Choose Initial Constants

From periodic time series $X_t, t \in \{1, ..., n\}$ with period, d,

$$K_n = [\lfloor n^{1/3} \rfloor, \lfloor n^{1/2} \rfloor], L_n = 4n^{1/4}$$

Optimal Block Lengths for Overall Mean

for $K_n \leftarrow K_{n,min} : K_{n,max}$ **do**

$\quad G(K_n) = \sum_{s=1}^{d} \sum_{k=1}^{K_n} \left| \lfloor \frac{k}{d} \rfloor \right| \text{Cov}(X_s, X_{s+k})$
$\quad + \sum_{s=1}^{d} \sum_{k=1}^{K_n} \left| \lfloor \frac{k}{d} \rfloor \right| \text{Cov}(X_{s-k}, X_s)$

end

$\hat{G}_n = \frac{1}{|K_n|} \sum_{K_n} G(K_n)$

for $k \leftarrow 1 : d$ **do**

$\quad \hat{f}_k(0) = 1/(2\pi d) \sum_{t=1}^{d} \sum_{\tau=-L_n}^{L_n} \text{Cov}(X_t, X_{t+\tau}) \exp(-2\pi i k t/d)$

end

$\hat{D}_n = \frac{4(2\pi d)^2}{3} \sum_{k=0}^{d-1} \left| \hat{f}_k(0) \right|^2$

$b_{GSBB}^{opt} = b_{CGSBB}^{opt} = \sqrt[3]{\frac{2\hat{G}_n^2}{\hat{D}_n}} \sqrt[3]{n}$

$b_{EMBB}^{opt} = b_{CEMBB}^{opt} = \sqrt[3]{\frac{2\hat{G}_n^2}{\hat{D}_n}} \sqrt[3]{n}$

Optimal Block Lengths for Overall Mean

for $K_n \leftarrow K_{n,min} : K_{n,max}$ **do**

$\quad G^s(K_n) = \sum_{k=1}^{K_n} d |k| \text{Cov}(X_s, X_{s+kd}) + \sum_{k=1}^{K_n} d |k| \text{Cov}(X_{s-kd}, X_s)$

end

$\hat{G}_n^s = \frac{1}{|K_n|} \sum_{K_n} G^s(K_n)$

$\hat{f}_0^s(0) = \frac{1}{2\pi} \sum_{k=1}^{K_n} \text{Cov}(X_s, X_{s+kd})$

$\hat{D}_n^s(d) = \frac{4}{3} \left[2\pi d \hat{f}_0^s(0) \right]^2$

$b_{GSBB}^{opt} = b_{CGSBB}^{opt} = \sqrt[3]{\frac{2(\hat{G}_n^s)^2}{\hat{D}_n^s}} \sqrt[3]{n}$

$b_{EMBB}^{opt} = b_{CEMBB}^{opt} = \sqrt[3]{\frac{2(\hat{G}_n^s)^2}{\hat{D}_n^s}} \sqrt[3]{n}$

GSBB: Generalized seasonal block bootstrap
CGSBB: Circular Generalized seasonal block bootstrap
EMBB: Extended moving block bootstrap
CEMBB: Circular extended moving block bootstrap

For all seasonal means:

$b_{all,GSBB}^{opt} = b_{all,CGSBB}^{opt} = \sqrt[3]{\frac{2\sum_{s=1}^{d}(\hat{G}_n^s)^2}{\sum_{s=1}^{d} \hat{D}_n^s}} \sqrt[3]{n}$

$b_{all,EMBB}^{opt} = b_{all,CEMBB}^{opt} = \sqrt[3]{\frac{2\sum_{s=1}^{d}(\hat{G}_n^s)^2}{\sum_{s=1}^{d} \hat{D}_n^s}} \sqrt[3]{n}$