Exercise 13.1

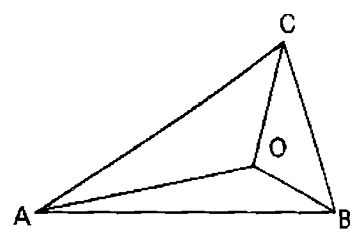
1. **Two sides of a triangle measure 10 cm and 15 cm. Which of the following measure is possible for the third side?**

(a)	5 cm	(b)	20 cm
(c)	25 cm	(d)	30 cm

Ans. 20cm.

2. **O is an interior point of the ABC. Show that**

$$m\overline{OA}+m\overline{OB}+m\overline{OC}>\frac{1}{2}(m\overline{AB}+m\overline{BC}+m\overline{CA})$$



Given: O is the interior point of ΔABC

To Prove:

$$m\overline{OA}+m\overline{OB}+m\overline{OC}>\frac{1}{2}\left(m\overline{AB}+m\overline{BC}+m\overline{CA}\right)$$

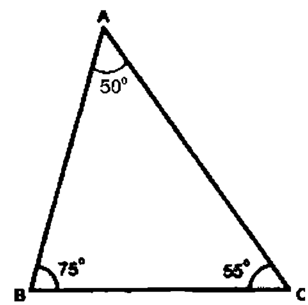
Construction:

Join O with A, B and C.

Proof:

Statements	Reasons
ΔOAB	
$m\overline{OA}+m\overline{OB}>m\overline{AB}$ ....(i)	Sum of two sides > third side
Similarly	
$m\overline{OB}+m\overline{OC}>m\overline{BC}$ ....(ii)	Sum of two sides > third side
and	
$m\overline{OC}+m\overline{OA}>m\overline{CA}$ ....(iii)	
$2m\overline{OA}+2m\overline{OB}+2m\overline{OC}>m\overline{AB}+m\overline{BC}+m\overline{CA}$	Adding (i), (ii) and (iii)
$2\left(m\overline{OA}+m\overline{OB}+m\overline{OC}\right)>m\overline{AB}+m\overline{BC}+m\overline{CA}$	
$m\overline{OA}+m\overline{OB}+m\overline{OC}>\frac{1}{2}\left(m\overline{AB}+m\overline{BC}+m\overline{CA}\right)$	

3. In the ΔABC, $m\angle B = 75^\circ$ and $m\angle C = 55^\circ$. Which of the sides of the triangle is longest and which is the shortest?



Ans: Given a ΔABC in which

$m\angle B = 75^0$

$m\angle C = 55^0$

As $m\angle A + m\angle B + m\angle C = 180^0$

$m\angle A + 75^0 + 55^0 = 180^0$

$m\angle A + 130^0 = 180^0$

$m\angle A = 180^0 - 130^0$

$m\angle A = 50^0$

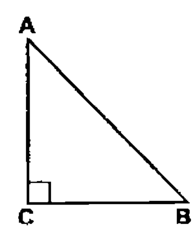
As we know in a triangle, the side opposite to greater angle is longer than the side opposite to smaller angle

So $m\overline{AC} > m\overline{BC}$

Hence longest side is $\overline{AC}$

and shortest side is $\overline{BC}$

4. Prove that in a right-angled triangle, the hypotenuse is longer than each of the other two sides.



Ans.

Given: ΔABC is a right angle triangle.

Hence AB is hypotenuse of ΔABC.

To prove:

$mAB > mAC$ and $mAB > mBC$

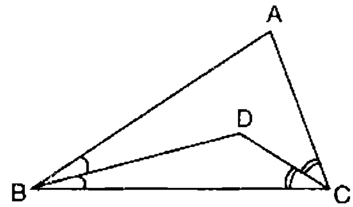
Proof:

As ΔABC is a right angle triangle. So $m\angle C = 90^0$ is the largest angle and the remaining angles $\angle A$ and $\angle B$ are acute. So $m\angle C > m\angle A$ and $m\angle C > m\angle B$

As the side opposite to the greater angle is longer than the side opposite to the smaller angle.

Hence $mAB > mAC$ and $mAB > mBC$

5. In the triangular figure, $\overline{AB} > \overline{AC}$. $\overline{BD}$ and $\overline{CD}$ are the bisectors of $\angle B$ and $\angle C$ respectively. Prove that $\overline{BC} > \overline{DC}$.



Given: $\overline{AB} > \overline{BC}$, $\overline{BD}$ and $\overline{CD}$ are the bisectors of the angles B and C

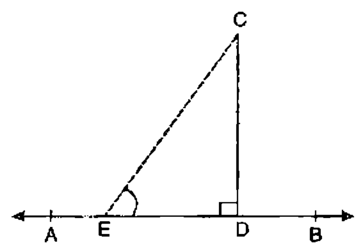
To Prove:

To prove = $\overline{BD} > \overline{CD}$

Proof

Statements	Reasons
∴ in ΔABC $\angle ACB > \angle ABC$ $\frac{1}{2}\angle ACB > \frac{1}{2}\angle ABC$	∴ $\overline{AB} > \overline{AC}$
∴ $\angle BCD > \angle DBC$ $\overline{BD} > \overline{CD}$	$\overline{CD}, \overline{BD}$ are bisectors of $\angle C$, $\angle B$. The bigger sides is opposite the bigger angle

Theorem From a point, outside a line, perpendicular is the shortest distance from the point to the line.



Given A line AB and a point C (not lying on $\overleftrightarrow{AB}$) and a point D on $\overleftrightarrow{AB}$ such that $\overline{CD} \perp \overleftrightarrow{AB}$.

To Prove

$m\overline{CD}$ is the shortest distance from the point C to $\overleftrightarrow{AB}$.

Construction

Take a point E on $\overleftrightarrow{AB}$. Join C and E to form a ΔCDE

Proof:

Statements	Reasons
In ΔCDE $m\angle CDB > m\angle CED$	(An exterior angle of a triangle is greater

	than non adjacent interior angle).
But $m\angle CDB = m\angle CDE$	Supplement of right angle.
$\therefore$ $m\angle CDE > m\angle CED$	
or $m\angle CED < m\angle CDE$	$a > b \Rightarrow b < a$
or $m\overline{CD} < m\overline{CE}$	Side opposite to greater angle is greater.
But E is any point on AB	
Hence $m\overline{CD}$ is the shortest distance from C to $\overline{AB}$.	

Note:

(i) The distance between a line and a point not on it, is the length of the perpendicular line segment from the point to the line.

(ii) The distance between a line and a point lying on it is zero