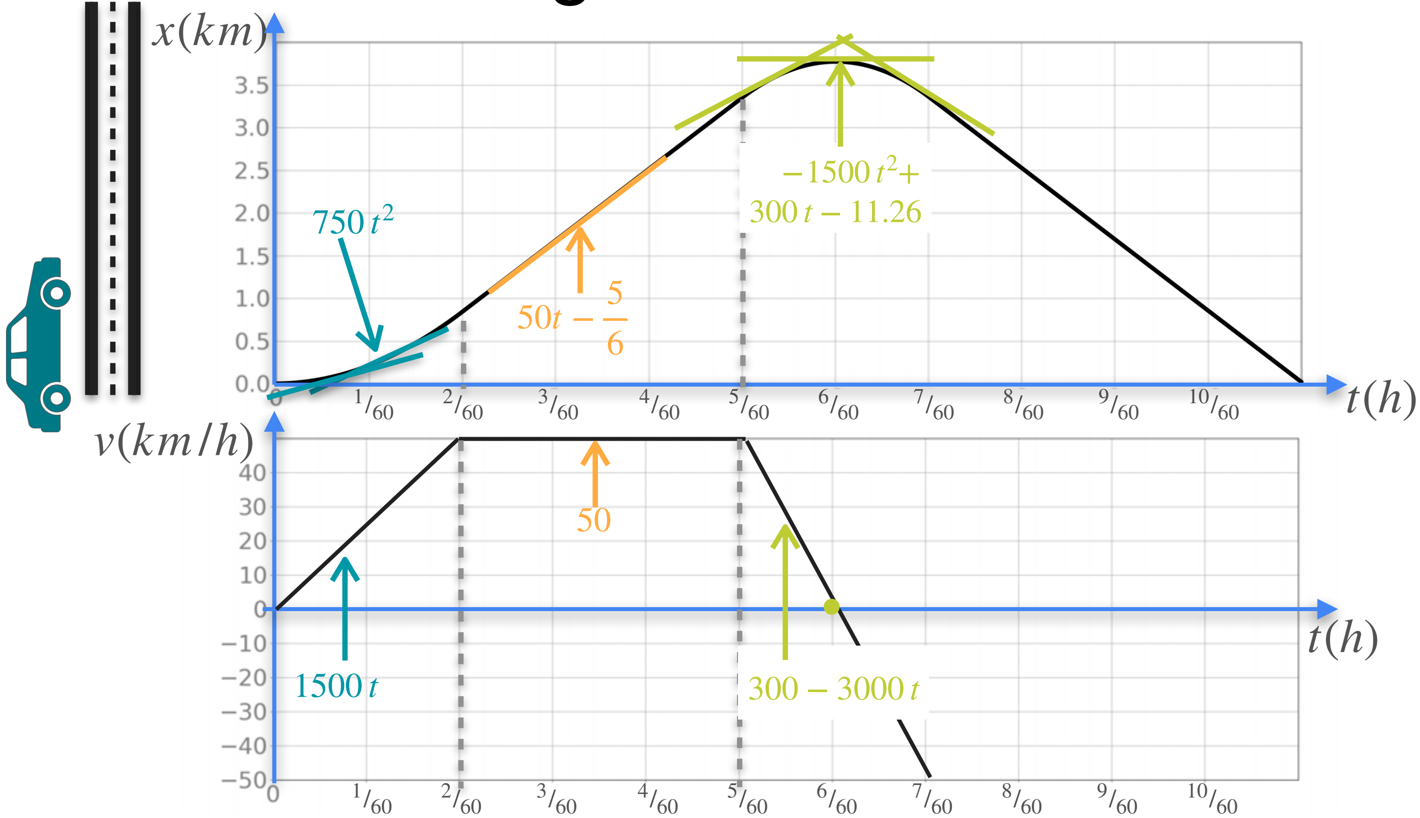
Understanding Second Derivative



x Distance

v Velocity $\frac{dx}{dt}$

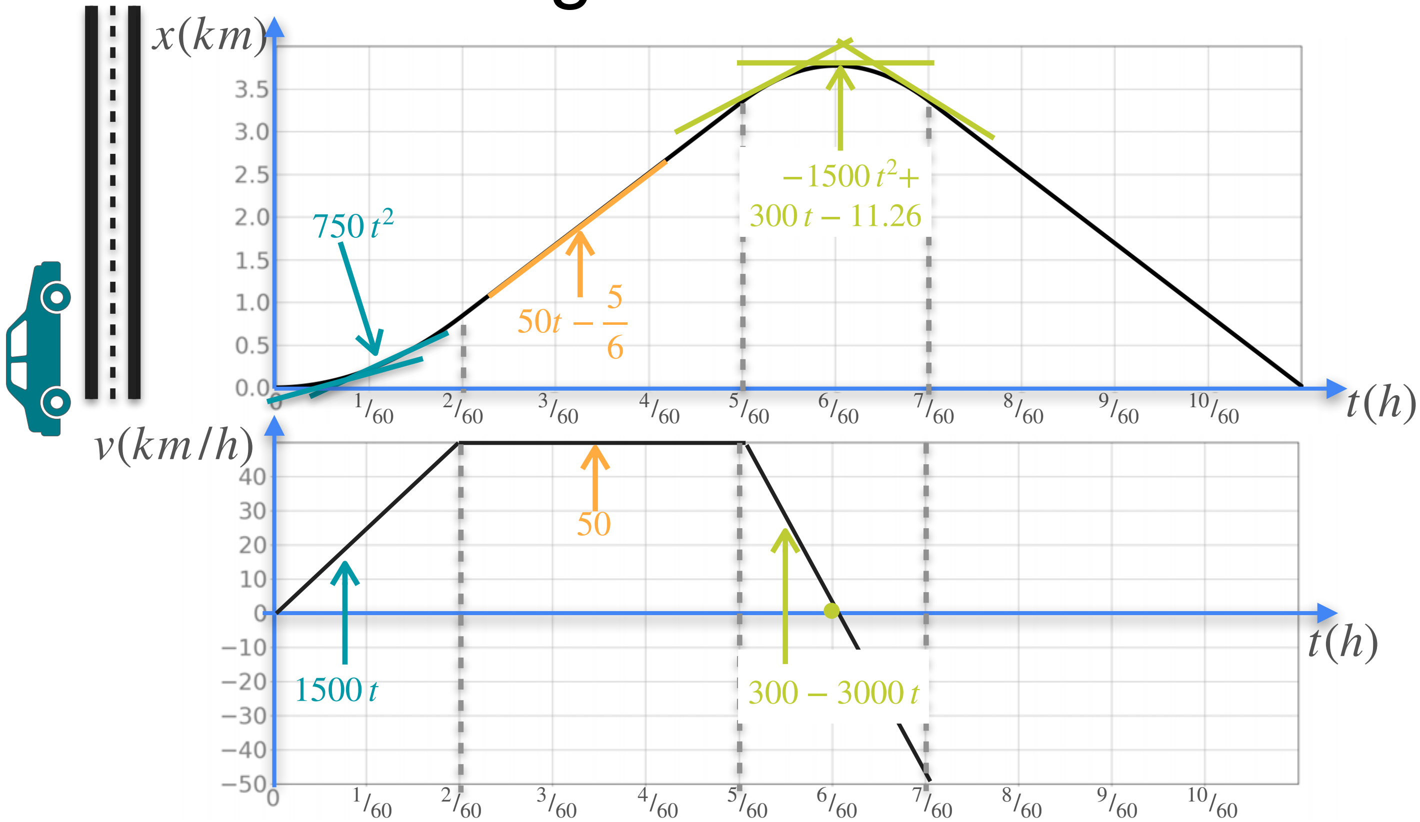
Understanding Second Derivative



x Distance

v Velocity $\frac{dx}{dt}$

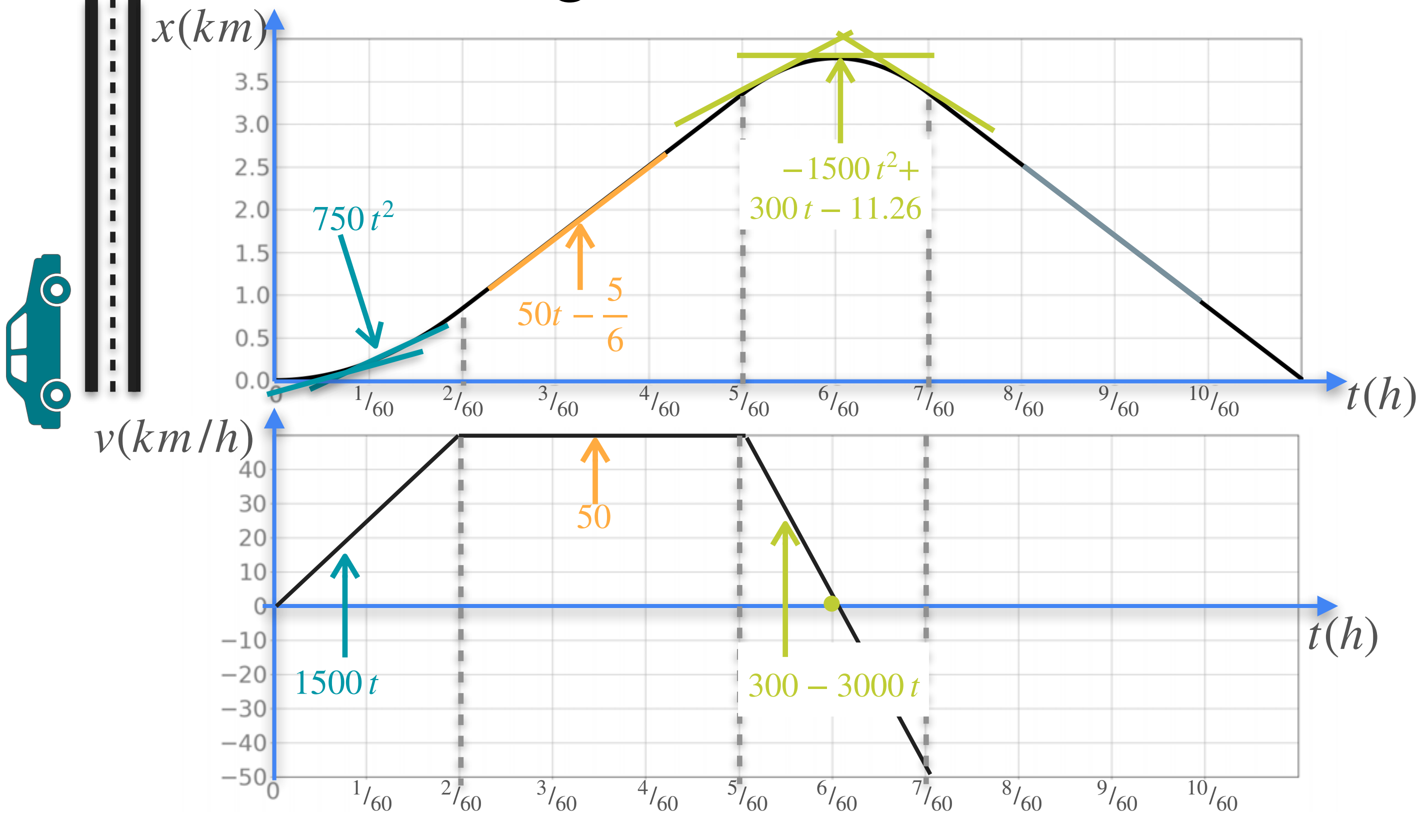
Understanding Second Derivative



x Distance

v Velocity $\frac{dx}{dt}$

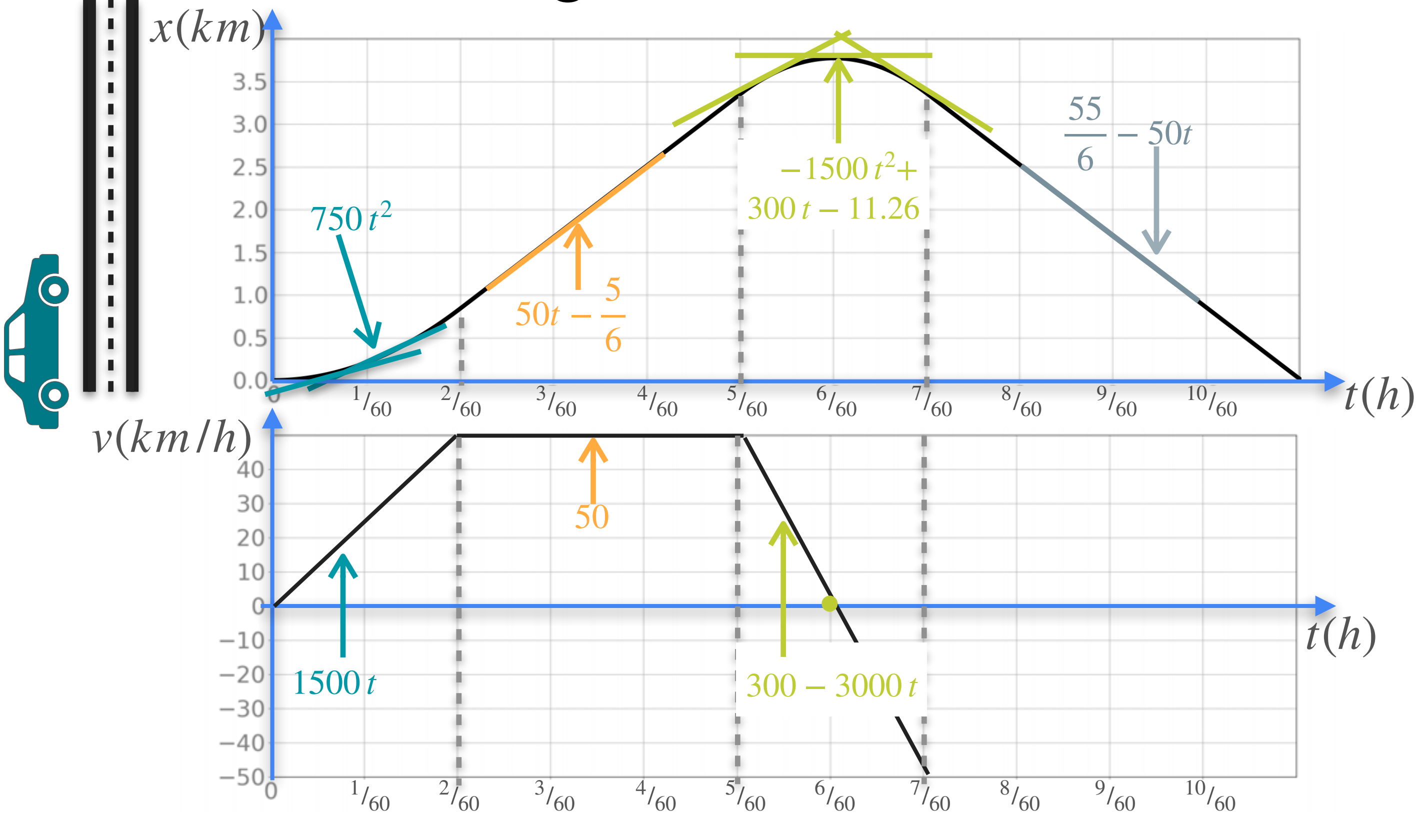
Understanding Second Derivative



x Distance

v Velocity $\frac{dx}{dt}$

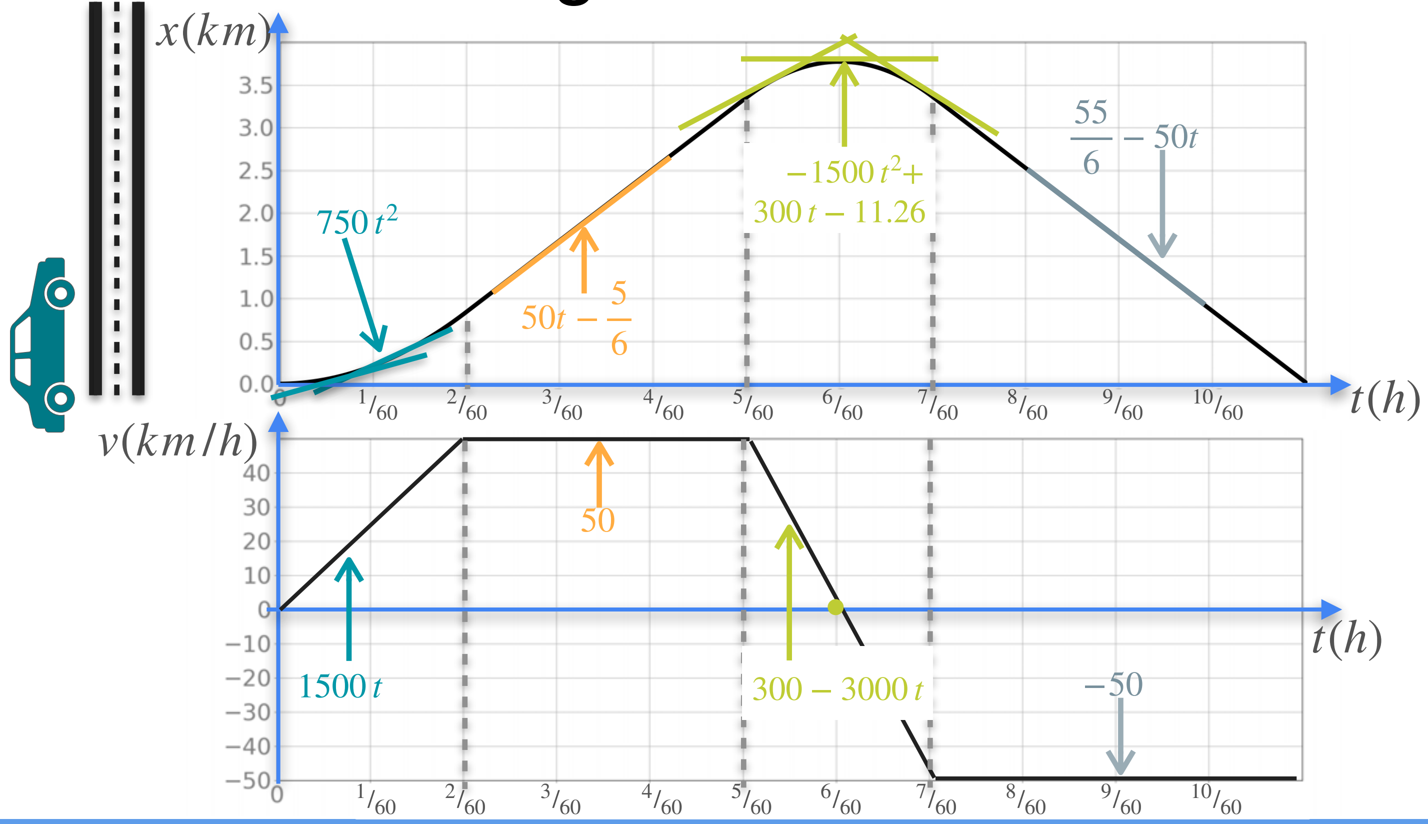
Understanding Second Derivative



x Distance

v Velocity $\frac{dx}{dt}$

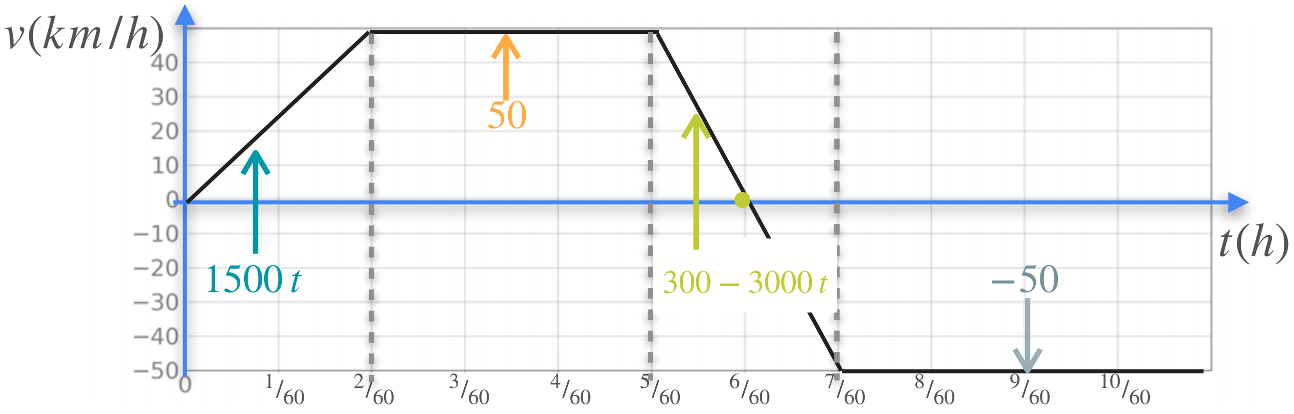
Understanding Second Derivative



x Distance

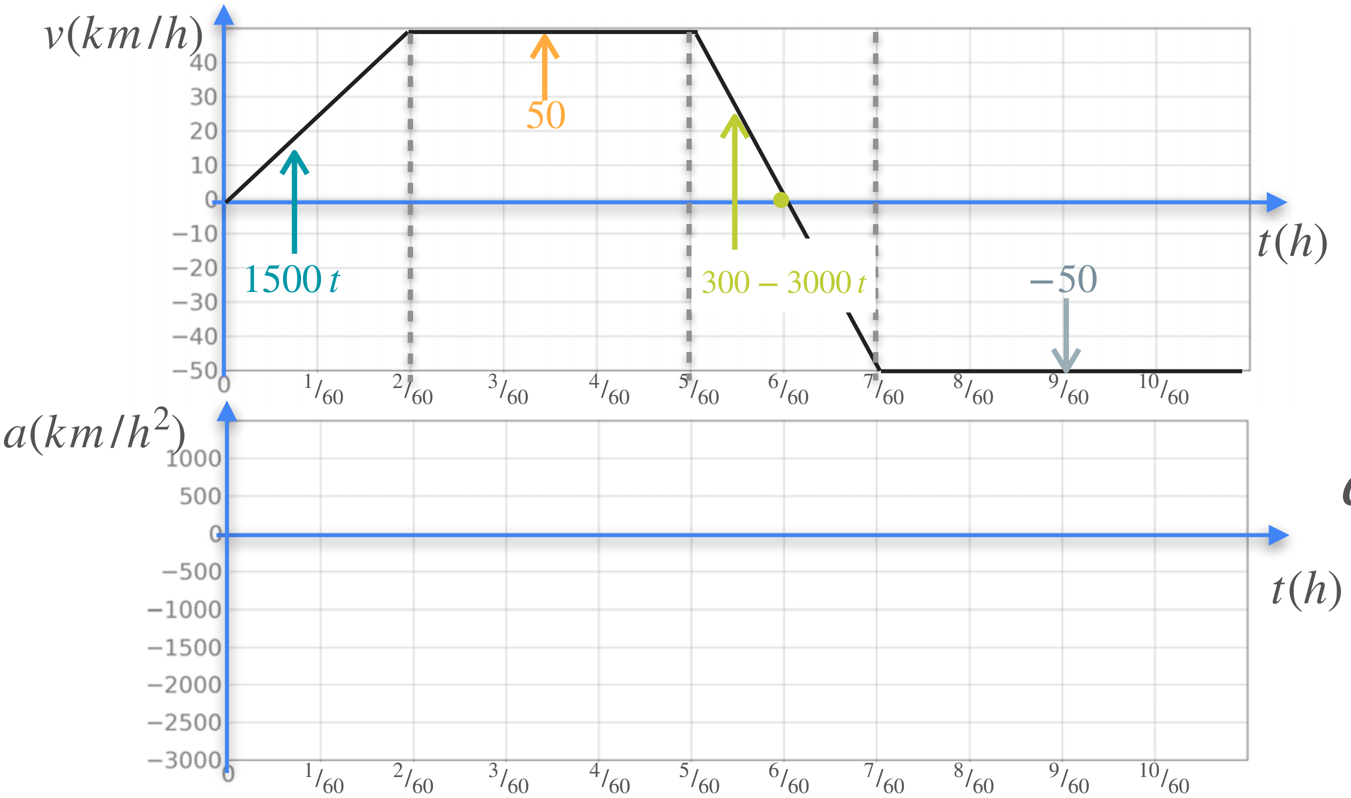
v Velocity $\frac{dx}{dt}$

Understanding Second Derivative



v Velocity $\frac{dx}{dt}$

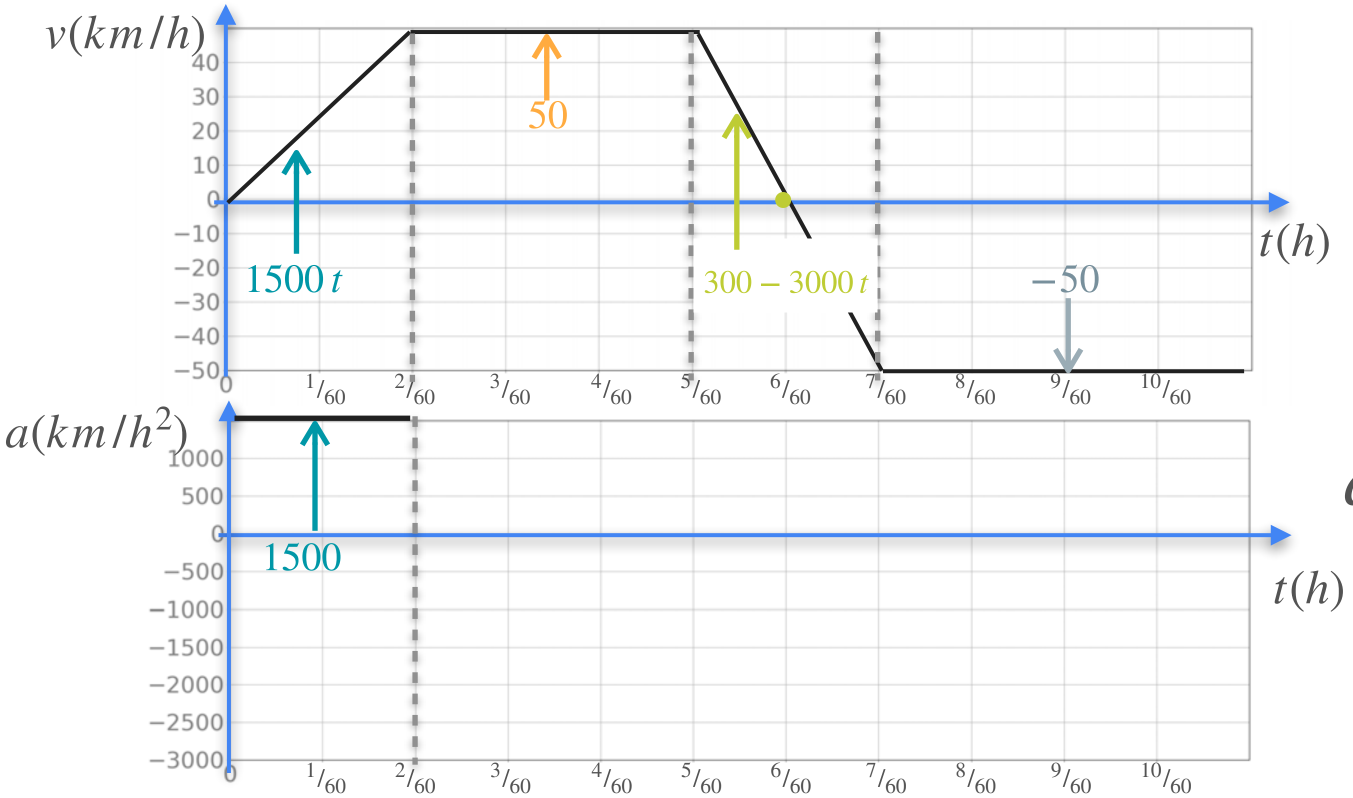
Understanding Second Derivative



v Velocity $\dfrac{dx}{dt}$

a Acceleration $\dfrac{dv}{dt}$

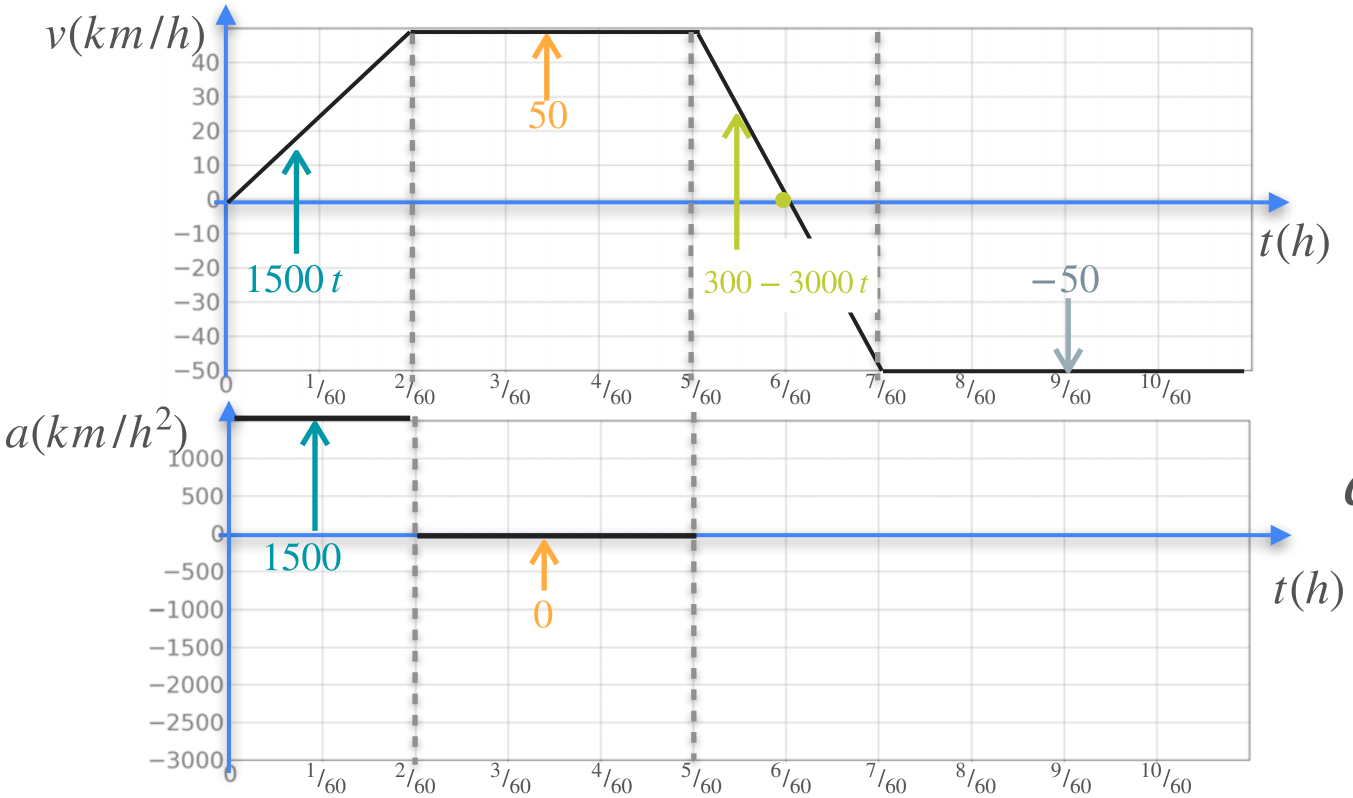
Understanding Second Derivative



v Velocity $\frac{dx}{dt}$

a Acceleration $\frac{dv}{dt}$

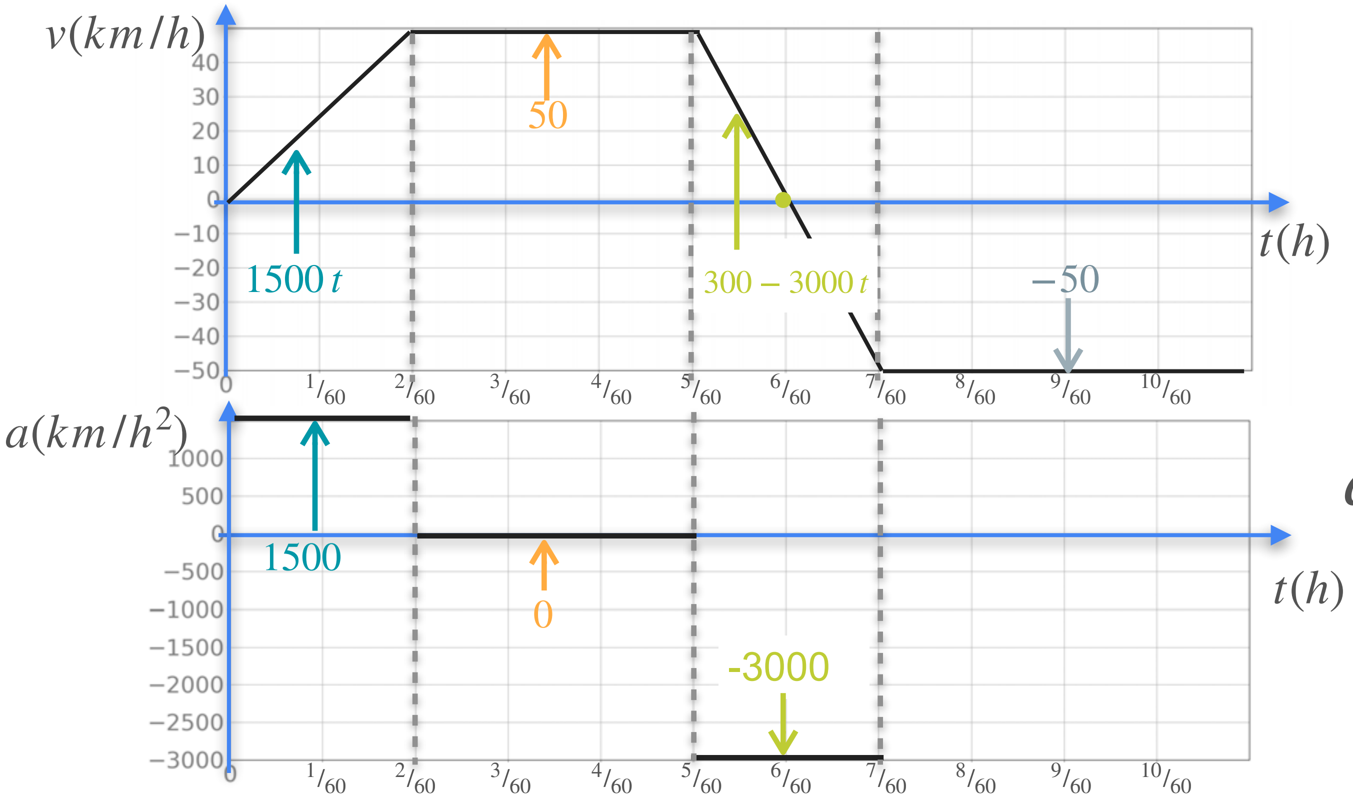
Understanding Second Derivative



v Velocity $\dfrac{dx}{dt}$

a Acceleration $\dfrac{dv}{dt}$

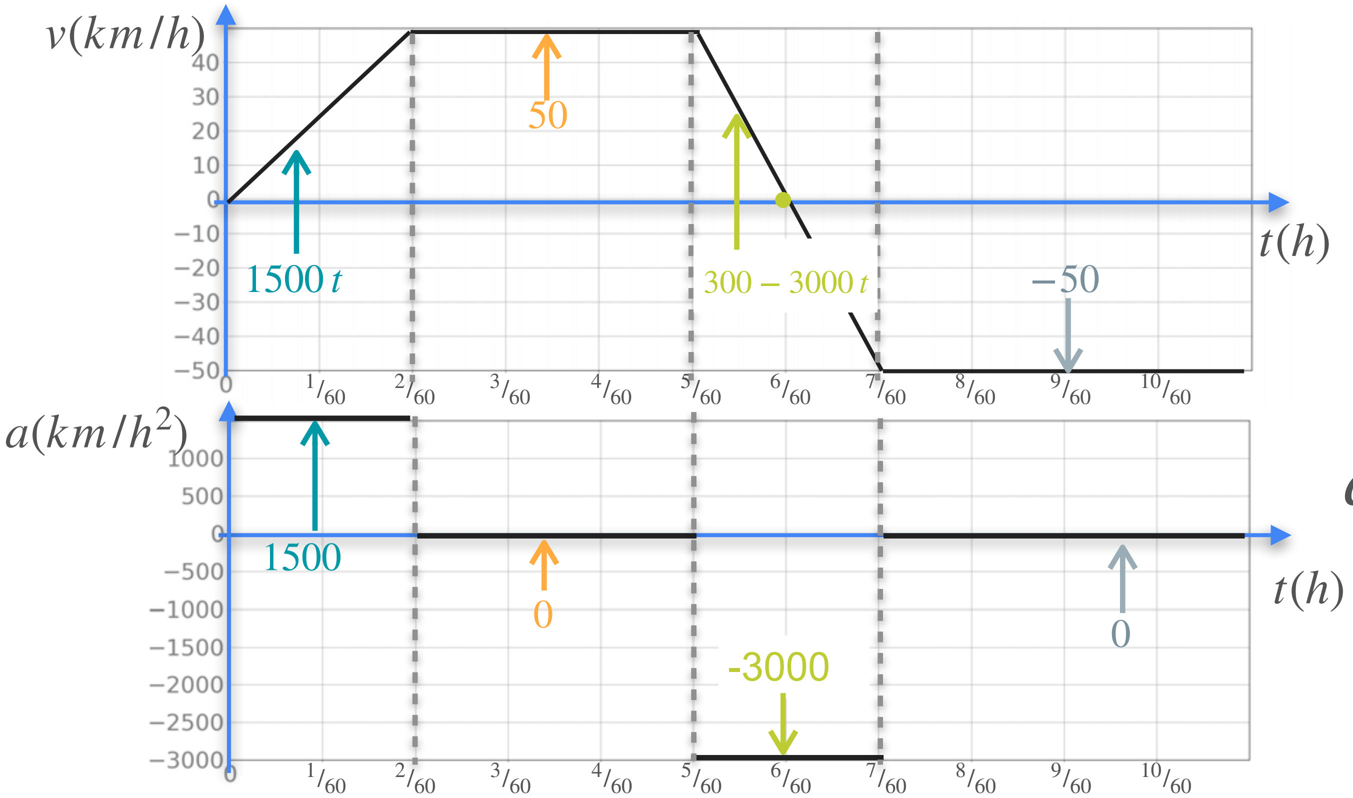
Understanding Second Derivative



v Velocity $\frac{dx}{dt}$

a Acceleration $\frac{dv}{dt}$

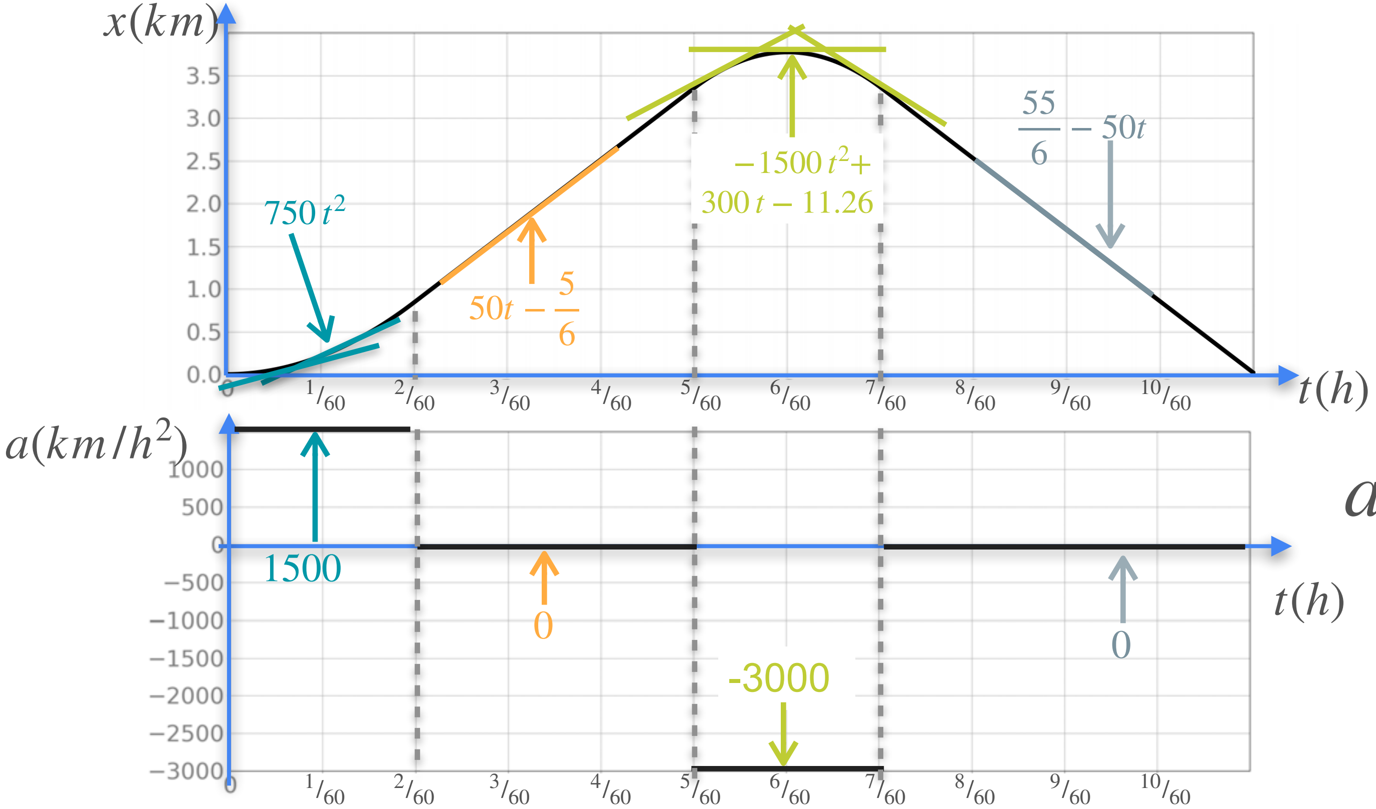
Understanding Second Derivative



v Velocity $\dfrac{dx}{dt}$

a Acceleration $\dfrac{dv}{dt}$

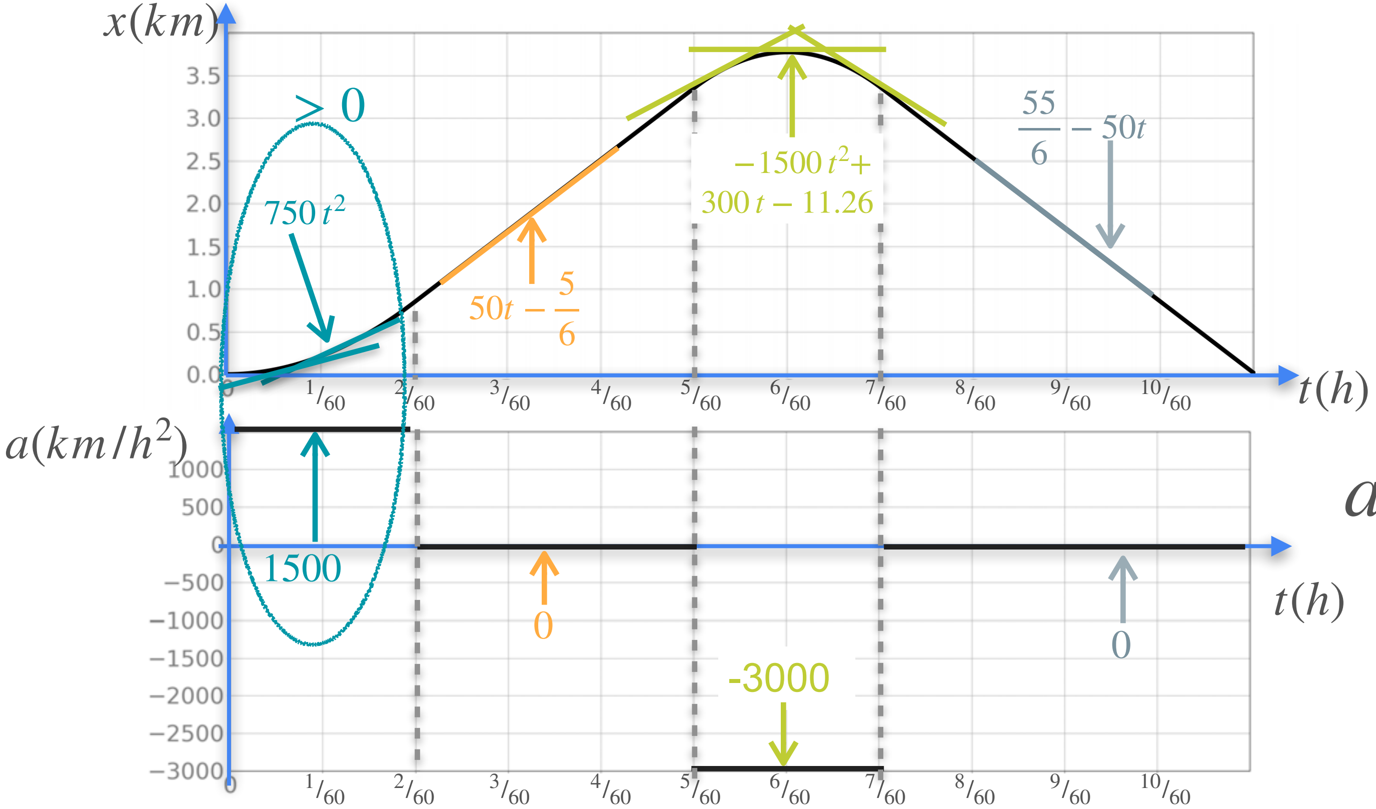
Understanding Second Derivative



x Distance

a Acceleration $\frac{d^2x}{dt^2}$

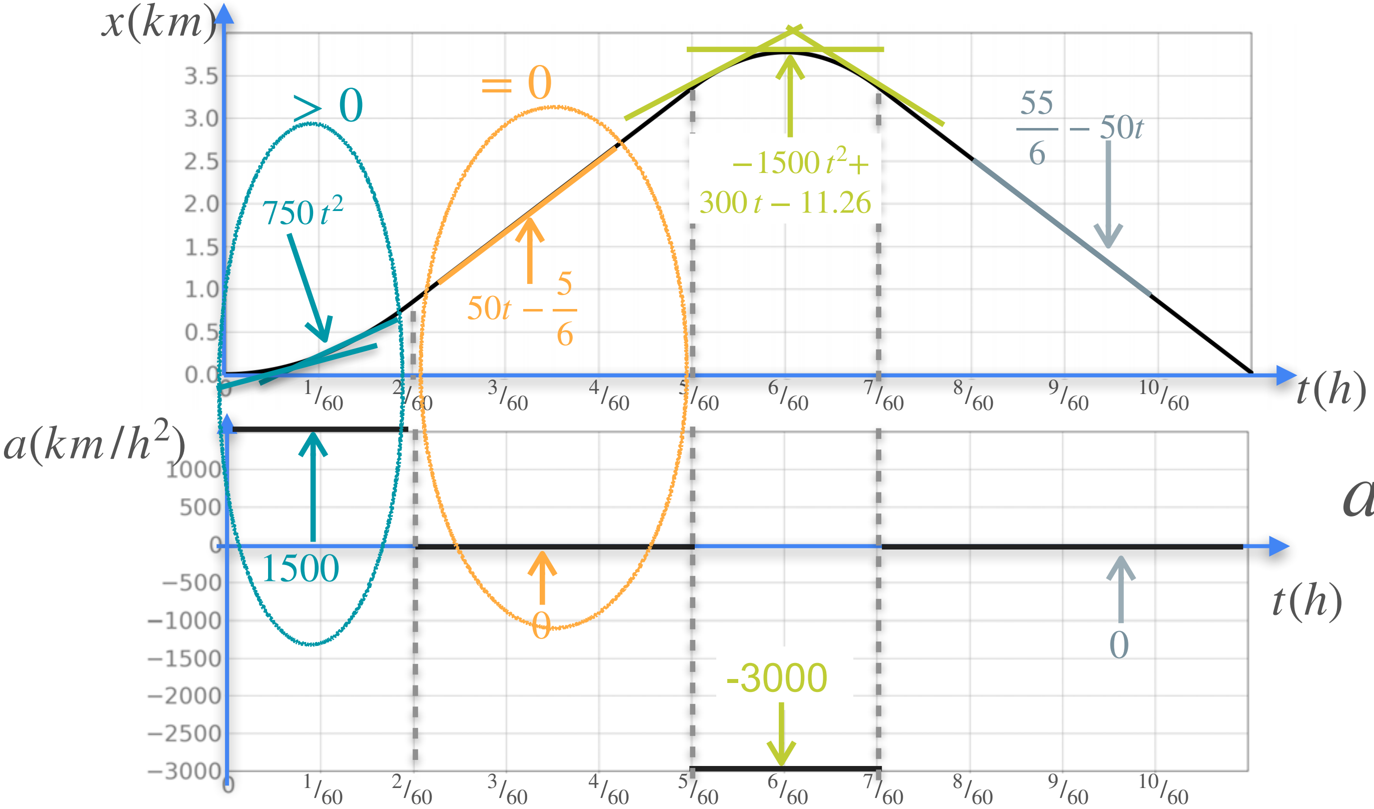
Understanding Second Derivative



x Distance

a Acceleration $\frac{d^2x}{dt^2}$

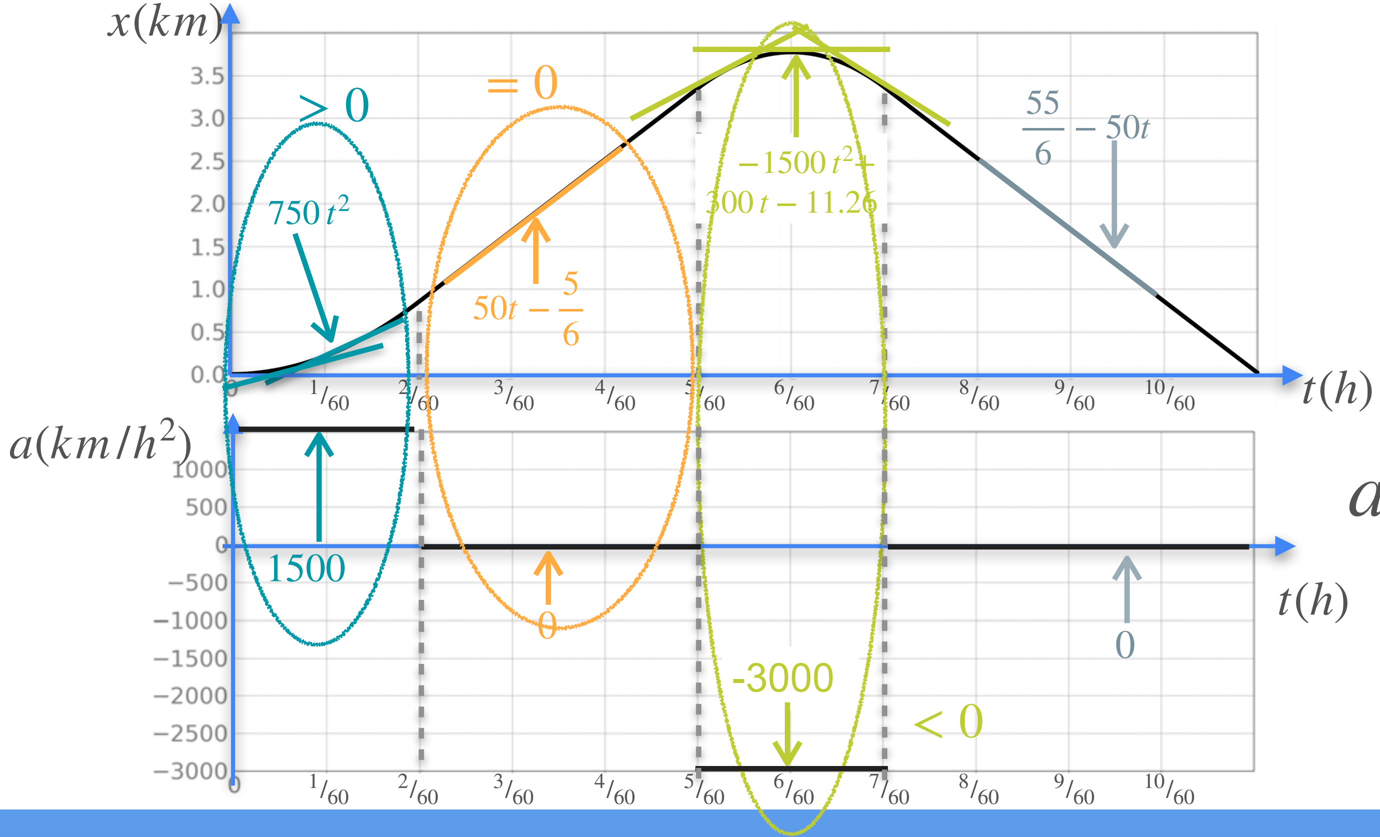
Understanding Second Derivative



x Distance

a Acceleration $\frac{d^2x}{dt^2}$

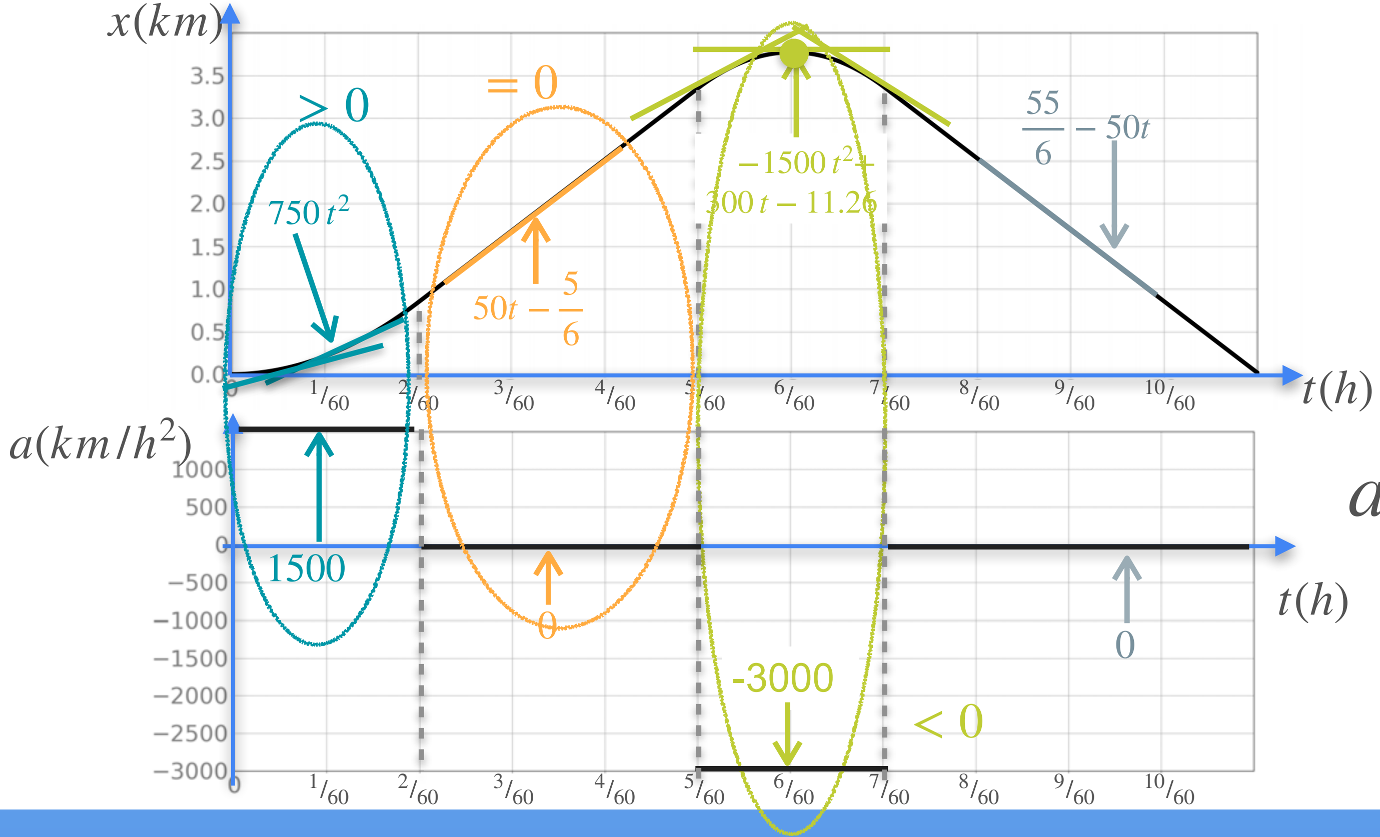
Understanding Second Derivative



x Distance

a Acceleration $\frac{d^2x}{dt^2}$

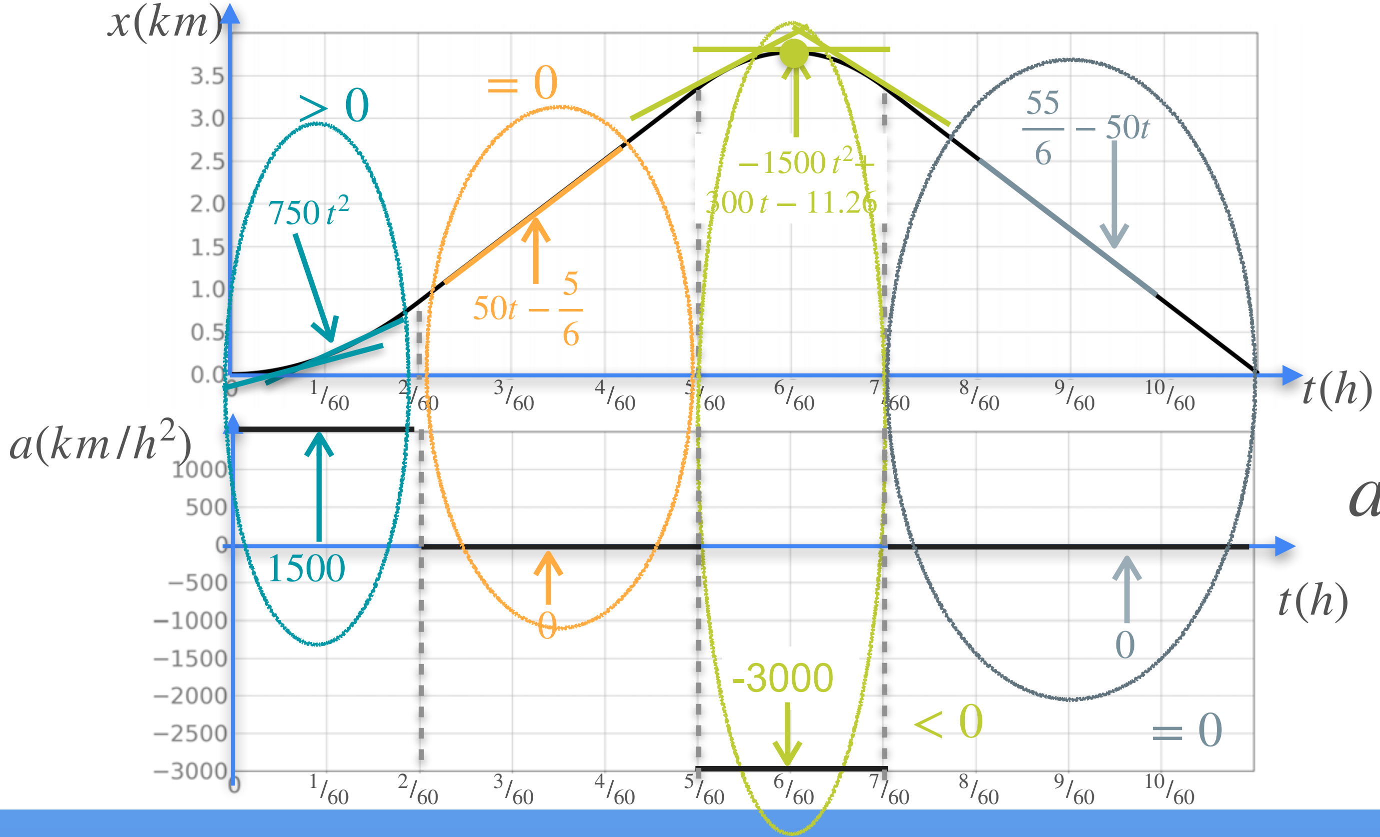
Understanding Second Derivative



x Distance

a Acceleration $\frac{d^2x}{dt^2}$

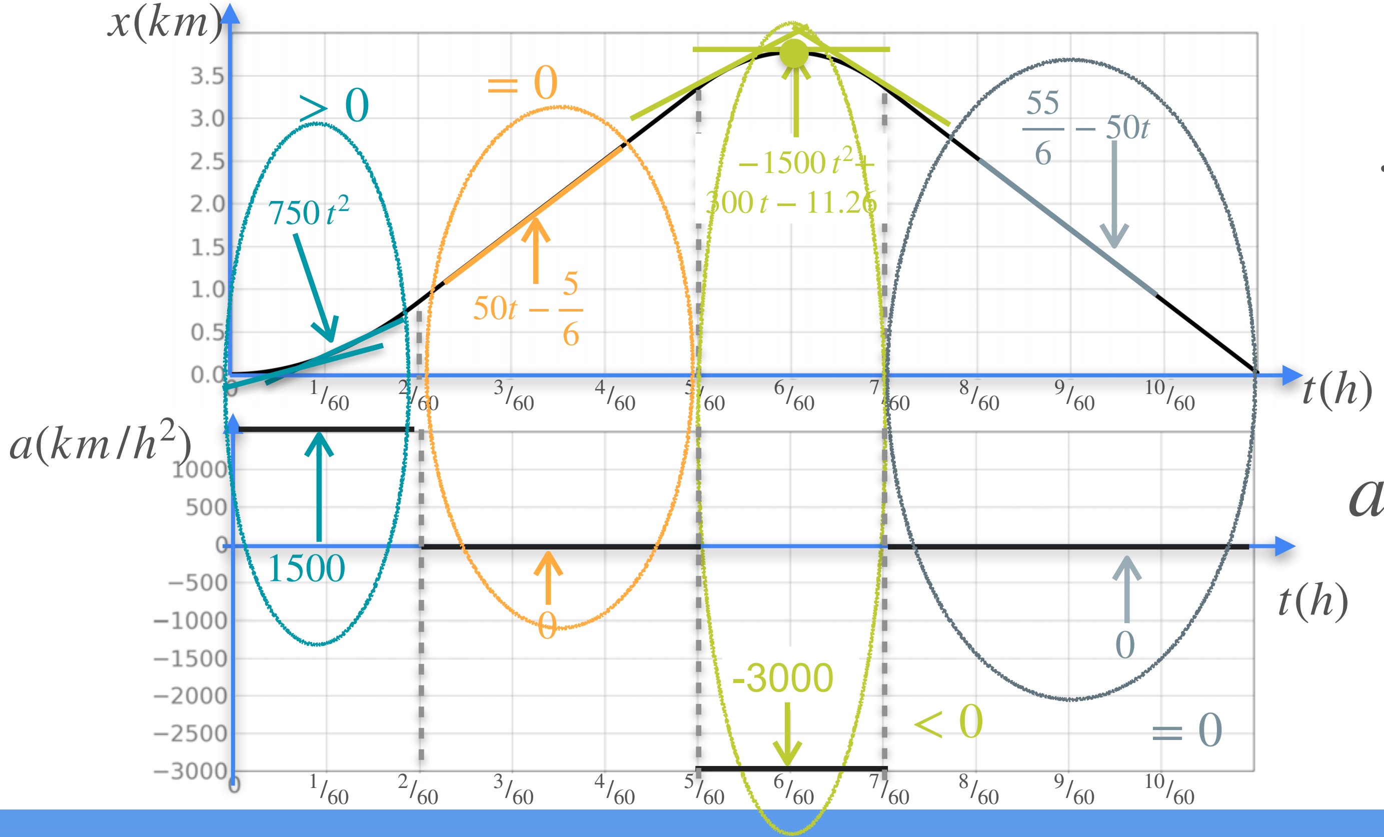
Understanding Second Derivative



x Distance

a Acceleration $\frac{d^2x}{dt^2}$

Understanding Second Derivative



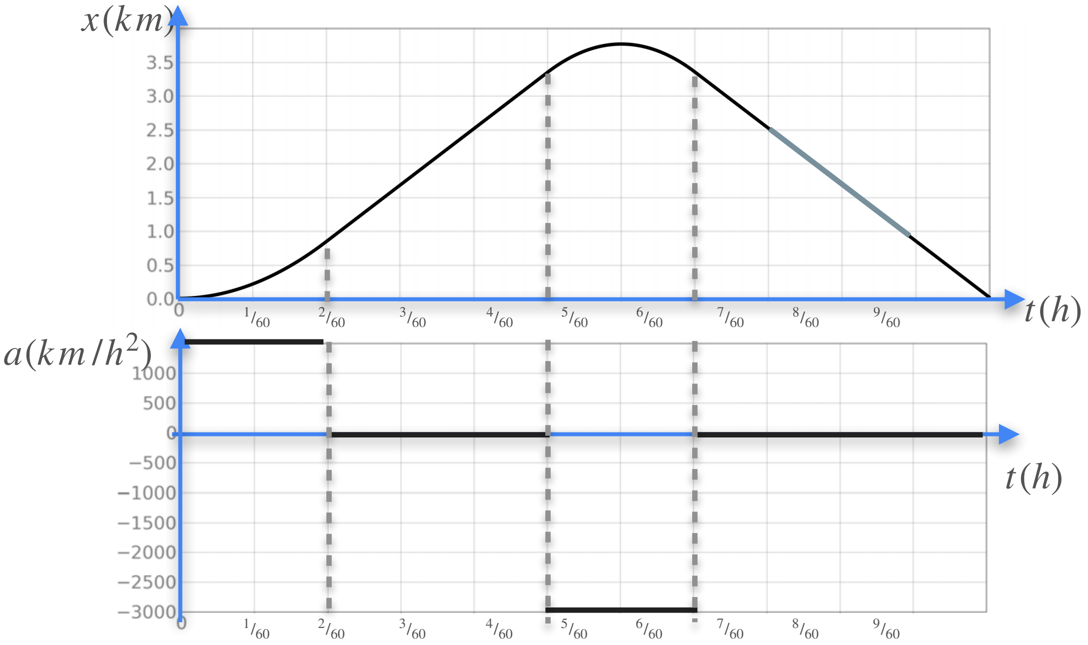
x Distance

Second derivative tells us about the curvature

a Acceleration $\frac{d^2x}{dt^2}$

Curvature

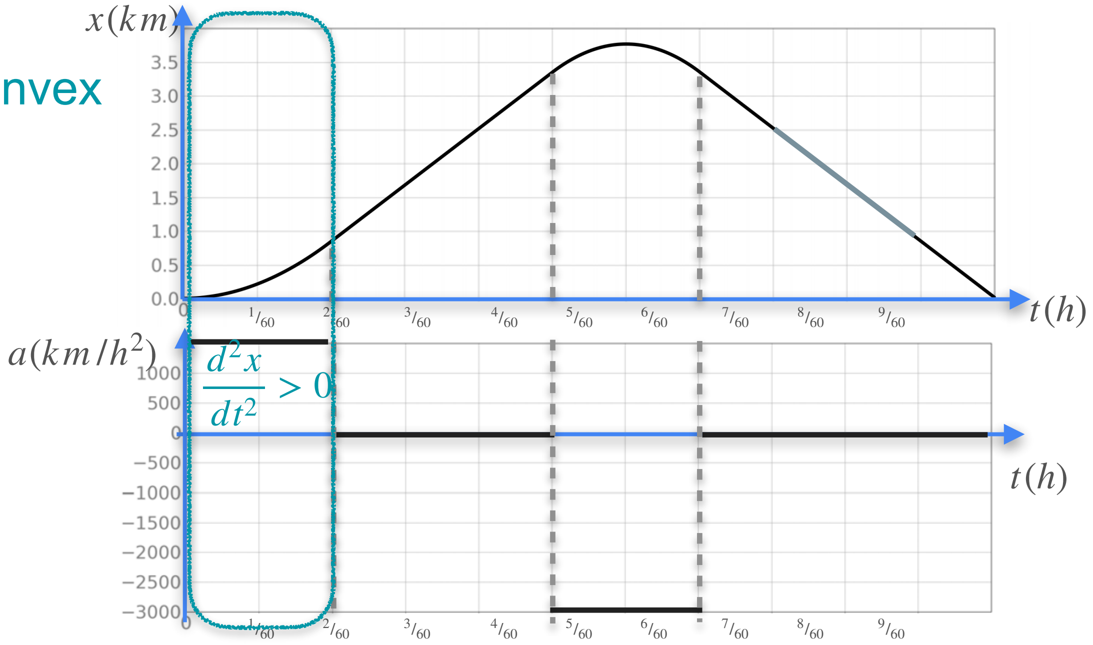
Curvature



$x(km)$
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
$^1/_{60}$
$^2/_{60}$
$^3/_{60}$
$^4/_{60}$
$^5/_{60}$
$^6/_{60}$
$^7/_{60}$
$^8/_{60}$
$^9/_{60}$
$t(h)$
$a(km/h^2)$
1000
500
0
−500
−1000
−1500
−2000
−2500
−3000
0
$^1/_{60}$
$^2/_{60}$
$^3/_{60}$
$^4/_{60}$
$^5/_{60}$
$^6/_{60}$
$^7/_{60}$
$^8/_{60}$
$^9/_{60}$
$t(h)$

Curvature

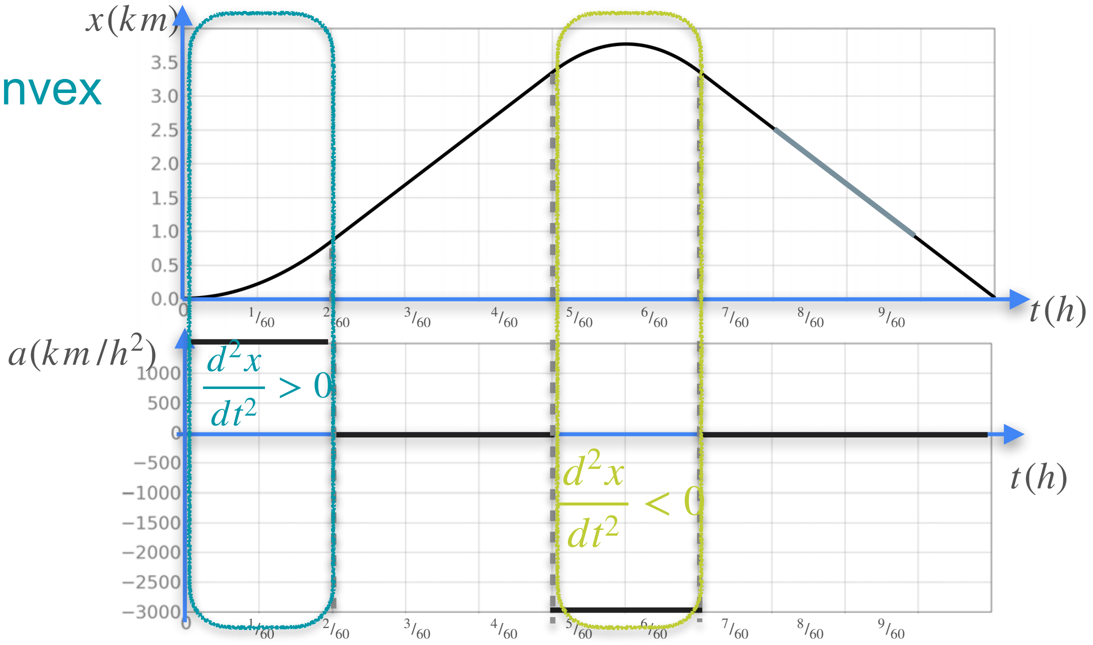
$$\frac{d^2x}{dt^2} > 0$$ Concave up or convex



Curvature

$$\frac{d^2x}{dt^2} > 0$$ Concave up or convex

$$\frac{d^2x}{dt^2} < 0$$ Concave down

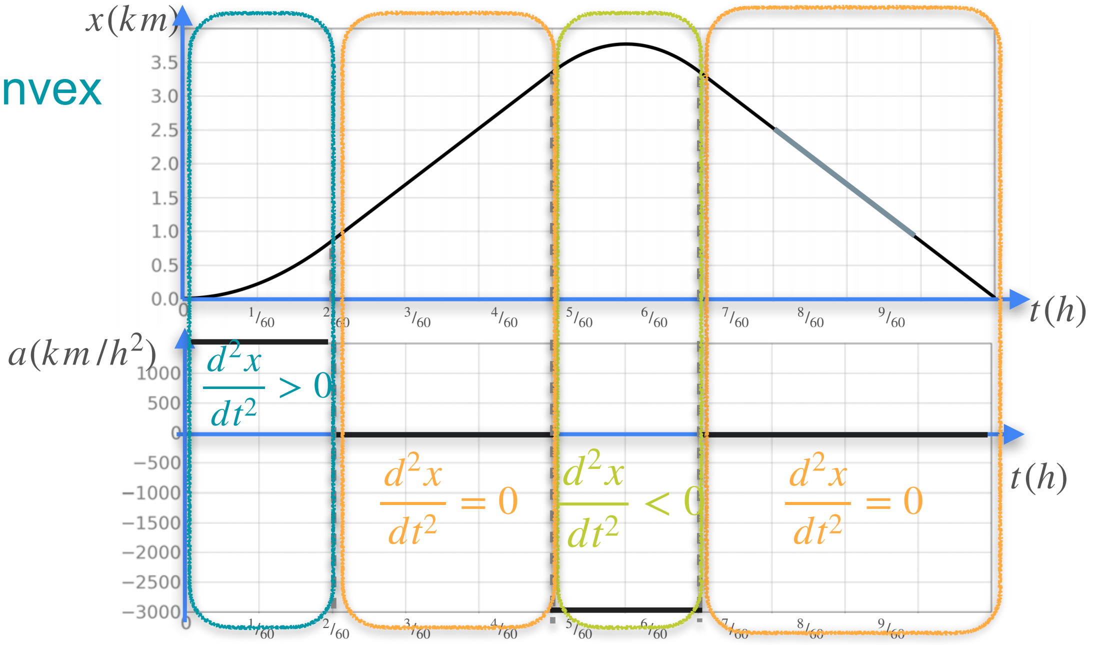


Curvature

$$\frac{d^2x}{dt^2} > 0$$ Concave up or convex

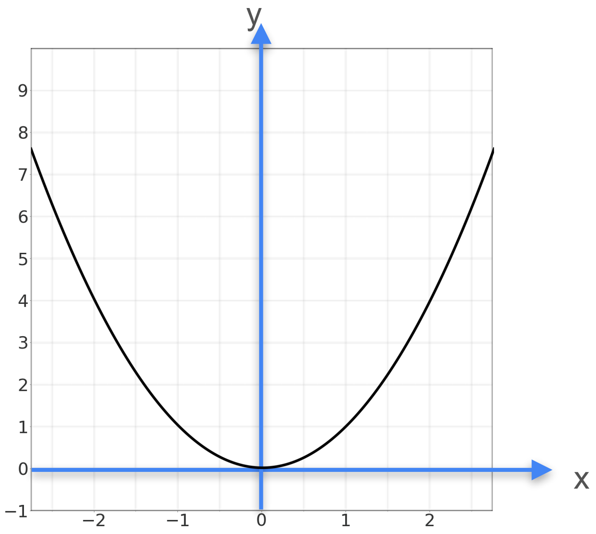
$$\frac{d^2x}{dt^2} < 0$$ Concave down

$$\frac{d^2x}{dt^2} = 0$$ Need more information

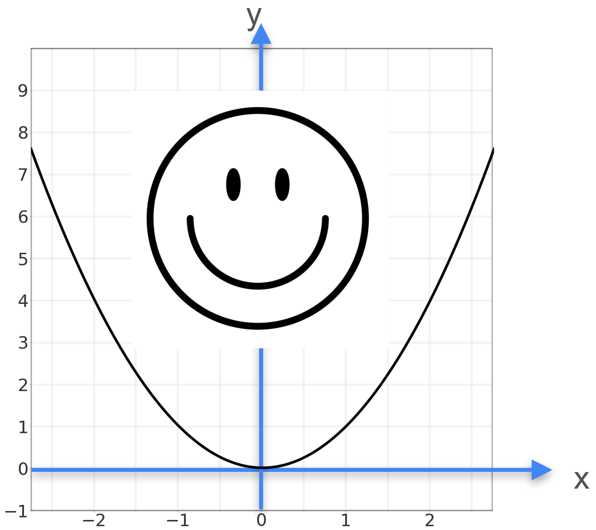


Curvature

Curvature



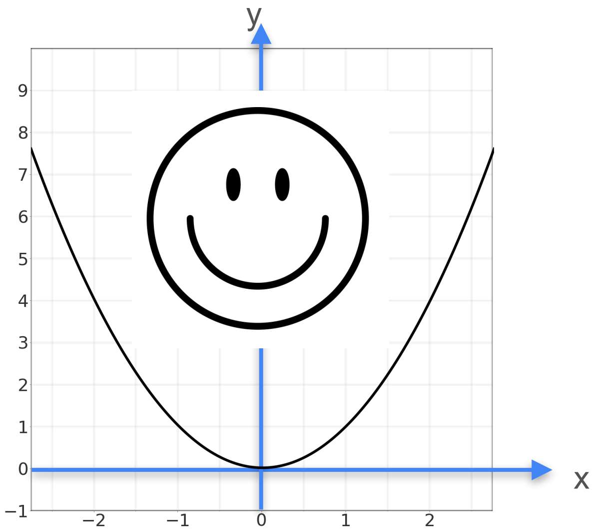
Curvature



Concave up or convex

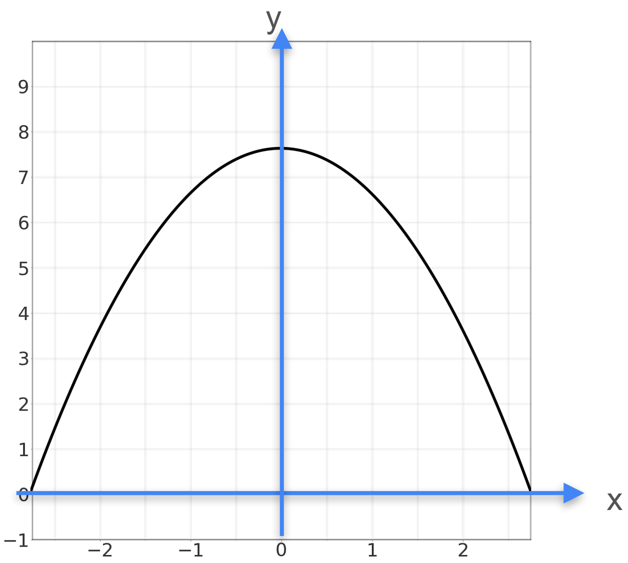
$$f''(0) > 0$$

Curvature

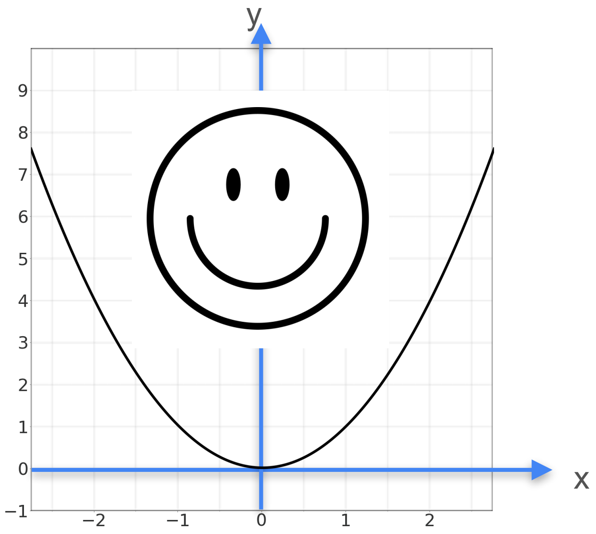


Concave up or convex

$$f''(0) > 0$$

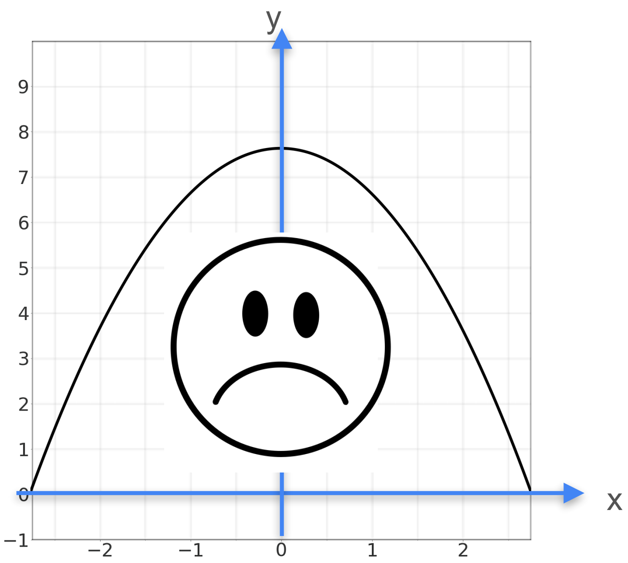


Curvature



Concave up or convex

$$f''(0) > 0$$



Concave down

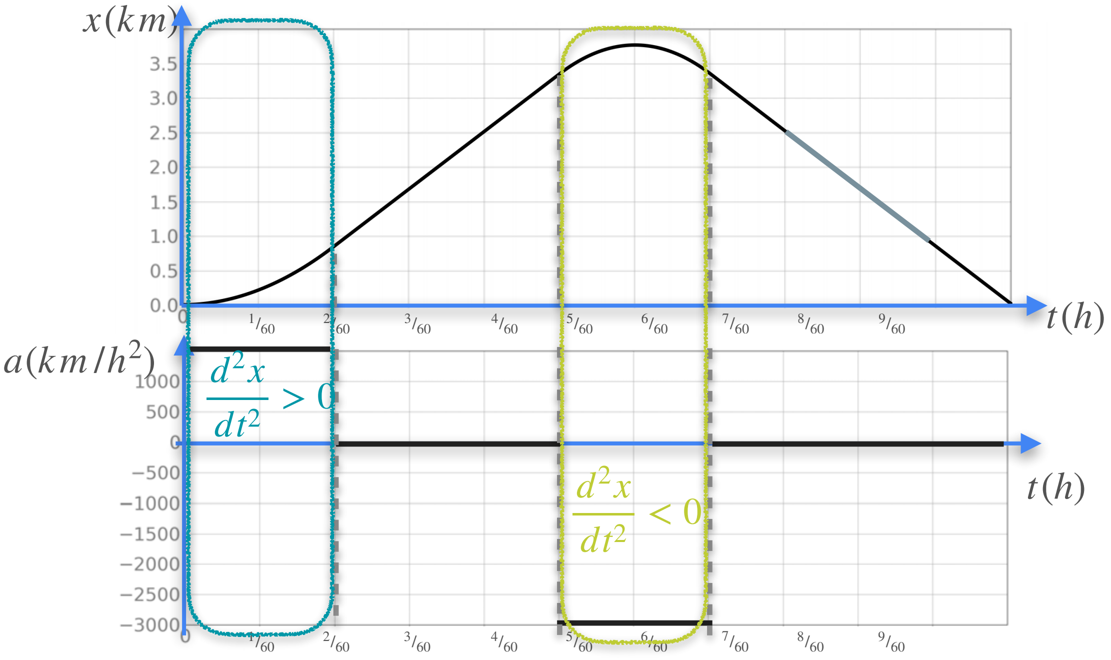
$$f''(0) < 0$$

Second Derivative and Optimization

$$\frac{d^2x}{dt^2} > 0$$

$$\frac{d^2x}{dt^2} < 0$$

$$\frac{d^2x}{dt^2} = 0$$

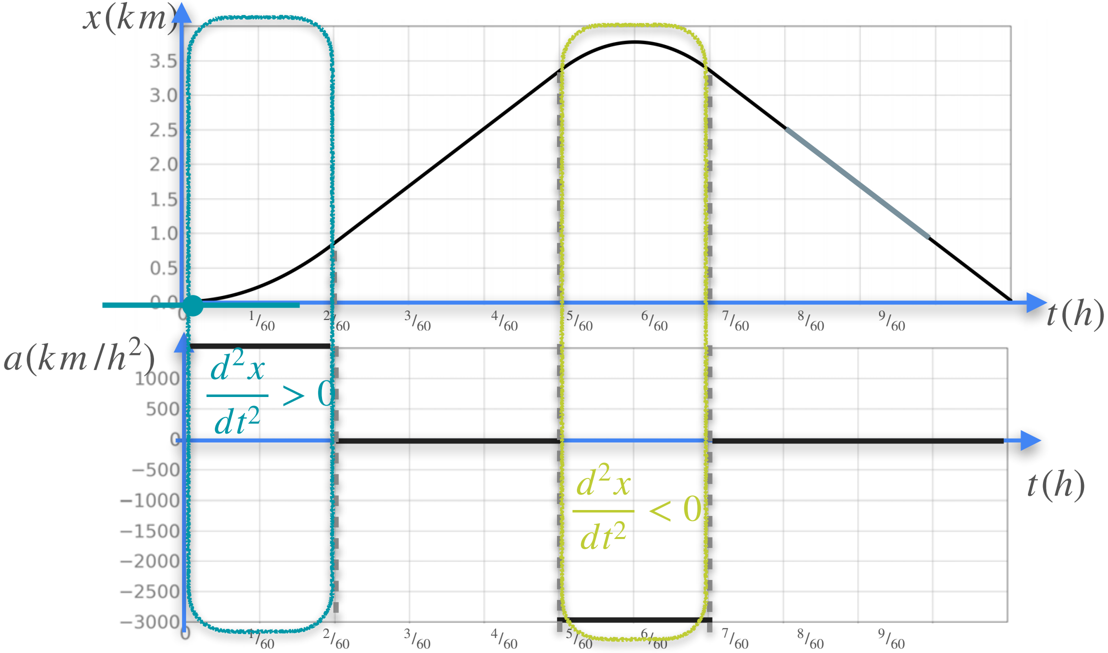


Second Derivative and Optimization

$$\frac{d^2x}{dt^2} > 0$$ (Local) Minimum

$$\frac{d^2x}{dt^2} < 0$$

$$\frac{d^2x}{dt^2} = 0$$



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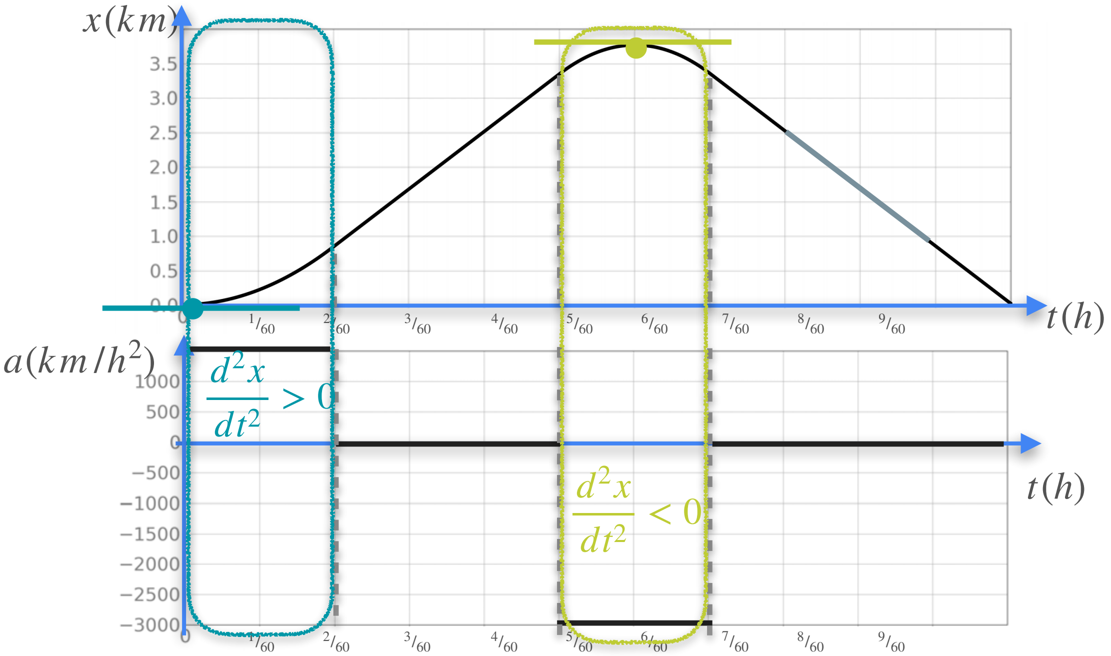
Second Derivative and Optimization

$$\frac{d^2x}{dt^2} > 0$$ (Local) Minimum

$$\frac{d^2x}{dt^2} < 0$$ (Local) maximum

$$\frac{d^2x}{dt^2} = 0$$

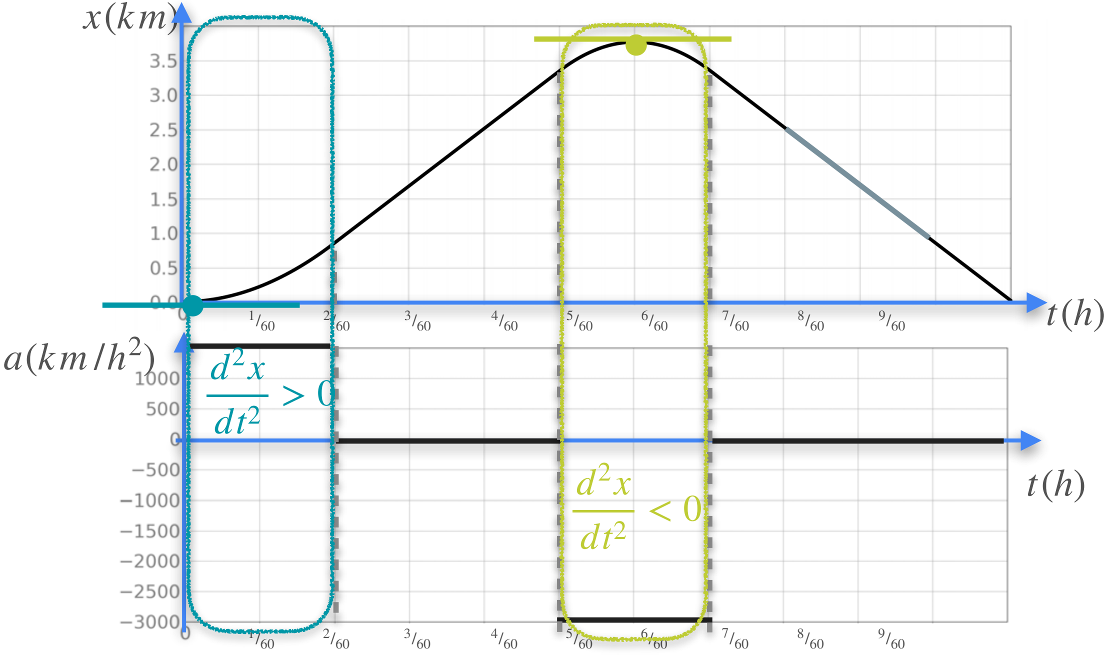


Second Derivative and Optimization

$$\frac{d^2x}{dt^2} > 0$$ (Local) Minimum

$$\frac{d^2x}{dt^2} < 0$$ (Local) maximum

$$\frac{d^2x}{dt^2} = 0$$ Inconclusive



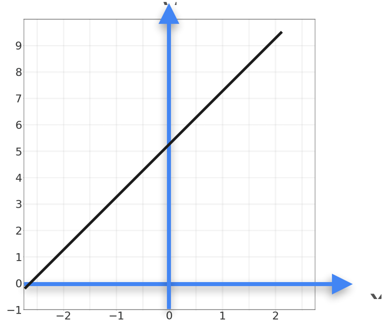
Curvature

First derivative

Second derivative

Curvature

First derivative



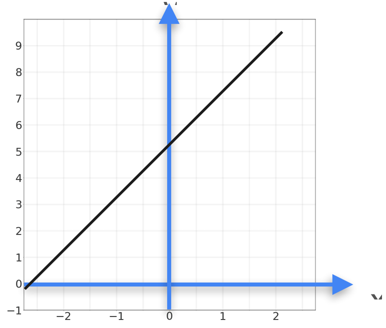
Increasing

$$f'(0) > 0$$

Second derivative

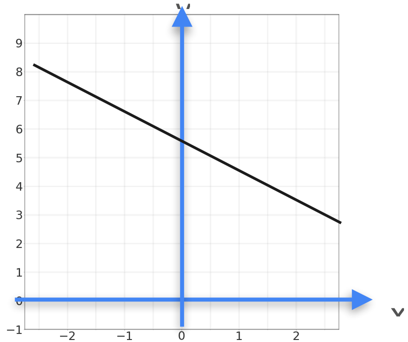
Curvature

First derivative



Increasing

$$f'(0) > 0$$



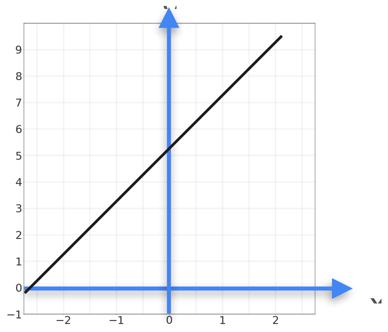
Decreasing

$$f'(0) < 0$$

Second derivative

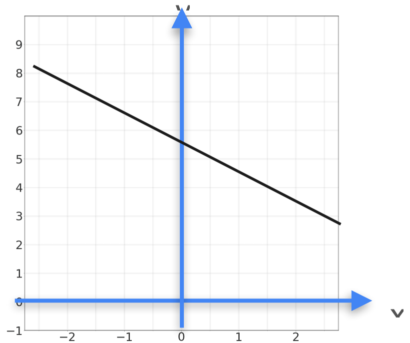
Curvature

First derivative



Increasing

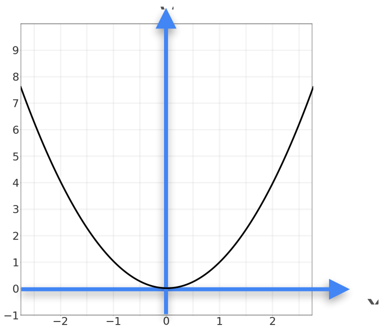
$$f'(0) > 0$$



Decreasing

$$f'(0) < 0$$

Second derivative

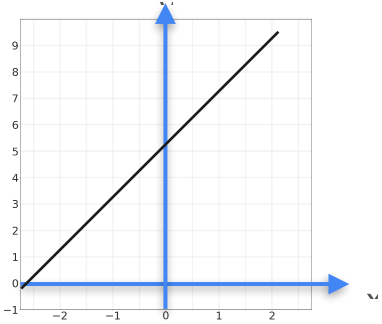


Concave up

$$f''(0) > 0$$

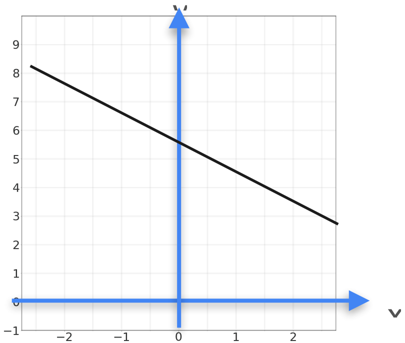
Curvature

First derivative



Increasing

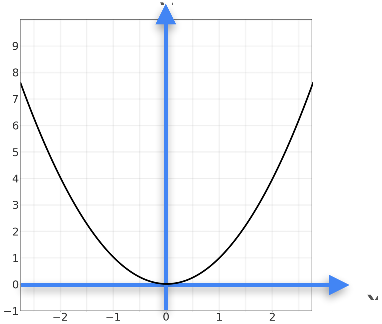
$$f'(0) > 0$$



Decreasing

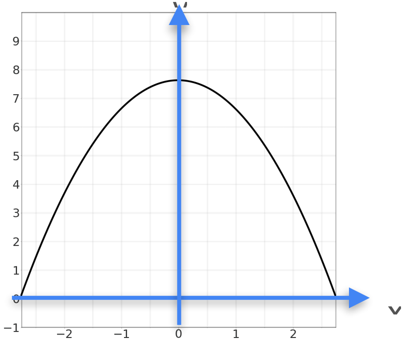
$$f'(0) < 0$$

Second derivative



Concave up

$$f''(0) > 0$$



Concave down

$$f''(0) < 0$$

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Optimization in Neural Networks and Newton's Method

The Hessian

Second Derivative

Second Derivative

	1 variable	2 variables

Second Derivative

	1 variable	2 variables
Function	$f(x)$	

Second Derivative

	1 variable	2 variables
Function	$f(x)$	$f(x, y)$

Second Derivative

	1 variable	2 variables
Function	$f(x)$	$f(x, y)$
First derivative	$f'(x)$ Rate of change of $f(x)$	

Second Derivative

	1 variable		2 variables	
Function	$f(x)$		$f(x, y)$	
First derivative	$f'(x)$	Rate of change of $f(x)$	$f_x(x, y)$	Rate of change w.r.t x

Second Derivative

	1 variable		2 variables	
Function	$f(x)$		$f(x, y)$	
First derivative	$f'(x)$	Rate of change of $f(x)$	$f_x(x, y)$	Rate of change w.r.t x
			$f_y(x, y)$	Rate of change w.r.t y

Second Derivative

	1 variable	2 variables
Function	$f(x)$	$f(x, y)$
First derivative	$f'(x)$ Rate of change of $f(x)$	$f_x(x, y)$ Rate of change w.r.t x $f_y(x, y)$ Rate of change w.r.t y $\nabla f = \begin{bmatrix} f_x(x, y) \\ f_y(x, y) \end{bmatrix}$

Second Derivative

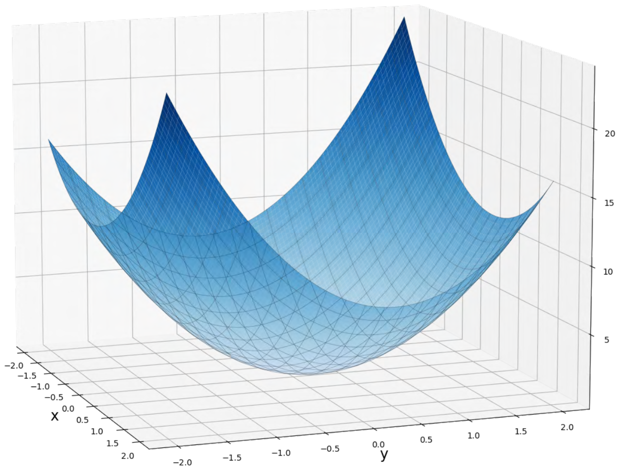
	1 variable		2 variables	
Function	$f(x)$		$f(x, y)$	
First derivative	$f'(x)$	Rate of change of $f(x)$	$f_x(x, y)$	Rate of change w.r.t x
			$f_y(x, y)$	Rate of change w.r.t y
			$\nabla f = \begin{bmatrix} f_x(x, y) \\ f_y(x, y) \end{bmatrix}$	
Second derivative	$f''(x)$	Rate of change of the rate of change of $f(x)$		

Second Derivative

	1 variable		2 variables	
Function	$f(x)$		$f(x, y)$	
First derivative	$f'(x)$	Rate of change of $f(x)$	$f_x(x, y)$ $f_y(x, y)$ $\nabla f = \begin{bmatrix} f_x(x, y) \\ f_y(x, y) \end{bmatrix}$	Rate of change w.r.t x Rate of change w.r.t y
Second derivative	$f''(x)$	Rate of change of the rate of change of $f(x)$	???	

Second Derivative

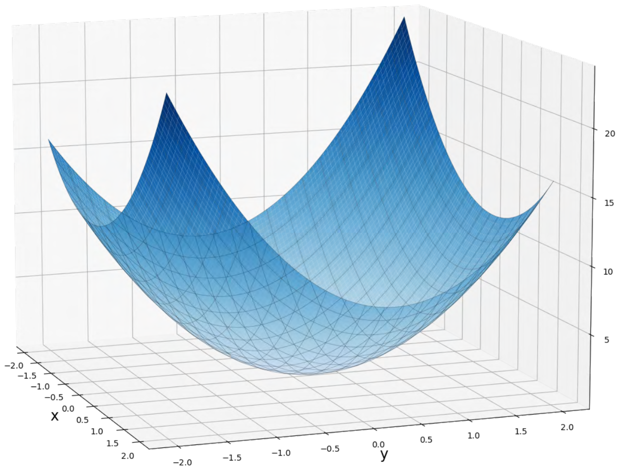
Second Derivative



$$f(x, y) =$$

$$2x^2 + 3y^2 - xy$$

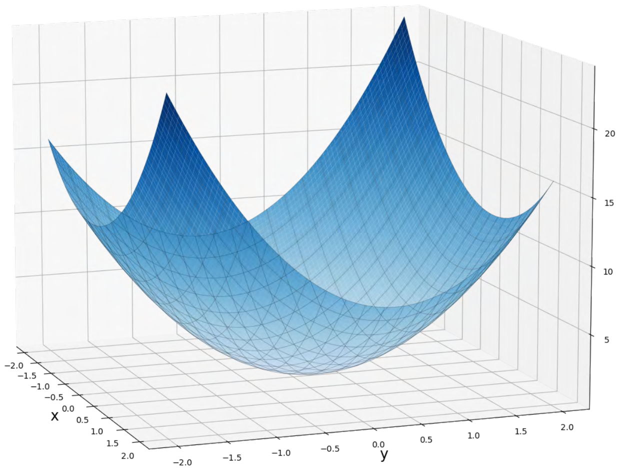
Second Derivative



$$f(x, y) =$$

$$2x^2 + 3y^2 - xy \xrightarrow{x} 4x - y$$

Second Derivative



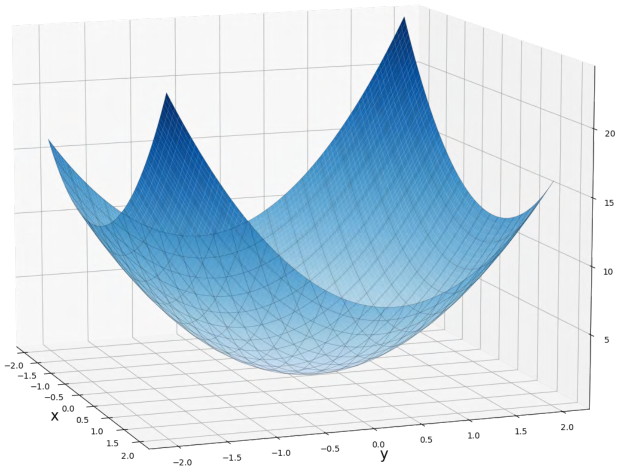
$$f(x, y) =$$

$$2x^2 + 3y^2 - xy$$

x: $4x - y$

y: $6y - x$

Second Derivative



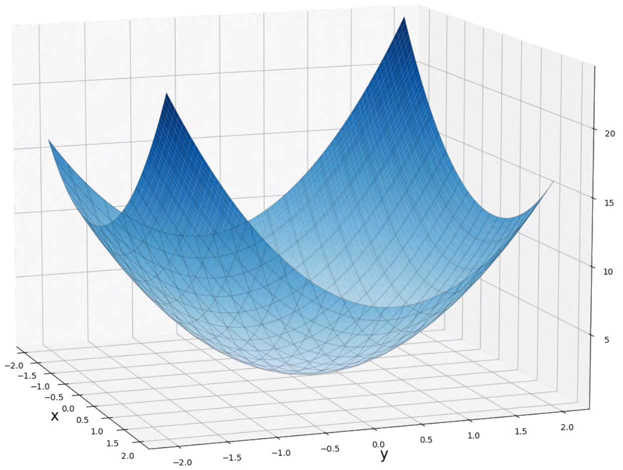
$f(x, y) =$

$2x^2 + 3y^2 - xy \xrightarrow{x} 4x - y \xrightarrow{x} 4$

$2x^2 + 3y^2 - xy \xrightarrow{y} 6y - x$

Rate of change of $f'_x(x, y)$ w.r.t x

Second Derivative



$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$2x^2 + 3y^2 - xy \xrightarrow{x} 4x - y$$

$$2x^2 + 3y^2 - xy \xrightarrow{y} 6y - x$$

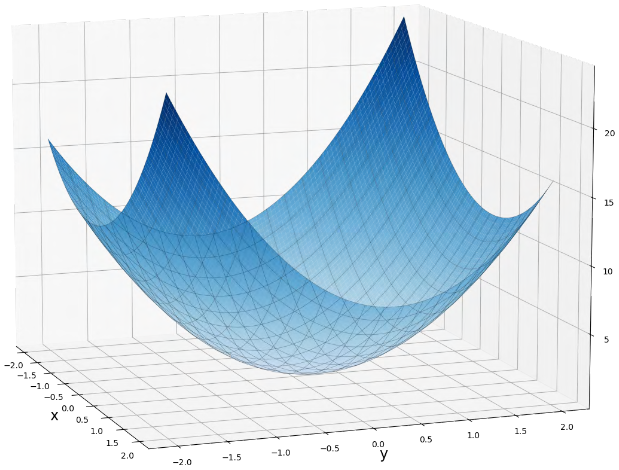
$$4x - y \xrightarrow{x} 4$$

Rate of change of $f'_x(x, y)$ w.r.t x

$$4x - y \xrightarrow{y} -1$$

Rate of change of $f'_x(x, y)$ w.r.t y

Second Derivative



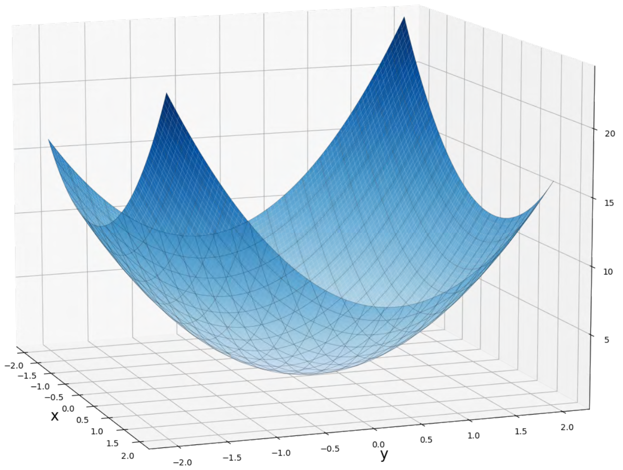
$$
\begin{tikzcd}[column sep=large]
 & & 4 \\
 & 4x - y \arrow[ur, "x"] \arrow[r, "y"] & -1 \\
f(x,y) = 2x^2 + 3y^2 - xy \arrow[ur, "x"] \arrow[dr, "y"'] & & -1 \\
 & 6y - x \arrow[ur, "x"] &
\end{tikzcd}
$$

4: Rate of change of $f'_x(x, y)$ w.r.t x

-1: Rate of change of $f'_x(x, y)$ w.r.t y

-1: Rate of change of $f'_y(x, y)$ w.r.t x

Second Derivative



$$
\begin{tikzcd}
 & & 4 \\
 & 4x-y \arrow[ur, "x"] \arrow[dr, "y"'] & \\
 & & -1 \\
f(x,y) = 2x^2+3y^2-xy \arrow[uur, "x"] \arrow[ddr, "y"'] & & \\
 & & -1 \\
 & 6y-x \arrow[ur, "x"] \arrow[dr, "y"'] & \\
 & & 6
\end{tikzcd}
$$

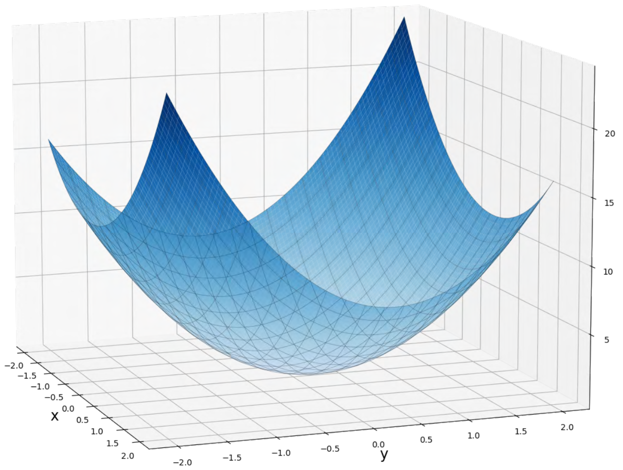
4: Rate of change of $f'_x(x,y)$ w.r.t x

-1: Rate of change of $f'_x(x,y)$ w.r.t y

-1: Rate of change of $f'_y(x,y)$ w.r.t x

6: Rate of change of $f'_y(x,y)$ w.r.t y

Second Derivative



$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$2x^2 + 3y^2 - xy \xrightarrow{x} 4x - y \xrightarrow{x} 4$$

Rate of change of $f'_x(x, y)$ w.r.t x

$$4x - y \xrightarrow{y} \boxed{-1}$$

Rate of change of $f'_x(x, y)$ w.r.t y

$$2x^2 + 3y^2 - xy \xrightarrow{y} 6y - x \xrightarrow{x} \boxed{-1}$$

Rate of change of $f'_y(x, y)$ w.r.t x

$$6y - x \xrightarrow{y} 6$$

Rate of change of $f'_y(x, y)$ w.r.t y

What Do These Mean?

Rate of change of $f_x(x, y)$ w.r.t x

Rate of change of $f_y(x, y)$ w.r.t y

Rate of change of $f_x(x, y)$ w.r.t y

Rate of change of $f_y(x, y)$ w.r.t x

What Do These Mean?

Rate of change of $f_x(x, y)$ w.r.t x

Rate of change of $f_y(x, y)$ w.r.t y

Change in the change in the function w.r.t tiny changes in x and y

Rate of change of $f_x(x, y)$ w.r.t y

Rate of change of $f_y(x, y)$ w.r.t x

What Do These Mean?

Rate of change of $f_x(x, y)$ w.r.t x

Rate of change of $f_y(x, y)$ w.r.t y

Change in the change in the function w.r.t tiny changes in x and y

Same idea as with one variable!

Rate of change of $f_x(x, y)$ w.r.t y

Rate of change of $f_y(x, y)$ w.r.t x

1. Change in the slope along one coordinate axis w.r.t tiny changes along an orthogonal coordinate axis

What Do These Mean?

Rate of change of $f_x(x, y)$ w.r.t x

Rate of change of $f_y(x, y)$ w.r.t y

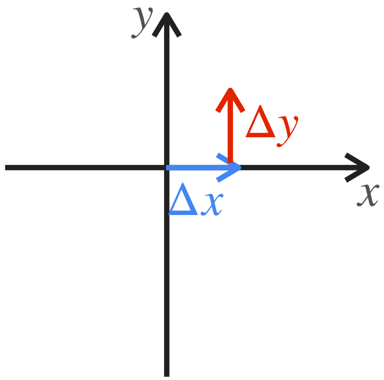
Change in the change in the function w.r.t tiny changes in x and y

Same idea as with one variable!

Rate of change of $f_x(x, y)$ w.r.t y

Rate of change of $f_y(x, y)$ w.r.t x

1. Change in the slope along one coordinate axis w.r.t tiny changes along an orthogonal coordinate axis



What Do These Mean?

Rate of change of $f_x(x, y)$ w.r.t x

Rate of change of $f_y(x, y)$ w.r.t y

Change in the change in the function w.r.t tiny changes in x and y

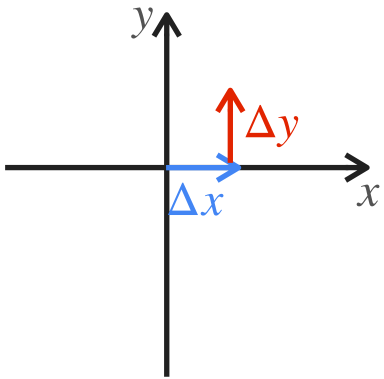
Same idea as with one variable!

Rate of change of $f_x(x, y)$ w.r.t y

Rate of change of $f_y(x, y)$ w.r.t x

1. Change in the slope along one coordinate axis w.r.t tiny changes along an orthogonal coordinate axis

2. They are the same!



What Do These Mean?

Rate of change of $f_x(x, y)$ w.r.t x

Rate of change of $f_y(x, y)$ w.r.t y

Change in the change in the function w.r.t tiny changes in x and y

Same idea as with one variable!

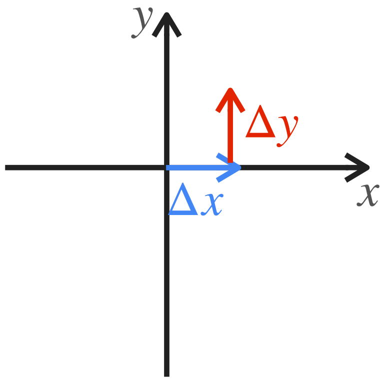
Rate of change of $f_x(x, y)$ w.r.t y

Rate of change of $f_y(x, y)$ w.r.t x

1. Change in the slope along one coordinate axis w.r.t tiny changes along an orthogonal coordinate axis

2. They are the same!

(In most cases)



Notation

Rate of change of
$f'_x(x, y)$ w.r.t x

Rate of change of
$f'_y(x, y)$ w.r.t y

Rate of change of
$f'_x(x, y)$ w.r.t y

Rate of change of
$f'_y(x, y)$ w.r.t x

Notation

Leibniz's notation

Rate of change of $f'_x(x, y)$ w.r.t x

Rate of change of $f'_y(x, y)$ w.r.t y

Rate of change of $f'_x(x, y)$ w.r.t y

Rate of change of $f'_y(x, y)$ w.r.t x

Notation

	Leibniz's notation
Rate of change of $f'_x(x, y)$ w.r.t x	$\frac{\partial^2 f}{\partial x^2}$
Rate of change of $f'_y(x, y)$ w.r.t y	$\frac{\partial^2 f}{\partial y^2}$
Rate of change of $f'_x(x, y)$ w.r.t y	
Rate of change of $f'_y(x, y)$ w.r.t x	

Notation

	Leibniz's notation
Rate of change of $f'_x(x, y)$ w.r.t x	$\dfrac{\partial^2 f}{\partial x^2}$
Rate of change of $f'_y(x, y)$ w.r.t y	$\dfrac{\partial^2 f}{\partial y^2}$
Rate of change of $f'_x(x, y)$ w.r.t y	$\dfrac{\partial^2 f}{\partial x \partial y}$
Rate of change of $f'_y(x, y)$ w.r.t x	$\dfrac{\partial^2 f}{\partial y \partial x}$

Notation

	Leibniz's notation	Lagrange's notation
Rate of change of $f'_x(x, y)$ w.r.t x	$\frac{\partial^2 f}{\partial x^2}$	
Rate of change of $f'_y(x, y)$ w.r.t y	$\frac{\partial^2 f}{\partial y^2}$	
Rate of change of $f'_x(x, y)$ w.r.t y	$\frac{\partial^2 f}{\partial x \partial y}$	
Rate of change of $f'_y(x, y)$ w.r.t x	$\frac{\partial^2 f}{\partial y \partial x}$	

Notation

	Leibniz's notation	Lagrange's notation
Rate of change of $f'_x(x, y)$ w.r.t x	$\frac{\partial^2 f}{\partial x^2}$	$f_{xx}(x, y)$
Rate of change of $f'_y(x, y)$ w.r.t y	$\frac{\partial^2 f}{\partial y^2}$	$f_{yy}(x, y)$
Rate of change of $f'_x(x, y)$ w.r.t y	$\frac{\partial^2 f}{\partial x \partial y}$	
Rate of change of $f'_y(x, y)$ w.r.t x	$\frac{\partial^2 f}{\partial y \partial x}$	

Notation

	Leibniz's notation	Lagrange's notation
Rate of change of $f'_x(x, y)$ w.r.t x	$\dfrac{\partial^2 f}{\partial x^2}$	$f_{xx}(x, y)$
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Rate of change of $f'_x(x, y)$ w.r.t y	$\dfrac{\partial^2 f}{\partial x \partial y}$	$f_{xy}(x, y)$
Rate of change of $f'_y(x, y)$ w.r.t x	$\dfrac{\partial^2 f}{\partial y \partial x}$	$f_{yx}(x, y)$

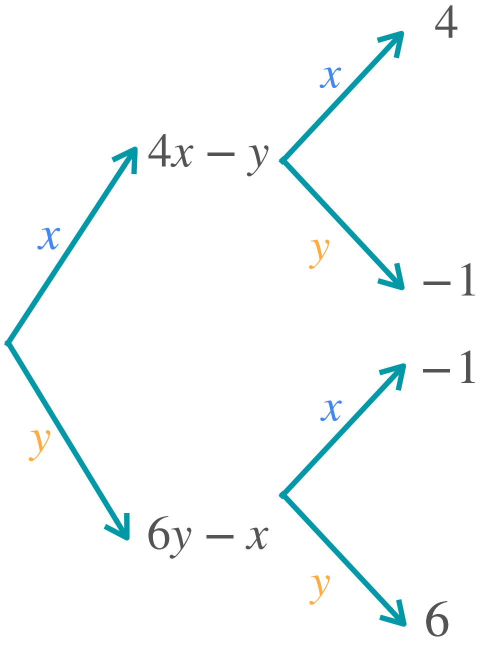
Hessian Matrix

Hessian Matrix

$$
\begin{tikzcd}
 & & 4 \\
 & 4x - y \arrow[ur, "x"] \arrow[dr, "y"'] & \\
 & & -1 \\
f(x, y) = 2x^2 + 3y^2 - xy \arrow[uur, "x"] \arrow[ddr, "y"'] & & \\
 & & -1 \\
 & 6y - x \arrow[ur, "x"] \arrow[dr, "y"'] & \\
 & & 6
\end{tikzcd}
$$

Hessian Matrix

$$f(x, y) =$$
$$2x^2 + 3y^2 - xy$$



$$\begin{matrix} 4 \\ -1 \end{matrix}$$

Hessian Matrix

$$
\begin{tikzcd}
 & & 4 \\
 & 4x - y \arrow[ur, "x"] \arrow[dr, "y"'] & \\
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 & & 6
\end{tikzcd}
$$

$$\nabla f_x = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

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Hessian Matrix

$f(x, y) =$

$$
\begin{tikzcd}
 & & 4 \\
 & 4x - y \arrow[ur, "x"] \arrow[dr, "y"'] & \\
 & & -1 \\
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Hessian Matrix

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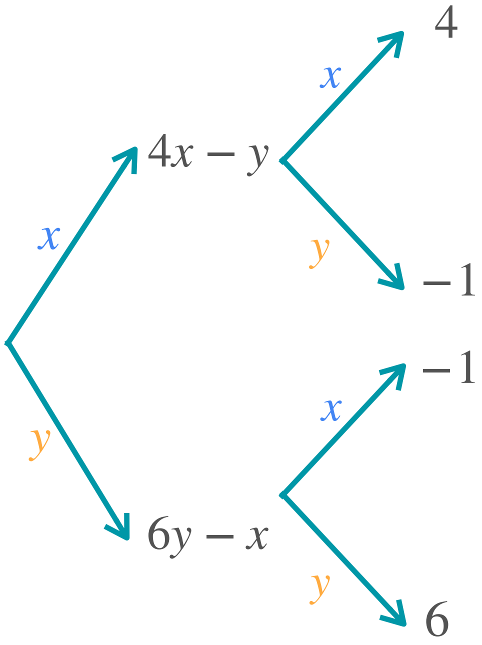

$$\nabla f_x = \begin{bmatrix} 4 \\ -1 \end{bmatrix} \qquad \nabla f_y = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$$

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\begin{tikzcd}
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$$\nabla f_x = \begin{bmatrix} 4 \\ -1 \end{bmatrix} \qquad \nabla f_y = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$$

$$\begin{matrix} 4 & -1 \\ -1 & 6 \end{matrix}$$

Hessian Matrix

$f(x, y) =$

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 & & 4 \\
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$$\nabla f_x = \begin{bmatrix} 4 \\ -1 \end{bmatrix} \qquad \nabla f_y = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -1 \\ -1 & 6 \end{bmatrix} = \begin{bmatrix} \nabla {f_x}^T \\ \nabla {f_y}^T \end{bmatrix}$$

Hessian Matrix

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\begin{tikzcd}
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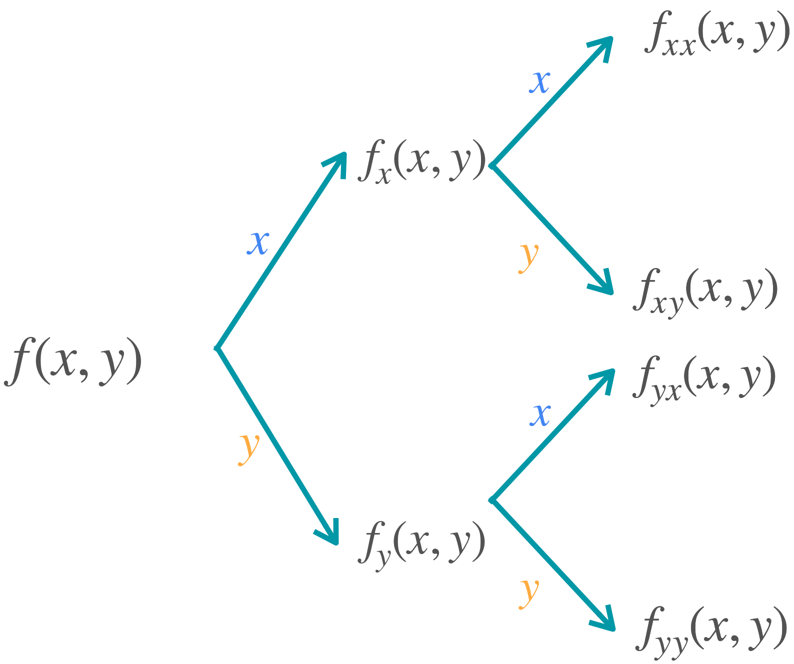
Hessian matrix

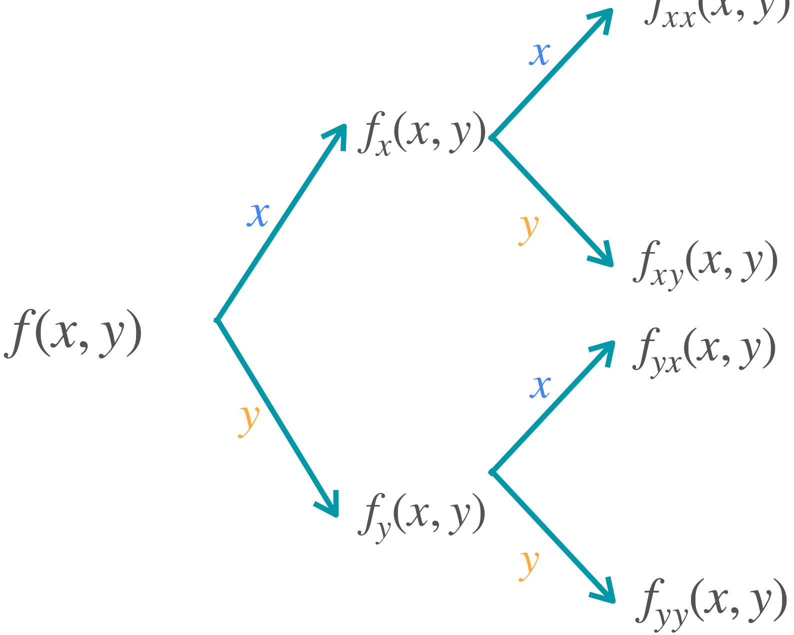
All information about second derivatives

Hessian Matrix - General Case

Hessian Matrix - General Case



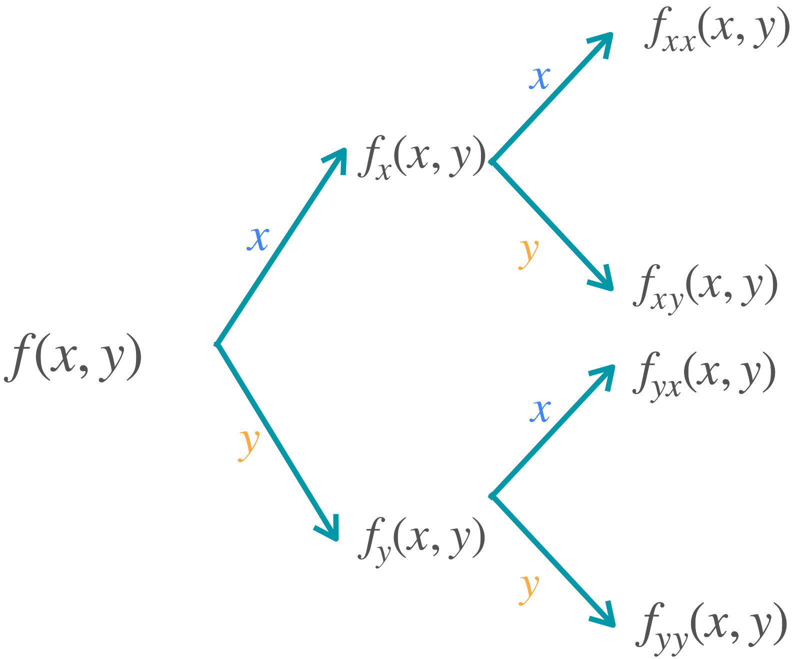
Hessian Matrix - General Case



$f_{xx}(x, y)$

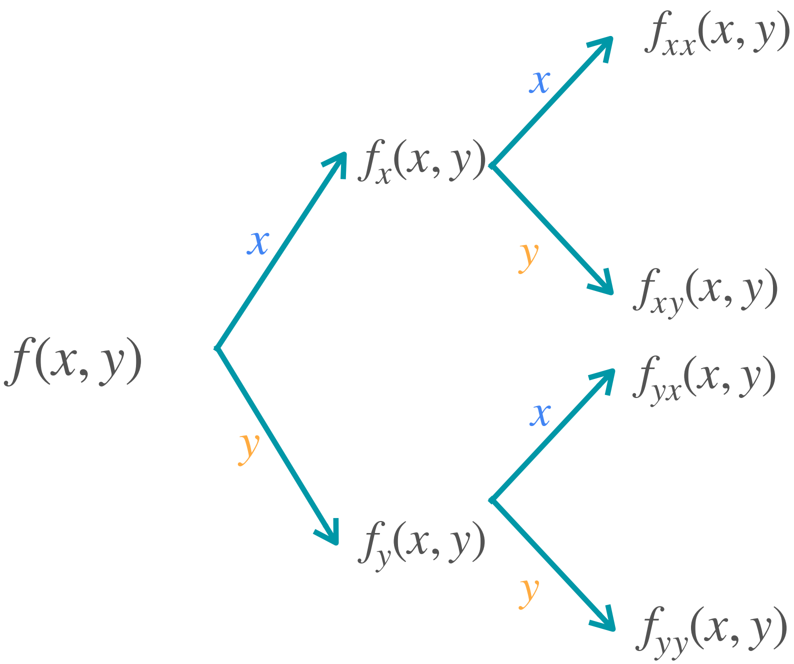
$f_{xy}(x, y)$

Hessian Matrix - General Case



$$\nabla f_x = \begin{bmatrix} f_{xx}(x, y) \\ f_{xy}(x, y) \end{bmatrix}$$

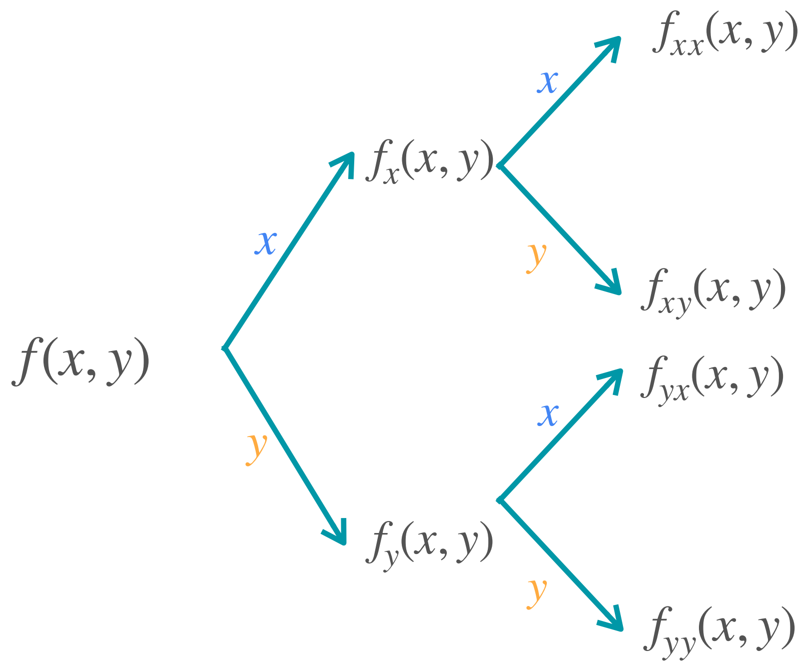
Hessian Matrix - General Case



$$\nabla f_x = \begin{bmatrix} f_{xx}(x, y) \\ f_{xy}(x, y) \end{bmatrix}$$

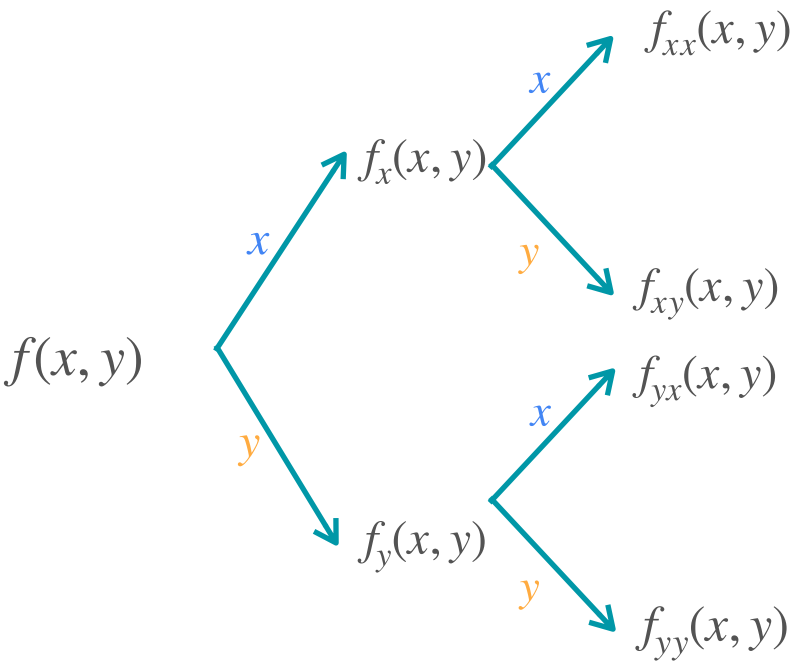
$$\begin{matrix} f_{yx}(x, y) \\ f_{yy}(x, y) \end{matrix}$$

Hessian Matrix - General Case



$$\nabla f_x = \begin{bmatrix} f_{xx}(x, y) \\ f_{xy}(x, y) \end{bmatrix} \qquad \nabla f_y' = \begin{bmatrix} f_{yx}(x, y) \\ f_{yy}(x, y) \end{bmatrix}$$

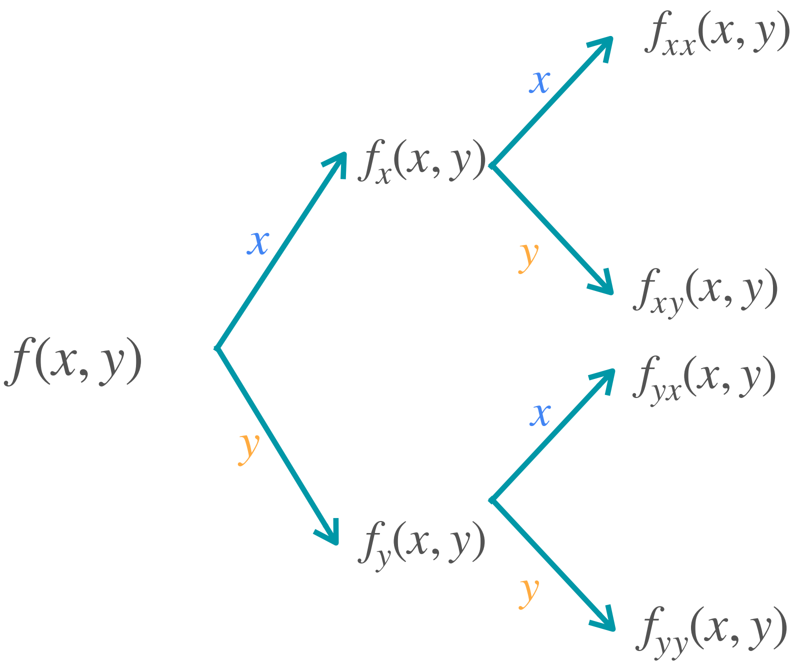
Hessian Matrix - General Case



$$\nabla f_x = \begin{bmatrix} f_{xx}(x, y) \\ f_{xy}(x, y) \end{bmatrix} \qquad \nabla f'_y = \begin{bmatrix} f_{yx}(x, y) \\ f_{yy}(x, y) \end{bmatrix}$$

$$\begin{matrix} f_{xx}(x, y) & f_{xy}(x, y) \\ f_{yx}(x, y) & f_{yy}(x, y) \end{matrix}$$

Hessian Matrix - General Case



$$\nabla f_x = \begin{bmatrix} f_{xx}(x, y) \\ f_{xy}(x, y) \end{bmatrix} \qquad \nabla f_y' = \begin{bmatrix} f_{yx}(x, y) \\ f_{yy}(x, y) \end{bmatrix}$$

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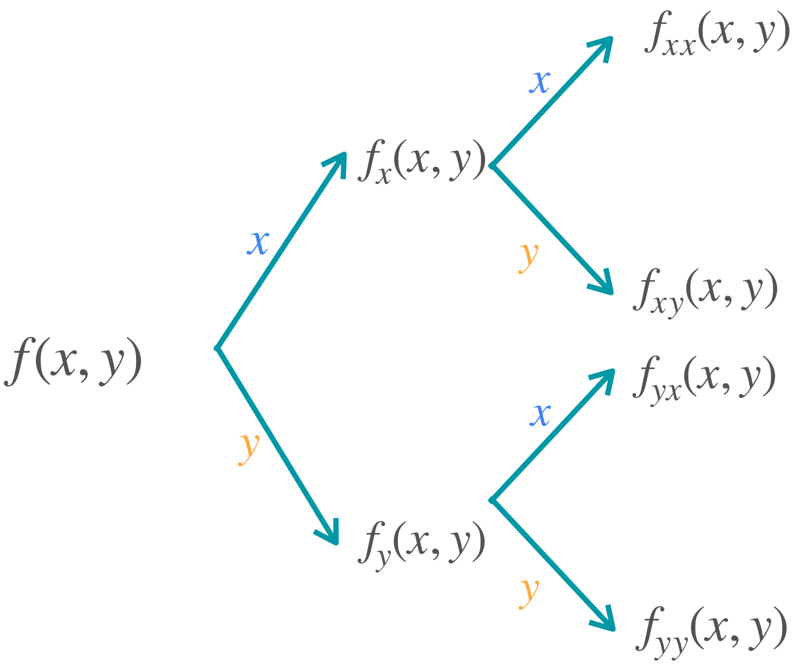
Hessian Matrix - General Case



$$\nabla f_x = \begin{bmatrix} f_{xx}(x, y) \\ f_{xy}(x, y) \end{bmatrix} \qquad \nabla f'_y = \begin{bmatrix} f_{yx}(x, y) \\ f_{yy}(x, y) \end{bmatrix}$$

$$H = \begin{bmatrix} f_{xx}(x, y) & f_{xy}(x, y) \\ f_{yx}(x, y) & f_{yy}(x, y) \end{bmatrix} = \begin{bmatrix} \nabla {f_x}^T \\ \nabla {f_y}^T \end{bmatrix}$$

Hessian matrix

All information about second derivatives

Second Derivative

Second Derivative

	1 variable		2 variables	
Function	$f(x)$		$f(x,y)$	
First derivative	$f'(x)$	Rate of change of $f(x)$	$f_x(x,y)$	Rate of change w.r.t x
			$f_y(x,y)$	Rate of change w.r.t y
			$\nabla f = \begin{bmatrix} f_x(x,y) \\ f_y(x,y) \end{bmatrix}$	
Second derivative	$f''(x)$	Rate of change of the rate of change of $f(x)$		

Second Derivative

	1 variable		2 variables	
Function	$f(x)$		$f(x, y)$	
First derivative	$f'(x)$	Rate of change of $f(x)$	$f_x(x, y)$	Rate of change w.r.t x
			$f_y(x, y)$	Rate of change w.r.t y
			$\nabla f = \begin{bmatrix} f_x(x, y) \\ f_y(x, y) \end{bmatrix}$	
Second derivative	$f''(x)$	Rate of change of the rate of change of $f(x)$	$H(x, y) = \begin{bmatrix} f_{xx}(x, y) & f_{xy}(x, y) \\ f_{yx}(x, y) & f_{yy}(x, y) \end{bmatrix}$	

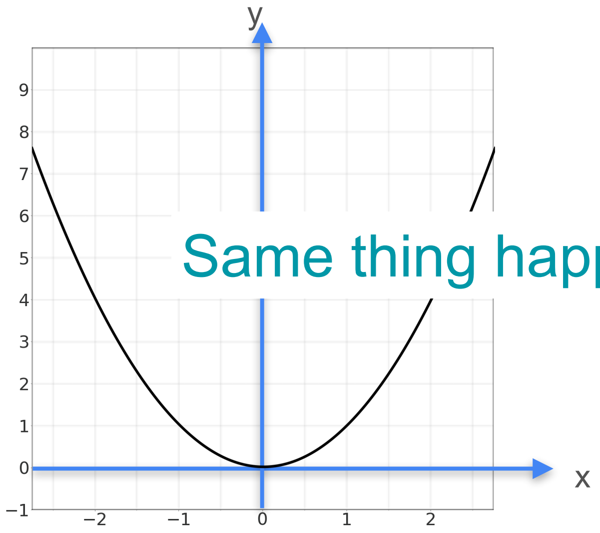
Optimization in Neural Networks and Newton's Method

Hessians and concavity

Remember…

Same thing happens for many variables!

Remember…

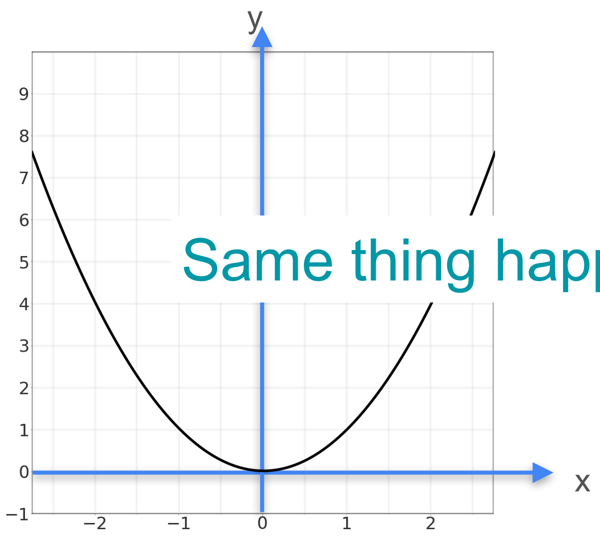


Same thing happens for many variables!

Concave up or convex

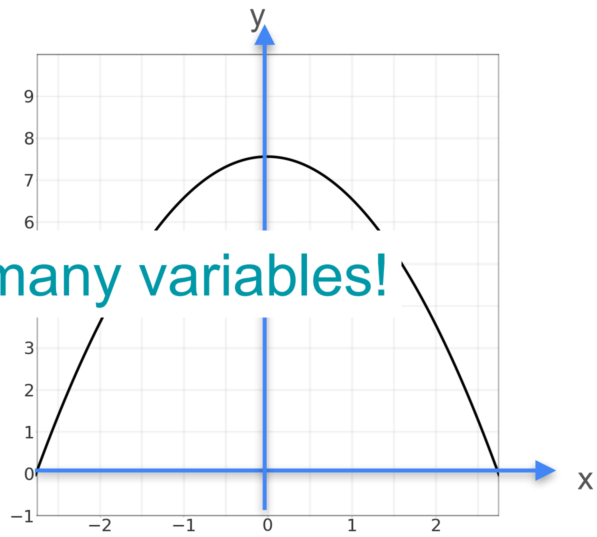
$$f''(0) > 0$$

Remember…



Concave up or convex

$$f''(0) > 0$$



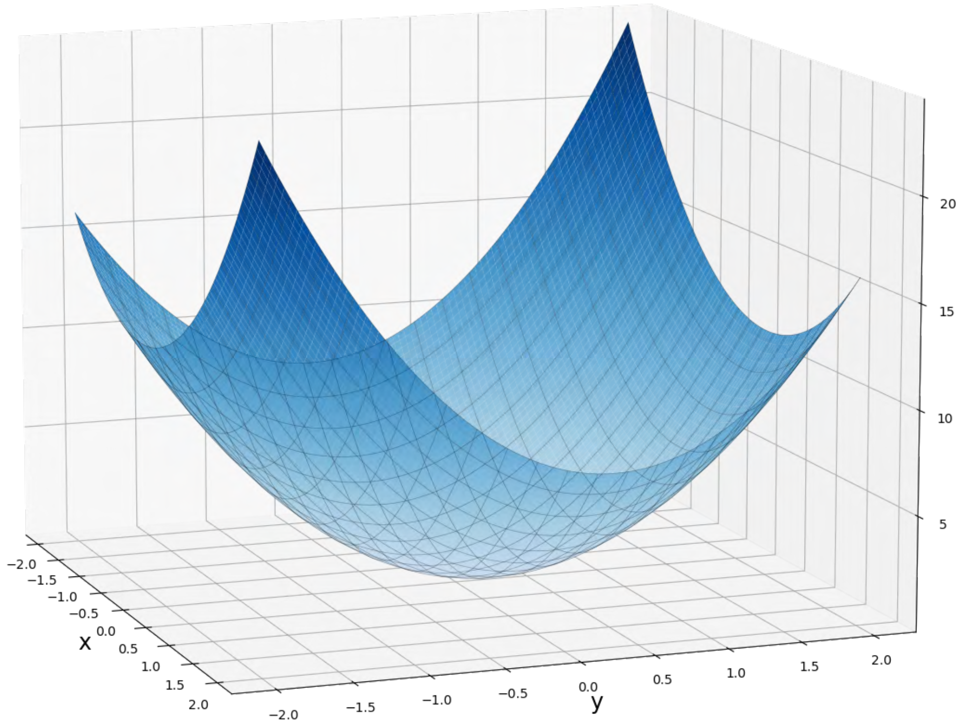
Concave down

$$f''(0) < 0$$

Same thing happens for many variables!

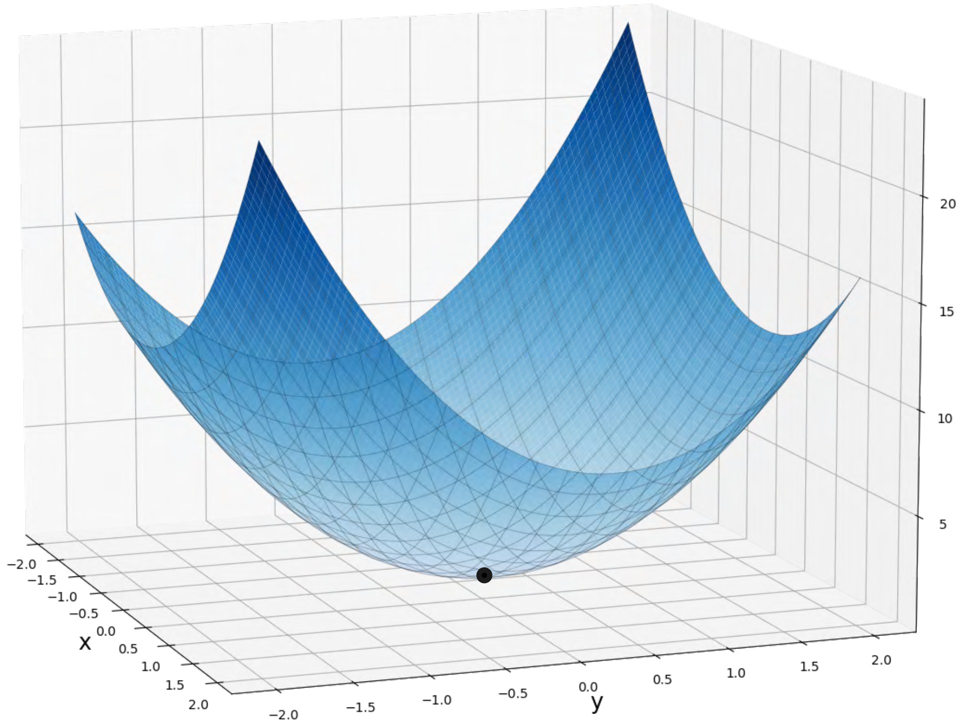
Concave Up

Concave Up



$$f(x, y) = 2x^2 + 3y^2 - xy$$

Concave Up



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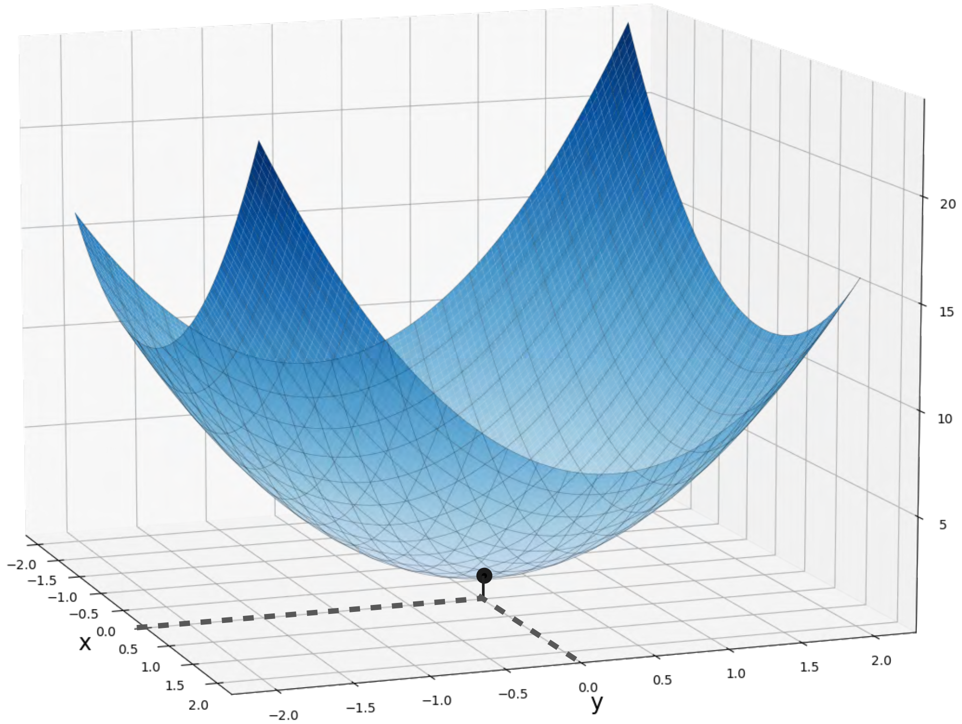
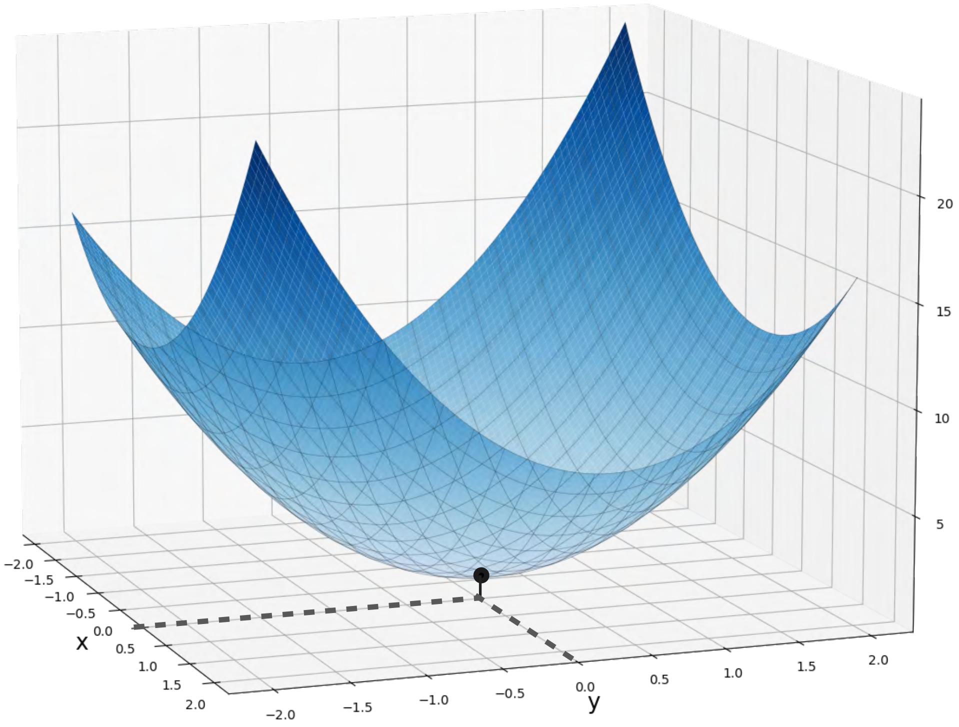
Concave Up



$$f(x, y) = 2x^2 + 3y^2 - xy$$

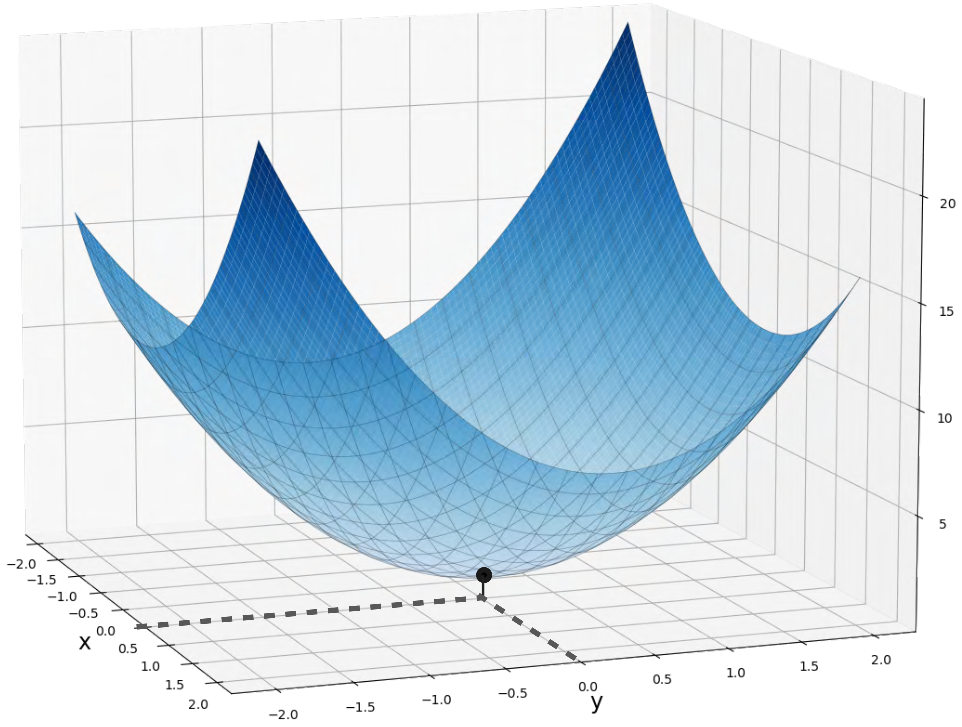
Concave Up



$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$H(0,0) = \begin{bmatrix} 4 & -1 \\ -1 & 6 \end{bmatrix}$$

Concave Up



$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$H(0,0) = \begin{bmatrix} 4 & -1 \\ -1 & 6 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) =$$

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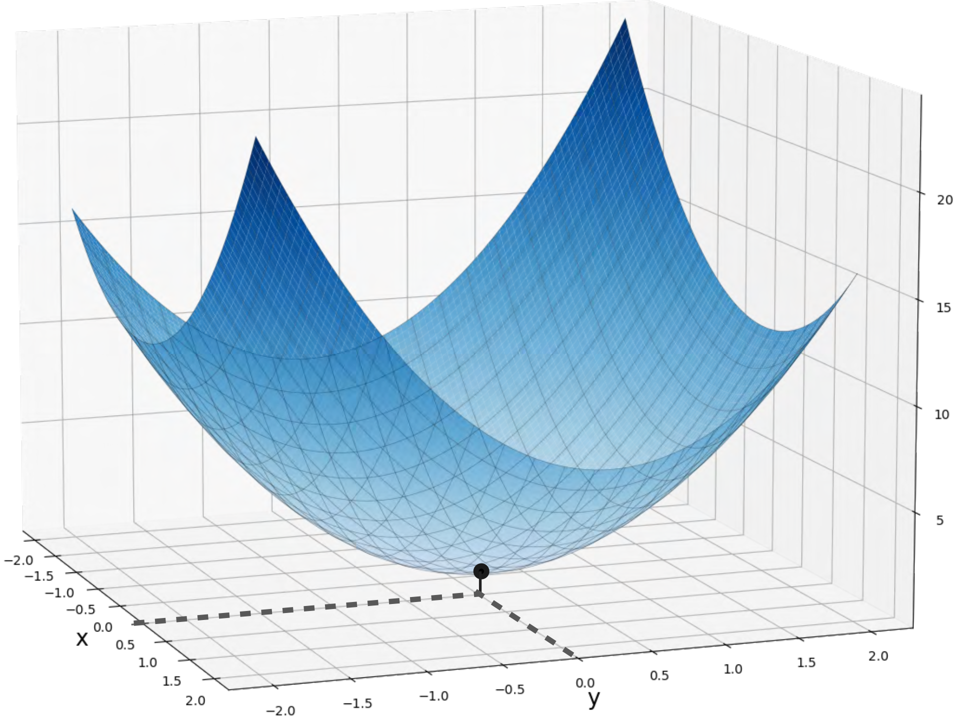
Concave Up



$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$H(0,0) = \begin{bmatrix} 4 & -1 \\ -1 & 6 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) = \det\left(\begin{bmatrix} 4-\lambda & -1 \\ -1 & 6-\lambda \end{bmatrix}\right)$$

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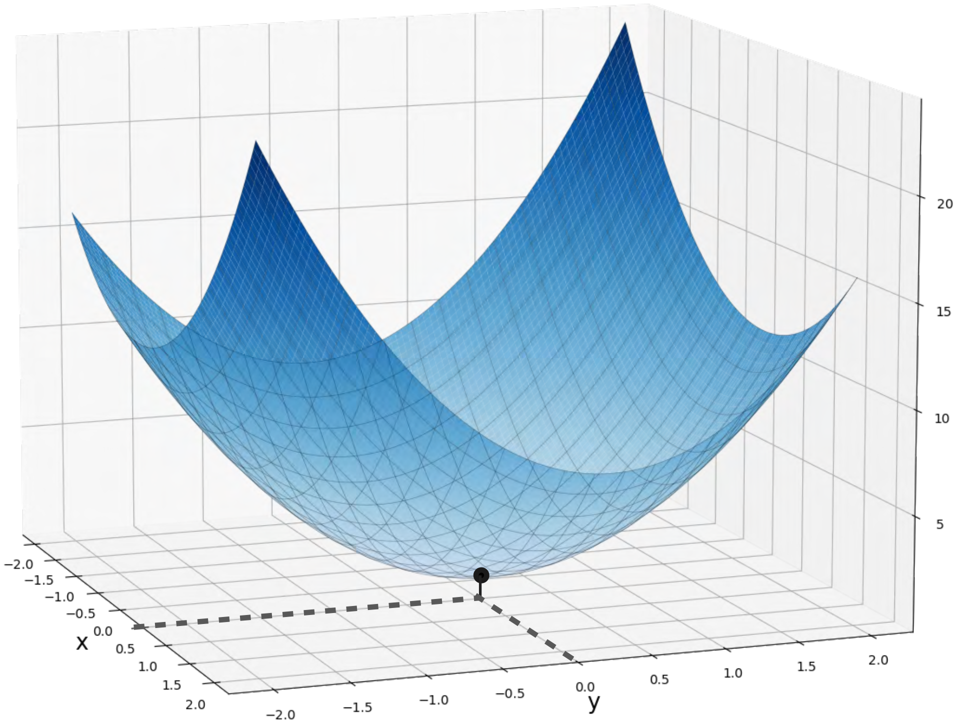
Concave Up



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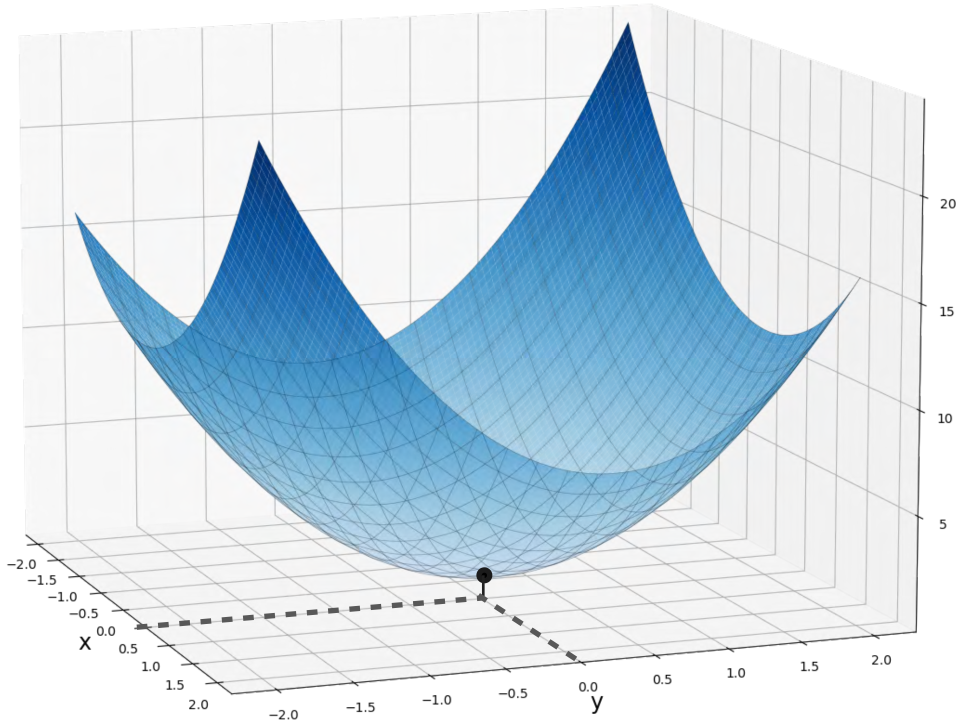
$$\det(H(0,0) - \lambda I) = \det\left(\begin{bmatrix} 4-\lambda & -1 \\ -1 & 6-\lambda \end{bmatrix}\right)$$

$$= (4-\lambda)(6-\lambda) - (-1)(-1)$$

Concave Up



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$$\det(H(0,0) - \lambda I) = \det\left(\begin{bmatrix} 4-\lambda & -1 \\ -1 & 6-\lambda \end{bmatrix}\right)$$

$$= (4-\lambda)(6-\lambda) - (-1)(-1)$$

$$= \lambda^2 - 10\lambda + 23$$

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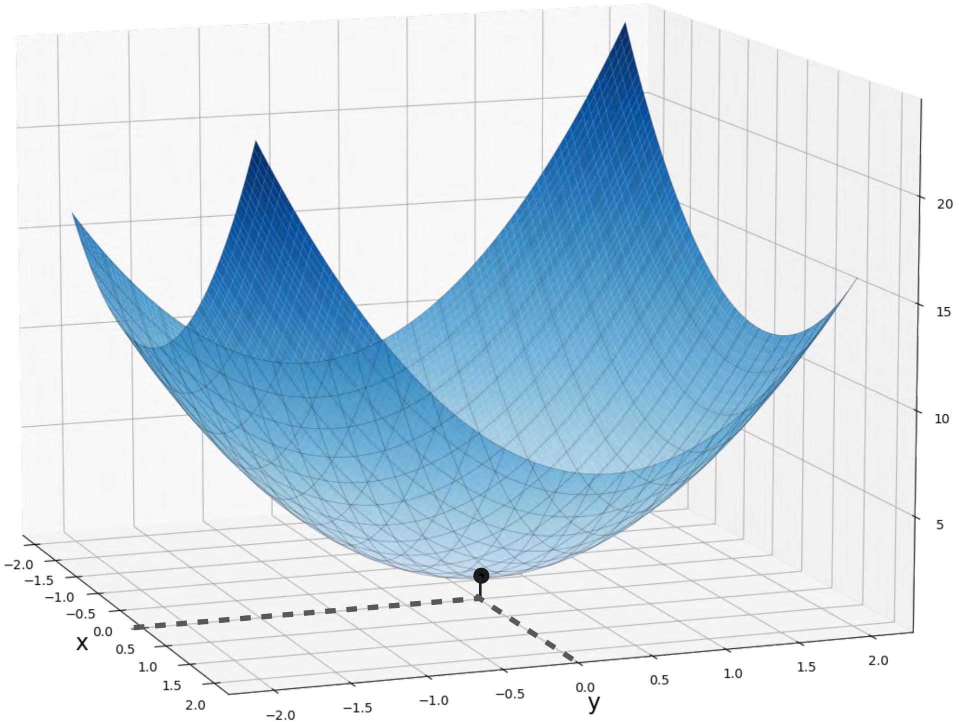
Concave Up



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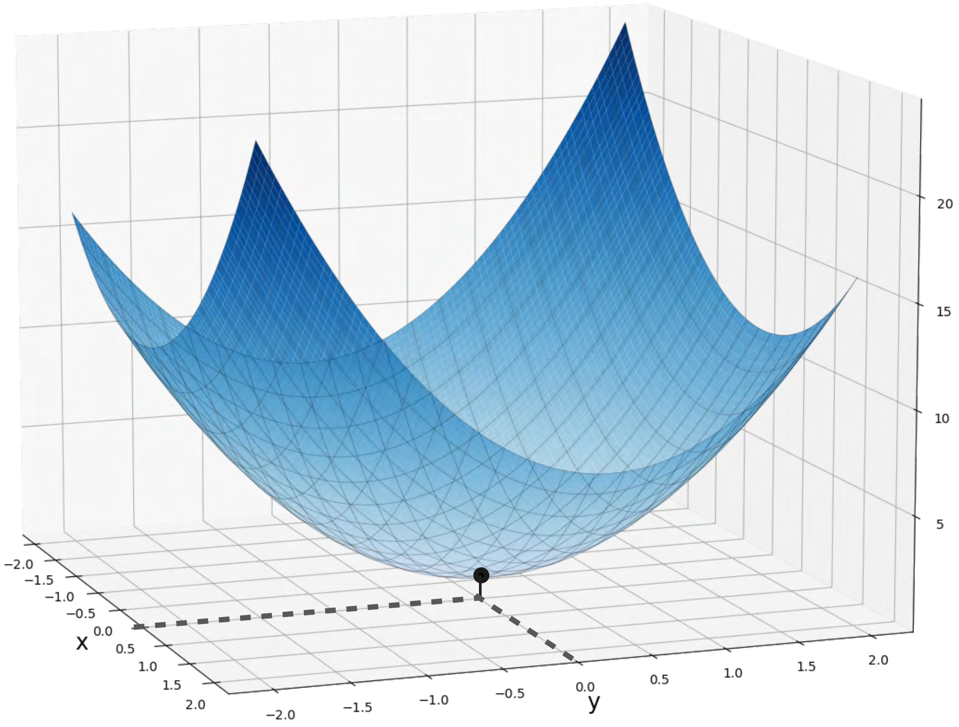
$$= (4-\lambda)(6-\lambda) - (-1)(-1)$$

$$= \lambda^2 - 10\lambda + 23 \rightarrow \lambda_1 = 6.41$$

Concave Up



$$f(x,y) = 2x^2 + 3y^2 - xy$$

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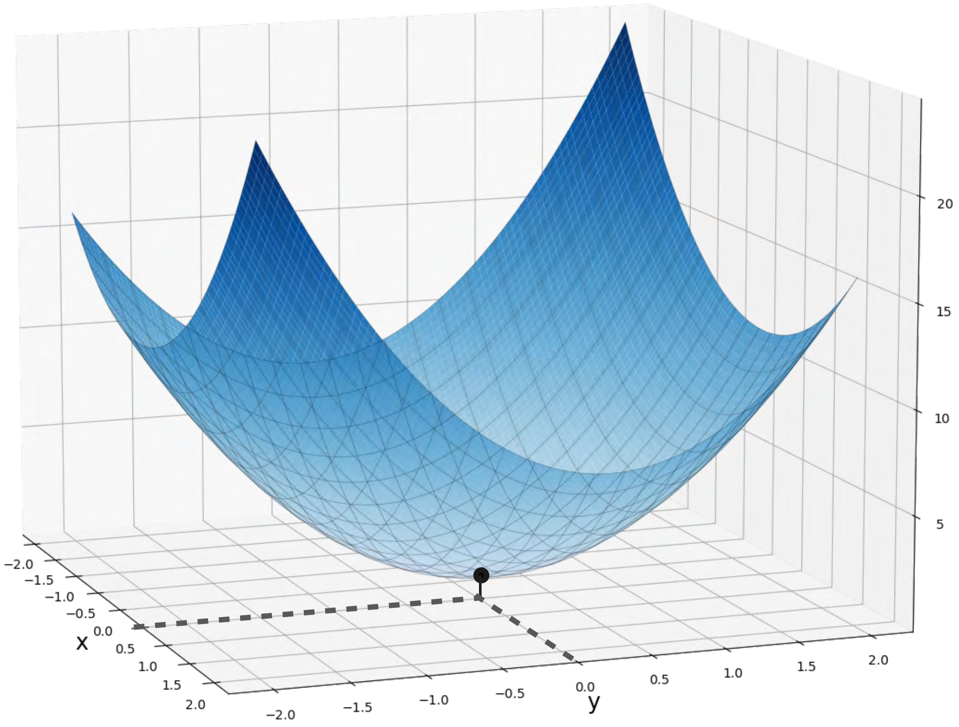
$$= (4-\lambda)(6-\lambda) - (-1)(-1)$$

$$= \lambda^2 - 10\lambda + 23$$

$$\lambda_1 = 6.41$$

$$\lambda_2 = 3.59$$

Concave Up



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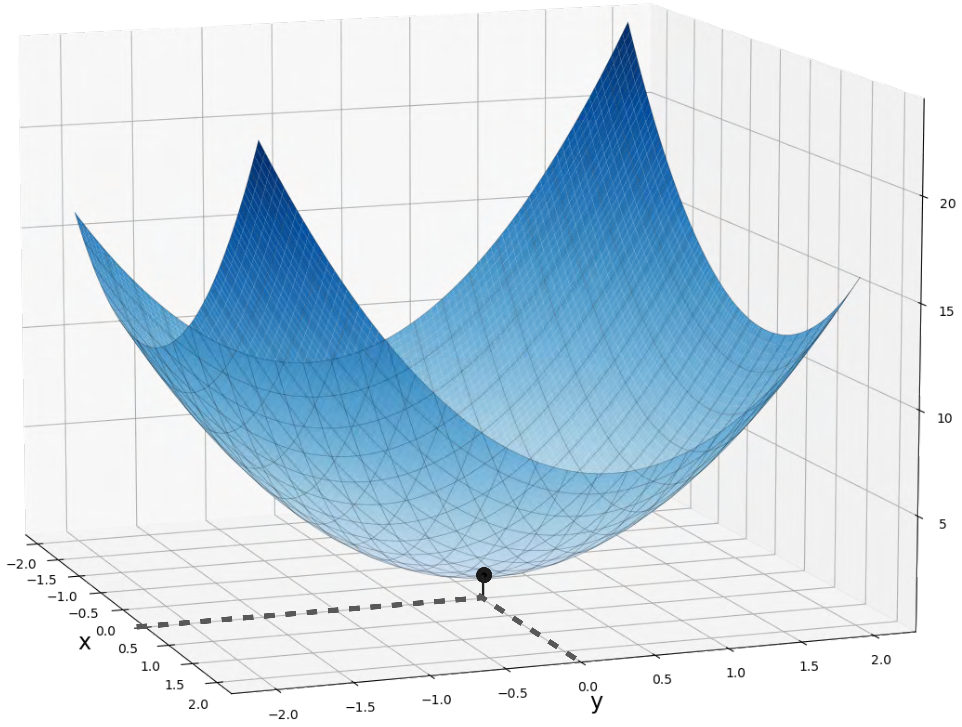
$$= (4-\lambda)(6-\lambda) - (-1)(-1)$$

$$= \lambda^2 - 10\lambda + 23$$

$$\boxed{\begin{matrix}\lambda_1 = 6.41 \\ \lambda_2 = 3.59\end{matrix}}$$

$$> 0$$

Concave Up



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$$= (4-\lambda)(6-\lambda) - (-1)(-1)$$

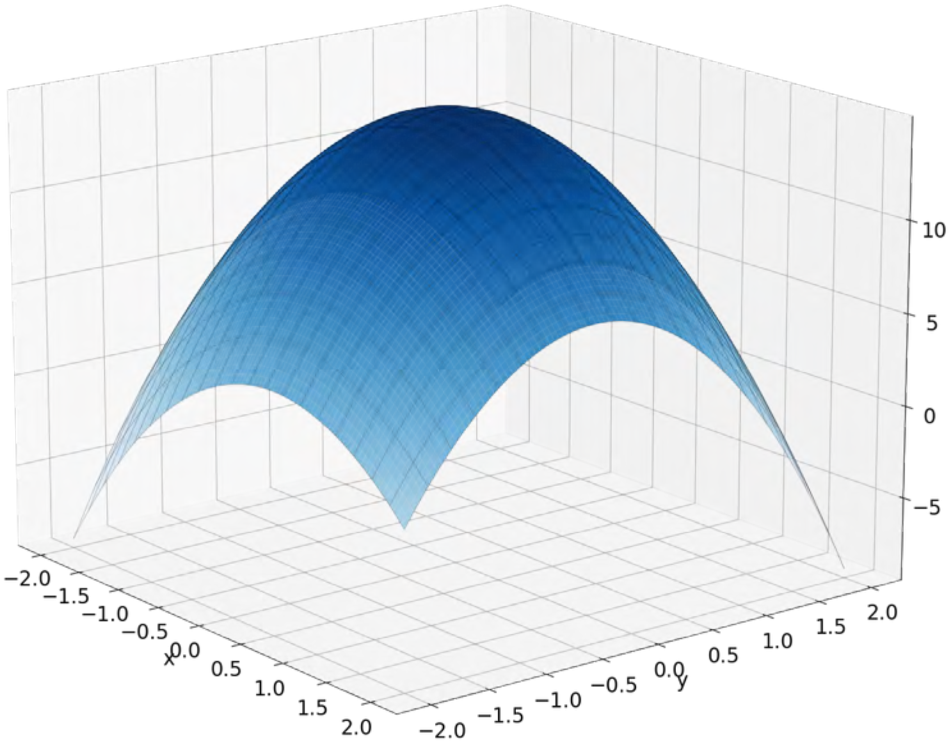
$$= \lambda^2 - 10\lambda + 23 \rightarrow \boxed{\begin{matrix} \lambda_1 = 6.41 \\ \lambda_2 = 3.59 \end{matrix}}$$

> 0

$(0,0)$ is a minimum!

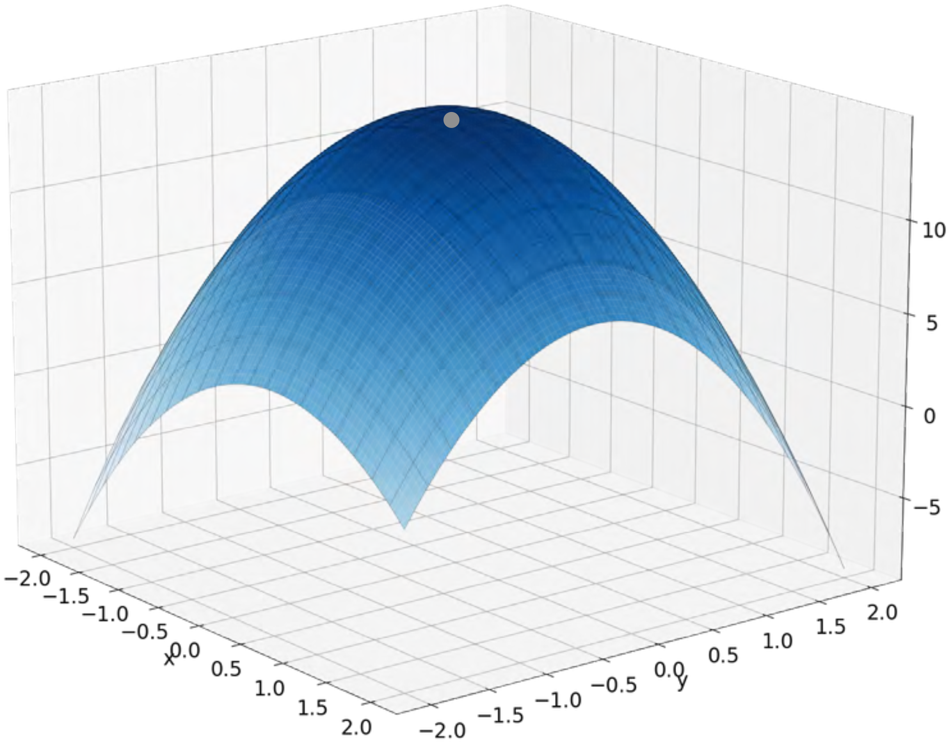
Concave Down

Concave Down



$$f(x, y) = -2x^2 + -3y^2 - xy + 15$$

Concave Down



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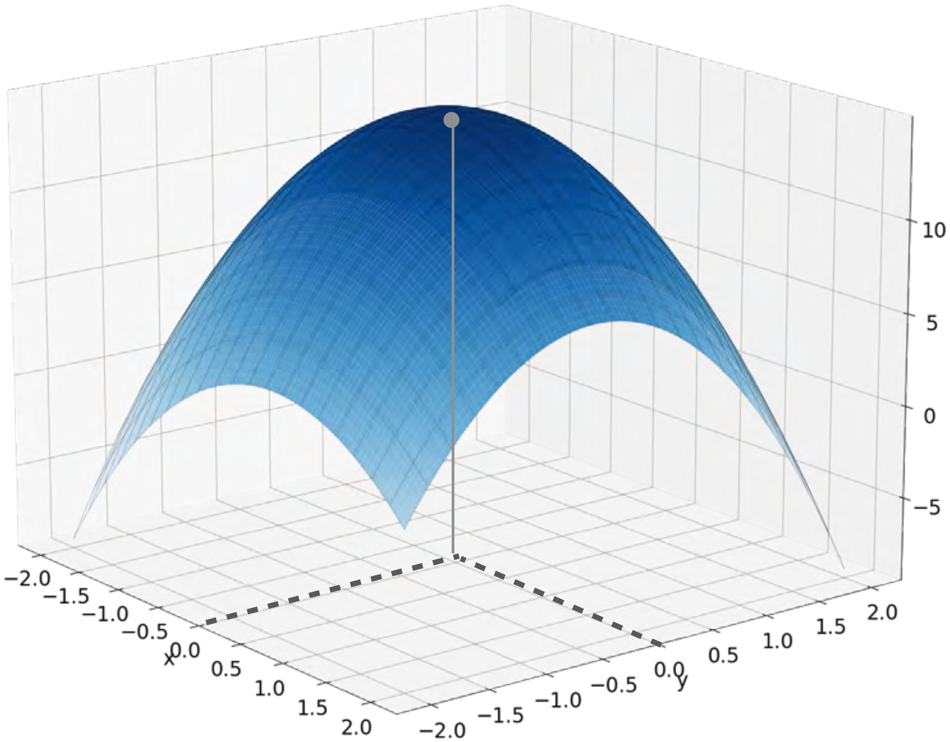
Concave Down



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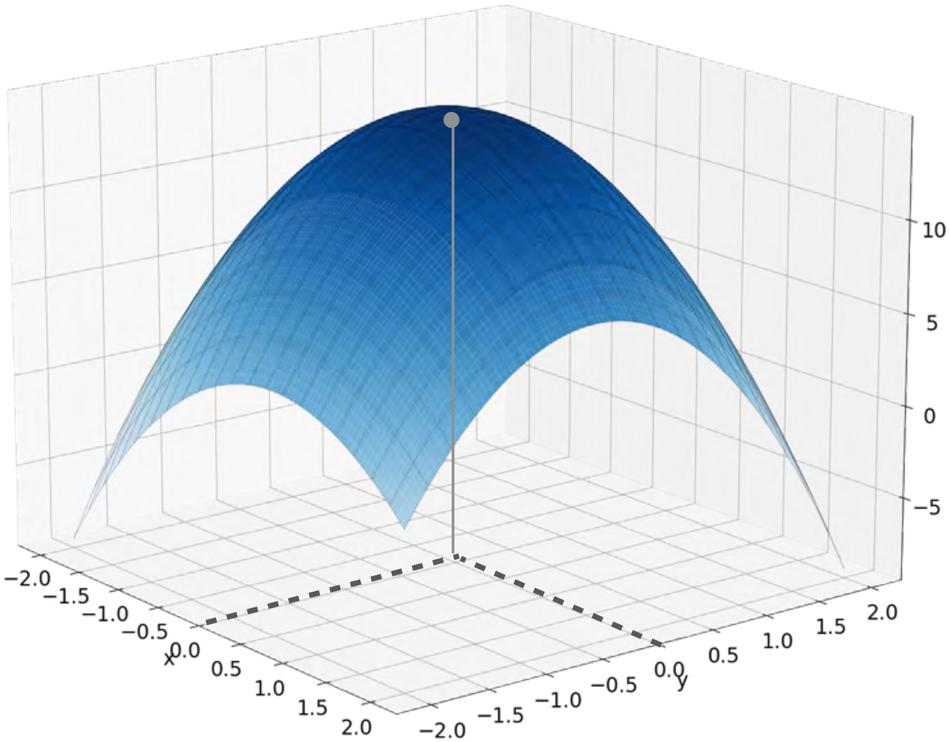
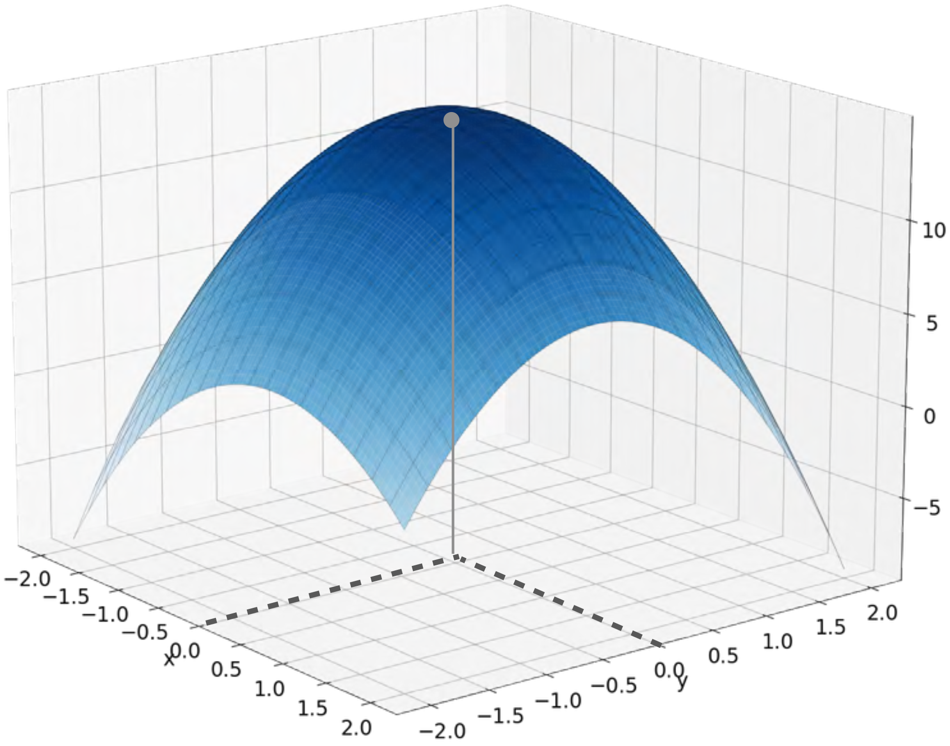
Concave Down



$$f(x, y) = -2x^2 + -3y^2 - xy + 15$$

$$\nabla f(x, y) = \begin{bmatrix} -4x - y \\ -x - 6y \end{bmatrix}$$

Concave Down



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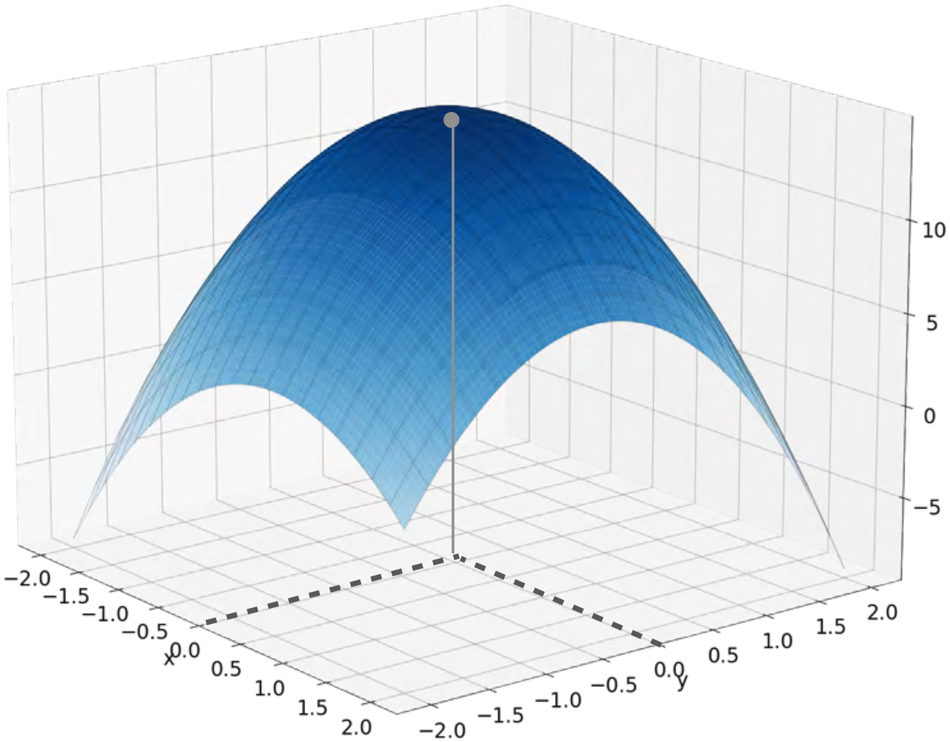
Concave Down



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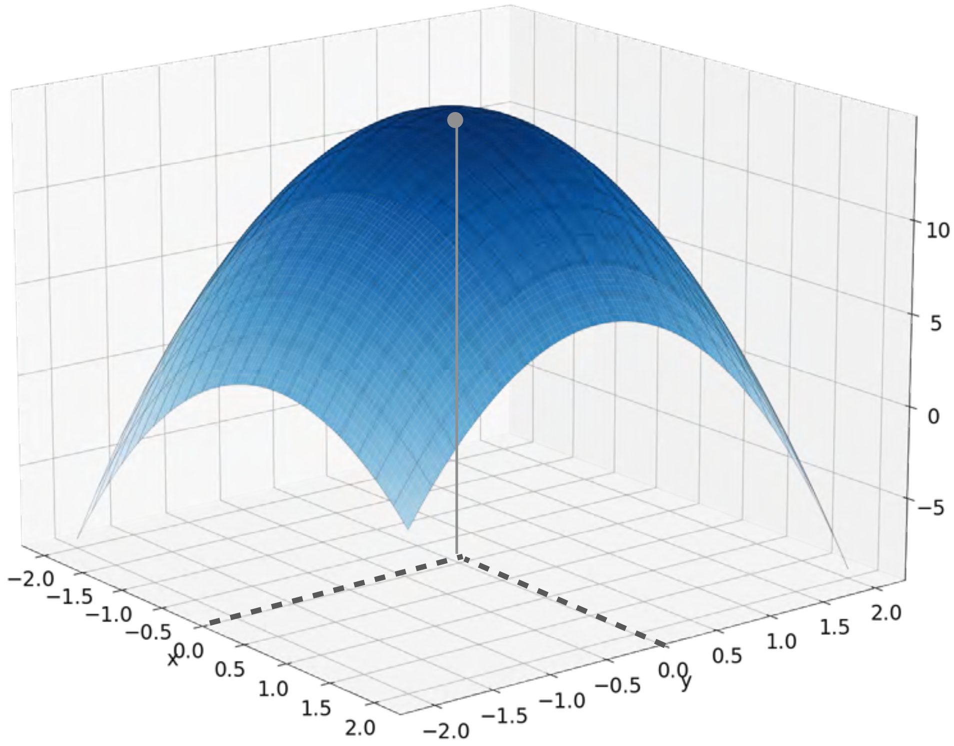
$$\det(H(0,0) - \lambda I) =$$

Concave Down



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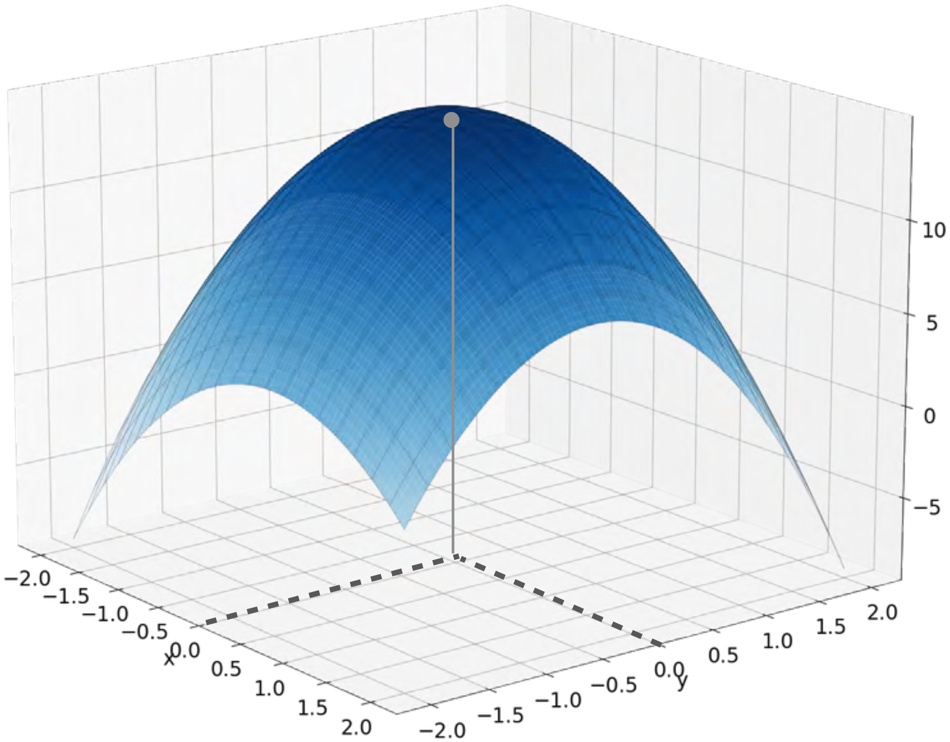
$$\det(H(0,0) - \lambda I) =$$

$$(-4 - \lambda)(-6 - \lambda) - (-1)(-1)$$

Concave Down



$$f(x, y) = -2x^2 + -3y^2 - xy + 15$$

$$\nabla f(x, y) = \begin{bmatrix} -4x - y \\ -x - 6y \end{bmatrix}$$

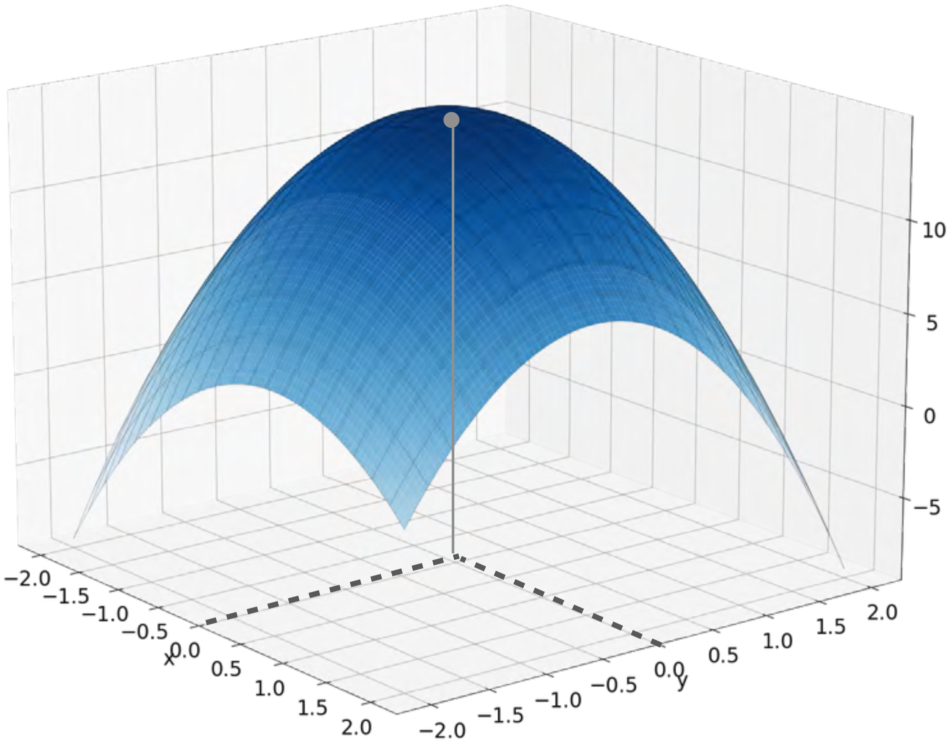
$$H(0,0) = \begin{bmatrix} -4 & -1 \\ -1 & -6 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) =$$

$$(-4 - \lambda)(-6 - \lambda) - (-1)(-1)$$

$$= \lambda^2 + 10\lambda + 23$$

Concave Down



$$f(x,y) = -2x^2 + -3y^2 - xy + 15$$

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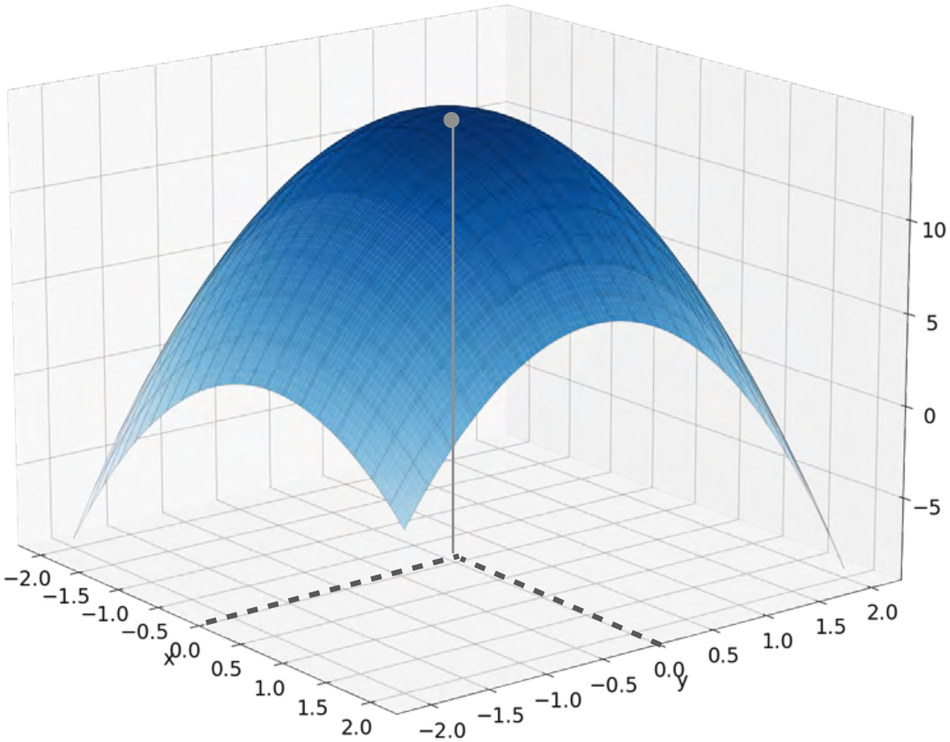
$$\det(H(0,0) - \lambda I) =$$

$$(-4-\lambda)(-6-\lambda) - (-1)(-1)$$

$$= \lambda^2 + 10\lambda + 23 \rightarrow \lambda_1 = -3.49$$

Concave Down



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$$\det(H(0,0) - \lambda I) =$$

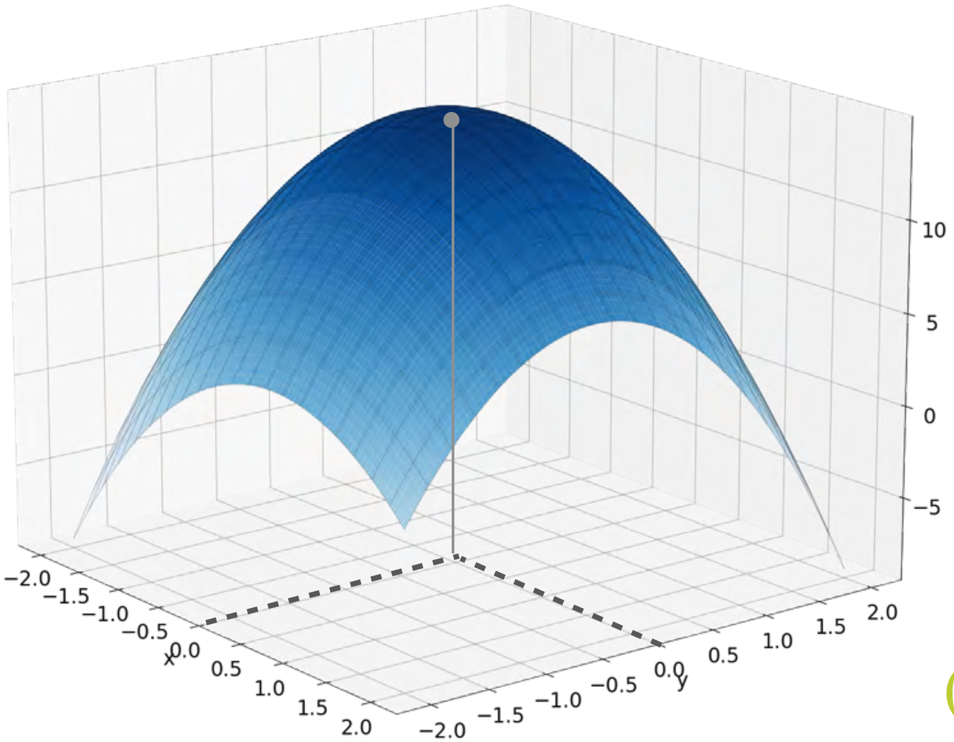
$$(-4 - \lambda)(-6 - \lambda) - (-1)(-1)$$

$$= \lambda^2 + 10\lambda + 23$$

$$\lambda_1 = -3.49$$

$$\lambda_2 = -6.41$$

Concave Down



$$f(x,y) = -2x^2 + -3y^2 - xy + 15$$

$$\nabla f(x,y) = \begin{bmatrix} -4x - y \\ -x - 6y \end{bmatrix}$$

$$H(0,0) = \begin{bmatrix} -4 & -1 \\ -1 & -6 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) =$$

$$(-4-\lambda)(-6-\lambda) - (-1)(-1)$$

$$= \lambda^2 + 10\lambda + 23$$

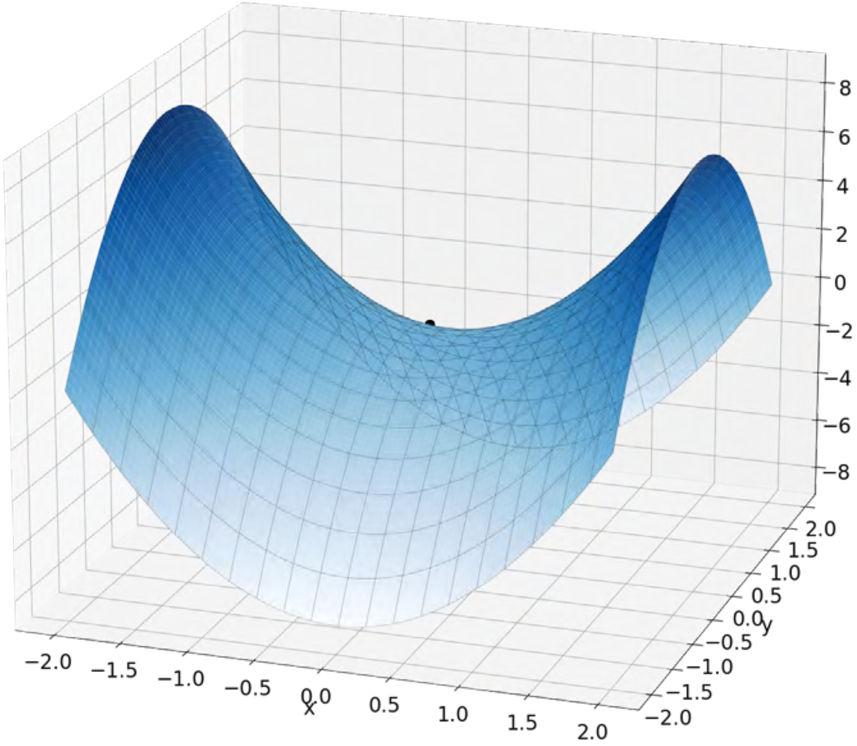
$$\boxed{\begin{aligned}\lambda_1 &= -3.49 \\ \lambda_2 &= -6.41\end{aligned}}$$

< 0

$(0,0)$ is a maximum!

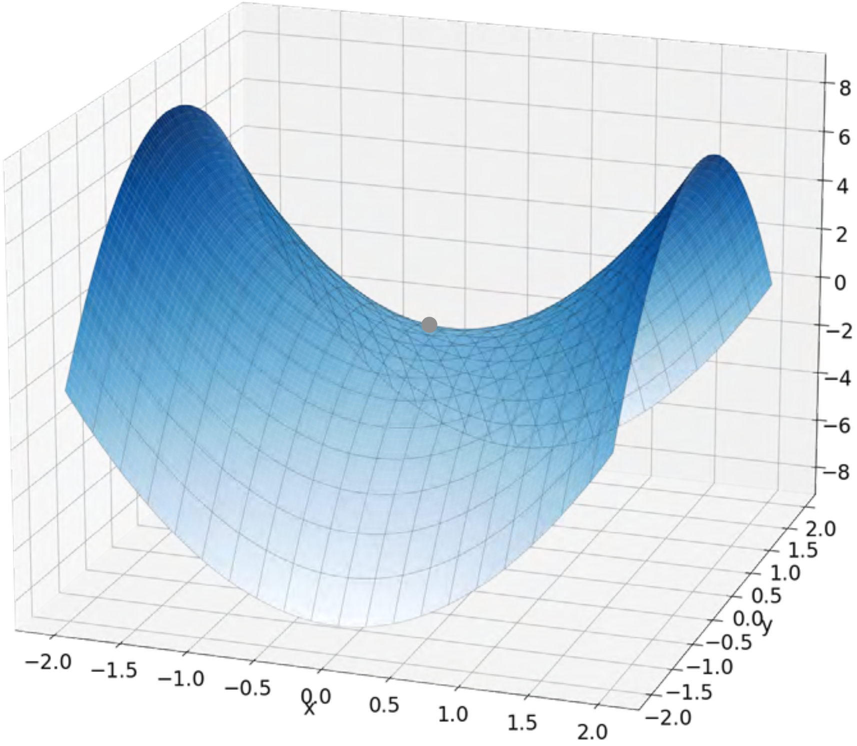
Saddle Point

Saddle Point



$$f(x, y) = 2x^2 - 2y^2$$

Saddle Point



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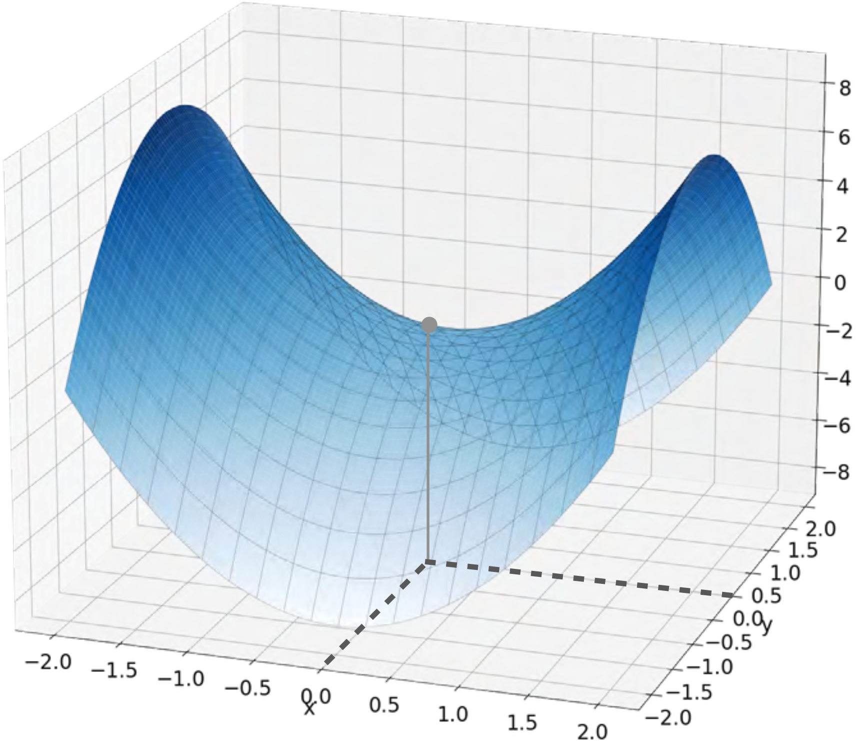
Saddle Point



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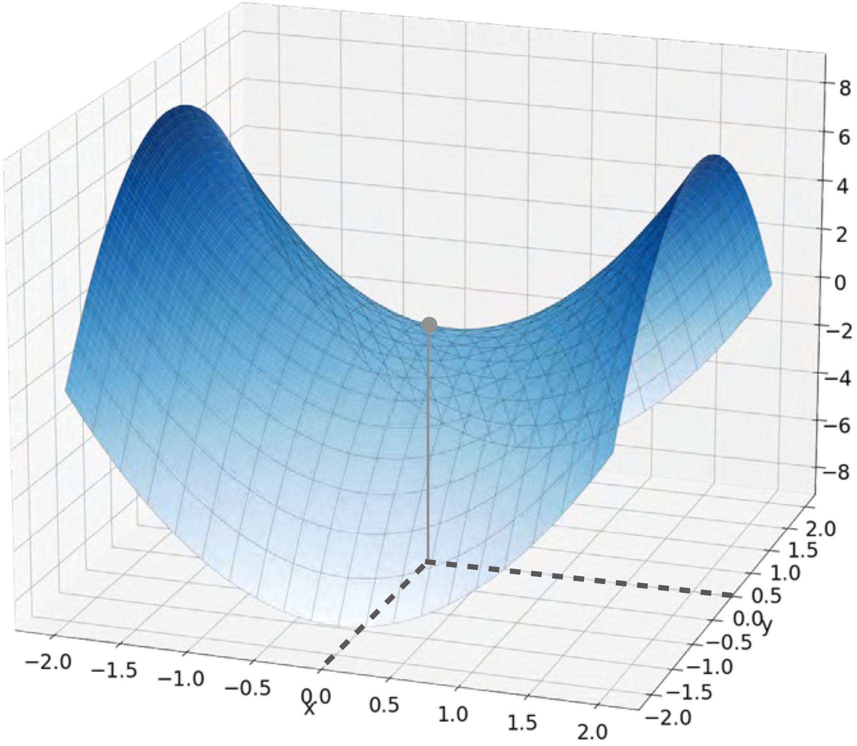
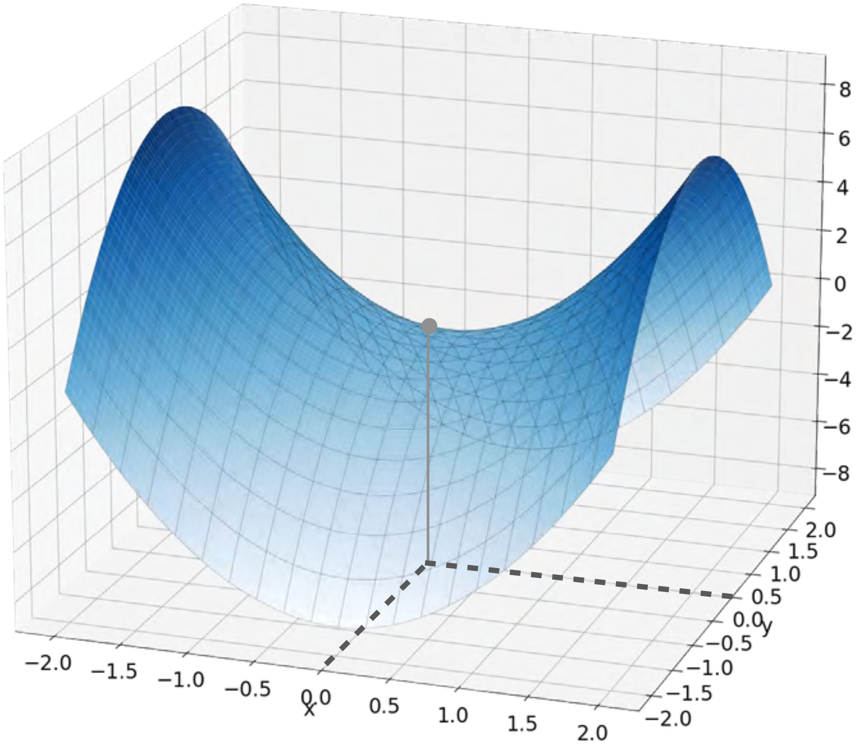
Saddle Point



$$f(x, y) = 2x^2 - 2y^2$$

$$\nabla f(x, y) = \begin{bmatrix} 4x \\ -4y \end{bmatrix}$$

Saddle Point

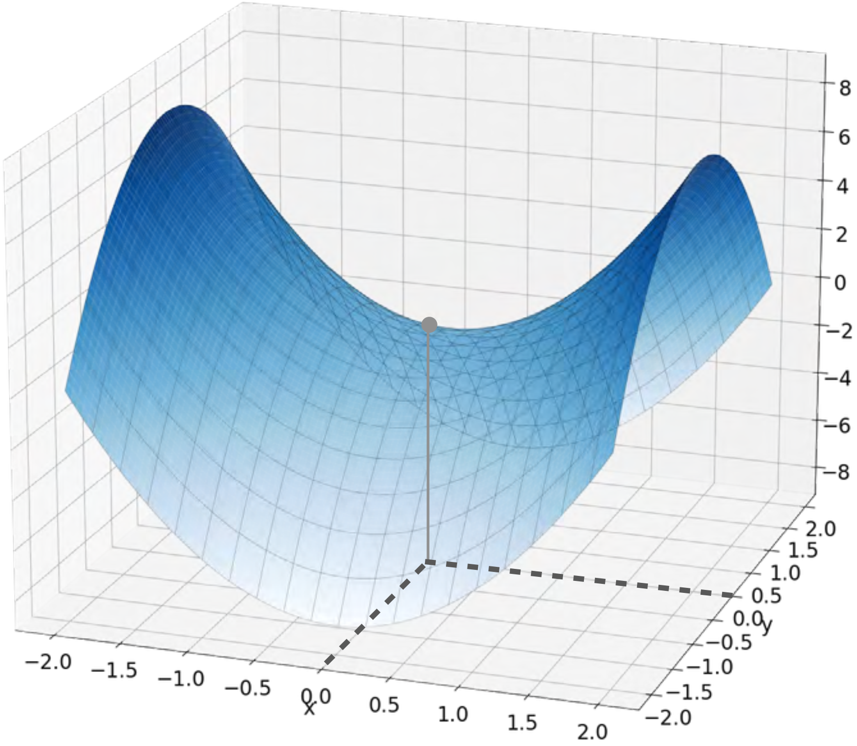


$$f(x, y) = 2x^2 - 2y^2$$

$$\nabla f(x, y) = \begin{bmatrix} 4x \\ -4y \end{bmatrix}$$

$$H(0,0) = \begin{bmatrix} 4 & 0 \\ 0 & -4 \end{bmatrix}$$

Saddle Point



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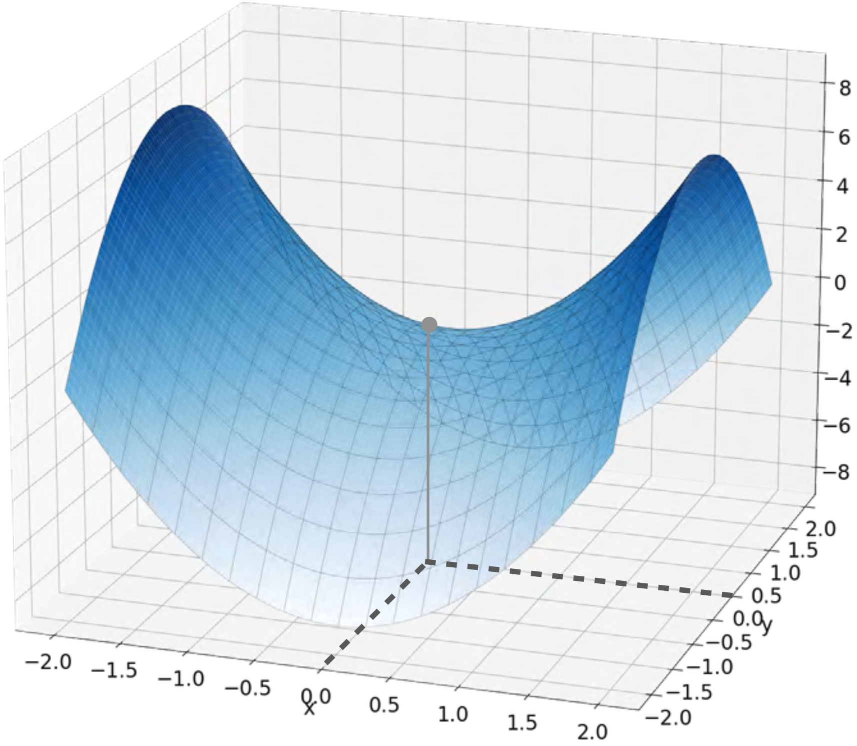
$$\det(H(0,0) - \lambda I) =$$

Saddle Point



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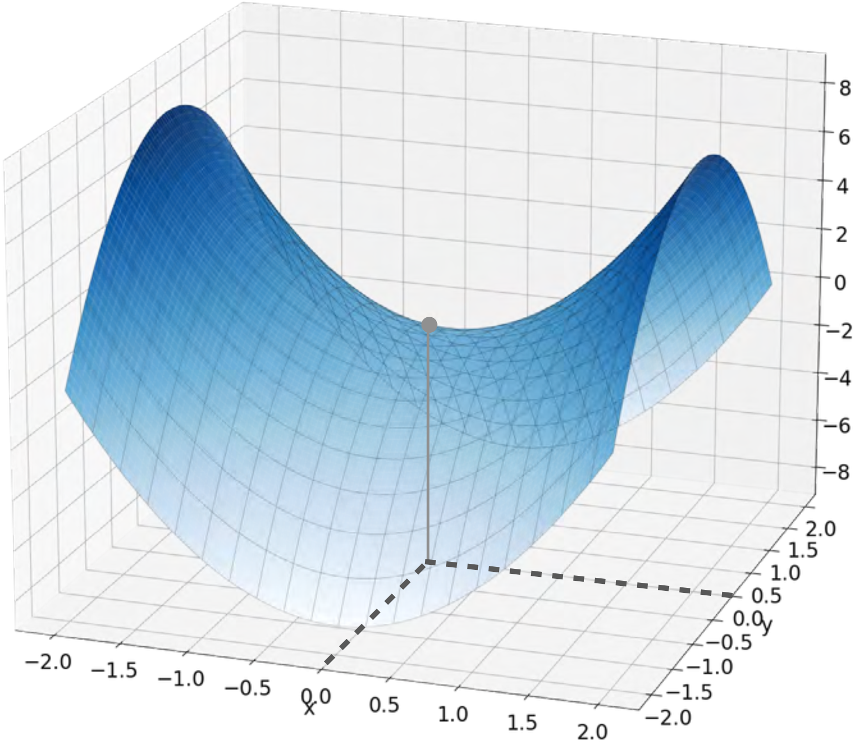
$$(4 - \lambda)(-4 - \lambda) - 0$$

$$\lambda_1 = -4$$

Saddle Point



$$f(x,y) = 2x^2 - 2y^2$$

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$$H(0,0) = \begin{bmatrix} 4 & 0 \\ 0 & -4 \end{bmatrix}$$

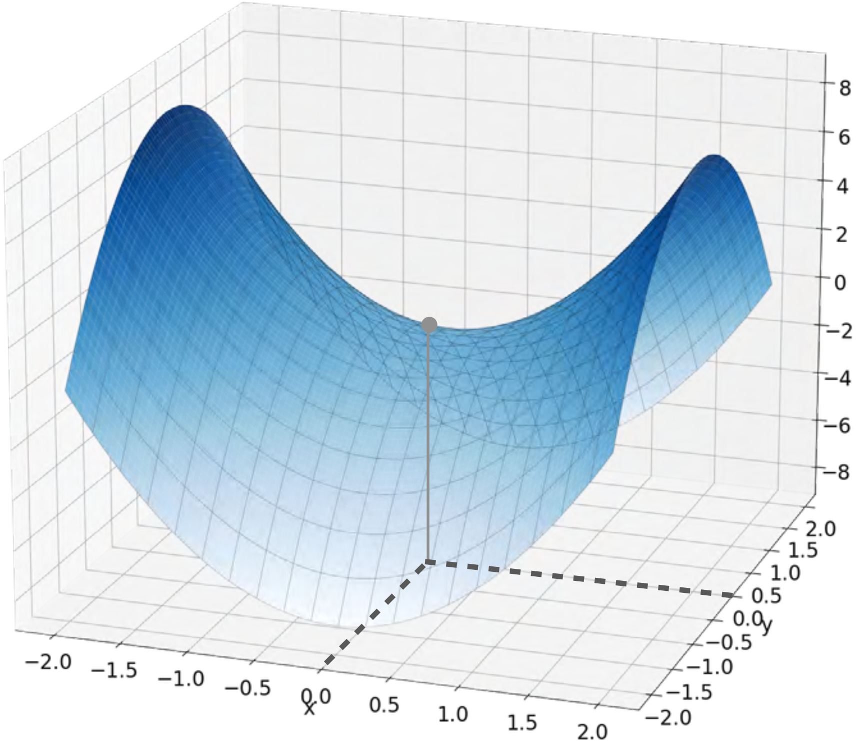
$$\det(H(0,0) - \lambda I) =$$

$$(4-\lambda)(-4-\lambda) - 0$$

$$\lambda_1 = -4$$

$$\lambda_2 = 4$$

Saddle Point



$$f(x, y) = 2x^2 - 2y^2$$

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$$H(0,0) = \begin{bmatrix} 4 & 0 \\ 0 & -4 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) =$$

$$(4 - \lambda)(-4 - \lambda) - 0$$

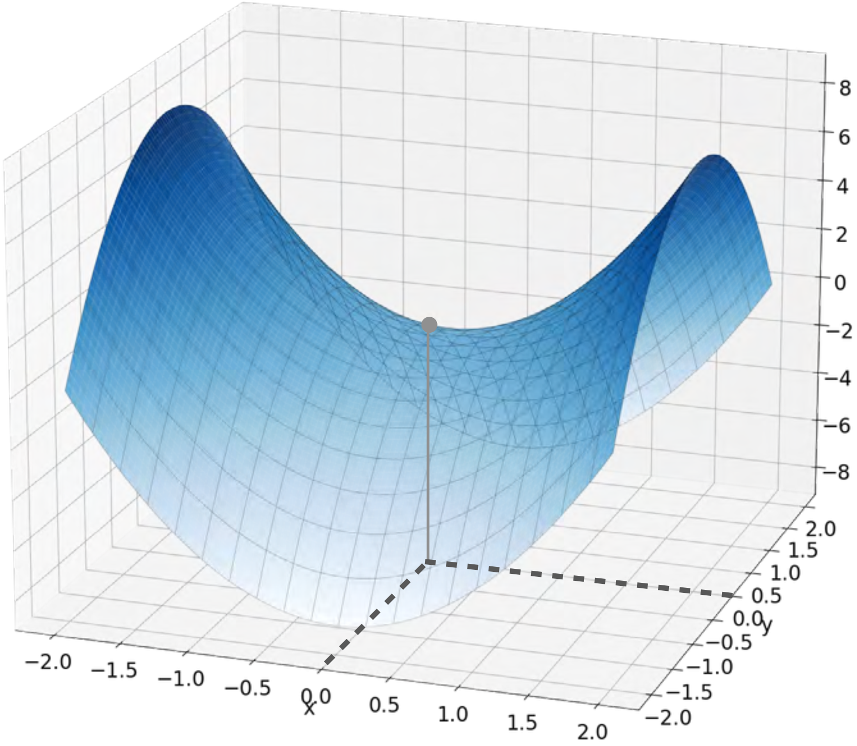
< 0

$$\boxed{\lambda_1 = -4}$$

$$\lambda_2 = 4$$

$(0,0)$ is saddle point

Saddle Point



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$$\det(H(0,0) - \lambda I) =$$

$$(4-\lambda)(-4-\lambda) - 0$$

< 0

$\boxed{\lambda_1 = -4}$

$\boxed{\lambda_2 = 4}$

> 0

$(0,0)$ is saddle point

Summary

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	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \ldots, x_n)$

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(Local) minima	Happy face $f''(x) > 0$		

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	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \ldots, x_n)$
(Local) minima	Happy face $f''(x) > 0$	Upper paraboloid $\lambda_1 > 0 \,\&\, \lambda_2 > 0$	

Summary

	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \ldots, x_n)$
(Local) minima	Happy face $f''(x) > 0$	Upper paraboloid $\lambda_1 > 0 \,\&\, \lambda_2 > 0$	All $\lambda_i > 0$

Summary

	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \ldots, x_n)$
(Local) minima	Happy face $f''(x) > 0$	Upper paraboloid $\lambda_1 > 0 \,\&\, \lambda_2 > 0$	All $\lambda_i > 0$
(Local) maxima	Sad face $f''(x) < 0$		

Summary

	1 variable $f(x)$	2 variables $f(x,y)$	More variables $f(x_1, x_2, \ldots, x_n)$
(Local) minima	Happy face $f''(x) > 0$	Upper paraboloid $\lambda_1 > 0 \,\&\, \lambda_2 > 0$	All $\lambda_i > 0$
(Local) maxima	Sad face $f''(x) < 0$	Down paraboloid $\lambda_1 < 0 \,\&\, \lambda_2 < 0$	

Summary

	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \ldots, x_n)$
(Local) minima	Happy face $f''(x) > 0$	Upper paraboloid $\lambda_1 > 0 \,\&\, \lambda_2 > 0$	All $\lambda_i > 0$
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Summary

	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \ldots, x_n)$
(Local) minima	Happy face $f''(x) > 0$	Upper paraboloid $\lambda_1 > 0 \,\&\, \lambda_2 > 0$	All $\lambda_i > 0$
(Local) maxima	Sad face $f''(x) < 0$	Down paraboloid $\lambda_1 < 0 \,\&\, \lambda_2 < 0$	All $\lambda_i < 0$
Need more information	$f''(x) = 0$		

Summary

	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \ldots, x_n)$
(Local) minima	Happy face $f''(x) > 0$	Upper paraboloid $\lambda_1 > 0 \,\&\, \lambda_2 > 0$	All $\lambda_i > 0$
(Local) maxima	Sad face $f''(x) < 0$	Down paraboloid $\lambda_1 < 0 \,\&\, \lambda_2 < 0$	All $\lambda_i < 0$
Need more information	$f''(x) = 0$	Saddle point $\lambda_1 > 0 \,\&\, \lambda_2 < 0$ $\lambda_1 < 0 \,\&\, \lambda_2 > 0$	

Summary

	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \ldots, x_n)$
(Local) minima	Happy face $f''(x) > 0$	Upper paraboloid $\lambda_1 > 0 \,\&\, \lambda_2 > 0$	All $\lambda_i > 0$
(Local) maxima	Sad face $f''(x) < 0$	Down paraboloid $\lambda_1 < 0 \,\&\, \lambda_2 < 0$	All $\lambda_i < 0$
Need more information	$f''(x) = 0$	Saddle point $\lambda_1 > 0 \,\&\, \lambda_2 < 0$ $\lambda_1 < 0 \,\&\, \lambda_2 > 0$ Or some $\lambda_i = 0$	

Summary

	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \ldots, x_n)$
(Local) minima	Happy face $f''(x) > 0$	Upper paraboloid $\lambda_1 > 0 \,\&\, \lambda_2 > 0$	All $\lambda_i > 0$
(Local) maxima	Sad face $f''(x) < 0$	Down paraboloid $\lambda_1 < 0 \,\&\, \lambda_2 < 0$	All $\lambda_i < 0$
Need more information	$f''(x) = 0$	Saddle point $\lambda_1 > 0 \,\&\, \lambda_2 < 0$ $\lambda_1 < 0 \,\&\, \lambda_2 > 0$ Or some $\lambda_i = 0$	Some $\lambda_i > 0$ and some $\lambda_j < 0$ OR At least one $\lambda_i = 0$

Optimization in Neural Networks and Newton's Method

DeepLearning.AI

Newton's method for two variables

Newton's Method

Newton's Method

1 variable

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$$

Newton's Method

1 variable

$$x_{k+1} = \boxed{x_k} - \frac{f'(x_k)}{f''(x_k)}$$

Newton's Method

1 variable

$$x_{k+1} = x_k - \frac{\boxed{f'(x_k)}}{f''(x_k)}$$

Newton’s Method

1 variable

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$$

Newton's Method

1 variable

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$$

$$x_{k+1} = x_k - f''(x_k)^{-1} f'(x_k)$$

Newton's Method

1 variable

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$$

$$x_{k+1} = x_k - f''(x_k)^{-1} f'(x_k)$$

2 variables

Newton's Method

1 variable

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$$

$$x_{k+1} = x_k - f''(x_k)^{-1} f'(x_k)$$

2 variables

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix}$$

Newton's Method

1 variable

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$$

$$x_{k+1} = x_k - f''(x_k)^{-1} f'(x_k)$$

2 variables

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} -$$

Newton's Method

1 variable

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$$

$$x_{k+1} = x_k - \boxed{f''(x_k)^{-1}} f'(x_k)$$

2 variables

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} - H^{-1}(x_k, y_k)$$

Newton's Method

1 variable

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$$

$$x_{k+1} = x_k - \boxed{f''(x_k)^{-1}}\,\boxed{f'(x_k)}$$

2 variables

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} - H^{-1}(x_k, y_k)\ \nabla f(x_k, y_k)$$

Newton's Method

2 variables

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} - H^{-1}(x_k, y_k) \; \nabla f(x_k, y_k)$$

Newton's Method

2 variables

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} - H^{-1}(x_k, y_k) \ \nabla f(x_k, y_k)$$

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Newton's Method

2 variables

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} - H^{-1}(x_k, y_k) \ \nabla f(x_k, y_k)$$

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} \cancel{- \nabla f(x_k, y_k) \ H^{-1}(x_k, y_k)}$$

Newton's Method

2 variables

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} - \underbrace{H^{-1}(x_k, y_k)}_{2\times 2} \; \underbrace{\nabla f(x_k, y_k)}_{2\times 1}$$

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} - \cancel{\nabla f(x_k, y_k) \, H^{-1}(x_k, y_k)}$$

Newton's Method

2 variables

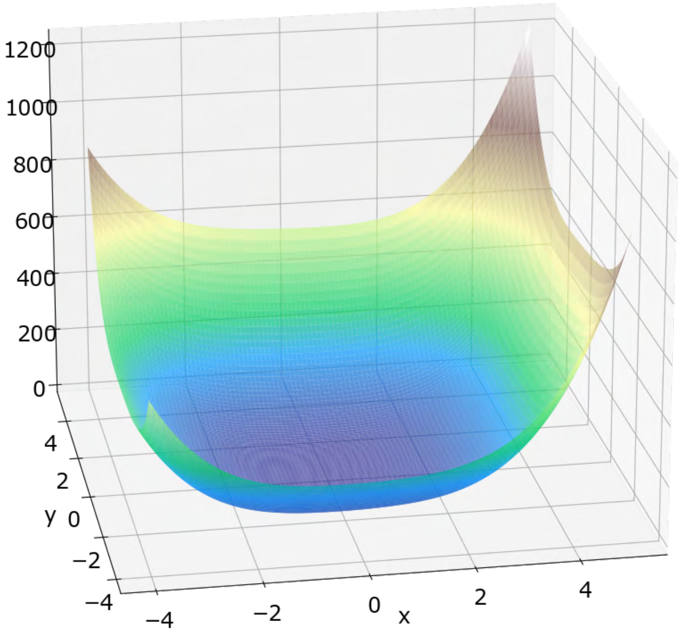
$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} - \underbrace{H^{-1}(x_k, y_k)}_{2\times 2} \; \underbrace{\nabla f(x_k, y_k)}_{2\times 1}$$

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} - \cancel{\nabla f(x_k, y_k) \, H^{-1}(x_k, y_k)}$$

When working with 2 variables the order is crucial!

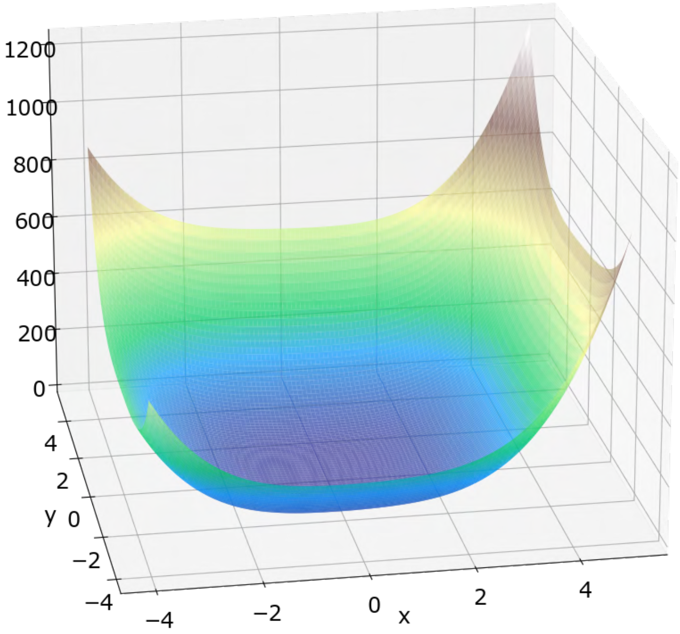
An Example

An Example



$$f(x, y) = x^4 + 0.8y^4 + 4x^2 + 2y^2 - xy - 0.2x^2y$$

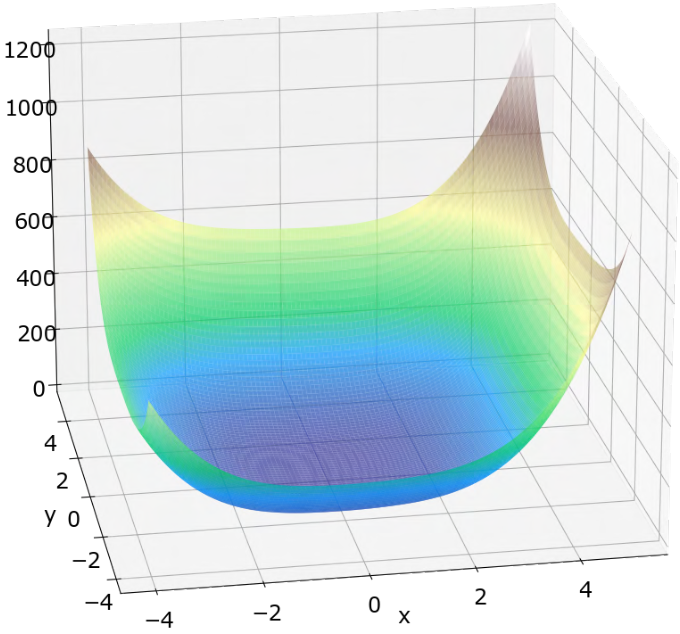
An Example



$$f(x, y) = x^4 + 0.8y^4 + 4x^2 + 2y^2 - xy - 0.2x^2y$$

$$f(x,y) \xrightarrow{x} 4x^3 + 8x - y - 0.4xy$$

An Example



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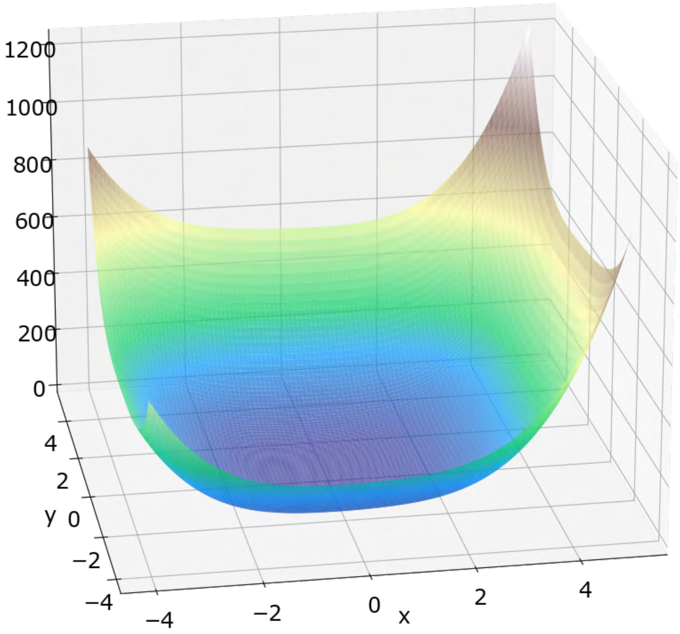
$$f(x, y) \xrightarrow{x} 4x^3 + 8x - y - 0.4xy$$

$$f(x, y) \xrightarrow{y} 3.2y^3 + 4y - x - 0.2x^2$$

An Example

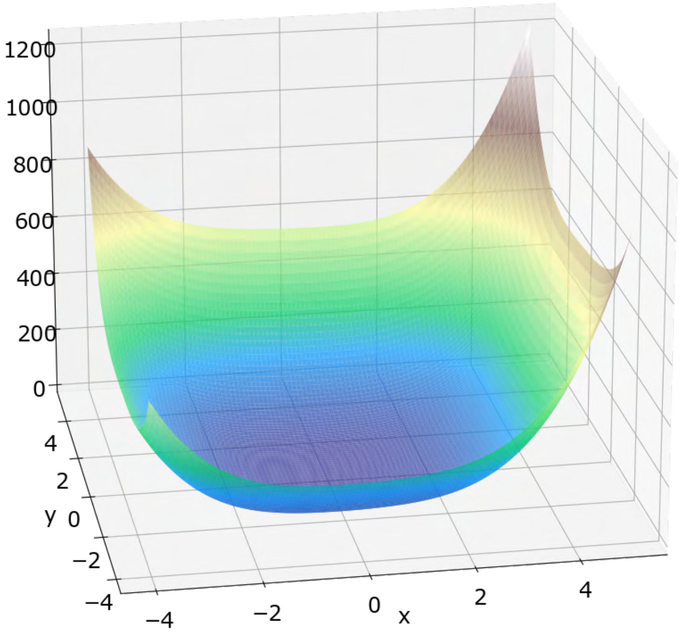


$$f(x,y) = x^4 + 0.8y^4 + 4x^2 + 2y^2 - xy - 0.2x^2y$$

$$f(x,y) \xrightarrow{x} 4x^3 + 8x - y - 0.4xy \xrightarrow{x} 12x^2 + 8 - 0.4y$$

$$f(x,y) \xrightarrow{y} 3.2y^3 + 4y - x - 0.2x^2$$

An Example



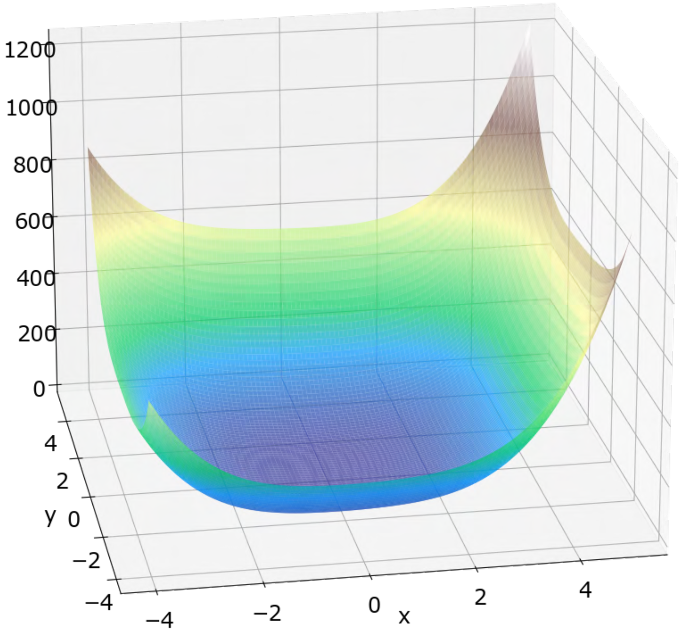
$$f(x, y) = x^4 + 0.8y^4 + 4x^2 + 2y^2 - xy - 0.2x^2y$$

$$f(x, y) \xrightarrow{x} 4x^3 + 8x - y - 0.4xy \xrightarrow{x} 12x^2 + 8 - 0.4y$$

$$4x^3 + 8x - y - 0.4xy \xrightarrow{y} -1 - 0.4x$$

$$f(x, y) \xrightarrow{y} 3.2y^3 + 4y - x - 0.2x^2$$

An Example



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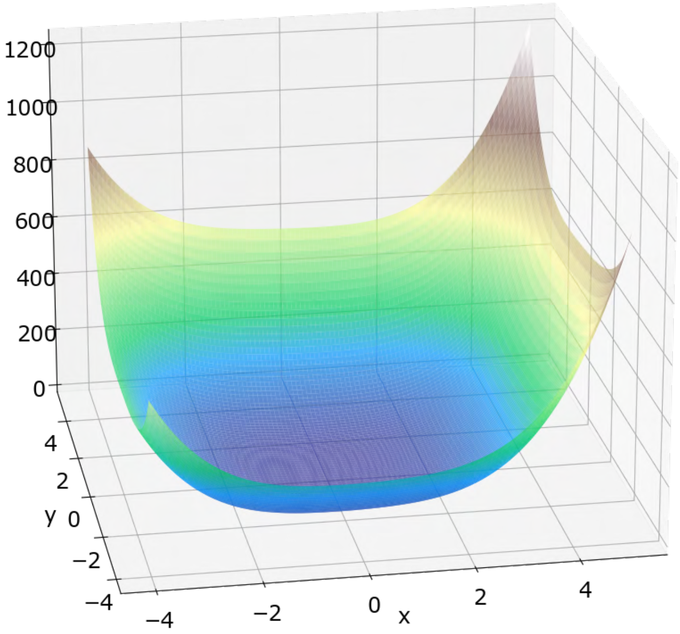
$$f(x, y) \xrightarrow{y} 3.2y^3 + 4y - x - 0.2x^2 \xrightarrow{x} -1 - 0.4x$$

An Example



$$f(x,y) = x^4 + 0.8y^4 + 4x^2 + 2y^2 - xy - 0.2x^2y$$

$$f(x,y) \xrightarrow{x} 4x^3 + 8x - y - 0.4xy$$

$$4x^3 + 8x - y - 0.4xy \xrightarrow{x} 12x^2 + 8 - 0.4y$$

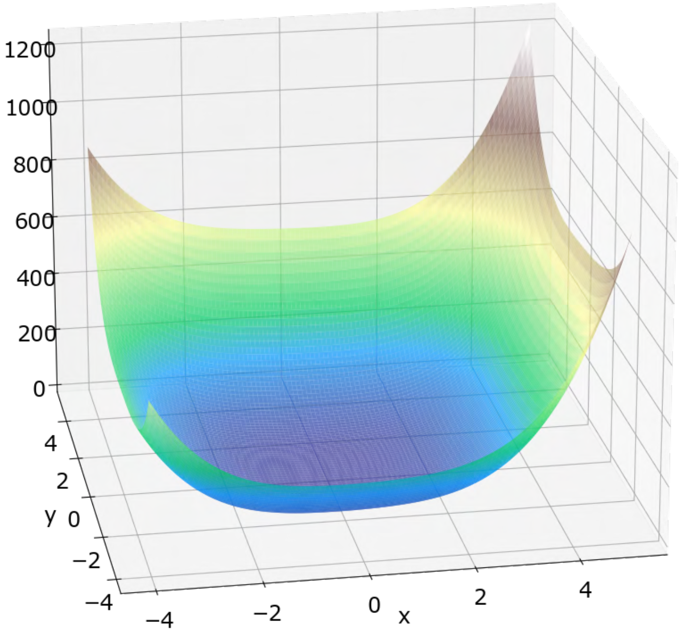
$$4x^3 + 8x - y - 0.4xy \xrightarrow{y} -1 - 0.4x$$

$$f(x,y) \xrightarrow{y} 3.2y^3 + 4y - x - 0.2x^2$$

$$3.2y^3 + 4y - x - 0.2x^2 \xrightarrow{x} -1 - 0.4x$$

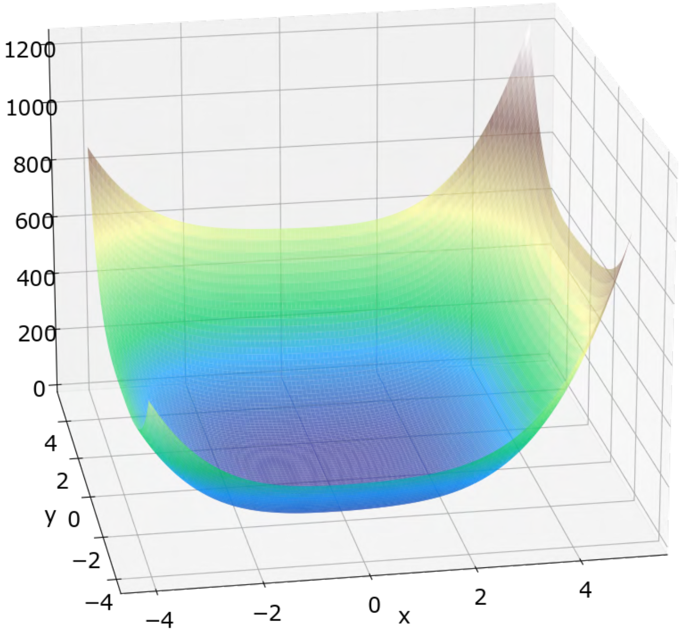
$$3.2y^3 + 4y - x - 0.2x^2 \xrightarrow{y} 9.6y^2 + 4$$

An Example



$$f(x, y) = x^4 + 0.8y^4 + 4x^2 + 2y^2 - xy - 0.2x^2y$$

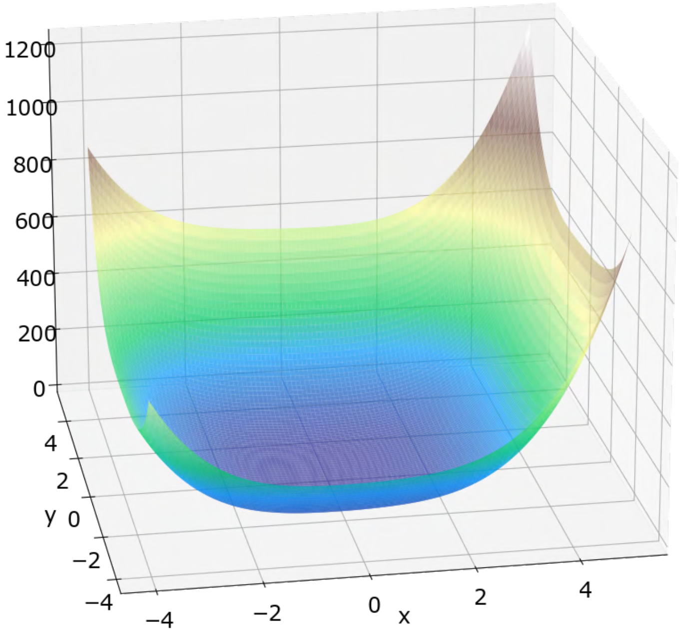
An Example



$$f(x, y) = x^4 + 0.8y^4 + 4x^2 + 2y^2 - xy - 0.2x^2y$$

$$\nabla f(x, y) = \begin{bmatrix} 4x^3 + 8x - y - 0.4xy \\ 3.2y^3 + 4y - x - 0.2x^2 \end{bmatrix}$$

An Example

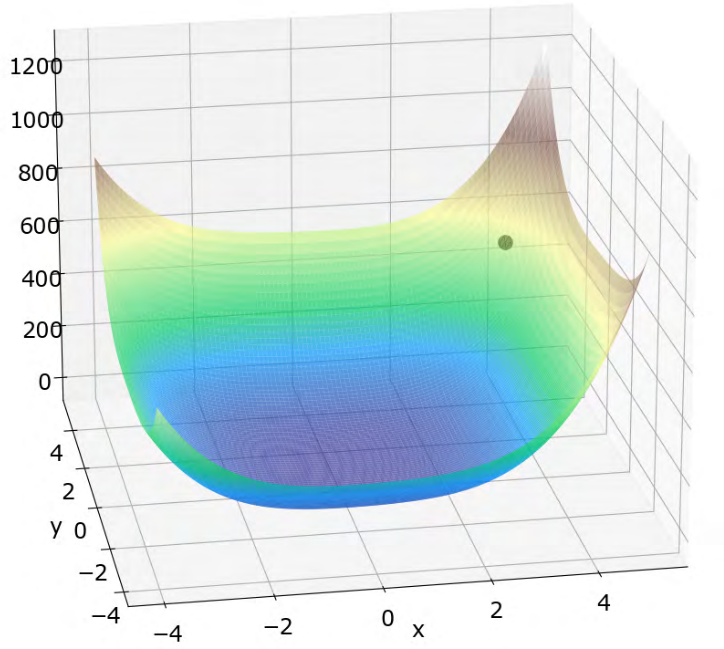


$$f(x, y) = x^4 + 0.8y^4 + 4x^2 + 2y^2 - xy - 0.2x^2y$$

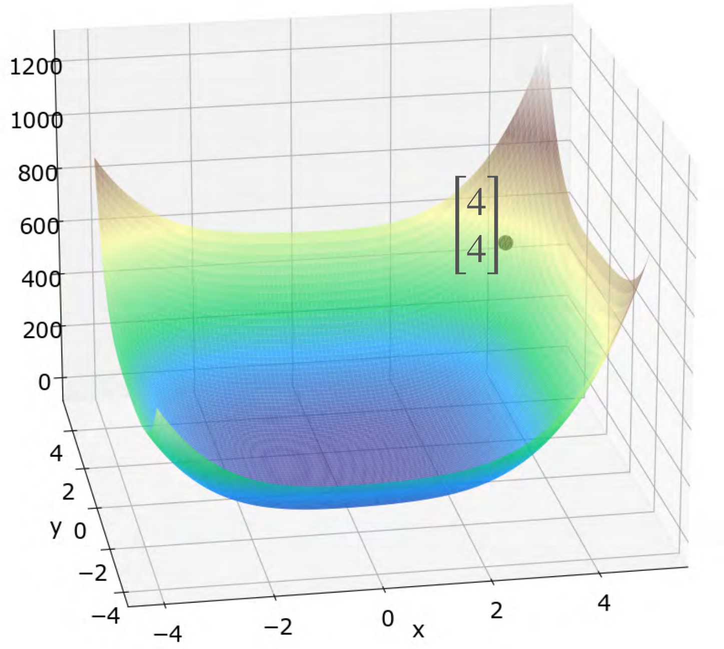
$$\nabla f(x, y) = \begin{bmatrix} 4x^3 + 8x - y - 0.4xy \\ 3.2y^3 + 4y - x - 0.2x^2 \end{bmatrix}$$

$$H(x, y) = \begin{bmatrix} 12x^2 + 8 - 0.4y & -1 - 0.4x \\ -1 - 0.4x & 9.6y^2 + 4 \end{bmatrix}$$

An Example

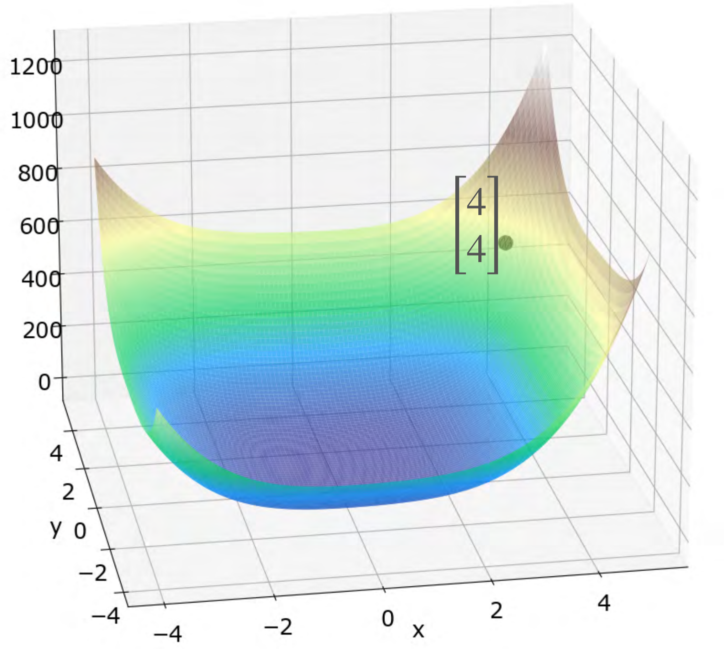


An Example



Start at some point $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$

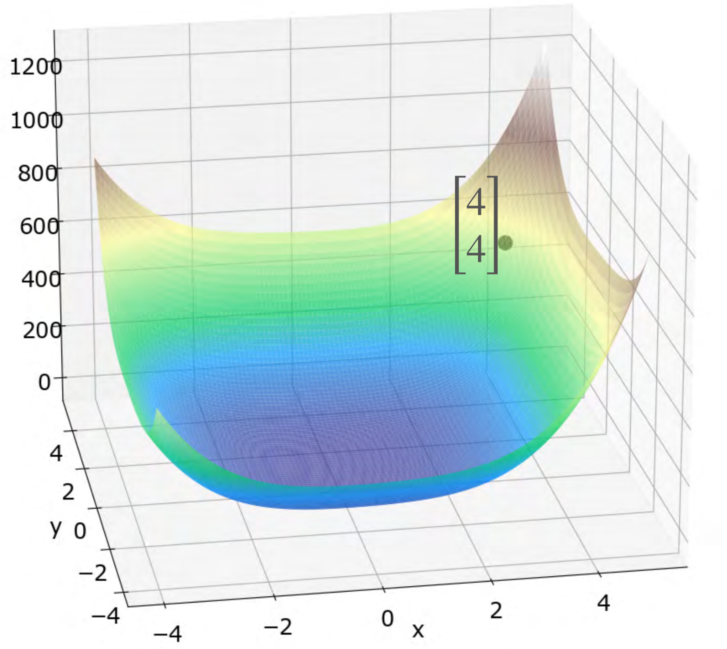
An Example



Start at some point $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$

$$\nabla f(4,4) = \begin{bmatrix} 277.6 \\ 213.6 \end{bmatrix}$$

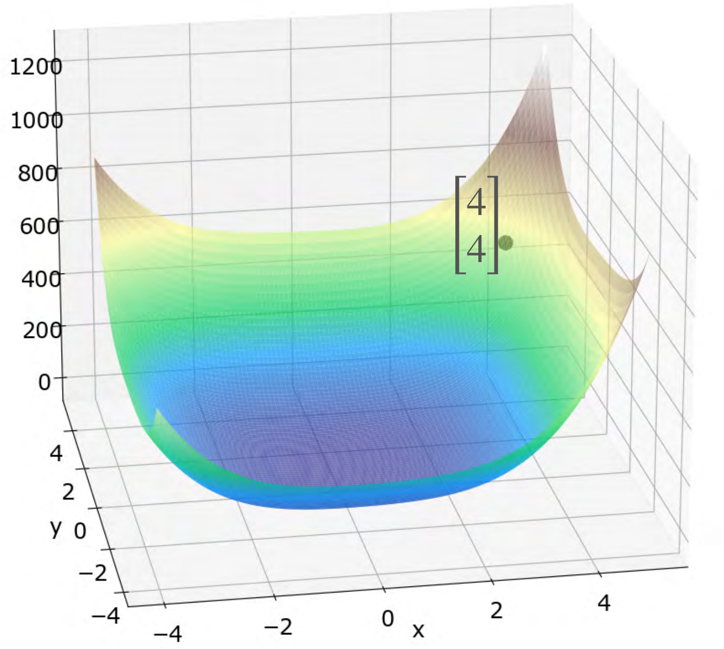
An Example



Start at some point $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$

$$\nabla f(4,4) = \begin{bmatrix} 277.6 \\ 213.6 \end{bmatrix} \qquad H(4,4) = \begin{bmatrix} 198.4 & -2.6 \\ -2.6 & 157.6 \end{bmatrix}$$

An Example



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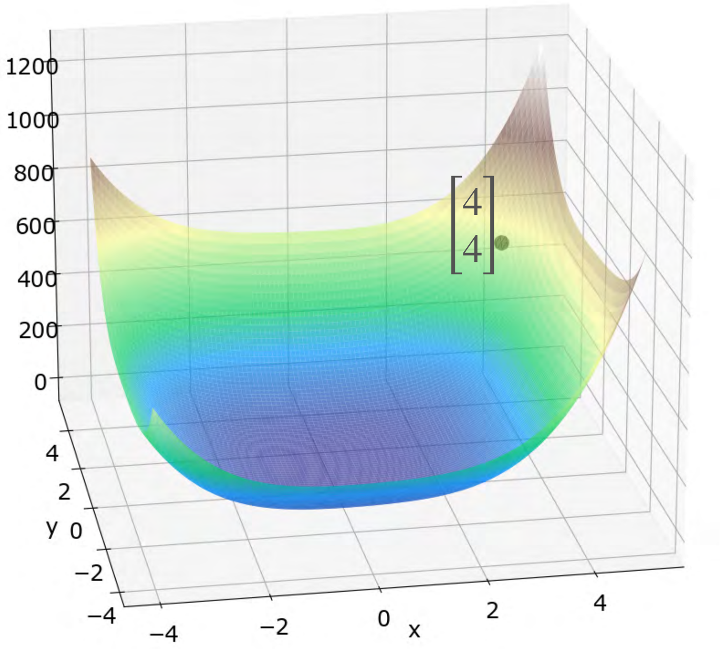
$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix} -$$

An Example



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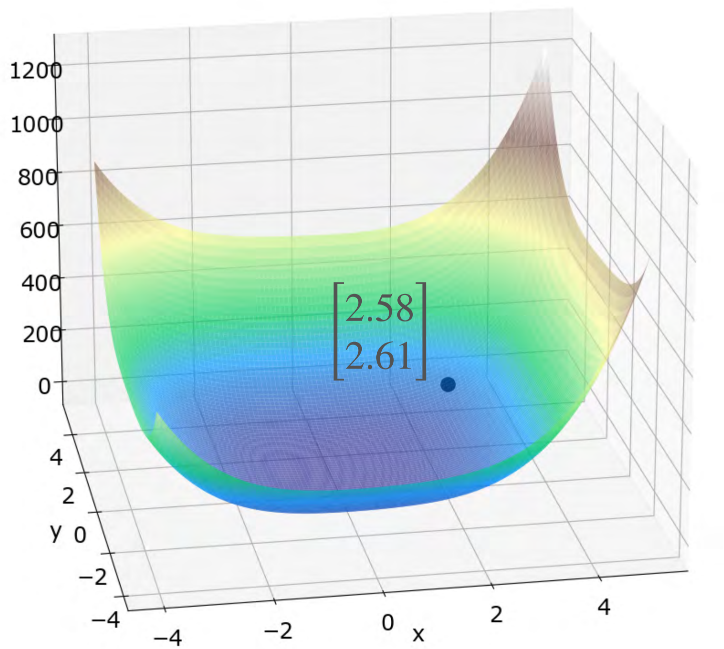
$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix} - \begin{bmatrix} 198.4 & -2.6 \\ -2.6 & 157.6 \end{bmatrix}^{-1} \begin{bmatrix} 277.6 \\ 213.6 \end{bmatrix}$$

An Example



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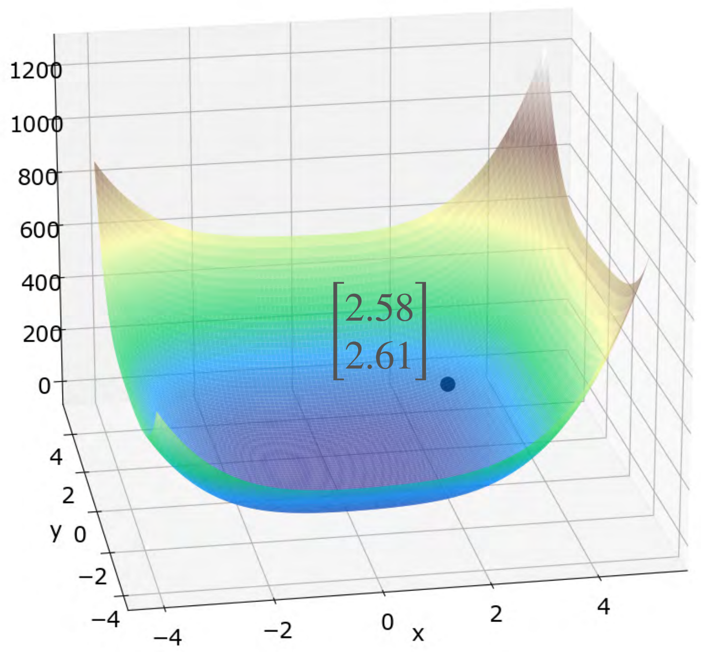
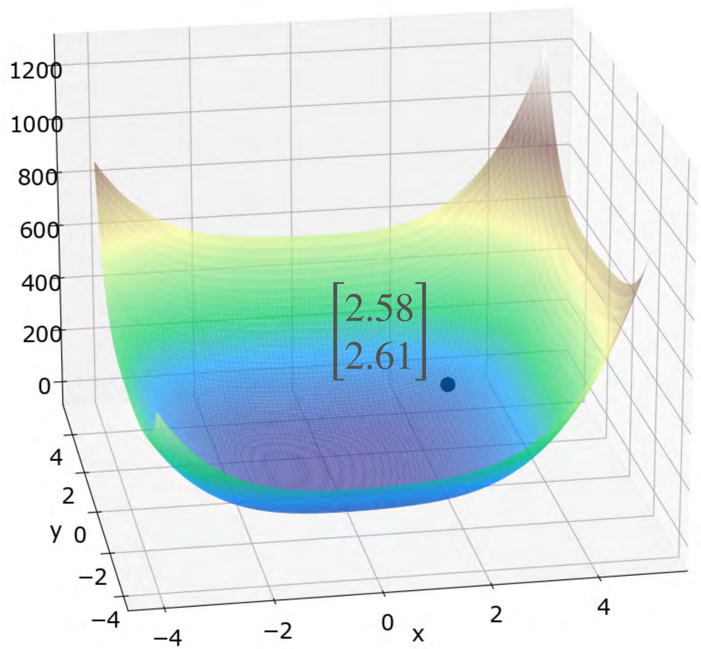
$$= \begin{bmatrix} 2.58 \\ 2.62 \end{bmatrix}$$

An Example



$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 2.58 \\ 2.61 \end{bmatrix}$$

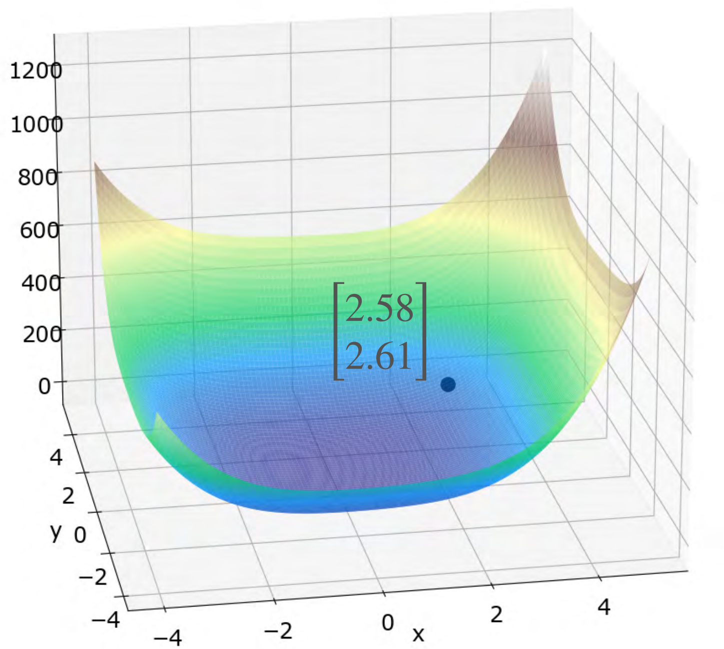
An Example



$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 2.58 \\ 2.61 \end{bmatrix}$$

$$\nabla f(2.58, 2.61) = \begin{bmatrix} 84.25 \\ 63.4 \end{bmatrix}$$

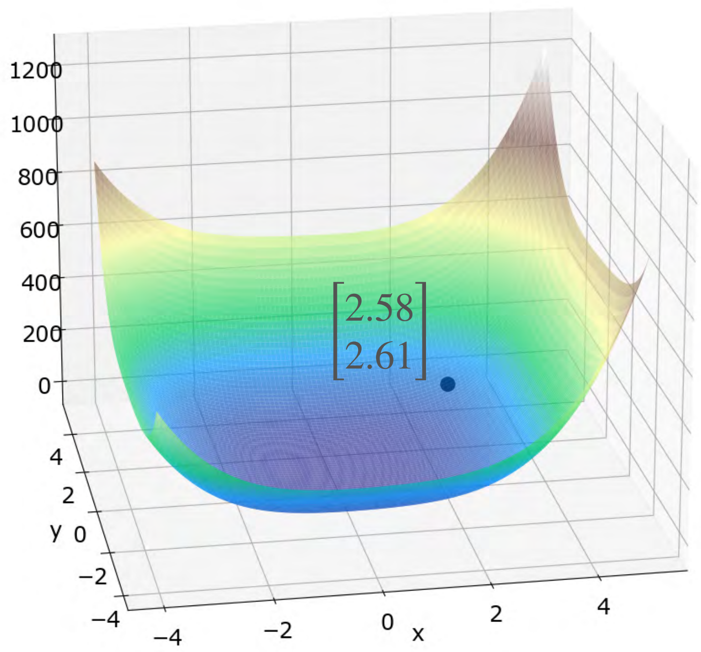
An Example



$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 2.58 \\ 2.61 \end{bmatrix}$$

$$\nabla f(2.58, 2.61) = \begin{bmatrix} 84.25 \\ 63.4 \end{bmatrix} \quad H(2.58, 2.61) = \begin{bmatrix} 86.83 & -2.032 \\ -2.032 & 69.39 \end{bmatrix}$$

An Example



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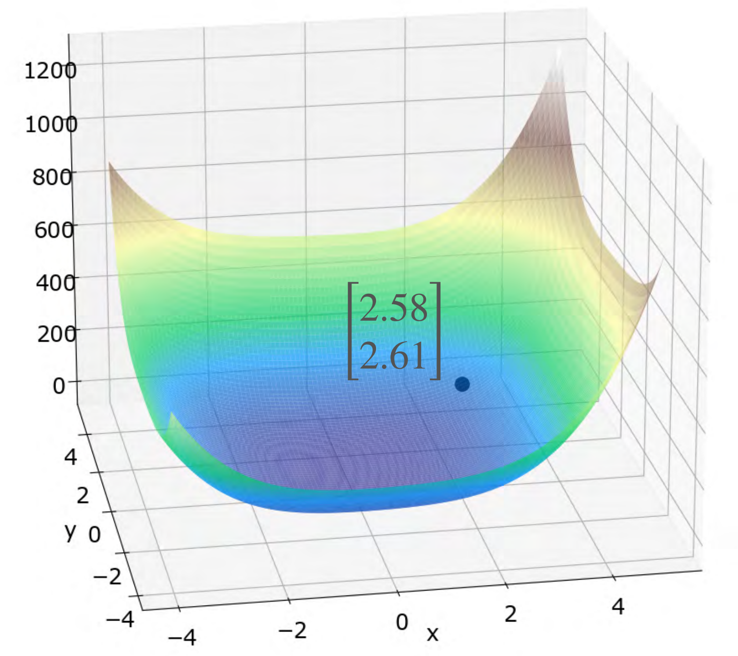
$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 2.58 \\ 2.61 \end{bmatrix} -$$

An Example



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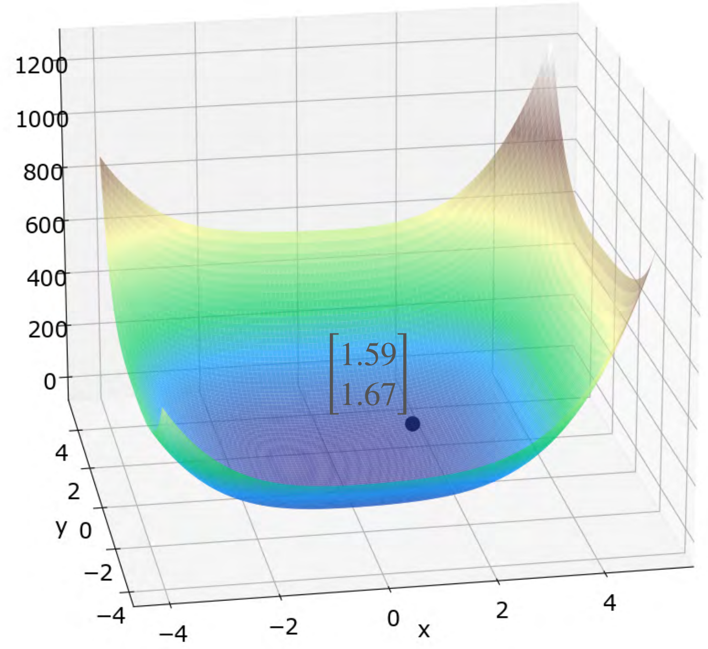
$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 2.58 \\ 2.61 \end{bmatrix} - \begin{bmatrix} 86.83 & -2.032 \\ -2.032 & 69.39 \end{bmatrix}^{-1} \begin{bmatrix} 84.25 \\ 63.4 \end{bmatrix}$$

An Example



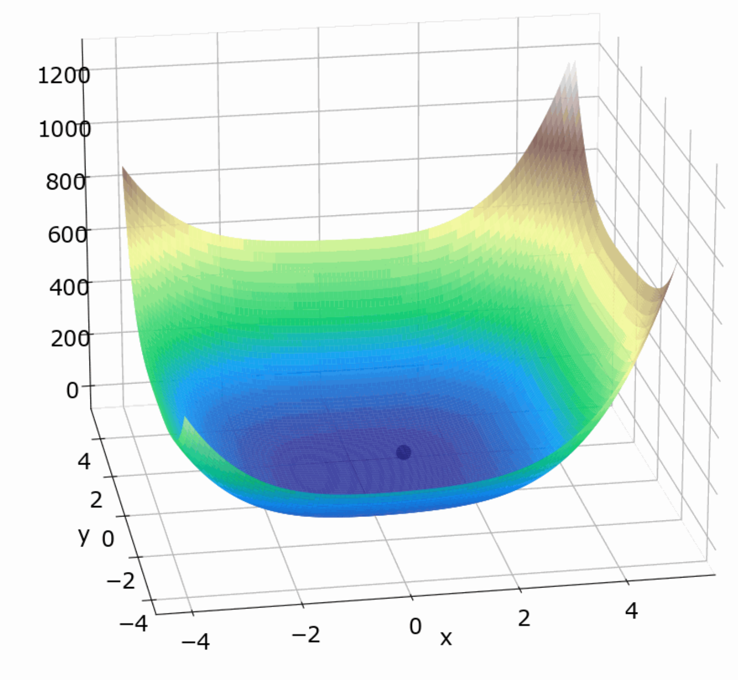
$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 2.58 \\ 2.61 \end{bmatrix}$$

$$\nabla f(2.58, 2.61) = \begin{bmatrix} 84.25 \\ 63.4 \end{bmatrix} \quad H(2.58, 2.61) = \begin{bmatrix} 86.83 & -2.032 \\ -2.032 & 69.39 \end{bmatrix}$$

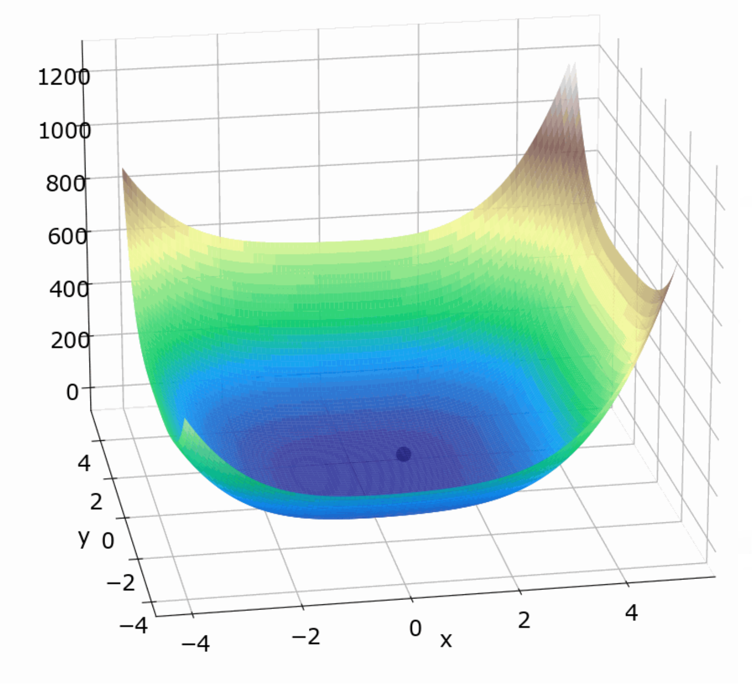
$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 2.58 \\ 2.61 \end{bmatrix} - \begin{bmatrix} 86.83 & -2.032 \\ -2.032 & 69.39 \end{bmatrix}^{-1} \begin{bmatrix} 84.25 \\ 63.4 \end{bmatrix}$$

$$= \begin{bmatrix} 1.59 \\ 1.67 \end{bmatrix}$$

An Example

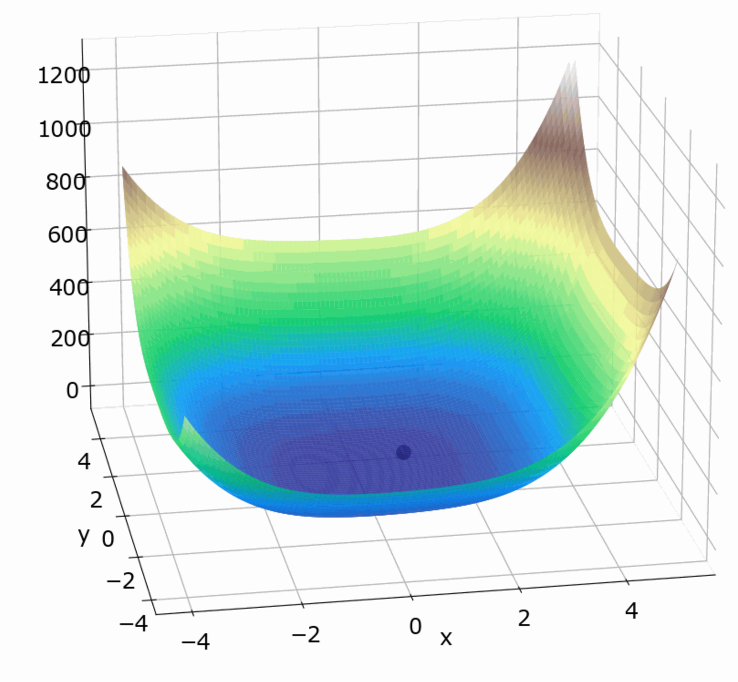


An Example



Repeat until you are close enough to the actual zero!

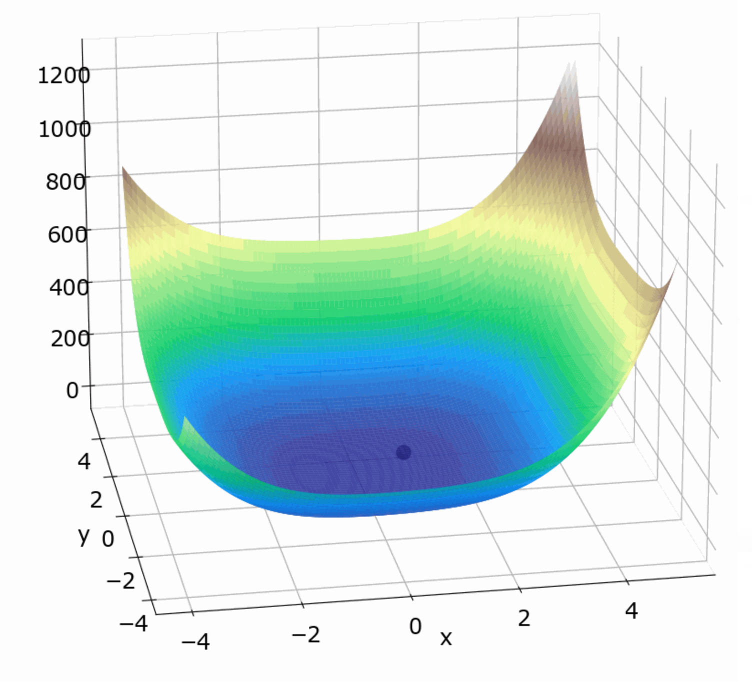
An Example



Repeat until you are close enough to the actual zero!

Needed $k = 8$ steps

An Example

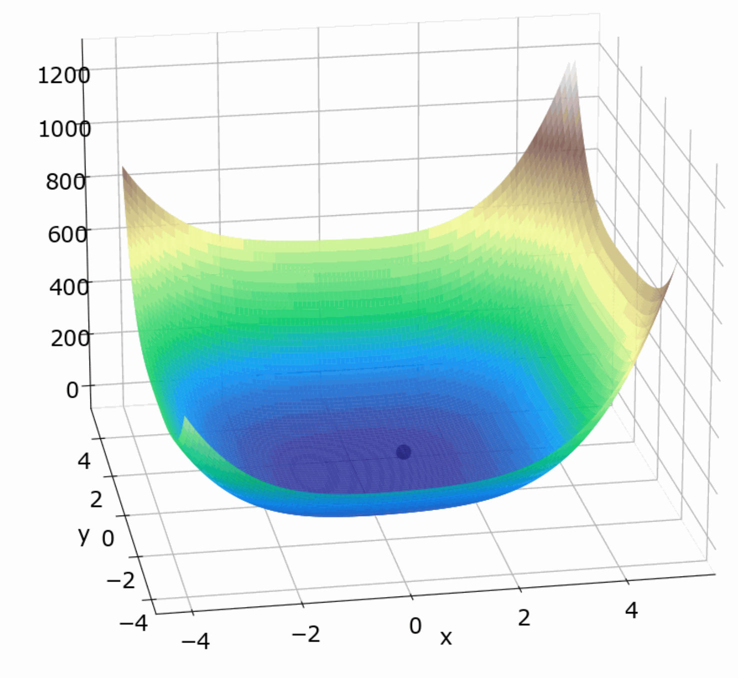


Repeat until you are close enough to the actual zero!

Needed $k = 8$ steps

$$\begin{bmatrix} x_8 \\ y_8 \end{bmatrix} = \begin{bmatrix} 4.15\ 10^{-17} \\ -2.05\ 10^{-17} \end{bmatrix}$$

An Example



Repeat until you are close enough to the actual zero!

Needed $k = 8$ steps

$$\begin{bmatrix} x_8 \\ y_8 \end{bmatrix} = \begin{bmatrix} 4.15\ 10^{-17} \\ -2.05\ 10^{-17} \end{bmatrix}$$

$$\begin{bmatrix} x^* \\ y^* \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Optimization in Neural Networks and Newton's Method

DeepLearning.AI

Conclusion