# ANGLE IN A SEGMENT OF A CIRCLE

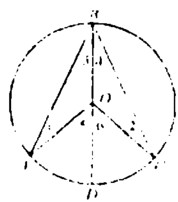
**In this unit, students will learn how to:**

Apply laws of logarithm to convert length processes, of multiplication.

- The measure of a central angle of a minor arc of a circle, is double that O of the angle subtended by the corresponding major arc.
- Any two angles in the same segment of a circle are equal.
- The angle
    - in a semi-circle is a right angle
    - in a segment, greater than a semi circle is less than a right angle
    - in a segment less than a semi-circle is greater than a right angle
- The opposite angles of any quadrilateral inscribed in a circle are supplementary.

## THEOREM 1

**12.1 (i) The measure of a central angle of a minor arc of a circle, is double that of the angle subtended by the corresponding major arc.**



**Given:**

AC is an arc of a circle with centre O.

Whereas $\angle AOC$ is the central angle

and $\angle ABC$ is circum angle.

**To prove:**

$m\angle AOC = 2m\angle ABC$

**Construction:**

Join B with O and produce it to meet the circle at D.

Write angles $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$ $\angle 5$ and $\angle 6$ as shown in the figure.

**Proof**

| Statements | Reasons |
|---|---|
| As $m\angle 1 = m\angle 3$ (i) | Angles opposite to equal sides in $\Delta OAB$ |
| and $m\angle 2 = m\angle 4$ (ii) | Angles opposite to equal sides in $\Delta OBC$ |
| Now $m\angle 5 = m\angle 1 + m\angle 3$ (iii) | External angle is the sum of internal opposite angles. |
| Similarly $m\angle 6 = m\angle 2 + m\angle 4$ (iv) | |
| Again $m\angle 5 = m\angle 3 + m\angle 3 = 2m\angle 3$ (v) | Using (i) and (iii) |
| and $m\angle 6 = m\angle 4 + m\angle 4 = 2m\angle 4$ (vi) | Using (ii) and (iv) |
| Then from figure | |

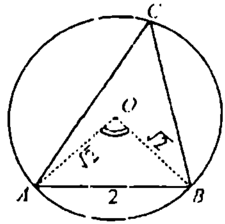
| | |
|---|---|
| $\Rightarrow m\angle 5 + m\angle 6 = 2m\angle 3 + 2m\angle 4$ <br> $\Rightarrow m\angle AOC = 2(m\angle 3 + m\angle 4) = 2m\angle ABC$ | Adding (v) and (vi) |

**Example:**

**The radius of a circle is $\sqrt{2}$ cm. A chord 2 cm in length divides the circle into two segments. Prove that the angle of large segment is 45°.**

**Given:**

In a circle with centre O and radius m OA = m OB = $\sqrt{2}$ cm,
The length of chord AB = 2 cm divides the circle info two segments with ACB as larger one.




**To prove:**

$m\angle ACB = 45°$

**Construction:**

Join O with A and O with B.

**Proof:**

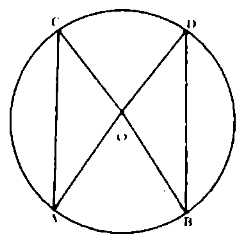
| Statements | Reasons |
|---|---|
| In Δ OAB | $m\overline{OA} = m\overline{OB} = \sqrt{2}$cm |
| $(\overline{OA})^2 + (\overline{OB})^2 = (\sqrt{2})^2 + (\sqrt{2})^2$ $= 2 + 2 = 4$ | Given m AB = 1 cm |
| $= (2)^2 = (\overline{AB})^2$ | Which being a central angle standing on an arc AB |
| ΔAOB is right angled triangle With $m\angle AOB = 90°$ | By theorem 1 |
| Then $m\angle ACB = \frac{1}{2} m\angle AOB$ $= \frac{1}{2}(90°) = 45°$ | Circum angle is half of the centre angle. |

# THEOREM 2

**12.1 (ii) Any two angles in the same segment of a circle are equal.**

**Given:**

$\angle ACB$ and $\angle ADB$ are the circum angles in the same segment of a circle with centre O.




**To prove:**

$m\angle ACB = m\angle ADB$

**Construction:**

Join O with A and O with B.
So that $\angle AOB$ is the central angle.

**Proof:**

| Statements | Reasons |
|---|---|
| Standing on the same arc AB of a circle. | Construction |
| $\angle AOB$ is the central angle whereas | |
| $\angle ACB$ and $\angle ADB$ are circum angles | Given |
| $m\angle AOB = 2m\angle ACB$ (i) | |
| and $m\angle AOB = 2m\angle ADB$ (ii) | By theorem 1 |
| $\Rightarrow 2\angle ACB = 2m\angle ADB$ | By theorem 1 |
| Hence, $m\angle ACB = m\angle ADB$ | Using (i) and (ii) |

# THEOREM 3

**12.1 (iii)The angle**

- **in a semi-circle is a right angle,**
- **in a segment greater than a semi circle is less than a right angle,**
- **in a segment less than a semi-circle is greater than a right angle;**

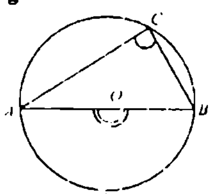


**Fig. I**

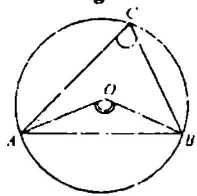


**Fig. II**

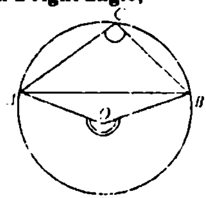


**Fig.III**

**Given:**

$\overline{AB}$ is the chord corresponding to an arc ADB

Whereas $\angle$ AOB is a central angle and $\angle$ ACB is a circum angle of a circle with centre O.

**To prove:**

In fig. (I) If sector ACB is a semi circle then $m\angle ACB = 1\angle rt$

In fig (II) If sector ACB is greater than a semi circle then $m\angle ACB < \angle 1\angle rt$

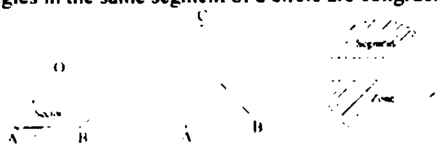
In fig (III) If sector ACB is less than a semi circle then $m\angle ACB > 1\angle rt$

**Proof:**

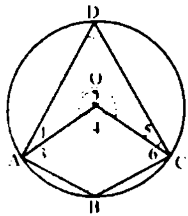
| Statements | Reasons |
|---|---|
| In each figure, $\overline{AB}$ is the chord of a circle with centre O. | Given |
| $\angle AOB$ is the central angle standing on an arc ADB. | |
| Whereas $\angle ACB$ is the circum angle | Given |
| Such that $m\angle AOB = 2m\angle ACB$ (i) | By theorem 1 |
| Now in fig (I) $m\angle AOB = 180°$ | A straight angle |

| | |
|---|---|
| $m\angle AOB = 2\angle rt$ (ii) | |
| $\Rightarrow m\angle ACB = 1\angle rt$ | Using (i) and (ii) |
| In fig (II) $m\angle AOB < 180^\circ$ | |
| $m\angle AOB < 2\angle rt$ (iii) | Using (i) and (iii) |
| $\Rightarrow m\angle ACB < 1\angle rt$ (iv) | |
| In fig (III) $m\angle AOB > 180^\circ$ | |
| $\therefore m\angle AOB > 2\angle rt$ | |
| $\Rightarrow m\angle ACB > 1\angle rt$ | Using (i) and (iv) |

**Corollary 1.** The angles subtended by an arc at the circumference of a circle are equal.
**Corollary 2.** The angles in the same segment of a circle are congruent.



**12.1 (iv)The opposite angles of any quadrilateral inscribed in a circle are supplementary.**




**Given:**
ABCD is a quadrilateral inscribed in a circle with centre O.

**To prove:**

$$\begin{cases} m\angle A + m\angle C = 2\angle rts \\ m\angle B + m\angle D = 2\angle rts \end{cases}$$

**Construction:**
Draw $\overline{OA}$ and $\overline{OC}$.

Write $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$, $\angle 5$ and $\angle 6$ as shown in the figure.

**Proof:**

| Statements | Reasons |
|---|---|
| Standing on the same arc ADC, $\angle 2$ is a central angle | Arc ADC of the circle with centre O. |

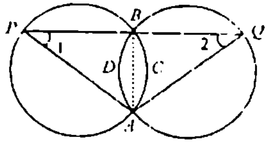
| | |
|---|---|
| Whereas $\angle B$ is the circum angle<br>$m\angle B = \frac{1}{2}(m\angle 2)$ (i) | By theorem 1 |
| Standing on the same arc ABC, $\angle 4$ is a central angle whereas $\angle D$) is the circum angle | Arc ABC of the circle with centre O. |
| $m\angle D = \frac{1}{2}(m\angle 4)$ (ii) | By theorem 1 |
| $\Rightarrow \quad m\angle B + m\angle D = \frac{1}{2}m\angle 2 + \frac{1}{2}m\angle 4$<br>$= \frac{1}{2}(m\angle 2 + m\angle 4) = \frac{1}{2}(\text{Total central angle})$<br>i.e., $m\angle B + m\angle D = \frac{1}{2}(4\angle rt) = 2\angle rt$<br>Similarly $m\angle A + m\angle C = 2\angle rt$ | Adding (i) and (ii) |

**Corollary 1.** In equal circles or in the same circle if two minor arcs are equal then angles inscribed by their corresponding major arcs are also equal.

**Corollary 2.** In equal circles or in the same circle, two equal arcs subtend equal angles at the circumference and vice versa.

**Example 1:**

**Two equal circles intersect in A and B. Through B, a straight line is drawn to meet the circumferences at P and Q respectively. Prove that mAP = mAQ.**




**Given:**

Two equal circles cut each other at points A and B. A straight the PBQ drawn through B meets the circles at P and Q respectively.

**To prove:**

$m\overline{AP} = m\overline{AQ}$

**Construction:**

Join the points A and B. Also $\overline{AP}$ and $\overline{AQ}$

Write $\angle 1$ and $\angle 2$ as shown in the figure.

**Proof:**

| Statements | Reasons |
|---|---|
| $\therefore \quad m\widehat{ACB} = m\widehat{ADB}$ | Arcs about the common chord AB. |
| $\therefore \quad m\angle 1 = m\angle 2$ | Corresponding angles made by opposite |

| | |
|---|---|
| | arcs. |
| So $m\overline{AQ} = m\overline{AP}$<br>or $m\overline{AP} = m\overline{AQ}$ | Sides opposite to equal angles in $\Delta APQ$. |

**Example 2:**

**ΔBCD is a quadrilateral circumscribed about a circle Show that**
**$m\overline{AB} + m\overline{CD} = m\overline{BC} + m\overline{DA}$**

**Given:**

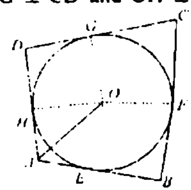
ΔBCD is a quadrilateral circumscribed about a circle with centre O.
So that each side becomes tangent to the circle.

**To prove:**

$m\overline{AB} + m\overline{CD} = m\overline{BC} + m\overline{DA}$

**Construction:**

Drawn $\overline{OE} \perp \overline{AB}$, $\overline{OF} \perp \overline{BC}$, $\overline{OG} \perp \overline{CD}$ and $\overline{OH} \perp \overline{DA}$




**Proof:**

| Statements | Reasons |
|---|---|
| $\therefore m\overline{AE} = m\overline{HA}$; $m\overline{EB} = m\overline{BF}$ ...(i)<br>$m\overline{CG} = m\overline{FC}$ and $m\overline{GD} = m\overline{DH}$ ...(ii) | Since tangents drawn from a point to the circle are equal in length |
| $\left(m\overline{AE} + m\overline{EB}\right) + \left(m\overline{CG} + m\overline{GD}\right)$<br>$= \left(m\overline{BF} + m\overline{FC}\right) + \left(m\overline{DH} + m\overline{HA}\right)$<br>or $m\overline{AB} + m\overline{CD} = m\overline{BC} + m\overline{DA}$ | Adding (i) & (ii) |

## SOLVED EXERCISE 12.1

1. **Prove that in a given cyclic quadrilateral, sum of opposite angles is two right angles and conversely.**

*Solution:*

**Given:**

A quadrilateral ABCD.