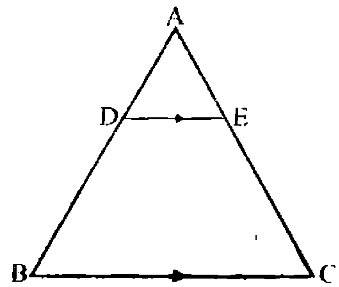
Exercise 14.1

1. In ΔABC, $\overline{DE} \parallel \overline{BC}$



i) $\overline{AD} = 1.5\text{cm}, \overline{BD} = 3\text{ cm},$
$\overline{AE} = 1.3\text{cm}$ then find $\overline{CE}$.

ii) If $\overline{AD} = 2.4\text{cm},\ \overline{AE} = 3.2\text{ cm},$
$\overline{EC} = 4.8\text{ cm}$, find $\overline{AB}$

iii) If $\dfrac{\overline{AD}}{\overline{DB}} = \dfrac{3}{5}, \overline{AC} = 4.8\text{ cm},$ find $\overline{AE}$

iv) If $\overline{AD} = 2.4\text{cm},\ \overline{AE} = 3.2\text{ cm},$
$\overline{DE} = 2\text{ cm},\ \overline{BC} = 5\text{ cm}$, find
$\overline{AB}, \overline{DB}, \overline{AC}, \overline{CE}$

v) If $\overline{AD} = 4x-3, \overline{AE} = 8x-7,$
$\overline{BD} = 3x-1$, and $\overline{CE} = 5x-3$, find the value of x

In ΔABC, $\overline{DE} \parallel \overline{BC}$

(i) $\dfrac{m\overline{AD}}{m\overline{BD}} = \dfrac{m\overline{AE}}{m\overline{EC}}$

$\dfrac{1.5}{3} = \dfrac{1.3}{m\overline{EC}}$

$m\overline{EC} = \dfrac{3 \times 1.3}{1.5}$

$= 2.6\text{cm}$

(ii) In ΔABC, $\overline{DE} \parallel \overline{BC}$

$m\overline{AB} = m\overline{AD} + m\overline{BD}$

Let $m\overline{DB} = x\text{ cm}$

Now $\dfrac{m\overline{AD}}{m\overline{DB}} = \dfrac{m\overline{AE}}{m\overline{EC}}$

$\dfrac{2.4}{x} = \dfrac{3.2}{4.8}$

$x = \dfrac{4.8 \times 2.4}{3.2}$

$x = \dfrac{48 \times 24}{10 \times 32}$

$x = 3.6\text{cm}.$

$\therefore$ $m\overline{AB} = m\overline{AD} + m\overline{BD}$

$m\overline{AB} = 2.4 + 3.6 = 6\text{cm}$

(iii) $\dfrac{m\overline{AD}}{m\overline{DB}} = \dfrac{3}{5}, m\overline{AC} = 4.8\text{cm}$

In ΔABC, $\overline{DE} \parallel \overline{BC}$

$\dfrac{m\overline{AD}}{m\overline{DB}} = \dfrac{m\overline{AE}}{m\overline{EC}}$

$\dfrac{m\overline{AD}}{m\overline{DB}} = \dfrac{m\overline{AC} - m\overline{CE}}{m\overline{CE}}$

$\dfrac{3}{5} = \dfrac{4.8 - m\overline{CE}}{m\overline{CE}}$

$3m\overline{CE} = 5(4.8 - m\overline{CE})$

$3m\overline{CE} = 24 - 5m\overline{CE}$

$3m\overline{CE} + 5m\overline{CE} = 24$

$8m\overline{CE} = 24$

$m\overline{CE} = \dfrac{24}{8} = 3\text{cm}$

$m\overline{AE} = m\overline{AC} - m\overline{CE}$

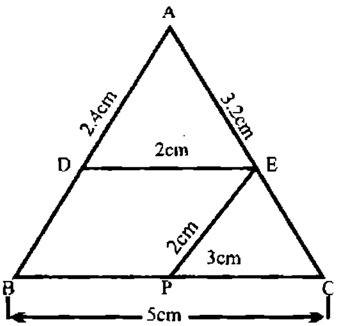
$= 4.8 - 3$

$m\overline{AE} = 1.8\text{cm}$

(iv) $m\overline{AD} = 2.4cm$,

$m\overline{AE} = 3.2\,cm, m\overline{DE} = 2cm,\ m\overline{BC} = 5cm.$

$m\overline{AB} = ?\ m\overline{DB} = ?\ m\overline{AC} = ?\ m\overline{CE} = ?$



$\overline{EP} \parallel \overline{AB}$

DEPB is a parallelogram, then

$m\overline{PB} = mDE = 2cm.$

$m\overline{CP} = 5 - 2 = 3cm$

In $\Delta ABC,\ \overline{EP} \parallel \overline{AB}$

$$\frac{m\overline{CE}}{m\overline{EA}} = \frac{m\overline{CP}}{m\overline{PB}}$$

$$\frac{m\overline{CE}}{3.2} = \frac{3}{2}$$

$$m\overline{CE} = \frac{3\times3.2}{2}$$

$$m\overline{CE} = 3\times1.6 = 4.8cm$$

Now $\overline{DE} \parallel \overline{BC}$, in ΔABC

$$\frac{m\overline{BD}}{m\overline{AD}} = \frac{m\overline{CE}}{m\overline{AE}}$$

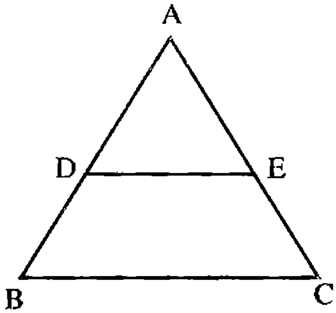
$$\frac{m\overline{BD}}{2.4} = \frac{4.8}{3.2}$$

$$m\overline{BD} = \frac{2.4\times4.8}{3.2} = 3.6cm$$

$$m\overline{AB} = m\overline{AD} + m\overline{DB}$$
$$= 2.4 + 3.6$$
$$= 6.0cm$$

$$m\overline{AC} = m\overline{AE} + m\overline{EC}$$
$$= 3.2 + 4.8$$
$$= 8.0cm.$$

(v) If $\overline{AD} = 4x - 3, \overline{AE} = 8x - 7, \overline{BD} = 3x - 1$ and $\overline{CE} = 5x - 3$, Find the value of x



In $\Delta ABC, \overline{DE} \parallel \overline{BC}$

$$\frac{m\overline{AD}}{m\overline{BD}} = \frac{m\overline{AE}}{m\overline{CE}}$$

$$\frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$$

$$(4x-3)(5x-3) = (8x-7)(3x-1)$$

$$20x^2 - 27x + 9 = 24x^2 - 29x + 7$$

$$20x^2 - 24x^2 - 27x + 29x + 9 - 7 = 0$$

$$-4x^2 + 2x + 2 = 0$$

$$2x^2 - x - 1 = 0$$

$$2x^2 - 2x + x - 1 = 0$$

$$2x(x-1) + 1(x-1) = 0$$

$$(x-1)(2x+1) = 0$$

$$x - 1 = 0 \text{ or } 2x + 1 = 0$$

$$x = 1 \text{ or } 2x = -1$$

$$x = 1 \text{ or } x = \frac{-1}{2}$$

But $x = \frac{-1}{2}$ not possible

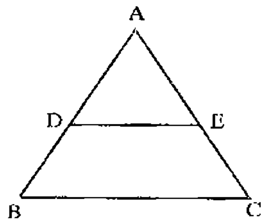
So $x = 1$

2. If ΔABC is an isosceles triangle, $\angle A$ is vertex angle and $\overrightarrow{DE}$ intersects the

sides $\overline{AB}$ and $\overline{AC}$ as shown in the figure so that.

$$m\overline{AD}:m\overline{DB}=m\overline{AE}:m\overline{EC}$$

Prove that ΔADE is also an isosceles triangle.



In ΔABC, $\angle A$ is vertical angle and $\overline{AB}\cong\overline{AC}$

$$\frac{m\overline{AD}}{m\overline{DB}}=\frac{m\overline{AE}}{m\overline{EC}}$$

$$\frac{m\overline{DB}}{m\overline{AD}}=\frac{m\overline{EC}}{m\overline{AE}}$$

$$\frac{m\overline{DB}+m\overline{AD}}{m\overline{AD}}=\frac{m\overline{EC}+m\overline{AE}}{m\overline{AE}}$$

$$\frac{m\overline{AB}}{m\overline{AD}}=\frac{m\overline{AC}}{m\overline{AE}}$$

Now $m\overline{AB}=m\overline{AC}$

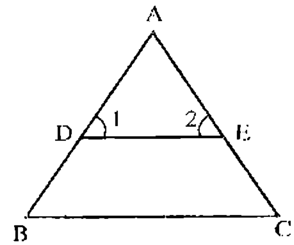
$m\overline{AD}=m\overline{AE}$

Δ ADE is an isosceles triangle.

3. In an equilateral triangle ABC shown in the figure.

$$m\overline{AE}:m\overline{AC}=m\overline{AD}:m\overline{AB}$$

Find all three angles of ΔADE and name it also.



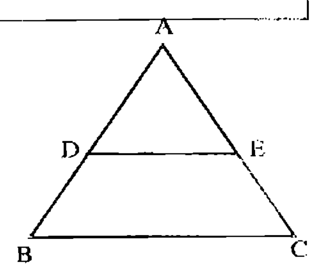
Given: ΔABC is an equilateral triangle.

$$\frac{m\overline{AE}}{m\overline{AC}}=\frac{m\overline{AD}}{m\overline{AB}}$$

To Prove: Find all angles of ΔADE

Statements	Reasons
$\frac{m\overline{AE}}{m\overline{AC}}=\frac{m\overline{AD}}{m\overline{AB}}$	Given
Then $\overline{DE}\parallel\overline{BC}$	Proved
ΔABC is equilateral triangle	Corresponding angle
Then $m\angle A=m\angle B=m\angle C=60^\circ$	
$\overline{DE}\parallel\overline{BC}$	
$m\angle 1=m\angle B=60^\circ$	
$m\angle 2=m\angle C=60^\circ$	
$m\angle A=60^\circ$	

4. Prove that the line segment drawn through the mid-point of one side of a triangle and parallel to another side bisects the third side.



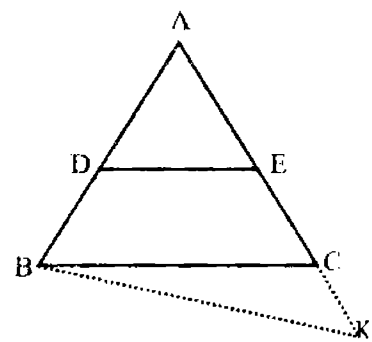
Given in $\Delta ABC, \overline{DE}$ is such that $m\overline{AD} = m\overline{DB}$ and $\overline{DE} \parallel \overline{BC}$

To Prove:

$m\overline{AE} = m\overline{EC}$

Statements	Reasons
In ΔABC	
$\overline{DE} \parallel \overline{BC}$	Given
$\dfrac{m\overline{AD}}{m\overline{BD}} = \dfrac{m\overline{AE}}{m\overline{EC}}$ .....(i)	
$m\overline{AD} = m\overline{DB}$	Given
$\dfrac{m\overline{DB}}{m\overline{DB}} = \dfrac{m\overline{AE}}{m\overline{EC}}$	Put $m\overline{AD} = m\overline{DB}$ in (i)
$1 = \dfrac{m\overline{AE}}{m\overline{EC}}$	
$m\overline{AE} = m\overline{EC}$	

5. Prove that the line segment joining the mid-points of any two sides of a triangle is parallel to the third side.



Given:

In ΔABC, points D, E are such that $m\overline{AD} = m\overline{DB}$

$m\overline{AE} = m\overline{EC}$

$\dfrac{m\overline{AD}}{m\overline{DB}} = \dfrac{m\overline{AE}}{m\overline{EC}}$

To Prove:

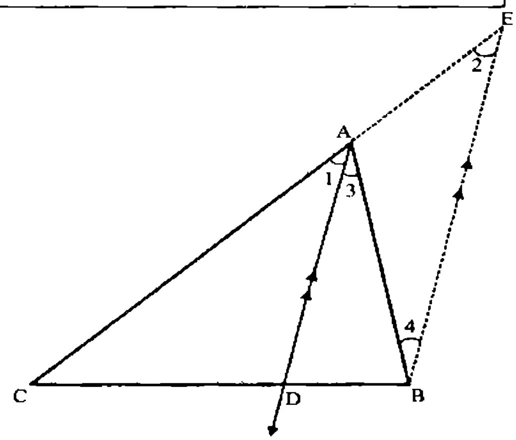
$\overline{DE} \parallel \overline{BC}$

Statements	Reasons
If $\overline{DE} \nparallel \overline{BC}$	
Then suppose $\overline{DE} \parallel \overline{BK}$	
Now $\dfrac{m\overline{AD}}{m\overline{DB}} = \dfrac{m\overline{AE}}{m\overline{EK}}$ .....(i)	
$\dfrac{m\overline{AD}}{m\overline{DB}} = \dfrac{m\overline{AE}}{m\overline{EC}}$ ......(ii)	Given
$\dfrac{m\overline{AE}}{m\overline{EK}} = \dfrac{m\overline{AE}}{m\overline{EC}}$	From (i) and (ii)
$m\overline{EK} = m\overline{EC}$	

It is possible only when point K lies on the point C. Thus $\overline{DE} \parallel \overline{BC}$	

Theorem

The internal bisector of an angle of a triangle divides the side opposite to it in the ratio of the lengths of the sides containing the angle.



Given: In ΔABC internal angle bisector of $\angle A$ meets $\overline{CB}$ at the point D.

To Prove: $m\overline{BD} : m\overline{DC} = m\overline{AB} : m\overline{AC}$

Construction:

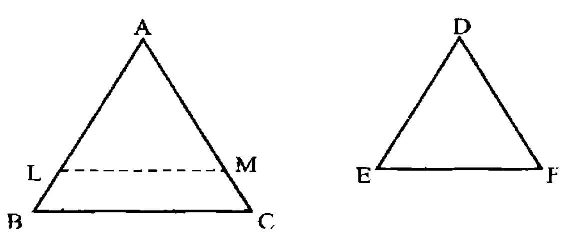
Draw a line segment $\overline{BE} \parallel \overline{DA}$ to meet $\overline{CA}$ produced at E.

Proof:

Statements	Reasons
$\because$ $\overline{AD} \parallel \overline{EB}$ and $\overline{EC}$ intersects them,	Construction
$\therefore$ $m\angle 1 = m\angle 2$ ....(i)	Corresponding angles
Again $\overline{AD} \parallel \overline{EB}$ and $\overline{AB}$ intersects them,	
$\therefore$ $m\angle 3 = m\angle 4$ ....(ii)	Alternate angles
But $m\angle 1 = m\angle 3$	Given
$\therefore$ $m\angle 2 = m\angle 4$	From (i) and (ii)
and $\overline{AB} \cong \overline{AE}$ or $\overline{AE} \cong \overline{AB}$	In a Δ, the sides opposite to congruent angles are also congruent.
Now $\overline{AD} \parallel \overline{EB}$	Construction
$\therefore$ $\dfrac{m\overline{BD}}{m\overline{DC}} = \dfrac{m\overline{EA}}{m\overline{AC}}$	By Theorem
or $\dfrac{m\overline{BD}}{m\overline{DC}} = \dfrac{m\overline{AB}}{m\overline{AC}}$	$m\overline{EA} = m\overline{AB}$ (proved)
Thus $m\overline{BD} : m\overline{DC} = m\overline{AB} : \overline{AC}$	

Theorem: If two triangles are similar, then the measures of their corresponding sides are proportional.

Given: $\Delta ABC \sim \Delta DEF$



i.e., $\angle A \cong \angle D$, $\angle B \cong \angle E$, and $\angle C \cong \angle F$

To Prove:

$$\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{AC}}{m\overline{DF}} = \frac{m\overline{BC}}{m\overline{EF}}$$

Construction:

i) Suppose that $m\overline{AB} > m\overline{DE}$

ii) $m\overline{AB} \leq m\overline{DE}$

On $\overline{AB}$ take a point L such that $m\overline{AL} = m\overline{DE}$

On $\overline{AC}$ take a point M such that $m\overline{AM} = m\overline{DF}$. Join L and M by the line segment LM.

Proof:

Statements	Reasons
i) In $\Delta ALM \longleftrightarrow \Delta DEF$	
$\angle A \cong \angle D$	Given
$\overline{AL} \cong \overline{DE}$	Construction
$\overline{AM} \cong \overline{DF}$	Construction
Thus $\Delta ALM \cong \Delta DEF$	S.A.S. Postulate
and $\angle L \cong \angle E$, $\angle M \cong \angle F$,	(Corresponding angles of congruent triangles)
Now $\angle E \cong \angle B$, and $\angle F \cong \angle C$	Given
$\therefore$ $\angle L \cong \angle B$, $\angle M \cong \angle C$,	Transitivity of congruence
Thus $\overline{LM} \parallel \overline{BC}$	Corresponding angles are equal.
Hence $\frac{m\overline{AL}}{m\overline{AB}} = \frac{m\overline{AM}}{m\overline{AC}}$	By Theorem
or $\frac{m\overline{DE}}{m\overline{AB}} = \frac{m\overline{DF}}{m\overline{AC}}$ .....(i)	$m\overline{AL} = m\overline{DE}$ and $m\overline{AM} = m\overline{DF}$ (construction)
Similarly by intercepting segments on $\overline{BA}$ and $\overline{BC}$, we can prove that $\frac{m\overline{DE}}{m\overline{AB}} = \frac{m\overline{EF}}{m\overline{BC}}$ .....(ii)	
Thus $\frac{m\overline{DE}}{m\overline{AB}} = \frac{m\overline{DF}}{m\overline{AC}} = \frac{m\overline{EF}}{m\overline{BC}}$	by (i) and (ii)
or $\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{AC}}{m\overline{DF}} = \frac{m\overline{BC}}{m\overline{EF}}$	by taking reciprocals
ii) If $m\overline{AB} < m\overline{DE}$, it can similarly be	

proved by taking intercepts on the sides of ΔDEF	
If $m\overline{AB} = m\overline{DE}$,	
then in $\Delta ABC \longleftrightarrow \Delta DEF$	
$\angle A \cong \angle D$	Given
$\angle B \cong \angle E$	Given
and $\overline{AB} \cong \overline{DE}$	
so $\Delta ABC \cong \Delta DEF$	$A.S.A \cong A.S.A$
Thus $\dfrac{m\overline{AB}}{m\overline{DE}} = \dfrac{m\overline{AC}}{m\overline{DF}} = \dfrac{m\overline{BC}}{m\overline{EF}} = 1$	$\overline{AC} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$
Hence the result is true for all the cases.	