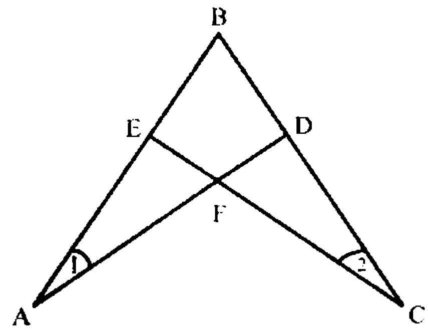
Exercise 10.1

1. In the given figure.

$\overline{AB} \cong \overline{CB}, \angle 1 \cong \angle 2.$

Prove that

$\Delta ABD \cong \Delta CBE$



Given

$\overline{AB} \cong \overline{CB}$

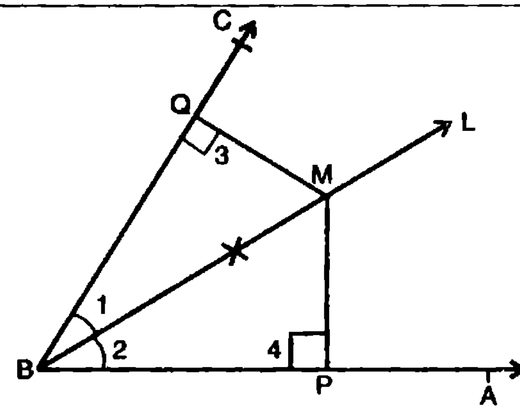
$\angle 1 = \angle 2$

To Prove

$\Delta ABD \cong \Delta CBE$

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta CBE$	
$\overline{AB} \cong \overline{CB}$	Given
$\angle 1 \cong \angle 2$	Given
$\angle ABD \cong \angle CBE$	Common angle
$\therefore$ $\Delta ABD \cong \Delta CBE$	A.S.A ≅ A.S.A

(2) From a point on the bisector of an angle, perpendiculars are drawn to the arms of the angle. Prove that these perpendiculars are equal in measure.



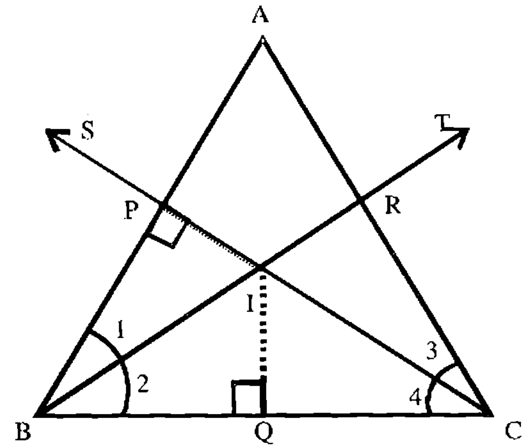
Given

$\angle ABC$, $\overrightarrow{BL}$ he bisector of $\angle ABC$, M any point on $\overrightarrow{BL}$, $\overline{MP}$ perpendicular on $\overline{AB}$, $\overline{MQ} \,||\, \overline{BC}$.

To Prove $\overline{MP} \cong \overline{MQ}$

Statements	Reasons
In $\Delta BMP \leftrightarrow \Delta BMQ$	
$\angle 1 \cong \angle 2$	$\overrightarrow{BL}$ bisects $\angle PBQ$
$\angle 3 \cong \angle 4$	Each = 90^{o}
$\overline{BM} \cong \overline{BM}$	Common
$\Delta BMP \cong \Delta BMQ$	A.S.A≅ A.S.A
$\overline{PM} \cong \overline{QM}$	Corresponding sides of the congruent triangles.

(3) In a triangle ABC, the bisectors of $\angle B$ and $\angle C$ meet in a point I. Prove that I is equidistant from the three sides of ΔABC.



Given

In ΔABC, $\overrightarrow{BT}, \overrightarrow{CS}$ are the bisectors of the angles B and C respectively.

To Prove

I is equidistant from the three sides of ΔABC i.e. $\overline{IP} \cong \overline{IQ} \cong \overline{IR}$

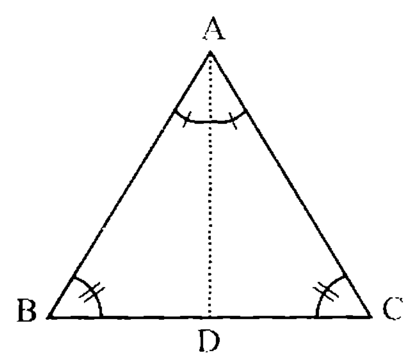
Construction

$\overline{IR} \perp \overline{AC}, \overline{IQ} \perp \overline{BC}, \overline{IP} \perp \overline{AB}$

Statements	Reasons
In $\Delta IPB \leftrightarrow \Delta IQB$	
$\angle 1 \cong \angle 2$	Given
$\angle P \cong \angle Q$	Each = 90°
$\overline{IB} \cong \overline{IB}$	Common
$\Delta IPB \cong \Delta IQB$	A.S.A $\cong$ A.S.A
$\overline{IP} \cong \overline{IQ}$ ....(i)	Corresponding sides of congruent triangles
Similarly $\Delta IRC \cong \Delta IQC$	
$\overline{IQ} \cong \overline{IR}$ ...(ii)	Corresponding sides of congruent triangles
$\overline{IP} \cong \overline{IQ} \cong \overline{IR}$	By (i) and (ii)

Theorem

If two angles of a triangle are congruent, then the sides opposite to them are also congruent.



Given

In ΔABC, $\angle B \cong \angle C$

To Prove

$\overline{AB} \cong \overline{AC}$

Construction

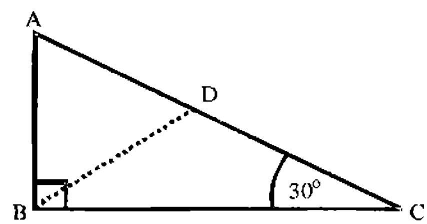
Draw the bisector of $\angle A$, meeting $\overline{BC}$ at the point D.

Proof

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta ACD$	
$\overline{AD} \cong \overline{AD}$	Common
$\angle B \cong \angle C$	Given
$\angle BAD \cong \angle CAD$	Construction
$\therefore$ $\Delta ABD \cong \Delta ACD$	S.A.A. $\cong$ S.A.A.
Hence $\overline{AB} \cong \overline{AC}$	Corresponding sides of congruent triangles

Example

If one angle of a right triangle is of 30°, the hypotenuse is twice as long as the side opposite to the angle.



Given

In ΔABC, $m\angle B = 90^\circ$ and $m\angle C = 30^\circ$

To Prove

$m\overline{AC} = 2m\overline{AB}$

Construction

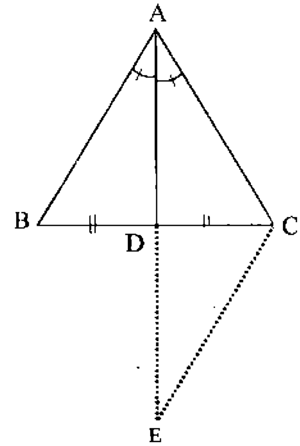
At B, construct $\angle CBD$ of 30°. Let $\overline{BD}$ cut $\overline{AC}$ at the point D.

Proof

Statements	Reasons
In ΔABD, $m\angle A = 60^\circ$	$m\angle ABC = 90^\circ$, $m\angle C = 30^\circ$
$m\angle ABD = m\angle ABC - m\angle CBD = 60^\circ$	$m\angle ABC = 90^\circ$, $m\angle CBD = 30^\circ$
$\therefore$ $m\angle ADB = 60^\circ$	Sum of measures of $\angle$s of aΔ is 180°
$\therefore$ ΔABD is equilateral	Each of its angles is equal to 60°
$\therefore$ $\overline{AB} \cong \overline{BD} \cong \overline{AD}$	Sides of equilateral Δ
InΔBCD, $\overline{BD} \cong \overline{CD}$	$\angle C = \angle CBD$ (each of 30°).
Thus	
$\left.\begin{aligned} m\overline{AC} &= m\overline{AD} + m\overline{CD} \\ &= m\overline{AB} + m\overline{AB} \\ &= 2(m\overline{AB}) \end{aligned}\right\}$	$\overline{AD} \cong \overline{AB}$ and $\overline{CD} \cong \overline{BD} \cong \overline{AB}$

Example

If the bisector of an angle of a triangle bisects the side opposite to it, the triangle is isosceles.



Given

In ΔABC, $\overline{AD}$ bisects $\angle A$ and $\overline{BD} \cong \overline{CD}$

To Prove

$\overline{AB} \cong \overline{AC}$

Construction

Produce $\overline{AD}$ to E, and take $\overline{ED} \cong \overline{AD}$.

joint C to E

Proof

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta EDC$	
$\overline{AD} \cong \overline{ED}$	Construction
$\angle ADB \cong \angle EDC$	Vertical angles
$\overline{BD} \cong \overline{CD}$	Given
$\therefore$ $\Delta ADB \cong \Delta EDC$	S.A.S. Postulate
$\therefore$ $\overline{AB} \cong \overline{EC}$ ....(1)	Corresponding sides of $\cong \Delta$s
and $\angle BAD \cong \angle E$	Corresponding angles of $\cong \Delta$s
But $\angle BAD \cong \angle CAD$	Given
$\therefore$ $\angle E \cong \angle CAD$	Each $\cong \angle BAD$
In ΔACE, $\overline{AC} \cong \overline{EC}$ ....(2)	$\angle E \cong \angle CAD$ (proved)
Hence $\overline{AB} \cong \overline{AC}$	From (1) and (2)