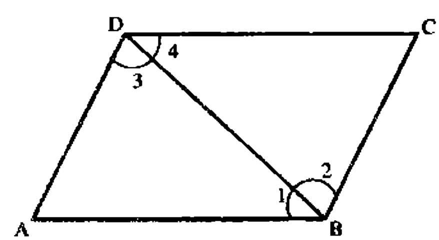
## Exercise 10.3

Q1. In the figure, $\overline{AB} \cong \overline{DC}$ , $\overline{AD} \cong \overline{BC}$.
Prove that $\angle A \cong \angle C$, $\angle ABC \cong \angle ADC$.




**Given** $\overline{AB} \cong \overline{DC}$

$\overline{AD} \cong \overline{BC}$

**To prove** $\angle A \cong \angle C$

$\angle ABC \cong \angle ADC$

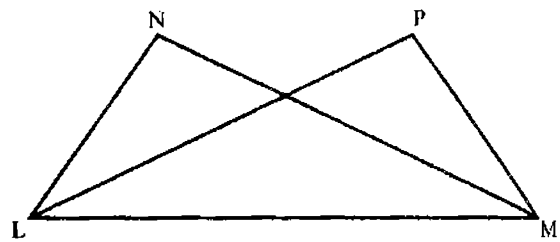
**Proof**

| Statements | Reasons |
|---|---|
| In $\Delta ABD \leftrightarrow \Delta CBD$ | |
| $\overline{AB} \cong \overline{DC}$ | Given |
| $\overline{AD} \cong \overline{BC}$ | Given |

| | |
|---|---|
| $\overline{BD} \cong \overline{BD}$ | Common |
| $\therefore$ $\Delta ABD \cong \Delta CBD$ | S.S.S ≅ S.S.S |
| $\angle A \cong \angle C$ | |
| $\angle 1 \cong \angle 4$ ....(i) | Corresponding angles of congruent triangles |
| $\angle 2 \cong \angle 3$ ....(ii) | Corresponding angles of congruent triangles |
| $\angle 1 + \angle 2 = \angle 3 + \angle 4$ | Adding (i) and (ii) |
| $\angle ABC \cong \angle ADC$ | |

**2.** In the figure, $\overline{LN} \cong \overline{MP}$, $\overline{MN} \cong \overline{LP}$.

Prove that $\angle N \cong \angle P, \angle NML \cong \angle PLM$.




**Given**

$\overline{LN} \cong \overline{MP}$

$\overline{LP} \cong \overline{MN}$

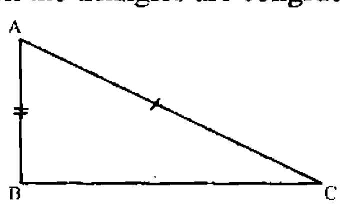
**To prove**

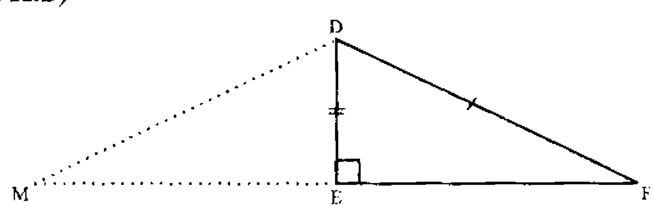
$\angle N \cong \angle P$, $\angle NML \cong \angle PLM$

| Statements | Reasons |
|---|---|
| In $\Delta LMN \leftrightarrow \Delta LMP$ | |
| $LM \cong \overline{MP}$ | Given |
| $\overline{LP} \cong \overline{MN}$ | Given |
| $\overline{LM} \cong LM$ | Common |
| $\Delta LMN \cong \Delta LPM$ | S.S.S ≅ S.S.S |
| $\angle N = \angle P$ | Corresponding angles of congruent triangles |
| $\angle NML \cong \angle PLM$ | Corresponding angles of congruent triangles |

**Theorem**

If in the correspondence of the two right-angled triangles, the hypotenuse and one side of one triangle are congruent to the hypotenuse and the corresponding side of the other, then the triangles are congruent. (H.S ≅ H.S)




**Given**

In $\Delta ABC \leftrightarrow \Delta DEF$

$\angle B \cong \angle E$ (right angles)

$\overline{CA} \cong \overline{FD}$, $\overline{AB} \cong \overline{DE}$

**To Prove** $\Delta ABC \cong \Delta DEF$

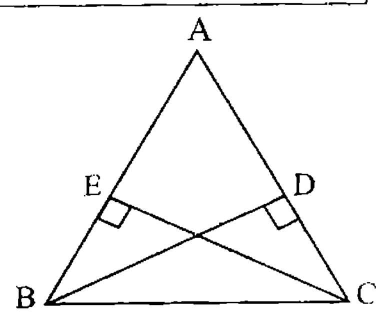
**Construction** Produce $\overline{FE}$ to a point M such that $\overline{EM} \cong \overline{BC}$ and join the points D and M.

**Proof**

| Statements | Reasons |
|---|---|
| In $m\angle DEF + m\angle DEM = 180^\circ \ldots$(i) | (Supplementary angles) |
| Now $m\angle DEF = 90^\circ \ldots\ldots\ldots$(ii) | (Given) |
| $\therefore$ $m\angle DEM = 90^\circ$ | {from (i) and (ii)} |
| In $\Delta ABC \leftrightarrow \Delta DEM$ | |
| $\overline{BC} \cong \overline{EM}$ | (construction) |
| $\angle ABC \cong \angle DEM$ | (each $\angle$ equal to $90^\circ$) |
| $\overline{AB} \cong \overline{DE}$ | (given) |
| $\therefore$ $\Delta ABC \cong \Delta DEM$ | S.A.S. postulate |
| And $\angle C \cong \angle M$ | (Corresponding angles of congruent triangles) |
| $\overline{CA} \cong \overline{MD}$ | (Corresponding sides of congruent triangles) |
| But $\overline{CA} \cong \overline{FD}$ | (given) |
| $\therefore$ $\overline{MD} \cong \overline{FD}$ | Each is congruent to $\overline{CA}$ |
| In $\Delta DMF$ | |
| $\angle F \cong \angle M$ | $\overline{FD} \cong \overline{MD}$ (Proved) |
| But $\angle C \cong \angle M$ | (proved) |
| $\angle C \cong \angle F$ | (each is congruent to $\angle M$) |
| In $\Delta ABC \leftrightarrow \Delta DEF$ | |
| $\overline{AB} \cong \overline{DE}$ | (given) |
| $\angle ABC \cong \angle DEF$ | (given) |
| $\angle C \cong \angle F$ | (proved) |
| Hence $\Delta ABC \cong \Delta DEF$ | (S.A.A $\cong$ S.A.A) |

**Example**

If perpendiculars from two vertices of a triangle to the opposite sides are congruent, then the triangle is isosceles.




**Given**

In $\Delta ABC$, $\overline{BD} \perp \overline{AC}$, $\overline{CE} \perp \overline{AB}$

Such that $\overline{BD} \cong \overline{CE}$

**To Prove** $\overline{AB} \cong \overline{AC}$

**Proof**

| Statements | Reasons |
|---|---|
| In $\Delta BCD \leftrightarrow \Delta CBE$ | |
| $\angle BDC \cong \angle BEC$ | $\overline{BD} \perp \overline{AC}, \overline{CE} \perp \overline{AB}$ (given) |
| | $\Rightarrow$ each angle = $90^\circ$ |
| $\overline{BC} \cong \overline{BC}$ | Common hypotenuse |
| $\overline{BD} \cong \overline{CE}$ | Given |
| $\Delta BCD \cong \Delta CBE$ | H.S. $\cong$ H.S. |
| $\angle BCD \cong \angle CBE$ | Corresponding angles of $\cong \Delta$s. |
| Thus $\angle BCA \cong \angle CBA$ | |
| Hence $\overline{AB} \cong \overline{AC}$ | In $\Delta ABC$, $\angle BCA \cong \angle CBA$ |