SOLVED EXERCISE 2.8

1. The product of two positive consecutive numbers is 182. Find the numbers.

Solution:

Let the numbers be x, x + 1

Then, $(x)(x+1) = 182$

$$x^2 + x - 182 = 0$$
$$x^2 + 14x - 13x - 182 = 0$$
$$x(x+14) - 13(x+14) = 0$$
$$(x-13)(x+14) = 0$$

$x - 13 = 0$ gives $x = 13$

Numbers are: $x = 13$

$x + 1 = 13 + 1 = 14$

Now, $x + 14 = 0$ gives $x = -14$

We ignore this value.

2. The sum of the squares of three positive consecutive numbers is 77. Find them.

Solution:

Let the numbers be x, x + 1, x + 2

Applying the given condition.

$$(x)^2 + (x+1)^2 + (x+2)^2 = 77$$
$$x^2 + x^2 + 2x + 1 + x^2 + 4x + 4 = 77$$
$$3x^2 + 6x + 5 = 77$$
$$3x^2 + 6x + 5 - 77 = 0$$
$$3x^2 + 6x - 72 = 0$$
$$x^2 + 2x - 24 = 0 \quad \text{(Dividing by 3)}$$
$$x^2 + 6x - 4x - 24 = 0$$
$$x(x+6) - 4(x+6) = 0$$
$$(x-4)(x+6) = 0$$

$x - 4 = 0$ gives $x = 4$

Numbers are: $x = 4$

$x + 1 = 4 + 1 = 5$

$x + 2 = 4 + 2 = 6$

4, 5, 6

$x + 6 = 0$

$x = -6$, we ignore it.

3. The sum of five times a number and the square of the number is 204. Find the number.

Solution:

Let the number be x

According to the given condition.

$$x^2 + 5x = 204$$
$$x^2 + 5x - 204 = 0$$
$$x^2 + 17x - 12x - 204 = 0$$
$$x(x + 17) - 12(x + 17) = 0$$
$$(x + 17)(x - 12) = 0$$
$x - 12 = 0$ gives $x = 12$

or $x + 17 = 0$ gives $x = -17$

Number is 12 or −17.

4. The product of five less than three times a certain number and one less than four times the number is 7. Find the number.

Solution:

Let the number be x.

According the given condition.

$$(3x - 5)(4x - 1) = 7$$
$$12x^2 - 23x + 5 = 7$$
$$12x^2 - 23x + 5 - 7 = 0$$
$$12x^2 - 23x - 2 = 0$$
$$12x^2 - 24x + x - 2 = 0$$
$$12x(x - 2) + 1(x - 2) = 0$$
$$(12x + 1)(x - 2) = 0$$
$x - 2 = 0$ gives $x = 2$

and $12x + 1 = 0$ gives $x = -\frac{1}{12}$

Number is 2 or $-\frac{1}{12}$

5. The difference of a number and its reciprocal is $\frac{15}{4}$.

Find the number.

Solution:

Let the number be x.

Then, $x - \frac{1}{x} = \frac{15}{4}$

$$4x^2 - 4 = 15x \quad \text{(Multiplying by 4x)}$$
$$4x^2 - 15x - 4 = 0$$
$$4x^2 - 16x + x - 4 = 0$$
$$4x(x - 4) + 1(x - 4) = 0$$
$$(x - 4)(4x + 1) = 0$$
$x - 4 = 0$ gives $x = 4$

and $4x + 1 = 0$ gives $x = -\frac{1}{4}$

Number is 4 and $-\frac{1}{4}$

6. The sum of the squares of two digits of a positive integral number is 65 and the number is 9 times the sum of its digits. Find the number.

Solution:

Let xy be the number, where unit digit is y and tens digit is x.
According the given condition.

$$x^2 + y^2 = 65 \text{............(A)}$$
$$\text{Number} = y + 10x$$
$$y + 10x = 9(x + y)$$
$$y + 10x = 9x + 9y$$
$$10x - 9x = 9y - y$$
$$x = 8y \text{ ....(B)}$$

Putting $x = 8y$ in (A)

$$(8y)^2 + y^2 = 65$$
$$64y^2 + y^2 = 65$$
$$65y^2 = 65$$
$$y^2 = 1$$
$$y = \pm 1$$
$$y = 1 \quad \text{(taking +ve value)}$$

put $y = 1$ in (B)

$$x = 8(1) = 8$$

Therefore, number is 8, 1

7. The sum of the co-ordinates of a point is 9 and sum of their squares is 45. Find the co-ordinates of the point.

Solution:

Let P(x, y) be the point.
According to the given conditions.

$$x + y = 9 \text{............(A)}$$

and $x^2 + y^2 = 45$...........(B)

From A $x = 9 - y$.......(C)

Putting $x = 9 - y$ in (B)

$$(9 - y)^2 + y^2 = 45$$
$$81 - 18y + y^2 + y^2 = 45$$
$$2y^2 - 18y + 81 - 45 = 0$$
$$2y^2 - 18y + 36 = 0$$
$$y^2 - 9y + 18 = 0 \quad \text{(Dividing by 2)}$$
$$y^2 - 6y - 3y + 18 = 0$$
$$y(y - 6) - 3(y - 6) = 0$$
$$(y - 6)(y - 3) = 0$$
$$y - 6 = 0 \quad \text{gives } y = 6$$

Then from (C)

$$x = 9 - 6 = 3$$

Point is P (3, 6)

When $y - 3 = 0$ then $y = 3$

From (C)

$$x = 9 - 3 = 6$$

Point is (6, 3)

8. Find two integers whose sum is 9 and the difference of their squares is also 9.

Solution:

Let the integers be x, y.

Then, $x + y = 9$ .... (A)

and $x^2 - y^2 = 9$ ....(B)

From A

$$x = 9 - y$$

Putting $x = 9 - y$ in (B)

$$(9 - y)^2 - y^2 = 9$$
$$81 - 18y + y^2 - y^2 = 9$$
$$-18y = 9 - 81$$
$$-18y = -72$$
$$y = -\frac{72}{18}$$
$$y = 4$$

Putting $y = 4$ in (A)

$$x + 4 = 9$$
$$x = 9 - 4 = 5$$

Integers are 5, 4

9. Find two integers whose difference is 4 and whose squares differ by 72.

Solution:

Let the integers by x and.

according to the given conditions.

$x - y = 4$ ....(A)

and $x^2 + y^2 = 72$ ....(B)

$x - y = 4$ from (A)

$$x = 4 + y$$

Putting $x = 4 + y$ in (B), we get

$$(4 + y)^2 - y^2 = 72$$
$$16 + y^2 + 8y - y^2 = 72$$
$$8y = 72 - 16$$
$$8y = 56$$

$$y = \frac{56}{8}$$

$$y = 7$$

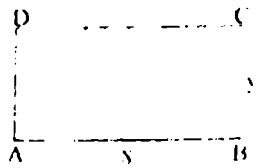
put $\quad y = 7$ in (A)

$$x - 7 = 4$$

$$x = 4 + 7 = 11$$

Integers are: 11 and 7

10. Find the dimensions of a rectangle, whose perimeter is 80cm and its area is 375cm^2.



Solution:

Let x and y be the length and width respectively of the rectangle.
According to the given conditions.

Perimeter: $\quad 2x + 2y = 80$

or $\quad x + y = 40$ ....(A)

Area: $\quad xy = 375$ ....(B)

$$x = 40 - y \text{ (from A)}$$

Putting $\quad x = 40 - y$ in (B)

$$(40 - y)y = 375$$

$$40y - y^2 = 375$$

$$\Rightarrow \quad y^2 - 40y + 375 = 0$$

$$y^2 - 25y - 15y + 375 = 0$$

$$y(y - 25) - 15(y - 25) = 0$$

$$(y - 15)(y - 25) = 0$$

$$y - 15 = 0 \quad \text{gives} \quad y = 15$$

Putting $\quad y = 15$ in (A)

$$x + 15 = 40$$

$$x = 40 - 15 = 25$$

Length = 25cm, Breadth = 15 cm.

SOLVED MISCELLANEOUS EXERCISE - 2

1. Multiple Choice Questions:
Four possible answers are given for the following questions. Tick (✓) the correct answer.

(i) If α, β are the roots of $3x^2 + 5x - 2 = 0$, then $\alpha + \beta$ is

(a) $\frac{5}{3}$ $\quad$ (b) $\frac{3}{5}$ $\quad$ (c) $\frac{-5}{3}$ $\quad$ (d) $\frac{-2}{3}$

(ii) If α, β are the roots of $7x^2 - x + 4 = 0$, then $\alpha\beta$ is

(a) $\frac{-1}{7}$ $\quad$ (b) $\frac{4}{7}$ $\quad$ (c) $\frac{7}{4}$ $\quad$ (d) $\frac{-4}{7}$

(iii) Roots of the equation $4x^2 - 5x + 2 = 0$ are