

مشق 3.6

1- درج ذیل کا جذر المربع معلوم کیجیے۔

1: $16x^2 + 24xy + 9y^2$

حل:

تجزی کرنے سے

$$16x^2 + 24xy + 9y^2$$

$$16x^2 + 24xy + 9y^2 = (4x)^2 + 2(4x)(3y) + (3y)^2$$

$$= (4x + 3y)^2$$

$$\sqrt{16x^2 + 24xy + 9y^2} = \pm \sqrt{(4x + 3y)^2} \quad \text{جذر لینے سے}$$

$$= \pm (4x + 3y)$$

2- $(x^2 - 7x + 12)(x^2 - 9x + 20)(x^2 - 8x + 15)$

حل: چونکہ

$$x^2 - 7x + 12 = x^2 - 4x - 3x + 12$$

$$= x(x - 4) - 3(x - 4)$$

$$= (x - 4)(x - 3)$$

$$x^2 - 9x + 20 = x^2 - 5x - 4x + 20$$

$$= x(x - 5) - 4(x - 5)$$

$$= (x - 5)(x - 4)$$

$$x^2 - 8x + 15 = x^2 - 5x - 3x + 15$$

$$= x(x - 5) - 3(x - 5)$$

$$= (x - 5)(x - 3)$$

$$(x^2 - 7x + 12)(x^2 - 9x + 20)(x^2 - 8x + 15)$$

$$= (x - 4)(x - 3) \times (x - 5)(x - 4) \times (x - 5)(x - 3)$$

$$= (x - 3)^2 (x - 4)^2 (x - 5)^2$$

جذر لینے سے

$$= \pm \sqrt{(x - 3)^2 (x - 4)^2 (x - 5)^2}$$

$$= \pm (x - 3)(x - 4)(x - 5)$$

پس

$$(x^2 + 8x + 7)(2x^2 - x - 3)(2x^2 + 11x - 21)$$

-3

حل: چونکہ

$$x^2 + 8x + 7$$

$$= x^2 + 7x + x + 7$$

$$= x(x + 7) + 1(x + 7)$$

$$= (x + 7)(x + 1)$$

$$2x^2 - x - 3$$

$$= 2x^2 - 3x + 2x - 3$$

$$= x(2x - 3) + 1(2x - 3)$$

$$= (2x - 3)(x + 1)$$

$$2x^2 + 11x - 21 = 2x^2 + 14x - 3x - 21$$

$$= 2x(x + 7) - 3(x + 7)$$

$$= (x + 7)(2x - 3)$$

لہذا

$$(x^2 + 8x + 7)(2x^2 - x - 3)(2x^2 + 11x - 21) = (x + 7)(x + 1) \times (2x - 3)(x + 1)$$

$$\times (x + 7)(2x - 3)$$

$$= (x + 1)^2 (x + 7)^2 (2x - 3)^2$$

جذر لینے سے

$$= \pm \sqrt{(x + 1)^2 (x + 7)^2 (2x - 3)^2}$$

$$= \pm (x + 1)(x + 7)(2x - 3)$$

$$x(x + 2)(x + 4)(x + 6) + 16$$

-4

$$x(x + 2)(x + 4)(x + 6) + 16$$

حل:

$$= [x(x + 6)][(x + 4)(x + 2)] + 16$$

$$= (x^2 + 6x)(x^2 + 6x + 8) + 16$$

اب
اگر

$$y = x^2 + 6x$$

$$= y(y + 8) + 16$$

$$= y^2 + 8y + 16$$

$$= y^2 + 2(4)(y) + 4^2$$

$$= (y + 4)^2$$

دو بارہ $y = x^2 + 6x$ رکھتے سے

$$\begin{aligned} &= (x^2 + 6x + 4)^2 \\ &= \pm \sqrt{(x^2 + 6x + 4)^2} \\ &= \pm (x^2 + 6x + 4) \end{aligned}$$

جذر لینے سے

$$(2x+1)(2x+3)(2x+5)(2x+7)+16$$

-5
حل:

$$\begin{aligned} &(2x+1)(2x+3)(2x+5)(2x+7)+16 \\ &= [(2x+1)(2x+7)][(2x+3)(2x+5)]+16 \\ &= (4x^2+14x+2x+7)(4x^2+10x+6x+15)+16 \\ &= (4x^2+16x+7)(4x^2+16x+15)+16 \\ &= (4x^2+16x+7)(4x^2+16x+7+8)+16 \end{aligned}$$

$$y = 4x^2 + 16x + 7$$

$$\begin{aligned} &= y(y+8)+16 \\ &= y^2+8y+16 \\ &= (y+4)^2 \end{aligned}$$

$$y = 4x^2 + 16x + 7$$

$$\begin{aligned} &= (4x^2+16x+7+4)^2 \\ &= (4x^2+16x+11)^2 \end{aligned}$$

جذر لینے سے

$$\begin{aligned} &= \pm \sqrt{(4x^2+16x+11)^2} \\ &= \pm (4x^2+16x+11) \end{aligned}$$

$$\left(x^2 + \frac{1}{x^2}\right) - 10\left(x + \frac{1}{x}\right) + 27, x \neq 0$$

-6

$$\left(x^2 + \frac{1}{x^2}\right) - 10\left(x + \frac{1}{x}\right) + 27$$

حل:

فرض کیا

$$x + \frac{1}{x} = y$$

$$\left(x + \frac{1}{x}\right)^2 = y^2$$

$$x^2 + \frac{1}{x^2} + 2 = y^2$$

$$x^2 + \frac{1}{x^2} = y^2 - 2$$

$$= (y^2 - 2) - 10y + 27$$

$$= y^2 - 10y + 25$$

$$= (y)^2 - 2(y)(5) + (5)^2$$

پس دیئے گئے اظہار یے میں قیمتیں رکھنے سے

$$= (y - 5)^2$$

$$y = x + \frac{1}{x} \text{ بدل کر لکھئے سے}$$

$$= \left(x + \frac{1}{x} - 5 \right)^2$$

جذر لینے سے

$$= \pm \sqrt{\left(x + \frac{1}{x} - 5 \right)^2}$$

$$= \pm \left(x + \frac{1}{x} - 5 \right)$$

$$\left(t - \frac{1}{t} \right)^2 - 4 \left(t + \frac{1}{t} \right) + 8 ; (t \neq 0) \quad -7$$

حل:

$$\left(t - \frac{1}{t} \right)^2 - 4 \left(t + \frac{1}{t} \right) + 8$$

$$= \left(t^2 + \frac{1}{t^2} - 2 \right) - 4 \left(t + \frac{1}{t} \right) + 8$$

$$= \left(t^2 + \frac{1}{t^2} \right) - 4 \left(t + \frac{1}{t} \right) - 2 + 8$$

$$= \left(t^2 + \frac{1}{t^2} \right) - 4 \left(t + \frac{1}{t} \right) + 6$$

$$t + \frac{1}{t} = u^2$$

فرض کیا

$$\left(t + \frac{1}{t} \right)^2 = u^2$$

$$t^2 + \frac{1}{t^2} + 2 = u^2$$

$$t^2 + \frac{1}{t^2} = u^2 - 2$$

قیمتیں درج کرنے سے

$$= (u^2 - 2) - 4u + 6$$

$$= u^2 - 4u + 4$$

$$= (u - 2)^2$$

$$t + \frac{1}{t} = u \text{ بدل کر لکھئے سے}$$

$$= \left(t + \frac{1}{t} - 2 \right)^2$$

جذر لینے سے

$$= \pm \sqrt{\left(t + \frac{1}{t} - 2\right)^2}$$

$$= \pm \left(t + \frac{1}{t} - 2\right)$$

پس

$$\left(x^2 + \frac{1}{x^2}\right)^2 - 4\left(x + \frac{1}{x}\right)^2 + 12; x \neq 0 \quad -8$$

$$\left(x^2 + \frac{1}{x^2}\right)^2 - 4\left(x + \frac{1}{x}\right)^2 + 12$$

حل:

فرض کیا

$$x + \frac{1}{x} = y$$

$$\left(x + \frac{1}{x}\right)^2 = y^2$$

$$x^2 + \frac{1}{x^2} + 2 = y^2$$

$$x^2 + \frac{1}{x^2} = y^2 - 2$$

قیمتیں درج کرنے سے

$$= (y^2 - 2)^2 - 4y^2 + 12$$

$$= y^4 - 4y^2 + 4 - 4y^2 + 12$$

$$= y^4 - 8y^2 + 16$$

$$= (y^2 - 4)^2$$

یہ کی قیمت درج کرنے سے

$$= \left(x^2 + \frac{1}{x^2} + 2 - 4\right)^2$$

$$= \left(x^2 + \frac{1}{x^2} - 2\right)^2$$

جذر لینے سے

$$= \pm \sqrt{\left(x^2 + \frac{1}{x^2} - 2\right)^2}$$

$$= \pm \left(x^2 + \frac{1}{x^2} - 2\right)$$

$$4x^4 + 12x^3 + 25x^2 + 24x + 16$$

-9
حل:

$$\begin{array}{r}
 2x^2 + 3x + 4 \quad - \\
 2x^2 \overline{) 4x^4 + 12x^3 + 25x^2 + 24x + 16} \\
 \underline{-4x^4} \\
 4x^2 + 3x \\
 \underline{12x^3 + 25x^2 + 24x + 16} \\
 \underline{-12x^3 + 9x^2} \\
 4x^2 + 6x + 4 \\
 \underline{16x^2 + 24x + 16} \\
 \underline{-16x^2 + 24x + 16} \\
 0
 \end{array}$$

پس

$$\begin{aligned}
 &= \sqrt{4x^4 + 12x^3 + 25x^2 + 24x + 16} \\
 &= \pm (2x^2 + 3x + 4)
 \end{aligned}$$

$$\frac{9x^2}{4y^2} - \frac{3x}{2y} - \frac{7}{4} + \frac{2y}{3x} + \frac{4x^2}{9y^2}; (x \neq 0, y \neq 0) \quad -10$$

حل: جذر بذریعہ تقسیم معلوم کرنے سے

$$\begin{array}{r}
 \frac{3x}{2y} - \frac{1}{2} - \frac{2y}{3x} \\
 \frac{3x}{2y} \overline{) \frac{9x^2}{4y^2} - \frac{3x}{2y} - \frac{7}{4} + \frac{2y}{3x} + \frac{4x^2}{9y^2}} \\
 \underline{-\frac{9x^2}{4y^2}} \phantom{- \frac{3x}{2y} - \frac{7}{4} + \frac{2y}{3x} + \frac{4x^2}{9y^2}} \\
 \frac{3x}{y} - \frac{1}{2} \phantom{+ \frac{2y}{3x} + \frac{4x^2}{9y^2}} \\
 \underline{-\frac{3x}{2y} - \frac{7}{4} + \frac{2y}{3x} + \frac{4x^2}{9y^2}} \\
 \phantom{\frac{3x}{y} - \frac{1}{2} -} \underline{+\frac{3x}{2y} + \frac{1}{4}} \\
 \frac{3x}{y} - 1 - \frac{2y}{3x} \phantom{+ \frac{4x^2}{9y^2}} \\
 \underline{-2 + \frac{2y}{3x} + \frac{4x^2}{9y^2}} \\
 \phantom{\frac{3x}{y} - 1 - \frac{2y}{3x} -} \underline{+2 + \frac{2y}{3x} + \frac{4x^2}{9y^2}} \\
 0
 \end{array}$$

$$\pm \left(\frac{3x}{2y} - \frac{1}{2} - \frac{2y}{3x} \right) = \text{پس جذر}$$

11- 'x' کی کس قیمت کے لیے $x^4 + 4x^2 + x + \frac{8}{x^2} + \frac{4}{x^4}$ ایک مکمل مربع ہے۔ جبکہ $x \neq 0$ ۔
حل:

$$\begin{array}{r}
 x^2 + 2 + \frac{2}{x^2} \\
 \hline
 x^2 \quad x^4 + 4x^2 + x + \frac{8}{x^2} + \frac{4}{x^4} \\
 \underline{-x^4} \\
 2x^2 + 2 \quad 4x^2 + x + \frac{8}{x^2} + \frac{4}{x^4} \\
 \underline{-4x^2 + 4} \\
 2x^2 + 4 + \frac{2}{x^2} \quad x - 4 + \frac{8}{x^2} + \frac{4}{x^4} \\
 \underline{+ 4 + \frac{8}{x^2} + \frac{4}{x^4}} \\
 x - 8
 \end{array}$$

چونکہ مکمل مربع کیلئے باقی کو صفر ہونا چاہیے

$$x - 8 = 0 \quad \text{اس لیے}$$

$$x = 8$$

پس $x = 8$ کیلئے $x^4 + 4x^2 + x + \frac{8}{x^2} + \frac{4}{x^4}$ مکمل مربع ہوگا۔

12- اگر $x^4 + lx^3 + mx^2 + 12x + 9$ ایک مکمل مربع ہو تو l اور m کی قیمتیں معلوم کریں۔
حل:

$$\begin{array}{r}
 x^2 + 2x + 3 \\
 \hline
 x^2 \quad x^4 + lx^3 + mx^2 + 12x + 9 \\
 \underline{\pm x^4} \\
 2x^2 + 2x \quad lx^3 + mx^2 + 12x + 9 \\
 \underline{+ 4x^3 + 4x^2} \\
 2x^2 + 4x + 3 \quad lx^3 - 4x^3 + mx^2 - 4x^2 + 12x + 9 \\
 \underline{+ 6x^2 + 12x + 9} \\
 lx^3 - 4x^3 + mx^2 - 10x^2
 \end{array}$$

چونکہ مکمل مربع کیلئے باقی کو صفر کے برابر ہونا چاہیے۔
اس لیے

$$lx^3 - 4x^3 + mx^2 - 10x^2 = 0$$

$$x^3 (l - 4) + x^2 (m - 10) = 0$$

$$l - 4 = 0 \quad \text{اور} \quad m - 10 = 0$$

$$l = 4 \quad \text{اور} \quad m = 10$$