To prove

ii) $\frac{1}{a} \cdot \frac{1}{b} = \frac{1}{ab}$

Consider

$(ab)\left(\frac{1}{a} \cdot \frac{1}{b}\right) = (ba)\left(\frac{1}{a} \cdot \frac{1}{b}\right)$ [Closure law]

$= b \cdot \left(a \cdot \frac{1}{a}\right) \cdot \frac{1}{b}$ (Assoc. law)

$= b \cdot 1 \cdot \frac{1}{b} = b \cdot \frac{1}{b} = 1$

$\Rightarrow ab$ and $\frac{1}{a} \cdot \frac{1}{b}$ are multiplicative inverse of each other.

$\therefore \frac{1}{a} \cdot \frac{1}{b} = \frac{1}{ab}$

iii) To prove $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$

L.H.S. $= \frac{a}{b} \cdot \frac{c}{d}$

$= \left(a \cdot \frac{1}{b}\right) \cdot \left(c \cdot \frac{1}{d}\right)$

$= a \cdot \left(\frac{1}{b} \cdot c\right) \cdot \frac{1}{d}$ (Assoc. law)

$= a \cdot \left(c \cdot \frac{1}{b}\right) \cdot \frac{1}{d}$ (Commutative law)

$= (ac)\left(\frac{1}{b} \cdot \frac{1}{d}\right)$ (Assoc. law)

$= ac \cdot \frac{1}{bd}$

$= \frac{ac}{bd} =$ R.H.S.

iv) To prove $\frac{a}{b} = \frac{ka}{kb}$ $(k \neq 0)$

L.H.S. $= \frac{a}{b}$

$= \frac{a}{b} \cdot 1$ (multiplicative identity)

$= \frac{a}{b} \cdot \left(k \cdot \frac{1}{k}\right)$ (multiplicative inverse)

$= \frac{a}{b} \cdot \frac{k}{k}$

$= \frac{ak}{bk} =$ R.H.S.

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v) To prove

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc}$$

L.H.S. $= \dfrac{\frac{a}{b}}{\frac{c}{d}}$

$= \dfrac{\frac{a}{b} \times 1}{1 \times \frac{c}{d}}$ [Multiplicative Identity]

$= \dfrac{\frac{a}{b} \times \left(d \times \frac{1}{d}\right)}{\left(b \times \frac{1}{b}\right) \times \frac{c}{d}}$ [Multiplicative Inverse]

$= \dfrac{\frac{a}{b} \times \frac{d}{d}}{\frac{b}{b} \times \frac{c}{d}} = \dfrac{\frac{ad}{bd}}{\frac{bc}{bd}}$

$= \dfrac{ad \times \frac{1}{bd}}{bc \times \frac{1}{bd}}$

$= \frac{ad}{bc}$ [Cancellation Law]

$=$ R.H.S.

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* EXERCISE 1.1 *

1) Which of the following sets have closure property w.r.t. '+' and '×'

i) $\{0\}$

Addition Table

+	0
0	0

$\because 0 + 0 = 0 \in \{0\} \Rightarrow \{0\}$ has closure property w.r.t. '+'

Multiplication Table

×	0
0	0

$\because 0 \times 0 = 0 \in \{0\} \Rightarrow \{0\}$ has closure property w.r.t. '×'

ii) $\{1\}$

Addition Table

+	1
1	2

$\because 1 + 1 = 2 \notin \{1\} \Rightarrow \{1\}$ does not have closure property w.r.t. '+'

Multiplication Table

×	1
1	1

$\because 1\times 1 = 1 \in \{1\} \Rightarrow \{1\}$ has closure property w.r.t. '×'

iii) $\{0, -1\}$

Addition Table

+	0	−1
0	0	−1
−1	−1	−2

$\because 0+0=0 \in \{0,-1\}$

$0+(-1)=-1 \in \{0,-1\}$

$(-1)+0=-1 \in \{0,-1\}$

$(-1)+(-1)=-2 \notin \{0,-1\}$

$\Rightarrow \{0,-1\}$ does not have closure property w.r.t. '+'

Multiplication Table

×	0	−1
0	0	0
−1	0	1

$\because 0\times 0=0 \in \{0,-1\}$

$0\times(-1)=0 \in \{0,-1\}$

$(-1)\times 0=0 \in \{0,-1\}$

$(-1)\times(-1)=1 \notin \{0,-1\}$

$\Rightarrow \{0,-1\}$ does not have closure property w.r.t. '×'

iv) $\{1,-1\}$

Addition Table

+	1	−1
1	2	0
−1	0	−2

$\because 1+1=2 \notin \{1,-1\}$

$1+(-1)=0 \notin \{1,-1\}$

$(-1)+1=0 \notin \{1,-1\}$

$(-1)+(-1)=-2 \notin \{1,-1\}$

$\Rightarrow \{1,-1\}$ does not have closure property w.r.t. '+'

Multiplication Table

×	1	−1
1	1	−1
−1	−1	1

$\because 1\times 1=1 \in \{1,-1\}$

$1\times(-1)=-1 \in \{1,-1\}$

$(-1)\times 1=-1 \in \{1,-1\}$

$(-1)\times(-1)=1 \in \{1,-1\}$

$\Rightarrow \{1,-1\}$ has closure property w.r.t. '×'

② Name the properties used in the following questions.

i) $4+9=9+4$ (Commutative property w.r.t. '+')

ii) $(a+1)+\frac{3}{4}=a+(1+\frac{3}{4})$ (Assoc. property w.r.t. '+')

iii) $(\sqrt{3}+\sqrt{5})+\sqrt{7}=\sqrt{3}+(\sqrt{5}+\sqrt{7})$ (")

iv) $100+0=100$ (Additive Identity)

v) $1000\times 1=1000$ (Multiplicative Identity)

vi) $4.1+(-4.1)=0$ (Additive inverse)

vii) $a-a=0$ (Additive inverse)

viii) $\sqrt{2}\times\sqrt{5}=\sqrt{5}\times\sqrt{2}$ (Commutative property w.r.t '×')

ix) $a(b-c)=ab-ac$ (Left distributive property)

x) $(x-y)z=xz-yz$ (Right distributive property)

xi) $4\times(5\times 8)=(4\times 5)\times 8$ (Associative property w.r.t. '×')

xii) $a(b+c-d)=ab+ac-ad$

(Left distributive property)

3. Name the properties used in the following inequalities.

i) $-3<-2 \Rightarrow 0<1$ (Additive property)

ii) $-5<-4 \Rightarrow 20>16$ (multiplicative property)

iii) $1>-1 \Rightarrow -3>-5$ (Additive property)

iv) $a<0 \Rightarrow -a>0$ (multiplicative property)

v) $a>b \Rightarrow \frac{1}{a}<\frac{1}{b}$ (")

vi) $a>b \Rightarrow -a<-b$ (")

4. Prove the following rules of addition.

i) $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$

$L.H.S. = \frac{a}{c}+\frac{b}{c}$

$= a\times\frac{1}{c}+b\times\frac{1}{c}$

$= (a+b)\times\frac{1}{c}$ (Right dist. property)

$= \frac{a+b}{c} = R.H.S.$

ii) $\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$

$L.H.S. = \frac{a}{b}+\frac{c}{d}$

$= \frac{a}{b}\times 1 + 1\times\frac{c}{d}$

$= \frac{a}{b}\times\left(d\times\frac{1}{d}\right)+\left(b\times\frac{1}{b}\right)\times\frac{c}{d}$

$= \frac{a}{b}\times\frac{d}{d} + \frac{b}{b}\times\frac{c}{d}$

$= \frac{ad}{bd}+\frac{bc}{bd}$

$= ad\times\frac{1}{bd} + bc\times\frac{1}{bd}$

$= (ad+bc)\times\frac{1}{bd}$

$= \frac{ad+bc}{bd} = R.H.S.$

5. Prove that

$-\frac{7}{12}-\frac{5}{18}=\frac{-21-10}{36}$

$L.H.S. = -\frac{7}{12}-\frac{5}{18}$

$= -\frac{7}{12}\times 1 - \frac{5}{18}\times 1$

$= -\frac{7}{12}\times\left(3\times\frac{1}{3}\right)-\frac{5}{18}\times\left(2\times\frac{1}{2}\right)$

$= -\frac{7}{12}\times\frac{3}{3}-\frac{5}{18}\times\frac{2}{2}$

$= -\frac{21}{36}-\frac{10}{36}$

$= -21\times\frac{1}{36}-10\times\frac{1}{36}$

$= (-21-10)\times\frac{1}{36}$

$= \frac{-21-10}{36} = R.H.S.$

6. Simplify by justifying each step.

i) $\frac{4+16x}{4} = \frac{1}{4}\times(4+16x)$ $\because \frac{a}{b}=\frac{1}{b}\times a$

$= \frac{1}{4}\times(4\times 1+4\times 4x)$ (multiplicative Identity)

$= \frac{1}{4}\times 4\times(1+4x)$ (Distributive Property)

$= 1\times(1+4x)$ (multiplicative inverse)

$= 1+4x$ (Multiplicative Identity)

ii) $\frac{\frac{1}{4}+\frac{1}{5}}{\frac{1}{4}-\frac{1}{5}} = \frac{\frac{1}{4}\times 1+\frac{1}{5}\times 1}{\frac{1}{4}\times 1-\frac{1}{5}\times 1}$ (mult. identity)

$= \frac{\frac{1}{4}\times\left(5\times\frac{1}{5}\right)+\frac{1}{5}\times\left(4\times\frac{1}{4}\right)}{\frac{1}{4}\times\left(5\times\frac{1}{5}\right)-\frac{1}{5}\times\left(4\times\frac{1}{4}\right)}$ (mult. inverse)

$= \frac{\frac{1}{4}\times\frac{5}{5}+\frac{1}{5}\times\frac{4}{4}}{\frac{1}{4}\times\frac{5}{5}-\frac{1}{5}\times\frac{4}{4}}$ $\because \frac{a}{b}=a\times\frac{1}{b}$

$= \frac{\frac{5}{20}+\frac{4}{20}}{\frac{5}{20}-\frac{4}{20}}$ $\because \frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$

$$= \frac{5\times\frac{1}{20}+4\times\frac{1}{20}}{5\times\frac{1}{20}-4\times\frac{1}{20}} \quad \because \frac{a}{b}=a\cdot\frac{1}{b}$$

$$= \frac{(5+4)\times\frac{1}{20}}{(5-4)\times\frac{1}{20}} \text{ (Dist. property)}$$

$$= \frac{5+4}{5-4} \text{ (Cancellation law)}$$

$$= \frac{9}{1} = 9$$

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iii) $$\frac{\frac{a}{b}+\frac{c}{d}}{\frac{a}{b}-\frac{c}{d}} = \frac{\frac{a}{b}\times 1+1\times\frac{c}{d}}{\frac{a}{b}\times 1-1\times\frac{c}{d}}$$

(multiplicative Identity)

$$= \frac{\frac{a}{b}\times(d\times\frac{1}{d})+(b\times\frac{1}{b})\times\frac{c}{d}}{\frac{a}{b}\times(d\times\frac{1}{d})-(b\times\frac{1}{b})\times\frac{c}{d}}$$

(multiplicative inverse)

$$= \frac{\frac{a}{b}\times\frac{d}{d}+\frac{b}{b}\times\frac{c}{d}}{\frac{a}{b}\times\frac{d}{d}-\frac{b}{b}\times\frac{c}{d}} \quad \because \frac{a}{b}=a\times\frac{1}{b}$$

$$= \frac{\frac{ad}{bd}+\frac{bc}{bd}}{\frac{ad}{bd}-\frac{bc}{bd}} \quad \because \frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$$

$$= \frac{ad\times\frac{1}{bd}+bc\times\frac{1}{bd}}{ad\times\frac{1}{bd}-bc\times\frac{1}{bd}} \quad \because \frac{a}{b}=a\times\frac{1}{b}$$

$$= \frac{(ad+bc)\times\frac{1}{bd}}{(ad-bc)\times\frac{1}{bd}} \text{ (Dist. Law)}$$

$$= \frac{ad+bc}{ad-bc} \text{ (Cancellation Law)}$$

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iv) $$\frac{\frac{1}{a}+\frac{1}{b}}{1-\frac{1}{a}\cdot\frac{1}{b}}$$

$$= \frac{\frac{1}{a}\times 1+1\times\frac{1}{b}}{1-\frac{1}{a}\cdot\frac{1}{b}} \text{ (Mult. Ident}$$

$$= \frac{\frac{1}{a}\times(b\times\frac{1}{b})+(a\times\frac{1}{a})\times\frac{1}{b}}{1-\frac{1}{a}\cdot\frac{1}{b}}$$

(Multiplicative inverse)

$$= \frac{\frac{1}{a}\times\frac{b}{b}+\frac{a}{a}\times\frac{1}{b}}{1-\frac{1}{a}\cdot\frac{1}{b}} \quad \because \frac{a}{b}=a\times\frac{1}{b}$$

$$= \frac{\frac{b}{ab}+\frac{a}{ab}}{1-\frac{1}{ab}} \quad \because \frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$$

$$= \frac{b\times\frac{1}{ab}+a\times\frac{1}{ab}}{1-\frac{1}{ab}} \quad \because \frac{a}{b}=a\times\frac{1}{b}$$

$$= \frac{b\times\frac{1}{ab}+a\times\frac{1}{ab}}{ab\times\frac{1}{ab}-1\cdot\frac{1}{ab}}$$

(Multiplicative inverse and multi. Identity)

$$= \frac{(b+a)\times\frac{1}{ab}}{(ab-1)\times\frac{1}{ab}} \text{ (Dist. property)}$$

$$= \frac{b+a}{ab-1} \text{ (Cancellation Law)}$$