

Initial Parameters

Initial block length, $\ell_1 = \lceil n^{\frac{1}{5}} \rceil$

No. of blocks to delete, $m = \lfloor n^{1/3} \ell_1^{2/3} \rfloor$

For bias calculation, $\ell_2 = 2\ell_1$

Calculate Bootstrap Estimates

for $\ell \leftarrow \{\ell_1, \ell_2\}$ **do**
 $b = \lfloor n/\ell \rfloor$

$$\pi_t^* = \frac{1}{b \|w_l\|_1} \sum_{j=1}^b \sum_{k=1}^{\ell} w_l(k) \mathbb{I}(I_j^* = t - k + 1),$$

$$t \in 1, \dots, n$$

Get $\tilde{\beta}_n$ for quantile τ from perturbed data (Y_t^*, X_t^*)
 and π_t^*

$$D_n^*(\tilde{\beta}_n) = \sum_{t=1}^n \pi_t^* X_t^* \text{sign}_{\theta}(Y_t^* - X_t^* \tilde{\beta}_n)$$

$$\tilde{D}_n = \mathbb{E}_* D_n^*(\tilde{\beta}_n)$$

$$\hat{\varphi}_n(\ell) = \text{tr}\{\text{Cov}_*[\sqrt{n} \tilde{D}_n(\tilde{\beta}_n)]\}$$

end

Estimate $\hat{\nu}$

Calculate the Jackknife-after-Bootstrap (JAB) Variance estimator $\widehat{\text{VAR}}$ using $\hat{\varphi}_n(\ell_1)$:

(i) Collect the indices set:

$$I_i^* = \{\{ {}_k B_1^*, \dots, {}_k B_b^* \} \cap \{B_i, \dots, B_{i+m-1}\} = \emptyset\}$$

with $k : 1 \leq k \leq K$

(ii) $\hat{\varphi}_n^{(i)}$: i -th block-deleted jackknife point value, calculated using $k \in I_i^*$

$$\textbf{(iii)} \quad \tilde{\varphi}_n^{(i)} = m^{-1} \left[N \hat{\varphi}_n - (N - m) \hat{\varphi}_n^{(i)} \right]$$

$$\textbf{(iv)} \quad \widehat{\text{VAR}}_{\text{JAB}}(\hat{\varphi}_n) = \frac{m}{N-m} \frac{1}{M} \sum_{i=1}^M (\tilde{\varphi}_n^{(i)} - \hat{\varphi}_n)^2$$

where, N = No. of all possible blocks, and $M = N - m + 1$

$$\textbf{(v)} \quad \hat{\nu} = \frac{n}{\ell_1} \widehat{\text{VAR}}_{\text{JAB}}[\hat{\varphi}_n(\ell_1)]$$

Tapered Block Bootstrap

$$\hat{B} = \frac{4}{3} \ell_1^2 [\hat{\varphi}_n(\ell_1) - \hat{\varphi}_n(\ell_2)]$$

$$\hat{\ell}_{\text{SETBB}}^{\text{opt}} = \left[\frac{4\hat{B}^2}{\hat{\nu}} \right]^{\frac{1}{5}} n^{\frac{1}{5}}$$

Moving Block Bootstrap

$$\hat{B} = 2\ell_1 [\hat{\varphi}_n(\ell_1) - \hat{\varphi}_n(\ell_2)]$$

$$\hat{\ell}_{\text{SETBB}}^{\text{opt}} = \left[\frac{2\hat{B}^2}{\hat{\nu}} \right]^{\frac{1}{3}} n^{\frac{1}{3}}$$