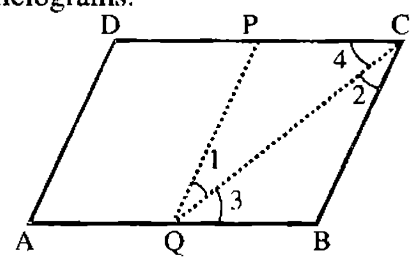
Exercise 16.1

(1) Show that the line segment joining the mid-points of opposite sides of a parallelogram, divides it into two equal parallelograms.



Given ABCD is parallelogram. point p is midpoint of side $\overline{DC}$ i.e. $\overline{DP} \cong \overline{PC}$ and point Q is midpoint of side $\overline{AB}$ i.e. $\overline{AQ} \cong \overline{QB}$.

To Prove

Parallelogram AQPD $\cong$ parallelogram QBCP

Construction

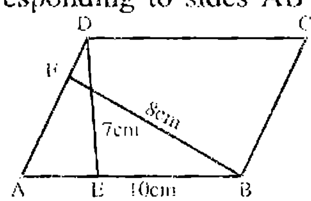
Join P to Q and Q to C.

Proof

Statements	Reasons
$m\overline{AB} = m\overline{DC}$ $\frac{1}{2}m\overline{AB} = \frac{1}{2}m\overline{DC}$ $m\overline{QB} = m\overline{PC}$	Dividing by 2

Now	
$\Delta PQC \leftrightarrow \Delta QBC$	
$\overline{QC} \cong \overline{QC}$	Common
$\overline{QB} \cong \overline{PC}$	Proved
$\angle 3 \cong \angle 4$	Alt. Angles $\overline{AB} \parallel \overline{DC}$
$\Delta PQC \cong \Delta QBC$	S.A.S = S.A.S
$\overline{PQ} \cong \overline{CB}$ ....(i)	Corresponding sides of congruent triangles
$\overline{AD} \cong \overline{CB}$ ....(ii)	
$\overline{PQ} \cong \overline{AD} \cong \overline{CB}$	
$\angle 1 \cong \angle 2$	Corresponding angles of congruent triangles
$m\angle 1 + m\angle 3 = m\angle 2 + m\angle 4$	Corresponding angles of $\parallel$gm
$\angle PQB \cong \angle PCB$	
$\angle A \cong \angle PCB$	
$\angle A \cong \angle PQB$	
Now	
$\parallel$gm AQPD and $\parallel$gm QBCP	
$\overline{AQ} \cong \overline{QB}$	Given
$\overline{AD} \cong \overline{PQ}$	Proved
$\angle A \cong \angle PQB$	
Thus $\parallel$gm AQPD $\cong$ $\parallel$gm QBCP	

(2) In a parallelogram ABCD, $m\overline{AB} = 10$cm. The altitudes corresponding to sides AB and AD are respectively 7 cm and 8 cm. Find $\overline{AD}$.



Given Parallelogram ABCD, $m\overline{AB} = 10$cm altitudes. Corresponding to the sides $\overline{AB}$ and $\overline{AD}$ are 7cm and 8cm.

To Prove: $m\overline{AD} = ?$

Construction Make $\parallel$gm ABCD and show the given altitudes $\overline{DE} = 7$cm, $\overline{BF} = 8$cm.

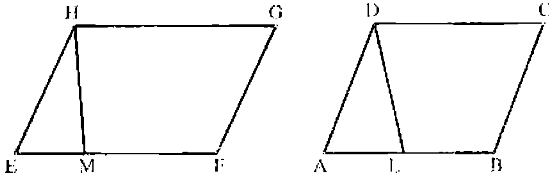
Proof The area of parallelogram = base x altitude

Statements	Reasons
$\therefore$ Area of parallelogram ABCD = 10 x 7 ....(i)	
Also area of the $\parallel$gm ABCD = $\overline{AD}$ x 8 ....(ii)	
$\therefore$ $m\overline{AD} \times 8 = 10 \times 7$	

$$m\overline{AD} = \frac{10 \times 7}{8}$$

$$m\overline{AD} = \frac{35}{4} = 8\frac{3}{4}\text{ cm}$$

(3) If two parallelograms of equal areas have the same or equal bases, their altitudes are equal.



Given Two parallelograms of same or equal bases and same areas.

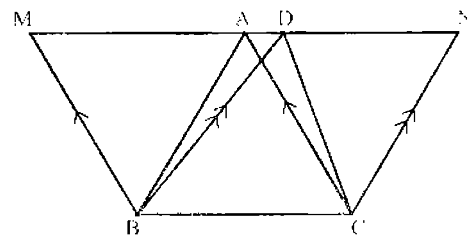
To Prove Their altitudes are equal.

Construction Make the llgm ABCD and EFGH. Draw $\overline{DL} \perp \overline{AB}$ and $\overline{HM} \perp \overline{EF}$

Proof

Statements	Reasons
Area of the llgm ABCD = area of the llgm EFGH	
base x altitude = base x altitude	
$m\overline{AB} \times m\overline{DL} = m\overline{EF} \times m\overline{HM}$	Area = base x altitude
But $m\overline{AB} = m\overline{EF}$	
$\therefore$ $m\overline{EF} \times m\overline{DL} = m\overline{EF} \times m\overline{HM}$	Dividing by $m\overline{EF}$ we get
$m\overline{DL} = m\overline{HM}$ so altitudes are equal	

Theorem Triangles on the same base and of the same (i.e., equal) altitudes are equal in area.



Given Δs ABC, DBC on the same base $\overline{BC}$ and having equal altitudes.

To Prove Area of (ΔABC) = area of (ΔDBC)

Construction Draw $\overline{BM}$ ll to $\overline{CA}$, $\overline{CN}$ ll to $\overline{BD}$ meeting $\overline{AD}$ produced in M, N.

Proof

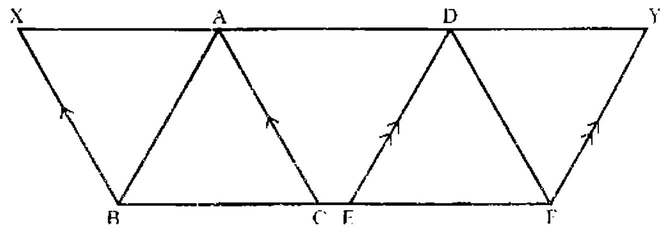
Statements	Reasons
Δ ABC and Δ DBC are between the same $\|^{s}$	Their altitudes are equal
Hence MADN is parallel to $\overline{BC}$	
$\therefore$ Area ($\|^{gm}$ BCAM)=Area ($\|^{gm}$ BCND)....(i)	These $\|^{gms}$ are on the same base $\overline{BC}$ and between the same $\|^{s}$
But $\Delta ABC = \frac{1}{2}(\|^{gm}\ BCAM)$ .....(ii)	Each diagonal of a $\|^{gm}$ bisects it into two congruent triangles

and $\Delta DBC = \frac{1}{2}(\|_{gm}\ BCND)$ .....(iii) Hence area (Δ ABC) = Area (Δ DBC)	From (i), (ii) and (iii)

Theorem **Triangles on equal bases and of equal altitudes are equal in area.**

Given

Δs ABC, DEF on equal bases $\overline{BC}$, $\overline{EF}$ and having altitudes equal.



To Prove

Area (Δ ABC) = Area (Δ DEF)

Construction

Place the Δs ABC and DEF so that their equal bases $\overline{BC}$ and $\overline{EF}$ are in the same straight line BCEF and their vertices on the same side of it. Draw BX || to CA and FY || to ED meeting AD produced in X, Y respectively

Proof

Statements	Reasons
Δ ABC and Δ DEF are between the same parallels	Their altitudes are equal (given)
$\therefore$ XADY is || to BCEF	
$\therefore$ Area ($\|^{gm}$ BCAX) = Area ($\|^{gm}$ EFYD) .....(i)	These $\|^{gms}$ are on equal bases and between the same parallels
But $\Delta ABC = \frac{1}{2}(\|^{gm}\ BCAX)$ .....(ii)	Diagonal of a $\|^{gm}$ bisects it
and $\Delta DEF = \frac{1}{2}(\|_{gm}\ EFYD)$ .....(iii)	
$\therefore$ area (ΔABC) = area (ΔDEF)	From (i), (ii) and (iii)

Corollaries

(1) Triangles on equal bases and between the same parallels are equal in area.

(2) Triangles having a common vertex and equal bases in the same straight line, are equal in area.