Exercise 9.3

Question # 1

Verify following: (i) $\sin 60^o \cos 30^o - \cos 60^o \sin 30^o = \sin 30^o$

(ii) $\sin^2\dfrac{\pi}{6} + \sin^2\dfrac{\pi}{3} + \tan^2\dfrac{\pi}{4} = 2$

(iii) $2\sin 45^\circ + \dfrac{1}{2}\csc 45^\circ = \dfrac{3}{\sqrt{2}}$

(iv) $\sin^2\dfrac{\pi}{6} : \sin^2\dfrac{\pi}{4} : \sin^2\dfrac{\pi}{3} : \sin^2\dfrac{\pi}{2} = 1:2:3:4$

Solution

(i) L.H.S = $\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$

$$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{1}{2}\cdot\frac{1}{2} \quad = \frac{3}{4} - \frac{1}{4} \quad = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2} \quad \ldots\ldots\ldots\ldots (i)$$

R.H.S = $\sin 30 = \dfrac{1}{2}$ ………… (ii)

From (i) and (ii)

L.H.S = R.H.S

(ii) L.H.S = $\sin^2\dfrac{\pi}{6} + \sin^2\dfrac{\pi}{3} + \tan^2\dfrac{\pi}{4}$

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + (1)^2 \quad = \frac{1}{4} + \frac{3}{4} + 1 = \frac{1+3+4}{4} \quad = \frac{8}{4} \quad = 2 \quad = \text{R.H.S}$$

(iii) L.H.S $= 2\sin 45^\circ + \dfrac{1}{2}\csc 45^\circ \quad = 2\sin 45^\circ + \dfrac{1}{2}\dfrac{1}{\sin 45^\circ}$

$$= 2\left(\frac{1}{\sqrt{2}}\right) + \frac{1}{2}\frac{1}{\left(\frac{1}{\sqrt{2}}\right)} \quad = \frac{2}{\sqrt{2}} + \frac{\sqrt{2}}{2} = \frac{\left(\sqrt{2}\right)^2}{\sqrt{2}} + \frac{\sqrt{2}}{\left(\sqrt{2}\right)^2}$$

$$= \sqrt{2} + \frac{1}{\sqrt{2}} \quad = \frac{2+1}{\sqrt{2}} \quad = \frac{3}{\sqrt{2}} \quad = \text{ R.H.S}$$

(iv) L.H.S = $\sin^2\dfrac{\pi}{6} : \sin^2\dfrac{\pi}{4} : \sin^2\dfrac{\pi}{3} : \sin^2\dfrac{\pi}{2}$

$$= \left(\frac{1}{2}\right)^2 : \left(\frac{1}{\sqrt{2}}\right)^2 : \left(\frac{\sqrt{3}}{2}\right)^2 : (1)^2 \quad = \frac{1}{4} : \frac{1}{2} : \frac{3}{4} : 1$$

$= 1:2:3:4 \quad$ = R.H.S $\qquad\qquad$ ×ing by 4

Question # 2

Evaluate the following

(i) $\dfrac{\tan\frac{\pi}{3}-\tan\frac{\pi}{6}}{1+\tan\frac{\pi}{3}\tan\frac{\pi}{6}}$ (ii) $\dfrac{1-\tan^2\frac{\pi}{3}}{1+\tan^2\frac{\pi}{3}}$

Solution

(i) $\dfrac{\tan\frac{\pi}{3}-\tan\frac{\pi}{6}}{1+\tan\frac{\pi}{3}\tan\frac{\pi}{6}} = \dfrac{\sqrt{3}-\frac{1}{\sqrt{3}}}{1+\sqrt{3}\cdot\frac{1}{\sqrt{3}}} = \dfrac{\frac{3-1}{\sqrt{3}}}{1+1}$

$= \dfrac{\frac{2}{\sqrt{3}}}{2} = \dfrac{2}{2\sqrt{3}} = \dfrac{1}{\sqrt{3}}$ Answer

(ii) $\dfrac{1-\tan^2\frac{\pi}{3}}{1+\tan^2\frac{\pi}{3}} = \dfrac{1-\left(\sqrt{3}\right)^2}{1+\left(\sqrt{3}\right)^2} = \dfrac{1-3}{1+3} = \dfrac{-2}{4} = -\dfrac{1}{2}$ Answer

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Question # 3

Verify the following when $\theta = 30^o, 45^o$

(i) $\sin 2\theta = 2\sin\theta\cos\theta$ (ii) $\cos 2\theta = 2\cos^2\theta - 1$

(iii) $\cos 2\theta = 2\cos^2\theta - 1$ (iv) $\cos 2\theta = 1-2\sin^2\theta$

(v) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

Solution

(i) When $\theta = 30^\circ$

L.H.S = $\sin 2\theta = \sin 2(30) = \sin 60 = \dfrac{\sqrt{3}}{2}$ .... (i)

R.H.S = $2\sin\theta\cos\theta = 2\sin 30^\circ\cos 30^\circ = 2\left(\dfrac{1}{2}\right)\left(\dfrac{\sqrt{3}}{2}\right) = \dfrac{\sqrt{3}}{2}$ .... (ii)

From (i) and (ii)

L.H.S = R.H.S

When $\theta = 45^\circ$

L.H.S = $\sin 2\theta = \sin 2(45) = \sin 90 = 1$ .... (i)

R.H.S = $2\sin\theta\cos\theta = 2\sin 45^\circ\cos 45^\circ = 2\left(\dfrac{1}{\sqrt{2}}\right)\left(\dfrac{1}{\sqrt{2}}\right) = \dfrac{2}{2} = 1$ .... (ii)

From (i) and (ii)

L.H.S = R.H.S

(ii) When $\theta = 30^\circ$

L.H.S = $\cos 2\theta = \cos 2(30) = \cos 60 = \frac{1}{2}$ .... (i)

R.H.S = $\cos^2\theta - \sin^2\theta = \cos^2 30^\circ - \sin^2 30^\circ$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = \frac{3}{4} - \frac{1}{4} = \frac{1}{2} \text{ .... } (ii)$$

From (i) and (ii)

L.H.S = R.H.S

When $\theta = 45^\circ$

L.H.S = $\cos 2\theta = \cos 2(45) = \cos 90 = 0$ .... (i)

R.H.S = $\cos^2\theta - \sin^2\theta = \cos^2 45^\circ - \sin^2 45^\circ$

$$= \left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} - \frac{1}{2} = 0 \text{ .... } (ii)$$

From (i) and (ii)

L.H.S = R.H.S

(iii) and (iv) *Do yourself as above*

(v) When $\theta = 30^\circ$

L.H.S = $\tan 2\theta = \tan 2(30) = \tan 60 = \sqrt{3}$ .... (i)

$$\text{R.H.S} = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\tan 30^\circ}{1-\tan^2 30^\circ} = \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{2/\sqrt{3}}{1-1/3} = \frac{2/\sqrt{3}}{2/3} = \frac{2}{\sqrt{3}}\cdot\frac{3}{2} = \frac{2}{\sqrt{3}}\cdot\frac{\left(\sqrt{3}\right)^2}{2} = \sqrt{3} \text{ .... } (ii)$$

From (i) and (ii)

L.H.S = R.H.S

When $\theta = 45^\circ$

L.H.S = $\tan 2\theta = \tan 2(45) = \tan 90 = \infty$ .... (i)

$$\text{R.H.S} = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\tan 45^\circ}{1-\tan^2 45^\circ} = \frac{2(1)}{1-(1)^2}$$

$$= \frac{2}{1-1} = \frac{2}{0} = \infty \text{ .... } (ii)$$

From (i) and (ii)

L.H.S = R.H.S

Question # 4

Find x, if $\tan^2 45^o - \cos^2 60^o = x\sin 45^o \cos 45^o \tan 60^o$

Solution

Since $\tan^2 45^o - \cos^2 60^o = x\sin 45^o \cos 45^o \tan 60^o$

This gives $(1)^2 - \left(\dfrac{1}{2}\right)^2 = x\left(\dfrac{1}{\sqrt{2}}\right)\left(\dfrac{1}{\sqrt{2}}\right)\left(\sqrt{3}\right)$

$$\Rightarrow 1 - \frac{1}{4} = x\left(\frac{\sqrt{3}}{2}\right) \frac{3}{4} = x\frac{\sqrt{3}}{2} \qquad \Rightarrow \frac{6}{4\sqrt{3}} = x$$

$$\Rightarrow x = \frac{3}{2\sqrt{3}} \qquad \Rightarrow \boxed{x = \frac{\sqrt{3}}{2}} \text{ Answer}$$

Question # 5

Find the values of the trigonometric functions of the following

(i) $-\pi$ (ii) -3π (iii) $\dfrac{5}{2}\pi$ (iv) $-\dfrac{9}{2}\pi$ (v) -15π

(vi) 1530^o (vii) -2430° (viii) $\dfrac{235}{2}\pi$ (ix) $\dfrac{407}{2}\pi$

Solutions

(i) Since $-\pi = -2\pi + \pi$, therefore

$\sin(-\pi) = \sin(-2\pi + \pi) = \sin\pi = 0$

$\cos(-\pi) = \cos(-2\pi + \pi) = \cos\pi = -1$

$\tan(-\pi) = \tan(-2\pi + \pi) = \tan\pi = 0$

$\csc(-\pi) = \csc(-2\pi + \pi) = \csc\pi = \infty$ (undefined)

$\sec(-\pi) = \sec(-2\pi + \pi) = \sec\pi = -1$

$\cot(-\pi) = \cot(-2\pi + \pi) = \cot\pi = \infty$ (undefined)

(ii) Since $-3\pi = -4\pi + \pi = -2(2\pi) + \pi$, therefore

$\sin(-3\pi) = \sin(-2(2\pi) + \pi) = \sin\pi = 0$

And now find other values yourself.

(iii) Since $\dfrac{5}{2}\pi = 2\dfrac{1}{2}\pi = 2\pi + \dfrac{\pi}{2}$, therefore

$\sin\left(\dfrac{5}{2}\pi\right) = \sin\left(2\pi + \dfrac{\pi}{2}\right) = \sin\dfrac{\pi}{2} = 1$, $\quad \cos\left(\dfrac{5}{2}\pi\right) = \cos\left(2\pi + \dfrac{\pi}{2}\right) = \cos\dfrac{\pi}{2} = 0$

$\tan\left(\dfrac{5}{2}\pi\right) = \tan\left(2\pi + \dfrac{\pi}{2}\right) = \tan\dfrac{\pi}{2} = \infty$, $\quad \csc\left(\dfrac{5}{2}\pi\right) = \csc\left(2\pi + \dfrac{\pi}{2}\right) = \csc\dfrac{\pi}{2} = 1$

$\sec\left(\dfrac{5}{2}\pi\right) = \sec\left(2\pi + \dfrac{\pi}{2}\right) = \sec\dfrac{\pi}{2} = \infty$, $\quad \cot\left(\dfrac{5}{2}\pi\right) = \cot\left(2\pi + \dfrac{\pi}{2}\right) = \cot\dfrac{\pi}{2} = 0$

(iv) Since $-\dfrac{9}{2}\pi = -4\dfrac{1}{2}\pi = -6\pi + \dfrac{3\pi}{2}$, therefore

$\sin\left(-\dfrac{9}{2}\pi\right) = \sin\left(-6\pi + \dfrac{3\pi}{2}\right) = \sin\dfrac{3\pi}{2} = -1$

And now find other values yourself.

(v) Since $-15\pi = -16\pi + \pi$, therefore

$$\sin(-15\pi) = \sin(-16\pi + \pi) = \sin\pi = 0$$

And now find other values yourself.

(vi) Since $1530^\circ = 1530 \times \dfrac{\pi}{180} = \dfrac{17}{2}\pi = 8\dfrac{1}{2}\pi = 8\pi + \dfrac{\pi}{2}$

So $\sin(1530^\circ) = \sin\left(8\pi + \dfrac{\pi}{2}\right) = \sin\dfrac{\pi}{2} = 1$

And now find other values yourself.

(vii) Since $-2430^\circ = -2430 \times \dfrac{\pi}{180} = -\dfrac{27}{2}\pi = -13\dfrac{1}{2}\pi = -14\pi + \dfrac{\pi}{2}$

Now do yourself

(viii) Since $\dfrac{235}{2}\pi = 116\dfrac{3}{2}\pi = 116\pi + \dfrac{3\pi}{2}$

Now do yourself

(*ix*) Since $\dfrac{407}{2}\pi = 202\dfrac{3}{2}\pi = 202\pi + \dfrac{3\pi}{2}$

Now do yourself

Question # 6

Find the values of the trigonometric functions of the following angles:

(i) 390^o	(ii) -330^o	(iii) 765^o
(iv) -675°	(v) $-\dfrac{17}{3}\pi$	(vi) $\dfrac{13}{3}\pi$
(vii) $\dfrac{25}{6}\pi$	(viii) $-\dfrac{71}{6}\pi$	(ix) -1035°

Solutions

(i) Since $390^\circ = 360 + 30$

So $\sin(390^\circ) = \sin(360+30) = \sin 30^\circ = \dfrac{1}{2}$

$\cos(390^\circ) = \cos(360+30) = \cos 30^\circ = \dfrac{\sqrt{3}}{2}$

$\tan(390^\circ) = \tan(360+30) = \tan 30^\circ = \dfrac{1}{\sqrt{3}}$

$\csc(390^\circ) = \csc(360+30) = \csc 30^\circ = 2$

$\sec(390^\circ) = \sec(360+30) = \sec 30^\circ = \dfrac{2}{\sqrt{3}}$

$\cot(390^\circ) = \cot(360+30) = \cot 30^\circ = \sqrt{3}$

(ii) Since $-330^\circ = -360 + 30$

So $\sin\left(-330^\circ\right)=\sin\left(-360+30\right)=\sin 30^\circ=\dfrac{1}{2}$

And now find other values yourself.

(iii) Since $765^\circ=720+45=2(360)+45$

So $\sin\left(760^\circ\right)=\sin\left(2(360)+45\right)=\sin 45^\circ=\dfrac{1}{\sqrt{2}}$

And now find other values yourself.

(iv) Since $-675^\circ=-720+45=-2(360)+45$ *Now do yourself*

(v) Since $-\dfrac{17}{3}\pi=-5\dfrac{2}{3}\pi=-6\pi+\dfrac{\pi}{3}$ *Now do yourself*

(vi) Since $\dfrac{13}{3}\pi=4\dfrac{1}{3}\pi=4\pi+\dfrac{\pi}{3}$ *Now do yourself.*

(vii) Since $\dfrac{25}{6}\pi=4\dfrac{1}{6}\pi=4\pi+\dfrac{\pi}{6}$

So $\sin\left(\dfrac{25}{6}\pi\right)=\sin\left(4\pi+\dfrac{\pi}{6}\right)=\sin\dfrac{\pi}{6}=\dfrac{1}{2}$ *Now do yourself*

(viii) Since $-\dfrac{71}{6}\pi=-11\dfrac{5}{6}\pi=-12\pi+\dfrac{\pi}{6}$ *Now do yourself.*

(ix) Since $-1035^\circ=-1035\cdot\dfrac{\pi}{180}=-\dfrac{23\pi}{4}=-5\dfrac{3\pi}{4}=-6\pi+\dfrac{\pi}{4}$

So $\sin\left(-1035^\circ\right)=\sin\left(-6\pi+\dfrac{\pi}{4}\right)=\sin\dfrac{\pi}{4}=\dfrac{1}{\sqrt{2}}$ *Now do yourself*
