Exercise 14

Question # 1

Find the solution set of the following equation which lies in $[0, 2\pi]$

(i) $\sin x = -\dfrac{\sqrt{3}}{2}$ (ii) $\operatorname{cosec}\theta = 2$ (iv) $\cot\theta = \dfrac{1}{\sqrt{3}}$

Solution

(i) Since $\sin x = -\dfrac{\sqrt{3}}{2}$ $\Rightarrow x = \sin^{-1}\left(-\dfrac{\sqrt{3}}{2}\right)$

$\Rightarrow x = \dfrac{5\pi}{3}, \dfrac{4\pi}{3}$ where $x \in [0, 2\pi]$

(ii) Since $\operatorname{cosec}\theta = 2$

$\Rightarrow \dfrac{1}{\sin\theta} = 2 \Rightarrow \sin\theta = \dfrac{1}{2}$

$\Rightarrow \theta = \sin^{-1}\left(\dfrac{1}{2}\right) = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$ where $\theta \in [0, 2\pi]$

(iii) *Do yourself*

(iv) Since $\cot\theta = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \dfrac{1}{\tan\theta} = \dfrac{1}{\sqrt{3}} \Rightarrow \tan\theta = \sqrt{3}$

$\Rightarrow \theta = \tan^{-1}\left(\sqrt{3}\right)$

$\Rightarrow \theta = \dfrac{\pi}{3}, \dfrac{4\pi}{3}$ where $\theta \in [0, 2\pi]$

Question # 2

Solve the following trigonometric equations:

(i) $\tan^2\theta = \dfrac{1}{3}$ (ii) $\operatorname{cosec}^2\theta = \dfrac{4}{3}$ (iii) $\sec^2\theta = \dfrac{4}{3}$ (iv) $\cot^2\theta = \dfrac{1}{3}$

Solution

(i) Since $\tan^2\theta = \dfrac{1}{3}$ $\Rightarrow \tan\theta = \pm\dfrac{1}{\sqrt{3}}$

$\Rightarrow \tan\theta = \dfrac{1}{\sqrt{3}}$ or $\tan\theta = -\dfrac{1}{\sqrt{3}}$

$\Rightarrow \theta = \tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$ or $\theta = \tan^{-1}\left(-\dfrac{1}{\sqrt{3}}\right)$

$\Rightarrow \theta = \dfrac{\pi}{6}$ or $\theta = \dfrac{5\pi}{6}$

Since period of $\tan\theta$ is π

Therefore general value of $\theta = \frac{\pi}{6}+n\pi \, , \frac{5\pi}{6}+n\pi$

So Solution Set $=\left\{\frac{\pi}{6}+n\pi\right\}\cup\left\{\frac{5\pi}{6}+n\pi\right\}$ where $n\in\mathbb{Z}$

(ii) Since $\operatorname{cosec}^2\theta=\frac{4}{3}$

$$\Rightarrow \operatorname{cosec}\theta=\pm\frac{2}{\sqrt{3}}$$

$$\Rightarrow \operatorname{cosec}\theta=\frac{2}{\sqrt{3}} \quad \text{or} \quad \operatorname{cosec}\theta=-\frac{2}{\sqrt{3}}$$

$$\Rightarrow \sin\theta=\frac{\sqrt{3}}{2} \quad \text{or} \quad \sin\theta=-\frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta=\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) \quad \text{or} \quad \theta=\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \theta=\frac{\pi}{3}, \frac{2\pi}{3} \quad \text{or} \quad \theta=\frac{4\pi}{3}, \frac{5\pi}{3}$$

Since period of $\operatorname{cosec}\theta$ is 2π

Therefore general value of $\theta = \frac{\pi}{3}+2n\pi \, , \frac{2\pi}{3}+2n\pi \, , \frac{4\pi}{3}+2n\pi \, , \frac{5\pi}{3}+2n\pi$

Solution set $=\left\{\frac{\pi}{3}+2n\pi\right\}\cup\left\{\frac{2\pi}{3}+2n\pi\right\}\cup\left\{\frac{4\pi}{3}+2n\pi\right\}\cup\left\{\frac{5\pi}{3}+2n\pi\right\}$ where $n\in\mathbb{Z}$.

(iii) Since $\sec^2\theta=\frac{4}{3}$

$$\Rightarrow \sec\theta=\pm\frac{2}{\sqrt{3}}$$

$$\Rightarrow \sec\theta=\frac{2}{\sqrt{3}} \quad \text{or} \quad \sec\theta=-\frac{2}{\sqrt{3}}$$

$$\Rightarrow \cos\theta=\frac{\sqrt{3}}{2} \quad \text{or} \quad \cos\theta=-\frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta=\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) \quad \text{or} \quad \theta=\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \theta=\frac{\pi}{6}, \frac{11\pi}{6} \quad \text{or} \quad \theta=\frac{5\pi}{6}, \frac{7\pi}{6}$$

$\because$ period of $\sec\theta$ is 2π

$\therefore$ general values of $\theta=\frac{\pi}{6}+2n\pi \, , \frac{5\pi}{6}+2n\pi \, , \frac{7\pi}{6}+2n\pi \, , \frac{11\pi}{6}+2n\pi$

S.Set $=\left\{\frac{\pi}{6}+2n\pi\right\}\cup\left\{\frac{5\pi}{6}+2n\pi\right\}\cup\left\{\frac{7\pi}{6}+2n\pi\right\}\cup\left\{\frac{11\pi}{6}+2n\pi\right\}$ where $n\in\mathbb{Z}$.

(iv)

Question # 3

Find the value of θ satisfying the following equation:

$3\tan^2\theta+2\sqrt{3}\tan\theta+1=0$

Solution $3\tan^2\theta+2\sqrt{3}\tan\theta+1=0$

$$\Rightarrow \left(\sqrt{3}\tan\theta\right)^2+2\left(\sqrt{3}\tan\theta\right)(1)+(1)^2=0$$

$$\Rightarrow \left(\sqrt{3}\tan\theta+1\right)^2=0$$

$$\Rightarrow \left(\sqrt{3}\tan\theta+1\right)=0$$

$$\Rightarrow \sqrt{3}\tan\theta=-1$$

$$\Rightarrow \tan\theta=-\frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta=\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$$

$$\Rightarrow \theta=\frac{5\pi}{6}$$

$\because$ period of $\tan\theta$ is π

$\therefore$ general value of $\theta=\dfrac{5\pi}{6}+n\pi$, $n\in\mathbb{Z}$

Question # 4

Find the value of θ satisfying the following equation:

$\tan^2\theta-\sec\theta-1=0$

Solution $\tan^2\theta-\sec\theta-1=0$

$$\Rightarrow \left(\sec^2\theta-1\right)-\sec\theta-1=0$$

$$\Rightarrow \sec^2\theta-1-\sec\theta-1=0$$

$$\Rightarrow \sec^2\theta-\sec\theta-2=0$$

$$\Rightarrow \sec^2\theta-2\sec\theta+\sec\theta-2=0$$

$$\Rightarrow \sec\theta\left(\sec\theta-2\right)+1\left(\sec\theta-2\right)=0$$

$$\Rightarrow \left(\sec\theta+1\right)\left(\sec\theta-2\right)=0$$

$$\Rightarrow \left(\sec\theta+1\right)=0 \quad \text{or} \quad \left(\sec\theta-2\right)=0$$

$$\Rightarrow \sec\theta=-1 \quad \text{or} \quad \sec\theta=+2$$

$$\Rightarrow \cos\theta=-1 \quad \text{or} \quad \cos\theta=\frac{1}{2}$$

$$\Rightarrow \theta=\cos^{-1}(-1) \quad \text{or} \quad \theta=\cos^{-1}\left(\frac{1}{2}\right)$$

$$\Rightarrow\ \theta=\frac{3\pi}{2} \qquad \text{or} \qquad \theta=\frac{\pi}{3}\ ,\ \frac{5\pi}{3}$$

$\because$ period of $\cos\theta$ is 2π

$\therefore$ general value of θ = $\dfrac{3\pi}{2}+2n\pi,\ \dfrac{\pi}{3}+2n\pi,\ \dfrac{5\pi}{3}+2n\pi$ where $n\in\mathbb{Z}$]

Question # 5

Find the value of θ satisfying the following equation:

$2\sin\theta+\cos^2\theta-1=0$

Solution $\quad 2\sin\theta+\cos^2\theta-1=0$

$$\Rightarrow\ 2\sin\theta+1-\sin^2\theta-1=0$$

$$\Rightarrow\ -\sin^2\theta+2\sin\theta=0$$

$$\Rightarrow\ -\sin\theta(\sin\theta-2)=0$$

$$\Rightarrow\ -\sin\theta=0 \qquad \text{or} \qquad \sin\theta-2=0$$

$$\Rightarrow\ \sin\theta=0 \qquad \text{or} \qquad \sin\theta=2$$

$\Rightarrow\ \theta=\sin^{-1}(0)$ $\qquad$ Which does not hold as $\sin\theta\in[-1,1]$

$$\Rightarrow\ \theta=0\ ,\ \pi$$

$\because$ period of $\sin\theta$ is 2π

$\therefore$ general value of θ $=0+2n\pi,\ \pi+2n\pi$

$= 2n\pi,\ \pi+2n\pi$ $\qquad$ where $n\in\mathbb{Z}$

Question # 6

Find the value of θ satisfying the following equation:

$2\sin^2\theta-\sin\theta=0$

Solution $\quad 2\sin^2\theta-\sin\theta=0$

$$\Rightarrow\ \sin\theta(2\sin\theta-1)=0\ \Rightarrow\ \sin\theta=0 \quad \text{or} \quad 2\sin\theta-1=0$$

Now do yourself

Question # 7

Find the value of θ satisfying the following equation:

$3\cos^2\theta-2\sqrt{3}\sin\theta\cos\theta-3\sin^2\theta=0$

Solution $\quad 3\cos^2\theta-2\sqrt{3}\sin\theta\cos\theta-3\sin^2\theta=0$

Dividing throughout by $\cos^2\theta$

$$\frac{3\cos^2\theta}{\cos^2\theta}-\frac{2\sqrt{3}\sin\theta\cos\theta}{\cos^2\theta}-\frac{3\sin^2\theta}{\cos^2\theta}=0$$

$$\Rightarrow\ 3-2\sqrt{3}\tan\theta-3\tan^2\theta=0$$

$$\Rightarrow\ -3\tan^2\theta-2\sqrt{3}\tan\theta+3=0$$

$\Rightarrow\ 3\tan^2\theta+2\sqrt{3}\tan\theta-3=0$ $\qquad$ ×ing by -1

$$\Rightarrow\ \tan\theta=\frac{-2\sqrt{3}\pm\sqrt{\left(2\sqrt{3}\right)^2-4(3)(-3)}}{2(3)}$$

$$\Rightarrow \tan\theta = \frac{-2\sqrt{3} \pm \sqrt{12+36}}{6} = \frac{-2\sqrt{3} \pm \sqrt{48}}{6}$$

$$= \frac{-2\sqrt{3} \pm \sqrt{16\times 3}}{6} = \frac{-2\sqrt{3} \pm 4\sqrt{3}}{6}$$

$$\Rightarrow \tan\theta = \frac{-2\sqrt{3} + 4\sqrt{3}}{6} = \frac{2\sqrt{3}}{6} \quad \text{or} \quad \tan\theta = \frac{-2\sqrt{3} - 4\sqrt{3}}{6} = -\frac{6\sqrt{3}}{6}$$

$$\Rightarrow \tan\theta = \frac{\sqrt{3}}{3} = \frac{\sqrt{3}}{\left(\sqrt{3}\right)^2} = \frac{1}{\sqrt{3}} \quad \text{or} \quad \Rightarrow \tan\theta = -\sqrt{3}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \quad \text{or} \quad \theta = \tan^{-1}\left(-\sqrt{3}\right)$$

$$\Rightarrow \theta = \frac{\pi}{6} \quad \text{or} \quad \theta = \frac{11\pi}{6}$$

$\because$ period of $\tan\theta$ is π

$\therefore$ general value of $\theta = \dfrac{\pi}{6} + n\pi\,,\ \dfrac{11\pi}{6} + n\pi$ where $n \in \mathbb{Z}$.

Question # 8

Find the value of θ satisfying the following equation:

$4\sin^2\theta - 8\cos\theta + 1 = 0$

Solution $\quad 4\sin^2\theta - 8\cos\theta + 1 = 0$

$$\Rightarrow 4\left(1-\cos^2\theta\right) - 8\cos\theta + 1 = 0$$

$$\Rightarrow 4 - 4\cos^2\theta - 8\cos\theta + 1 = 0$$

$$\Rightarrow -4\cos^2\theta - 8\cos\theta + 5 = 0$$

$$\Rightarrow 4\cos^2\theta + 8\cos\theta - 5 = 0$$

$$\Rightarrow 4\cos^2\theta + 10\cos\theta - 2\cos\theta - 5 = 0$$

$$\Rightarrow 2\cos\theta\left(2\cos\theta + 5\right) - 1\left(2\cos\theta + 5\right) = 0$$

$$\Rightarrow \left(2\cos\theta + 5\right)\left(2\cos\theta - 1\right) = 0$$

$$\Rightarrow 2\cos\theta + 5 = 0 \quad \text{or} \quad 2\cos\theta - 1 = 0$$

$$\Rightarrow 2\cos\theta = -5 \quad \text{or} \quad 2\cos\theta = 1$$

$$\Rightarrow \cos\theta = \frac{-5}{2} \quad \text{or} \quad \cos\theta = \frac{1}{2}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{-5}{2}\right) \quad \text{or} \quad \theta = \cos^{-1}\left(\frac{1}{2}\right)$$

Which is not possible as $\cos\theta \in [-1,1]$ or $\theta = \dfrac{\pi}{3},\ \dfrac{5\pi}{3}$

$\because$ period of $\cos\theta$ is 2π

$\therefore$ general value of $\theta = \dfrac{\pi}{3} + 2\pi n,\ \dfrac{5\pi}{3} + 2n\pi$ where $n \in \mathbb{Z}$.

Question # 9

Find the solution set; $\sqrt{3}\tan x-\sec x-1=0$

Solution $\sqrt{3}\tan x-\sec x-1=0$ ......(i)

$$\Rightarrow \sqrt{3}\frac{\sin x}{\cos x}-\frac{1}{\cos x}-1=0$$

$$\Rightarrow \sqrt{3}\sin x-1-\cos x=0 \qquad \times\text{ing by } \cos\theta.$$

$$\Rightarrow \sqrt{3}\sin x-1=\cos x$$

On squaring both sides.

$$\left(\sqrt{3}\sin x-1\right)^2=\left(\cos x\right)^2$$

$$\Rightarrow 3\sin^2 x-2\sqrt{3}\sin x+1=\cos^2 x$$

$$\Rightarrow 3\sin^2 x-2\sqrt{3}\sin x+1=1-\sin^2 x$$

$$\Rightarrow 3\sin^2 x-2\sqrt{3}\sin x+1-1+\sin^2 x=0$$

$$\Rightarrow 4\sin^2 x-2\sqrt{3}\sin x=0$$

$$\Rightarrow 2\sin x\left(2\sin x-\sqrt{3}\right)=0$$

$$\Rightarrow 2\sin x=0 \qquad \text{or} \qquad 2\sin x=\sqrt{3}$$

$$\Rightarrow \sin x=0 \qquad \text{or} \qquad \sin x=\frac{\sqrt{3}}{2}$$

$$\Rightarrow x=\sin^{-1}(0) \qquad \text{or} \qquad x=\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow x=0\ ,\ \pi \qquad \text{or} \qquad x=\frac{\pi}{3}\ ,\ \frac{2\pi}{3}$$

Now to check extraneous roots put $x=0$ in (i)

$$\text{L.H.S}=\sqrt{3}\tan(0)-\sec(0)-1=0-1-1=-2\neq 0=\text{R.H.S}$$

Implies that $x=0$ is an extraneous root of given equation.

Now put $x=\pi$ in (i)

$$\text{L.H.S}=\sqrt{3}\tan(\pi)-\sec(\pi)-1=0-(-1)-1=0=\text{R.H.S}$$

Implies that $x=\pi$ is a root of the equation.

Now put $x=\frac{\pi}{3}$ in (i)

$$\text{L.H.S}=\sqrt{3}\tan\left(\frac{\pi}{3}\right)-\sec\left(\frac{\pi}{3}\right)-1=\sqrt{3}\left(\sqrt{3}\right)-2-1=1-2-1=0=\text{R.H.S}$$

Implies that $x=\frac{\pi}{3}$ is a root of given equation.. Since period of tan is π.

Now put $x=\frac{2\pi}{3}$ in (i)

$$\text{L.H.S}=\sqrt{3}\tan\left(\frac{2\pi}{3}\right)-\sec\left(\frac{2\pi}{3}\right)-1$$

$$=\sqrt{3}\left(-\sqrt{3}\right)-(-2)-1=-3+2-1=-2=\text{R.H.S}$$

Implies $x=\frac{2\pi}{3}$ is an extraneous root of given equation.

$\because$ period of $\sin x$ is 2π

$\therefore$ general values of $x = \pi+2n\pi\ ,\ \frac{\pi}{3}+2n\pi$

Solution Set $=\{\pi+2n\pi\}\cup\left\{\frac{\pi}{3}+2n\pi\right\}$ where $n\in\mathbb{Z}$.

Question # 10

Find the solution set; $\cos 2x=\sin 3x$

Solution $\cos 2x=\sin 3x$

$\Rightarrow \cos^2 x-\sin^2 x=3\sin x-4\sin^3 x$ $\qquad\because \cos 2x=\cos^2 x-\sin^2 x$

$\Rightarrow \cos^2 x-\sin^2 x-3\sin x+4\sin^3 x=0$ $\qquad \sin 3x=3\sin x-4\sin^3 x$

$\Rightarrow \left(1-\sin^2 x\right)-\sin^2 x-3\sin x+4\sin^3 x=0$

$\Rightarrow 1-2\sin^2 x-3\sin x+4\sin^3 x=0$

$\Rightarrow 4\sin^3 x-2\sin^2 x-3\sin x+1=0$

Take $\sin x=1$ as a root then by synthetic division

$$\begin{array}{c|cccc} 1 & 4 & -2 & -3 & 1 \\ & \downarrow & 4 & 2 & -1 \\ \hline & 4 & 2 & -1 & \underline{|0} \end{array}$$

$\Rightarrow (\sin x-1)\left(4\sin^2 x+2\sin x-1\right)=0$

$\Rightarrow \sin x-1=0$ or $4\sin^2 x+2\sin x-1=0$

$\Rightarrow \sin x=1$ or $\sin x=\frac{-2\pm\sqrt{(2)^2-4(4)(-1)}}{2(4)}$

$\Rightarrow x=\sin^{-1}(1)$ or $\sin x=\frac{-2\pm\sqrt{4+16}}{8}=\frac{-2\pm\sqrt{20}}{8}$

$\Rightarrow x=\frac{\pi}{2}$ or $\sin x=\frac{-2+\sqrt{20}}{8}$ or $\sin x=\frac{-2-\sqrt{20}}{8}$

$\sin x=0.309$ or $\sin x=-0.809$

$\Rightarrow x=\sin^{-1}(0.309)$ or $x=\sin^{-1}(-0.809)$

$\approx 18, 162$ or $\approx 234\ ,\ 306$

$\Rightarrow x=18\cdot\frac{\pi}{180}, 162\cdot\frac{\pi}{180}$ or $x=234\cdot\frac{\pi}{180}\ ,\ 306\cdot\frac{\pi}{180}$

$=\frac{\pi}{10}, \frac{9\pi}{10}$ or $=\frac{13\pi}{10}\ ,\ \frac{17\pi}{10}$

$\because$ period of $\sin x$ is 2π

$\therefore$ general value of $x = \frac{\pi}{10} + 2n\pi, \frac{9\pi}{10} + 2n\pi, \frac{13\pi}{10} + 2n\pi, \frac{17\pi}{10} + 2n\pi, \frac{\pi}{2} + 2n\pi$

S.Set $= \left\{\frac{\pi}{10} + 2n\pi\right\} \cup \left\{\frac{9\pi}{10} + 2n\pi\right\} \cup \left\{\frac{13\pi}{10} + 2n\pi\right\} \cup \left\{\frac{17\pi}{10} + 2n\pi\right\} \cup \left\{\frac{\pi}{2} + 2n\pi\right\}$

Question # 11

Find the solution set; $\sec 3\theta = \sec\theta$

Solution $\sec 3\theta = \sec\theta$

$$\Rightarrow \frac{1}{\cos 3\theta} = \frac{1}{\cos\theta}$$

$$\Rightarrow \cos 3\theta = \cos\theta$$

$$\Rightarrow 4\cos^3\theta - 3\cos\theta = \cos\theta \qquad \because \cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

$$\Rightarrow 4\cos^3\theta - 3\cos\theta - \cos\theta = 0$$

$$\Rightarrow 4\cos^3\theta - 4\cos\theta = 0$$

$$\Rightarrow 4\cos\theta\left(\cos^2\theta - 1\right) = 0$$

$$\Rightarrow 4\cos\theta = 0 \quad \text{or} \quad \cos^2\theta - 1 = 0$$

$$\Rightarrow \cos\theta = 0 \quad \text{or} \quad \cos^2\theta = 1$$

$$\Rightarrow \theta = \cos^{-1}(0) \quad \text{or} \quad \cos\theta = \pm 1$$

$$\Rightarrow \theta = \cos^{-1}(1) \;,\; \theta = \cos^{-1}(-1)$$

$$\Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2} \quad \text{or} \quad \theta = 0, \pi$$

$\because$ period of $\cos\theta$ is 2π

$$\therefore \text{ general values of } \theta = \frac{\pi}{2} + 2n\pi, \frac{3\pi}{2} + 2n\pi, 0 + 2n\pi, \pi + 2n\pi$$

$$= \frac{\pi}{2} + 2n\pi, \frac{3\pi}{2} + 2n\pi, n\pi$$

S. Set $= \left\{\frac{\pi}{2} + 2n\pi\right\} \cup \left\{\frac{3\pi}{2} + 2n\pi\right\} \cup \{n\pi\}$ where $n \in \mathbb{Z}$.

Question # 12

Find the solution set; $\tan 2\theta + \cot\theta = 0$

Solution $\tan 2\theta + \cot\theta = 0$

$$\Rightarrow \tan 2\theta = -\cot\theta$$

$$\Rightarrow \frac{\sin 2\theta}{\cos 2\theta} = -\frac{\cos\theta}{\sin\theta}$$

$$\Rightarrow \frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta} = -\frac{\cos\theta}{\sin\theta}$$

$$\Rightarrow (2\sin\cos\theta)(\sin\theta) = (-\cos\theta)\left(\cos^2\theta - \sin^2\theta\right)$$

$$\Rightarrow 2\sin^2\theta\cos\theta = -\cos^3\theta + \sin^2\theta\cos\theta$$

$$\Rightarrow 2\sin^2\theta\cos\theta + \cos^3\theta - \sin^2\theta\cos\theta = 0$$

$$\Rightarrow \sin^2\theta\cos\theta + \cos^3\theta = 0$$

$$\Rightarrow \cos\theta\left(\sin^2\theta+\cos^2\theta\right)=0 \qquad \Rightarrow \cos\theta(1)=0$$

$$\Rightarrow \theta=\cos^{-1}(0)$$

$$=\frac{\pi}{2}\ ,\ \frac{3\pi}{2}$$

$\because$ period of $\cos\theta$ is 2π

$\therefore$ general values of $\theta=\frac{\pi}{2}+2n\pi\ ,\ \frac{3\pi}{2}+2n\pi$

S. Set = $\left\{\frac{\pi}{2}+2n\pi\right\}\cup\left\{\frac{3\pi}{2}+2n\pi\right\}$ where $n\in\mathbb{Z}$.

Question # 13

Find the solution set; $\sin 2x+\sin x=0$

Solution

$$\sin 2x+\sin x=0$$

$$\Rightarrow 2\sin x\cos x+\sin x=0 \qquad \because \sin 2\theta=2\sin\theta\cos\theta$$

$$\Rightarrow \sin x\left(2\cos x+1\right)=0$$

$$\Rightarrow \sin x=0 \quad \text{or} \quad 2\cos x+1=0$$

$$\Rightarrow x=\sin^{-1}(0) \quad \text{or} \quad \cos x=-\frac{1}{2}$$

$$\Rightarrow x=\cos^{-1}\left(-\frac{1}{2}\right)$$

$$\Rightarrow x=0\ ,\ \pi \quad \text{or} \quad x=\frac{2\pi}{3}\ ,\ \frac{4\pi}{3}$$

$\because$ period of $\sin x$ and $\cos x$ is 2π

$\therefore$ general values of $x=0+2n\pi\ ,\ \pi+2n\pi\ ,\ \frac{2\pi}{3}+2n\pi\ ,\ \frac{4\pi}{3}+2n\pi$

$$=n\pi\ ,\ \frac{2\pi}{3}+2n\pi\ ,\ \frac{4\pi}{3}+2n\pi$$

So solution set $=\{\,n\pi\}\cup\left\{\frac{2\pi}{3}+2n\pi\right\}\cup\left\{\frac{4\pi}{3}+2n\pi\right\}$ where $n\in\mathbb{Z}$.

Question # 14

Find the solution set; $\sin 4x-\sin 2x=\cos 3x$

Solution

$$\sin 4x-\sin 2x=\cos 3x$$

$$\Rightarrow 2\cos\left(\frac{4x+2x}{2}\right)\sin\left(\frac{4x-2x}{2}\right)=\cos 3x$$

$$\Rightarrow 2\cos 3x\sin x-\cos 3x=0$$

$$\Rightarrow \cos 3x\left(2\sin x-1\right)=0$$

$$\Rightarrow \cos 3x=0 \quad \text{or} \quad 2\sin x-1=0$$

$$\Rightarrow 3x=\cos^{-1}(0) \quad , \quad \sin x=\frac{1}{2}$$

$$\Rightarrow 3x=\frac{\pi}{2}\ ,\ \frac{3\pi}{2} \quad , \quad x=\sin^{-1}\left(\frac{1}{2}\right)$$

$$\Rightarrow\ x=\frac{\pi}{6},\frac{\pi}{2} \qquad , \qquad x=\frac{\pi}{6},\frac{5\pi}{6}$$

Since period of $\cos 3x$ is $\frac{2\pi}{3}$ and period of $\sin x$ is 2π

$\therefore$ general values of $x=\frac{\pi}{6}+\frac{2n\pi}{3}, \frac{\pi}{2}+\frac{2n\pi}{3}, \frac{\pi}{6}+2n\pi, \frac{5\pi}{6}+2n\pi$

So solution set $=\left\{\frac{\pi}{6}+\frac{2n\pi}{3}\right\}\cup\left\{\frac{\pi}{2}+\frac{2n\pi}{3}\right\}\cup\left\{\frac{\pi}{6}+2n\pi\right\}\cup\left\{\frac{5\pi}{6}+2n\pi\right\}$ where $n\in\mathbb{Z}$.

Question # 15

Find the solution set; $\sin x+\cos 3x=\cos 5x$

Solution $\sin x+\cos 3x=\cos 5x$

$$\Rightarrow\ \sin x=\cos 5x-\cos 3x$$
$$\Rightarrow\ \sin x=-2\sin\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)$$
$$\Rightarrow\ \sin x=-2\sin 4x\sin x$$
$$\Rightarrow\ \sin x+2\sin 4x\sin x=0$$
$$\Rightarrow\ \sin x(1+2\sin 4x)=0$$
$$\Rightarrow\ \sin x=0 \quad \text{or} \quad 1+2\sin 4x=0$$
$$\Rightarrow\ x=\sin^{-1}(0) \quad \text{or} \quad \sin 4x=-\frac{1}{2} \Rightarrow 4x=\sin^{-1}\left(-\frac{1}{2}\right)$$
$$\Rightarrow\ x=0\ ,\ \pi \quad \text{or} \quad 4x=\frac{7\pi}{6},\frac{11\pi}{6} \Rightarrow x=\frac{7\pi}{24},\frac{11\pi}{24}$$

Since period of $\sin x$ is 2π and period of $\sin 4x$ is $\frac{2\pi}{4}=\frac{\pi}{2}$

$\therefore$ general values of $x=0+2n\pi\ ,\ \pi+2n\pi\ ,\ \frac{7\pi}{24}+\frac{n\pi}{2}\ ,\ \frac{11\pi}{24}+\frac{n\pi}{2}$

So solution set $=\{2n\pi\}\cup\{\pi+2n\pi\}\cup\left\{\frac{7\pi}{24}+\frac{n\pi}{2}\right\}\cup\left\{\frac{11\pi}{24}+\frac{n\pi}{2}\right\}$ where $n\in\mathbb{Z}$.

Question # 16

Find the solution set; $\sin 3x+\sin 2x+\sin x=0$

Solution $\sin 3x+\sin 2x+\sin x=0$

$$\Rightarrow\ (\sin 3x+\sin x)+\sin 2x=0$$
$$\Rightarrow\ 2\sin\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)+\sin 2x=0$$
$$\Rightarrow\ 2\sin 2x\cos x+\sin 2x=0$$
$$\Rightarrow\ \sin 2x(2\cos x+1)=0$$
$$\Rightarrow\ \sin 2x=0 \quad \text{or} \quad 2\cos x+1=0$$
$$\Rightarrow\ 2x=\sin^{-1}(0) \quad \text{or} \quad \cos x=-\frac{1}{2}$$

$$\Rightarrow\ 2x=0\ ,\ \pi \qquad \text{or} \qquad x=\cos^{-1}\left(-\frac{1}{2}\right)$$

$$\Rightarrow\ x=0\ ,\ \frac{\pi}{2} \qquad \text{or} \qquad x=\frac{2\pi}{3}\ ,\ \frac{4\pi}{3}$$

Since period of $\sin 2x$ is $\frac{2\pi}{2}=\pi$ and period of $\cos x$ is 2π

$\therefore$ general values of $x=0+n\pi\ ,\ \frac{\pi}{2}+n\pi\ ,\ \frac{2\pi}{3}+2n\pi\ ,\ \frac{4\pi}{3}+2n\pi$

S. Set $=\{n\pi\}\cup\left\{\frac{\pi}{2}+n\pi\right\}\cup\left\{\frac{2\pi}{3}+2n\pi\right\}\cup\left\{\frac{4\pi}{3}+2n\pi\right\}$ where $n\in\mathbb{Z}$.

Question # 17

Find the solution set; $\sin 7x-\sin x=\sin 3x$

Solution

$$\sin 7x-\sin x=\sin 3x$$

$$\Rightarrow\ 2\cos\left(\frac{7x+x}{2}\right)\sin\left(\frac{7x-x}{2}\right)=\sin 3x$$

$$\Rightarrow\ 2\cos 4x\sin 3x-\sin 3x=0$$

$$\Rightarrow\ \sin 3x(2\cos 4x-1)=0$$

$$\Rightarrow\ \sin 3x=0 \qquad \text{or} \qquad 2\cos 4x-1=0$$

$$\Rightarrow\ 3x=\sin^{-1}(0) \qquad \text{or} \qquad \cos 4x=\frac{1}{2}$$

$$\Rightarrow\ 3x=0\ ,\ \pi \qquad \text{or} \qquad 4x=\cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}\ ,\ \frac{5\pi}{3}$$

$$\Rightarrow\ x=0\ ,\ \frac{\pi}{3} \qquad \text{or} \qquad x=\frac{\pi}{12}\ ,\ \frac{5\pi}{12}$$

Since period of $\sin 3x$ is $\frac{2\pi}{3}$ and period of $\cos 4x$ is $\frac{2\pi}{4}=\frac{\pi}{2}$

$\therefore$ general values of $x=0+\frac{2n\pi}{3}\ ,\ \frac{\pi}{3}+\frac{2n\pi}{3}\ ,\ \frac{\pi}{12}+\frac{n\pi}{2}\ ,\ \frac{5\pi}{12}+\frac{n\pi}{2}$

So S. set $=\left\{\frac{2n\pi}{3}\right\}\cup\left\{\frac{\pi}{3}+\frac{2n\pi}{3}\right\}\cup\left\{\frac{\pi}{12}+\frac{n\pi}{2}\right\}\cup\left\{\frac{5\pi}{12}+\frac{n\pi}{2}\right\}$ where $n\in\mathbb{Z}$.

Question # 18

Find the solution set; $\sin x+\sin 3x+\sin 5x=0$

Solution

$$\sin x+\sin 3x+\sin 5x=0$$

$$\Rightarrow\ (\sin 5x+\sin x)+\sin 3x=0$$

$$\Rightarrow\ 2\sin\left(\frac{5x+x}{2}\right)\cdot\cos\left(\frac{5x-x}{2}\right)+\sin 3x=0$$

$$\Rightarrow\ 2\sin 3x\cos 2x+\sin 3x=0$$

$$\Rightarrow\ \sin 3x(2\cos 2x+1)=0$$

$$\Rightarrow\ \sin 3x=0 \qquad \text{or} \qquad 2\cos 2x+1=0$$

$$\Rightarrow\ 3x=\sin^{-1}(0) \quad \text{or} \quad 2\cos 2x=-1\ \Rightarrow\ 2x=\cos^{-1}\left(-\frac{1}{2}\right)$$

$$\Rightarrow\ 3x=0\,,\ \pi \qquad \text{or} \qquad 2x=\frac{2\pi}{3}\,,\ \frac{4\pi}{3}$$

$$\Rightarrow\ x=0\,,\ \frac{\pi}{3} \qquad \text{or} \qquad x=\frac{\pi}{3}\,,\ \frac{2\pi}{3}$$

Since period of $\sin 3x$ is $\frac{2\pi}{3}$ and period of $\cos 2x$ is $\frac{2\pi}{2}=\pi$

$\therefore$ general values of $x=0+\frac{2n\pi}{3}\,,\ \frac{\pi}{3}+\frac{2n\pi}{3}\,,\ \frac{\pi}{3}+n\pi\,,\ \frac{2\pi}{3}+n\pi$

S.Set = $\left\{\frac{2n\pi}{3}\right\}\cup\left\{\frac{\pi}{3}+\frac{2n\pi}{3}\right\}\cup\left\{\frac{\pi}{3}+n\pi\right\}\cup\left\{\frac{2\pi}{3}+n\pi\right\}$ where $n\in\mathbb{Z}$

Question # 19

Find the solution set; $\sin\theta+\sin 3\theta+\sin 5\theta+\sin 7\theta=0$

Solution $\quad \sin\theta+\sin 3\theta+\sin 5\theta+\sin 7\theta=0$

$$\Rightarrow\ (\sin 7\theta+\sin\theta)+(\sin 5\theta+\sin 3\theta)=0$$

$$\Rightarrow\ 2\sin\left(\frac{7\theta+\theta}{2}\right)\cos\left(\frac{7\theta-\theta}{2}\right)+2\sin\left(\frac{5\theta+3\theta}{2}\right)\cos\left(\frac{5\theta-3\theta}{2}\right)=0$$

$$\Rightarrow\ 2\sin 4\theta\,\cos 3\theta+2\sin 4\theta\,\cos\theta=0$$

$$\Rightarrow\ 2\sin 4\theta\,(\cos 3\theta+\cos\theta)=0$$

$$\Rightarrow\ 2\sin 4\theta\left(2\cos\left(\frac{3\theta+\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right)\right)=0$$

$$\Rightarrow\ 4\sin 4\theta\ \cos 2\theta\ \cos\theta=0$$

$$\Rightarrow\ \sin 4\theta=0 \quad \text{or} \quad \cos 2\theta=0 \quad \text{or} \quad \cos\theta=0$$

$$\Rightarrow\ 4\theta=\sin^{-1}(0)\ , \quad 2\theta=\cos^{-1}(0)\ , \quad \theta=\cos^{-1}(0)$$

$$\Rightarrow\ 4\theta=0\ ,\ \pi \quad \text{or} \quad 2\theta=\frac{\pi}{2}\,,\ \frac{3\pi}{2} \quad \text{or} \quad \theta=\frac{\pi}{2}\,,\ \frac{3\pi}{2}$$

$$\Rightarrow\ \theta=0\ ,\ \frac{\pi}{4} \quad \text{or} \quad \theta=\frac{\pi}{4}\,,\ \frac{3\pi}{4}$$

Since period of $\sin 4\theta$ is $\frac{2\pi}{4}=\frac{\pi}{2}$, $\cos 2\theta$ is $\frac{2\pi}{2}=\pi$ and $\cos\theta$ is 2π

$\therefore$ general values of $\theta=0+\frac{n\pi}{2}\,,\ \frac{\pi}{4}+\frac{n\pi}{2}\,,\ \frac{\pi}{4}+n\pi\,,\ \frac{3\pi}{4}+n\pi\,,\ \frac{\pi}{2}+2n\pi\,,\ \frac{3\pi}{2}+2n\pi$

S. Set $=\left\{\frac{n\pi}{2}\right\}\cup\left\{\frac{\pi}{4}+\frac{n\pi}{2}\right\}\cup\left\{\frac{\pi}{4}+n\pi\right\}\cup\left\{\frac{3\pi}{4}+n\pi\right\}\cup\left\{\frac{\pi}{2}+2n\pi\right\}\cup\left\{\frac{3\pi}{2}+2n\pi\right\}$.

Question # 20

Find the solution set; $\cos\theta+\cos 3\theta+\cos 5\theta+\cos 7\theta=0$

Solution $\quad \cos\theta+\cos 3\theta+\cos 5\theta+\cos 7\theta=0$

$$\Rightarrow\ (\cos 7\theta+\cos\theta)+(\cos 5\theta+\cos 3\theta)=0$$

$$\Rightarrow\ 2\cos\left(\frac{7\theta+\theta}{2}\right)\cos\left(\frac{7\theta-\theta}{2}\right)+2\cos\left(\frac{5\theta+3\theta}{2}\right)\cos\left(\frac{5\theta-3\theta}{2}\right)=0$$

$$\Rightarrow\ 2\cos 4\theta\,\cos 3\theta+2\cos 4\theta\,\cos\theta=0$$

$$\Rightarrow\ 2\cos 4\theta\,(\cos 3\theta+\cos\theta)=0$$

$$\Rightarrow\ 2\cos 4\theta\left(2\cos\left(\frac{3\theta+\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right)\right)=0$$

$$\Rightarrow\ 4\cos 4\theta\,\cos 2\theta\,\cos\theta=0$$

Now do yourself as above question.