EXERCISE 4.10

Q.1

Let x be certain positive number, then one less than x means $x-1$

Two less than three times x means $3x-2$, Now According to given condition

(one less than x)(Two less than Three times x) $= 14$

i.e. $(x-1)(3x-2) = 14$

$$3x^2-2x-3x+2-14=0$$

$$3x^2-5x-12=0$$

$$3x^2-9x+4x-12=0$$

$$3x(x-3)+4(x-3)=0$$

$$(x-3)(3x+4)=0$$

$x-3=0$, $3x+4=0 \Rightarrow x=3$, $x=\frac{-4}{3}$

$x=\frac{-4}{3}$ (impossible being negative)

Hence $x=3$ is required positive number.

Q.2 Let x be the positive number its square will be x^2. Now according to given condition.

$$x+x^2=380$$

$$\Rightarrow x^2+x-380=0$$

$$x^2+20x-19x-380=0$$

$$x(x+20)-19(x+20)=0$$

$$(x-19)(x+20)=0$$

$x-19=0$, $x+20=0$

$x=19$, $x=-20$ (impossible being negative)

Hence $x=19$ is required positive number.

Q.3 Let x be one part then other part will be $40-x$

Sum of squares of parts $= x^2+(40-x)^2$

Product of the parts $= x(40-x)$

According to given condition

$$\left[x^2+(40-x)^2\right]-2\left[x(40-x)\right]=100$$

$$x^2+(1600-80x+x^2)-2x(40-x)=100$$

$$x^2+1600-80x+x^2-80x+2x^2-100=0$$

$$4x^2-160x+1500=0$$

Dividing by 4.

$$x^2-40x+375=0$$

$$x^2-25x-15x+375=0$$

$$x(x-25)-15(x-25)=0$$

$$(x-15)(x-25)=0$$

$x-15=0$, $x-25=0$

$x=15$, $x=25$

If one part is 15 then other part $= 40-15=25$

If one part is 25 then other part $= 40-25=15$

Q.4 Let x be positive number According to given condition

$$x+\frac{1}{x}=\frac{26}{5}$$

Multiply by $5x$ we get

$$5x^2+5=26x$$
$$5x^2-26x+5=0$$
$$5x^2-25x-x+5=0$$
$$5x(x-5)-1(x-5)=0$$
$$(x-5)(5x-1)=0$$
$$x-5=0 \quad , \quad 5x-1=0$$
$$x=5 \quad , \quad x=\frac{1}{5}$$

Hence $x=5$ and $x=\frac{1}{5}$ are required numbers.

Q.5 Let x be the number then

Its square root $=\sqrt{x}$

Now according to given condition.

$$x=\sqrt{x}+56$$
$$x-56=\sqrt{x}$$

Squaring both sides

$$(x-56)^2=(\sqrt{x})^2$$
$$x^2-112x+3136=x$$
$$x^2-112x-x+3136=0$$
$$x^2-113x+3136=0$$
$$x^2-64x-49x+3136=0$$
$$x(x-64)-49(x-64)=0$$
$$(x-64)(x-49)=0$$
$$x-64=0 \quad , \quad x-49=0$$
$$x=64 \quad , \quad x=49$$

$x=49$ does not satisfy given condition

Hence required number is $x=64$

Q.6 Let x and $x+1$ be two consective numbers then according to given condition.

$$x(x+1)=132$$
$$x^2+x-132=0$$
$$x^2+12x-11x-132=0$$
$$x(x+12)-11(x+12)=0$$
$$(x-11)(x+12)=0$$
$$x-11=0 \quad , \quad x+12=0$$
$$x=11 \quad , \quad x=-12$$

If $x=11$ then $x+1=11+1=12$

If $x=-12$ then $x+1=-12+1=-11$

Hence two consective numbers are $11, 12$ or $-12, -11$

Q.7 Let x and $x+2$ be two consective even numbers then according to given condition;

$$(x+2)^3-x^3=296$$
$$x^3+8+3(x^2)(2)+3(x)(2)^2-x^3-296=0$$
$$6x^2+12x-288=0$$

Dividing by 6 we get.

$$x^2+2x-48=0$$
$$x^2+8x-6x-48=0$$
$$x(x+8)-6(x+8)=0$$
$$(x-6)(x+8)=0$$
$$x-6=0 \quad , \quad x+8=0$$
$$x=6 \quad , \quad x=-8$$

If $x=6$ then $x+2=6+2=8$

If $x=-8$ then $x+2=(-8)+2=-6$

Hence two consective numbers are $6, 8$ or $-8, -6$.

Q.8 Let x be number of sheep.

Amount for x sheep $=9000$

Amount for 1 sheep $=\frac{9000}{x}$

Amount for $x+3$ sheep $=\frac{9000}{x+3}$

According to given condition.

$$\frac{9000}{x}-100=\frac{9000}{x+3}$$

Multiply by $x(x+3)$ we get

$$x(x+3)\cdot\frac{9000}{x} - x(x+3)100 = x(x+3)\frac{9000}{x+3}$$

$$9000(x+3) - 100x(x+3) = 9000x$$

Dividing by 100

$$90(x+3) - x(x+3) = 90x$$
$$90x + 270 - x^2 - 3x = 90x$$
$$0 = x^2 + 3x + 90x - 90x - 270$$
$$x^2 + 3x - 270 = 0$$
$$x^2 + 18x - 15x - 270 = 0$$
$$x(x+18) - 15(x+18) = 0$$
$$(x-15)(x+18) = 0$$
$$x - 15 = 0 \quad , \quad x + 18 = 0$$
$$x = 15 \quad , \quad x = -18 \text{ (impossible)}$$

Hence $x = 15$ is number of sheep.

Q.9 Let total dozen eggs to be sold $= x$

Amount for x dozen eggs $= 240$

Amount for 1 dozen egg $= \frac{240}{x}$

Amount for $x+2$ dozen eggs $= \frac{240}{x+2}$

According to given condition.

$$\frac{240}{x} - 0.50 = \frac{240}{x+2}$$

Multiplying by $x(x+2)$ we get

$$x(x+2)\cdot\frac{240}{x} - 0.50x(x+2) = \frac{240}{x+2}x(x+2)$$
$$240(x+2) - 0.50x(x+2) = 240x$$
$$240x + 480 - 0.50x^2 - x = 240x$$
$$-0.50x^2 - x + 480 = 0$$
$$0.50x^2 + x - 480 = 0$$

Multiplying by 2

$$x^2 + 2x - 960 = 0$$
$$x^2 + 32 - 30x - 960 = 0$$
$$x(x+32) - 30(x+32) = 0$$
$$(x-30)(x+32) = 0$$
$$x - 30 = 0 \quad , \quad x + 32 = 0$$
$$x = 30 \quad , \quad x = -32 \text{ (impossible)}$$

Hence $x = 30$ dozen eggs were sold by the stockist.

Q.10 Let speed to cover 48km $= x$

Time to cover 48 km $= t$

As Distance = speed $\times$ time

So $48 = xt$ or $xt = 48 \longrightarrow (1)$

Now speed to cover 48 km by travelling 2km/hr slower $= x - 2$

Time taken with this speed $= t + 2$

Distance = speed $\times$ time

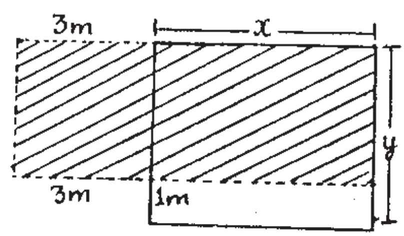
$$48 = (x-2)(t+2)$$
$$\Rightarrow 48 = xt + 2x - 2t - 4$$
$$48 = 48 + 2x - 2t - 4$$
$$2x - 2t - 4 = 0$$
$$x - t - 2 = 0$$
$$x = t + 2$$

Putting value of x in (1) we get

$$(t+2)t = 48$$
$$t^2 + 2t - 48 = 0$$
$$t^2 - 6t + 8t - 48 = 0$$
$$t(t-6) + 8(t-6) = 0$$
$$(t-6)(t+8) = 0$$
$$t - 6 = 0, t + 8 = 0 \Rightarrow t = 6, t = -8 \text{ (impossible)}$$

So $t = 6$ hours is required time.

Q.11



Let length of original rectangle $= x$

width of original rectangle $= y$

$\because$ Area = length $\times$ width

So $297 = xy \longrightarrow (1)$

After changing length and width

Now, Length of new rectangle $= x+3$

width of new rectangle $= y-1$

$\because$ Area of new rectangle $= (x+3)(y-1)$

But given that area $= 297+3 = 300$

So $300 = (x+3)(y-1)$

$300 = xy - x + 3y - 3$

$300 = 297 - x + 3y - 3$

$300 - 294 + x - 3y = 0$

$x - 3y + 6 = 0$

$\Rightarrow x = 3y - 6 \longrightarrow (2)$

Putting value of x in (1)

$297 = (3y-6)y$

$3y^2 - 6y = 297 \Rightarrow y^2 - 2y - 99 = 0$

$y^2 - 9y - 11y - 99 = 0$

$y(y+9) - 11(y+9) = 0$

$(y-11)(y+9) = 0$

$y - 11 = 0,\ y + 9 = 0 \rightarrow y = 11, y = -9$

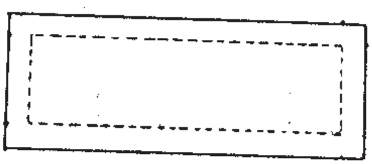
$y = -9$ (impossible) If $y = 11$ then from (2)

$x = 3(11) - 6 = 33 - 6 = 27$

So length of original rectangle $= x = 27$ m

width of original rectangle $= y = 11$ m

Q.12



let breadth (width) of original rectangle $= x$

length of original rectangle $= x+5$

After cutting a strip of 0.5 cm from all arround.

Change in breadth $= x - 2(0.5) = x - 1$

Change in length $= x+5-2(0.5) = x+5-1$
$= x+4$

Now breadth of new rectangle $= x-1$

Length of new rectangle $= x+4$

$\because$ Area $=$ Length $\times$ breadth

So Area $= (x+4)(x-1)$

But Area $= 500\ cm^2$ (given)

$500 = (x+4)(x-1)$

$x^2 - x + 4x - 4 = 500$

$x^2 + 3x - 504 = 0$

$x^2 + 24x - 21x - 504 = 0$

$x(x+24) - 21(x+24) = 0$

$(x-21)(x+24) = 0$

$x - 21 = 0,\ x + 24 = 0$

$x = 21,\ x = -24$ (impossible)

If $x = 21$ then $x+5 = 21+5 = 26$

So length of original rectangle $= 26$ cm

breadth of original rectangle $= 21$ cm.

Q.13 Let

unit digit $= x$

tens digit $= y$

then number $= x + 10y$

According to given condition.

$xy = 18 \longrightarrow (1)$

$x + 10y - 27 = y + 10x$

$x + 10y - 27 - y - 10x = 0$

$9y - 9x - 27 = 0$

$y - x - 3 = 0$

$y = x + 3 \longrightarrow (2)$

Putting value of y in (1)

$x(x+3) = 18$

$x^2 + 3x - 18 = 0$

$x^2 - 3x + 6x - 18 = 0$

$x(x-3) + 6(x-3) = 0$

$(x-3)(x+6) = 0$

$x - 3 = 0,\ x + 6 = 0$

$x = 3,\ x = -6$

If $x=3$ then from ② $y=3+3=6$

then number $= x+10y$

$= 3+10(6)$

$= 63$

If $x=-6$ then from ② $y=-6+3=-3$

then number $= x+10y$

$= -6+10(-3)$

$= -36$

Hence required number is

63 or -36

Q.14

Let unit digit $= x$

tens digits $= y$

Then number $= x+10y$

According to given condition.

$$xy = 14 \longrightarrow ①$$

$$x+10y+45 = y+10x$$

$$x+10y+45-y-10x = 0$$

$$9y-9x+45 = 0$$

$$y-x+5 = 0$$

$$y = x-5$$

Putting value of y in eq. ①

$$x(x-5) = 14$$

$$x^2-5x-14 = 0$$

$$x^2+2x-7x-14 = 0$$

$$x(x+2)-7(x+2) = 0$$

$$(x-7)(x+2) = 0$$

$$x-7=0,\ x+2=0 \Rightarrow x=7,\ x=-2$$

If $x=7$ then from ② $y=7-5=2$

Then number $= x+10y$

$= 7+10(2)$

$= 27$

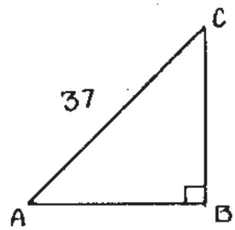
If $x=-2$ then from ② $y=-2-5=-7$

Then number $= x+10y$

$= -2+10(-7) = -72$

Hence number is 27 or -72

Q.15



Given that in right angled triangle

Area $= 210\,m^2$, Hypoteneous $= 37$

Let Base $= x$, Perpendicular $= y$

We know that

Area of triangle $= \frac{1}{2}$(Base)(altitude)

$$210 = \frac{1}{2}(x)(y)$$

$$xy = 420 \Rightarrow 2xy = 840 \longrightarrow ①$$

By Pythagora's theorem

$$(Hyp)^2 = (Base)^2 + (Prep)^2$$

$$Hyp = \sqrt{(Base)^2 + (Prep)^2}$$

Putting values we get.

$$37 = \sqrt{x^2+y^2}$$

$$\text{or } x^2+y^2 = (37)^2$$

$$x^2+y^2 = 1369 \longrightarrow ②$$

Subtracting eq ① from eq ②

$$x^2+y^2-2xy = 1369-840$$

$$x^2+y^2-2xy = 529$$

$$(x-y)^2 = (23)^2$$

$$\Rightarrow x-y = 23$$

$$x = y+23 \longrightarrow ③$$

Putting value of x in ①

$$2(y+23)y = 840$$

$$y^2+23y = 420$$

$$y^2+23y-420 = 0$$

$$y^2-12y+35y-420 = 0$$

$$y(y-12)+35(y-12) = 0$$

$$(y+35)(y-12) = 0$$

$$y+35=0,\ y-12=0$$

$y = -35$ (impossible), $y = 12$

If $y = 12$ then from ③

$x = 12 + 23 = 35$

So, Base $= 35m$, Perpendicular $= 12m$

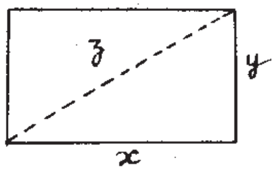
Q.16

Let

Length of reclanale $= x$

width of rectangle $= y$

diagonal of rectangle $= z$



$\because$ Area = length x width

So $1680 = xy \longrightarrow ①$

Given that $z = 58$

By Pythagora's theorem

$(Hyp)^2 = (Base)^2 + (prep)^2$

By the figure

$z^2 = x^2 + y^2$

$\Rightarrow (58)^2 = x^2 + y^2$

$x^2 + y^2 = 3364 \longrightarrow ②$

From ① $xy = 1680$

$\Rightarrow 2xy = 3360 \longrightarrow ③$

Subtracting eq. ③ from eq. ②

$x^2 + y^2 - 2xy = 3364 - 3360$

$x^2 + y^2 - 2xy = 4$

$(x - y)^2 = (2)^2$

$\Rightarrow x - y = 2$

$\Rightarrow x = y + 2 \longrightarrow ④$

Putting value of x in ①

$(y+2)y = 1680$

$y^2 + 2y - 1680 = 0$

$y^2 + 42y - 40y - 1680 = 0$

$y(y+42) - 40(y+42) = 0$

$(y-40)(y+42) = 0$

$y - 40 = 0$, $y + 42 = 0$

$y = 40$, $y = -42$ (impossible)

If $y = 40$ then from ④

$x = 40 + 2 = 42$

Hence length of rectangle $= x = 42m$

Breadth (width) of rectangle $= y = 40m$

Q.17

Let B can do work in days $= x$

Work done by B in one day $= \frac{1}{x}$

A can do work in days $= x + 10$

Work done by A in one day $= \frac{1}{x+10}$

Work done by both A and B in one day $= \frac{1}{x} + \frac{1}{x+10}$

Given that

A and B both can do work in one day $= 12$

$\Rightarrow$ Work done by both A and B in one day $= \frac{1}{12}$

So $\frac{1}{x} + \frac{1}{x+10} = \frac{1}{12}$

Multiplying by $12x(x+10)$ we get

$12x(x+10)\frac{1}{x} + 12x(x+10)\frac{1}{(x+10)} = 12x(x+10)\frac{1}{12}$

$12(x+10) + 12x = x(x+10)$

$12x + 120 + 12x = x^2 + 10x$

$24x + 120 = x^2 + 10x$

$x^2 + 10x - 24x - 120 = 0$

$x^2 - 14x - 120 = 0$

$x^2 - 20x + 6x - 120 = 0$

$x(x-20) + 6(x-20) = 0$

$(x-20)(x+6) = 0$

$x - 20 = 0$, $x + 6 = 0$

$x = 20$, $x = -6$ (impossible)

Hence B can finish his work alone in 20 days.

Q.18

Let

B can do the job in days $= x$

Work done by B in one day $= \frac{1}{x}$

A can do the job in days $= 2x$

Work done by A in one day $= \frac{1}{2x}$

Work done by both A and B in one day =

$$= \frac{1}{x} + \frac{1}{2x}$$

Given that

A and B both can do the job in days = 4

work done by both A and B in one day = $\frac{1}{4}$

So $\frac{1}{x} + \frac{1}{2x} = \frac{1}{4}$

Multiplying by $4x$ we get

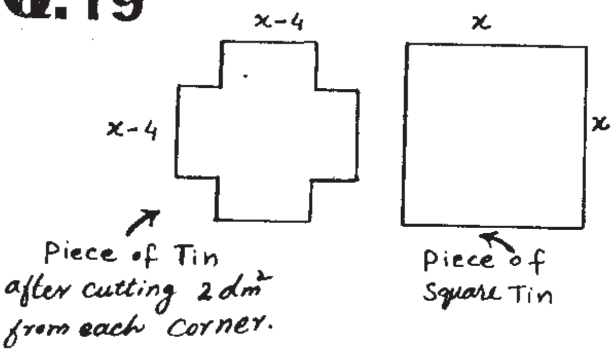
$$4x \cdot \frac{1}{x} + 4x \cdot \frac{1}{2x} = 4x \cdot \frac{1}{4}$$

$$4 + 2 = x \rightarrow x = 6$$

If $x = 6$ then $2x = 2(6) = 12$

So B can do job in 6 days.
while A can do job in 12 days.

Q.19



Let Length of piece of square tin = x dm

width of piece of square tin = x dm

After cutting $2 dm^2$ from each corner

Length of box = $x - 4$ dm

width of box = $x - 4$ dm

Height of box = 2 dm

We know that

Volume of box = length × width × Height

So $128 = (x-4)(x-4) \cdot 2$

$$(x-4)^2 = 64$$

$$(x-4)^2 = (8)^2$$

$$x - 4 = 8 \rightarrow x = 8 + 4 = 12$$

So $x = 12$ dm is length of square tin piece.

Q.20

Let A and B be the two companies. Now let

Investment in company A = x Rs.

Investment in company B = $100000 - x$ Rs.

Profit rate in company A = $y\%$

Profit rate in company B = $(y+1)\%$

As we know that

$$\text{Profit} = \frac{\text{Amount} \times \text{Rate} \times \text{Period}}{100}$$

So $1980 = \frac{x \times y \times 1}{100} \rightarrow xy = 198000 \rightarrow (1)$

Also $3080 = \frac{(100000 - x)(y+1) \times 1}{100}$

$$(100000 - x)(y+1) = 308000$$

$$100000y + 100000 - xy - x = 308000$$

$$100000y - xy - x = 308000 - 100000$$

$$100000y - 198000 - x = 208000$$

$$100000y - x = 208000 + 198000$$

$$100000y - x = 406000 \longrightarrow (2)$$

From (1) $x = \frac{198000}{y} \longrightarrow (3)$

Putting value of x in (2)

$$100000y - \frac{198000}{y} = 406000$$

$$100000y^2 - 198000 = 406000y$$

$50y^2 - 99 = 203y$ ∵ Dividing by 2000

$50y^2 - 203y - 99 = 0$, using $y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$y = \frac{203 \pm \sqrt{(-203)^2 - 4(50)(-99)}}{2(50)} \rightarrow y = \frac{203 \pm \sqrt{61009}}{100}$$

$$y = \frac{203 \pm 247}{100} \rightarrow y = \frac{450}{100}, \; y = \frac{-44}{100}$$

$y = 4.5$, $y = -0.44$ (impossible)

Putting value of y in (3)

$$x = \frac{198000}{4.5} \rightarrow x = 44{,}000$$

Investment in company A = 44,000 Rs.

Investment in company B = 100000 − 44000

= 56,000 Rs.