Exercise 6.10

QNo1 i)

$\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots\ldots$ is in H.P

$3, 5, 7, \ldots\ldots$ is in A.P

$a_1 = 3, \ d = 5-3 = 2, \ n = 9$

Since $a_n = a_1 + (n-1)d$

$\Rightarrow a_9 = 3 + (9-1)(2)$

$= 3 + (8)(2) = 3 + 16$

$= 19$

so 9th term of A.P is 19

hence 9th term of H.P is $\frac{1}{19}$

ii) Do yourself

QNo 2 i)

$\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \ldots\ldots$ is in H.P

$2, 5, 8, \ldots\ldots$ is in A.P

$a_1 = 2, \ d = 5-2 = 3, \ n = 12$

Since $a_n = a_1 + (n-1)d$

$\Rightarrow a_{12} = 2 + (12-1)(3)$

$= 2 + (11)(3) = 2 + 33$

$= 35$

so 12 term of A.P is 35

hence 12 term of H.P is $\frac{1}{35}$

ii) Do yourself

QNo3 i)

Let H_1, H_2, H_3, H_4, H_5 are five H.Ms between $-2/5$ and $2/13$

then

$-2/5, H_1, H_2, H_3, H_4, H_5, \frac{2}{13}$ are in H.P

So

$-\frac{5}{2}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, \frac{1}{H_4}, \frac{1}{H_5}, \frac{13}{2}$ are in A.P

$a_1 = -5/2, \quad a_7 = \frac{13}{2}$

$\Rightarrow a_1 + 6d = \frac{13}{2}$

$\Rightarrow -\frac{5}{2} + 6d = \frac{13}{2}$

$\Rightarrow 6d = \frac{13}{2} + \frac{5}{2} = 9$

$\Rightarrow d = \frac{9}{6} = \frac{3}{2}$

Now

$\frac{1}{H_1} = a_2 = a_1 + d = -\frac{5}{2} + \frac{3}{2} = -1$

$\Rightarrow H_1 = -1$

$\frac{1}{H_2} = a_3 = a_1 + 2d = -\frac{5}{2} + 2\left(\frac{3}{2}\right)$

$= -\frac{5}{2} + 3 = \frac{1}{2}$

$\Rightarrow H_2 = 2$

$\frac{1}{H_3} = a_4 = a_1 + 3d = -\frac{5}{2} + 3\left(\frac{3}{2}\right)$

$= -\frac{5}{2} + \frac{9}{2} = 2$

$\Rightarrow H_3 = \frac{1}{2}$

$\frac{1}{H_4} = a_5 = a_1 + 4d = -\frac{5}{2} + 4\left(\frac{3}{2}\right)$

$= -\frac{5}{2} + 6 = \frac{7}{2}$

$\Rightarrow H_4 = \frac{2}{7}$

$\frac{1}{H_5} = a_6 = a_1 + 5d = -\frac{5}{2} + 5\left(\frac{3}{2}\right)$

$= -\frac{5}{2} + \frac{15}{2} = 5$

$\Rightarrow H_5 = \frac{1}{5}$

Hence $-1, 2, \frac{1}{2}, \frac{2}{7}, \frac{1}{5}$ are five H.Ms between $-2/5$ and $2/13$

ii) Do yourself as (i)

QNo4 Hint (i) (ii) & (iii)

Consider H_1, H_2, H_3, H_4 are four H.Ms between $1/3$ and $1/23$

Now Do yourself as QNo 3

QNo5

$a_7 = 1/3$ in H.P

So $a_7 = 3$ in A.P

$\Rightarrow a_1 + 6d = 3$ —— (i)

Also $a_{10} = \frac{5}{21}$ in H.P

$\Rightarrow$ So $a_{10} = \frac{21}{5}$ in A.P

$\Rightarrow a_1 + 9d = \frac{21}{5}$ —— (ii)

Subtracting (i) and (ii)

$\cancel{a_1} + 6d = 3$

$\underline{\cancel{a_1} + 9d = 21/5}$

$-3d = -\frac{6}{5} \Rightarrow 3d = \frac{6}{5}$

$\Rightarrow d = \left(\frac{6}{5}\right)\left(\frac{1}{3}\right) \Rightarrow \boxed{d = \frac{2}{5}}$

putting value of d in eq (i)

$a_1 + 6\left(\frac{2}{5}\right) = 3$

$\Rightarrow a_1 + \frac{12}{5} = 3 \Rightarrow a_1 = 3 - \frac{12}{5}$

$\Rightarrow \boxed{a_1 = \frac{3}{5}}$

Now $a_{14} = a_1 + 13d$ in A.P

$\Rightarrow a_{14} = \frac{3}{5} + 13\left(\frac{2}{5}\right)$

$= \frac{3}{5} + \frac{26}{5} = \frac{29}{5}$

So $a_{14} = \frac{5}{29}$ in H.P.

Thus 14th term of H.P is $\frac{5}{29}$

Q.No.6 $a_1 = -\frac{1}{3}$ in H.P

So $a_1 = -3$ in A.P

Also $a_5 = \frac{1}{5}$ in H.P

So $a_5 = 5$ in A.P

$\Rightarrow a_1 + 4d = 5$

put $a_1 = -3$ in above

$-3 + 4d = 5 \Rightarrow 4d = 5 + 3$

$\Rightarrow 4d = 8 \Rightarrow \boxed{d = 2}$

Now $a_9 = a_1 + 8d$ in A.P

$\Rightarrow a_9 = -3 + 8(2) = -3 + 16$

$= 13$

So $a_9 = \frac{1}{13}$ is in H.P

Thus 9th term of H.P is $\frac{1}{13}$

Q.No.7 here

H.M = 5 , a = 2 , b = b

Since $H.M = \frac{2ab}{a+b}$

$\Rightarrow 5 = \frac{2(2)b}{2+b}$

$\Rightarrow 5(2+b) = 4b$

$\Rightarrow 10 + 5b = 4b$

$\Rightarrow 5b - 4b = -10$

$\Rightarrow b = -10$ Answer

Q.No.8

Since $\frac{1}{k}, \frac{1}{2k+1}, \frac{1}{4k-1}$ are in H.P

So $k, 2k+1, 4k-1$ are in A.P.

So $d = 2k+1-k = 4k-1-2k-1$

$\Rightarrow k+1 = 2k-2$

$\Rightarrow k - 2k = -2-1$

$\Rightarrow -k = -3 \Rightarrow \boxed{k = 3}$ Ans

Q.No.9

Since H.M between a & $b = \frac{2ab}{a+b}$ —(i)

but we have given

$H.M = \frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ —(ii)

Comparing (i) and (ii)

$\frac{2ab}{a+b} = \frac{a^{n+1}+b^{n+1}}{a^n+b^n}$

$\Rightarrow 2ab(a^n+b^n) = (a^{n+1}+b^{n+1})(a+b)$

$\Rightarrow 2a^{n+1}b + 2ab^{n+1} = a^{n+2} + ab^{n+1} + a^{n+1}b + b^{n+2}$

$\Rightarrow 2a^{n+1}b + 2ab^{n+1} - ab^{n+1} - a^{n+1}b = a^{n+2} + b^{n+2}$

$\Rightarrow a^{n+1}b + ab^{n+1} = a^{n+2} + b^{n+2}$

$\Rightarrow a^{n+1}b - a^{n+2} = b^{n+2} - ab^{n+1}$

$\Rightarrow a^{n+1}(b-a) = b^{n+1}(b-a)$

$\Rightarrow a^{n+1} = b^{n+1}$

$\Rightarrow \frac{a^{n+1}}{b^{n+1}} = 1$

$\Rightarrow \left(\frac{a}{b}\right)^{n+1} = \left(\frac{a}{b}\right)^0 \quad \because \left(\frac{a}{b}\right)^0 = 1$

$\Rightarrow n+1 = 0 \Rightarrow \boxed{n = -1}$ Answer

Q.No 10 Since a^2, b^2, c^2 are in A.P

therefore $d = b^2 - a^2 = c^2 - b^2$

$$\Rightarrow (b-a)(b+a) = (c-b)(c+b) \text{ — (i)}$$

Now to prove

$a+b,\ c+a,\ b+c$ are in H.P.

we will prove

$\frac{1}{a+b}, \frac{1}{c+a}, \frac{1}{b+c}$ are in A.P

Now

$$d = \frac{1}{c+a} - \frac{1}{a+b}$$

$$= \frac{a+b-c-a}{(c+a)(a+b)}$$

$$= \frac{b-c}{(c+a)(a+b)} \text{ —— (ii)}$$

Also

$$d = \frac{1}{b+c} - \frac{1}{c+a}$$

$$= \frac{c+a-b-c}{(b+c)(c+a)}$$

$$= \frac{a-b}{(b+c)(c+a)}$$

from eq (i) $c+b = \frac{(b-a)(b+a)}{(c-b)}$

putting in above

$$d = \frac{a-b}{\frac{(b-a)(b+a)}{c-b}(c+a)}$$

$$= \frac{\cancel{(a-b)}(c-b)}{-\cancel{(a-b)}(b+a)(c+a)}$$

$$= \frac{-(c-b)}{(b+a)(c+a)}$$

$$= \frac{b-c}{(c+a)(a+b)} \text{ —— (iii)}$$

from (ii) & (iii)

$$d = d$$

i.e $\frac{1}{a+b}, \frac{1}{c+a}, \frac{1}{b+c}$ are in A.P.

$\Rightarrow a+b,\ c+a,\ b+c$ are in H.P proved

Q.No 11 Suppose the harmonic sequence

$$\frac{1}{a_1}, \frac{1}{a_1+d}, \frac{1}{a_1+2d}, \ldots\ldots$$

By given condition

$$\frac{1}{a_1} + \frac{1}{a_1+4d} = \frac{4}{7} \text{ —— (i)}$$

Also we have given

$$\frac{1}{a_1} = \frac{1}{2} \Rightarrow \boxed{a_1 = 2}$$

putting in (i)

$$\frac{1}{2} + \frac{1}{2+4d} = \frac{4}{7}$$

$$\Rightarrow \frac{1}{2+4d} = \frac{4}{7} - \frac{1}{2}$$

$$\Rightarrow \frac{1}{2+4d} = \frac{1}{14}$$

$$\Rightarrow 2+4d = 14$$

$$\Rightarrow 4d = 14-2 \Rightarrow 4d = 12$$

$$\Rightarrow d = \frac{12}{4} \Rightarrow \boxed{d = 3}$$

Now

$$\frac{1}{a_1+d} = \frac{1}{2+3} = \frac{1}{5}$$

$$\frac{1}{a_1+2d} = \frac{1}{2+2(3)} = \frac{1}{2+6} = \frac{1}{8}$$

$$\frac{1}{a_1+3d} = \frac{1}{2+3(3)} = \frac{1}{2+9} = \frac{1}{11}$$

Thus the required sequence is

$$\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \frac{1}{11}, \ldots\ldots\ldots$$

Q.No 12 Since $A = \frac{a+b}{2}$

$$G = \pm\sqrt{ab}$$

$$H = \frac{2ab}{a+b}$$

Now

$$G^2 = (\pm\sqrt{ab})^2 = ab \text{ —— (i)}$$

$$AH = \left(\frac{a+b}{2}\right)\left(\frac{2ab}{a+b}\right) = ab \text{ —— (ii)}$$

from (i) and (ii)

$$G^2 = AH \quad \text{proved}$$

~~Q.No 13 (i)~~

~~$a = 2i \;;\; b = 4i$~~

Q No 13 ii) $a = 2i$, $b = 4i$

$A = \frac{a+b}{2} = \frac{2i+4i}{2} = \frac{6i}{2} = 3i$

$G = \pm\sqrt{ab} = \pm\sqrt{(2i)(4i)}$

$= \pm\sqrt{8i^2} = \pm 2\sqrt{2}\, i$

$H = \frac{2ab}{a+b} = \frac{2(2i)(4i)}{2i+4i}$

$= \frac{16i^2}{6i} = \frac{8}{3} i$

Now

$G^2 = (\pm 2\sqrt{2}\, i)^2 = 4(2)(-1) = -8$ —— (i)

$AH = (3i)(\frac{8}{3} i) = 8i^2 = -8$ —— (ii)

From (i) and (ii)

$G^2 = AH$.

Q No 14 i) $a = 2$, $b = 8$

$A = \frac{a+b}{2} = \frac{2+8}{2} = \frac{10}{2} = 5$

$G = \sqrt{ab}$ $\because G > 0$

$= \sqrt{(2)(8)} = \sqrt{16} = 4$

$H = \frac{2ab}{a+b} = \frac{2(2)(8)}{2+8} = \frac{32}{10} = \frac{16}{5}$

$= 3\frac{1}{5}$

Since $5 > 4 > 3\frac{1}{5}$

$\Rightarrow A > G > H$ proved

Q No 15 ii) $a = -\frac{2}{5}$, $b = -\frac{8}{5}$

$A = \frac{a+b}{2} = \frac{-2/5 - 8/5}{2} = \frac{-10/5}{2}$

$= \frac{-10}{10} = -1$

$G = -\sqrt{ab}$ $\because G < 0$

$= -\sqrt{(-\frac{2}{5})(-\frac{8}{5})} = -\sqrt{\frac{16}{25}} = -\frac{4}{5}$

$H = \frac{2ab}{a+b} = \frac{2(-\frac{2}{5})(-\frac{8}{5})}{-\frac{2}{5}-\frac{8}{5}} = \frac{32/25}{-10/5}$

$= \frac{32/25}{-2} = -\frac{32}{50} = -\frac{16}{25}$

Since $-1 < -4/5 < -16/25$

$\Rightarrow A < G < H$ proved

Q No 16 Let a & b be two number

Since $H \cdot M = 4$

$\Rightarrow \frac{2ab}{a+b} = 4$

$\Rightarrow 2ab = 4(a+b)$

$\Rightarrow ab = 2(a+b)$ —— (i)

Also $A \cdot M = \frac{9}{2}$

$\Rightarrow \frac{a+b}{2} = \frac{9}{2}$

$\Rightarrow a+b = 9$ —— (ii)

putting value of $a+b$ in (i)

$ab = 2(9) \Rightarrow ab = 18$

$\Rightarrow a = \frac{18}{b}$ —— (iii)

putting in (ii)

$\frac{18}{b} + b = 9$

$\Rightarrow \frac{18+b^2}{b} = 9$

$\Rightarrow 18 + b^2 = 9b$

$\Rightarrow b^2 - 9b + 18 = 0$

$\Rightarrow b^2 - 6b - 3b + 18 = 0$

$\Rightarrow b(b-6) - 3(b-6) = 0$

$\Rightarrow (b-6)(b-3) = 0$

$b - 6 = 0$ or $b - 3 = 0$

$b = 6$, $b = 3$

putting in (iii)

$a = \frac{18}{6} = 3$ | $a = \frac{18}{3} = 6$

Thus 3, 6 OR 6, 3 are required numbers

Q No. 17

Let a & b be two number.

Since $G.M = 4$

$\Rightarrow \sqrt{ab} = 4$

$\Rightarrow ab = 16$ —— (i) on squaring

Also

$H \cdot M = \frac{16}{5}$

$\Rightarrow \frac{2ab}{a+b} = \frac{16}{5}$

Do your self

The End