مشق 3.2

تقسیم کے طریقے سے عادِاعظم HCF معلوم کریں۔

1۔ $x^4 + x^2 + 1, x^4 + x^3 + x + 1$

حل:

$$\begin{array}{r|l} & 1 \\ \hline x^4 + x^2 + 1 & x^4 + x^3 + x + 1 \\ & \underline{-x^4 \quad \pm x^2 \pm 1} \\ & x^3 - x^2 + x = x\,(x^2 - x + 1) \end{array}$$

$x(x^2 - x + 1)$ کو $x^2 - x + 1$ پر تقسیم کرنے سے

$$\begin{array}{r|l} & x^2 + x + 1 \\ \hline x^2 - x + 1 & x^4 + x^2 + 1 \\ & \underline{-x^4 \pm x^2 \quad \mp x^3} \\ & x^3 + 1 \\ & \underline{-x^3 \mp x^2 \pm x} \\ & x^2 - x + 1 \\ & \underline{-x^2 \mp x \pm 1} \\ & 0 \end{array}$$

پس عادِاعظم $= x^2 - x + 1$

2۔ $6x^3 + 7x^2 - 9x + 2, 8x^4 + 6x^3 - 15x^2 + 9x - 2$

حل:

$$
\begin{array}{r|l}
6x^3 + 7x^2 - 9x + 2 & 4x + 5 \\
\hline
8x^4 + 6x^3 - 15x^2 + 9x - 2 & \\
\times\ 3 & \\
\hline
24x^4 + 18x^3 - 45x^2 + 27x - 6 & \\
\underline{-}24x^4 \pm 28x^3 \mp 36x^2 \pm 8x & \\
\hline
-10x^3 - 9x^2 + 19x - 6 & \\
\times -3 & \\
\hline
30x^3 + 27x^2 - 57x + 18 & \\
\underline{-}30x^3 \pm 35x^2 \underline{+} 45x \pm 10 & \\
\hline
-8x^2 - 12x + 8 &
\end{array}
$$

4- پر تقسیم کرنے سے $2x^2 + 3x - 2$

اب

$$
\begin{array}{r|l}
2x^2 + 3x - 2 & 3x \\
\hline
6x^3 + 7x^2 - 9x + 2 & \\
\underline{-}6x^3 \pm 9x^2 \underline{+} 6x & \\
\hline
-2x^2 - 3x + 2 = -(2x^2 + 3x - 2) &
\end{array}
$$

اب

$$
\begin{array}{r|l}
2x^2 + 3x - 2 & 1 \\
\hline
2x^2 + 3x - 2 & \\
\underline{-}2x^2 \pm 3x \underline{+} 2 & \\
\hline
0 &
\end{array}
$$

پس، عادِ اعظم $= 2x^2 + 3x - 2$

3- $4x^3 + 2x^2 - 6x$, $4x^3 - 8x + 4$

حل:

$$
\begin{array}{r|l}
4x^3 - 8x + 4 & 1 \\
\hline
4x^3 + 2x^2 - 6x & \\
\underline{-}4x^3 \quad \underline{+} 8x \pm 4 & \\
\hline
2 \left| \begin{array}{l} 2x^2 + 2x - 4 \end{array} \right. & x^2 + x - 2
\end{array}
$$

اب

$$
\begin{array}{l}
\qquad\qquad 4x \\
x^2 - x - 2 \;\overline{)\; 4x^3 - 8x + 4} \\
\quad \underline{-4x^3 \mp 8x \qquad \pm 4x^2} \\
\qquad\qquad -4x^2 + 4 = -4\cdot(x^2 - 1)
\end{array}
$$

4 پر تقسیم کرنے سے حاصل ہوا $x^2 - 1$

$$
\begin{array}{l}
\qquad\qquad\quad 1 \\
x^2 - 1 \;\overline{)\; x^2 + x - 2} \\
\quad \underline{-x^2 \quad \mp 1} \\
\qquad\qquad x - 1 \;\overline{)\; x^2 - 1 \;}\; x \\
\qquad\qquad\quad \underline{-x^2 \quad \pm x} \\
\qquad\qquad\qquad\quad x - 1 \;\overline{)\; 1} \\
\qquad\qquad\qquad\quad \underline{-x \pm 1} \\
\qquad\qquad\qquad\qquad 0
\end{array}
$$

پس، عاد اعظم $= x - 1$

4- $x^3 + 7x^2 + 12x \;,\; x^3 - 2x^2 - 15x$

حل:

$$
\begin{array}{l}
\qquad\qquad\quad 1 \\
x^3 - 2x^2 - 15x \;\overline{)\; x^3 + 7x^2 + 12x} \\
\quad \underline{-x^3 \mp 2x^2 \mp 15x} \\
\qquad\qquad\quad \overline{)\; 9x^2 + 27x = 9 \;}\; (x^2 + 3x)
\end{array}
$$

اب

$$
\begin{array}{l}
\qquad\qquad\quad x \\
x^2 + 3x \;\overline{)\; x^3 - 2x^2 - 15x} \\
\quad \underline{-x^3 - 3x^2} \\
\qquad\qquad -5x^2 - 15x = -5(x^2 + 3x)
\end{array}
$$

$5-$ سے تقسیم کرنے سے $x^2 + 3x$ حاصل ہوا

$$
\begin{array}{l}
\qquad\quad 1 \\
x^2 + 3x \;\overline{)\; x^2 + 3x} \\
\quad \underline{-x^2 \pm 3x} \\
\qquad\quad 0
\end{array}
$$

پس، عاد اعظم $= x^2 + 3x$ یا $x\,(x + 3)$

5۔ $x^3 - x^2 - x + 1,\ x^4 - 2x^3 + 2x - 1$

حل:

$$x^3 - x^2 - x + 1 \overline{)\ x^4 - 2x^3 + 0x^2 + 2x - 1\ } \quad (x - 1)$$

$$\pm x^4 \mp x^3 \mp x^2 \pm x$$

$$-x^3 + x^2 + x - 1$$

$$\mp x^3 \pm x^2 \mp x \pm 1$$

$$0$$

عادِ اعظم $= x^3 - x^2 - x + 1$

$= x^2(x-1) - 1(x-1)$

$= (x^2-1)(x-1)$

$= [(x)^2 - (1)^2](x-1)$

$= (x+1)(x-1)(x-1)$

$= (x+1)(x-1)^2$

6۔ $x^3 - x^2 - x - 2,\ x^3 + 3x^2 - 6x - 8$

حل:

$$x^3 - x^2 - x - 2 \overline{)\ x^3 + 3x^2 - 6x - 8\ } \quad (1)$$

$$x^3 - x^2 - x - 2$$

$$4x^2 - 5x - 6$$

اب

$$4x^2 - 5x - 6 \overline{)\ x^3 - x^2 - x - 2\ } \quad (x + 1)$$

$$\times 4$$

$$4x^3 - 4x^2 - 4x - 8$$

$$4x^3 - 5x^2 - 6x$$

$$x^2 + 2x - 8$$

$$\times 4$$

$$4x^2 + 8x - 32$$

$$4x^2 - 5x - 6$$

$$13x - 26 = 13(x - 2)$$

$$x - 2 \overline{)\ 4x^2 - 5x - 6\ } \quad (4x)$$

$$4x^2 - 8x$$

$$3x - 6 = 3(x - 2)$$

$$
\begin{array}{r|l}
x-2 & 1 \\
\hline
x-2 & \\
\mp x \pm 2 & \\
\hline
0 &
\end{array}
$$

پس، عاد اعظم $= x-2$

7۔ $x^2+3x-4,\ x^3-2x^2-2x+3$

حل:

$$
\begin{array}{r|l}
x^2+3x-4 & x-5 \\
\hline
x^3-2x^2-2x+3 & \\
\mp x^3 \pm 3x^2 \mp 4x & \\
\hline
-5x^2+2x+3 & \\
\pm 5x^2 \mp 15x \pm 20 & \\
\hline
17x-17=17(x-1) &
\end{array}
$$

اب

$$
\begin{array}{r|l}
x-1 & x+4 \\
\hline
x^2+3x-4 & \\
\mp x^2 \pm x & \\
\hline
4x-4 & \\
\mp 4x \pm 4 & \\
\hline
0 &
\end{array}
$$

پس، عاد اعظم $= x-1$

8۔ $3x^3-14x^2+9x+10,\ 15x^3-34x^2+21x-10$

حل:

$$
\begin{array}{r|l}
3x^3-14x^2+9x+10 & 5 \\
\hline
15x^3-34x^2+21x-10 & \\
\mp 15x^3 \pm 70x^2 \pm 45x \pm 50 & \\
\hline
36x^2-24x-60=12(3x^2-2x-5) &
\end{array}
$$

اب

$$
\begin{array}{r|l}
3x^2-2x-5 & x \\
\hline
3x^3-14x^2+9x+10 & \\
\mp 3x^3 \pm 2x^2 \pm 5x & \\
\hline
-12x^2+14x+10=-2(6x^2-7x-5) &
\end{array}
$$

اب

$$3x^2 - 2x - 5 \;\overline{)\; 6x^2 - 7x - 5 \;}\;\; 2$$

$$\underline{-6x^2 \mp 4x \mp 10}$$

$$-3x + 5 = -(3x - 5)$$

اب

$$3x - 5 \;\overline{)\; 3x^2 - 2x - 5 \;}\;\; x + 1$$

$$\underline{-3x^2 \mp 5x}$$

$$3x - 5$$

$$\underline{-3x \mp 5}$$

$$0$$

پس، عاداعظم $= 3x - 5$

9۔ $2x^4 + x^3 + 4x + 2,\ 6x^3 + 5x^2 + x,\ 2x^4 + 3x^3 + x^2 + 2x + 1$

حل:

$$2x^4 + x^3 + 4x + 2 \;\overline{)\; 2x^4 + 3x^3 + x^2 + 2x + 1 \;}\;\; 1$$

$$\underline{-2x^4 \mp x^3 \quad \mp 4x \mp 2}$$

$$2x^3 + x^2 - 2x - 1$$

اب

$$2x^3 + x^2 - 2x - 1 \;\overline{)\; 2x^4 + x^3 + 4x + 2 \;}\;\; x$$

$$\underline{-2x^4 \mp x^3 \mp x \quad \mp 2x^2}$$

$$2x^2 + 5x + 2$$

$$2x^2 + 5x + 2 \;\overline{)\; 2x^3 + x^2 - 2x - 1 \;}\;\; x - 2$$

$$\underline{-2x^3 \mp 5x^2 \mp 2x}$$

$$-4x^2 - 4x - 1$$

$$\underline{\mp 4x^2 \mp 10x \mp 4}$$

$$6x + 3 = 3(2x + 1)$$

$$
\begin{array}{r|l}
 & x + 2 \\
\hline
2x + 1 & 2x^2 + 5x + 2 \\
 & \underline{-2x^2 \pm x} \\
 & 4x + 2 \\
 & \underline{-4x \pm 2} \\
 & 0
\end{array}
$$

پس $2x+1$، $2x^4+3x^3+x^2+2x+1$ اور $2x^4 + x^3 + 4x + 2$ کا عاد اعظم ہے۔ اب ہم $6x^3 + 5x^2 + x$ کو $2x + 1$ پر تقسیم کریں گے۔

$$
\begin{array}{r|l}
 & 3x^2 + x \\
\hline
2x + 1 & 6x^3 + 5x^2 + x \\
 & \underline{-6x^3 \pm 3x^2} \\
 & 2x^2 + x \\
 & \underline{-2x^2 \pm x} \\
 & 0
\end{array}
$$

پس $2x+1$ تمام اظہاریوں کا عاد اعظم ہے

10۔ $x^3 + x^2 - 5x + 3, x^3 - 7x + 6, x^3 + 2x^2 - 2x + 3$

حل:

$$
\begin{array}{r|l}
 & 1 \\
\hline
x^3 + x^2 - 5x + 3 & x^3 + 2x^2 - 2x + 3 \\
 & \underline{-x^3 \pm x^2 \mp 5x \pm 3} \\
 & x^2 + 3x
\end{array}
$$

$$
\begin{array}{r|l}
x^3 + x^2 - 5x + 3 & x - 2 \\
\hline
\underline{-x^3 \pm 3x^2} & \\
- 2x^2 - 5x + 3 & \\
\underline{\mp 2x^2 \mp 6x} & \\
x + 3 &
\end{array}
$$

اب

$$
\begin{array}{r|l}
 & x \\
\hline
x + 3 & x^2 + 3x \\
 & \underline{-x^2 \pm 3x} \\
 & 0
\end{array}
$$

پس $x^3 + x^2 - 5x + 3$ اور $x^3 + 2x^2 - 2x + 3$ کا عاد اعظم $x + 3$ ہے۔ اب ہم $x^3 + 7x + 6$ کو $x + 3$ پر تقسیم کریں گے۔

$$
\begin{array}{r|l}
 & x^2 - 3x + 2 \\
\hline
x + 3 & x^3 - 7x + 6 \\
 & \underline{-x^3 \quad \pm 3x^2} \\
 & \quad -3x^2 - 7x + 6 \\
 & \quad \underline{\mp 3x^2 \mp 9x} \\
 & \quad\quad 2x + 6 \\
 & \quad\quad \underline{-2x \pm 6} \\
 & \quad\quad\quad 0
\end{array}
$$

پس، مشترک عاد اعظم $= x + 3$