# Exercise: 6.6

① i) Centre $(0,0)$, Focus $(6,0)$
vertex $(4,0)$

Here $c=6,\quad a=4$

Now using $c^2=a^2+b^2$

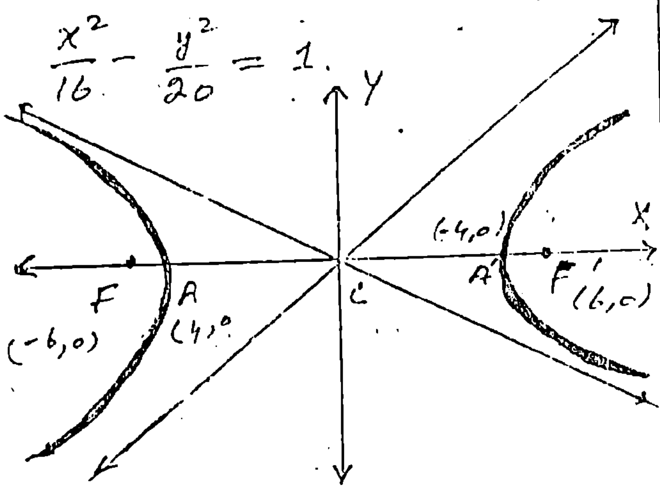
$\Rightarrow 36=16+b^2 \Rightarrow b^2=36-16$

$\Rightarrow \boxed{b^2=20}$

Also X-Axis is the Transverse Axis of the hyperbola.
∴ Required Equation is

$y=\pm\frac{\sqrt5}{2}x$ Asymptotes

$$\frac{x^2}{16}-\frac{y^2}{20}=1.$$




ii) Foci $(\pm5,0)$ vertex $(3,0)$

Here $c=5,\ a=3$

using $c^2=a^2+b^2 \Rightarrow b^2=c^2-a^2$

$\Rightarrow b^2=25-9=16 \Rightarrow b=4$

∴ Equation of the Hyperbola

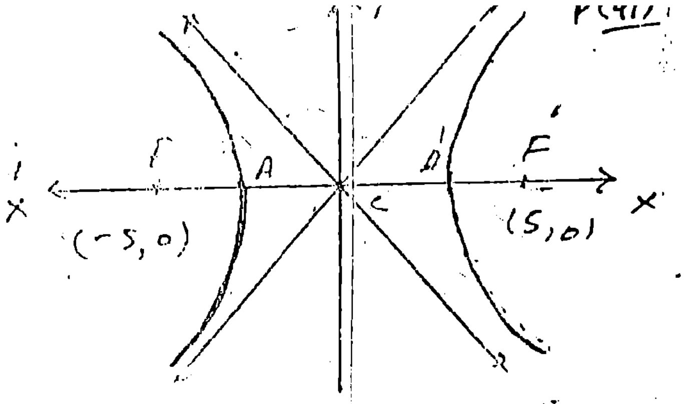
$$\frac{x^2}{9}-\frac{y^2}{16}=1 \quad —①$$

Asymptotes are

$y=\pm\frac{4}{3}x$

Centre $(0,0)$

Transverse Axis is X-Axis




iii) Foci $(2\pm5\sqrt2,-7)$

$F(2+5\sqrt2,-7)$, $F'(2-5\sqrt2,-7)$

Mid point of Foci is the centre

∴ Centre $=\left(\frac{2+5\sqrt2+2-5\sqrt2}{2},\ \frac{-7-7}{2}\right)$

$=(2,-7)$

Given that $2a=10 \Rightarrow a=5$

Now $|FF'|=2c=\sqrt{(2+5\sqrt2-2+5\sqrt2)^2+(-7+7)^2}$

$\Rightarrow 2c=10\sqrt2 \Rightarrow c=5\sqrt2$

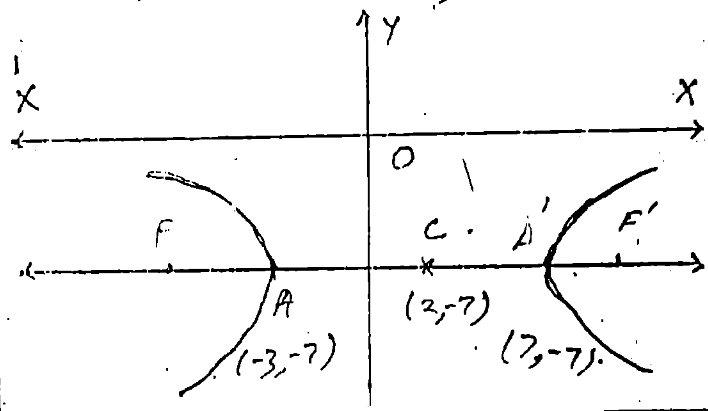
using $c^2=a^2+b^2 \Rightarrow 50=25+b^2$

$\Rightarrow b^2=25$

Transverse Axis is along the Horizontal line $y=-7$

and $a=5$ So vertices are

$(2\pm5,-7) \Rightarrow (7,-7)$, and $(-3,-7)$




v) Foci $(0,\pm9)$; Directrices $y=\pm4$.

Transverse Axis is Y-Axis

$c=9 \Rightarrow ae=9 \quad —①$

and $\frac{a}{e}=4 \Rightarrow e=\frac{a}{4} \quad —②$

$$\frac{x^2}{9}-\frac{1}{b^2}=1 \Rightarrow \frac{4}{b^2}=\frac{25}{9}-1$$

$$\Rightarrow \frac{4}{b^2}=\frac{16}{9} \Rightarrow 16b^2=36 \Rightarrow b^2=\frac{36}{16}$$

$$\Rightarrow b^2=\frac{9}{4}$$

Thus equation ① becomes

$$\frac{y^2}{9}-\frac{(x-2)^2}{9/4}=1.$$

which of the form

$$\frac{Y^2}{9}-\frac{X^2}{9/4}=1$$

where $Y=y$, $X=x-2$

$a^2=9$, $b^2=9/4$.

$c^2=a^2+b^2=9+\frac{9}{4}=\frac{45}{4}$

$c=\frac{3\sqrt{5}}{2}$

Foci $(0, \pm c)$

i.e $X=0$, $Y=\pm c$

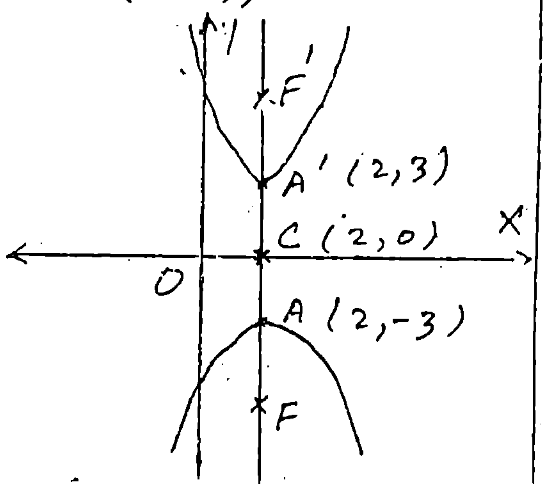
$x-2=0$ $\quad y=\pm\frac{3\sqrt{5}}{2}$

$x=2$

$\therefore$ Foci are $(2, \frac{3\sqrt{5}}{2}), (2, -\frac{3\sqrt{5}}{2})$

Centre $(2,0)$

vertices $(2,3), (2,-3)$




vii: Foci $(5,-2), (5,4)$

one vertex $(5,3)$

Transverse Axis is parallel to the Y-Axis.

Centre = mid point of $FF'$

$$=\left(\frac{5+5}{2}, \frac{-2+4}{2}\right)=(5,1)$$

$a$ = length between the centre & the vertex $(5,3)$

$$=\sqrt{(5-5)^2+(3-1)^2}=2.$$

$\boxed{a=2}$

$2c=|FF'|=\sqrt{(5-5)^2+(4+2)^2}$

$2c=6 \Rightarrow \boxed{c=3}$

using $c^2=a^2+b^2$

$9=4+b^2 \Rightarrow b^2=5$

$\boxed{b^2=5}$

Now required equation of the Hyperbola is

$$\frac{(y-1)^2}{4}-\frac{(x-5)^2}{5}=1.$$

which is of the form.

$$\frac{Y^2}{4}-\frac{X^2}{5}=1.$$

where $X=x-5$, $Y=y-1$

$a^2=4$, $b^2=5$

centre $(5,1)$

vertices $(0, \pm a)$

i.e $X=0$ $\quad Y=\pm a$

$x-5=0$ $\quad y-1=\pm 2$

$x=5$ $\quad y=1\pm 2=3,-1$

$\therefore$ vertices are $(5,3)$ & $(5,-1)$

Foci $(0, \pm c)$

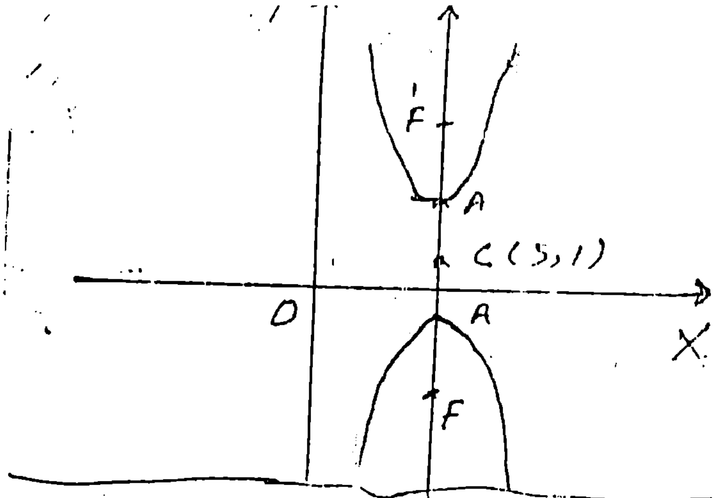
i.e $X=0$, $Y=\pm c$

$x-5=0$ $\quad y-1=\pm 3$

$x=5$ $\quad y=1\pm 3=4,-2$

$\therefore$ Foci are $(5,-2), (5,4)$

Now graph of the Hyperbola is




2) (i) $x^2 - y^2 = 9$ $\quad c = ae$

$$\Rightarrow \frac{x^2}{9} - \frac{y^2}{9} = 1 \quad \text{—①}$$

Here
$a^2 = 9 \Rightarrow a = 3$
$b^2 = 9 \Rightarrow b = 3$ } Transverse Axis is along X-Axis

using
$c^2 = a^2 + b^2 = 9 + 9 = 18 \Rightarrow c = 3\sqrt{2}$

Now Centre of ① is $(0,0)$

Foci $(\pm c, 0) \Rightarrow (\pm 3\sqrt{2}, 0)$

Eccentricity $e = \frac{c}{a} = \frac{3\sqrt{2}}{3} = \sqrt{2}$

vertices $(\pm a, 0) = (\pm 3, 0)$

Directrices $x = \pm \frac{c}{e^2} = \pm \frac{3\sqrt{2}}{2}$

$\Rightarrow x = \pm \frac{3}{\sqrt{2}}$

(ii) $\frac{x^2}{4} - \frac{y^2}{9} = 1$ —①

Here
$a^2 = 4 \Rightarrow a = 2$
$b^2 = 9 \Rightarrow b = 3$ } Transverse Axis is along the X-Axis

using $c^2 = a^2 + b^2 = 4 + 9 = 13$

$c = \sqrt{13}$

Now Centre is $(0,0)$

Foci are $(\pm 2, 0)$

Eccentricity $e = \frac{c}{a} = \frac{\sqrt{13}}{2}$

vertices $(\pm 2, 0)$

Directrices $x = \pm \frac{c}{e^2} = \pm \frac{\sqrt{13}}{\frac{13}{4}}$

$\Rightarrow x = \pm \frac{4\sqrt{13}}{13} = \pm \frac{4}{\sqrt{13}}$ An.

(iii) $\frac{y^2}{16} - \frac{x^2}{9} = 1$ —①

Transverse Axis is along y-Axis

Here $a^2 = 16 \Rightarrow a = 4$
$b^2 = 9 \Rightarrow b = 3$

using $c^2 = a^2 + b^2 \Rightarrow c^2 = 16 + 9$

$c^2 = 25 \Rightarrow c = 5$

Now Centre of ① is $(0,0)$

Foci are $(0, \pm 5)$

Eccentricity $e = \frac{c}{a} = \frac{5}{4}$

vertices $(0, \pm 4)$

Directrices $y = \pm \frac{c}{e^2} = \pm \frac{5}{\frac{25}{16}}$

$\Rightarrow y = \pm 5 \times \frac{16}{25} = \pm \frac{16}{5}$

(iv) $\frac{y^2}{4} - \frac{x^2}{1} = 1$ —①

Transverse Axis of ① is along Y-Axis

Here $a^2 = 4 \Rightarrow a = 2$
$b^2 = 1 \Rightarrow b = 1$

using $c^2 = a^2 + b^2 \Rightarrow c^2 = 4 + 1 = 5$

$\Rightarrow c = \sqrt{5}$

Now Centre $(0,0)$

Foci are $(0, \pm\sqrt{5})$

Eccentricity $e = \frac{c}{a} \Rightarrow e = \frac{\sqrt{5}}{2}$

vertices are $(0, \pm 2)$

Equations of Directrices

$y = \pm \frac{c}{e^2} = \pm \frac{\sqrt{5}}{\frac{5}{4}} = \pm \frac{4\sqrt{5}}{5}$

$y = \pm \frac{4}{\sqrt{5}}$

(v) $\frac{(x-1)^2}{2} - \frac{(y-1)^2}{9} = 1$ —①

① is of the form

$\frac{X^2}{2} - \frac{Y^2}{9} = 1$ —②

where $X = x - 1$, $Y = y - 1$

and $a^2 = 2 \Rightarrow a = \sqrt{2}$, $b^2 = 9 \Rightarrow b = 3$

For centre $(0,0)$ $c=\sqrt{2+9}=\sqrt{11}$

$X=0$ , $Y=0$

$\Rightarrow x-1=0$, $y-1=0$

$x=1$ $y=1$

Centre $(1,1)$.

Transverse Axis is // to X-Axis

For Foci $(\pm c, 0)$

$X=\pm\sqrt{11}$ , $Y=0$

$x-1=\pm\sqrt{11}$ $y-1=0$

$y=1$

$x=1\pm\sqrt{11}$

$\therefore$ Foci are $(1\pm\sqrt{11}, 1)$

For Eccentricity

$e=\frac{c}{a} \Rightarrow e=\frac{\sqrt{11}}{\sqrt{2}}$

$\Rightarrow e=\sqrt{\frac{11}{2}}$

For vertices $(\pm a, 0)$

$X=\pm\sqrt{2}$ , $Y=0$

$x-1=\pm\sqrt{2}$ , $y-1=0$

$x=1\pm\sqrt{2}$ , $y=1$

$\therefore$ vertices are $(1\pm\sqrt{2}, 1)$

Equations of directrices

$X=\pm\frac{c}{e^2}=\pm\frac{ae}{e^2}=\pm\frac{a}{e}$

$\Rightarrow x-1=\pm\frac{\sqrt{2}}{\sqrt{\frac{11}{2}}} \Rightarrow x=1\pm\frac{2}{\sqrt{11}}$

---

vi $\frac{(y+2)^2}{9}-\frac{(x-2)^2}{16}=1$ ---- ①

which is of the form

$\frac{Y^2}{9}-\frac{X^2}{16}=1$ ---- ②

where $Y=y+2$, $X=x-2$

$a^2=9 \Rightarrow a=3$

$b^2=16 \Rightarrow b=4$

Transverse is // to the Y-Axis

Now using. $c^2=a^2+b^2$

$\Rightarrow c^2=9+16=25 \Rightarrow c=5$

For Centre of ①

$X=0$, $Y=0$

$x-2=0$ $y+2=0$

$x=2$ $y=-2$

$\therefore$ Centre is $(2,-2)$.

For Foci $(0, \pm c)$

$\Rightarrow X=0$ $Y=\pm 5$

$\Rightarrow x-2=0$ $\Rightarrow y+2=\pm 5$

$x=2$ $y=-2\pm 5$

$y=-2+5, -2-5$

$y=3, -7$

$\therefore$ Foci are

$(2,3), (2,-7)$

Eccentricity $e=\frac{c}{a}$

$\Rightarrow e=\frac{5}{3}$

For vertices $(0, \pm a)$

$\Rightarrow X=0$ , $Y=\pm a$

$\Rightarrow x-2=0$ $y+2=\pm 3$

$x=2$ $y=-2\pm 3$

$y=-2+3, -2-3$

$y=1, -5$

$\therefore$ vertices are $(2,1)$, $(2,-5)$

Equations of Directrices

$Y=\pm\frac{c}{e^2} \Rightarrow y+2=\pm\frac{5}{\frac{25}{9}}$

$\Rightarrow y+2=\pm\frac{45}{25}$

$y=-2\pm\frac{9}{5}$

---

vii $9x^2-12x-y^2-2y+2=0$

$\Rightarrow 9\left(\frac{9x^2}{9}-\frac{12x}{9}\right)-(y^2+2y)=-2$

$9\left(x^2-\frac{4}{3}x\right)-(y^2+2y)=-2$

$9\left(x^2-\frac{4}{3}x+\frac{4}{9}-\frac{4}{9}\right)-(y^2+2y+1-1)=-2$

$9\left[\left(x-\frac{2}{3}\right)^2-\frac{4}{9}\right]-\left[(y+1)^2-1\right]=-2$

$9\left(x-\frac{2}{3}\right)^2-4-(y+1)^2+1=-2$

$9\left(x-\frac{2}{3}\right)^2-(y+1)^2=-2-1+4$

$9\left(x-\frac{2}{3}\right)^2-(y+1)^2=1$

$$\frac{\left(x-\frac{2}{3}\right)^2}{\frac{1}{9}}-\frac{(y+1)^2}{1}=1 \quad \text{—(1)}$$

which is of the form

$$\frac{X^2}{1/9}-\frac{Y^2}{1}=1 \quad \text{—(2)}$$

where $X=x-\frac{2}{3}$ , $Y=y+1$

$a^2=\frac{1}{9} \Rightarrow a=1/3$

$b^2=1 \Rightarrow b=1$

using $c^2=a^2+b^2=\frac{1}{9}+1=\frac{10}{9}$

$c=\frac{\sqrt{10}}{3}$

Now for centre of (1)

$X=0$ , $Y=0$

$\Rightarrow x-\frac{2}{3}=0$ , $y+1=0$

$x=\frac{2}{3}$ , $y=-1$

$\therefore$ Centre is $\left(\frac{2}{3},-1\right)$

For Foci

$X=\pm c$ , $Y=0$

$\Rightarrow x-\frac{2}{3}=\pm\frac{\sqrt{10}}{3}$ , $y+1=0$

$x=\frac{2}{3}\pm\frac{\sqrt{10}}{3}$ , $y=-1$

$\therefore$ Required Foci $\left(\frac{2}{3}\pm\frac{\sqrt{10}}{3},-1\right)$

Eccentricity: $e=\frac{c}{a}$

$\Rightarrow e=\frac{\frac{\sqrt{10}}{3}}{1/3} \Rightarrow e=\sqrt{10}$

For vertices

$X=\pm a$, $Y=0$

$\Rightarrow x-\frac{2}{3}=\pm\frac{1}{3}$ , $y+1=0$

$x=\frac{2}{3}\pm\frac{1}{3}$ , $y=-1$

$x=1, \frac{1}{3}$

$\therefore$ Foci are $\left(\frac{1}{3},-1\right)$ & $(1,-1)$

Equations of directrices:

$X=\pm\frac{c}{e^2} \Rightarrow x-\frac{2}{3}=\pm\frac{\frac{\sqrt{10}}{3}}{\frac{10}{1}}$

$x=\frac{2}{3}\pm\frac{\sqrt{10}}{3}\cdot\frac{1}{10}=\frac{2}{3}\pm\frac{1}{3\sqrt{10}}$

$x=\frac{2}{3}\pm\frac{1}{3\sqrt{10}}$

---

viii $4y^2+12y-x^2+4x+1=0$

$4(y^2+3y)-(x^2-4x)=-1$

$4\left[y^2+3y+\frac{9}{4}-\frac{9}{4}\right]-\left\{x^2-4x+4-4\right\}=-1$

$4\left[\left(y+\frac{3}{2}\right)^2-\frac{9}{4}\right]-\left[(x-2)^2-4\right]=-1$

$4\left(y+\frac{3}{2}\right)^2-9-(x-2)^2+4=-1$

$4\left(y+\frac{3}{2}\right)^2-(x-2)^2=4$

$$\Rightarrow \frac{\left(y+\frac{3}{2}\right)^2}{1}-\frac{(x-2)^2}{4}=1 \quad \text{—(1)}$$

which is of the form

$$\frac{Y^2}{1}-\frac{X^2}{4}=1 \quad \text{—(2)}$$

where $Y=y+\frac{3}{2}$, $X=x-2$

$a^2=1 \Rightarrow a=1$

$b^2=4 \Rightarrow b=2$

Transverse Axis is along $X=0$ $\Rightarrow x-2=0$ i.e. along $x=2$

Now using $c^2=a^2+b^2$

$\Rightarrow c^2=1+4 \Rightarrow c^2=5 \Rightarrow c=\sqrt{5}$

For Centre

$X=0$ , $Y=0$

$\Rightarrow x-2=0$ , $y+\frac{3}{2}=0$

$x=2$ , $y=-\frac{3}{2}$

$\therefore$ Centre is $\left(2,-\frac{3}{2}\right)$

For Foci

$X=0$ , $Y=\pm c$

$x-2=0$ , $y+\frac{3}{2}=\pm\sqrt{5}$

$x = 2$ $\qquad y = -\frac{3}{2} \pm \sqrt{5}$

$\therefore$ Foci $\left(2, -\frac{3}{2} + \sqrt{5}\right)$

Eccentricity $e = \frac{c}{a} \Rightarrow e = \frac{\sqrt{5}}{1}$

$\Rightarrow e = \sqrt{5}$

For vertices

$X = 0, \qquad Y = \pm a$

$\Rightarrow x - 2 = 0 \qquad y + \frac{3}{2} = \pm 1$

$x = 2 \qquad y = -\frac{3}{2} \pm 1$

$y = -\frac{3}{2} + 1, -\frac{3}{2} - 1 = -\frac{1}{2}, -\frac{5}{2}$

$\therefore$ vertices are

$\left(2, -\frac{5}{2}\right), \left(2, -\frac{1}{2}\right)$

Directrices $Y = \pm \frac{c}{e^2}$

$\Rightarrow y + \frac{3}{2} = \pm \frac{\sqrt{5}}{5}$

$\Rightarrow y = -\frac{3}{2} \pm \frac{1}{\sqrt{5}}$

---

(ix) $x^2 - y^2 + 8x - 2y - 10 = 0$

$x^2 + 8x - y^2 - 2y = 10$

$x^2 + 8x + 16 - 16 - (y^2 + 2y + 1 - 1) = 10$

$(x+4)^2 - 16 - (y+1)^2 + 1 = 10$

$(x+4)^2 - (y+1)^2 = 25$

$\Rightarrow \frac{(x+4)^2}{25} - \frac{(y+1)^2}{25} = 1$ —— (1)

which is of the form

$\frac{X^2}{25} - \frac{Y^2}{25} = 1$ —— (2)

where $X = x + 4, \quad Y = y + 1$

$a^2 = 25 \Rightarrow a = 5$

$b^2 = 25 \Rightarrow b = 5$

using $c^2 = a^2 + b^2 = 25 + 25 = 50$

$c = 5\sqrt{2}$

For centre

$X = 0 \qquad , \qquad Y = 0$

$\Rightarrow x + 4 = 0 \qquad y + 1 = 0$

$x = -4 \qquad y = -1$

$\therefore$ centre $(-4, -1)$.

For Foci

$X = \pm c \qquad Y = 0$

$x + 4 = \pm 5\sqrt{2} \qquad y + 1 = 0$

$x = -4 \pm 5\sqrt{2} \qquad y = -1$

$\therefore$ Foci are $(-4 \pm 5\sqrt{2}, -1)$

Eccentricity $e = \frac{c}{a} \Rightarrow e = \frac{5\sqrt{2}}{5}$

$\Rightarrow e = \sqrt{2}$

For vertices

$X = \pm a \qquad Y = 0$

$y + 1 = 0$

$x + 4 = \pm 5 \qquad y = -1$

$x = -4 \pm 5$

$x = 1, -9$

$\therefore$ vertices are $(1, -1), (-9, -1)$

Directrices $X = \pm \frac{c}{e^2}$

$\Rightarrow x + 4 = \pm \frac{5\sqrt{2}}{2} \Rightarrow y = -4 \pm \sqrt{\frac{5}{2}}$

---

(x) $9x^2 - y^2 - 36x - 6y + 18 = 0$

$9x^2 - 36x - y^2 - 6y + 18 = 0$

$9(x^2 - 4x) - (y^2 + 6y) = -18$

$9(x^2 - 4x + 4 - 4) - (y^2 + 6y + 9 - 9) = -18$

$9[(x-2)^2 - 4] - [(y+3)^2 - 9] = -18$

$\Rightarrow 9(x-2)^2 - 36 - (y+3)^2 + 9 = -18$

$9(x-2)^2 - (y+3)^2 = -18 + 27$

$9(x-2)^2 - (y+3)^2 = 9$

$\Rightarrow \frac{(x-2)^2}{1} - \frac{(y+3)^2}{9} = 1$ —— (1)

which is of the form.

$\frac{X^2}{1} - \frac{Y^2}{9} = 1$ —— (2)

where $X = x - 2, \quad Y = y + 3$

$a^2 = 1 \Rightarrow a = 1$

$b^2 = 9 \Rightarrow b = 3$

using $c^2 = a^2 + b^2 \Rightarrow c^2 = 1 + 9 = 10$

$\Rightarrow c = \sqrt{10}$

Now for centre

$X = 0, \quad Y = 0$

$\Rightarrow x - 2 = 0, \quad y + 3 = 0$

$x = 2 \quad y = -3$

$\therefore$ Required centre is $(2, -3)$.

for Foci

$X = \pm c, \quad Y = 0$

$x - 2 = \pm\sqrt{10} \quad y + 3 = 0$

$x = 2 \pm \sqrt{10} \quad y = -3$

$\therefore$ Foci are $(2 \pm \sqrt{10}, -3)$

Eccentricity $e = \frac{c}{a} \Rightarrow e = \frac{\sqrt{10}}{1}$

$\Rightarrow e = \sqrt{10}$

for vertices

$X = \pm a, \quad Y = 0$

$x - 2 = \pm 1, \quad y + 3 = 0$

$y = -3$

$x = 2 \pm 1$

$x = 3, 1$

$\therefore$ vertices are $(1, -3), (3, -3)$

Directrices $X = \pm \frac{c}{e^2}$

$\Rightarrow x - 2 = \pm \frac{\sqrt{10}}{10} \Rightarrow x = 2 \pm \frac{1}{\sqrt{10}}$

---

(3) $0 < a < c$

$F(-c, 0), F'(c, 0), P(x, y)$

Given that

$|PF| - |PF'| = \pm 2a.$

$\Rightarrow \sqrt{(x+c)^2 + y^2} - \sqrt{(x-c)^2 + y^2} = \pm 2a$

$\Rightarrow \sqrt{(x+c)^2 + y^2} = \pm 2a + \sqrt{(x-c)^2 + y^2}$ —(1)

Squaring both sides of the above (1) Equation we have

$(x+c)^2 + y^2 = 4a^2 + (x-c)^2 + y^2 \pm 4a\sqrt{(x-c)^2 + y^2}$

$x^2 + 2cx + c^2 + y^2 = 4a^2 + x^2 - 2cx + c^2 + y^2 \pm 4a\sqrt{(x-c)^2 + y^2}.$

$4cx - 4a^2 = \pm 4a\sqrt{(x-c)^2 + y^2}.$

$\Rightarrow cx - a^2 = \pm a\sqrt{(x-c)^2 + y^2}$ —(2)

Squaring both sides of (2)

$c^2x^2 - 2a^2cx + a^4 = a^2[x^2 - 2cx + c^2 + y^2]$

$c^2x^2 - 2a^2cx + a^4 = a^2x^2 - 2a^2cx + a^2c^2 + a^2y^2$

$\Rightarrow c^2x^2 + a^4 - a^2x^2 - a^2y^2 = a^2c^2$

$(c^2 - a^2)x^2 - a^2y^2 = a^2c^2 - a^4$

$= a^2(c^2 - a^2)$

$\frac{(c^2 - a^2)x^2}{a^2(c^2 - a^2)} - \frac{a^2y^2}{a^2(c^2 - a^2)} = \frac{a^2(c^2 - a^2)}{a^2(c^2 - a^2)}$

$\frac{x^2}{a^2} - \frac{y^2}{c^2 - a^2} = 1.$

Hence proved.

---

(4) $F(-5, -5), F'(5, 5)$

$A(-3\sqrt{2}, -3\sqrt{2}), A'(3\sqrt{2}, 3\sqrt{2})$

Let $P(x, y)$ be any point on the hyperbola.

Now $2a = |AA'| = \sqrt{(3\sqrt{2} + 3\sqrt{2})^2 + (3\sqrt{2} + 3\sqrt{2})^2}$

$\Rightarrow 2a = \sqrt{(6\sqrt{2})^2 + (6\sqrt{2})^2} = \sqrt{72 + 72} = \sqrt{144}$

$\Rightarrow 2a = 12$

using $|PF| - |PF'| = \pm 2a$

$\Rightarrow |PF| = \pm 2a + |PF'|$

$\sqrt{(x+5)^2 + (y+5)^2} = \pm 12 + \sqrt{(x-5)^2 + (y-5)^2}$

Squaring both sides of the above

$(x+5)^2 + (y+5)^2 = 144 + (x-5)^2 + (y-5)^2 \pm 24\sqrt{(x-5)^2 + (y-5)^2}$

$\Rightarrow x^2 + y^2 + 10x + 10y + 50 = 144 + x^2 + y^2 - 10x - 10y + 50 \pm 24\sqrt{(x-5)^2 + (y-5)^2}$

$\Rightarrow 20x + 20y = 144 \pm 24\sqrt{(x-5)^2 + (y-5)^2}$

$5x+5y=36\pm 6\sqrt{(x-5)^2+(y-5)^2}$

$5x+5y-36=\pm 6\sqrt{(x-5)^2+(y-5)^2}$

$\Rightarrow \pm 6\sqrt{(x-5)^2+(y-5)^2}=5x+5y-36$

Squaring both sides of the above

$36[x^2+y^2-10x-10y+50]=25x^2+25y^2+1296+50xy-360x-360y$

$\Rightarrow 36x^2+36y^2-360x-360y+1800=25x^2+25y^2+1296+50xy-360x-360y$

$\Rightarrow 36x^2-25x^2+36y^2-25y^2-50xy+1800-1296=0$

$11x^2-50xy+11y^2+504=0$

which is the required equation of the hyperbola.

(5) Given points $(2,2)$, $(10,2)$

Let $P(x,y)$ be any point on the hyperbola. Then given that

$\sqrt{(x-2)^2+(y-2)^2}-\sqrt{(x-10)^2+(y-2)^2}=6$

$\Rightarrow \sqrt{(x-2)^2+(y-2)^2}=6+\sqrt{(x-10)^2+(y-2)^2}$

Squaring both sides we have

$(x-2)^2+(y-2)^2=36+(x-10)^2+(y-2)^2+12\sqrt{(x-10)^2+(y-2)^2}$

$\Rightarrow x^2+y^2-4x-4y+8=36+x^2+y^2-20x-4y+104+12\sqrt{(x-10)^2+(y-2)^2}$

$\Rightarrow -4x+20x+8-36-104=12\sqrt{x^2+y^2-20x-4y+104}$

$\Rightarrow 16x-132=12\sqrt{x^2+y^2-20x-4y+104}$

$4x-33=3\sqrt{x^2+y^2-20x-4y+104}$

Squaring both sides

$(4x-33)^2=9(x^2+y^2-20x-4y+104)$

$16x^2+1089-264x=9x^2+9y^2-180x-36y+936$

$\Rightarrow 7x^2-9y^2-84x+36y+153=0$

which is the required equation.

⑥ Let two listening $F_1$ and $F_2$ hear the sound of enemy gun after $t$ and $t-1$ seconds respectively. Here listening posts are 1400m apart

i.e $2c=1400 \Rightarrow \boxed{c=700}$

If $P$ is the position of enemy gun. Given that sound travels at 1080 ft/sec so we have

$|PF_1|-|PF_2|=2a$

$\Rightarrow 1080t-(1080)(t-1)=2a$

$1080t-1080t+1080=2a$

$\Rightarrow 2a=1080 \Rightarrow \boxed{a=540}$

Now using $c^2=a^2+b^2$

$\Rightarrow b^2=c^2-a^2$

$=(700)^2-(540)^2$

$=490000-291600$

$b^2=198400$

Thus equation of Hyperbola is

$\frac{x^2}{(540)^2}-\frac{y^2}{198400}=1.$

$\Rightarrow \frac{x^2}{291600}-\frac{y^2}{198400}=1.$