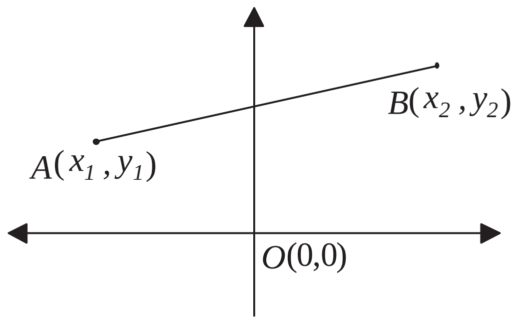
EXERCISE 4.1

◈ Distance Formula

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two points in a plane and d be a distance between A and B then



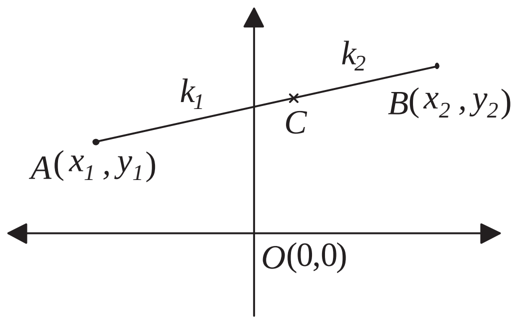
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

or $$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

See proof on book at page181

◈ Ratio Formula

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two points in a plane. The coordinates of the point C dividing the line segment AB in the ratio $k_1 : k_2$ are



$$\left(\frac{k_1x_2 + k_2x_1}{k_1 + k_2}, \frac{k_1y_2 + k_2y_1}{k_1 + k_2}\right)$$

See proof on book at page 182

If C be the midpoint of AB i.e. $k_1 : k_2 = 1:1$ then coordinate of C becomes

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

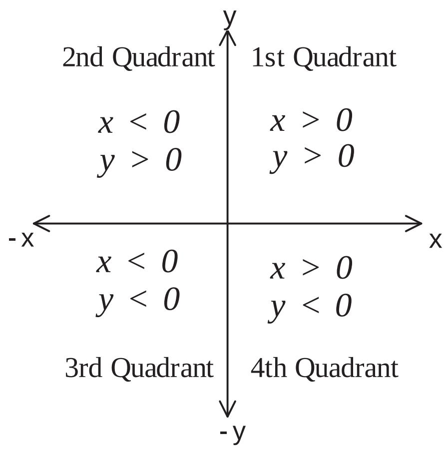
◈ Question # 1

(*i*) $x > 0$

Right half plane

(*ii*) $x > 0$ and $y > 0$

The 1st quadrant.



(*iii*) $x = 0$

y-axis

(*iv*) $y = 0$

x-axis

(*v*) $x < 0$ and $y \geq 0$

2nd quadrant & negative x-axis

(*vi*) $x = y$

It is a line bisecting 1st and 3rd quadrant.

(*vii*) $|x| = -|y|$

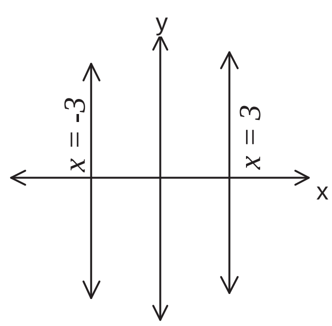
A positive value can't equal to a negative value, except number zero, so origin, $(0,0)$, is the only point which satisfies $|x| = -|y|$

(*viii*) $|x| \geq 3$

$\Rightarrow \pm x \geq 3 \quad \Rightarrow x \geq 3$ or $-x \geq 3$

$\Rightarrow x \geq 3$ or $x \leq -3$

which is the set of points lying on right side of the line $x = 3$ and the points lying on left side of the line $x = -3$.



(*ix*) $x > 2$ and $y = 2$

The set of all points on the line $y = 2$ for which $x > 2$.

(*x*) x and y have opposite signs.

It is the set of points lying in 2nd and 4th quadrant.

◈ Question # 2

(a) $A(3,1)$; $B(-2,-4)$

(i) $|AB|=\sqrt{(-2-3)^2+(-4-1)^2}=\sqrt{(-5)^2+(-5)^2}$
$=\sqrt{25+25}=\sqrt{50}=\sqrt{25\times 2}=5\sqrt{2}$

(ii) Midpoint of $AB=\left(\frac{3-2}{2},\frac{1-4}{2}\right)=\left(\frac{1}{2},\frac{-3}{2}\right)$

> **Review:**
> The midpoint of $A(x_1,y_1)$ and $B(x_2,y_2)$ is $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$.

(b) $A(-8,3)$; $B(2,-1)$

Do yourself as above.

(c) $A\left(-\sqrt{5},-\frac{1}{3}\right)$; $B\left(-3\sqrt{5},5\right)$

(i) $|AB|=\sqrt{\left(-3\sqrt{5}+\sqrt{5}\right)^2+\left(5+\frac{1}{3}\right)^2}=\sqrt{\left(2\sqrt{5}\right)^2+\left(\frac{16}{3}\right)^2}$
$=\sqrt{20+\frac{256}{9}}=\sqrt{\frac{436}{9}}=\sqrt{\frac{4\times 109}{9}}=\frac{4\sqrt{109}}{3}$

(ii) Midpoint of $AB=\left(\frac{-\sqrt{5}-3\sqrt{5}}{2},\frac{-1/3+5}{2}\right)=\left(\frac{-4\sqrt{5}}{2},\frac{14/3}{2}\right)=\left(-2\sqrt{5},\frac{7}{3}\right)$

◈ Question # 3

(a) Distance of $\left(\sqrt{176},7\right)$ from origin $=\sqrt{\left(\sqrt{176}-0\right)^2+(7-0)^2}$
$=\sqrt{(176)+(49)}$
$=\sqrt{(176)+(49)}=\sqrt{225}=15$

$\Rightarrow$ the point $\left(\sqrt{176},7\right)$ is at 15 unit away from origin.

(b) Distance of $(10,-10)$ from origin $=\sqrt{(10-0)^2+(-10-0)^2}$
$=\sqrt{100+100}=\sqrt{200}$
$=\sqrt{100\times 2}=10\sqrt{2}\neq 15$

$\Rightarrow$ the point $(10,-10)$ is not at distance of 15 unit from origin.

(c) *Do yourself as above*

(d) Distance of $\left(\frac{15}{2},\frac{15}{2}\right)$ from origin $=\sqrt{\left(\frac{15}{2}-0\right)^2+\left(\frac{15}{2}-0\right)^2}$
$=\sqrt{\frac{225}{4}+\frac{225}{4}}=\sqrt{\frac{225}{2}}=\frac{15}{\sqrt{2}}\neq 15$

Hence the point $\left(\frac{15}{2},\frac{15}{2}\right)$ is not at distance of 15 unit from origin.

◈ Question # 4

(i) Given: $A(0,2)$, $B(\sqrt{3},-1)$ and $C(0,-2)$

$|AB|=\sqrt{\left(\sqrt{3}-0\right)^2+(-1-2)^2}=\sqrt{\left(\sqrt{3}\right)^2+(-3)^2}$
$=\sqrt{3+9}=\sqrt{12}$ $\Rightarrow$ $|AB|^2=12$

$|BC|=\sqrt{\left(0-\sqrt{3}\right)^2+(-2+1)^2}=\sqrt{\left(-\sqrt{3}\right)^2+(-1)^2}$
$=\sqrt{3+1}=\sqrt{4}=2$ $\Rightarrow$ $|BC|^2=4$

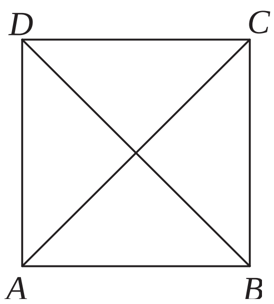
$$|CA| = \sqrt{(0-0)^2 + (2+2)^2} = \sqrt{0 + (4)^2}$$
$$= \sqrt{16} = 4 \qquad \Rightarrow \ |CA|^2 = 16$$
$$\because \ |AB|^2 + |BC|^2 = 12 + 4 = 16 = |CA|^2$$
$\therefore$ by Pythagoras theorem A, B & C are vertices of a right triangle.

(ii) Given: $A(3,1)$, $B(-2,-3)$ and $C(2,2)$

$$|AB| = \sqrt{(-2-3))^2 + (-3-1)^2} = \sqrt{(-5)^2 + (-4)^2} = \sqrt{25+16} = \sqrt{41}$$
$$|BC| = \sqrt{(2-(-2))^2 + (2-(-3))^2} = \sqrt{(4)^2 + (5)^2} = \sqrt{16+25} = \sqrt{41}$$
$$|CA| = \sqrt{(3-2)^2 + (1-2)^2} = \sqrt{(1)^2 + (-1)^2}$$
$$= \sqrt{1+1} = \sqrt{2}$$
$\because \ |AB| = |BC| \quad \Rightarrow \ A, B$ & C are vertices of an isosceles triangle.

(iii) Given: $A(5,2)$, $B(-2,3)$ & $C(2,2)$

$$|AB| = \sqrt{(-2-5)^2 + (3-2)^2} = \sqrt{(-7)^2 + (1)^2}$$
$$= \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$$



$$|BC| = \sqrt{(-3+2)^2 + (-4-3)^2} = \sqrt{(-1)^2 + (-7)^2}$$
$$= \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$$
$$|CD| = \sqrt{(4+3)^2 + (-5+4)^2} = \sqrt{(7)^2 + (-1)^2}$$
$$= \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$$
$$|DA| = \sqrt{(5-4)^2 + (2+5)^2} = \sqrt{(1)^2 + (7)^2}$$
$$= \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$$
$\because \ |AB| = |CD|$ and $|BC| = |DA| \quad \Rightarrow \ A, B, C$ and D are vertices of parallelogram.

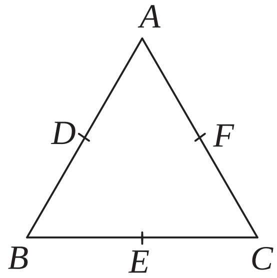
Now $$|AC| = \sqrt{(-3-5)^2 + (-4-2)^2} = \sqrt{(-8)^2 + (-6)^2}$$
$$= \sqrt{64+36} = \sqrt{100} = 10$$
$$|BD| = \sqrt{(4+2)^2 + (-5-3)^2} = \sqrt{(6)^2 + (-8)^2}$$
$$= \sqrt{36+64} = \sqrt{100} = 10$$

Since all sides are equals and also both diagonals are equal therefore A, B, C, D are vertices of a square.

◈ Question # 5

Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are vertices of triangle ABC, and let $D(1,-1)$, $E(-4,-3)$ and $F(-1,1)$ are midpoints of sides AB, BC and CA respectively.

Then



$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = (1,-1)$$
$\Rightarrow \ x_1 + x_2 = 2 \ ....(i)$ and $y_1 + y_2 = -2 \ ....(ii)$
$$\left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right) = (-4,-3)$$
$\Rightarrow \ x_2 + x_3 = -8 ....(iii)$ and $y_2 + y_3 = -6 \ ....(iv)$
$$\left(\frac{x_3 + x_1}{2}, \frac{y_3 + y_1}{2}\right) = (-1,1)$$
$\Rightarrow \ x_1 + x_3 = -2 \ ....(v)$, and $y_1 + y_3 = 2 \ ....(vi)$

Subtracting (i) and (iii)

$$\begin{array}{rl} x_1 + x_2 &= 2 \\ \underline{-\; x_2 \pm x_3} &= \underline{-8} \\ x_1 \quad - x_3 &= 10 \text{ ....}(vii) \end{array}$$

Adding (v) and (vii)

$$\begin{array}{rl} x_1 + x_3 &= -2 \\ \underline{x_1 - x_3} &= \underline{10} \\ 2x_1 &= 8 \quad \Rightarrow \boxed{x_1 = 4} \end{array}$$

Putting value of x_1 in (i)

$4 + x_2 = 2$

$\Rightarrow\; x_2 = 2 - 4 \;\Rightarrow\; \boxed{x_2 = -2}$

Putting value of x_1 in (v)

$4 + x_3 = -2$

$\Rightarrow\; x_3 = -2 - 4 \;\Rightarrow\; \boxed{x_3 = -6}$

Subtracting (ii) and (iv)

$$\begin{array}{rl} y_1 + y_2 &= -2 \\ \underline{-\; y_2 \pm y_3} &= \underline{\mp 6} \\ y_1 \quad - y_3 &= 4 \text{ ....}(viii) \end{array}$$

Adding (vi) and ($viii$)

$$\begin{array}{rl} y_1 + y_3 &= 2 \\ \underline{y_1 - y_3} &= \underline{4} \\ 2y_1 &= 6 \quad \Rightarrow \boxed{y_1 = 3} \end{array}$$

Putting value of y_1 in (ii)

$3 + y_2 = -2$

$\Rightarrow\; y_2 = -2 - 3 \;\Rightarrow\; \boxed{y_2 = -5}$

Putting value of y_1 in (v)

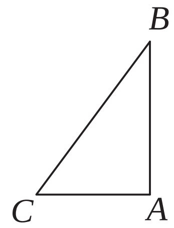
$3 + y_3 = 2$

$\Rightarrow\; y_3 = 2 - 3 \;\Rightarrow\; \boxed{y_3 = -1}$

Hence vertices of triangle are $(4,3), (-2,-5)$ & $(-6,-1)$.

◈ *Question # 6*

Since ABC is a right triangle therefore by Pythagoras theorem



$$|AB|^2 + |CA|^2 = |BC|^2$$

$$\Rightarrow \left[\left(0-\sqrt{3}\right)^2 + (2+1)^2\right] + \left[\left(\sqrt{3}-h\right)^2 + (-1+2)^2\right] = (h-0)^2 + (-2-2)^2$$

$$\Rightarrow [3+9] + \left[3 - 2\sqrt{3}h + h^2 + 1\right] = h^2 + 16$$

$$\Rightarrow 12 + 4 - 2\sqrt{3}\,h + h^2 = h^2 + 16$$

$$\Rightarrow -2\sqrt{3}\,h = h^2 + 16 - 12 - 4 - h^2 \quad \Rightarrow -2\sqrt{3}\,h = 0 \quad \Rightarrow \boxed{h = 0}.$$

◈ *Question # 7*

Points (x_1, y_1), (x_2, y_2) and (x_3, y_3) are collinear if

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Since given points are collinear therefore

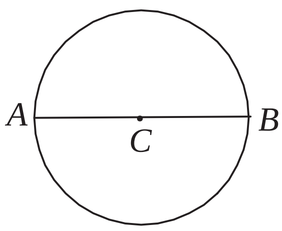
$$\begin{vmatrix} -1 & h & 1 \\ 3 & 2 & 1 \\ 7 & 3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow -1(2-3) - h(3-7) + 1(9-14) = 0 \quad \Rightarrow -1(-1) - h(-4) + 1(-5) = 0$$

$$\Rightarrow 1 + 4h - 5 = 0 \quad \Rightarrow 4h - 4 = 0 \quad \Rightarrow 4h = 4 \quad \Rightarrow \boxed{h = 1}$$

◈ *Question # 8*

The centre of the circle is mid point of AB



i.e. centre 'C' $= \left(\dfrac{-5+5}{2}, \dfrac{-2-4}{2}\right) = \left(\dfrac{0}{2}, \dfrac{-6}{2}\right) = (0,-3)$

Now radius $= |AC|$

$$= \sqrt{(0+5)^2 + (-3+2)^2}$$

$$= \sqrt{25+1} \quad = \sqrt{26}$$

◈ *Question # 9*

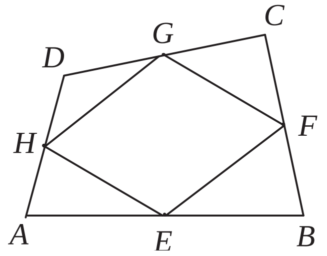
Do yourself as Question # 6

Hint: you will get a equation $h^2+4h-60=0$
Solve this quadratic equation to get two values of h.

◈ *Question # 10*

Given: $A(9,3)$, $B(-7,7)$, $C(-3,-7)$ and $D(5,-5)$

Let E, F, G and H be the mid-points of sides of quadrilateral



Coordinate of $E = \left(\frac{9-7}{2},\frac{3+7}{2}\right) = \left(\frac{2}{2},\frac{10}{2}\right) = (1,5)$

Coordinate of $F = \left(\frac{-7-3}{2},\frac{7-7}{2}\right) = \left(\frac{-10}{2},\frac{0}{2}\right) = (-5,0)$

Coordinate of $G = \left(\frac{-3+5}{2},\frac{-7-5}{2}\right) = \left(\frac{2}{2},\frac{-12}{2}\right) = (1,-6)$

Coordinate of $H = \left(\frac{9+5}{2},\frac{3-5}{2}\right) = \left(\frac{14}{2},\frac{-2}{2}\right) = (7,-1)$

Now $|EF| = \sqrt{(-5-1)^2+(0-5)^2} = \sqrt{36+25} = \sqrt{61}$

$|FG| = \sqrt{(1+5)^2+(-6-0)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$

$|GH| = \sqrt{(7-1)^2+(-1+6)^2} = \sqrt{36+25} = \sqrt{61}$

$|HE| = \sqrt{(1-7)^2+(5+1)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$

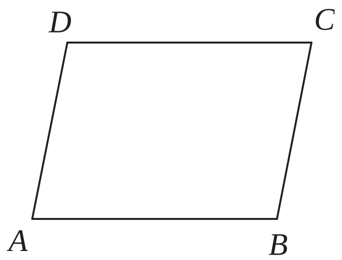
Since $|EF| = |GH|$ and $|FG| = |HE|$

Therefore $EFGH$ is a parallelogram.

◈ *Question # 11*

Given: $A(-3,0)$, $B(1,-2)$, $C(5,0)$, $D(1,h)$

Quadrilateral $ABCD$ is a parallelogram if



$|AB| = |CD|$ & $|BC| = |AD|$

when $|AB| = |CD|$

$\Rightarrow \sqrt{(1+3)^2+(-2-0)^2} = \sqrt{(1-5)^2+(h-0)^2}$

$\Rightarrow \sqrt{16+4} = \sqrt{16+h^2} \quad \Rightarrow \sqrt{20} = \sqrt{16+h^2}$

On squaring

$20 = 16+h^2 \quad \Rightarrow h^2 = 20-16 \quad \Rightarrow h^2 = 4 \quad \Rightarrow h = \pm 2$

When $h=2$, then $D(1,h) = D(1,2)$

Then $|AB| = \sqrt{(1+3)^2+(-2-0)^2} = \sqrt{16+4} = \sqrt{20}$

$|BC| = \sqrt{(5-1)^2+(0+2)^2} = \sqrt{16+4} = \sqrt{20}$

$|CA| = \sqrt{(1-5)^2+(2-0)^2} = \sqrt{16+4} = \sqrt{20}$

$|DA| = \sqrt{(-3-1)^2+(-0-2)^2} = \sqrt{16+4} = \sqrt{20}$

Now for diagonals

$|AC| = \sqrt{(5+3)^2+(0-0)^2} = \sqrt{64+0} = 8$

$|BD| = \sqrt{(1-1)^2+(2-0)^2} = \sqrt{0+4} = 2$

Since all sides are equal but diagonals $|AC| \neq |BD|$

Therefore $ABCD$ is not a square.

Now when $h=-2$, then $D(1,h) = D(1,-2)$ but we also have $B(1,-2)$

i.e. B and D represents the same point, which can not happened in quadrilateral so we can not take $h=-2$.

Question # 12

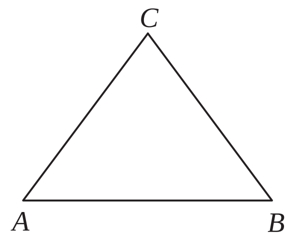
Given: $A(-3,0)$, $B(3,0)$

Let $C(x,y)$ be a third vertex of an equilateral triangle ABC.

Then $|AB| = |BC| = |CA|$

$$\Rightarrow \sqrt{(3+3)^2+(0-0)^2} = \sqrt{(x-3)^2+(y-0)^2} = \sqrt{(x+3)^2+(y-0)^2}$$

$$\Rightarrow \sqrt{36+0} = \sqrt{x^2-6x+9+y^2} = \sqrt{x^2+6x+9+y^2}$$



On squaring

$$36 = x^2+y^2-6x+9 = x^2+y^2+6x+9 \quad ....(i)$$

From equation (i)

$$x^2+y^2-6x+9 = x^2+y^2+6x+9$$

$$\Rightarrow x^2+y^2-6x+9-x^2-y^2-6x-9 = 0$$

$$\Rightarrow -12x = 0 \quad \Rightarrow x=0$$

Again from equation (i)

$$36 = x^2+y^2-6x+9$$

$$\Rightarrow 36 = (0)^2+y^2-6(0)+9 \qquad \because x=0$$

$$\Rightarrow 36 = y^2+9 \quad \Rightarrow y^2 = 36-9 = 27 \quad \Rightarrow y = \pm 3\sqrt{3}$$

so coordinate of C is $\left(0,3\sqrt{3}\right)$ or $\left(0,-3\sqrt{3}\right)$.

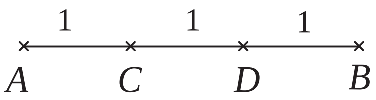
And hence two triangle can be formed with vertices $A(-3,0), B(3,0), C\left(0,3\sqrt{3}\right)$ and $A(-3,0), B(3,0), C\left(0,-3\sqrt{3}\right)$.

Question # 13

Given: $A(-1,4)$, $B(6,2)$

Let C and D be points trisecting A and B

Then $AC:CB = 1:2$



So coordinate of $C = \left(\dfrac{1(6)+2(-1)}{1+2}, \dfrac{1(2)+2(4)}{1+2}\right)$

$= \left(\dfrac{6-2}{3}, \dfrac{2+8}{3}\right) = \left(\dfrac{4}{3}, \dfrac{10}{3}\right)$

Also $AD:DB = 2:1$

So coordinate of $D = \left(\dfrac{2(6)+1(-1)}{2+1}, \dfrac{2(2)+1(4)}{2+1}\right)$

$= \left(\dfrac{12-1}{3}, \dfrac{4+4}{3}\right) = \left(\dfrac{11}{3}, \dfrac{8}{3}\right)$

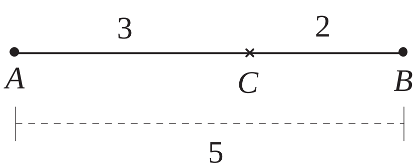
Hence $\left(\dfrac{4}{3}, \dfrac{10}{3}\right)$ and $\left(\dfrac{11}{3}, \dfrac{8}{3}\right)$ are points trisecting A and B.

Question # 14

Given: $A(-5,8)$, $B(5,3)$

Let $C(x,y)$ be a required point

$\because AC:CB = 3:2$



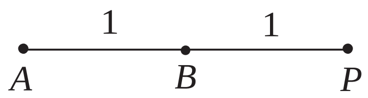
$\therefore$ Co-ordinate of $C = \left(\dfrac{3(5)+2(-5)}{3+2}, \dfrac{3(3)+2(8)}{3+2}\right)$

$$= \left(\frac{15-10}{5}, \frac{9+16}{5}\right) = \left(\frac{5}{5}, \frac{25}{5}\right) = (1, 5)$$

Question # 15

Given: $A(1,4)$, $B(5,6)$

(i) Let $P(x,y)$ be required point, then



$$AB:AP = 1:2$$

$$\Rightarrow AB:BP = 1:1 \quad \text{i.e. } B \text{ is midpoint of } AP$$

Then $B(5,6) = \left(\dfrac{1+x}{2},\dfrac{4+y}{2}\right)$

$$\Rightarrow 5 = \frac{1+x}{2} \quad \text{and} \quad 6 = \frac{4+y}{2}$$

$$\Rightarrow 10 = 1+x \quad \text{and} \quad 12 = 4+y$$

$$\Rightarrow x = 10-1 \quad , \quad y = 12-4$$

$$= 9 \quad , \quad = 8$$

Hence $P(9,8)$ is required point.

(ii) Since $PA:AB = 2:1$

$$\Rightarrow A(1,4) = \left(\frac{2(5)+1(x)}{2+1},\frac{2(6)+1(y)}{2+1}\right)$$

$$= \left(\frac{10+x}{3},\frac{12+y}{3}\right)$$

$$\Rightarrow 1 = \frac{10+x}{3} \quad \text{and} \quad 4 = \frac{12+y}{3}$$

$$\Rightarrow 3 = 10+x \quad \text{and} \quad 12 = 12+y$$

$$\Rightarrow x = 3-10 \quad \text{and} \quad y = 12-12$$

$$= -7 \quad , \quad = 0$$

Hence $P(-7,0)$ is required point.

Question # 16

Given: $A(5,3)$, $B(-2,2)$ and $C(4,2)$

Let $D(x,y)$ be a point equidistance from A , B and C then

$$\left|\overline{DA}\right| = \left|\overline{DB}\right| = \left|\overline{DC}\right|$$

$$\Rightarrow \left|\overline{DA}\right|^2 = \left|\overline{DB}\right|^2 = \left|\overline{DC}\right|^2$$

$$\Rightarrow (x-5)^2+(y-3)^2 = (x+2)^2+(y-2)^2 = (x-4)^2+(y-2)^2 \quad \text{........} (i)$$

From eq. (i)

$$(x-5)^2+(y-3)^2 = (x+2)^2+(y-2)^2$$

$$\Rightarrow x^2-10x+25+y^2-6y+9 = x^2+4x+4+y^2-4y+4$$

$$\Rightarrow x^2-10x+25+y^2-6y+9-x^2-4x-4-y^2+4y-4 = 0$$

$$\Rightarrow -14x-2y+26 = 0 \quad \Rightarrow 7x+y-13 = 0 \quad \text{.........} (ii)$$

Again from equation (i)

$$(x+2)^2+(y-2)^2 = (x-4)^2+(y-2)^2$$

$$\Rightarrow x^2+4x+4+y^2-4y+4 = x^2-8x+16+y^2-4y+4$$

$$\Rightarrow 12x-12 = 0 \quad \Rightarrow 12x = 12 \quad \Rightarrow x = 1$$

Put $x=1$ in eq. (ii)

$$7(1)+y-13 = 0 \quad \Rightarrow y-6 = 0 \quad \Rightarrow y = 6$$

Hence $(1,6)$ is required point.

Now radius of circumcircle $= \left|\overline{DA}\right|$

$$= \sqrt{(5-1)^2+(3-6)^2} = \sqrt{16+9} = \sqrt{25} = 5\,\text{units}$$

Intersection of Median

Let $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ are vertices of triangle.

Intersection of median is called centroid of triangle and can be determined as

$$\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$$

See proof at page 184

Centre of In-Circle (In-Centre)

Let $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ are vertices of triangle.

And $|AB| = c$, $|BC| = a$, $|CA| = b$

Then incentre of triangle $= \left(\dfrac{ax_1+bx_2+cx_3}{a+b+c},\dfrac{ay_1+by_2+cy_3}{a+b+c}\right)$

See proof at page 184

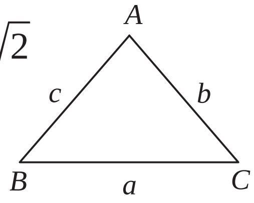
Question # 17

Let $A(4,-2)$, $B(-2,4)$, $C(5,5)$ are vertices of triangle then

$$a = |BC| = \sqrt{(5+2)^2+(5-4)^2} = \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$$

$$b = |CA| = \sqrt{(4-5)^2+(-2-5)^2} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$$

$$c = |AB| = \sqrt{(-2-4)^2+(4+2)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$$



Now

$$\text{In-centre} = \left(\frac{ax_1+bx_2+cx_3}{a+b+c},\frac{ay_1+by_2+cy_3}{a+b+c}\right)$$

$$= \left(\frac{5\sqrt{2}(4)+5\sqrt{2}(-2)+6\sqrt{2}(5)}{5\sqrt{2}+5\sqrt{2}+6\sqrt{2}},\frac{5\sqrt{2}(-2)+5\sqrt{2}(4)+6\sqrt{2}(5)}{5\sqrt{2}+5\sqrt{2}+6\sqrt{2}}\right)$$

$$= \left(\frac{20\sqrt{2}-10\sqrt{2}+30\sqrt{2}}{16\sqrt{2}},\frac{-10\sqrt{2}+20\sqrt{2}+30\sqrt{2}}{16\sqrt{2}}\right)$$

$$= \left(\frac{40\sqrt{2}}{16\sqrt{2}},\frac{40\sqrt{2}}{16\sqrt{2}}\right) = \left(\frac{5}{2},\frac{5}{2}\right)$$

Question # 18

Given: $A(x_1,y_1)$, $B(x_2,y_2)$

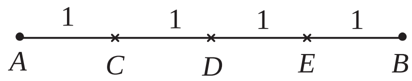
Let C, D and E are points dividing AB into four equal parts.

$\because\ AC:CB = 1:3$

$\Rightarrow$ Co-ordinates of $C = \left(\dfrac{1(x_2)+3(x_1)}{1+3},\dfrac{1(y_2)+3(y_1)}{1+3}\right) = \left(\dfrac{3x_1+x_2}{4},\dfrac{3y_1+y_2}{4}\right)$

Now $AD:DB = 2:2$

$= 1:1$ i.e. D is midpoint of AB.



$\Rightarrow$ Co-ordinates of $D = \left(\dfrac{x_1+x_2}{2},\dfrac{y_1+y_2}{2}\right)$

Now $AE:EB = 3:1$

$\Rightarrow$ Co-ordinates of $E = \left(\dfrac{3(x_2)+1(x_1)}{3+1},\dfrac{3(y_2)+1(y_1)}{3+1}\right) = \left(\dfrac{x_1+3x_2}{4},\dfrac{y_1+3y_2}{4}\right)$

Hence $\left(\dfrac{3x_1+x_2}{4},\dfrac{3y_1+y_2}{4}\right)$, $\left(\dfrac{x_1+x_2}{2},\dfrac{y_1+y_2}{2}\right)$ and $\left(\dfrac{x_1+3x_2}{4},\dfrac{y_1+3y_2}{4}\right)$ are the points dividing AB into four equal parts.
