Exercise 5.2

Q No. 1 $\quad \dfrac{2x^2-3x+4}{(x-1)^3}$

Resolving it into partial fraction.

$$\frac{2x^2-3x+4}{(x-1)^3} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3}$$

xing both sides by $(x-1)^3$

$$2x^2-3x+4 = A(x-1)^2 + B(x-1) + C \quad \text{——— (i)}$$

put $x-1=0 \Rightarrow x=1$ in eq (i)

$$2(1)^2-3(1)+4 = A(0)^2 + B(0) + C$$

$$\Rightarrow \quad 2-3+4 = 0+0+C \quad \Rightarrow \quad \boxed{C=3}$$

Now eq (i) implises

$$2x^2-3x+4 = A(x^2-2x+1) + B(x-1) + C$$

Comparing the coeffients of x^2, x and x^0

$$2 = A \quad \text{——— (ii)}$$

$$-3 = -2A+B \quad \text{——— (iii)}$$

$$4 = A-B+C \quad \text{——— (iv)}$$

from eq (i) $\boxed{A=2}$

putting value of A in eq (iii)

$$-3 = -2(2)+B \Rightarrow -3 = -4+B$$

$$\Rightarrow -3+4 = B \quad \Rightarrow \quad \boxed{B=1}$$

So

$$\frac{2x^2-3x+4}{(x-1)^3} = \frac{2}{x-1} + \frac{1}{(x-1)^2} + \frac{3}{(x-1)^3}$$

Answer

Q No. 2 $\quad \dfrac{5x^2-2x+3}{(x+2)^3}$ $\quad$ * Correction

Resolving it into partial fraction.

$$\frac{5x^2-2x+3}{(x+2)^3} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{(x+2)^3}$$

[Do yourself as above, You will get $A=5$, $B=-22$, $C=27$]

Q No. 3 $\dfrac{4x}{(x+1)^2(x-1)}$

Resolving it into partial fraction.

$$\frac{4x}{(x+1)^2(x-1)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-1}$$

xing both sides by $(x+1)^2(x-1)$

$$4x = A(x+1)(x-1) + B(x-1) + C(x+1)^2 \quad \text{——— (i)}$$

put $x+1=0 \Rightarrow x=-1$ in eq (i)

$$4(-1) = A(0) + B(-1-1) + C(0)^2$$

$$\Rightarrow -4 = 0 - 2B + 0 \Rightarrow -4 = -2B \Rightarrow \boxed{B=2}$$

put $x-1=0 \Rightarrow x=1$ in eq (i)

$$4(1) = A(0) + B(0) + C(1+1)^2$$

$$\Rightarrow 4 = 0 + 0 + C(2)^2 \Rightarrow 4 = 4C \Rightarrow \boxed{C=1}$$

Now from eq (i)

$$4x = A(x^2-1) + B(x-1) + C(x^2+2x+1)$$

Comparing co-effients of x^2 only

$$0 = A + C \quad \text{——— (ii)}$$

put $C=1$ in eq (ii)

$$0 = A + 1 \Rightarrow \boxed{A=-1}$$

So $\dfrac{4x}{(x+1)^2(x-1)} = \dfrac{-1}{x+1} + \dfrac{2}{(x+1)^2} + \dfrac{1}{x-1}$ Answer

———— x ———— x ————

Q No. 4 Do yourself. $A=-1$, $B=-3$, $C=1$

———— x ———— x ————

Q No. 5 Do yourself, $A=-\frac{1}{16}$, $B=\frac{1}{4}$, $C=\frac{1}{16}$

———— x ———— x ————

Q No. 6 Do yourself, $A=4$, $B=-3$, $C=1$

———— x ———— x ————

Q No. 7 Do yourself, $A=-\frac{1}{4}$, $B=\frac{1}{2}$, $C=\frac{1}{4}$

———— x ———— x ————

Q No 8 $\dfrac{x^2}{(x-1)^3(x+1)}$

Resolving it into partial fraction.

$$\frac{x^2}{(x-1)^3(x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} + \frac{D}{x+1}$$

xing both sides by $(x-1)^3(x+1)$

$$x^2 = A(x-1)^2(x+1) + B(x-1)(x+1) + C(x+1) + D(x-1)^3 \quad \text{——(i)}$$

put $x-1=0 \Rightarrow x=1$ in eq (i)

$$(1)^2 = A(0) + B(0) + C(1+1) + D(0)^3$$

$$\Rightarrow 1 = 0+0+C(2)+0 \Rightarrow 1=2C \Rightarrow \boxed{C=\frac{1}{2}}$$

put $x+1=0 \Rightarrow x=-1$ in eq (i).

$$(-1)^2 = A(0) + B(0) + C(0) + D(-1-1)^3$$

$$\Rightarrow 1 = 0+0+0+D(-2)^3 \Rightarrow 1=-8D \Rightarrow \boxed{D=-\frac{1}{8}}$$

From eq (i)

$$x^2 = A(x^2-2x+1)(x+1) + B(x^2-1) + C(x+1) + D(x^3-3x^2+3x-1)$$

$$\Rightarrow x^2 = A(x^3-2x^2+x+x^2-2x+1) + B(x^2-1) + C(x+1) + D(x^3-3x^2+3x-1)$$

$$\Rightarrow x^2 = A(x^3-x^2-x+1) + B(x^2-1) + C(x+1) + D(x^3-3x^2+3x-1)$$

Comparing coefficients of x^3, x^2, x and x^0.

$$0 = A + D \quad \text{——(ii)}$$

$$1 = -A + B - 3D \quad \text{——(iii)}$$

$$0 = -A + C + 3D \quad \text{——(iv)}$$

$$0 = A - B + C - D \quad \text{——(v)}$$

put $D=-\frac{1}{8}$ in eq (ii)

$$0 = A - \frac{1}{8} \Rightarrow \boxed{A=\frac{1}{8}}$$

put $A=\frac{1}{8}$ and $D=-\frac{1}{8}$ in eq (iii)

$$1 = -\frac{1}{8} + B - 3\left(-\frac{1}{8}\right) \Rightarrow 1 = -\frac{1}{8} + B + \frac{3}{8}$$

$$\Rightarrow 1 + \frac{1}{8} - \frac{3}{8} = B \Rightarrow \boxed{B=\frac{3}{4}}$$

So

$$\frac{x^2}{(x-1)^3(x+1)} = \frac{1/8}{x-1} + \frac{3/4}{(x-1)^2} + \frac{1/2}{(x-1)^3} + \frac{-1/8}{x+1}$$

$$= \frac{1}{8(x-1)} + \frac{3}{4(x-1)^2} + \frac{1}{2(x-1)^3} - \frac{1}{8(x+1)}$$ Answer

Q No. 9 Do yourself $A = \frac{1}{27}$, $B = -\frac{1}{27}$, $C = -\frac{1}{9}$, $D = \frac{2}{3}$

Q No. 10 $$\frac{4x^3}{(x^2-1)(x+1)^2} = \frac{4x^3}{(x-1)(x+1)(x+1)^2}$$

$$= \frac{4x^3}{(x-1)(x+1)^3}$$

Now Resolving it into partial fraction.

$$\frac{4x^3}{(x-1)(x+1)^3} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2} + \frac{D}{(x+1)^3}$$

[Now Do yourself you will get $A = \frac{1}{2}$, $B = \frac{7}{2}$, $C = -5$, $D = 2$]

Q No. 11 $$\frac{2x+1}{(x+3)(x-1)(x+2)^2}$$

Resolving it into partial fraction.

$$\frac{2x+1}{(x+3)(x-1)(x+2)^2} = \frac{A}{x+3} + \frac{B}{x-1} + \frac{C}{x+2} + \frac{D}{(x+2)^2}$$

Multiplying both sides by $(x+3)(x-1)(x+2)^2$

$$2x+1 = A(x-1)(x+2)^2 + B(x+3)(x+2)^2 + C(x+3)(x-1)(x+2) + D(x+3)(x-1) \quad \text{---(i)}$$

put $x+3=0 \Rightarrow x=-3$ in eq (i)

$$2(-3)+1 = A(-3-1)(-3+2)^2 + B(0) + C(0) + D(0)$$

$$\Rightarrow -6+1 = A(-4)(-1)^2 + 0+0+0 \Rightarrow -5 = -4A \Rightarrow \boxed{A = \frac{5}{4}}$$

Put $x-1=0 \Rightarrow x=1$ in eq (i)

$$2(1)+1 = A(0) + B(1+3)(1+2)^2 + C(0) + D(0)$$

$$\Rightarrow 3 = 0 + B(4)(3)^2 + 0+0 \Rightarrow 3 = 36B$$

$$\Rightarrow B = \frac{3}{36} \Rightarrow \boxed{B = \frac{1}{12}}$$

Put $x+2=0 \Rightarrow x=-2$ in eq (i)

$$2(-2)+1 = A(0) + B(0) + C(0) + D(-2+3)(-2-1)$$

$$\Rightarrow -4+1 = 0+0+0+D(1)(-3) \Rightarrow -3 = -3D$$

$$\Rightarrow \boxed{D = 1}$$

Now from eq (i)

$$2x+1 = A(x-1)(x^2+4x+4) + B(x+3)(x^2+4x+4) + C(x+3)(x^2-x+2x-2) + D(x^2+3x-x-3)$$

$$\Rightarrow 2x+1 = A(x-1)(x^2+4x+4) + B(x+3)(x^2+4x+4) + C(x+3)(x^2+x-2) + D(x^2+2x-3)$$

$$\Rightarrow 2x+1 = A(x^3+4x^2+4x-x^2-4x-4) + B(x^3+4x^2+4x+3x^2+12x+12) + C(x^3+x^2-2x+3x^2+3x-6) + D(x^2+2x-3)$$

$$\Rightarrow 2x+1 = A(x^3+3x^2-4) + B(x^3+7x^2+16x+12) + C(x^3+4x^2+x-6) + D(x^2+2x-3)$$

Comparing the coeffients of x^3 only.

$$0 = A+B+C \quad \text{——— (ii)}$$

put $A = \frac{5}{4}$ and $B = \frac{1}{12}$ in above

$$0 = \frac{5}{4} + \frac{1}{12} + C \Rightarrow 0 = \frac{4}{3} + C \Rightarrow \boxed{C = -\frac{4}{3}}$$

So

$$\frac{2x+1}{(x+3)(x-1)(x+2)^2} = \frac{5/4}{x+3} + \frac{1/12}{x-1} + \frac{-4/3}{x+2} + \frac{1}{(x+2)^2}$$

$$= \frac{5}{4(x+3)} + \frac{1}{2(x-1)} - \frac{4}{3(x+2)} + \frac{1}{(x+2)^2}$$

Answer

Q No 12

$$\frac{2x^4}{(x-3)(x+2)^2} = \frac{2x^4}{(x-3)(x^2+4x+4)}$$

$$= \frac{2x^4}{x^3+4x^2+4x-3x^2-12x-12}$$

$$= \frac{2x^4}{x^3+x^2-8x-12}$$

$$= 2x-2+\frac{18x^2+8x-24}{x^3+x^2-8x-12}$$

$$= 2x-2+\frac{18x^2+8x-24}{(x-3)(x+2)^2}$$

$$x^3+x^2-8x-12 \;\overline{)\; 2x^4 \;\; 2x-2}$$

$$2x^4 \mp 2x^3 \mp 16x^2 \mp 24x$$

$$-2x^3+16x^2+24x$$

$$\mp 2x^3 \mp 2x^2 \mp 16x \mp 2$$

$$18x^2+8x-24$$

Now consider

$$\frac{18x^2+8x-24}{(x-3)(x+2)^2} = \frac{A}{x-3} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

xing both sides by $(x-3)(x+2)^2$

$18x^2+8x-24 = A(x+2)^2 + B(x-3)(x+2) + C(x-3)$ ——— (i)

put $x-3=0 \Rightarrow x=3$ in eq (i)

$18(3)^2+8(3)-24 = A(3+2)^2 + B(0) + C(0)$

$\Rightarrow 162+24-24 = A(5)^2 + 0 + 0 \Rightarrow 162 = 25A \Rightarrow \boxed{A = \frac{162}{25}}$

put $x+2=0 \Rightarrow x=-2$ in eq (i)

$18(-2)^2+8(-2)-24 = A(0)^2 + B(0) + C(-2-3)$

$\Rightarrow 72-16-24 = 0+0+C(-5) \Rightarrow 32 = -5C \Rightarrow \boxed{C = -\frac{32}{5}}$

Now eq (i) can be written as

$18x^2+8x-24 = A(x^2+4x+4) + B(x^2-3x+2x-6) + C(x-3)$

$\Rightarrow 18x^2+8x-24 = A(x^2+4x+4) + B(x^2-x-6) + C(x-3)$

Comparing coeffients of x^2 only.

$18 = A + B$ ——— (ii)

put $A = \frac{162}{25}$ in above

$18 = \frac{162}{25} + B \Rightarrow 18 - \frac{162}{25} = B \Rightarrow \boxed{B = \frac{288}{25}}$

So $\frac{18x^2+8x-24}{(x-3)(x+2)^2} = \frac{162/25}{x-3} + \frac{288/25}{x+2} + \frac{-32/5}{(x+2)^2}$

$= \frac{162}{25(x-3)} + \frac{288}{25(x+2)} - \frac{32}{5(x+2)^2}$

Hence

$\frac{2x^4}{(x-3)(x+2)^2} = 2x-2 + \frac{162}{25(x-3)} + \frac{288}{25(x+2)} - \frac{32}{5(x+2)^2}$

Answer

——— x ——— x ———