Exercise 7.2

Q1. Identify the following statements as True or False.

i) $|x| = 0$ has only one solution. (True)

ii) All absolute value equations have two solutions. (False)

iii) The equation $|x| = 2$ is equivalent to $x = 2$ or $x = -2$. (True)

iv) The equation $|x-4|=-4$ has no solution. (True)

v) The equation $|2x-3|=5$ is equivalent to $2x-3=5$ or $2x+3=5$ (False.)

Q2. Solve for 'x'.

i) $|3x-5|=4$

$\Rightarrow \quad +(3x-5)=4 \quad$ or $\quad -(3x-5)=4$

$3x-5=4 \quad$ or $\quad 3x-5=-4$

$3x=4+5 \quad$ or $\quad 3x=-4+5$

$3x=9 \quad$ or $\quad 3x=1$

$x=3 \quad$ or $\quad x=\frac{1}{3}$

Check:

Substituting $x=3$ in given equation

$|3(3)-5|=4$

$|9-5|=4$

$|4|=4$

$4=4$ which is true

Putting $x=\frac{1}{3}$ in given equation

$\left|3\left(\frac{1}{3}\right)-5\right|=4$

$|1-5|=4$

$|-4|=4$

$4=4$ which is true, so

Solution Set $=\left\{3,\frac{1}{3}\right\}$

ii) $\frac{1}{2}|3x+2|-4=11$

$\frac{1}{2}|3x+2|=11+4$

$\frac{1}{2}|3x+2|=15$

$|3x+2|=15\times 2$

$|3x+2|=30$

$+(3x+2)=30 \quad$ or $\quad -(3x+2)=30$

$3x+2=30 \quad$ or $\quad 3x+2=-30$

$3x=30-2 \quad$ or $\quad 3x=-30-2$

$3x=28 \quad$ or $\quad 3x=-32$

$x=\frac{28}{3} \quad$ or $\quad x=\frac{-32}{3}$

Check:

Putting $x=\frac{28}{3}$ in the given equation

$\frac{1}{2}\left|\cancel{3}\left(\frac{28}{\cancel{3}}\right)+2\right|-4=11$

$\frac{1}{2}|28+2|-4=11$

$\frac{1}{2}|30|-4=11$

$\frac{1}{2}(30)-4=11$

$15-4=11$

$11=11$ which is true

Now putting $x=-\frac{32}{3}$ in the given equation.

$\frac{1}{2}\left|\cancel{3}\left(-\frac{32}{\cancel{3}}\right)+2\right|-4=11$

$\frac{1}{2}|-32+2|-4=11$

$\frac{1}{2}|-30|-4=11$

$\frac{1}{2}(30)-4=11$

$15-4=11$

$11=11$ which is true, so

Hence Solution Set $=\left\{\frac{28}{3}, -\frac{32}{3}\right\}$

iii) $|2x+5|=11$

$+(2x+5)=11$ or $-(2x+5)=11$

$2x+5=11$ or $2x+5=-11$

$2x=11-5$ or $2x=-11-5$

$2x=6$ or $2x=-16$

$x=\frac{6}{2}$ or $x=\frac{-16}{2}$

$x=3$ or $x=-8$

Check:

Putting $x=3$ in the given equation.

$|2(3)+5|=11$

$|6+5|=11$

$|11|=11$

$11=11$ which is true

Now putting $x=-8$ in the given equation.

$|2(-8)+5|=11$

$|-16+5|=11$

$|-11|=11$

$11=11$ which is true, so

Solution Set $=\{3, -8\}$

iv) $|3+2x|=|6x-7|$

$\frac{|3+2x|}{|6x-7|}=1$

$\left|\frac{3+2x}{6x-7}\right|=1$

$+\left(\frac{3+2x}{6x-7}\right)=1$ or $-\left(\frac{3+2x}{6x-7}\right)=1$

$\frac{3+2x}{6x-7}=1$ or $\frac{3+2x}{6x-7}=-1$

$3+2x=6x-7$ or $3+2x=-6x+7$

$3+7=6x-2x$ or $2x+6x=7-3$

$10=4x$ or $8x=4$

$\Rightarrow x=\frac{10}{4}$ or $x=\frac{4}{8}$

$x=\frac{5}{2}$ or $x=\frac{1}{2}$

Check:

Putting $x=\frac{5}{2}$ in the given equation

$\left|3+2\left(\frac{5}{2}\right)\right|=\left|3(6)\left(\frac{5}{2}\right)-7\right|$

$|3+5|=|15-7|$

$|8|=|8|$

$8=8$ which is true

Now putting $x=\frac{1}{2}$ in the given equation

$\left|3+2\left(\frac{1}{2}\right)\right|=\left|6\left(\frac{1}{2}\right)-7\right|$

$|3+1|=|3-7|$

$|4|=|-4|$

$4=4$ which is true, so

Solution Set $=\left\{\frac{5}{2}, \frac{1}{2}\right\}$

v) $|x+2|-3=5-|x-2|$

$|x+2|+|x+2|=5+3$

$2|x+2|=8$

$|x+2|=\frac{8}{2}$

$|x+2|=4$

$+(x+2)=4$ or $-(x+2)=4$

$x+2=4$ or $x+2=-4$

$x=4-2$ or $x=-4-2$

$x=2$ or $x=-6$

Check:

Putting $x=2$ in the give equation

$|2+2|-3=5-|2+2|$

$|4|-3=5-|4|$

$4-3=5-4$

$1=1$ which is true

Now putting $x=-6$ in the given equation.

$|-6+2|-3=5-|-6+2|$

$|-4|-3=5-|-4|$

$4-3=5-4$

$1=1$ which is true, so

Solution Set $=\{2,-6\}$

vi) $\frac{1}{2}|x+3|+21=9$

$\frac{1}{2}|x+3|=9-21$

$\frac{1}{2}|x+3|=-12$

$|x+3|=-24$

As the value of absolute cannot be negative, so Solution Set $=\{\ \}$

vii) $\left|\frac{3-5x}{4}\right|-\frac{1}{3}=\frac{2}{3}$

$\left|\frac{3-5x}{4}\right|=\frac{2}{3}+\frac{1}{3}$

$\left|\frac{3-5x}{4}\right|=\frac{\cancel{3}}{\cancel{3}}$

$\left|\frac{3-5x}{4}\right|=1$

$+\left(\frac{3-5x}{4}\right)=1$ or $-\left(\frac{3-5x}{4}\right)=1$

$\frac{3-5x}{4}=1$ or $\frac{3-5x}{4}=-1$

$3-5x=4$ or $3-5x=-4$

$3-4=5x$ or $3+4=5x$

$-1=5x$ or $7=5x$

$x=-\frac{1}{5}$ or $x=\frac{7}{5}$

Check:

Putting $x=-\frac{1}{5}$ in the given equation

$$\left|\frac{3-5\left(-\frac{1}{5}\right)}{4}\right|-\frac{1}{3}=\frac{2}{3}$$

$\left|\frac{3+1}{4}\right|-\frac{1}{3}=\frac{2}{3}$

$\left|\frac{4}{4}\right|-\frac{1}{3}=\frac{2}{3}$

$|1|-\frac{1}{3}=\frac{2}{3}$

$1-\frac{1}{3}=\frac{2}{3}$

$\frac{3-1}{3}=\frac{2}{3}$

$\frac{2}{3}=\frac{2}{3}$ which is true,

Now putting $x=\frac{7}{5}$ in the given equation

$$\left|\frac{3-\cancel{5}\left(\frac{7}{\cancel{5}}\right)}{4}\right|-\frac{1}{3}=\frac{2}{3}$$

$$\left|\frac{3-7}{4}\right|-\frac{1}{3}=\frac{2}{3}$$

$$\left|-\frac{4}{4}\right|-\frac{1}{3}=\frac{2}{3}$$

$$|-1|-\frac{1}{3}=\frac{2}{3}$$

$$1-\frac{1}{3}=\frac{2}{3}$$

$$\frac{2}{3}=\frac{2}{3}$$ which is true

So, solution set = $\left\{-\frac{1}{5},\frac{7}{5}\right\}$

viii) $\left|\frac{x+5}{2-x}\right|=6$

$+\left(\frac{x+5}{2-x}\right)=6$	or	$-\left(\frac{x+5}{2-x}\right)=6$
$\frac{x+5}{2-x}=6$	or	$\frac{x+5}{2-x}=-6$
$x+5=12-6x$	or	$x+5=12+6x$
$x+6x=12-5$	or	$5+12=6x-x$
$7x=7$	or	$17=5x$
$x=1$	or	$x=\frac{17}{5}$

Check:

Putting $x=1$ in the given equation.

$$\left|\frac{1+5}{2-1}\right|=6$$

$$\left|\frac{6}{1}\right|=6$$

$$|6|=6$$

$$6=6$$

Now putting $x=\frac{17}{5}$ in the given equation

$$\left|\frac{\frac{17}{5}+5}{2-\frac{17}{5}}\right|=6$$

$$\left|\frac{\frac{17+25}{5}}{\frac{10-17}{5}}\right|=6$$

$$\left|\frac{42}{-7}\right|=6$$

$$|-6|=6$$

$6=6$ which is true

So, solution set = $\left\{1,\frac{17}{5}\right\}$