Exercise 10.1

Question # 1

Without using the tables, find the value of :

(i) $\sin(-780^\circ)$ (ii) $\cot(-855^\circ)$ (iii) $\csc(2040^\circ)$

(iv) $\csc(2040^\circ)$ (v) $\tan(1110^\circ)$ (iv) $\sin(-300^\circ)$

Solution

(i) $\sin(-780^\circ) = -\sin 780^\circ = -\sin\left(8(90)+60\right)$

$$= -\sin(60) = -\frac{\sqrt{3}}{2} \qquad \because\ 780 \text{ is in the Ist quad.}$$

(ii) $\cot(-855^\circ) = -\cot 855^\circ = -\cot\left(9(90)+45\right)$

$$= -\left(-\tan 45^\circ\right) = \tan 45^\circ = 1 \qquad \because\ 855 \text{ is in the IInd quad.}$$

(iii) $\csc(2040^\circ) = \csc\left(22(90)+60\right) = -\csc(60)$ $\qquad \because\ 2040^\circ$ is in the Ist quad.

$$= -\frac{1}{\sin(60)} = -\frac{1}{\sqrt{3}/2} = -\frac{2}{\sqrt{3}}$$

(iv) $\sec(-960) = \sec(960) = \sec\left(10(90)+60\right) = -\sec 60^\circ$ $\qquad \because\ 960^\circ$ is in the IIIrd quad.

$$= -\frac{1}{\cos 60^\circ} = -\frac{1}{1/2} = -2$$

(v) $\tan(1110) = \tan\left(12(90)+30\right) = \tan(30) = \frac{1}{\sqrt{3}}$ $\qquad \because\ 1110^\circ$ is in the Ist quad

(vi) $\sin(-300) = -\sin(300) = -\sin\left(3(90)+30\right)$

$$= -(-\cos 30^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2} \qquad \because\ 300^\circ \text{ is in the IIIrd quad.}$$

Question # 2

Express each of the following as a trigonometric function of an angle of positive degree measure of less than 45°.

(i) $\sin 196^\circ$ (ii) $\cos 147^\circ$ (iii) $\sin 319^\circ$ (iv) $\cos 254^\circ$

(v) $\tan 294^\circ$ (vi) $\cos 728^\circ$ (vii) $\sin(-625^\circ)$ (viii) $\cos(-435^\circ)$

Solution

(i) $\sin 196^\circ = \sin(180+16) = \sin 180^\circ \cos 16^\circ + \cos 180^\circ \sin 16^\circ$

$$= (0)\cos 16^\circ + (-1)\sin 16^\circ = -\sin 16^\circ$$

(ii) $\cos 147^\circ = \cos(180-33) = \cos 180^\circ \cos 33^\circ + \sin 180^\circ \sin 33^\circ$

$$= (-1)\cos 33^\circ + (0)\sin 33^\circ = -\cos 33^\circ$$

(iii) $\sin 319^\circ = \sin(360-41) = \sin 360^\circ \cos 41^\circ - \cos 360^\circ \sin 41^\circ$ ***Now Do yourself***

(iv) $\cos 254^\circ = \cos(270-16)$ ***Do yourself***

(v) $$\tan 294^\circ = \frac{\sin 294^\circ}{\cos 294^\circ} = \frac{\sin(270+24)}{\cos(270+24)}$$
$$= \frac{\sin 270^\circ \cos 24^\circ + \cos 270^\circ \sin 24^\circ}{\cos 270^\circ \cos 24^\circ - \sin 270^\circ \sin 24^\circ} = \frac{(-1)\cos 24^\circ + (0)\sin 24^\circ}{(0)\cos 24^\circ - (-1)\sin 24^\circ}$$
$$= \frac{-\cos 24^\circ + 0}{0+\sin 24^\circ} = \frac{-\cos 24^\circ}{\sin 24^\circ} = -\cot 24^\circ$$

Alternative Method:

$$\tan 294^\circ = \tan(270+24) = \frac{\tan 270^\circ + \tan 24^\circ}{1-\tan 270^\circ \tan 24^\circ}$$
$$= \frac{\tan 270^\circ\left(1+\dfrac{\tan 24^\circ}{\tan 270^\circ}\right)}{\tan 270^\circ\left(\dfrac{1}{\tan 270^\circ} - \tan 24^\circ\right)} = \frac{\left(1+\dfrac{\tan 24^\circ}{\infty}\right)}{\left(\dfrac{1}{\infty} - \tan 24^\circ\right)}$$
$$= \frac{(1+0)}{(0-\tan 24^\circ)} = -\frac{1}{\tan 24^\circ} = -\cot 24^\circ \quad \square$$

(vi) $\cos 728^\circ = \cos(720+8)$ ***Now Do yourself***

(vii) $$\sin(-625^\circ) = -\sin 625^\circ = -\sin(630-5)$$
$$= -\left(\sin 630^\circ \cos 5^\circ - \cos 630^\circ \sin 5^\circ\right) = -\left((-1)\cos 5^\circ - (0)\sin 5^\circ\right)$$
$$= -\left(-\cos 5^\circ - 0\right) = \cos 5^\circ$$

(viii) $$\cos(-435^\circ) = \cos 435^\circ$$
$$= \cos(450-15)$$ ***Now Do yourself***

Question # 3

Prove the following:

(i) $\sin(180+\alpha)\sin(90-\alpha) = -\sin\alpha\cos\alpha$

(ii) $\sin 780^\circ \sin 480^\circ + \cos 120^\circ \sin 30^\circ = \dfrac{1}{2}$

(iii) $\sin 306^\circ + \cos 234^\circ + \cos 162^\circ + \cos 18^\circ = 0$

(iv) $\cos 330^\circ \sin 600^\circ + \cos 120^\circ \sin 150^\circ = -1$

Solution

(i) L.H.S $= \sin(180+\alpha)\sin(90-\alpha)$
$$= \left(\sin 180^\circ \cos\alpha + \cos 180^\circ \sin\alpha\right)\left(\sin 90^\circ \cos\alpha - \cos 90^\circ \sin\alpha\right)$$
$$= \left((0)\cos\alpha + (-1)\sin\alpha\right)\left((1)\cos\alpha - (0)\sin\alpha\right)$$

$$=\left(0-\sin\alpha\right)\right)\left(\cos\alpha-0\right)=-\sin\alpha\cos\alpha = \text{R.H.S} \quad \square$$

(ii) First we calculate

$$\sin 780^\circ = \sin(720+60) = \sin(2\times 360+60) = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\sin 480^\circ = \sin(450+30) = \sin 450^\circ \cos 30^\circ + \cos 450^\circ \sin 30^\circ$$

$$= (1)\cos 30 + (0)\sin 30 = \cos 30 + 0 = \frac{\sqrt{3}}{2}$$

$$\cos 120^\circ = -\frac{1}{2} \quad \text{and} \quad \sin 30^\circ = \frac{1}{2}.$$

So L.H.S $= \sin 780^\circ \sin 480^\circ + \cos 120^\circ \sin 30^\circ$

$$=\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)+\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)=\frac{3}{4}-\frac{1}{4}=\frac{1}{2} = \text{R.H.S} \quad \square$$

(iii) First we calculate

$$\cos 306^\circ = \cos(270+36) = \cos 270^\circ \cos 36^\circ - \sin 270^\circ \sin 36^\circ$$

$$= (0)\cos 36^\circ - (-1)\sin 36^\circ = 0 + \sin 36^\circ = \sin 36^\circ$$

$$\cos 234^\circ = \cos(270-36) = \cos 270 \cos 36 + \sin 270 \cos 36$$

$$= (0)\cos 36^\circ + (-1)\sin 36^\circ = 0 - \sin 36^\circ = -\sin 36^\circ$$

$$\cos 162^\circ = \cos(180-18) = \cos 180^\circ \cos 18^\circ + \sin 180^\circ \sin 18^\circ$$

$$= (-1)\cos 18 + (0)\sin 18 = -\cos 18 + 0 = -\cos 18$$

So L.H.S $= \sin 306^\circ + \cos 234^\circ + \cos 162^\circ + \cos 18^\circ$

$$= \sin 36^\circ - \sin 36^\circ - \cos 18^\circ + \cos 18^\circ = 0 = \text{R.H.S} \quad \square$$

(iv) First we calculate (*Alternative Method*)

$$\cos 330^\circ = \cos(360-30) = \cos(-30^\circ) = \cos(30^\circ) = \frac{\sqrt{3}}{2}$$

$$\sin 600 = \sin(6\times 90+60) = -\sin 60 = -\frac{\sqrt{3}}{2} \qquad \because 600^\circ \text{ is in the IIIrd quad}$$

$$\cos 120^\circ = \cos(90+30) = -\sin 30 = -\frac{1}{2} \qquad \because 120^\circ \text{ is in the IInd quad}$$

$$\sin 150^\circ = \sin(90+60) = \cos 60^\circ = \frac{1}{2} \qquad \because 150^\circ \text{ is in the IInd quad}$$

So L.H.S $= \cos 330^\circ \sin 600^\circ + \cos 120^\circ \sin 150^\circ$

$$=\left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right)+\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)=-\frac{3}{4}-\frac{1}{4}=-\frac{4}{4}=-1 = \text{R.H.S}$$

Question # 4

Prove that;

(i) $$\frac{\sin^2(\pi+\theta)\tan\left(\frac{3\pi}{2}+\theta\right)}{\cot^2\left(\frac{3\pi}{2}-\theta\right)\cos^2(\pi-\theta)\operatorname{cosec}(2\pi-\theta)}=\cos\theta$$

(ii) $$\frac{\cos\left(90^\circ+\theta\right)\ \sec(-\theta)\ \tan\left(180^\circ-\theta\right)}{\sec\left(360^\circ-\theta\right)\ \sin\left(180^\circ+\theta\right)\ \cot\left(90^\circ-\theta\right)}=-1$$

Solution

(i) First we calculate

$$\sin(\pi+\theta)=\sin\pi\ \cos\theta+\cos\pi\ \sin\theta=(0)\cos\theta+(-1)\sin\theta$$
$$=0-\sin\theta=-\sin\theta$$

$$\tan\left(\frac{3\pi}{2}+\theta\right)=\tan\left(3\cdot\frac{\pi}{2}+\theta\right)=-\cot\theta \qquad \because \frac{3\pi}{2}+\theta \text{ is in the IVth quad}$$

$$\cot\left(\frac{3\pi}{2}-\theta\right)=\cot\left(3\cdot\frac{\pi}{2}-\theta\right)=\tan\theta \qquad \because \frac{3\pi}{2}-\theta \text{ is in the IIIrd quad}$$

$$\cos(\pi-\theta)=\cos\pi\ \cos\theta+\sin\pi\ \sin\theta=(-1)\cos\theta+(0)\sin\theta$$
$$=-\cos\theta+0=-\cos\theta$$

$$\csc(2\pi-\theta)=\csc(-\theta)=-\csc\theta$$

Now

$$\text{L.H.S}=\frac{\sin^2(\pi+\theta)\ \tan\left(\frac{3\pi}{2}+\theta\right)}{\cot\left(\frac{3\pi}{2}-\theta\right)\cos^2(\pi-\theta)\ \csc(2\pi-\theta)}$$

$$=\frac{(-\sin\theta)^2\ (-\cot\theta)}{(\tan\theta)^2\ (-\cos\theta)^2\ (-\csc\theta)}=\frac{\sin^2\theta\ (-\cot\theta)}{\tan^2\theta\ \cos^2\theta\ (-\csc\theta)}$$

$$=\frac{\sin^2\theta\ \frac{\cos\theta}{\sin\theta}}{\frac{\sin^2\theta}{\cos^2\theta}\ \cos^2\theta\ \frac{1}{\sin\theta}}=\frac{\sin\theta\ \cos\theta}{\sin\theta}=\cos\theta=\text{R.H.S}$$

(ii) First we calculate

$$\cos(90+\theta)=-\sin\theta \qquad \because 90+\theta \text{ is in the IInd quad.}$$

$$\sec(-\theta)=\sec\theta$$

$$\tan(180-\theta)=\tan\left(2(90)-\theta\right)=-\tan\theta \qquad \because 180-\theta \text{ is in the IInd quad.}$$

$$\sec(360-\theta)=\sec(-\theta)=\sec\theta$$

$$\sin(180+\theta)=\sin\left(2(90)+\theta\right)=-\sin\theta \qquad \because 180+\theta \text{ is in the IIIrd quad.}$$

$$\cot(90-\theta)=\tan\theta \qquad \because 90-\theta \text{ is in the Ist quad.}$$

Now

$$\text{L.H.S} = \frac{\cos(90+\theta)\sec(-\theta)\tan(180-\theta)}{\sec(360-\theta)\sin(180+\theta)\cot(90-\theta)}$$
$$= \frac{(-\sin\theta)\sec\theta\,(-\tan\theta)}{\sec\theta\,(-\sin\theta)\,(-\tan\theta)} = 1 = \text{R.H.S}$$

Question # 5

If α, β, γ are the angles of a triangle ABC, then prove that;

(i) $\sin(\alpha+\beta) = \sin\gamma$ (ii) $\cos\left(\dfrac{\alpha+\beta}{2}\right) = \sin\dfrac{\gamma}{2}$

(iii) $\cos(\alpha+\beta) = \cos\gamma$ (iv) $\tan(\alpha+\beta) + \tan\gamma = 0$

Solution

(i) Since α, β and γ are angels of triangle therefore

$$\alpha+\beta+\gamma = 180 \quad \Rightarrow \alpha+\beta = 180-\gamma$$

Now L.H.S $= \sin(\alpha+\beta) = \sin(180-\gamma)$

$$= \sin 180 \cos\gamma - \cos 180 \sin\gamma$$
$$= (0)\cos\gamma - (-1)\sin\gamma = 0 + \sin\gamma = \sin\gamma = \text{R.H.S}$$

(ii) Since α, β and γ are angels of triangle therefore

$$\alpha+\beta+\gamma = 180$$
$$\Rightarrow \alpha+\beta = 180-\gamma \quad \Rightarrow \frac{\alpha+\beta}{2} = \frac{180-\gamma}{2}$$

Now L.H.S $= \cos\left(\dfrac{\alpha+\beta}{2}\right) = \cos\left(\dfrac{180-\gamma}{2}\right) = \cos\left(\dfrac{180}{2} - \dfrac{\gamma}{2}\right)$

$$= \cos\left(90 - \frac{\gamma}{2}\right) = \cos 90 \cos\frac{\gamma}{2} + \sin 90 \sin\frac{\gamma}{2}$$
$$= (0)\cos\frac{\gamma}{2} + (1)\sin\frac{\gamma}{2} = 0 + \sin\frac{\gamma}{2} = \sin\frac{\gamma}{2} = \text{R.H.S} \quad \square$$

(iii) Since α, β and γ are angels of triangle therefore

$$\alpha+\beta+\gamma = 180 \quad \Rightarrow \alpha+\beta = 180-\gamma$$

Now L.H.S $= \cos(\alpha+\beta) = \cos(180-\gamma)$

$$= \cos 180 \cos\gamma + \sin 180 \sin\gamma$$
$$= (-1)\cos\gamma + (0)\sin\gamma = -\cos\gamma + 0 = -\cos\gamma = \text{R.H.S}$$

(iv) Since α, β and γ are angels of triangle therefore

$$\alpha+\beta+\gamma = 180 \quad \Rightarrow \alpha+\beta = 180-\gamma$$

Now L.H.S $= \tan(\alpha+\beta) + \tan\gamma = \tan(180-\gamma) + \tan\gamma$

$$= \frac{\tan 180 - \tan\gamma}{1 + \tan 180 \tan\gamma} + \tan\gamma$$

$$= \frac{(0) - \tan\gamma}{1 + (0)\tan\gamma} + \tan\gamma = \frac{-\tan\gamma}{1+0} + \tan\gamma$$

$$= -\tan\gamma + \tan\gamma = 0 = \text{R.H.S} \quad \square$$

Remember:

- $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
- $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
- $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
- $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
- $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$
- $\tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

Three Steps to solve $\sin\left(n\cdot\frac{\pi}{2} \pm \theta\right)$

Step I: First check that *n* is even or odd

Step II: If *n* is even then the answer will be in *sin* and if the *n* is odd then *sin* will be converted to *cos* and vice virsa (i.e. *cos* will be converted to *sin*).

Step III: Now check in which quadrant $n\cdot\frac{\pi}{2} \pm \theta$ is lying if it is in *Ist* or *IInd* quadrant the answer will be positive as *sin* is positive in these quadrant and if it is in the *IIIrd* or *IVth* quadrant the answer will be negative.

e.g. $\sin 667^\circ = \sin\left(7(90) + 37\right)$

Since n = 7 is odd so answer will be in *cos* and 667 is in *IVth* quadrant and *sin* is –ive in *IVth* quadrant therefore answer will be in negative. i.e $\sin 667^\circ = -\cos 37$

Similar technique is used for other trigonometric rations. i.e $\tan \rightleftarrows \cot$ and $\sec \rightleftarrows \csc$.
