Exercise 1.6

1. Use matrices, if possible, to solve the following systems of linear equations by:

(i) the matrix inverse method

(ii) the Cramer's rule.

(i) $2x-2y=4$

$3x+2y=6$

Matrix inverse method

$$\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$A=\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}; X=\begin{bmatrix} x \\ y \end{bmatrix}; B=\begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$AX=B$

$X=A^{-1}B$..........(i)

$$|A|=\begin{vmatrix} 2 & -2 \\ 3 & 2 \end{vmatrix}$$

$$=2(2)-(-2)(3)$$

$$=4+6=10\neq 0$$

As $|A|\neq 0$ so solution is possible

$$\text{Adj } A=\begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}AdjA$$

$$A^{-1}=\frac{1}{10}\begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}$$

Putting the values of A^{-1} and B in equation (i)

$$X=\frac{1}{10}\begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$X=\frac{1}{10}\begin{bmatrix} 2(4)+2(6) \\ -3(4)+2(6) \end{bmatrix}$$

$$X=\frac{1}{10}\begin{bmatrix} 8+12 \\ -12+12 \end{bmatrix}$$

$$X=\frac{1}{10}\begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

$$X=\begin{bmatrix} \cancel{20}^{2}\times\frac{1}{\cancel{10}} \\ 0\times\frac{1}{10} \end{bmatrix}$$

$$X=\begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$\Rightarrow$ $x=2$

$y=0$

$$S.S.=\{(x,y)\}=\{(2,0)\}$$

$$S.S.=\{(2,0)\}$$

(ii) $2x+y=3$

$6x+5y=1$

In matrices form

$$\begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Let

$$A = \begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix};\ X = \begin{bmatrix} x \\ y \end{bmatrix};B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$AX = B$

$X = A^{-1}B$

$$|A| = \begin{vmatrix} 2 & 1 \\ 6 & 5 \end{vmatrix}$$

$= 2(5) - 6(1)$

$= 10 - 6$

$|A| = 4 \neq 0$

As $|A| \neq 0$, so solution is possible

$$Adj\,A = \begin{bmatrix} 5 & -1 \\ -6 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times Adj\,A$$

$$A^{-1} = \frac{1}{4}\begin{bmatrix} 5 & -1 \\ -6 & 2 \end{bmatrix}$$

Putting the value of A^{-1} & B in equation i.

$$X = A^{-1}B$$

$$= \frac{1}{4}\begin{bmatrix} 5 & -1 \\ -6 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 5(3) + (-1)(1) \\ -6(3) + 2(1) \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 15 - 1 \\ -18 + 2 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 14 \\ -16 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{14}{4} \\ \frac{16}{4} \end{bmatrix}$$

$$x = \begin{bmatrix} \frac{7}{2} \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{7}{2} \\ -4 \end{bmatrix}$$

$\Rightarrow \quad x = \frac{7}{2}$

$y = -4$

Solution set $S.S. = \left\{\left(\frac{7}{2}, -4\right)\right\}$

(iii) $4x + 2y = 8$

$3x - y = -1$

In matrices form

$$\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

Let

$$A = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix};X = \begin{bmatrix} x \\ y \end{bmatrix};B = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

$AX = B$

$X = A^{-1}B$

$$|A| = \begin{vmatrix} 4 & 2 \\ 3 & -1 \end{vmatrix}$$

$= 4(-1) - 3(2)$

$= -4 - 6$

$|A| = -10 \neq 0$

As $|A| \neq 0$, so solution is possible

$$\text{Adj } A=\begin{bmatrix}-1 & -2\\ -3 & 4\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}\times \text{Adj}\,A$$

$$A^{-1}=\frac{1}{-10}\begin{bmatrix}-1 & -2\\ -3 & 4\end{bmatrix}$$

Putting values of A^{-1} & B in equation.

$$X=A^{-1}B$$

$$X=\frac{1}{-10}\begin{bmatrix}-1 & -2\\ -3 & 4\end{bmatrix}\begin{bmatrix}8\\ -1\end{bmatrix}$$

$$X=\frac{1}{-10}\begin{bmatrix}-1(8)+(-2)(-1)\\ -3(8)+4(-1)\end{bmatrix}$$

$$X=\frac{1}{-10}\begin{bmatrix}-8+2\\ -24-4\end{bmatrix}$$

$$=\frac{1}{-10}\begin{bmatrix}-6\\ -28\end{bmatrix}$$

$$=\begin{bmatrix}-\cancel{6}^{3}\times\frac{1}{\cancel{-10}_5}\\ -\cancel{28}^{14}\times\frac{1}{\cancel{-10}_5}\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}\frac{3}{5}\\ \frac{14}{5}\end{bmatrix}$$

$$\Rightarrow \quad x=\frac{3}{5}$$

$$y=\frac{14}{5}$$

$$S.S=\left\{\left(\frac{3}{5},\frac{14}{5}\right)\right\}$$

(iv) $3x-2y=-6$

$5x-2y=-10$

In matrices form

$$\begin{bmatrix}3 & -2\\ 5 & -2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}-6\\ -10\end{bmatrix}$$

Let

$$A=\begin{bmatrix}3 & -2\\ 5 & -2\end{bmatrix};\ X=\begin{bmatrix}x\\ y\end{bmatrix};B=\begin{bmatrix}-6\\ -10\end{bmatrix}$$

$$AX=B$$

$$\Rightarrow \quad X=A^{-1}B$$

$$|A|=\begin{vmatrix}3 & -2\\ 5 & -2\end{vmatrix}$$

$$=3(-2)-(5)(2)$$

$$=-6+10$$

$$|A|=4\neq 0$$

As $|A|\neq 0$, so solution is possible

$$\text{Adj } A=\begin{bmatrix}-2 & 2\\ -5 & 3\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}\times \text{Adj}\,A$$

$$A^{-1}=\frac{1}{4}\begin{bmatrix}-2 & 2\\ -5 & 3\end{bmatrix}$$

Putting the values of A^{-1} & B in equation i.

$$X=A^{-1}B$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}-2 & 2\\ -5 & 3\end{bmatrix}\begin{bmatrix}-6\\ -10\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}-2(-6)+2(-10)\\ -5(-6)+3(-10)\end{bmatrix}$$

$$=\frac{1}{4}\begin{bmatrix}12+(-20)\\ 30-30\end{bmatrix}$$

$$=\frac{1}{4}\begin{bmatrix}-8\\ 0\end{bmatrix}$$

$$=\begin{bmatrix} \cancel{8}^{2}\times\frac{1}{\cancel{4}} \\ 0\times\frac{1}{4} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

$\Rightarrow \quad x=-2$

$y=0$

$S.S=\{(-2,0)\}$

(v) $3x-2y=4$

$-6x+4y=7$

In matrices form

$$\begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 4 \\ 7 \end{bmatrix}$$

Let

$$A=\begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix}; X=\begin{bmatrix} x \\ y \end{bmatrix}; B=\begin{bmatrix} 4 \\ 7 \end{bmatrix}$$

AX = B

$X=A^{-1}B$

$$|A|=\begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix}$$

$=3(4)-(6)(-2)$

$=12-12$

$=0$

As $|A|=0$, so solution is not possible

(vi) $4x+y=9$

$-3x-y=-5$

In matrices form

$$\begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

Let

$$A=\begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix}; X=\begin{bmatrix} x \\ y \end{bmatrix}; B=\begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

AX = B

$X=A^{-1}B$

$$|A|=\begin{vmatrix} 4 & 1 \\ -3 & -1 \end{vmatrix}$$

$=4(-1)-(-3)(1)$

$=-4+3$

$=-1\neq 0$

As $|A|\neq 0$, so solution is possible

$$\text{Adj } A=\begin{bmatrix} -1 & -1 \\ 3 & 4 \end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}\times Adj\,A$$

$$=\frac{1}{-1}\times\begin{bmatrix} -1 & -1 \\ 3 & 4 \end{bmatrix}$$

Putting the values in equation (i) of A^{-1} and B

$$X=A^{-1}B$$

$$X=\frac{1}{-1}\begin{bmatrix} -1 & -1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

$$=\frac{1}{-1}\begin{bmatrix} -1(9)+(-1)(-5) \\ 3(9)+4(-5) \end{bmatrix}$$

$$=\frac{1}{-1}\begin{bmatrix} -9+5 \\ 27-20 \end{bmatrix}$$

$$=\frac{1}{-1}\begin{bmatrix} -4 \\ 7 \end{bmatrix}$$

$$=\begin{bmatrix} \frac{1}{-1}\times-4 \\ \frac{1}{-1}\times 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -7 \end{bmatrix}$$

$$\Rightarrow \quad x = 4$$

$$y = -7$$

$$S.S. = \{(4, -7)\}$$

(vii) $2x - 2y = 4$

$-5x - 2y = -10$

In matrices form

$$\begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

A X = B

$\Rightarrow$ X = A^{-1} B

Let $A = \begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix}; X = \begin{bmatrix} x \\ y \end{bmatrix}; B = \begin{bmatrix} 4 \\ -10 \end{bmatrix}$

AX = B

X = A^{-1}B

$$|A| = \begin{vmatrix} 2 & -2 \\ -5 & -2 \end{vmatrix}$$

$$= 2(-2) - (-5)(-2)$$

$$= -4 - 10$$

$$|A| = -14 \neq 0$$

As $|A| \neq 0$, so solution is possible

$$\text{Adj } A = = \begin{bmatrix} -2 & 2 \\ 5 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{Adj } A$$

$$= \frac{1}{-14} \times \begin{bmatrix} -2 & 2 \\ 5 & 2 \end{bmatrix}$$

Putting the values of A^{-1} and B in equation

(i) $X = A^{-1} B$

$$X = \frac{1}{-14} \times \begin{bmatrix} -2 & 2 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

$$X = \frac{1}{-14} \begin{bmatrix} -2(4) + 2(-10) \\ 5(4) + 2(-10) \end{bmatrix}$$

$$= \frac{1}{-14} \begin{bmatrix} -8 - 20 \\ 20 - 20 \end{bmatrix}$$

$$= \frac{1}{-14} \begin{bmatrix} -28 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -\cancel{28}^{2} \times \frac{1}{\cancel{-14}} \\ 0 \times \frac{1}{-14} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad x = 2$$

$$y = 0$$

$$S.S. = \{(2, 0)\}$$

(viii) $3x - 4y = 4$

$x + 2y = 8$

In matrices form

$$\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

Let

$$A = \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}; X = \begin{bmatrix} x \\ y \end{bmatrix}; B = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

$AX = B$

$\Rightarrow \quad X = A^{-1} B$............$i$

$$|A| = \begin{vmatrix} 3 & -4 \\ 1 & 2 \end{vmatrix}$$

$$= 3(2) - (1)(-4)$$

$$= 6 + 4$$

$$|A| = 10 \neq 0$$

As $|A| \neq 0$, so solution is possible

$$\text{Adj}\,A=\begin{bmatrix}2 & 4\\-1 & 3\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}\times\text{Adj}\,A$$

$$A^{-1}=\frac{1}{10}\times\begin{bmatrix}2 & 4\\-1 & 3\end{bmatrix}$$

Putting the values of A^{-1} & B in equation (i)

$$X=A^{-1}B$$

$$X=\frac{1}{10}\begin{bmatrix}2 & 4\\-1 & 3\end{bmatrix}\begin{bmatrix}4\\8\end{bmatrix}$$

$$X=\frac{1}{10}\begin{bmatrix}2(4)+4(8)\\-1(4)+3(8)\end{bmatrix}$$

$$X=\frac{1}{10}\begin{bmatrix}8+32\\-4+24\end{bmatrix}$$

$$X=\frac{1}{10}\begin{bmatrix}40\\20\end{bmatrix}$$

$$X=\begin{bmatrix}\cancel{40}^{4}\times\frac{1}{\cancel{10}}\\ \cancel{20}^{2}\times\frac{1}{\cancel{10}}\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}4\\2\end{bmatrix}$$

$$\Rightarrow\quad x=4$$

$$y=2$$

$$S.S.=\{(4,2)\}$$

Cramer's rule

(i) $2x-2y=4$

$3x+2y=6$

In matrices form

$$\begin{bmatrix}2 & -2\\3 & 2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}4\\6\end{bmatrix}$$

Let $A=\begin{bmatrix}2 & -2\\3 & 2\end{bmatrix}$; $X=\begin{bmatrix}x\\y\end{bmatrix}$; $B=\begin{bmatrix}4\\6\end{bmatrix}$

$$|A|=\begin{vmatrix}2 & -2\\3 & 2\end{vmatrix}$$

$$=2(2)-3(-2)$$

$$=4+6$$

$$|A|=10\neq 0$$

As $|A|\neq 0$, so solution is possible.

A_x; - (Determinant No. 1)

In determinant 1 we change first column to constant matrix.

$$|A_x|=\begin{vmatrix}4 & -2\\6 & 2\end{vmatrix}$$

$$=4(2)-6(-2)$$

$$=8+12$$

$$|A_x|=20$$

$$x=\frac{|A_x|}{|A|}=\frac{20}{10}=2$$

$$x=2$$

$|A_y|$ (Determinant No.2)

In determinant 2 we change 2nd column to constant matrix.

$$|A_y|=\begin{vmatrix}2 & 4\\3 & 6\end{vmatrix}$$

$$=2(6)-3(4)$$

$$=12-12$$

$$|A_y|=0$$

$$y=\frac{|A_y|}{|A|}=\frac{0}{10}=0$$

$$y=0$$

S.S={(2, 0)} .ans.

(ii) $2x+y=3$

$6x+5y=1$

In matrices form

$$\begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Let

$$A=\begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix};X=\begin{bmatrix} x \\ y \end{bmatrix};B=\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$|A|=\begin{vmatrix} 2 & 1 \\ 6 & 5 \end{vmatrix}$$

$$=2(5)-6(1)$$

$$=10-6$$

$$|A|=4\neq 0$$

As $|A|\neq 0$, so solution is possible.

$$|A_x|=\begin{vmatrix} 3 & 1 \\ 1 & 5 \end{vmatrix}$$

$$=3(5)-1(1)$$

$$|A_x|=15-1$$

$$|A_x|=14$$

$$x=\frac{|A_x|}{|A|}=\frac{\cancel{14}^{7}}{\cancel{4}_{2}}$$

$$x=\frac{7}{2}$$

$$|A_y|=\begin{vmatrix} 2 & 3 \\ 6 & 1 \end{vmatrix}$$

$$=2(1)-6(3)$$

$$|A_y|=2-18$$

$$|A_y|=-16$$

$$y=\frac{|A_y|}{|A|}=\frac{-16}{4}=-4$$

$$y=-4$$

$$S.S=\left\{\left(\frac{7}{2},-4\right)\right\}$$

(iii) $4x+2y=8$

$3x-y=-1$

In matrices form

$$\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

Let

$$A=\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix};X=\begin{bmatrix} x \\ y \end{bmatrix};B=\begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

$$|A|=4(-1)-3(2)$$

$$=-4-6$$

$$|A|=-10\neq 0$$

As $|A|\neq 0$, so solution is possible.

$$|A_x|=\begin{vmatrix} 8 & 2 \\ -1 & -1 \end{vmatrix}$$

$$=8(-1)-2(-1)$$

$$=-8+2$$

$$=-6$$

$$x=\frac{|A_x|}{|A|}$$

$$x=\frac{-\cancel{6}^{3}}{-\cancel{10}_{5}}=\frac{3}{5}$$

$$|A_y|=\begin{vmatrix} 4 & 8 \\ 3 & -1 \end{vmatrix}$$

$$=4(-1)-(3)(8)$$

$$=-4-24$$

$$=-28$$

$$y=\frac{|A_y|}{|A|}$$

$$=\frac{-28}{-10}=\frac{14}{5}$$

$S.S.=\left\{\left(\frac{3}{5},\frac{14}{5}\right)\right\}$

(iv) $3x-2y=-6$

$5x-2y=-10$

In matrices form

$$\begin{bmatrix}3 & -2\\5 & -2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-6\\-10\end{bmatrix}$$

Let

$A=\begin{bmatrix}3 & -2\\5 & -2\end{bmatrix};X=\begin{bmatrix}x\\y\end{bmatrix};B=\begin{bmatrix}-6\\-10\end{bmatrix}$

$|A|=\begin{vmatrix}3 & -2\\5 & -2\end{vmatrix}$

$=3(-2)-5(-2)$

$=-6+10$

$|A|=4\neq 0$

As $|A|\neq 0$, so solution is possible.

$|A_x|=\begin{vmatrix}-6 & -2\\-10 & -2\end{vmatrix}$

$=-6(-2)-(-2)(-10)$

$=12-20$

$|A_x|=-8$

$x=\frac{|A_x|}{|A|}=\frac{-\cancel{8}^{2}}{\cancel{4}}$

$x=-2$

$|A_y|=\begin{vmatrix}3 & -6\\5 & -10\end{vmatrix}$

$=3(-10)-(5)(-6)$

$=-30+30$

$=0$

$y=\frac{|A_y|}{|A|}=\frac{0}{4}$

$y=0$

$S.S.=\{(-2,0)\}$

(v) $3x-2y=4$

$-6x+4y=7$

In matrices form

$$\begin{bmatrix}3 & -2\\-6 & 4\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}4\\7\end{bmatrix}$$

Let

$A=\begin{bmatrix}3 & -2\\-6 & 4\end{bmatrix};X=\begin{bmatrix}x\\y\end{bmatrix};B=\begin{bmatrix}4\\7\end{bmatrix}$

$|A|=\begin{vmatrix}3 & -2\\-6 & 4\end{vmatrix}$

$=3(4)-(-6)(-2)$

$=12-12$

$|A|=0$

As $|A|=0$, so solution is not possible

(vi) $4x+y=9$

$-3x-y=-5$

In matrices form

$$\begin{bmatrix}4 & 1\\-3 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}9\\-5\end{bmatrix}$$

Let

$A=\begin{bmatrix}4 & 1\\-3 & -1\end{bmatrix};X=\begin{bmatrix}x\\y\end{bmatrix};B=\begin{bmatrix}9\\-5\end{bmatrix}$

$|A|=\begin{vmatrix}4 & 1\\-3 & -1\end{vmatrix}$

$=4(-1)-(-3)(1)$

$=-4+3$

$|A|=-1\neq 0$

As $|A|\neq 0$, so solution is possible.

$|A_x|=\begin{vmatrix}9 & 1\\-5 & -1\end{vmatrix}$

$=9(-1)-1(-5)$
$=-4$
$$x=\frac{|A_x|}{|A|}=\frac{\cancel{-}4}{\cancel{-}1}$$
$x=4$
$$|A_y|=\begin{vmatrix}4 & 9\\ -3 & -5\end{vmatrix}$$
$=4(-5)-9(-3)$
$=-20+27$
$=7$
$$y=\frac{|A_y|}{|A|}=\frac{7}{-1}$$
$y=-7$

$S.S=\{(4,-7)\}$

(vii) $2x-2y=4$
$-5x-2y=-10$

In matrices form
$$\begin{bmatrix}2 & -2\\ -5 & -2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}4\\ -10\end{bmatrix}$$
Let
$$A=\begin{bmatrix}2 & -2\\ -5 & -2\end{bmatrix};X=\begin{bmatrix}x\\ y\end{bmatrix};B=\begin{bmatrix}4\\ -10\end{bmatrix}$$
$$|A|=\begin{vmatrix}2 & -2\\ -5 & -2\end{vmatrix}$$
$=2(-2)-(-5)(-2)$
$=-4-10$
$|A|=-14\neq 0$

As $|A|\neq 0$, so solution is possible.
$$|A_x|=\begin{vmatrix}4 & -2\\ -10 & -2\end{vmatrix}$$
$=4(-2)-(-10)(-2)$
$=-8-20$
$=-28$
$$x=\frac{|A_x|}{|A|}=\frac{\cancel{-}\cancel{28}^{2}}{\cancel{-}\cancel{14}}$$
$x=2$
$$|A_y|=\begin{vmatrix}2 & 4\\ -5 & -10\end{vmatrix}$$
$=2(-10)-(-5)(4)$
$=-20+20$
$=0$
$$y=\frac{|A_y|}{|A|}=\frac{0}{-14}$$
$y=0$

$S.S=\{(2,0)\}$ ans.

(viii) $3x-4y=4$
$x+2y=8$

In matrices form
$$\begin{bmatrix}3 & -4\\ 1 & 2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}4\\ 8\end{bmatrix}$$
Let
$$A=\begin{bmatrix}3 & -4\\ 1 & 2\end{bmatrix};X=\begin{bmatrix}x\\ y\end{bmatrix};B=\begin{bmatrix}4\\ 8\end{bmatrix}$$
$$|A|=\begin{vmatrix}3 & -4\\ 1 & 2\end{vmatrix}$$
$=3(2)-1(-4)$
$=6+4$
$|A|=10\neq 0$

As $|A|\neq 0$, so solution is possible.
$$|A_x|=\begin{vmatrix}4 & -4\\ 8 & 2\end{vmatrix}$$
$=4(2)-8(-4)$
$=8+32$
$=40$

$$x=\frac{|A_x|}{|A|}=\frac{\cancel{40}\,4}{\cancel{10}}$$

$$x=4$$

$$|A_y|=\begin{vmatrix}3 & 4\\1 & 8\end{vmatrix}$$

$$=3(8)-1(4)$$

$$=24-4$$

$$=20$$

$$y=\frac{|A_y|}{|A|}=\frac{\cancel{20}^{\,2}}{\cancel{10}}$$

$$y=2$$

$S.S.=\{(4,2)\}$ ans.

Q.2. The length of a rectangle is 4 times its width. The perimeter of the rectangle is 150cm. Find dimensions of the rectangle?

Let width of rectangle = x.

and length of rectangle = y

According to first condition

$$y=4x$$

$$4x-y=0.........(i)$$

According to 2nd condition

Perimeter =150cm.

$$2(x+y)=150$$

$$x+y=\frac{150}{2}$$

$$x+y=75\ ....(ii)$$

In matrices form

$$\begin{bmatrix}1 & 1\\4 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}75\\0\end{bmatrix}$$

$$AX=B$$

$$\Rightarrow \quad X=A^{-1}B$$

Now

$$A=\begin{bmatrix}1 & 1\\4 & -1\end{bmatrix}$$

$$|A|=\begin{vmatrix}1 & 1\\4 & -1\end{vmatrix}$$

$$=1(-1)-4(1)$$

$$=-1-4$$

$$=-5\neq 0$$

$$Adj\ A=\begin{bmatrix}-1 & -1\\-4 & 1\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}\times \text{Adj}\,A$$

$$A^{-1}=-\frac{1}{5}\begin{bmatrix}-1 & -1\\-4 & 1\end{bmatrix}$$

$$=\frac{1}{5}\begin{bmatrix}1 & 1\\4 & -1\end{bmatrix}$$

$$\Rightarrow \quad X=A^{-1}B$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{5}\begin{bmatrix}1 & 1\\4 & -1\end{bmatrix}\begin{bmatrix}75\\0\end{bmatrix}$$

$$=\frac{1}{5}\begin{bmatrix}1(75)+1(0)\\4(75)+(-1)(0)\end{bmatrix}$$

$$=\frac{1}{5}\begin{bmatrix}75\\300\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}\frac{75}{5}\\\frac{300}{5}\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}15\\60\end{bmatrix}$$

$$\Rightarrow \quad x=15cm$$

$$\Rightarrow \quad y=60cm$$

Q.3. Two sides of rectangle differ by 3.5cm. Find the dimensions of the rectangle if its perimeter is 67cm.

Let required sides of rectangle are x and y.

According to first condition

$$x - y = 3.5 \longrightarrow (i)$$

According to 2^{nd} condition

Perimeter $= 67$

$2(x+y) = 67$

$$\Rightarrow \quad x + y = 33.5 \longrightarrow (ii)$$

In matrices form

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3.5 \\ 33.5 \end{bmatrix}$$

We have

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}; A_x = \begin{bmatrix} 3.5 & -1 \\ 33.5 & 1 \end{bmatrix},$$

$$A_y = \begin{bmatrix} 1 & 3.5 \\ 1 & 33.5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}$$

$$= 1(1) - 1(-1)$$

$$= 1 + 1 = 2 \neq 0$$

$$x = \frac{|A_x|}{|A|}$$

$$= \frac{\begin{vmatrix} 3.5 & -1 \\ 33.5 & 1 \end{vmatrix}}{2}$$

$$= \frac{3.5(1) - 33.5(-1)}{2}$$

$$= \frac{3.5 + 33.5}{2}$$

$$= \frac{37}{2} = 18.5$$

$$y = \frac{|A_y|}{|A|}$$

$$= \frac{\begin{vmatrix} 1 & 3.5 \\ 1 & 33.5 \end{vmatrix}}{2}$$

$$= \frac{1(33.5) - 1(3.5)}{2}$$

$$= \frac{33.5 - 3.5}{2}$$

$$= \frac{30}{2} = 15$$

$$\Rightarrow \quad x = 18.5, \quad y = 15$$

Q.4. The third angle of an isosceles triangle is 16° less than the sum of the two equal angles. Find three angles of the triangle.

Let third angle of triangle = y

and two equal angle of triangle = x

we know that

$$x + x + y = 180^o$$

$$2x + y = 180^o ....(i)$$

According to given condition.

$$y = 2x - 16$$

$$2x - y = 16$$

In matrices form

$$\begin{bmatrix} 2 & 1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 180 \\ 16 \end{bmatrix}$$

$$AX = B$$

$$\Rightarrow \quad X = A^{-1}B$$

Now

$$A = \begin{bmatrix} 2 & 1 \\ 2 & -1 \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} 2 & 1 \\ 2 & -1 \end{vmatrix}$$

$$|A| = 2(-1) - 2(1)$$

$$= -2 - 2$$

$$= -4 \neq 0$$

$$\text{Adj} A = \begin{bmatrix} -1 & -1 \\ -2 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{Adj A}$$

$$= \frac{1}{-4} \begin{bmatrix} -1 & -1 \\ -2 & 2 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix}$$

$$\Rightarrow \quad X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} 180 \\ 16 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 1(180) + 1(16) \\ 2(180) + (-2)(16) \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 180 + 16 \\ 360 - 32 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 196 \\ 328 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 49 \\ 82 \end{bmatrix}$$

Hence: $x = 49^\circ$, $y = 82^\circ$

Required angles are 49°, 49°, 82°.

Q.5. One acute angle of a right triangle is 12° more than twice the other acute angle. Find the acute angles of the right triangle?

Let acute angles of right angled triangle are x and y

We know that

$$x + y = 90^o \; (i)$$

According to given condition

$$x = 2y + 12^o$$

$$x - 2y = 12^o \longrightarrow (ii)$$

In matrix form

$$\begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 90 \\ 12 \end{bmatrix}$$

We have

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}, A_x = \begin{bmatrix} 90 & 1 \\ 12 & -2 \end{bmatrix}$$

$$A_y = \begin{bmatrix} 1 & 90 \\ 1 & 12 \end{bmatrix}$$

Now $$A = \begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}$$

$$|A| = 1(-2) - 1(1)$$

$$= -2 - 1$$

$$= -3 \neq 0$$

$$x = \frac{|A_x|}{|A|}$$

$$= \frac{\begin{vmatrix} 90 & 1 \\ 12 & -2 \end{vmatrix}}{-3}$$

$$= \frac{90(-2) - 1(12)}{-3}$$

$$x = \frac{-180 - 12}{-3}$$

$$= \frac{-192}{-3} = 64^o$$

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{\begin{vmatrix} 1 & 90 \\ 1 & 12 \end{vmatrix}}{-3}$$

$$= \frac{1(12)-1(90)}{-3}$$

$$= \frac{12-90}{-3}$$

$$= \frac{-78}{-3}$$

$$= 26^o$$

$\therefore$ Required angles are 26° and 64°

$\Rightarrow \quad x = 64^o$

$\Rightarrow \quad y = 26^o$

Q6. Two cars that are 600 km apart are moving towards each other. Their speeds differ by 6km per hour and the cars are 123 km apart after $4\frac{1}{2}$ hours. Find the speed of each car.

Solution:

Let required speed of two cars are x and y

According to given condition

$$x - y = 6$$

$$\frac{9}{2}x - \frac{9}{2}y = 600 - 123 = 477$$

$$x - y = 6$$

$$9x + 9y = 477 \times 2 = 954$$

$\Rightarrow \quad x - y = 6$

$9x + 9y = 954$

In matrix form

$$\begin{bmatrix} 1 & -1 \\ 9 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 954 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 \\ 9 & 9 \end{bmatrix}, A_x = \begin{bmatrix} 6 & -1 \\ 954 & 9 \end{bmatrix}$$

$$A_y = \begin{bmatrix} 1 & 6 \\ 9 & 954 \end{bmatrix}$$

Now

$$A = \begin{bmatrix} 1 & -1 \\ 9 & 9 \end{bmatrix}$$

$$|A| = 1(9) - (-1)(9)$$

$$= 9 + 9 = 0$$

$$= 18 \neq 0$$

$$x = \frac{|A_x|}{|A|} = \frac{\begin{vmatrix} 6 & -1 \\ 954 & 9 \end{vmatrix}}{18}$$

$$= \frac{6(9) - (-1)(954)}{18} = \frac{54 + 954}{18} = \frac{1008}{18} = 56\text{km}/\text{h}$$

$$y = \frac{|A_y|}{|A|} = \frac{\begin{vmatrix} 1 & 6 \\ 9 & 954 \end{vmatrix}}{18}$$

$$= \frac{1(954) - 6(9)}{18}$$

$$= \frac{954 - 54}{18}$$

$$= \frac{900}{18} = 50km/h$$