

Initial Parameters

Initial block length, $\ell_1 = \left\lfloor n^{\frac{1}{5}} \right\rfloor$

No. of blocks to delete, $m = \lfloor n^{1/3} \ell_1^{2/3} \rfloor$

For bias calculation, $\ell_2 = 2\ell_1$

Calculate Bootstrap Estimates

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for  $\ell \leftarrow \{\ell_1, \ell_2\}$  do
     $b = \lfloor n/\ell \rfloor$ 
     $\pi_t^* = \frac{1}{b\|w_\ell\|_1} \sum_{j=1}^b \sum_{k=1}^\ell w_\ell(k) \mathbb{I}(I_j^* = t - k + 1),$ 
     $t \in 1, \dots, n$ 
    Get  $\tilde{\beta}_n$  for quantile  $\tau$  from perturbed data  $(Y_t^*, X_t^*)$  and  $\pi_t^*$ 
     $D_n^*(\tilde{\beta}_n) = \sum_{t=1}^n \pi_t^* X_t^* \text{sign}_\theta(Y_t^* - X_t^* \tilde{\beta}_n)$ 
     $\tilde{D}_n = \mathbb{E}_* D_n^*(\tilde{\beta}_n)$ 
     $\hat{\varphi}_n(\ell) = \text{tr}\{\text{Cov}_*[\sqrt{n}\tilde{D}_n(\tilde{\beta}_n)]\}$ 
end

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Estimate $\hat{\nu}$

Calculate the Jackknife-after-Bootstrap (JAB) Variance estimator $\widehat{\text{VAR}}$ using $\hat{\varphi}_n(\ell_1)$:

- Collect the indices set:
 $I_i^* = \{\{{}_k B_1^*, \dots, {}_k B_b^*\} \cap \{B_i, \dots, B_{i+m-1}\} = \emptyset\}$
with $k : 1 \leq k \leq K$
- $\hat{\varphi}_n^{(i)}$: i -th block-deleted jackknife point value, calculated using $k \in I_i^*$
- $\tilde{\varphi}_n^{(i)} = m^{-1} [N\hat{\varphi}_n - (N-m)\hat{\varphi}_n^{(i)}]$
- $\widehat{\text{VAR}}_{\text{JAB}}(\hat{\varphi}_n) = \frac{m}{N-m} \frac{1}{M} \sum_{i=1}^M (\tilde{\varphi}_n^{(i)} - \hat{\varphi}_n)^2$
where, N = No. of all possible blocks, and $M = N - m + 1$
- $\hat{\nu} = \frac{n}{\ell_1} \widehat{\text{VAR}}_{\text{JAB}}[\hat{\varphi}_n(\ell_1)]$

Tapered Block Bootstrap

$$\hat{B} = \frac{4}{3} \ell_1^2 [\hat{\varphi}_n(\ell_1) - \hat{\varphi}_n(\ell_2)]$$

$$\hat{\ell}_{SETBB}^{opt} = \left[\frac{4\hat{B}^2}{\hat{\nu}} \right]^{\frac{1}{5}} n^{\frac{1}{5}}$$

Moving Block Bootstrap

$$\hat{B} = 2\ell_1 [\hat{\varphi}_n(\ell_1) - \hat{\varphi}_n(\ell_2)]$$

$$\hat{\ell}_{SETBB}^{opt} = \left[\frac{2\hat{B}^2}{\hat{\nu}} \right]^{\frac{1}{3}} n^{\frac{1}{3}}$$