Exercise 5.3

Q.1 Use the remainder theorem to find the remainder, when.

(i) $3x^3 - 10x^2 + 13x - 6$ is divided by $(x-2)$

Sol:

Let $P(x) = 3x^3 - 10x^2 + 13x - 6$

When $P(x)$ is divided by $x - 2$ by remainder theorem, the remainder is:

$$R = P(2) = 3(2)^3 - 10(2)^2 + 13(2) - 6$$
$$= 3(8) - 10(4) + 26 - 6$$
$$= 24 - 40 + 26 - 6$$
$$= 50 - 46$$
$$= 4$$

(ii) $4x^3 - 4x + 3$ is divided by $(2x-1)$

Sol:

Let $P(x) = 4x^3 - 4x + 3$ when $P(x)$ is divided by $2x - 1$ by remainder theorem, the remainder is

$$R = P\left(\frac{1}{2}\right) \qquad = 4\left(\frac{1}{2}\right)^3 - 4\left(\frac{1}{2}\right) + 3$$
$$= 4\left(\frac{1}{8}\right) - 2 + 3$$
$$= \frac{1}{2} + 1$$
$$= \frac{1+2}{2}$$

$R \quad = \frac{3}{2}$

(iii) $6x^4+2x^3-x+2$ is divided by $(x+2)$

Sol:

Let P (x) = $6x^4+2x^3-x+2$ when P(x) is divided by x + 2 by remainder theorem, the remainder is

$$R = P(-2) = 6(-2)^4+2(-2)^3-(-2)+2$$
$$= 6(16)+2(-8)+2+2$$
$$= 96-16+4$$
$$= 80+4$$
$$R = 84$$

(iv) $(2x-1)^3+6(3+4x)^2-10$ is divided by 2x + 1

Sol:

Let p(x) = $(2x-1)^3+6(3+4x)^2-10$ when P(x) is divided by 2x + 1 by remainder theorem, then remainder is

$$R = p\left(-\frac{1}{2}\right) = \left[2\left(-\frac{1}{2}\right)-1\right]^3+6\left[3+4\left(-\frac{1}{2}\right)\right]^2-10$$
$$= (-1-1)^3+6(3-2)^2-10$$
$$= (-2)^3+6(1)^2-10$$
$$= -8+6-10$$
$$= -12$$

(v) $x^3-3x^2+4x-14$ is divided by x + 2

Sol:

Let P(x) = $x^3-3x^2+4x-14$ when P(x) is divided by x + 2 by remainder theorem, then remainder is

$$R = P(-2) = (-2)^3-3(-2)^2+4(-2)-14$$
$$= -8-3(4)-8-14$$
$$= -8-12-8-14$$
$$= -42$$

Q.2.

(i) If (x+2) is a factor of $3x^2-4kx-4k^2$, then find the value(s) of k.

Sol:

Let $P(x)=3x^2-4kx-4k^2$

As given that x + 2 is a factor of P(x), so

$R = 0$

i.e. $P(-2) = 0$

So $3(-2)^2-4k(-2)-4k^2=0$

$12+8k-4k^2=0$

Dividing by 4

$3+2k-k^2=0$

$3+3k-k-k^2=0$

$3(1+k)-k(1+k)=0$

$(1+k)(3-k)=0$

$\Rightarrow 1+k=0$ or $3-k=0$

$\Rightarrow k=-1$ or $k=3$

(ii) If (x −1) is factor of $x^3-kx^2+11x-6$ then find the value of k.

Sol:

$P(x)=x^3-kx^2+11x-6$

As given that x − 1 is a factor of P(x), so

$R = 0$

$P(1) = 0$

$(1)^3-k(1)^2+11(1)-6 = 0$

$1-k+11-6 = 0$

$6-k = 0$

$\Rightarrow k = 6$

Q.3 **Without actual long division determine whether**

(i) (x − 2) and (x − 3) are factors of P

$P(x)=x^3-12x^2+44x-48$

Sol:

$P(x)=x^3-12x^2+44x-48$

Taking $x-2$

$R=P(2)$

$=(2)^3-12(2)^2+44(2)-48$

$=8-12(4)+88-48$

$=8-48+88-48$

$=0$

As the remainder is zero, so $(x-2)$ is a factor of $P(x)$

Now $P(x)=x^3-12x^2+44x-48$

Taking $x-3$

$R=P(3)$

$=(3)^3-12(3)^2+44(3)-48$

$=27-12(9)+132-48$

$=27-108+132-148$

$=3\neq 0$

As the remainder is not equal to zero, so $(x-3)$ is not a factor of $P(x)$.

(ii) $(x-2)$, $(x+3)$ and $(x-4)$ are factors of $q(x)=x^3+2x^2-5x-6$

Sol:

$q(x)=x^3+2x^2-5x-6$

Taking $x-2$

$R=q(2)=(2)^3+2(2)^2-5(2)-6$

$=8+2(4)-10-6$

$R=0$

As the remainder is zero so $(x-2)$ is a factor of $P(x)$

Now $q(x)=x^3+2x^2-5x-6$

Taking $x+3$

$R=q(-3)$

$=(-3)^3+2(-3)^2-5(-3)-6$

$=-27+2(9)+15-6$

$=-27+18+15-6$

$=0$

As the remainder is zero, so $(x+3)$ is a factor of $P(x)$

Now $q(x)=x^3+2x^2-5x-6$

Taking $x-4$

$R=q(4)$

$=(4)^3+2(4)^2-5(4)-6$

$=64+2(16)-20-6$

$=64+32-20-6$

$=70\neq 0$

As remainder is not equal to zero, so $x-4$ is not a factor of $P(x)$

Q.4 For what value of m is the polynomial $P(x)=4x^3-7x^2+6x-3m$ exactly divisible by $x+2$?

Sol:

$m=?$

$P(x)=4x^3-7x^2+6x-3m$

Taking $x+2$

As $p(x)$ is exactly divisible by $(x+2)$, so

$R=0$

$P(-2)=0$

$4(-2)^3-7(-2)^2+6(-2)-3m=0$

$4(-8)-7(4)-12-3m=0$

$-32-28-12-3m=0$

$-72-3m=0$

$-3m=+72$

$$m = \frac{72}{-3}$$

$$m = -24$$

Q.5 Determine the value of k if $P(x) = kx^3 + 4x^2 + 3x - 4$ and $q(x) = x^3 - 4x + k$. Leaves the same remainder when divided by x – 3.

Sol:

K = ?

When p(x) is divided by (x–3) by remainder theorem then remainder is

$$R_1 = P(3)$$

$$= k(3)^3 + 4(3)^2 + 3(3) - 4$$

$$= 27k + 36 + 9 - 4$$

$$= 27k + 41$$

When q(x) is divided by (x–3) by remainder theorem then remainder is

$$R_2 = q(3)$$

$$q(x) = x^3 - 4x + k$$

$$= (3)^3 - 4(3) + k$$

$$= 27 - 12 + k$$

$$= 15 + k$$

As given that when P(x) and q(x) are divided by x – 3, then remainder is same, so

$$R_1 = R_2$$

$$27k + 41 = 15 + k$$

$$27k - k = 15 - 41$$

$$26k = -26$$

$$k = \frac{-26}{26}$$

$$\boxed{k = -1}$$

Q.6

The remainder of dividing the polynomial $P(x) = x^3 + ax^2 + 7$ by (x + 1) is 2b. calculate the value of 'a' and 'b' if this expression leaves a remainder of (b + 5) on being divided by (x – 2)

Sol:

$$P(x) = x^3 + ax^2 + 7$$

The remainder by dividing $P(x)$ by $x + 1$ is $2b$, so

$$P(-1) = 2b$$

$$(-1)^3 + a(-1)^2 + 7 = 2b$$

$$-1 + a + 7 = 2b$$

$$a + 6 = 2b$$

$$a - 2b = -6 \quad \text{......(i)}$$

Taking $x - 2$

The remainder by dividing $P(x)$ by $(x-2)$ is $(b+5)$, so

$$P(2) = b + 5$$

$$(2)^3 + a(2)^2 + 7 = b + 5$$

$$8 + 4a + 7 = b + 5$$

$$4a + 15 = b + 5$$

$$4a - b = 5 - 15$$

$$4a - b = -10 \quad \text{.....(ii)}$$

Multiplying (ii) by 2

$$8a - 2b = -20 \quad \text{..... (iii)}$$

By Subtracting , (iii) from (i)

$$\begin{array}{rrr} a & -2b & = -6 \\ 8a & \mp 2b & = \mp 20 \\ \hline -7a & & = 14 \end{array}$$

$$a = -\frac{14}{7} = -2$$

Putting (1)

$a-2b=-6$

$-2-2b=-6$

$-2b=-6+2$

$-2b=-4$

$b=2$

Q.7 The polynomial $x^3+\ell x^2+mx+24$ has a factor (x + 4) and it leaves a remainder of 36 when divided by (x–2). Find the value of ℓ and m.

Sol:

Let $P(x)=x^3+\ell x^2+mx+24$

As $(x+4)$ is a factor of $P(x)$,

So remainder will be zero. i.e

$R=P(-4)=0$

$P(-4)=0$

$(-4)^3+\ell(-4)^2+m(-4)+24=0$

$-64+16\ell-4m+24=0$

$16\ell-4m-40=0$

$16\ell-4m=40$

Dividing by 4

$4\ell-m=10$.....(i)

Now as given that P(x) is divided by (x – 2) leaves a remainder 36, so

$R=36$

i.e. $P(2)=36$

$(2)^3+\ell(2)^2+m(2)+24=36$

$8+4\ell+2m+24=36$

$4\ell+2m+32=36$

$4\ell+2m=36-32$

$4\ell+2m=4$

Dividing by 2

$2\ell+m=2$.......(ii)

Adding (i) and (ii)

$$\begin{array}{l} 4\ell-m=10 \\ \underline{2\ell+m=2} \\ 6\ell \quad =12 \end{array}$$

$\ell = \dfrac{12}{6}$

$\ell=2$

Putting value of 'ℓ' in (ii)

$2\ell+m=2$

$2(2)+m=2$

$m=2-4$

$m=-2$

Q.8. The Expression ℓx^3+mx^2-4 leaves remainder of –3 and 12 when divided by (x–1) and (x+2) respectively. Calculate the values of ℓ and m.

Sol:

Let $P(x)=\ell x^3+mx^2-4$

As given that P(x) when divided by x – 1 leaves remainder –3, so

$R=-3$

$P(1)=-3$

$\ell(1)^3+m(1)^2-4=-3$

$\ell+m-4=-3$

$\ell+m=4-3$

$\ell+m=1$...........(i)

As given that P(x) when divided by (x + 2) leaves the remainder 12, so

$R=12$

$P(-2)=12$

$\ell(-2)^3+m(-2)^2-4=12$

$-8\ell+4m-4=12$

$-8\ell+4m=12+4$

$-8\ell+4m=16$

Dividing by 4

$-2\ell+m=4$........(ii)

Subtracting (ii) from (i)

$$\begin{array}{l} \ell+m=1 \\ -2\ell+m=4 \\ \underline{+\quad -\quad -} \\ 3\ell \quad =-3 \end{array}$$

$$\ell=\frac{-3}{3}$$

$$\ell=-1$$

Putting value of 'ℓ' in (i)

$\ell+m=1$

$-1+m=1$

$m=1+1$

$m=2$

Q.9 The expression $ax^3-9x^2+bx+3a$ **is exactly divisible by** x^2-5x+6**. Find the values of a and b**

Sol:

Let $P(x)=ax^3-9x^2+bx+3a$

Taking x^2-5x+6

$=x^2-2x-3x+6$

$=x(x-2)-3(x-2)$

$=(x-2)(x-3)$

As given that P(x) is exactly divisible by (x – 2), so $P(2)=0$

$a(2)^3-9(2)^2+b(2)+3a=0$

$8a-36+2b+3a=0$

$11a+2b=36$.........(i)

As given that P(x) is exactly divisible by x–3, so

$P(3)=0$

$a(3)^3-9(3)^2+b(3)+3a=0$

$27a-81+3b+3a=0$

$30a+3b=81$

Dividing by 3

$10a+b=27$......(ii)

Multiplying (ii) by 2 and subtracting (i) from it.

$$\begin{array}{l} 20a+2b=54 \\ 11a+2b=36 \\ \underline{-\quad -\quad -} \\ 9a \quad =18 \end{array}$$

$$a=\frac{18}{9}$$

$$a=2$$

Putting value of 'a' in (ii)

$10a+b=27$

$10(2)+b=27$

$b=27-20$

$b=7$