Ex 6.5

① (i) Foci $(\pm 3, 0)$

Length of Minor Axis $= 10$

Here $c = 3$ and $2b = 10 \Rightarrow b = 5$

Now $c^2 = a^2 - b^2 \Rightarrow 9 = a^2 - 25 \Rightarrow a^2 = 34$

Thus Required Equation of the Ellipse is $\dfrac{x^2}{34} + \dfrac{y^2}{25} = 1$ —— ①

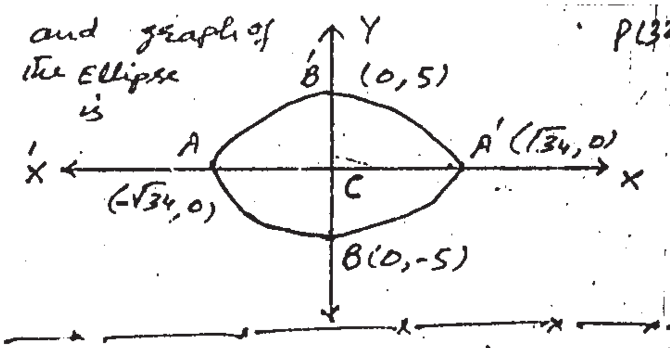
From ①

$a^2 = 34 \Rightarrow a = \pm\sqrt{34}$

$b^2 = 25 \Rightarrow b = \pm 5$

$\therefore$ vertices of the Ellipse on the X-Axis are $(\pm\sqrt{34}, 0)$ and Co-vertices are $(0, \pm 5)$

and graph of the Ellipse is



(ii) Foci $(0, -1)$ & $(0, -5)$

Length of major Axis $= 6$

Here Centre of the Ellipse

$= \left(\dfrac{0+0}{2}, \dfrac{-1-5}{2}\right) = (0, -3)$

and c is the distance from the centre to each focus. So

$c = \sqrt{(0-0)^2 + (-3+1)^2} = \sqrt{0+4} = 2 \Rightarrow \boxed{c = 2}$

Also given that $2a = 6$

$\Rightarrow \boxed{a = 3}$

Now using $c^2 = a^2 - b^2$

$\Rightarrow 4 = 9 - b^2 \Rightarrow b^2 = 9 - 4 \Rightarrow b^2 = 5$

From the foci we see that Major Axis is along the Y-Axis

Thus Required equation of the Ellipse is

$\dfrac{(y+3)^2}{9} + \dfrac{x^2}{5} = 1$ —— ①

From ① $a^2 = 9 \Rightarrow a = \pm 3$

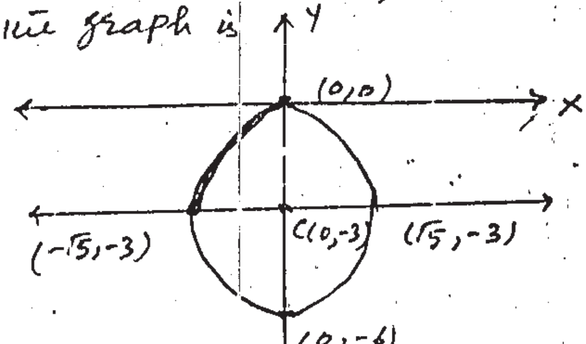
$b^2 = 5 \Rightarrow b = \pm\sqrt{5}$

$\therefore$ vertices are $(0, -3 \pm 3)$

i.e $(0, 0)$, $(0, -6)$ and

covertices are $(\sqrt{5}, -3)$, $(-\sqrt{5}, -3)$

and the graph is



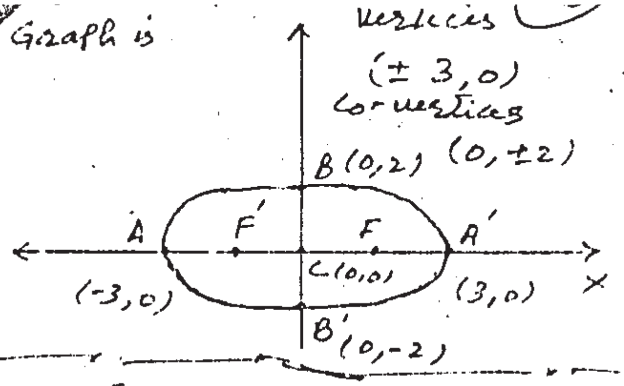
(iii) Foci $(\pm 3\sqrt{3}, 0)$ vertices $(\pm 6, 0)$

Here $c = 3\sqrt{3}$ and $\boxed{a = 6}$

Now using $c^2 = a^2 - b^2$

$\Rightarrow 27 = 36 - b^2 \Rightarrow \boxed{b^2 = 9}$

Graph is



vi) vertices $(0, \pm 5)$, $e = \frac{3}{5}$

Here $\boxed{a = 5}$, $e = \frac{3}{5}$, centre $(0,0)$

Now $\because c = ae \Rightarrow c = 5\left(\frac{3}{5}\right) = 3$

$c = 3$

Again using $c^2 = a^2 - b^2$

$\Rightarrow 9 = 25 - b^2 \Rightarrow b^2 = 25 - 9$

$\Rightarrow \boxed{b^2 = 9}$

From the vertices $(0,5), (0,-5)$ we see that Axis of the Ellipse is along Y-Axis

$\therefore$ Required Equation of the Ellipse is

$$\frac{x^2}{9} + \frac{y^2}{25} = 1$$

i.e $$\frac{y^2}{25} + \frac{x^2}{9} = 1 \quad \text{—— (1)}$$

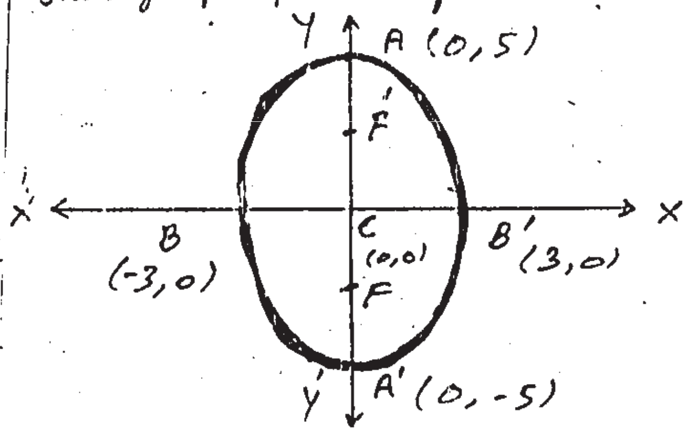
From (1) $a^2 = 25 \Rightarrow a = \pm 5$

$\therefore$ vertices are $(0,5), (0,-5)$

and $b^2 = 9 \Rightarrow b = \pm 3$

$\therefore$ Co-vertices $(3,0), (-3,0)$

Thus graph of the Ellipse (1) is



vii Centre $(0,0)$ Focus $(0,-3)$

vertex $(0,4)$

Here $c = 3$, $a = 4$

Now using $c^2 = a^2 - b^2$

$\Rightarrow b^2 = a^2 - c^2 \Rightarrow b^2 = 16 - 9 = 7$

$\boxed{b^2 = 7}$

Thus Required eq. of the Ellipse

$$\frac{y^2}{16} + \frac{x^2}{7} = 1 \quad \text{—— (1)}$$

$\because a^2 = 16 \Rightarrow a = \pm 4$

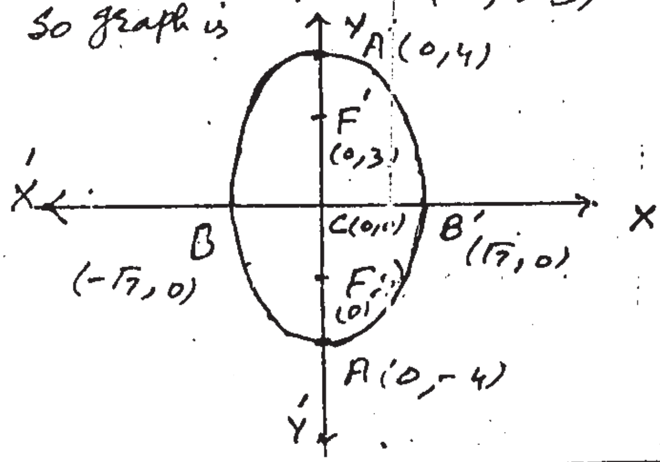
$\therefore$ vertices $(0,4), (0,-4)$

and $b^2 = 7 \Rightarrow b = \pm\sqrt{7}$.

$\therefore$ Co-vertices $(\sqrt{7}, 0), (-\sqrt{7}, 0)$

Centre $(0,0)$, Foci $(0, \pm 3)$

So graph is



viii Centre $(2,2)$

$2a = 8 \Rightarrow a = 4$ $\parallel$ to Y-Axis

and $2b = 6 \Rightarrow b = 3$ $\parallel$ to X-Axis

Required equation is

$$\frac{(y-2)^2}{16} + \frac{(x-2)^2}{9} = 1 \quad \text{—— (1)}$$

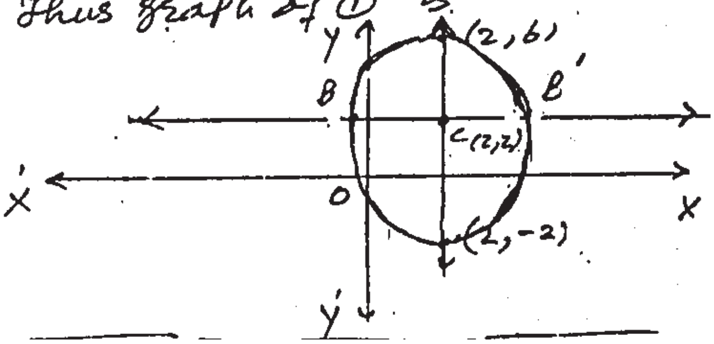
vertices are $(2, 2 \pm 4)$

i.e $(2,6), (2,-2)$

Co-vertices are $(2 \pm 3, 2)$

i.e $(5,2), (-1,2)$, Centre $(2,2)$

Thus graph of (1) is



Thus Required Equation of the Ellipse is $\frac{x^2}{36}+\frac{y^2}{9}=1$. —— ①

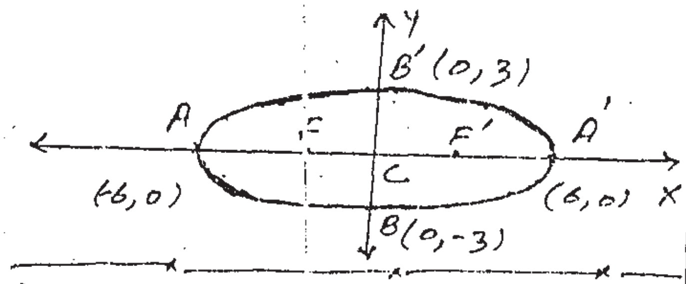
Here $a^2=36 \Rightarrow a=\pm 6$

$\therefore$ vertices $(6,0), (-6,0)$

and $b^2=9 \Rightarrow b=\pm 3$

So Co-vertices are $(0,3), (0,-3)$

and the graph of ① is



(iv) vertices $(-1,1), (5,1)$

Foci $(4,1), (0,1)$

Here Mid point of the Foci = Centre of the ellipse

$= \left(\frac{4+0}{2}, \frac{1+1}{2}\right) = (2,1)$

Distance between the vertices

$= 2a = \sqrt{(5+1)^2+(1-1)^2} = \sqrt{36} = 6$

$\Rightarrow 2a = 6 \Rightarrow \boxed{a=3}$

Distance between the foci $= 2c$

$= \sqrt{(4-0)^2+(1-1)^2} = \sqrt{16} = 4$

$\Rightarrow 2c = 4 \Rightarrow \boxed{c=2}$

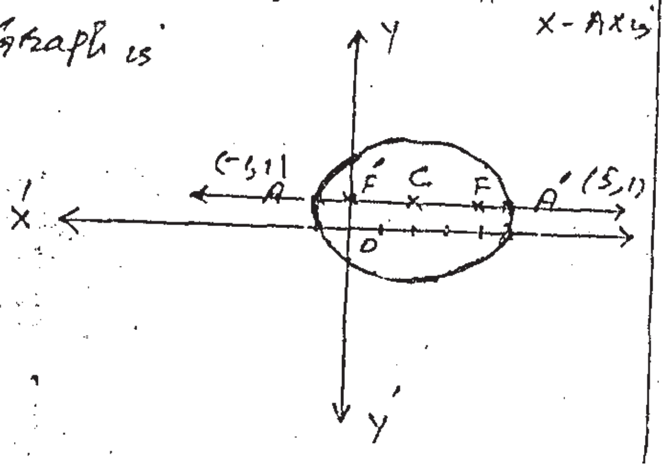
Now using $c^2=a^2-b^2 \Rightarrow b^2=a^2-c^2$

$\Rightarrow b^2 = 9-4 = 5 \Rightarrow \boxed{b^2=5}$

Thus required equation of the Ellipse

$\frac{(x-2)^2}{9}+\frac{(y-1)^2}{5}=1$ $\because$ Axis is $\parallel$ to the X-Axis

Graph is



(v) Foci $(\pm\sqrt{5}, 0)$ and Through $\left(\frac{3}{2}, \sqrt{3}\right)$.

Here Foci are $(\sqrt{5},0)$ & $(-\sqrt{5},0)$

Centre is $= \left(\frac{\sqrt{5}-\sqrt{5}}{2}, \frac{0+0}{2}\right) = (0,0)$

and $c=\sqrt{5}$.

Now using $c^2=a^2-b^2 \Rightarrow 5=a^2-b^2$

$\Rightarrow \boxed{b^2=a^2-5}$ —— (i)

From the foci we see that Major Axis of the Ellipse is along the X-Axis

So the Required equation is of the form

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$\Rightarrow \frac{x^2}{a^2}+\frac{y^2}{a^2-5}=1$ —(ii) $\because b^2=a^2-5$

If this Ellipse passes through $\left(\frac{3}{2}, \sqrt{3}\right)$

Then $\frac{9}{4a^2}+\frac{3}{a^2-5}=1$

$\Rightarrow 9(a^2-5)+12a^2 = 4a^2(a^2-5)$

$\Rightarrow 9a^2-45+12a^2 = 4a^4-20a^2$

$21a^2-45-4a^4+20a^2=0$

$-4a^4+41a^2-45=0$

$\Rightarrow 4a^4-41a^2+45=0$

which is quadratic in a^2

$\therefore a^2 = \frac{41\pm\sqrt{1681-720}}{8} = \frac{41\pm 31}{8}$

$a^2 = \frac{41+31}{8}, \frac{41-31}{8} = \frac{72}{8}, \frac{10}{8}$

$a^2 = 9, \frac{5}{4}$

For $a^2=9$

From (i) $b^2 = 9-5 \Rightarrow \boxed{b^2=4}$

For $a^2=\frac{5}{4}$

From (i) $b^2 = \frac{5}{4}-5 = \frac{5-20}{4} = -\frac{15}{4} < 0$

i.e $b^2 = -\frac{15}{4} < 0$ Neglecting (which is not possible)

$\therefore$ Required equation is

$\frac{x^2}{9}+\frac{y^2}{4}=1$

$a^2=9 \Rightarrow a=\pm 3$

$b^2=4 \Rightarrow b=\pm 2$

1, (ix) $C(0,0)$, $(2,3)$, $(6,1)$

Let Equation of the Ellipse is

$$\frac{x^2}{d_1^2}+\frac{y^2}{d_2^2}=1 \quad \text{--- (1)}$$

∵ The Ellipse passes through $(2,3)$ and $(6,1)$

$$\therefore \frac{4}{d_1^2}+\frac{9}{d_2^2}=1 \quad \text{--- (2)}$$

and

$$\frac{36}{d_1^2}+\frac{1}{d_2^2}=1 \quad \text{--- (3)}$$

By multiplying Eq (2) by 9 and then subtracting (3) from it.

$$\frac{36}{d_1^2}+\frac{81}{d_2^2}=9$$

$$\underline{\mp\frac{36}{d_1^2}\pm\frac{1}{d_2^2}=\mp 1}$$

$$\frac{80}{d_2^2}=8 \Rightarrow 8d_2^2=80$$

$$\Rightarrow \boxed{d_2^2=10}$$

putting $d_2^2=10$ in (2) we get

$$\frac{4}{d_1^2}+\frac{9}{10}=1 \Rightarrow \frac{4}{d_1^2}=1-\frac{9}{10}$$

$$\Rightarrow \frac{4}{d_1^2}=\frac{1}{10} \Rightarrow \boxed{d_1^2=40}$$

Thus Required equation of the Ellipse is

$$\frac{x^2}{40}+\frac{y^2}{10}=1.$$

Here $C(0,0)$

$a^2=40 \Rightarrow a=2\sqrt{10}$

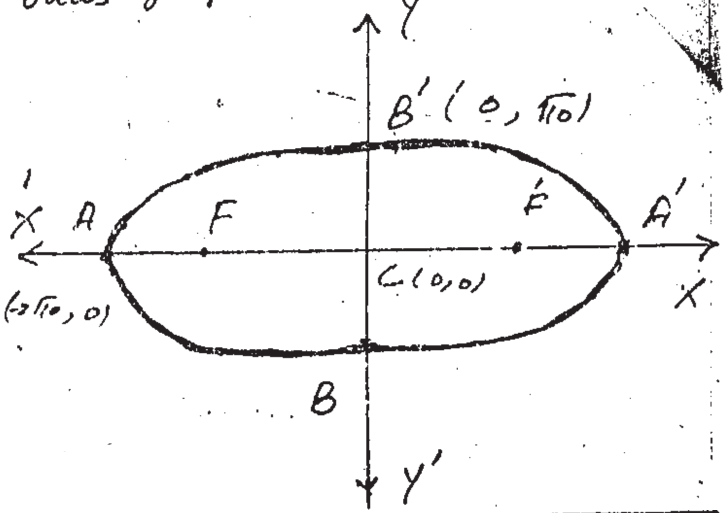
∴ vertices $(\pm 2\sqrt{10}, 0)$

and $b^2=10 \Rightarrow b=\pm\sqrt{10}$

∴ co-vertices $(0, \pm\sqrt{10})$

points on the Ellipse are $(2,3)$ & $(6,1)$

Thus graph is



(x) Centre $(0,0)$, $(3,1)$, $(4,0)$

Let the required equation is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad \text{--- (1)}$$

As the points $(3,1)$ and $(4,0)$ lie on (1)

$$\therefore \frac{9}{a^2}+\frac{1}{b^2}=1 \quad \text{--- (2)}$$

and $\frac{16}{a^2}+0=1 \Rightarrow \frac{16}{a^2}=1$

$$\Rightarrow \boxed{a^2=16}$$

putting $a^2=16$ in (2) we get

$$\frac{9}{16}+\frac{1}{b^2}=1 \Rightarrow \frac{1}{b^2}=1-\frac{9}{16}=\frac{7}{16}$$

$$7b^2=16 \Rightarrow \boxed{b^2=\frac{16}{7}}$$

∴ Required equation of the Ellipse is

$$\frac{x^2}{16}+\frac{y^2}{\frac{16}{7}}=1$$

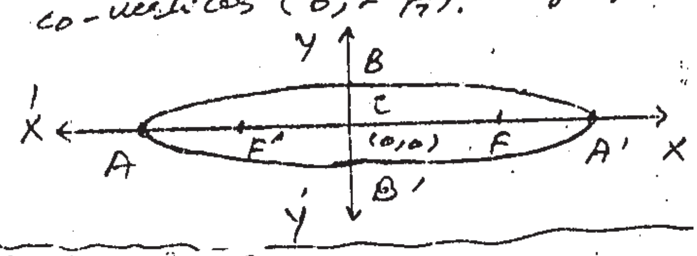
$$\Rightarrow \frac{x^2}{16}+\frac{7y^2}{16}=1 \quad \text{Ans.}$$

∵ $a^2=16 \Rightarrow a=\pm 4$

So vertices $(\pm 4, 0)$.

$b^2=16/7 \Rightarrow b=\pm\frac{4}{\sqrt{7}}$

co-vertices $(0, \pm\frac{4}{\sqrt{7}})$. graph is



(36)

(i) $x^2+4y^2=16$ case

$\Rightarrow \frac{x^2}{16}+\frac{4y^2}{16}=\frac{16}{16}$ $e=\frac{c}{a}$

$\Rightarrow \frac{x^2}{16}+\frac{y^2}{4}=1$ ——(1)

Here $a^2=16 \Rightarrow a=4$

$b^2=4 \Rightarrow b=2$

and $c^2=a^2-b^2=16-4 \Rightarrow c^2=12$

$\Rightarrow c=2\sqrt{3}$

Now Centre is $(0,0)$

Foci are $(\pm 2\sqrt{3},0)$

Eccentricity: $e=\frac{c}{a}=\frac{2\sqrt{3}}{4}=\frac{\sqrt{3}}{2}$

vertices: $(\pm 4,0)$

Directrices: $x=\pm\frac{c}{e^2}=\pm\frac{ae}{e^2}$

$\Rightarrow x=\pm\frac{a}{e}=\pm\frac{4}{\frac{\sqrt{3}}{2}}$

$\Rightarrow x=\pm\frac{8}{\sqrt{3}}$

(ii) $9x^2+y^2=18$ case

$\Rightarrow \frac{9x^2}{18}+\frac{y^2}{18}=\frac{18}{18}$

$\frac{x^2}{2}+\frac{y^2}{18}=1$ ——(1)

Here $a^2=18 \Rightarrow a=3\sqrt{2}$

$b^2=2 \Rightarrow b=\sqrt{2}$

and $c^2=a^2-b^2=18-2=16$

$c=4$, $e=\frac{c}{a}=\frac{4}{3\sqrt{2}}$

Now Centre: $(0,0)$

Foci: $(0,\pm c)=(0,\pm 4)$

Eccentricity: $e=\frac{c}{a}=\frac{4}{3\sqrt{2}}$

$\Rightarrow e=\frac{4}{3\sqrt{2}}$

vertices: $(0,\pm a)=(0,\pm 3\sqrt{2})$

Directrices $y=\pm\frac{c}{e^2}=\pm\frac{ae}{e^2}$

$\Rightarrow y=\pm\frac{a}{e} \Rightarrow y=\pm\frac{3\sqrt{2}}{\frac{4}{3\sqrt{2}}}=\pm\frac{18}{4}$

$\Rightarrow y=\pm\frac{9}{2}$.

(iii) $25x^2+9y^2=225$ case

$\Rightarrow \frac{25x^2}{225}+\frac{9y^2}{225}=\frac{225}{225}$

$\frac{x^2}{9}+\frac{y^2}{25}=1$ ——(1)

Here $a^2=25 \Rightarrow a=5$

$b^2=9 \Rightarrow b=3$

Using $c^2=a^2-b^2$

$\Rightarrow c^2=25-9=16 \Rightarrow c=4$

Now Centre of the Ellipse (1) is $(0,0)$

Foci: $(0,\pm 4)$

Eccentricity: $e=\frac{c}{a}=\frac{4}{5}$

Vertices: $(0,\pm a)=(0,\pm 5)$

Directrices: $y=\pm\frac{c}{e^2}=\pm\frac{ae}{e^2}$

$\Rightarrow y=\pm\frac{a}{e}=\pm\frac{5}{\frac{4}{5}}=\pm\frac{25}{4}$

$\Rightarrow y=\pm\frac{25}{4}$

(iv) $\frac{(2x-1)^2}{4}+\frac{(y+2)^2}{16}=1$ ——(1)

$\Rightarrow \frac{X^2}{4}+\frac{Y^2}{16}=1$ ——(1') $X=2x-1$, $Y=y+2$

$a^2=16 \Rightarrow a=4$, $b^2=4 \Rightarrow b=2$

and $c^2=a^2-b^2=16-4=12 \Rightarrow c=2\sqrt{3}$

For Centre $X=0$, $Y=0$

$\Rightarrow 2x-1=0$ $\Rightarrow y+2=0$.

$x=1/2$ $y=-2$

$\therefore$ Centre is $(\frac{1}{2},-2)$

For Foci: $X=0$ $Y=\pm c$

$\Rightarrow 2x-1=0$ $y+2=\pm 2\sqrt{3}$

$x=\frac{1}{2}$ $y=-2\pm 2\sqrt{3}$

$\therefore$ Foci: $(\frac{1}{2},-2\pm 2\sqrt{3})$

Eccentricity $e=\frac{c}{a}=\frac{2\sqrt{3}}{4}$

$\Rightarrow e=\frac{\sqrt{3}}{2}$

Directrices $Y=\pm\frac{c}{e^2}=\pm\frac{ae}{e^2}=\pm\frac{a}{e}$

$\Rightarrow y+2=\pm\frac{4}{\frac{\sqrt{3}}{2}} \Rightarrow y=-2\pm\frac{8}{\sqrt{3}}$

(v) $x^2+16x+4y^2-16y+76=0$ — ①

$\Rightarrow x^2+16x+64+4(y^2-4y)+76=64$

$\Rightarrow (x+8)^2+4(y^2-4y+4-4)=64-76$

$(x+8)^2+4(y-2)^2-16=-12$

$(x+8)^2+4(y-2)^2=-12+16$

$(x+8)^2+4(y-2)^2=4$

$\Rightarrow \frac{(x+8)^2}{4}+\frac{4(y-2)^2}{4}=\frac{4}{4}$

$\frac{(x+8)^2}{4}+\frac{(y-2)^2}{1}=1$ — ①

which is of the form

$\frac{X^2}{4}+\frac{Y^2}{1}=1$] Major Axis (‖ to x-Axis)

where $X=x+8$, $Y=y-2$

$a^2=4 \Rightarrow a=2$

$b^2=1 \Rightarrow b=1$

Now using $c^2=a^2-b^2$

$=4-1=3$

$c=\sqrt{3}$

For Centre $X=0$, $Y=0$

$\Rightarrow x+8=0$, $y-2=0$

$x=-8$, $y=2$

$\therefore$ Required Centre of the Ellipse is $(-8,2)$

For Foci

$X=\pm c$, $Y=0$

$\Rightarrow x+8=\pm\sqrt{3}$, $y-2=0$

$x=-8\pm\sqrt{3}$, $y=2$

$\therefore$ Foci $(-8\pm\sqrt{3}, 2)$.

for Eccentricity $e=\frac{c}{a}=\frac{\sqrt{3}}{2}$

For vertices

$X=\pm a$, $Y=0$

$\Rightarrow x+8=\pm 2$, $y-2=0$

$x=-8\pm 2=-6,-10$ $\boxed{y=2}$

$\therefore$ vertices are $(-6,2)$, $(-10,2)$

Directrices $X=\pm\frac{c}{e^2}=\pm\frac{\sqrt{3}}{\frac{3}{4}}$

$\Rightarrow x+8=\pm\frac{4\sqrt{3}}{3}=\pm\frac{4}{\sqrt{3}}$

$\Rightarrow x=-8\pm\frac{4}{\sqrt{3}}$

---

vi) $25x^2+4y^2-250x-16y+541=0$

$\Rightarrow 25x^2-250x+4y^2-16y=-541$

$25(x^2-10x)+4(y^2-4y)=-541$

$25[x^2-10x+25-25]+4[y^2-4y+4-4]=-541$

$25[(x-5)^2-25]+4[(y-2)^2-4]=-541$

$25(x-5)^2-625+4(y-2)^2-16=-541$

$25(x-5)^2+4(y-2)^2=-541+625+16$

$25(x-5)^2+4(y-2)^2=100$

$\Rightarrow \frac{25(x-5)^2}{100}+\frac{4(y-2)^2}{100}=\frac{100}{100}$

$\frac{(x-5)^2}{4}+\frac{(y-2)^2}{25}=1$ — ①

which is of the form

$\frac{X^2}{4}+\frac{Y^2}{25}=1$ — ①

where $X=x-5$, $Y=y-2$

$a^2=25 \Rightarrow a=5$

$b^2=4 \Rightarrow b=2$

Now using $c^2=a^2-b^2$

$\Rightarrow c^2=25-4=21 \Rightarrow c=\sqrt{21}$

Now for Centre

$X=0$ and $Y=0$

$\Rightarrow x-5=0$, $y-2=0$

$x=5$, $y=2$

$\therefore$ Centre of the Ellipse $(5,2)$.

For Foci

$X=0$, $Y=\pm c$

$\Rightarrow x-5=0$, $y-2=\pm\sqrt{21}$

$x=5$, $y=2\pm\sqrt{21}$

$\therefore$ Foci : $(5, 2 \pm \sqrt{21})$.

Eccentricity $e = \frac{c}{a} = \frac{\sqrt{21}}{5}$

For vertices

$X = 0, \quad Y = \pm a$

$\Rightarrow x - 5 = 0 \quad y - 2 = \pm 5$

$x = 5 \quad y = 2 \pm 5 = 7, -3$

$\therefore$ vertices are

$(5, -3), (5, 7)$.

Directrices $Y = \pm \frac{c}{e^2}$

$\Rightarrow y - 2 = \pm \frac{\sqrt{21}}{\frac{21}{25}} = \pm \frac{25\sqrt{21}}{21}$

$y = 2 \pm \frac{25}{\sqrt{21}}$

(3) $0 < c < a$, $F(-c, 0)$, $F'(c, 0)$

$P(x, y)$

Given That $|PF| + |PF'| = 2a$

$\Rightarrow \sqrt{(x+c)^2 + (y-0)^2} + \sqrt{(x-c)^2 + (y-0)^2} = 2a$

$\sqrt{(x+c)^2 + y^2} = 2a - \sqrt{(x-c)^2 + y^2}$ — (1)

Squaring both sides of (1)

$(x+c)^2 + y^2 = 4a^2 - 4a\sqrt{(x-c)^2 + y^2} + (x-c)^2 + y^2$

$x^2 + 2cx + c^2 + y^2 = 4a^2 - 4a\sqrt{(x-c)^2 + y^2} + x^2 + c^2 - 2cx + y^2$

$\Rightarrow 2cx + 2cx - 4a^2 = -4a\sqrt{(x-c)^2 + y^2}$

$4cx - 4a^2 = -4a\sqrt{x^2 - 2cx + c^2 + y^2}$

$4(cx - a^2) = -4a\sqrt{x^2 + y^2 + c^2 - 2cx}$

$cx - a^2 = -a\sqrt{x^2 + y^2 + c^2 - 2cx}$

$\Rightarrow a^2 - cx = a\sqrt{x^2 + y^2 + c^2 - 2cx}$ — (2)

Again Squaring both sides of (2) we get

$a^4 + c^2x^2 - 2a^2cx = a^2x^2 + a^2y^2 + a^2c^2 - 2a^2cx$

$\Rightarrow c^2x^2 - a^2x^2 - a^2y^2 = a^2c^2 - a^4$

$\Rightarrow a^2x^2 - c^2x^2 + a^2y^2 = a^4 - a^2c^2$

P(38)

$(a^2 - c^2)x^2 + a^2y^2 = a^2(a^2 - c^2)$

Dividing both sides by $a^2(a^2 - c^2)$

$$\frac{(a^2-c^2)x^2}{a^2(a^2-c^2)} + \frac{a^2y^2}{a^2(a^2-c^2)} = \frac{a^2(a^2-c^2)}{a^2(a^2-c^2)}$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{a^2 - c^2} = 1$$

i.e $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where $a^2 - c^2 = b^2$

which is an Ellipse

(4) $P(x, y)$ $(0,0)$, $(1,1)$, 2

We Name the given points

$O(0,0)$ & $A(1,1)$

Now Given That

$|OP| + |AP| = 2$

i.e $\sqrt{(x-0)^2 + (y-0)^2} + \sqrt{(x-1)^2 + (y-1)^2} = 2$

$\Rightarrow \sqrt{(x-1)^2 + (y-1)^2} = 2 - \sqrt{x^2 + y^2}$ — (1)

Squaring both sides of (1) we have

$x^2 - 2x + 1 + y^2 - 2y + 1 = 4 + x^2 + y^2 - 4\sqrt{x^2 + y^2}$

$-2x - 2y + 2 - 4 = -4\sqrt{x^2 + y^2}$

$-2x - 2y - 2 = -4\sqrt{x^2 + y^2}$

$\Rightarrow x + y + 1 = 2\sqrt{x^2 + y^2}$

or $2\sqrt{x^2 + y^2} = x + y + 1$ — (2)

Again Squaring both sides of (2)

$4(x^2 + y^2) = x^2 + y^2 + 1 + 2xy + 2x + 2y$

$4x^2 + 4y^2 - x^2 - y^2 - 2xy - 2x - 2y - 1 = 0$

$3x^2 + 3y^2 - 2xy - 2x - 2y - 1 = 0$

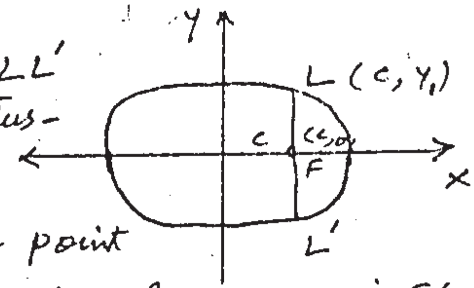
$3x^2 - 2xy + 3y^2 - 2x - 2y - 1 = 0$ Ans

(5) Latusrectum: The focal chord perpendicular to the Major Axis is called Latusrectum of the Ellipse.

Let us consider the Ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ — (1) $a > b$

Suppose LL' be the latus-rectum



Then the point L is $L(c, y_1)$ when Focus is $F(c, 0)$

$\because L(c, y_1)$ lies on ①

$\therefore \dfrac{c^2}{a^2} + \dfrac{y_1^2}{b^2} = 1 \Rightarrow y_1^2 = b^2\left(1 - \dfrac{c^2}{a^2}\right)$

$\Rightarrow y_1^2 = b^2\left(\dfrac{a^2 - c^2}{a^2}\right)$

$y_1^2 = b^2\left(\dfrac{b^2}{a^2}\right) \quad \because a^2 - c^2 = b^2$

$\Rightarrow y_1^2 = \dfrac{b^4}{a^2} \Rightarrow y_1 = \pm \dfrac{b^2}{a}$

$\therefore$ The points L and L' are $L\left(c, \dfrac{b^2}{a}\right)$ and $L'\left(c, -\dfrac{b^2}{a}\right)$

Now $|LL'| = \sqrt{(c-c)^2 + \left(\dfrac{b^2}{a} + \dfrac{b^2}{a}\right)^2}$

$= \sqrt{\left(\dfrac{b^2 + b^2}{a}\right)^2} = \dfrac{2b^2}{a}$

Thus $|LL'| = \dfrac{2b^2}{a}$ Hence proved.

(6) Given that $2a = 4\sqrt{2}$

$\Rightarrow a = 2\sqrt{2} \Rightarrow \boxed{a^2 = 8}$

Also given that $2c = 2b$

$\Rightarrow c = b$

Now using $c^2 = a^2 - b^2$

$\Rightarrow b^2 = a^2 - b^2 \Rightarrow 2b^2 = a^2$

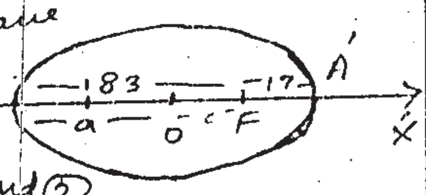
$\Rightarrow 2b^2 = 8 \Rightarrow \boxed{b^2 = 4}$

Thus required equation of the Ellipse is $\dfrac{x^2}{8} + \dfrac{y^2}{4} = 1$ Ans

(7) Let the sun be at F. Then we have

$a - c = 17$ — ①

$a + c = 183$ — ②



Adding ① and ②

$2a = 200 \Rightarrow \boxed{a = 100}$

putting $a = 100$ in ② we get

$100 + c = 183 \Rightarrow c = 83$

Now using $c^2 = a^2 - b^2$

$\Rightarrow b^2 = a^2 - c^2 = (100)^2 - (83)^2$

$= 10000 - 6889 = 3111$

$\boxed{b^2 = 3111}$

$\therefore$ The equation is

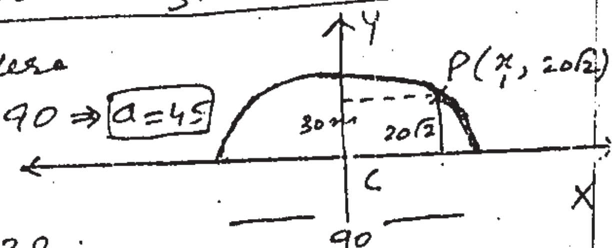
$\dfrac{x^2}{100^2} + \dfrac{y^2}{3111} = 1$ Ans.

(8) Here

$2a = 90 \Rightarrow \boxed{a = 45}$

and

$b = 30$



$\therefore$ Equation of the ellipse is

$$\frac{x^2}{(45)^2} + \frac{y^2}{(30)^2} = 1 \quad \text{—— ①}$$

At the height $20\sqrt{2}$ m let x_1 be the distance from the centre. Then the point $(x_1, 20\sqrt{2})$ lies on the Ellipse ①

$\therefore \dfrac{x_1^2}{(45)^2} + \dfrac{(20\sqrt{2})^2}{(30)^2} = 1$

$\dfrac{x_1^2}{2025} + \dfrac{800}{900} = 1 \Rightarrow \dfrac{x_1^2}{2025} = 1 - \dfrac{8}{9}$

$\Rightarrow \dfrac{x_1^2}{2025} = \dfrac{1}{9} \Rightarrow 9x_1^2 = 2025$

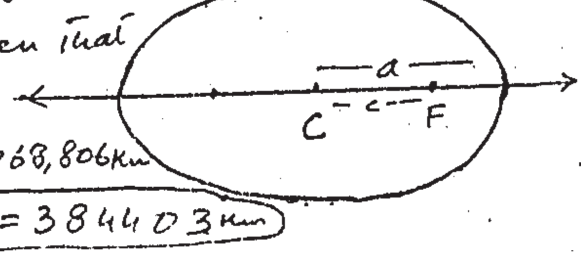
$\Rightarrow x_1^2 = 225 \Rightarrow x_1 = \pm 15$

$\Rightarrow x_1 = 15\text{m}$ (Neglecting negative value of x_1)

$\therefore$ Required distance from the Centre $= 15$m.

(9) Let the earth be at F.

Given that



$2a = 768{,}806$ Km

$\Rightarrow \boxed{a = 384403 \text{ Km}}$

$2b = 767{,}746$ Km

$b = 383873$ km

Using $c^2 = a^2 - b^2$

$\Rightarrow c^2 = (a-b)(a+b)$

$\Rightarrow c^2 = 530(768276)$

$c^2 = 407186280$

$c = 20178.86$

Now Required greatest distance

$= a + c = 404582$ Km (Approx.)

and

Least distance $= a - c$

$= 364224$ Km (Approx.)