EXERCISE 3.1

1. **If** $A = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$**, and** $B = \begin{bmatrix} 1 & 7 \\ 6 & 4 \end{bmatrix}$**, then show that**

(i) $4A - 3A = A$ **(ii)** $3B - 3A = 3(B - A)$.

Solution. $A = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} \Rightarrow 4A = 4\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 2(4) & 3(4) \\ 1(4) & 5(4) \end{bmatrix} = \begin{bmatrix} 8 & 12 \\ 4 & 20 \end{bmatrix}$

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} \Rightarrow 3A = 3\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 2(3) & 3(3) \\ 1(3) & 5(3) \end{bmatrix} = \begin{bmatrix} 6 & 9 \\ 3 & 15 \end{bmatrix}$$

$$4A - 3A = \begin{bmatrix} 8 & 12 \\ 4 & 20 \end{bmatrix} - \begin{bmatrix} 6 & 9 \\ 3 & 15 \end{bmatrix} = \begin{bmatrix} 8-6 & 12-9 \\ 4-3 & 20-15 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} = A$$

(ii) Now $3B = 3\begin{bmatrix} 1 & 7 \\ 6 & 4 \end{bmatrix} = \begin{bmatrix} 1(3) & 7(3) \\ 6(3) & 4(3) \end{bmatrix} = \begin{bmatrix} 3 & 21 \\ 18 & 12 \end{bmatrix}$

$$\therefore\ 3B-3A=\begin{bmatrix}3 & 21\\ 18 & 12\end{bmatrix}-\begin{bmatrix}6 & 9\\ 3 & 15\end{bmatrix}=\begin{bmatrix}3-6 & 21-9\\ 18-3 & 12-15\end{bmatrix}=\begin{bmatrix}-3 & 12\\ 15 & -3\end{bmatrix}\ldots(1)$$

$$3(B-A)\ =3\left\{\begin{bmatrix}1 & 7\\ 6 & 4\end{bmatrix}-\begin{bmatrix}2 & 3\\ 1 & 5\end{bmatrix}\right\}\ =3\begin{bmatrix}1-2 & 7-3\\ 6-1 & 4-5\end{bmatrix}=3\begin{bmatrix}-1 & 4\\ 5 & -1\end{bmatrix}$$

$$=\begin{bmatrix}-1(3) & 4(3)\\ 5(3) & -1(3)\end{bmatrix}=\begin{bmatrix}-3 & 12\\ 15 & -3\end{bmatrix}\ \ldots\ (2)$$

From (1) and (2), we get : $3B-3A = 3(B-A)$.

2. If $A=\begin{bmatrix}i & 0\\ 1 & -i\end{bmatrix}$, **show that** $A^4 = I_2$.

Solution. $A=\begin{bmatrix}i & 0\\ 1 & -i\end{bmatrix} \Rightarrow A^2 = A.A = \begin{bmatrix}i & 0\\ 1 & -i\end{bmatrix}\begin{bmatrix}i & 0\\ 1 & -i\end{bmatrix}$

$$=\begin{bmatrix}i(i)+0(1) & i(0)+0(-i)\\ 1(i)+(-i)(1) & 1(0)+(-i)(-i)\end{bmatrix}=\begin{bmatrix}i^2 & 0\\ 0 & i^2\end{bmatrix}=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$$

$$A^4 = A^2 . A^2 = \begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$$

$$=\begin{bmatrix}(-1)(-1)+0(0) & (-1)(0)+0(-1)\\ 0(-1)+(-1)(0) & 0(0)+(-1)(-1)\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I_2$$

3. Find x and y if

Solution. (i) $\begin{bmatrix}x+3 & 1\\ -3 & 3y-4\end{bmatrix}=\begin{bmatrix}2 & 1\\ -3 & 2\end{bmatrix}$

Using definition of equality of two matrices, we have

$$x+3 = 2 \Rightarrow x = 2-3 = -1$$

and $3y-4 = 2 \Rightarrow 3y = 6 \Rightarrow y = 2 \quad \therefore \boxed{x=-1, y=2}$

Solution.(ii) $\begin{bmatrix}x+3 & 1\\ -3 & 3y-4\end{bmatrix}=\begin{bmatrix}y & 1\\ -3 & 2x\end{bmatrix}$

Using definition of equality of two matrices, we have

$$\begin{bmatrix}x+3 & 1\\ -3 & 3y-4\end{bmatrix}=\begin{bmatrix}y & 1\\ -3 & 2x\end{bmatrix}\text{ implies}$$

$$x+3 = y \qquad 3y-4 = 2x$$

or $\quad x-y=-3 \ \ldots\ (1), \qquad 2x-3y=-4\ \ldots\ (2)$

Multiplying (1) by 3 gives $3x - 3y = -9$ ... (3)

Subtracting (2) from (3), we get : $x = -5$,

put in (1) then $y = x + 3 = -5 + 3 = -2$

$\therefore \quad \boxed{x = -5, y = -2}$

4. **If** $A = \begin{bmatrix} -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix}$ **and** $B = \begin{bmatrix} 0 & 3 & 2 \\ 1 & -1 & 2 \end{bmatrix}$, **find the following:**

(i) $4A - 3B$ **(ii)** $A + 3(B - A)$.

Solution. (i) $4A = 4\begin{bmatrix} -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 4(-1) & 4(2) & 4(3) \\ 4(1) & 4(0) & 4(2) \end{bmatrix} = \begin{bmatrix} -4 & 8 & 12 \\ 4 & 0 & 8 \end{bmatrix}$

$$3B = 3\begin{bmatrix} 0 & 3 & 2 \\ 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 3(0) & 3(3) & 3(2) \\ 3(1) & 3(-1) & 3(2) \end{bmatrix} = \begin{bmatrix} 0 & 9 & 6 \\ 3 & -3 & 6 \end{bmatrix}$$

$$\therefore \quad 4A - 3B = \begin{bmatrix} -4 & 8 & 12 \\ 4 & 0 & 8 \end{bmatrix} - \begin{bmatrix} 0 & 9 & 6 \\ 3 & -3 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} -4-0 & 8-9 & 12-6 \\ 4-3 & 0-(-3) & 8-6 \end{bmatrix} = \begin{bmatrix} -4 & -1 & 6 \\ 1 & 3 & 2 \end{bmatrix}$$

(ii) $B - A = \begin{bmatrix} 0 & 3 & 2 \\ 1 & -1 & 2 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 0-(-1) & 3-2 & 2-3 \\ 1-1 & -1-0 & 2-2 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 1 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$A + 3(B - A) = \begin{bmatrix} -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix} + 3\begin{bmatrix} 1 & 1 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 3(1) & 3(1) & 3(-1) \\ 3(0) & 3(-1) & 3(0) \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 3 & -3 \\ 0 & -3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1+3 & 2+3 & 3-3 \\ 1+0 & 0-3 & 2+0 \end{bmatrix} = \begin{bmatrix} 2 & 5 & 0 \\ 1 & -3 & 2 \end{bmatrix}$$

5. **Find x and y if** $\begin{bmatrix} 2 & 0 & x \\ 1 & y & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & x & y \\ 0 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 6 & 1 \end{bmatrix}$.

Solution. $\begin{bmatrix} 2 & 0 & x \\ 1 & y & 3 \end{bmatrix} + \begin{bmatrix} 2 & 2x & 2y \\ 0 & 4 & -2 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 6 & 1 \end{bmatrix}$

$$\begin{bmatrix} 2+2 & 0+2x & x+2y \\ 1+0 & y+4 & 3-2 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 6 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2x & x+2y \\ 1 & y+4 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 6 & 1 \end{bmatrix}$$

Now, using the equality of matrices , we have

$\therefore \quad 2x = -2 \quad \Rightarrow \quad x = -1$

$y + 4 = 6 \quad \Rightarrow \quad y = 6-4 = 2 \quad \therefore \quad x = -1,\ y = 2$

6. If $A = [a_{ij}]_{3\times3}$, show that

(i) $\lambda(\mu A) = (\lambda\mu)A$ (ii) $(\lambda+\mu)A = \lambda A + \mu A$ (iii) $\lambda A - A = (\lambda-1)A$

Solution. (i) $\lambda(\mu A) = \lambda(\mu[a_{ij}]_{3\times3}) = \lambda([\mu a_{ij}]_{3\times3})$

$= [\lambda\mu a_{ij}]_{3\times3} = \lambda\mu[a_{ij}]_{3\times3} = (\lambda\mu)A$

(ii) $(\lambda+\mu)A = (\lambda+\mu)[a_{ij}]_{3\times3} = [(\lambda+\mu)a_{ij}]_{3\times3} = [\lambda a_{ij} + \mu a_{ij}]_{3\times3}$

$= [\lambda a_{ij}]_{3\times3} + [\mu a_{ij}]_{3\times3} = \lambda[a_{ij}]_{3\times3} + \mu[a_{ij}]_{3\times3} = \lambda A + \mu A$

(iii) $\lambda A - A = \lambda[a_{ij}]_{3\times3} - 1[a_{ij}]_{3\times3}$. Taking $[a_{ij}]_{3\times3}$ common, we get

$= (\lambda-1)[a_{ij}]_{3\times3} = (\lambda-1)A$

7. If $A = [a_{ij}]_{2\times3}$, $B = [b_{ij}]_{2\times3}$, show that $\lambda(A+B) = \lambda A + \lambda B$.

Solution. $\lambda(A+B) = \lambda\left([a_{ij}]_{2\times3} + [b_{ij}]_{2\times3}\right)$

$= \lambda[a_{ij}]_{2\times3} + \lambda[b_{ij}]_{2\times3} = \lambda A + \lambda B$.

8. If $A = \begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix}$ and $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, find the values of a and b.

Solution. $A = \begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix}$, then

$$A^2 = A.A = \begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix}\begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix} = \begin{bmatrix} 1+2a & 2+2b \\ a+ab & 2a+b^2 \end{bmatrix}$$

Given that $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1+2a & 2+2b \\ a+ab & 2a+b^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Equality of matrices $\Rightarrow 1 + 2a = 0 \Rightarrow a = -\frac{1}{2}$

and $2 + 2b = 0 \Rightarrow b = -1 \quad \therefore \quad \boxed{a = -1/2,\ b = -1}$

9. If $A = \begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix}$ and $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find the values of a and b.

Solution. $A = \begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix}\begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix} = \begin{bmatrix} 1-a & -1-b \\ a+ab & -a+b^2 \end{bmatrix}$

Given : $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1-a & -1-b \\ a+ab & a+b^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Equality of matrices $\Rightarrow 1-a = 1 \Rightarrow a = 0$

and $-1-b = 0 \Rightarrow b = -1 \quad \therefore \boxed{a = 0,\ b = -1}$

Definition. A matrix obtained by interchanging rows and columns is said to be the *transpose* of the original matrix and it is denoted by A^t. In other words, let A be any matrix. If rows and columns of A are interchanged then the resulting matrix is called the transpose of the matrix A.

For example, $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 1 \end{bmatrix}$ then $A^t = \begin{bmatrix} 1 & 0 \\ -1 & 3 \\ 2 & 1 \end{bmatrix}$.

Note the formula : $(A+B)^t = A^t + B^t$ & $(AB)^t = B^t A^t$.

10. If $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & -1 \end{bmatrix}$, show that $(A+B)^t = A^t + B^t$.

Solution. $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 1 \end{bmatrix} \Rightarrow A^t = \begin{bmatrix} 1 & 0 \\ -1 & 3 \\ 2 & 1 \end{bmatrix}$,

$B = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & -1 \end{bmatrix} \Rightarrow B^t = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 0 & -1 \end{bmatrix}$

Adding: $A + B = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 2 \\ 1 & 5 & 0 \end{bmatrix}$

$A+B = \begin{bmatrix} 3 & 2 & 2 \\ 1 & 5 & 0 \end{bmatrix} \Rightarrow (A+B)^t = \begin{bmatrix} 3 & 1 \\ 2 & 5 \\ 2 & 0 \end{bmatrix}$ ... (1)

Also $A^t + B^t = \begin{bmatrix} 1 & 0 \\ -1 & 3 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 2 & 5 \\ 2 & 0 \end{bmatrix}$ ... (2)

From (1) and (2) $(A+B)^t = A^t + B^t$.

11. **Find A^3 if $A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$.**

Solution. We have $A^2 = AA = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$

$$= \begin{bmatrix} 1(1)+1(5)+3(-2) & 1(1)+1(2)+3(-1) & 1(3)+1(6)+3(-3) \\ 5(1)+2(5)+6(-2) & 5(1)+2(2)+6(-1) & 5(3)+2(6)+6(-3) \\ (-2)1+(-1)5+(-3)(-2) & (-2)1+(-1)2+(-3)(-1) & (-2)3+(-1)6+(-3)(-3) \end{bmatrix}$$

$$= \begin{bmatrix} 1+5-6 & 1+2-3 & 3+6-9 \\ 5+10-12 & 5+4-6 & 15+12-18 \\ -2-5+6 & -2-2+3 & -6-6+9 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 3 & 3 & 9 \\ -1 & -1 & -3 \end{bmatrix}$$

$$\therefore \quad A^3 = A^2 . A = \begin{bmatrix} 0 & 0 & 0 \\ 3 & 3 & 9 \\ -1 & -1 & -3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 0(1)+0(5)+0(-2) & 0(1)+0(2)+0(-1) & 0(3)+0(6)+0(-3) \\ 3(1)+3(5)+9(-2) & 3(1)+3(2)+9(-1) & 3(3)+3(6)+9(-3) \\ (-1)1+(-1)5+(-3)(-2) & (-1)1+(-1)2+(-3)(-1) & (-1)3+(-1)6+(-3)(-3) \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+0 & 0+0+0 & 0+0+0 \\ 3+15-18 & 3+6-9 & 9+18-27 \\ -1-5+6 & -1-2+3 & -3-6+9 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O_3$$

12. **Find the matrix X if** (i) $X\begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix}$,

(ii) $\begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix} X = \begin{bmatrix} 2 & 1 \\ 5 & 10 \end{bmatrix}$.

Solution. (i) $X\begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix}$

Let $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 5a-2b & 2a+b \\ 5c-2d & 2c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix}$$

Comparing the corresponding elements, we get

$5a-2b = -1$ ... (i), $\quad 2a+b = 5$ ... (ii)

$5c-2d = 12$ ... (iii) , $\quad 2c+d = 3$ ... (iv)

To solve (i) and (ii), put $b = 5-2a$ from (ii) in (i), then

$5a - 2(5-2a) = -1 \Rightarrow 5a - 10 + 4a = -1 \Rightarrow 9a = 9 \Rightarrow \boxed{a = 1}$

Put $a = 1$ in (ii), then $b = 5-2a = 5-2(1) = 5-2 = 3 \Rightarrow \boxed{b = 3}$

To solve (iii) and (iv), put $d = 3-2c$ from (iv) in (iii), then

$5c-2(3-2c) = 12 \Rightarrow 5c-6+4c = 12 \Rightarrow 9c = 18 \Rightarrow \boxed{c = 2}$

Put $c = 2$ in (iv), then $d = 3-2c = 5-2(2) = 3-4 = -1 \Rightarrow \boxed{d = -1}$

Thus $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$

Solution. (ii) $\begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix} X = \begin{bmatrix} 2 & 1 \\ 5 & 10 \end{bmatrix}$.

Let $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then

$$\begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 5 & 10 \end{bmatrix}$$

$$\begin{bmatrix} 5a+2c & 5b+2d \\ -2a+c & -2b+d \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 5 & 10 \end{bmatrix}$$

Comparing the corresponding elements, we get

$5a+2c = 2$ ... (i), $\quad 5b+2d = 1$ ... (ii)

$-2a+c = 5$ ... (iii) , $\quad -2b+d = 10$ ... (iv)

To solve (i) and (iii), put $c = 5+2a$ from (iii) in (i), then

$5a + 2(5+2a) = 2 \Rightarrow 5a+10+4a = 2 \Rightarrow 9a = -8 \Rightarrow \boxed{a = -\frac{8}{9}}$

Put $a = -\frac{8}{9}$ in (iii), then $c = 5+2a = 5+2\left(-\frac{8}{9}\right) = 5-\frac{16}{9} = \frac{45-16}{9} = \frac{29}{9}$

$$\Rightarrow \quad \boxed{c = \frac{29}{9}}$$

To solve (ii) and (iv), put $d = 10 + 2b$ from (iv) in (ii), then

$$5b + 2(10+2b) = 1 \Rightarrow 5b + 20 + 4b = 1 \Rightarrow 9b = -19 \Rightarrow \boxed{b = -\frac{19}{9}}$$

Put $b = -\frac{19}{9}$ in (ii), then

$$d = 10 + 2b = 10 + 2\left(-\frac{19}{9}\right) = 10 - \frac{38}{9} = \frac{90-38}{9} = \frac{52}{9} \quad \Rightarrow \quad \boxed{d = \frac{52}{9}}$$

Thus $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -\frac{8}{9} & -\frac{19}{9} \\ \frac{29}{9} & \frac{52}{9} \end{bmatrix}$

13. Find the matrix A if (i) $\begin{bmatrix} 5 & -1 \\ 0 & 0 \\ 3 & 1 \end{bmatrix} A = \begin{bmatrix} 3 & -7 \\ 0 & 0 \\ 7 & 2 \end{bmatrix}$,

(ii) $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} A = \begin{bmatrix} 0 & -3 & 8 \\ 3 & 3 & -7 \end{bmatrix}$.

Solution. (i) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then

$$\begin{bmatrix} 5 & -1 \\ 0 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 3 & -7 \\ 0 & 0 \\ 7 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 5a - c & 5b - d \\ 0+0 & 0+0 \\ 3a + c & 3b + d \end{bmatrix} = \begin{bmatrix} 3 & -7 \\ 0 & 0 \\ 7 & 2 \end{bmatrix}$$

Comparing the corresponding elements, we get

$5a - c = 3$ ... (i), $\quad 5b - d = -7$ ... (ii)

$3a + c = 7$ ... (iii) , $3b + d = 2$ ... (iv)

Adding (i) and (iii), we get

$$8a = 10 \Rightarrow a = \frac{10}{8} = \frac{5}{4}$$

(iii) gives $c = 7 - 3a = 7 - \frac{15}{4} = \frac{28-15}{4} = \frac{13}{4}$

Adding (ii) and (iv), we get $8b = -5 \Rightarrow b = -\frac{5}{8}$

(iv) gives $d = 2 - 3b = 2 + \frac{15}{8} = \frac{16+15}{8} = \frac{31}{8}$

Hence required matrix is $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 5/4 & -5/8 \\ 13/4 & 31/8 \end{bmatrix}$.

Solution. (ii) $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} A = \begin{bmatrix} 0 & -3 & 8 \\ 3 & 3 & -7 \end{bmatrix}$.

Let $A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$, then

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} 0 & -3 & 8 \\ 3 & 3 & -7 \end{bmatrix}$$

$$\begin{bmatrix} 2a-d & 2b-e & 2c-f \\ -a+2d & -b+2e & -c+2f \end{bmatrix} = \begin{bmatrix} 0 & -3 & 8 \\ 3 & 3 & -7 \end{bmatrix}$$

Comparing the corresponding elements, we get

$2a - d = 0$ ... (i), $2b - e = -3$ ... (ii), $2c - f = 8$ ... (iii)

$-a + 2d = 3$ ... (iv), $-b + 2e = 3$ ... (v), $-c + 2f = -7$ ... (vi)

To solve (i) and (iv), put $d = 2a$ from (i) in (iv), then

$-a + 2d = 3 \Rightarrow -a + 2(2a) = 3 \Rightarrow -a + 4a = 3 \Rightarrow 3a = 3 \Rightarrow \boxed{a = 1}$

Put $a = 1$ in (iv), then $d = 2a = 2(1) = 2 \Rightarrow \boxed{d = 2}$

To solve (ii) and (v), put $e = 2b + 3$ from (ii) in (v), then

$-b + 2(2b+3) = 3 \Rightarrow -b + 4b + 6 = 3 \Rightarrow 3b = -3 \Rightarrow \boxed{b = -1}$

Put $b = -1$ in (v), then $e = 2b + 3 = 2(-1) + 3 = -2 + 3 = 1 \Rightarrow \boxed{e = 1}$

To solve (iii) and (vi), put $f = 2c - 8$ from (iii) in (vi), then

$-c + 2f = -7 \Rightarrow -c + 2(2c - 8) = -7$

$\Rightarrow -c + 4c - 16 = -7 \Rightarrow 3c = 9 \Rightarrow \boxed{c = 3}$

Put $c = 3$ in (vi), then $f = 2c - 8 = 2(3) - 8 = 6 - 8 = -2 \Rightarrow \boxed{f = -2}$

Hence, $A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & -2 \end{bmatrix}$

14 Show that $\begin{bmatrix} r\cos\phi & 0 & -\sin\phi \\ 0 & r & 0 \\ r\sin\phi & 0 & \cos\phi \end{bmatrix} \cdot \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -r\sin\phi & 0 & r\cos\phi \end{bmatrix} = r\,I_3.$

Solution. $$\begin{bmatrix} r\cos^2\phi+0+r\sin^2\phi & 0+0+0 & r\cos\phi\sin\phi+0-r\cos\phi\sin\phi \\ 0+0+0 & 0+r+0 & 0+0+0 \\ r\cos\phi\sin\phi+0-r\cos\phi\sin\phi & 0+0+0 & r\sin^2\phi+0+r\cos^2\phi \end{bmatrix}$$

$$= \begin{bmatrix} r & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r \end{bmatrix} = r\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = r\,I_3$$

because $r\sin^2\phi + r\cos^2\phi = r(\sin^2\phi + \cos^2\phi) = r(1) = r.$