Exercise 5.4

Q.No.1 $\dfrac{x^3+2x+2}{(x^2+x+1)^2}$

Consider $\dfrac{x^3+2x+2}{(x^2+x+1)^2} = \dfrac{Ax+B}{x^2+x+1} + \dfrac{Cx+D}{(x^2+x+1)^2}$

xing both sides by $(x^2+x+1)^2$

$$x^3+2x+2 = (Ax+B)(x^2+x+1) + Cx+D.$$

$$\Rightarrow x^3+2x+2 = A(x^3+x^2+x) + B(x^2+x+1) + Cx+D.$$

Comparing co-effients of x^3, x^2, x and x^0.

$1 = A$ ——— (i)

$0 = A+B$ ——— (ii)

$2 = A+B+C$ ——— (iii)

$2 = B+D$ ——— (iv)

from eq. (i) $\boxed{A=1}$ putting in (ii)

$0 = 1+B \Rightarrow \boxed{B=-1}$

putting value of A and B in (iii)

$2 = 1-1+C \Rightarrow \boxed{C=2}$

putting value of B in eq. (iv)

$2 = -1+D \Rightarrow 2+1 = D \Rightarrow \boxed{D=3}$

hence $\dfrac{x^3+2x+2}{(x^2+x+1)^2} = \dfrac{(1)x-1}{x^2+x+1} + \dfrac{2x+3}{(x^2+x+1)^2}$

$= \dfrac{x-1}{x^2+x+1} + \dfrac{2x+3}{(x^2+x+1)^2}$ Answer

Q.No 2 $\dfrac{x^2}{(x^2+1)^2(x-1)}$

Consider $\dfrac{x^2}{(x^2+1)^2(x-1)} = \dfrac{Ax+B}{x^2+1} + \dfrac{Cx+D}{(x^2+1)^2} + \dfrac{E}{x-1}$

Multiplying both sides by $(x^2+1)^2(x-1)$

$$x^2 = (Ax+B)(x^2+1)(x-1) + (Cx+D)(x-1) + E(x^2+1)^2 \text{ —— (i)}$$

put $x-1=0 \Rightarrow x=1$ in eq (i)

$(1)^2 = 0+0+E((1)^2+1)^2 \Rightarrow 1 = E(2)^2 \Rightarrow 1 = 4E$

$\Rightarrow \boxed{E = 1/4}$

equation (i) can be written as

$$x^2 = (Ax+B)(x^3+x-x^2-1) + (Cx+D)(x-1) + E(x^4+2x^2+1)$$

$$\Rightarrow x^2 = A(x^4+x^2-x^3-x) + B(x^3+x-x^2-1)$$
$$+ C(x^2-x) + D(x-1) + E(x^4+2x^2+1)$$

Comparing co-effients of x^4, x^3, x^2, x and x^0.

$$0 = A + E \quad \text{———— (ii)}$$
$$0 = -A + B \quad \text{———— (iii)}$$
$$1 = A - B + C + 2E \quad \text{———— (iv)}$$
$$0 = -A + B - C + D \quad \text{———— (v)}$$
$$0 = -B \mp D + E \quad \text{———— (vi)}$$

putting $E = \frac{1}{4}$ in eq (ii)

$$0 = A + \frac{1}{4} \Rightarrow \boxed{A = -\frac{1}{4}}$$

putting value of A in (iii)

$$0 = -\left(-\frac{1}{4}\right) + B \Rightarrow 0 = \frac{1}{4} + B \Rightarrow \boxed{B = \frac{-1}{4}}$$

putting value of A, B and E in (iv)

$$1 = -\frac{1}{4} - \left(\frac{-1}{4}\right) + C + 2\left(\frac{1}{4}\right)$$

$$\Rightarrow 1 = -\frac{1}{4} + \frac{1}{4} + C + \frac{1}{2} \Rightarrow 1 + \frac{1}{4} \mp \frac{1}{4} - \frac{1}{2} = C$$

$$\Rightarrow \boxed{C = \frac{1}{2}}$$

putting value of B and E in (vi)

$$0 = -\left(-\frac{1}{4}\right) \mp D + \frac{1}{4} \Rightarrow 0 = \frac{1}{4} \mp D + \frac{1}{4}$$

$$\Rightarrow 0 = \frac{1}{2} \mp D \Rightarrow \boxed{D = \frac{1}{2}}$$

Hence
$$\frac{x^2}{(x^2+1)^2(x-1)} = \frac{-\frac{1}{4}x - \frac{1}{4}}{x^2+1} + \frac{\frac{1}{2}x + \frac{1}{2}}{(x^2+1)^2} + \frac{\frac{1}{4}}{x-1}$$

$$= \frac{\frac{-x-4}{4}}{x^2+1} + \frac{\frac{x+2}{2}}{(x^2+1)^2} + \frac{\frac{1}{4}}{x-1}$$

$$= -\frac{x+4}{4(x^2+1)} + \frac{x+2}{2(x^2+1)^2} + \frac{1}{4(x-1)}$$

Answer

———— x ———— x ————

Q No. 3 $\dfrac{2x-5}{(x^2+2)^2(x-2)}$

Consider

$$\frac{2x-5}{(x^2+2)^2(x-2)} = \frac{Ax+B}{x^2+2} + \frac{Cx+D}{(x^2+2)^2} + \frac{E}{x-2}$$

[Do yourself same as Q No. 2 you will get
$A = \frac{1}{36}$, $B = \frac{1}{18}$, $C = \frac{1}{6}$, $D = \frac{7}{3}$, $E = -\frac{1}{36}$]

Q No. 4 $\dfrac{8x^2}{(x^2+1)^2(1-x^2)}$

$$= \frac{8x^2}{(x^2+1)^2(1-x)(1+x)}$$

Resolving it into partial fraction.

$$\frac{8x^2}{(x^2+1)^2(1-x)(1+x)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{(x^2+1)^2} + \frac{E}{1-x} + \frac{F}{1+x}$$

$$\Rightarrow 8x^2 = (Ax+B)(x^2+1)(1-x)(1+x) + (Cx+D)(1-x)(1+x)$$
$$+E(x^2+1)^2(1+x) + F(x^2+1)^2(1-x) \text{ ——— (i)}$$

put $1-x=0 \Rightarrow x=1$ in eq (i)

$$8(1)^2 = 0 + 0 + E((1)^2+1)^2(1+1) + 0$$

$$\Rightarrow 8 = E(2)^2(2) \Rightarrow 8 = 8E \Rightarrow \boxed{E = 1}$$

put $1+x=0 \Rightarrow x=-1$ in eq (i)

$$8(-1)^2 = 0 + 0 + 0 + F((-1)^2+1)^2(1-(-1))$$

$$\Rightarrow 8(1) = F(1+1)^2(1+1) \Rightarrow 8 = F(4)(2) \Rightarrow \boxed{F = 1}$$

Eq. (i) can be written as

$$8x^2 = (Ax+B)(x^2+1)(1-x^2) + (Cx+D)(1-x^2)$$
$$+E(x^4+2x^2+1)(1+x) + F(x^4+2x^2+1)(1-x)$$

$$\Rightarrow 8x^2 = (Ax+B)(x^2-x^4+1-x^2) + (Cx+D)(1-x^2)$$
$$+E(x^4+2x^2+1+x^5+2x^3+x) + F(x^4+2x^2+1-x^5-2x^3-x)$$

$$\Rightarrow 8x^2 = (Ax+B)(-x^4+1) + (Cx+D)(1-x^2)$$
$$+E(x^5+x^4+2x^3+2x^2+x+1) + F(-x^5+x^4-2x^3+2x^2-x+1)$$

$$\Rightarrow 8x^2 = A(-x^5+x) + B(-x^4+1) + C(x-x^3) + D(1-x^2)$$
$$+E(x^5+x^4+2x^3+2x^2+x+1) + F(-x^5+x^4-2x^3+2x^2-x+1)$$

Comparing co-efficients of x^5, x^4, x^3, x^2, x and x^0

$0 = -A + E - F$ ———— (ii)

$0 = -B + E + F$ ———— (iii)

$0 = -C + 2E - 2F$ ———— (iv)

$8 = -D + 2E + 2F$ ———— (v)

$0 = A + C + E - F$ ———— (vi)

$0 = B + D + E + F$ ———— (vii)

putting value of E and F in (ii)

$0 = -A + 1 - 1 \Rightarrow 0 = -A \Rightarrow \boxed{A = 0}$

putting value of E and F in (iv)

$0 = -C + 2(1) - 2(1) \Rightarrow 0 = -C \Rightarrow \boxed{C = 0}$

putting value of E and F in (iii)

$0 = -B + 1 + 1 \Rightarrow 0 = -B + 2 \Rightarrow \boxed{B = 2}$

putting value of E and F in (v)

$8 = -D + 2(1) + 2(1) \Rightarrow 8 = -D + 4$

$\Rightarrow 8 - 4 = -D \Rightarrow 4 = -D \Rightarrow \boxed{D = -4}$

~~putting value of~~

hence $\dfrac{8x^2}{(x^2+1)^2(1-x)(1+x)} = \dfrac{0.x + 2}{x^2+1} + \dfrac{0.x - 4}{(x^2+1)^2} + \dfrac{1}{1-x} + \dfrac{1}{1+x}$

$$= \frac{0+2}{x^2+1} + \frac{0-4}{(x^2+1)^2} + \frac{1}{1-x} + \frac{1}{1+x}$$

$$= \frac{2}{x^2+1} - \frac{4}{(x^2+1)^2} + \frac{1}{1-x} + \frac{1}{1+x}$$

Answer

———— x ———— x ————

Q No. 5 $\dfrac{4x^4+3x^3+6x^2+5x}{(x-1)(x^2+x+1)^2}$

Consider

$$\frac{4x^4+3x^3+6x^2+5x}{(x-1)(x^2+x+1)^2} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1} + \frac{Dx+E}{(x^2+x+1)^2}$$

Multiplying both sides by $(x-1)(x^2+x+1)^2$

$4x^4+3x^3+6x^2+5x = A(x^2+x+1)^2 + (Bx+C)(x-1)(x^2+x+1) +$

$+ (Dx+E)(x-1)$ ———— (i)

Put $x - 1 = 0 \Rightarrow x = 1$ in eq (i)

$4(1)^4 + 3(1)^3 + 6(1)^2 + 5(1) = A((1)^2+1+1)^2 + 0 + 0$

$\Rightarrow 4+3+6+5 = A(1+1+1)^2$

$\Rightarrow 18 = 9A \Rightarrow \boxed{A=2}$

eq. (i) can be written as

$4x^4+3x^3+6x^2+5x = A(x^4+x^2+1+2x^3+2x+2x^2)$
$+(Bx+C)(x^3+x^2+x-x^2-x-1)+(Dx+E)(x-1)$

$\Rightarrow 4x^4+3x^3+6x^2+5x = A(x^4+2x^3+3x^2+2x+1)+(Bx+C)(x^3-1)$
$+(Dx+E)(x-1)$

$\Rightarrow 4x^4+3x^3+6x^2+5x = A(x^4+2x^3+3x^2+2x+1)+B(x^4-x)$
$+C(x^3-1)+D(x^2-x)+E(x-1)$

Comparing co-effients of x^4, x^3, x^2, x and x^0

$4 = A+B$ ——— (ii)

$3 = 2A+C$ ——— (iii)

$6 = 3A+D$ ——— (iv)

$5 = 2A-B-D+E$ ——— (v)

$0 = A-C-E$ ——— (vi)

put $A=2$ in eq (ii)

$4=2+B \Rightarrow 4-2=B \Rightarrow \boxed{B=2}$

put $A=2$ in eq (iii)

$3=2(2)+C \Rightarrow 3-4=C \Rightarrow \boxed{C=-1}$

Put $A=2$ in eq (iv)

$6=3(2)+D \Rightarrow 6-6=D \Rightarrow \boxed{D=0}$

Put $A=2$ and $C=-1$ in eq (vi)

$0=2-(-1)-E \Rightarrow 0=2+1-E \Rightarrow \boxed{E=3}$

So

$$\frac{4x^4+3x^3+6x^2+5x}{(x-1)(x^2+x+1)^2} = \frac{2}{x-1}+\frac{2x-1}{x^2+x+1}+\frac{0.x+3}{(x^2+x+1)^2}$$

$$= \frac{2}{x-1}+\frac{2x-1}{x^2+x+1}+\frac{3}{(x^2+x+1)^2}$$ Answer

Question # 6:

Do yourself.