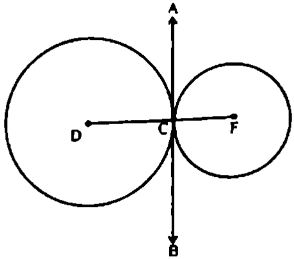
Two circles with centres D and F respectively touch each other externally at point. So that $\overline{CD}$ and $\overline{CF}$ are respectively the radii of the two circles.



To prove:

(i) Point C lies on the join of centres D and F.

(ii) $m\overline{DF} = m\overline{DC} + m\overline{CF}$

Construction:

Draw $\overline{ACB}$ as a common tangent to the pair of circles at C.

Join C with D and C with F.

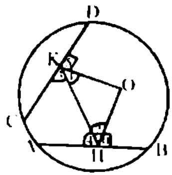
Proof:

Statements	Reasons
Both circles touch externally at C whereas $\overline{CD}$ is radial segment and ACB is the common tangent.	
$\therefore$ $m\angle ACD = 90°$ (i)	Radial segment CD $\perp$ the tangent line AB
Similarly CF is radial segment and ACS is the common tangent	
$m\angle ACF = 90°$ (ii)	Radial segment CF $\perp$ the tangent line AB
$m\angle ACD + m\angle ACF\ 90° + 90°$	Adding (i) and (ii)
$m\angle DCF = 180°$ (iii)	Sum of supplementary adjacent angles
Hence DCF is a straight line with point C between D and F	
so that $m\overline{DF} = m\overline{DC} + m\overline{CF}$	

SOLVED EXERCISE 10.2

1. AB and CD are two equal chords in a circle with centre 0. H and K are respectively the mid points of the chords. Prove that HK makes equal angles with AB and CD.

Solution:



Given:

$\overline{AB}$ and $\overline{CD}$ are equal chords of a circle with centre O.

To prove:

(i) $m\angle AHK = m\angle CKH$

(ii) $m\angle BHK = m\angle DKH$

Proof:

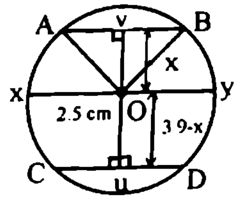
	Statements	Reasons
In	Δ HOK	
	$m\overline{OH} = m\overline{OK}$	radii of the circle
$\therefore$	$m\angle 1 = m\angle 2$ _______(i)	
And	$m\angle 5 = m\angle 6$ _______(ii)	Each of 90°
	$m\angle 1 + m\angle 5 = m\angle 2 + m\angle 6$	adding (i) and (ii)
	$m\angle DKH = m\angle BHK$	Proved
	$m\angle AHO = m\angle CKO$ __(iii)	Each of 90°
	$m\angle 2 = m\angle 1$ _______(iv)	Subtract (iv) from (iii)
	$m\angle AHO - m\angle 2 = m\angle CKO - m\angle 1$	Proved
	$m\angle AHK = m\angle CKH$	

2. The radius of a circle is 2.5 cm. 7S and CD are two chords 3.9cm apart. If mAB = 1.4 cm, then measure' the other chord.

Solution:

Given:

$m\overline{OY} = m\overline{OX} = 2.5\text{cm}$



$m\overline{UV} = 3.9\text{cm}$

$m\overline{AB} = 1.4\text{cm}$

Required:

$m\overline{CD} = ?$

In triangle OAV

$m\overline{OA}^2 = m\overline{OV}^2 + m\overline{VA}^2$

$2.5^2 = x^2 + (0.7)^2$

$\Rightarrow \quad x^2 = 2.5^2 - 0.7^2$

$= 625 - 0.49 = 5.76$

$x = 2.4$ cm

$m\overline{OU} = 3.9 - 2.4 = 1.5$ cm

In ΔOUC

$m\overline{OC}^2 = m\overline{OU}^2 + m\overline{CU}^2$

$2.5^2 = 1.5^2 + m\overline{CU}^2$

$\Rightarrow \quad m\overline{CU}^2 = 2.5^2 - 1.5^2$

$= 6.25 - 2.25 = 4$

$\overline{CU}^2 = 4 \Rightarrow \overline{CU} = \sqrt{4} = 2$

$m\overline{CD} = m\overline{CU} + m\overline{UD}$

$m\overline{CD} = 2 + 2$

$\overline{CD} = 4cm$

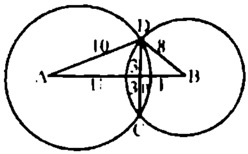
3. The radii of two intersecting circles are 10cm and 8cm. If the length of their common chord is 6cm then find the distance between the centres.

Solution:

Given:

$m\overline{AT} = 10cm$, $m\overline{BU}$ = sum, $m\overline{DC}$ = 6cm

and $m\overline{DC} = 6cm$



Required:

$m\overline{AP} = ?,\ m\overline{PB} \doteq ?$

In Δ ADP.

$m\overline{AD}^2 = m\overline{DP}^2 + m\overline{AP}^2$

$(10)^2 = (3)^2 + m AP^2$. $\quad \left[\because m\overline{AD} = m\overline{AP}\right]$

$\Rightarrow \quad mAP^2 = 100 - 9 = 91$

$\Rightarrow \quad mAP^2 = \sqrt{91}cm = 9.54cm(\text{approx})$

and In ΔDCB.

$m\overline{BD}^2 = m\overline{DP}^2 + m\overline{PB}^2$

$8^2 = 3^2 + m\overline{BP}^2$

$\Rightarrow \quad m\overline{BP}^2 = 64 - 9 = 55$

$m\overline{BP}^2 = \sqrt{55}cm = 7.42cm(\text{approx})$

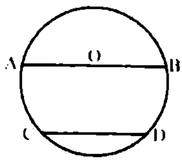
So, the distance between the centres = $m\overline{AP} + m\overline{BP}$ = 9.54 + 7.42 = 16.96 cm

4. Show that greatest chord in a circle is its diameter.

Solution:

Given:

A diameter AB and a chord CD in a circle with centre O.



To prove:

AB > CD

Or greater than any other chord.

Proof:

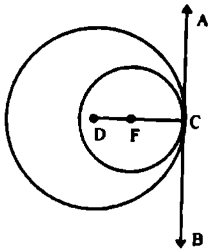
$\therefore$ AB is nearer the canter the CD.

$\therefore$ AB > CD

Hence, AB, being nearest the centre then all chords. So, AB is greater than any one of them.

THEOREM 4 (B)

10.1 (v) If two circles touch each other internally, then the point of contact lies on the straight line through their centres and distance between their centres is equal to the difference of their radii.



Given:

Two circles with centres D and F touch each other internally at point C.

So that $\overline{CD}$ and $\overline{CF}$ are the radii of two circles.

To prove:

(i) Point C lies on the join of centres; and .F extended,

(ii) $m\overline{DF} = m\overline{DC} - m\overline{CF}$

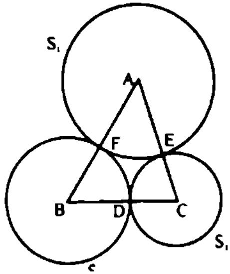
Construction:

Draw $\overline{ACB}$ as the common tangent to the pair of circles at C.

Proof:

Statements	Reasons
Both circles touch internally at C whereas $\overline{ACB}$ is the common tangent and $\overline{CD}$ is the radial segment of the first circle.	
$\therefore$ $m\angle ACD = 90°$ (i)	Radial segment $\overline{CD} \perp$ the tangent line $\overline{AB}$
Similarly ACB is the common tangent and CF is the radial segment of the second circle.	
$m\angle ACF = 90°$ (ii)	Radial segment $\overline{CF} \perp$ the tangent line AB.
$\Rightarrow$ $m\angle ACD = m\angle ACF = 90°$	Using (i) and (ii)
Where $\angle ACD$ and $\angle ACF$ coincide each other with point F between D and C.	
Hence $m\overline{DC} = m\overline{DF} + m\overline{FC}$ (iii)	
i.e., $m\overline{DC} = m\overline{FC} = m\overline{DF}$	
or $m\overline{DF} = m\overline{DC} = m\overline{FC}$	

Corollary: If two congruent circles touch each other internally the distance between their centres is eqi.al to zero.



Example 1:

Three circles touch in pairs externally. Prove that the perimeter of a triangle formed by joining centres is equal to the sum of their diameters.

Given:

Three circles have centres A, B and C respectively.

They touch in pairs externally at D, E and F. So that ΔABC is formed by joining the centres of these circles.

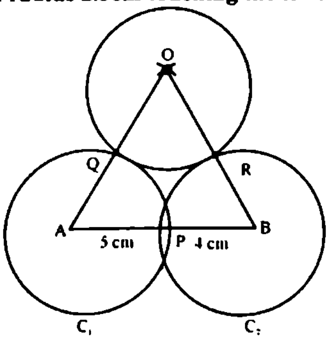
To prove:
Perimeter of ΔABC Sum of the diameters of these circles.

Proof:

Statements	Reasons
Three circles with centres A, B and C touch in pairs externally at the points, D, E and F.	Given
$m\overline{AB} = m\overline{AF} + m\overline{FB}$ (i)	
$m\overline{BC} = m\overline{BD} + m\overline{DC}$ (ii)	
and $m\overline{CA} = m\overline{CE} + m\overline{EA}$ (iii)	Adding (i), (ii) and (iii)
$m\overline{AB} + m\overline{BC} + m\overline{CA} = m\overline{AF} + m\overline{FB} + m\overline{BD}$	
$+ m\overline{DC} + m\overline{CE} + m\overline{EA}$	
$= (m\overline{AF} + m\overline{EA}) + (m\overline{FB} + m\overline{BD})$	
$+ (m\overline{CD} + m\overline{CE})$	$d_1 = 2r_1, d_2 = 2r_2$ and $d_3 = 2r_3$ are diameters of the circles.
Perimeter of $\Delta BC = 2r_1 + 2r_2 + 2r_3$	
$= d_1 + d_2 + d_3$	
= Sum of diameters of the circles.	

SOLVED EXERCISE 10.3

1. Two circles with radii 5cm and 4cm touch each other externally. Draw another circle with radius 2.5cm touching the first pair, externally.



Solution:

Construction:

1. Draw two circles C_1 and C_2 heaving radius 5cm and 4cm touch each. Other at point P.