Choose Initial Constants

From periodic time series $X_t, t \in \{1, ..., n\}$ with period, *d*,

$$K_n = [\lfloor n^{1/3} \rfloor, \lfloor n^{1/2} \rfloor], \; L_n = 4n^{1/4}$$

Optimal Block Lengths for Overall Mean

for $K_n \leftarrow K_{n,min} : K_{n,max}$ **do**

$$G(K_n) = \sum_{s=1}^{d} \sum_{k=1}^{K_n} \left| \lfloor \frac{k}{d} \rfloor \right| \mathrm{Cov}(X_s, X_{s+k}) + \sum_{s=1}^{d} \sum_{k=1}^{K_n} \left| \lfloor \frac{k}{d} \rfloor \right| \mathrm{Cov}(X_{s-k}, X_s)$$

end

$$\hat{G}_n = \frac{1}{|K_n|} \sum_{K_n} G(K_n)$$

for $k \leftarrow 1 : d$ **do**

$$\hat{f}_k(0) = 1/(2\pi d) \sum_{t=1}^{d} \sum_{\tau=-L_n}^{L_n} \mathrm{Cov}(X_t, X_{t+\tau}) \exp(-2\pi i k t/d)$$

end

$$\hat{D}_n = \frac{4(2\pi d)^2}{3} \sum_{k=0}^{d-1} \left| \hat{f}_k(0) \right|^2$$

$$b^{opt}_{GSBB} = b^{opt}_{CGSBB} = \sqrt[3]{\frac{2\hat{G}_n^2}{\hat{D}_n}} \sqrt[3]{n}$$

$$b^{opt}_{EMBB} = b^{opt}_{CEMBB} = \sqrt[3]{\frac{2\hat{G}_n^2}{\hat{D}_n}} \sqrt[3]{n}$$

Optimal Block Lengths for Overall Mean

for $K_n \leftarrow K_{n,min} : K_{n,max}$ **do**

$$G^s(K_n) = \sum_{k=1}^{K_n} d\,|k| \,\mathrm{Cov}(X_s, X_{s+kd}) + \sum_{k=1}^{K_n} d\,|k| \,\mathrm{Cov}(X_{s-kd}, X_s)$$

end

$$\hat{G}^s_n = \frac{1}{|K_n|} \sum_{K_n} G^s(K_n)$$

$$\hat{f}^s_0(0) = \frac{1}{2\pi} \sum_{k=1}^{K_n} \mathrm{Cov}(X_s, X_{s+kd})$$

$$\hat{D}^s_n(d) = \frac{4}{3} \left[2\pi d \hat{f}^s_0(0) \right]^2$$

$$b^{opt}_{GSBB} = b^{opt}_{CGSBB} = \sqrt[3]{\frac{2(\hat{G}^s_n)^2}{\hat{D}^s_n}} \sqrt[3]{n}$$

$$b^{opt}_{EMBB} = b^{opt}_{CEMBB} = \sqrt[3]{\frac{2(\hat{G}^s_n)^2}{\hat{D}^s_n}} \sqrt[3]{n}$$

For all seasonal means:

$$b^{opt}_{all,GSBB} = b^{opt}_{all,CGSBB} = \sqrt[3]{\frac{2\sum_{s=1}^{d} (\hat{G}^s_n)^2}{\sum_{s=1}^{d} \hat{D}^s_n}} \sqrt[3]{n}$$

$$b^{opt}_{all,EMBB} = b^{opt}_{all,CEMBB} = \sqrt[3]{\frac{2\sum_{s=1}^{d} (\hat{G}^s_n)^2}{\sum_{s=1}^{d} \hat{D}^s_n}} \sqrt[3]{n}$$

GSBB: Generalized seasonal block bootstrap
CGSBB: Circular Generalized seasonal block bootstrap
EMBB: Extended moving block bootstrap
CEMBB: Circular extended moving block bootstrap