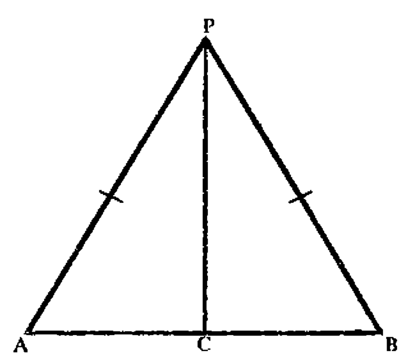
## Exercise 10.2

Prove that a point, which is equidistant from the end points of a line segment, is on the right bisector of the line segment.




**Given**

$\overline{AB}$ is a line segment. Point P is such that $\overline{PA} \cong \overline{PB}$

**To Prove**

Point P is on the right bisector of $\overline{AB}$.

**Construction**

Join P to C, the midpoint of $\overline{AB}$

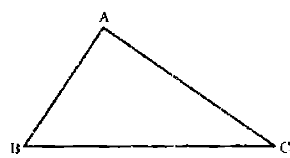
**Proof**

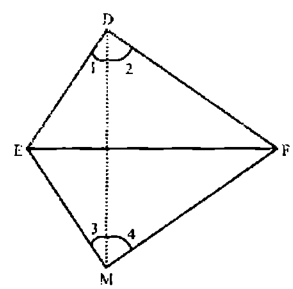
| Statements | Reasons |
|---|---|
| In $\Delta ACP \leftrightarrow \Delta BCP$ | |
| $\overline{PA} \cong \overline{PB}$ | Given |
| $\overline{PC} \cong \overline{PC}$ | Common |
| $\overline{AC} \cong \overline{BC}$ | Construction |
| $\Delta ACP \cong \Delta BCP$ | $S.S.S \cong S.S.S$ |
| $\angle ACP \cong \angle BCP$ ...(i) | Corresponding angles of congruent triangles |
| But $m\angle ACP + m\angle BCP = 180^{\circ}$...(ii) | supplementary angles, |
| $m\angle ACP = m\angle BCP = 90^{\circ}$ | From (i) and (ii) |
| or $\overline{PC} \perp \overline{AB}$ ....(iii) | $m\angle ACP = 90^{\circ}$ (proved) |
| Also $\overline{CA} \cong \overline{CB}$ ....(iv) | construction |
| $\therefore$ $\overline{PC}$ is a right bisector Of $\overline{AB}$ i.e, the point P is on the right bisector of $\overline{AB}$. | from (iii) and (vi) |

**Theorem**

In a correspondence of two triangles, if three sides of one triangle are congruent to the corresponding three sides of the other, then the two triangles are congruent.

($S.S.S. \cong S.S.S.$)




**Given**

In $\Delta ABC \leftrightarrow \Delta DEF$

$\overline{AB} \cong \overline{DE}, \overline{BC} \cong \overline{EF}$ and $\overline{CA} \cong \overline{FD}$

**To Prove**

$\Delta ABC \cong \Delta DEF$

**Construction**

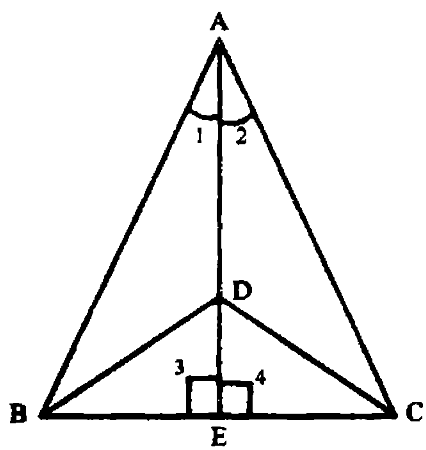
Suppose that in $\Delta DEF$ the side $\overline{EF}$ is not smaller than any of the remaining two sides. On $\overline{EF}$ construct a $\Delta MEF$ in which, $\angle FEM \cong \angle B$ and $\overline{ME} \cong \overline{AB}$. Join D and M. As shown in the above figures we label some of the angles as 1,2,3 and 4.

**Proof**

| Statements | Reasons |
|---|---|
| In $\Delta ABC \leftrightarrow \Delta MEF$ | |
| $\overline{BC} \cong \overline{EF}$ | Given |
| $\angle B \cong \angle FEM$ | Construction |
| $\overline{AB} \cong \overline{ME}$ | Construction |
| $\therefore$ $\Delta ABC \cong \Delta MEF$ | S.A.S postulate |
| and $\overline{CA} \cong \overline{FM}$ ..................(i) | (Corresponding sides of congruent triangles) |
| Also $\overline{CA} \cong \overline{FD}$ ..................(ii) | Given |
| $\therefore$ $\overline{FM} \cong \overline{FD}$ | From (i) and (ii) |
| In $\Delta FDM$ | |
| $\angle 2 \cong \angle 4$..................(iii) | $\overline{FM} \cong \overline{FD}$ (proved) |
| Similarly $\angle 1 \cong \angle 3$ ................(iv) | |
| $\therefore$ $m\angle 2 + m\angle 1 = m\angle 4 + m\angle 3$ | {from (iii) and (iv)} |
| $\therefore$ $m\angle EDF = m\angle EMF$ | |
| Now, In$\Delta DEF \leftrightarrow \Delta MEF$ | |
| $\overline{FD} \cong \overline{FM}$ | Proved |
| And $m\angle EDF \cong m\angle EMF$ | Proved |
| $\overline{DE} \cong \overline{ME}$ | Each one $\cong \overline{AB}$ |
| $\therefore$ $\Delta DEF \cong \Delta MEF$ | S.A.S postulate |
| Also $\Delta ABC \cong \Delta MEF$ | Proved |
| Hence $\Delta ABC \cong \Delta DEF$ | Each $\Delta \cong \Delta MEF$ (Proved) |

**Example**

If two isosceles triangles are formed on the same side of their common base, the line through their vertices would be the right bisector of their common base.




**Given**

$\Delta ABC$ and $\Delta DBC$ are formed on the same side of $\overline{BC}$ such that

$\overline{AB} \cong \overline{AC}, \overline{DB} \cong \overline{DC}$, $\overline{AD}$ meets $\overline{BC}$ at E.

**To prove**

$\overline{BE} \cong \overline{CE}, \overline{AE} \perp \overline{BC}$

**Proof**

| Statements | Reasons |
|---|---|
| In $\Delta ADB \leftrightarrow \Delta ADC$ | |
| $\overline{AB} \cong \overline{AC}$ | Given |
| $\overline{DB} \cong \overline{DC}$ | Given |
| $\overline{AD} \cong \overline{AD}$ | Common |
| $\therefore$ $\Delta ADB \cong \Delta ADC$ | S.S.S $\cong$ S.S.S. |
| $\therefore$ $\angle 1 \cong \angle 2$ | Corresponding angles of $\cong$ $\Delta$s |
| In $\Delta ABE \leftrightarrow \Delta ACE$ | |
| $\overline{AB} \cong \overline{AC}$ | Given |
| $\angle 1 \cong \angle 2$ | Proved |
| $\overline{AE} \cong \overline{AE}$ | Common |
| $\therefore$ $\Delta ABE \cong \Delta ACE$ | S.A.S. postulate |
| $\therefore$ $\overline{BE} \cong \overline{CE}$ | Corresponding sides of $\cong$ $\Delta$s |
| $\angle 3 \cong \angle 4$ .......I | Corresponding angles of $\cong$ $\Delta$s |
| $m\angle 3 + m\angle 4 = 180^\circ$.........II | Supplementary angles Postulate |
| $\therefore$ $m\angle 3 = m\angle 4 = 90^\circ$ | From I and II |
| Hence $\overline{AE} \perp \overline{BC}$ | |

**Corollary: An equilateral triangle is an equiangular triangle.**