

Exercise 4.1

1. Identify whether the following algebraic expression are polynomials (yes or no).

(i) $3x^2 + \frac{1}{x} - 5$ No

(ii) $3x^3 - 4x^2 - x\sqrt{x} + 3$ No

(iii) $x^2 - 3x + \sqrt{2}$ Yes

(iv) $\frac{3x}{2x-1} + 8$ No

2. State whether each of the following expression is a rational expression or not.

(i) $\frac{3\sqrt{x}}{3\sqrt{x} + 5}$ No

(ii) $\frac{x^3 - 2x^2 + \sqrt{3}}{2 + 3x - x^2}$ Yes

(iii) $\frac{x^2 + 6x + 9}{x^2 - 9}$ Yes

$$(iv) \frac{2\sqrt{x}+3}{2\sqrt{x}-3} \quad \text{No}$$

3. Reduce the following rational expression to the lowest forms.

$$(i) \frac{120x^2y^3z^5}{30x^3yz^2}$$

$$= 4x^{2-3}y^{3-1}z^{5-2}$$

$$= 4x^{-1}y^2z^3$$

$$= \frac{4y^2z^3}{x}$$

$$(ii) \frac{8a(x+1)}{2(x^2-1)} = \frac{4a(\cancel{x+1})}{(x-1)(\cancel{x+1})} = \frac{4a}{x-1}$$

$$(iii) \frac{(x+y)^2-4xy}{(x-y)^2} = \frac{x^2+y^2+2xy-4xy}{(x-y)(x-y)}$$

$$= \frac{x^2+y^2-2xy}{(x-y)(x-y)}$$

$$= \frac{(x-y)^2}{(x-y)(x-y)}$$

$$= \frac{(\cancel{x-y})^2}{(\cancel{x-y})^2} = 1$$

$$(iv) \frac{(x^3-y^3)(x^2-2xy+y^2)}{(x-y)(x^2+xy+y^2)}$$

$$= \frac{(\cancel{x^3-y^3})(x-y)^2}{\cancel{x^3-y^3}} = (x-y)^2$$

$$(v) \frac{(x+2)(x^2-1)}{(x+1)(x^2-4)}$$

$$= \frac{(\cancel{x+2})(x-1)(\cancel{x+1})}{(\cancel{x+1})(x-2)(\cancel{x+2})} = \frac{x-1}{x-2}$$

$$(vi) \frac{x^2-4x+4}{2x^2-8} = \frac{(x-2)^2}{2(x^2-4)}$$

$$= \frac{(x-2)^2}{2(x-2)(x+2)}$$

$$= \frac{(\cancel{x-2})(x-2)}{2(\cancel{x-2})(x+2)}$$

$$= \frac{x-2}{2(x+2)}$$

$$(vii) \frac{64x^5-64x}{(8x^2+8)(2x+2)}$$

$$= \frac{64x(x^4-1)}{8(x^2+1) \cdot 2(x+1)}$$

$$= \frac{64x(x^4-1)}{16(x^2+1)(x+1)}$$

$$= \frac{4x(x^2+1)(x^2-1)}{(x^2+1)(x+1)}$$

$$= \frac{4x(\cancel{x^2+1})(x-1)(\cancel{x+1})}{(\cancel{x^2+1})(\cancel{x+1})}$$

$$= 4x(x-1)$$

$$\frac{9x^2-(x^2-4)^2}{4+3x-x^2} = \frac{(3x)^2-(x^2-4)^2}{4+3x-x^2}$$

$$= \frac{(3x+x^2-4)(\cancel{3x-x^2}+4)}{(\cancel{4+3x-x^2})}$$

$$= 3x+x^2-4$$

$$= x^2+3x-4$$

4. Evaluate (a) $\frac{x^3y-2z}{xz}$ for (i) $x=3$

$$y=-1, z=-2.$$

$$(a) \frac{(3)^3(-1)-2(-2)}{3(-2)} = \frac{-27+4}{-6}$$

$$= \frac{-23}{-6} = \frac{23}{6} = 3\frac{5}{6}$$

$$\begin{aligned}
 \text{(b)} \quad & \frac{x^2y^3 - 5z^4}{xyz} \text{ for } x = 4, y = -2, z = -1 \\
 &= \frac{(4)^2(-2)^3 - 5(-1)^4}{(4)(-2)(-1)} = \frac{-16(8) - 5}{8} \\
 &= \frac{-128 - 5}{8} = \frac{-133}{8} = -16\frac{5}{8}
 \end{aligned}$$

5. Perform the indicated operation and simplify

$$\begin{aligned}
 \text{(i)} \quad & \frac{15}{2x-3y} - \frac{4}{3y-2x} \\
 &= \frac{15(3y-2x) - 4(2x-3y)}{(2x-3y)(3y-2x)} \\
 &= \frac{45y - 30x - 8x + 12y}{(2x-3y)(3y-2x)} \\
 &= \frac{57y - 38x}{(2x-3y)(3y-2x)} \\
 &= \frac{19(3y-2x)}{(2x-3y)(3y-2x)} = \frac{19}{2x-3y}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad & \frac{1+2x}{1-2x} - \frac{1-2x}{1+2x} \\
 &= \frac{(1+2x)^2 - (1-2x)^2}{(1-2x)(1+2x)} \\
 &= \frac{(1+4x^2+4x) - (1+4x^2-4x)}{(1-2x)(1+2x)} \\
 &= \frac{1+4x^2+4x-1-4x^2+4x}{(1-2x)(1+2x)} \\
 &= \frac{8x}{(1-2x)(1+2x)} = \frac{8x}{1-4x^2}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad & \frac{x^2-25}{x^2-36} - \frac{x+5}{x+6} \\
 &= \frac{(x-5)(x+5)}{(x-6)(x+6)} - \frac{x+5}{x+6} \\
 &= \frac{(x-5)(x+5) - (x+5)(x-6)}{(x-6)(x+6)}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{(x-5)(x+5) - (x+5)(x-6)}{(x+6)(x-6)} \\
 &= \frac{(x+5)[(x-5) - (x-6)]}{(x+6)(x-6)} \\
 &= \frac{(x+5)(\cancel{x-5} + \cancel{x+6})}{(x+6)(x-6)} \\
 &= \frac{(x+5)(1)}{(x+6)(x-6)} = \frac{x+5}{x^2-36}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad & \frac{x}{x-y} - \frac{y}{x+y} - \frac{2xy}{x^2-y^2} \\
 &= \frac{x(x+y) - y(x-y)}{(x-y)(x+y)} - \frac{2xy}{x^2-y^2} \\
 &= \frac{x^2 + \cancel{xy} - \cancel{xy} + y^2}{x^2-y^2} - \frac{2xy}{x^2-y^2} \\
 &= \frac{x^2+y^2-2xy}{x^2-y^2} \\
 &= \frac{(x-y)^2}{(x-y)(x+y)} = \frac{x-y}{x+y}
 \end{aligned}$$

$$\begin{aligned}
 \text{(v)} \quad & \frac{x-2}{x^2+6x+9} - \frac{x+2}{2x^2-18} \\
 &= \frac{x-2}{x^2+3x+3x+9} - \frac{x+2}{2(x^2-9)} \\
 &= \frac{x-2}{x(x+3)+3(x+3)} - \frac{x+2}{2(x-3)(x+3)} \\
 &= \frac{x-2}{(x+3)(x+3)} - \frac{x+2}{2(x-3)(x+3)} \\
 &= \frac{2(x-3)(x-2) - (x+3)(x+2)}{2(x-3)(x+3)(x+3)} \\
 &= \frac{2(x^2-2x-3x+6) - (x^2+2x+3x+6)}{2(x-3)(x+3)^2}
 \end{aligned}$$

$$= \frac{2(x^2 - 5x + 6) - (x^2 + 5x + 6)}{2(x-3)(x+3)^2}$$

$$= \frac{2x^2 - 10x + 12 - x^2 - 5x - 6}{2(x-3)(x+3)^2}$$

$$= \frac{x^2 - 15x + 6}{2(x-3)(x+3)^2}$$

(vi)

$$\frac{1}{x-1} - \frac{1}{x+1} - \frac{2}{x^2+1} - \frac{4}{x^4-1}$$

$$= \frac{x+1-(x-1)}{(x-1)(x+1)} - \frac{2}{x^2+1} - \frac{4}{x^4-1}$$

$$= \frac{\cancel{x+1} - \cancel{x+1}}{x^2-1} - \frac{2}{x^2+1} - \frac{4}{x^4-1}$$

$$= \frac{2}{x^2-1} - \frac{2}{x^2+1} - \frac{4}{x^4-1}$$

$$= \frac{2(x^2+1)-2(x^2-1)}{(x^2-1)(x^2+1)} - \frac{4}{x^4-1}$$

$$= \frac{\cancel{2x^2} + 2 - \cancel{2x^2} + 2}{x^4-1} - \frac{4}{x^4-1}$$

$$= \frac{4}{x^4-1} - \frac{4}{x^4-1}$$

$$= \frac{4-4}{x^4-1}$$

$$= \frac{0}{x^4-1}$$

$$= 0$$

6. Perform the indicated operation and simplify:

(i) $(x^2 - 49) \frac{5x+2}{x+7}$

$$= (x-7) \cancel{(x+7)} \frac{5x+2}{\cancel{x+7}}$$

$$= (x-7)(5x+2)$$

(ii)

$$\frac{4x-12}{x^2-9} \div \frac{18-2x^2}{x^2+6x+9}$$

$$= \frac{4(x-3)}{(x-3)(x+3)} \div \frac{2(9-x^2)}{x^2+3x+3x+9}$$

$$= \frac{4(x-3)}{(x-3)(x+3)} \div \frac{2(3-x)(3+x)}{x(x+3)+3(x+3)}$$

$$= \frac{4(x-3)}{(x-3)(x+3)} \div \frac{2(3-x)(3+x)}{(x+3)(x+3)}$$

$$= \frac{4(x-3)}{(x+3)(x-3)} \times \frac{(x+3)(x+3)}{2(3+x)(3-x)}$$

$$= \frac{2}{3-x}$$

(iii)

$$\frac{x^6-y^6}{x^2-y^2} \div (x^4+x^2y^2+y^4)$$

$$= \frac{(x^3)^2 - (y^3)^2}{x^2-y^2} \div (x^4+x^2y^2+y^4)$$

$$= \frac{(x^3-y^3)(x^3+y^3)}{x^2-y^2} \div (x^4+x^2y^2+y^4)$$

$$= \frac{(x-y)(x^2+xy+y^2)(x+y)(x^2-xy+y^2)}{x^2-y^2}$$

$$\times \frac{1}{x^4+x^2y^2+y^4}$$

$$= \frac{\cancel{(x^2-y^2)}(x^2+xy+y^2)(x^2-xy+y^2)}{\cancel{x^2-y^2}}$$

$$\times \frac{1}{x^4+x^2y^2+y^4}$$

$$= \frac{x^4+x^2y^2+y^4}{x^4+x^2y^2+y^4} = 1$$

(iv)

$$\frac{x^2-1}{x^2+2x+1} \cdot \frac{x+5}{1-x}$$

$$= \frac{-(\cancel{x-1})(x+1)}{x^2+x+x+1} \cdot \frac{x+5}{(x-1)}$$

$$= \frac{-(x+1)(x+5)}{x(x+1)+1(x+1)}$$

$$= \frac{\cancel{-(x+1)}(x+5)}{(x+1)\cancel{(x+1)}} = -\frac{x+5}{x+1}$$

(v)
$$\frac{x^2+xy}{y(x+y)} \cdot \frac{x^2+xy}{y(x+y)} + \frac{x^2-x}{xy-2y}$$

$$= \frac{\cancel{x}(\cancel{x+y})}{y(\cancel{x+y})} \cdot \frac{\cancel{x}(\cancel{x+y})}{\cancel{y}(\cancel{x+y})} \times \frac{\cancel{y}(x-2)}{\cancel{x}(x-1)}$$

$$= \frac{x(x-2)}{y(x-1)}$$