COMP 301 Analysis of Algorithms, Fall 2021
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HW 4

Submit your answers to Canvas for the problems given below.

1.

Indicate, for each pair of expressions $(A, B)$ in the table below, whether $A$ is $O$, $o$, $\Omega$, $\omega$, or $\Theta$ of $B$. Assume that $k \geq 1$, $\epsilon > 0$, and $c > 1$ are constants. Your answer should be in the form of the table with "yes" or "no" written in each box.

| | $A$ | $B$ | $O$ | $o$ | $\Omega$ | $\omega$ | $\Theta$ |
|---|---|---|---|---|---|---|---|
| ***a.*** | $\lg^k n$ | $n^\epsilon$ | | | | | |
| ***b.*** | $n^k$ | $c^n$ | | | | | |
| ***c.*** | $\sqrt{n}$ | $n^{\sin n}$ | | | | | |
| ***d.*** | $2^n$ | $2^{n/2}$ | | | | | |
| ***e.*** | $n^{\lg c}$ | $c^{\lg n}$ | | | | | |
| ***f.*** | $\lg(n!)$ | $\lg(n^n)$ | | | | | |

2. Consider a modification to merge sort in which $n/k$ sublists of length $k$ are sorted using insertion sort and then merged using the standard merging mechanism (i.e. you only do insertion sort at one level of recursion tree), where $k$ is a value to be determined.

(a) Show that the insertion sort can sort the $n/k$ sublists each of length $k$ in $\Theta(nk)$ worst-case time.

(b) Show that the $n/k$ sublists can be merged in $\Theta(n \lg(n/k))$ worst-case time

(c) Given that the modified algorithm runs in $\Theta(nk + n\lg(n/k))$ worst-case time, what is the largest value of $k$ as a function of $n$ and in $\Theta$-notation for which the modified algorithm has the same running time as merge sort in $\Theta$-notation?

(d) How should we choose $k$ in practice? Hint: consider the list lengths for which insertion sort is better than merge sort, which is a range of integers to choose from. Then consider which of these values is the best option to start with when combined with merge sort.

3. Show that if $f(n)$ and $g(n)$ are monotonically increasing functions, then so is the function $f(g(n))$, and if $f(n)$ and $g(n)$ are in addition nonnegative, then $f(n)g(n)$ is monotonically increasing.