MATRICES AND DETERMINANTS

Idea of matrices:

The idea of matrices was given by Arthur Cayley, an English mathematician of nineteenth century who first developed, "Theory of Matrices" in 1858.

Q1. Define the following terms.

(i) Matrix

"A rectangular array or a formation of a collection of real numbers, say 0, 1, 2, 3, 4 and 7, such as: $\begin{matrix} 1 & 3 & 4 \\ 7 & 2 & 0 \end{matrix}$ and then enclosed by brackets '[ ]' is said to form a matrix $\begin{bmatrix} 1 & 3 & 4 \\ 7 & 2 & 0 \end{bmatrix}$. Similarly $\begin{bmatrix} 0 & 1 \\ 3 & 4 \end{bmatrix}$ is another matrix.

The matrices are denoted conventionally by the capital letters A,B,C,....,M,N etc. of the English alphabet.

(ii) Order of a Matrix

The number of rows and columns in a matrix specifies its order. If a matrix M has m rows and n columns then M is said to be of order, m-by-n, For example, $M=\begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix}$ is order 2-by-3,

(iii) Equal Matrices

Let A and B be two matrices. Then A is said to be equal to B, and is denoted by A = B, if and only if;

(i) The order of A = The order of B

(ii) Their corresponding entries are equal.

Examples

(i) $A=\begin{bmatrix} 1 & 3 \\ -4 & 2 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & 2+1 \\ -4 & 4-2 \end{bmatrix}$

are equal matrices.

We see that:

(a) The order of matrix A = The order of matrix B

(b) Their corresponding elements are equal.

Thus A = B

(ii) $L=\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ and $M=\begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$ are not equal matrices.

We see that: order of L = order of M but entries in the second row and second column are not same, so L ≠ M.

(iii) $P=\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ and $Q=\begin{bmatrix} 2 & 3 & 4 \\ -1 & 2 & 0 \end{bmatrix}$

are not equal matrices.

We see that order of P ≠ order of Q, so P ≠ Q.

Exercise 1.1

1. Find the order of the following matrices.

$A=\begin{bmatrix} 2 & 3 \\ -5 & 6 \end{bmatrix}$, order of A is 2-by-2

$B=\begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$, order of B is 2-by-2

$C=\begin{bmatrix} 2 & 4 \end{bmatrix}$ order of C is 1-by-2

$D=\begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}$, order of D is 3-by-1

$E = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$, order of E is 3-by-2

$F = [2]$ order of F is 1-by-1

$G = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix}$, order of G is 3-by-3

$H = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 6 \end{bmatrix}$ order of H is 2-by-3

2. Which of the following matrices are equal?

$A = [3]$,

$B = [3 \quad 5], C = [5-2]$

$D = [5 \quad 3], E = \begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}$,

$F = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$

$G = \begin{bmatrix} 3-1 \\ 3+3 \end{bmatrix}$,

$H = \begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}, I = [3 \quad 3+2]$

$J = \begin{bmatrix} 2+2 & 2-2 \\ 2+4 & 2+0 \end{bmatrix}$

Ans. Equal matrices are

$A = C$ $\quad B = I$

$E = H = J$ $\quad F = G$

3. Find the values of a, b, c and d which satisfy the matrix equation.

$$\begin{bmatrix} a+c & a+2b \\ c-1 & 4d-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2d \end{bmatrix}$$

Ans. $a + c = 0$ ....(i)

$a + 2b = -7$ ....(ii)

$c - 1 = 3$ ....(iii)

$4d - 6 = 2d$ ......(iv)

From (iii)

$c = 3+1$

$\boxed{c=4}$

From (iv)

$4d - 2d = 6$

$2d = 6$

$d = \frac{6}{2}$

$\boxed{d=3}$

Put value of c = 4 in (i)j

$a + 4 = 0$

$\boxed{a=-4}$

Put value of a = –4 in (ii)

$-4 + 2b = -7$

$2b = -7 + 4$

$2b = -3$

$\boxed{b = \frac{-3}{2}}$