Exercise: 5.2

Q.1: Given inequalities are

$2x - 3y \leq 6 \rightarrow (i)$

$2x + 3y \leq 12 \rightarrow (ii)$

$x \geq 0 \rightarrow (iii)$, $y \geq 0 \rightarrow (iv)$

Associated equations of (i) and (ii) are

$2x - 3y = 6 \rightarrow (1)$

$2x + 3y = 12 \rightarrow (2)$

For $y = 0$, (1) $\Rightarrow 2x - 0 = 6 \quad \therefore x = 3$

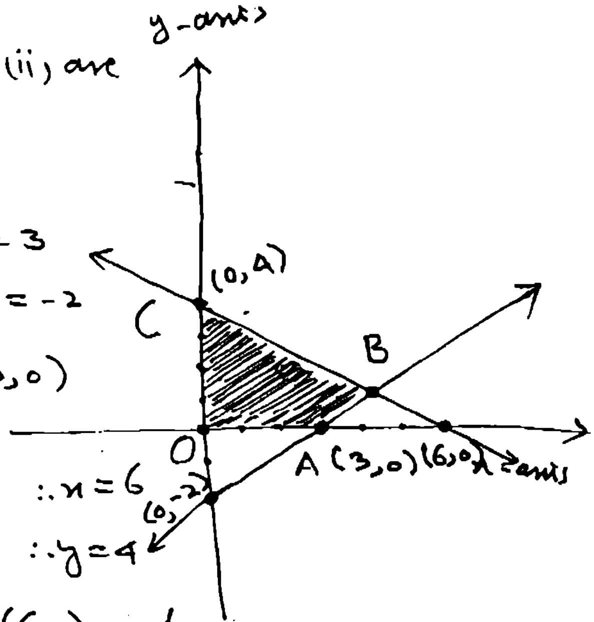
for $x = 0$, (1) $\Rightarrow 0 - 3y = 6 \quad \therefore y = -2$

$\therefore$ The line (1) cuts x-axis at (3, 0) and y-axis at (0, −2).

Now for $y = 0$, (2) $\Rightarrow 2x + 0 = 12 \quad \therefore x = 6$

for $x = 0$, (2) $\Rightarrow 0 + 3y = 12 \quad \therefore y = 4$

$\therefore$ The line (2) cuts x-axis at (6, 0) and y-axis at (0, 4)



We take (0, 0) as test point.

(0, 0) satisfy both inequalities (i) and (ii).

$\therefore$ The graphs of (i) and (ii) are the closed half planes on the side of (0, 0).

The graph of $x \geq 0$ is the closed right half plane of xy-plane and the graph of $y \geq 0$ is the closed upper half plane of xy-plane.

The intersection of graphs of (i), (ii), (iii) and (iv) is the feasible region of the given system of linear inequalities as shown in the figure by shading the region.

Here OABC is the feasible region.

where O(0, 0), A(3, 0), B(4.5, 1) or $B(\frac{9}{2}, 1)$ and C(0, 4) are corner points of feasible region.

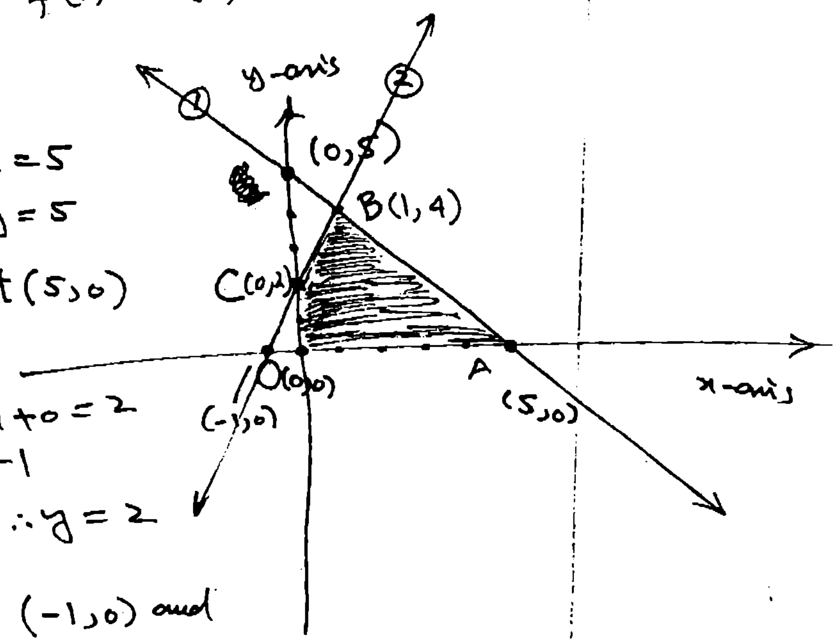
$B(\frac{9}{2}, 1)$ is the point of intersection of lines (1) and (2).

(ii) Given inequalities are

$$x + y \le 5 \rightarrow (i)$$
$$-2x + y \le 2 \rightarrow (ii)$$
$$x \geq 0 \rightarrow (iii), \quad y \geq 0 \rightarrow (iv)$$

The associated equations of (i) and (ii) are

$$x + y = 5 \rightarrow ①$$
$$-2x + y = 2 \rightarrow ②$$



For $y = 0$, ① $\Rightarrow x + 0 = 5 \quad \therefore x = 5$

for $x = 0$, ① $\Rightarrow 0 + y = 5 \quad \therefore y = 5$

$\therefore$ The line ① cuts x-axis at $(5,0)$ and y-axis at $(0,5)$.

Now for $y = 0$, (ii) $\Rightarrow -2x + 0 = 2$
$\therefore x = -1$

for $x = 0$, ② $\Rightarrow -0 + y = 2 \quad \therefore y = 2$

$\therefore$ The line ② cuts x-axis at $(-1,0)$ and y-axis at $(0,2)$

We take $(0,0)$ as test point.

As $(0,0)$ satisfies both (i) and (ii)

$\therefore$ The graphs of (i) and (ii) are the closed half planes on the side of $(0,0)$.

The graph of $x \geq 0$ is the closed right half plane of xy-plane and the graph of $y \geq 0$ is the closed upper half plane of xy-plane.

$\therefore$ The intersection of graphs of (i), (ii), (iii) and (iv) is the feasible region of the given system of linear inequalities and is shown in the diagram by shading the region.

Here OABC is the feasible region.

where $O(0,0)$, $A(5,0)$, $B(1,4)$, $C(0,2)$ are corner point of feasible region OABC.

B is point of intersection of lines ① and ②.

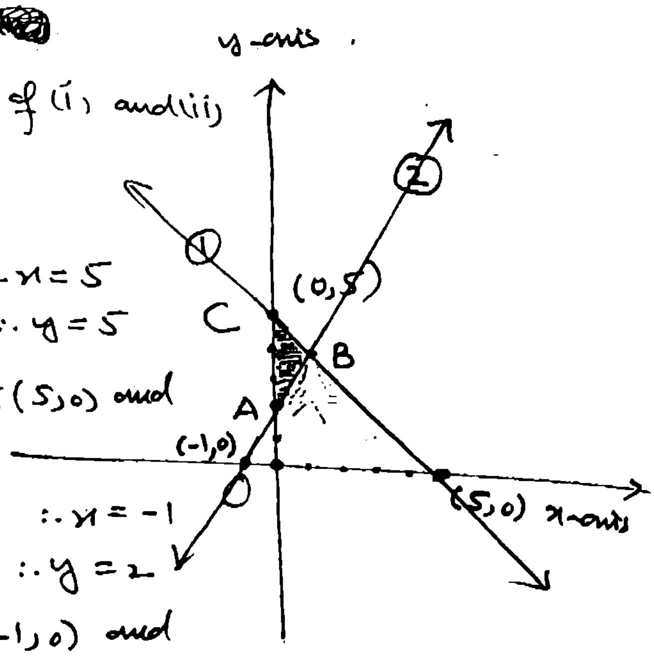
③

(iii) Given inequalities are

$x+y \le 5 \rightarrow (i)$

$-2x+y \ge 2 \rightarrow (ii)$

$x \ge 0$, $\rightarrow (iii)$



The associated equations of (i) and (ii) are $x+y=5 \rightarrow ①$

$-2x+y=2 \rightarrow ②$

For $y=0$, $① \Rightarrow x+0=5 \quad \therefore x=5$

for $x=0$, $① \Rightarrow 0+y=5 \quad \therefore y=5$

$\therefore$ The line ① cuts x-axis at $(5,0)$ and y-axis at $(0,5)$.

For $y=0$, $② \Rightarrow -2x+0=2 \quad \therefore x=-1$

for $x=0$, $② \Rightarrow 0+y=2 \quad \therefore y=2$

$\therefore$ Line ② cuts x-axis at $(-1,0)$ and y-axis at $(0,2)$.

We take test point as origin $(0,0)$

$(0,0)$ satisfies (i).

$\therefore$ Graph of (i) is the closed half plane on the side of $(0,0)$

As $(0,0)$ does not satisfy (ii)

$\therefore$ Graph of (ii) is the closed half plane (made by line ②) not on the side of $(0,0)$.

Graph of (iii) or graph of $x \ge 0$ is the closed right half plane of xy-plane.

~~Graph of (iv) or graph of $y \ge 0$ is the closed upper half plane.~~

The intersection of graphs of (i), (ii), (iii) is the feasible region ABC as shown in the figure by shading the region.

$A(0,0)$, $B(1,4)$, $C(0,5)$ are corner points of the feasible region ABC.

(iv) Given inequalities are

$3x+7y \le 21 \rightarrow$ (i)

$x-y \le 3 \rightarrow$ (ii)

$x \ge 0 \rightarrow$ (iii)

$y \ge 0 \rightarrow$ (iv)

$3x+7y=21$
$7x-7y=21$
$10x=42$
$x=\frac{21}{5}$

$y=x-3=\frac{21}{5}-3$
$=\frac{6}{5}$

The associated equations of (i) and (ii) are $3x+7y=21 \rightarrow$ (1)

$x-y=3 \rightarrow$ (2)

For $y=0$, (1) $\Rightarrow 3x+0=21 \therefore x=7$

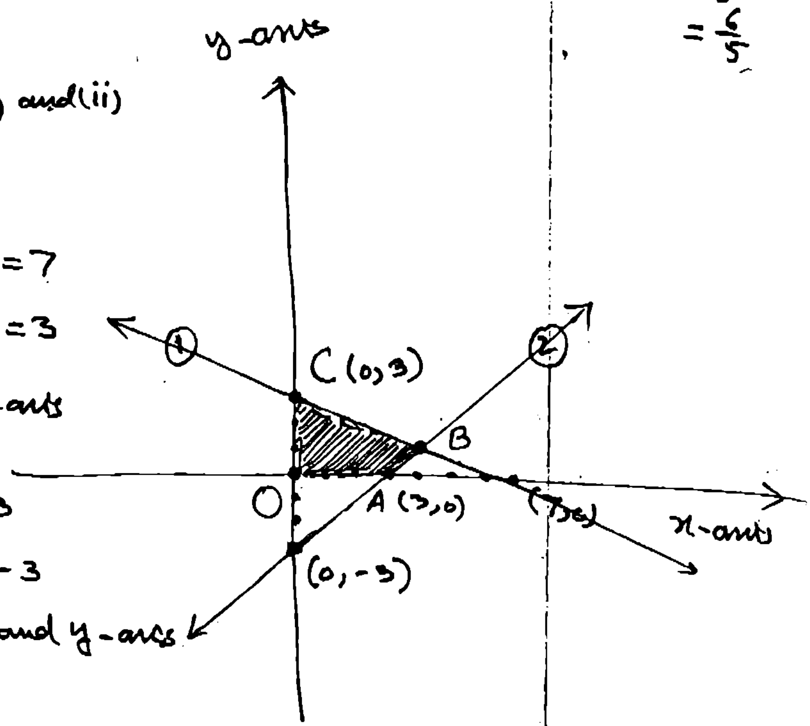
for $x=0$, (1) $\Rightarrow 0+7y=21 \therefore y=3$

$\therefore$ (1) cuts x-axis at (7,0) and y-axis at (0,3).

For $y=0$, (2) $\Rightarrow x-0=3 \therefore x=3$

For $x=0$, (2) $\Rightarrow 0-y=3 \therefore y=-3$

$\therefore$ Line (2) cuts x-axis at (3,0) and y-axis at (0,-3).



We take (0,0) as test point.

As (0,0) satisfies (i) and (ii)

$\therefore$ The graphs of (i) and (ii) are the closed half planes on the side of origin (0,0).

The graph of $x \ge 0$ is the closed right half plane of xy-plane. and the graph of $y \ge 0$ is the closed upper half plane of xy-plane.

The intersection of graphs of (i), (ii), (iii) and (iv) is the feasible region of the given system of linear inequalities ~~system~~ and is shown in the figure by shading the region.

Here OABC is the feasible region.

where $O(0,0)$, $A(3,0)$, $B(\frac{21}{5}, \frac{6}{5})$, $C(0,3)$ are the corner of feasible region.

(v) Given inequalities are

$$\left.\begin{array}{l} 3x+2y \geq 6 \rightarrow (i) \\ x+y \leq 4 \rightarrow (ii) \\ x \geq 0 \rightarrow (iii) \\ y \geq 0 \rightarrow (iv) \end{array}\right] \rightarrow (I)$$

The associated equations of (i) and (ii) are

$$3x+2y=6 \rightarrow (1)$$
$$x+y=4 \rightarrow (2)$$

For $y=0$, (1) $\Rightarrow 3x+0=6 \quad \therefore x=2$

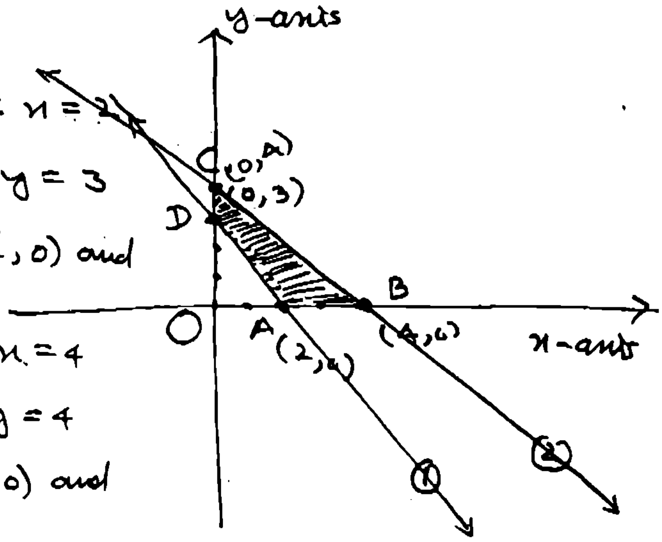
for $x=0$, (1) $\Rightarrow 0+2y=6 \quad \therefore y=3$

$\therefore$ Line (1) cuts x-axis at $(2,0)$ and y-axis at $(0,3)$.

For $y=0$, (2) $\Rightarrow x+0=4 \quad \therefore x=4$

for $x=0$, (2) $\Rightarrow 0+y=4 \quad \therefore y=4$

$\therefore$ Line (2) cuts x-axis at $(4,0)$ and y-axis at $(0,4)$.



We take $(0,0)$ as test point.

$(0,0)$ does not satisfy (i)

$\therefore$ The graph of (i) is the closed half plane (made by line (1)) not on the side of $(0,0)$.

$(0,0)$ satisfies (ii)

$\therefore$ graph of (ii) is the closed half plane (made by line (2)) on the side of $(0,0)$.

Graph of (iii) is the closed right half plane of xy-plane and graph of (iv) is the closed upper half plane of xy-plane.

The intersection of graphs of (i), (ii), (iii) and (iv) is the feasible region ABCD as shown in the figure by shading the region.

$\therefore$ $A(2,0)$, $B(4,0)$, $C(0,4)$, $D(0,3)$ are the corners of the feasible region.

(vi) Given inequalities are

$$\left.\begin{array}{l} 5x+7y \le 35 \to (i) \\ x-2y \le 4 \to (ii) \\ x \ge 0 \to (iii) \quad , \; y \ge 0 \to (iv) \end{array}\right\} \to (I)$$

$$\begin{array}{r} 5x+7y=35 \\ 5x-10y=20 \\ \hline 17y=15 \end{array}$$

$\therefore y = \frac{15}{17}$

$x = 4 + \frac{30}{17} = \frac{98}{17}$

The associated equations of (i) and (ii)

are $\quad 5x+7y=35 \to (1)$

$\quad\quad x-2y=4 \to (2)$

For $y=0$, (1) $\Rightarrow 5x+0=35 \quad \therefore x=7$

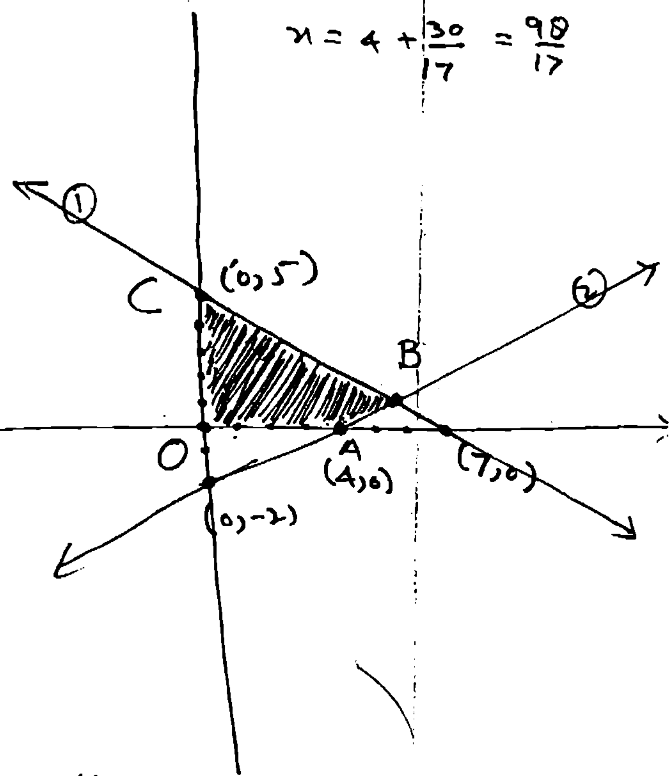
for $x=0$, (1) $\Rightarrow 0+7y=35 \quad \therefore y=5$

$\therefore$ Line (1) cuts x-axis at $(7,0)$ and y-axis at $(0,5)$.

For $y=0$, (2) $\Rightarrow x-0=4 \quad \therefore x=4$

for $x=0$, (2) $\Rightarrow 0-2y=4 \quad \therefore y=-2$

$\therefore$ The line (2) cuts x-axis at $(4,0)$ and y-axis at $(0,-2)$



We take $(0,0)$ as the test point.

As $(0,0)$ satisfies both inequalities (i) and (ii)

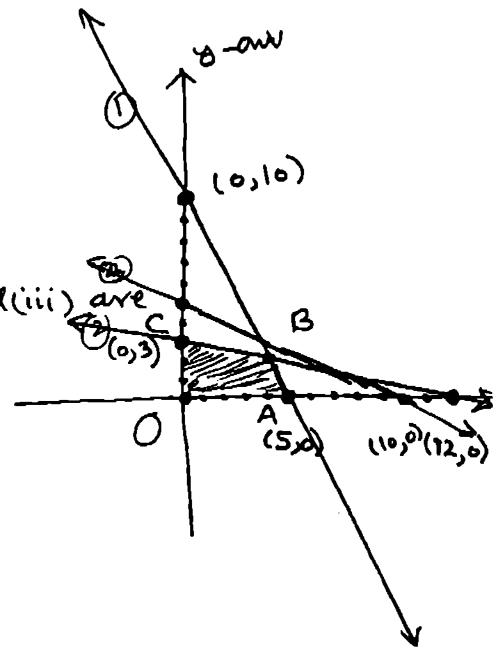
$\therefore$ The graphs of (i) and (ii) are the closed half planes on the side of $(0,0)$.

The graph of $x \ge 0$ is the closed right half plane and the graph of $y \ge 0$ is the closed upper half plane.

The intersection of graphs of (i), (ii), (iii) and (iv) is the feasible region of the system of given inequalities as shown in the figure by shading the region.

Here OABC is the feasible region.

$O(0,0)$, $A(4,0)$, $B\left(\frac{98}{17}, \frac{15}{17}\right)$ and $C(0,5)$ are the corner points of the feasible region OABC.



Q.2 (i) Given inequalities are

$2x+y \le 10 \rightarrow$ (i)
$x+4y \le 12 \rightarrow$ (ii)
$x+2y \le 10 \rightarrow$ (iii)
$x \geq 0 \rightarrow$ (iv), $y \geq 0 \rightarrow$ (v)

$\rightarrow$ (I)

The associated equations of (i), (ii) and (iii) are

$2x+y = 10 \rightarrow$ ①
$x+4y = 12 \rightarrow$ ②
$x+2y = 10 \rightarrow$ ③

For $y=0$, ① $\Rightarrow 2x+0=10 \quad \therefore x=5$
for $x=0$, ① $\Rightarrow 0+y=10 \quad \therefore y=10$
$\therefore$ Lines ① cuts x-axis at $(5,0)$ and y-axis at $(0,10)$.
for $y=0$, ② $\Rightarrow x+0=12 \quad \therefore x=12$
for $x=0$, ② $\Rightarrow 0+4y=12 \quad \therefore y=3$
$\therefore$ Line ② cuts x-axis at $(12,0)$ and y-axis at $(0,3)$
for $y=0$, ③ $\Rightarrow x+0=10 \quad \therefore x=10$
for $x=0$, ③ $\Rightarrow 0+2y=10 \quad \therefore y=5$
$\therefore$ Line ③ cuts x-axis at $(10,0)$ and y-axis at $(0,5)$.

We take origin $(0,0)$ as test point.
As $(0,0)$ satisfies (i), (ii) and (iii)
$\therefore$ The graphs of (i), (ii), (iii) are the closed half plane on the origin side of ①, ② and ③.

The graph of $x \geq 0$ is the closed right half plane and the graph of $y \geq 0$ is the closed upper half plane.

The intersection of graphs of (i), (ii), (iii), (iv) and (v) is the feasible region OABC of the system of inequalities, and is as shown in the figure by shading the region.

The corners of feasible region are

$O(0,0)$, $A(5,0)$, $B(4,2)$, $C(0,3)$.

$B(4,2)$ is point of intersection of lines ① and ②.

(ii) Given inequalities are

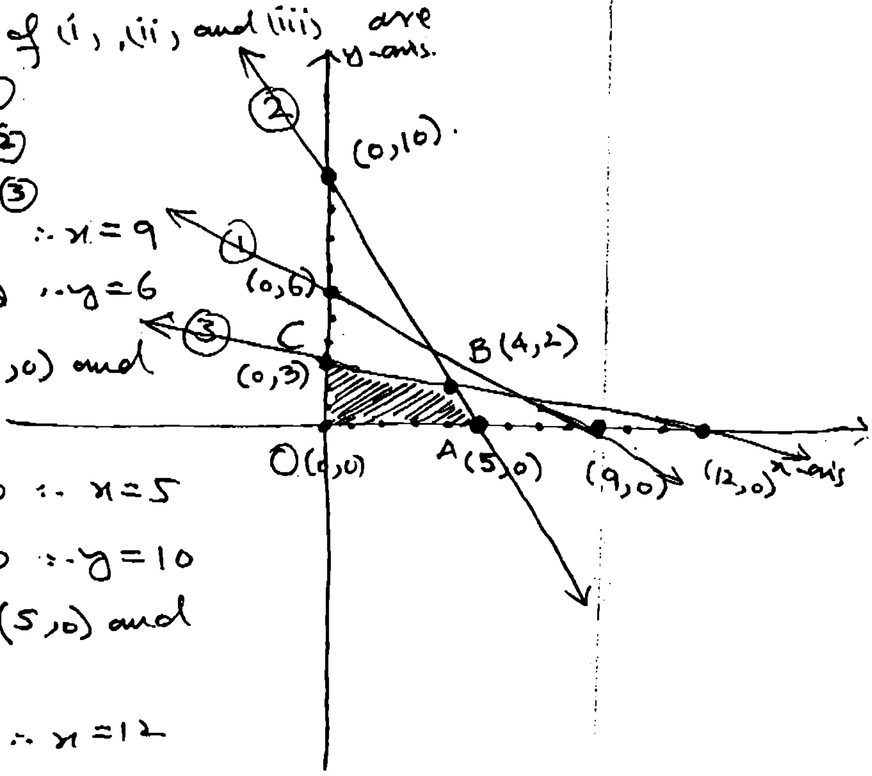
$$\left.\begin{array}{l} 2x+3y \le 18 \rightarrow \text{(i)} \\ 2x+y \le 10 \rightarrow \text{(ii)} \\ x+4y \le 12 \rightarrow \text{(iii)} \\ x \ge 0 \rightarrow \text{(iv)};\quad y \ge 0 \rightarrow \text{(v)} \end{array}\right\} \rightarrow (I)$$

The associated equations of (i), (ii) and (iii) are

$2x+3y = 18 \rightarrow (1)$

$2x+y = 10 \rightarrow (2)$

$x+4y = 12 \rightarrow (3)$



For $y=0$, (1) $\Rightarrow 2x+0=18$ $\therefore x=9$

for $x=0$, (1) $\Rightarrow 0+3y=18$ $\therefore y=6$

$\therefore$ Line (1) cuts x-axis at (9,0) and y-axis at (0,6).

for $y=0$, (2) $\Rightarrow 2x+0=10$ $\therefore x=5$

for $x=0$, (2) $\Rightarrow 0+y=10$ $\therefore y=10$

$\therefore$ Line (2) cuts x-axis at (5,0) and y-axis at (0,10)

For $y=0$, (3) $\Rightarrow x+0=12$ $\therefore x=12$

for $x=0$, (3) $\Rightarrow 0+4y=12$ $\therefore y=3$

$\therefore$ Line (3) cuts x-axis at (12,0) and y-axis at (0, 3).

We take origin (0,0) as the test point.

As (0,0) satisfies (i), (ii) and (iii).

$\therefore$ The graphs of (i), (ii) and (iii) are the closed half planes on the side of origin (0,0) of the lines (1), (2) and (3).

The graph of $x \ge 0$ is the closed right half plane of xy-plane.

The graph of $y \ge 0$ is the closed upper half plane of xy-plane.

The intersection of graphs of (i), (ii), (iii), (iv), (v) is the feasible region OABC of the system of linear inequalities as shown in the figure by shading the region.

O(0,0), A(5,0), B(4,2), C(0,3) are corners of feasible region OABC.

B(4,2) is the point of intersection of lines (2) and (3)

(iii) Given inequalities are

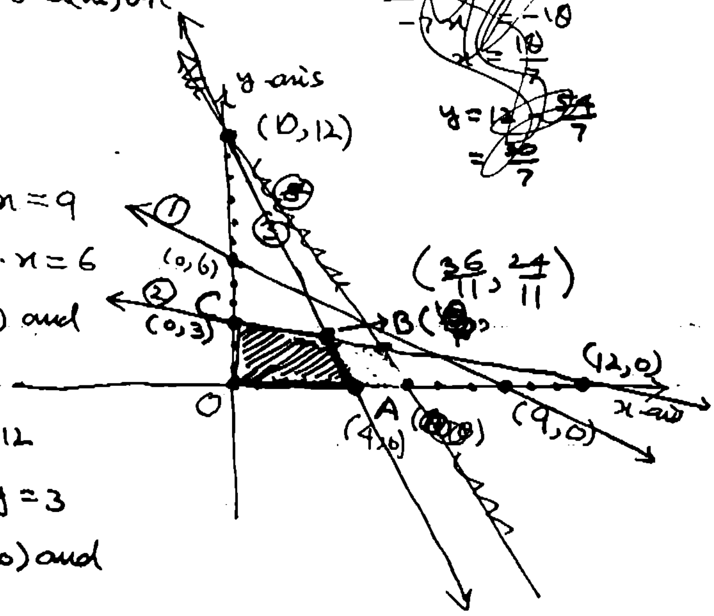
$$\left.\begin{aligned} 2x+3y &\le 18 \rightarrow (i)\\ x+4y &\le 12 \rightarrow (ii)\\ 3x+y &\le 12 \rightarrow (iii)\\ x\ge 0 \rightarrow (iv)&,\ y\ge 0 \rightarrow (v) \end{aligned}\right\} \rightarrow I$$

Associated equations of (i), (ii) and (iii) are

$2x+3y = 18 \rightarrow ①$

$x+4y = 12 \rightarrow ②$

$3x+y = 12 \rightarrow ③$



For $y=0$, ① $\Rightarrow 2x+0=18 \quad \therefore x=9$

for $x=0$, ① $\Rightarrow 0+3y=18 \quad \therefore x=6$

$\therefore$ Line ① cuts x-axis at (9, 0) and y-axis at (0, 6)

for $y=0$, ② $\Rightarrow x+0=12 \quad \therefore x=12$

for $x=0$, ② $\Rightarrow 0+4y=12 \quad \therefore y=3$

$\therefore$ Line ② cuts x-axis at (12, 0) and y-axis at (0, 3).

for $y=0$, ③ $\Rightarrow 3x+0=12 \quad \therefore x=4$

for $x=0$, ③ $\Rightarrow 0+y=12 \quad \therefore y=12$

$\therefore$ Line ③ cuts x-axis at (4, 0) and y-axis at (0, 12)

We take origin (0,0) as test point.

As (0,0) satisfies (i), (ii), (iii)

$\therefore$ The graphs of (i), (ii) and (iii) are the closed half planes on the origin side of ①, ② and ③.

The graph of $x\ge 0$ is the closed right half plane of xy-plane.

and the graph of $y\ge 0$ is the closed upper half plane of xy-plane.

The intersection of graphs of (i), (ii), (iii), (iv) and (v) is the feasible region OABC of the system of linear inequalities as shown in the figure by shading the region.

O(0,0), A(6,0), B($\frac{36}{11}$, $\frac{24}{11}$) and C(0, 3) are corners of feasible region.

B($\frac{36}{11}$, $\frac{24}{11}$) is the point of intersection of ② and ③

(iv) Given inequalities are

$$\left.\begin{array}{l} x+2y \le 14 \rightarrow (i) \\ 3x+4y \le 36 \rightarrow (ii) \\ 2x+y \le 10 \rightarrow (iii) \\ x \ge 0 \rightarrow (iv),\ y \ge 0 \rightarrow (v) \end{array}\right\} \rightarrow (I)$$

The associated equations of (i), (ii) and (iii) are

$$x+2y = 14 \rightarrow ①$$
$$3x+4y = 36 \rightarrow ②$$
$$2x+y = 10 \rightarrow ③$$

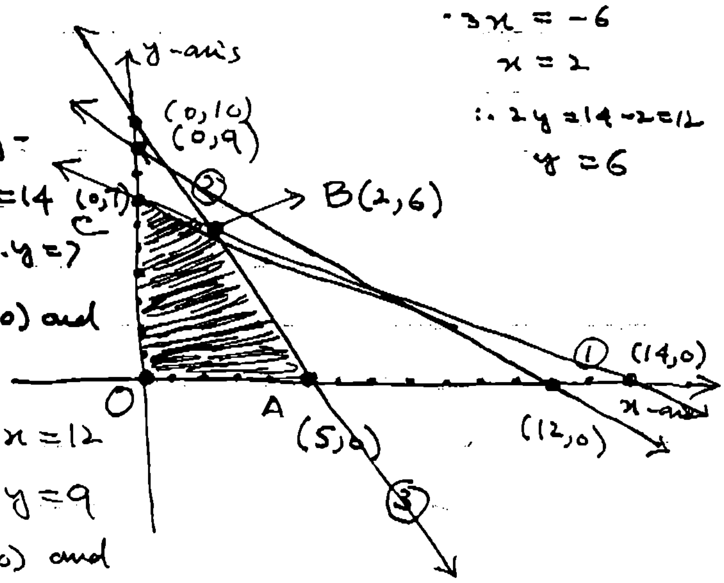
$x+2y=14$
$4x+2y=20$
$-3x=-6$
$x=2$
$\therefore 2y = 14-2 = 12$
$y = 6$

~~For y = 0, ① ⇒ 0+2y = 14 ∴ y =~~

For $y=0$, ① $\Rightarrow x+0=14 \therefore x=14$

for $x=0$, ① $\Rightarrow 0+2y=14 \therefore y=7$

$\therefore$ Line ① cuts x-axis at $(14,0)$ and y-axis at $(0,7)$



For $y=0$, ② $\Rightarrow 3x+0=36 \therefore x=12$

for $x=0$, ② $\Rightarrow 0+4y=36 \therefore y=9$

$\therefore$ line ② cuts x-axis at $(12,0)$ and y-axis at $(0,9)$

For $y=0$, ③ $\Rightarrow 2x+0=10 \therefore x=5$

for $x=0$, ③ $\Rightarrow 0+y=10 \therefore y=10$

$\therefore$ line ③ cuts x-axis at $(5,0)$ and y-axis at $(0,10)$.

We take $(0,0)$ as test point.

As $(0,0)$ satisfies (i), (ii) and (iii)

$\therefore$ The graphs of (i), (ii) and (iii) are the closed half planes on the origin side of ①, ②, ③.

Graph of $x \ge 0$ is the closed right half plane of xy-plane. and graph of $y \ge 0$ is the closed upper half plane of xy-plane.

The intersection of graphs of (i), (ii), (iii), (iv) and (v) is the feasible region OABC of system of linear inequalities as shown in the figure by shading the region.

$O(0,0)$, $A(5,0)$, $B(2,6)$, $C(0,7)$ are corners of feasible region.

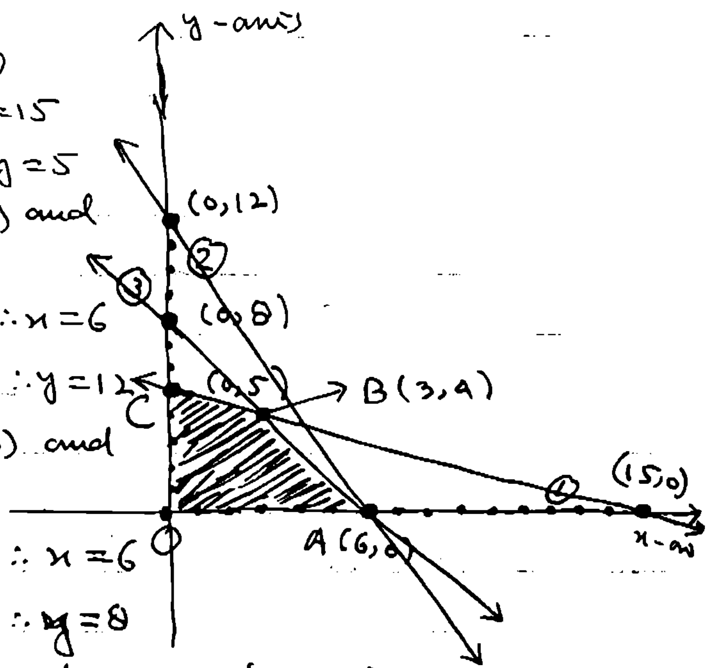
$B(2,6)$ is point of intersection of lines ① and ③.

(v) Given inequalities are

$$\left.\begin{array}{l} x+3y \le 15 \rightarrow (i) \\ 2x+y \le 12 \rightarrow (ii) \\ 4x+3y \le 24 \rightarrow (iii) \\ x \ge 0 \rightarrow (iv),\ y \ge 0 \rightarrow (v) \end{array}\right\} \rightarrow (I)$$

The associated equations of (i), (ii) and (iii) are

$$x+3y=15 \rightarrow (1)$$
$$2x+y=12 \rightarrow (2)$$
$$4x+3y=24 \rightarrow (3)$$



For $y=0$, (1) $\Rightarrow x+0=15 \therefore x=15$

for $x=0$, (1) $\Rightarrow 0+3y=15 \therefore y=5$

$\therefore$ Line (1) cuts x-axis at (15,0) and y-axis at (0,5)

For $y=0$, (2) $\Rightarrow 2x+0=12 \therefore x=6$

for $x=0$, (2) $\Rightarrow 0+y=12 \therefore y=12$

$\therefore$ Line (2) cuts x-axis at (6,0) and y-axis at (0,12)

for $y=0$, (3) $\Rightarrow 4x+0=24 \therefore x=6$

for $x=0$, (3) $\Rightarrow 0+3y=24 \therefore y=8$

$\therefore$ Line (3) cuts x-axis at (6,0) and y-axis at (0,8)

We take (0,0) as test point.

As (0,0) satisfies (i), (ii) and (iii)

$\therefore$ Graphs of (i), (ii) and (iii) are closed half plane on the origin side of (1), (2) and (3).

Graph of $x \ge 0$ is the closed right half plane of xy-plane. and graph of $y \ge$ is the closed upper half plane of xy-plane.

The intersection of graphs of (i), (ii), (iii), (iv) and (v) is the feasible region OABC of system of linear inequalities as shown in the figure by shading the region.

O(0,0), A(6,0), B(3,4) and C(0,5) are corners of the feasible region.

The point B(3,4) is the intersection of lines (1) and (3).

(vi) Given inequalities are

$$\left.\begin{aligned} 2x + y &\le 20 \rightarrow (i) \\ 8x + 15y &\le 120 \rightarrow (ii) \\ x + y &\le 11 \rightarrow (iii) \\ x \ge 0 \rightarrow (iv)&,\ y \ge 0 \rightarrow (v) \end{aligned}\right\} \rightarrow (I)$$

The associated equations of (i), (ii) and (iii) are

$2x + y = 20 \rightarrow (1)$

$8x + 15y = 120 \rightarrow (2)$

$x + y = 11 \rightarrow (3)$

$$\begin{array}{r} 8x + 15y = 120 \\ \underline{8x + 8y = 88} \\ 7y = 32 \end{array}$$

$x = 11 - \frac{32}{7}$

$= \frac{45}{7}$

For $y = 0$, (1) $\Rightarrow 2x + 0 = 20 \therefore x = 10$

for $x = 0$, (1) $\Rightarrow 0 + y = 20 \therefore y = 20$

$\therefore$ Line (1) cuts x-axis at $(10, 0)$ and y-axis at $(0, 20)$.

For $y = 0$, (2) $\Rightarrow 8x + 0 = 120 \therefore x = 15$

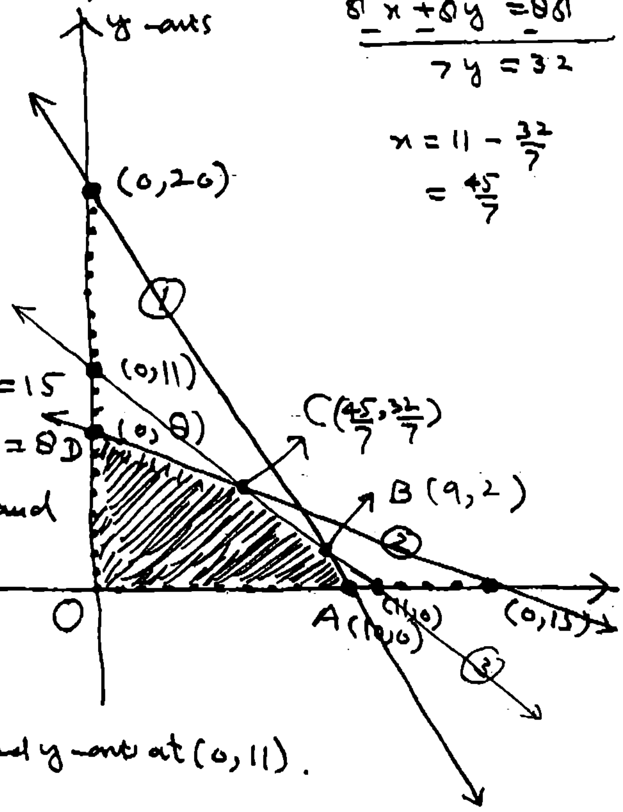
for $x = 0$, (2) $\Rightarrow 0 + 15y = 120 \therefore y = 8$

$\therefore$ Line (2) cuts x-axis at $(15, 0)$ and y-axis at $(0, 8)$.

for $y = 0$, (3) $\Rightarrow x + 0 = 11 \therefore x = 11$

for $x = 0$, (3) $\Rightarrow 0 + y = 11 \therefore y = 11$

$\therefore$ Line (3) cuts x-axis at $(11, 0)$ and y-axis at $(0, 11)$.



We take $(0,0)$ as test point.

As $(0,0)$ satisfies (i), (ii) and (iii).

$\therefore$ The graphs of (i), (ii) and (iii) are the closed half planes on the origin sides of (1), (2) and (3).

The graph of $x \ge 0$ is the closed right half plane of xy-plane and the graph of $y \ge 0$ is the closed upper half plane of xy-plane.

The intersection of graphs of (i), (ii), (iii), (iv) and (v) is the feasible region OABCD of system of linear inequalities as shown in the figure by shading the region.

$O(0,0)$, $A(10,0)$, $B(9,2)$, $C(\frac{45}{7}, \frac{32}{7})$, $D(0,8)$ are corners of feasible region OABCD.

$B(9,2)$ is point of intersection of lines (1) and (3)

and $C(\frac{45}{7}, \frac{32}{7})$ is point of intersection of lines (2) and (3).