COMP 301 Analysis of Algorithms
Instructor: Zafer Aydın
HW 3

Submit your answers to Canvas for the problems given below.

1. Describe a Θ(n logn)-time algorithm that, given a set S of n integers and another integer x, determines whether or not there exist two elements in S whose sum is exactly x. Hint: you can use binary search.

2. Use mathematical induction to show that when n is an exact power of 2, the solution of the recurrence

$$T(n) = \begin{cases} 2 & \text{if } n = 2\,, \\ 2T(n/2) + n & \text{if } n = 2^k, \text{ for } k > 1 \end{cases}$$

is $T(n) = n \lg n$. That is start with $n = 2$ and show that it holds and then assuming that it is true for $n = 2^k, k > 1$ show that it holds for $n = 2^{k+1}$, which is the next n value in input sequence. Beware that $T(n)$ is expressed as a function directly and you are not required to do asymptotic analysis on this question.

3. In this question you will analyze the Horner's rule

The following code fragment implements Horner's rule for evaluating a polynomial

$$\begin{aligned} P(x) &= \sum_{k=0}^{n} a_k x^k \\ &= a_0 + x(a_1 + x(a_2 + \cdots + x(a_{n-1} + x a_n) \cdots))\,, \end{aligned}$$

given the coefficients $a_0, a_1, \ldots, a_n$ and a value for x:

```
y = 0
for i = n downto 0
    y = a_i + x · y
```

a. In terms of Θ-notation, what is the running time of this code fragment for Horner's rule?

b. Write pseudocode to implement the naive polynomial-evaluation algorithm that computes each term of the polynomial from scratch. What is the running time of this algorithm? How does it compare to Horner's rule?

c. Consider the following loop invariant:

At the start of each iteration of the **for** loop of lines 2–3,

$$y = \sum_{k=0}^{n-(i+1)} a_{k+i+1} x^k .$$

Interpret a summation with no terms as equaling 0. Following the structure of the loop invariant proof presented in this chapter, use this loop invariant to show that, at termination, $y = \sum_{k=0}^{n} a_k x^k$.

d. Conclude by arguing that the given code fragment correctly evaluates a polynomial characterized by the coefficients $a_0, a_1, \ldots, a_n$.