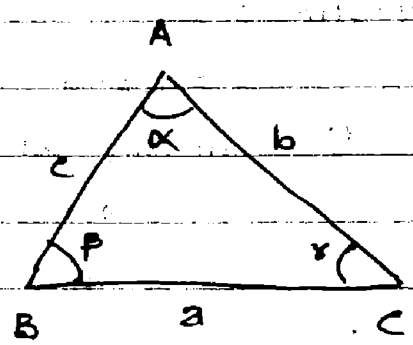
Exercise 12.7

Area of Triangle :- (only Formulas).

i) Area of triangle ABC $= \frac{1}{2} bc \sin\alpha$

$= \frac{1}{2} ca \sin\beta$

$= \frac{1}{2} ab \sin\gamma$



See proof on page 373

ii) Area of triangle denoted by Δ.

$$\Delta = \frac{c^2 \sin\alpha \sin\beta}{2\sin\gamma}$$

$$= \frac{a^2 \sin\beta \sin\gamma}{2\sin\alpha} \quad \text{OR} \quad = \frac{b^2 \sin\gamma \sin\alpha}{2\sin\beta}$$

See proof at page 374.

iii) If Δ denotes area of triangle then

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)} \qquad \because s = \frac{a+b+c}{2}$$

This formula is known as "Hero's Formula"

EXERCISE : 12.7

Question # 1

i) $a = 200$, $b = 120$, $\gamma = 150^\circ$

Since

$$\Delta = \frac{1}{2} ab \sin\gamma$$

$$= \frac{1}{2}(200)(120) \sin 150^\circ$$

$$= \frac{1}{2}(200)(120)(0.5)$$

$= 6000$ sq. unit Answer

ii) & iii) Do yourself.

Question # 2

i) $b = 25.4$, $\gamma = 36°41'$, $\alpha = 45°17'$

Since in any triangle

$$\alpha + \beta + \gamma = 180$$

$$\Rightarrow \beta = 180 - \alpha - \gamma$$

$$= 180 - 45°17' - 36°41' = 98°2'$$

Now

$$\text{Area of triangle} = \frac{b^2 \sin\gamma \sin\alpha}{2\sin\beta}$$

$$= \frac{(25.4)^2 \sin 36°41' . \sin 45°17'}{2 \sin 98°2'}$$

$$= \frac{(645.16)(0.5974)(0.2924)}{2(0.9902)}$$

$$= 138.293 \text{ sq. unit}$$ Answer

Question # 3

i) $a = 18$, $b = 24$, $c = 30$

$$s = \frac{a+b+c}{2} = \frac{18+24+30}{2}$$

$$= \frac{72}{2} = 36$$

Now

$$s - a = 36 - 18 = 18$$

$$s - b = 36 - 24 = 12$$

$$s - c = 36 - 30 = 6$$

Now

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{36(18)(12)(6)}$$

$$= \sqrt{46656}$$

$$= 216 \text{ sq. unit}$$ Answer

ii) & iii) Do yourself

Question # 4

Area of triangle = 2437 sq. unit

$a = 79$

$c = 97$, $\beta = ?$

Since

Area of triangle $= \frac{1}{2} ac \sin\beta$

$\Rightarrow \quad 2437 = \frac{1}{2}(79)(97)\sin\beta$

$\Rightarrow \quad 2437 = 3831.5 \sin\beta$

$\Rightarrow \quad \frac{2437}{3831} = \sin\beta \quad \Rightarrow \quad 0.636 = \sin\beta$

$\Rightarrow \quad \beta = \sin^{-1}(0.636)$

$= 39^\circ 30'$ Answer

Question # 5

Area of triangle $= \Delta = 121.34$

$\alpha = 32^\circ 15'$, $\beta = 65^\circ 37'$, $c = ?$, $\gamma = ?$

Since in any triangle

$\alpha + \beta + \gamma = 180$

$\Rightarrow \quad \gamma = 180 - \alpha - \beta$

$\cancel{180 - 32 - 65} \quad \cancel{\Rightarrow} \quad \boxed{\cancel{\gamma = 83^\circ}}$

$= 180 - 32^\circ 15' - 65^\circ 37'$

$\Rightarrow \quad \boxed{\gamma = 82^\circ 8'}$

Now

Area of triangle $= \dfrac{c^2 \sin\alpha \sin\beta}{2\sin\gamma}$

$\Rightarrow \quad 121.34 = \dfrac{c^2 \sin 32^\circ 15' \cdot \sin 65^\circ 37'}{2 \sin 82^\circ 8'}$

$= \dfrac{c^2 (0.5336)(0.9108)}{2(0.9906)}$

$= c^2 (0.2453)$

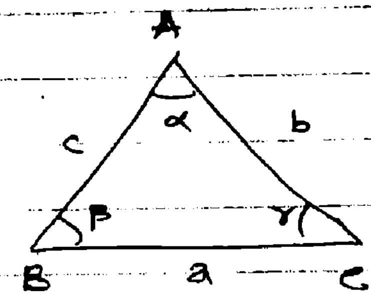
$\Rightarrow \quad \dfrac{121.34}{0.2453} = c^2$

$\Rightarrow c^2 = 494.66$

$\Rightarrow c = \sqrt{494.66}$

$\Rightarrow \boxed{c = 22.24}$

Question # 6



Suppose ABC be a triangular garden such that

$a = 30m \quad , \beta = 22^\circ\frac{1}{2} \quad , \gamma = 112^\circ\frac{1}{2}$

Since in any triangle

$$\alpha + \beta + \gamma = 180$$

$$\alpha = 180 - \beta - \gamma = 180 - 22^\circ\tfrac{1}{2} - 112^\circ\tfrac{1}{2}$$

$$= 45^\circ$$

Now Area of triangular garden $= \dfrac{a^2 \sin\beta \sin\gamma}{2\sin\alpha}$

$$= \frac{(30)^2 \sin 22^\circ\frac{1}{2} \cdot \sin 112^\circ\frac{1}{2}}{2 \sin 45}$$

$$= \frac{(900)(0.3827)(0.9239)}{2(0.7072)} = 224.99\ m^2$$

$$\approx 225\ m^2$$

Since

~~Plant~~ Cost of planting per square meter $= 5$ Rs

therefore

Cost of planting $225\ m^2 = 5 \times 225$

$= 1125$ Rs

Answer