**Q1 Roger and Caleb's Party**

**(a) Payoff Matrix**

The players are Roger (hiring a clown (H) or not (N)) and Caleb (going to the party (G) or not (NG)). Here's the payoff matrix:

|  |  |  |
| --- | --- | --- |
|  | **Caleb: G** | **Caleb: NG** |
| Roger: H | (8x, 0) | (3x, 1) |
| Roger: N | (4, 4) | (2, 3) |

**(b) Suppose x = 0**

When the cost of hiring a clown (x) is zero, the payoff matrix changes to:

|  |  |  |
| --- | --- | --- |
|  | **Caleb: G** | **Caleb: NG** |
| Roger: H | (0, 0) | (0, 1) |
| Roger: N | (4, 4) | (2, 3) |

Here, Roger's strategy of not hiring a clown (N) dominates hiring a clown (H) because his payoff is always higher with N regardless of Caleb's choice. Similarly, for Caleb, going to the party (G) is better than not going (NG) if Roger doesn't hire a clown (N). This scenario leads to a pure strategy Nash equilibrium where Roger doesn't hire a clown (N) and Caleb goes to the party (G), with both players receiving a payoff of (4, 4).

**(c) Suppose x = 2**

With a cost of hiring a clown (x) of 2, there's no dominated strategy. Neither hiring a clown (H) nor not hiring (N) is always better for Roger. We can analyze a mixed strategy Nash equilibrium using probabilities, but for simplicity, we'll focus on pure strategies here. In this case, there's no pure strategy Nash equilibrium where neither player has an incentive to change their strategy.

**Q2 Constable and Pickpocket**

**(a) Pure Strategy Nash Equilibrium**

The Constable (patrol (P) or relax (R)) and the Pickpocket (prowl the market (M) or stay home (H)) face this payoff matrix:

|  |  |  |
| --- | --- | --- |
|  | **Pickpocket: M** | **Pickpocket: H** |
| Constable: P | (30, -15) | (0, 0) |
| Constable: R | (10, 10) | (10, 0) |

Here, the Constable prefers patrolling (P) if the Pickpocket is prowling (M) to catch them. But the Constable relaxing (R) is just as good as patrolling (P) if the Pickpocket stays home (H). The Pickpocket benefits from prowling (M) only if the Constable relaxes (R). This situation leads to a situation where no player has a dominant strategy, and their best choices depend on each other's actions.

**Q3 Market Entry for Two Firms**

**(a) Payoff Matrix**

Two firms (F1 and F2) decide to produce product A (A), product B (B), or not enter the market (D). Here's the payoff matrix:

|  |  |  |  |
| --- | --- | --- | --- |
|  | **F2: A** | **F2: B** | **F2: D** |
| F1: A | (-10, -10) | (10, 10) | (15, 0) |
| F1: B | (10, 10) | (5, 5) | (30, 0) |
| F1: D | (0, 15) | (0, 30) | (0, 0) |

**(b) Evaluating Dominance of B**

For F1, entering the market and producing B (B) seems favorable:

* Against F2 producing A, the payoff is (10, 10).
* Against F2 producing B, the payoff is (5, 5).
* Against F2 not entering, the payoff is (30, 0).

Compared to not entering at all (D), where the payoff is always zero, producing B guarantees a non-negative outcome. This makes B a dominant strategy for F1.

**(c) Nash Equilibrium with B**

If both firms choose B based on the logic above, the outcome is (5, 5). Neither firm can improve their payoff by unilaterally changing their strategy. Therefore, (B, B) is a higher payoff.

## Q4 Second-Price Sealed-Bid Auctions

This document explores bidding strategies in second-price sealed-bid auctions with two bidders.

### Dominant Strategy for Bidder A

In a second-price auction, the winner pays the second-highest bid, not their own. Regardless of whether bidder B makes mistakes about their value, bidder A's dominant strategy is to always bid their true value.

Here's why:

* Bidding their true value ensures they pay no more than their own valuation for the item.
* This holds true even if bidder B makes a mistake.

Therefore, bidding truthfully is the best approach for bidder A.

### Seller's Expected Revenue

Assuming bidders have equal probabilities (50%) of valuing the item at 0 or 1, the seller's expected revenue depends on the bids:

* **Both value it at 1:** (Probability: 0.25) The seller gets the highest bid (1) as the second-highest bid.
* **One at 1, one at 0:** (Probability: 0.5) The seller gets the highest bid (1) again.
* **Both value it at 0:** (Probability: 0.25) There's no second-highest bid, so the revenue is 0.

Considering these probabilities, the seller's expected revenue is 0.25.

### Q5 Sealed-Bid Auction for Rare Wine

Here's how bidding strategies work in a second-price auction for rare wine:

**(a) Collusion**

If bidders collude, they can try to lower the price:

* One bidder submits a real bid reflecting their true value.
* Others submit bids of 0.

This aims to make the second-highest bid 0, allowing the chosen bidder to win for free.

**(b) Seller Deception**

If the seller might deceive bidders:

* The seller could claim a fictional, higher bid.
* The highest real bidder might then be offered the wine at their bid price.

To counter this, bidders can:

* **Be Cautious:** If deception is suspected, adjust bids to account for the risk.
* **Bid Their True Value (or Slightly Lower):** If the wine is worth their true value, bidders can still bid truthfully or slightly lower to avoid overpaying if aware of deception.

However, this second approach can be risky if not all bidders are aware.

**Recommendation:**

For fair auctions, bidding your true value is optimal. If deception is a concern:

* Report and verify bids to ensure transparency.
* Bid sincerely if the auction seems fair.
* Consider slightly lowering bids if deception is likely, but be cautious as this can backfire.

pen\_spark

tuneshare

more\_vert