Stochastic Processes HW01

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1 Coin flipping

$$P[HH|B] = (\frac{3}{4})^2 = 0.5625$$

$$P[HH] = P[A]P[HH|A] + P[B]P[HH|B] = \frac{1}{2}(\frac{1}{4})^2 + \frac{1}{2}(\frac{3}{4})^2 = 0.3125$$

$$P[B|HH] = \frac{P[HH|B]P[B]}{P[HH]} = \frac{0.5625 \times \frac{1}{2}}{0.3125} = 0.9$$

2 Bernouli trials and binomially distribution

2.1 A

Lets define event A as at least 1 success in n trials and event B as failure in all the trials.

$$P(B) = (1 - P)^n$$

$$P(A) = 1 - P(B) = 1 - (1 - P)^n$$

2.2 B

Lets define event A as K successes in n trials.

$$P(A) = \binom{n}{k} p^K (1 - p)^{n-K}$$

3 Tea Time with De Morgan

3.1 A

$$A \subset B \implies A \cap \bar{B} = \emptyset \implies P[A \cap \bar{B}] = P[A] + P[\bar{B}]$$

$$P[B] - P[\bar{A} \cap B] = P[B] - (1 - P[\overline{\bar{A} \cap B}]) = P[B] - (1 - P[A \cup \bar{B}])$$
$$= P[B] - (1 - P[A] - P[\bar{B}]) = P[B] + P[\bar{B}] - 1 + P[A] = P[A]$$

3.2 B

$$(I) A \subset B \implies B = A \cup (B - A)$$
$$(II)(B - A) \cap A = \emptyset \implies P[A \cup (B - A)] = P[A] + P[(B - A)]$$
$$(III) P[(B - A)] \geq 0$$

$$P[B] \overset{I}{=} P[A \cup (B - A)] \overset{II}{=} P[A] + P[(B - A)] \overset{III}{\implies} P[B] \geq P[A]$$

4 Independence

$$(I) P[A \cap B] + P[A \cap \bar{B}] = P[A] \implies P[A \cap B] = P[A] - P[A \cap \bar{B}]$$
$$(II) P[A]P[B] = P[A](1 - P[\bar{B}]) = P[A] - P[A]P[\bar{B}]$$
$$P[A \cap B] = P[A]P[B] \overset{II}{=} P[A] - P[A]P[\bar{B}] \overset{I}{\implies} P[A] - P[A \cap \bar{B}] = P[A] - P[A]P[\bar{B}]$$
$$\implies P[A \cap \bar{B}] = P[A]P[\bar{B}]$$

5 PDF Validity

5.1 Binomial distribution:

$$f(x) = \binom{n}{x} p^x (1 - p)^{n-x}$$
$$1) 0 < p < 1, f(x) \geq 0$$
$$2) \sum_{x=0}^{n} f(x) = \sum_{x=0}^{n} \binom{n}{x} p^x (1 - p)^{n-x} = [1 - p + p]^n = 1^n = 1$$
$$note : (a + b)^n = \sum_{x=0}^{n} \binom{n}{x} b^x a^{n-x}$$

5.2 Poisson distribution:

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \text{ for [x = 0, 1,2,3,...] } \lambda > 0$$

$$1) x! > 0, \lambda^x \geq 0, e^{-\lambda} \geq 0 \implies f(x) \geq 0$$

$$2) \sum_{x=0}^{n} f(x) = \sum_{x=0}^{\infty} \frac{\lambda^x e^{-\lambda}}{x!} = e^{-\lambda} \sum_{x=0}^{\infty} \frac{\lambda^x}{x!} = e^{-\lambda} e^{\lambda} = 1$$

$$\text{note: Maclaurin series : } [1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + ... = e^{\lambda}]$$

5.3 Exponential distribution:

$$f(x) = \lambda e^{-\lambda x}, \lambda > 0$$

$$1) f(x) \geq 0$$

$$2) \int_0^{\infty} f(x) = \int_0^{\infty} \lambda e^{-\lambda x} = [0 - (-1)] = 1$$

5.4 Uniform distribution:

$$f(x) = \frac{1}{b-a} \ , a \leq x \leq b$$

$$1) f(x) \geq 0$$

$$2) \int_a^b f(x) = \int_a^b \frac{1}{b-a} = 1$$

5.5 $\mathbf{P[\{n\}] = 2^{-n}, N = \{1, 2, \ldots\}}$

$$f(x) = 2^{-x} \text{ for x= [1, 2, 3, ...]}$$

$$1) f(x) \geq 0$$

$$2) \sum_{x=1}^{n} 2^{-x} = \frac{\frac{1}{2}}{1 - \frac{1}{2}} = 1$$