Exercise 4.2

1.(i) If $a + b = 10$ and $a - b = 6$ then find value of $a^2 + b^2$.

Solution:

$$2(a^2+b^2) = (a+b)^2 + (a-b)^2$$
$$2(a^2+b^2) = (10)^2 + (6)^2$$
$$2(a^2+b^2) = 100 + 36$$
$$a^2 + b^2 = \frac{136}{2} = 68$$

(ii) If $a + b = 5$, $a - b = \sqrt{17}$ then find value of ab.

Solution:

$$4ab = (a+b)^2 - (a-b)^2$$
$$4ab = (5)^2 - \left(\sqrt{17}\right)^2$$
$$4ab = 25 - 17$$
$$4ab = 8$$

$$ab=\frac{8}{4}=2$$

2. If $a^2+b^2+c^2=45$ and $a+b+c=-1$ find value of $ab+bc+ca$.

Solution:

$a+b+c=-1$

Squaring

$(a+b+c)^2=(-1)^2$

$a^2+b^2+c^2+2ab+abc+2ca=1$

$a^2+b^2+c^2+2(ab+bc+ca)=1$

$45+2(ab+bc+ca)=1$

$2(ab+bc+ca)=1-45$

$2(ab+bc+(a)=-44$

$ab+bc+ca=\frac{-44}{2}=-22$

3. If $m+n+p=10$, $mn+np+pm=27$ find value of $m^2+n^2+p^2$.

Solution:

$m+n+p=10$

Squaring both sides

$(m+n+p)^2=(10)^2$

$m^2+n^2+p^2+2mn+2np+2mp=100$

$m^2+n^2+p^2+2(mn+np+mp)=100$

$m^2+n^2+p^2+2(27)=100$

$m^2+n^2+p^2+54=100$

$m^2+n^2+p^2=100-54$

$m^2+n^2+p^2=46$

4. If $x^2+y^2+z^2=78$ and $y+yz+zx=59$ find $x+y+z$.

Solution:

$(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$

$=x^2+y^2+z^2+2(xy+yz+zx)$

$=78+2(59)$

$=78+118$

$=196$

$\sqrt{(x+y+z)^2}=\sqrt{196}=\sqrt{(\pm 14)^2}$

$x+y+z=\pm 14$

5. If $x+y+z=12$ and $x^2+y^2+z^2=64$ find value of $xy+yz+zx$.

Solution:

$x+y+z=12$

Squaring both sides

$(x+y+z)^2=(12)^2$

$x^2+y^2+z^2+2xy+2yz+2zx=144$

$x^2+y^2+z^2+2(xy+yz+zx)=144$

$64+2(xy+yz+zx)=144$

$2(xy+yz+zx)=144-64$

$2(xy+yz+zx)=80$

$xy+yz+zx=\frac{80}{2}=40.$

6. If $x+y=7$ and $xy=12$ then find value of x^3+y^3.

Solution:

$x+y=7$

$(x+y)^3=(7)^3$

$x^3+y^3+3xy(x+y)=343$

$x^3+y^3+3(12)(7)=343$

$x^3+y^3+252=343$

$x^3+y^3=343-252$

$x^3+y^3=91$

7. If $3x+4y=11$ and $xy=12$ then find value of $27x^3+64y^3$.

Solution:

$3x+4y=11$

$(3x+4y)^3=(11)^3$

$(3x)^3+(4y)^3+3(3x)(4x)(3x+4y)=$

1331

$27x^3+64y^3+36xy(3x+4y)=1331$

$27x^3+64y^3+36(12)(11)=1331$

$27x^3+64y^3+4752=1331$

$27x^3+64y^3=1331-4752=-3421$

8. If $x-y=4$ and $xy=21$ then find value of x^3-y^3.

Solution:

$$x - y = 4$$
$$(x-y)^3 = (4)^3$$
$$x^3-y^3-3xy(x-y) = 64$$
$$x^3-y^3-3(21)(4) = 64$$
$$x^3-y^3-252 = 64$$
$$x^3 - y^3 = 64 + 252$$
$$x^3 - y^3 = 316$$

9. If $5x - 6y = 13$ and $xy = 6$ then find value of $125x^3 - 216y^3$.

Solution:

$$5x - 6y = 13$$
$$\Rightarrow (5x-6y)^3 = (13)^3$$
$$\Rightarrow (5x)^3-(6y)^3-3(5x)(6y)(5x-6y)=2197$$
$$125x^3-216y^3-90xy(5x-6y) = 2197$$
$$125x^3-216y^3-90(6)(13) = 2197$$
$$125x^3-216y^3-7020 = 2197$$
$$125x^3-216y^3 = 2197 + 7020$$
$$125x^3-216y^3 = 9217$$

10. If $x+\frac{1}{x}=3$ then find $x^3+\frac{1}{x^3}$.

$x+\frac{1}{x}=3$ Cubing both sides

$$\left(x+\frac{1}{x}\right)^3 = (3)^3$$
$$x^3+\frac{1}{x^3}+3(x)\left(\frac{1}{x}\right)\left(x+\frac{1}{x}\right)=27$$
$$x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)=27$$
$$x^3+\frac{1}{x^3}+3(3)=27$$
$$x^3+\frac{1}{x^3}=27-9$$
$$x^3+\frac{1}{x^3}=18$$

11. If $x-\frac{1}{x}=7$, then find value of $x^3-\frac{1}{x^3}$

$x-\frac{1}{x}=7$ Taking cube of both sides

$$\left(x-\frac{1}{x}\right)^3=(7)^3$$
$$x^3-\frac{1}{x^3}-3(x)\left(\frac{1}{x}\right)\left(x-\frac{1}{x}\right)=343$$
$$x^3-\frac{1}{x^3}-3\left(x-\frac{1}{x}\right)=343$$
$$x^3-\frac{1}{x^3}-3(7)=343$$
$$x^3-\frac{1}{x^3}-21=343$$
$$x^3-\frac{1}{x^3}=343+21$$
$$x^3-\frac{1}{x^3}=364$$

12. If $3x+\frac{1}{3x}=5$, then find value of $27x^3+\frac{1}{27x^3}$

$$\left(3x+\frac{1}{3x}\right)^3=(5)^3$$
$$(3x)^3+\left(\frac{1}{3x}\right)^3+3(3x)\left(\frac{1}{3x}\right)\left(3x+\frac{1}{3x}\right)=125$$
$$27x^3+\frac{1}{27x^3}+3\left(3x+\frac{1}{3x}\right)=125$$
$$27x^3+\frac{1}{27x^3}+3(5)=125$$
$$27x^3+\frac{1}{27x^3}+15=125$$

$$27x^3+\frac{1}{27x^3}=125-15$$

$$27x^3+\frac{1}{27x^3}=110$$

13. If $\left(5x-\frac{1}{5x}\right)=6$**, then find value of** $125x^3-\frac{1}{25x^3}$.

$$\left(5x-\frac{1}{5x}\right)=6$$

Taking cube of both sides

$$\left(5x-\frac{1}{5x}\right)^3=(6)^3$$

$$(5x)^3-\left(\frac{1}{5x}\right)^3-3(\cancel{5x})\left(\frac{1}{\cancel{5x}}\right)\left(5x-\frac{1}{5x}\right)=216$$

$$125x^3-\frac{1}{125x^3}-3\left(5x-\frac{1}{5x}\right)=216$$

$$125x^3-\frac{1}{125x^3}-3(6)=216$$

$$125x^3-\frac{1}{25x^3}-18=216$$

$$125x^3-\frac{1}{125x^3}=216+18$$

$$125x^3-\frac{1}{125x^3}=234$$

14. Factorize (i) x^3-y^3-x+y

(i) x^3-y^3-x+y

$$=(x-y)(x^2+xy+y^2)-1(x-y)$$

$$=(x-y)\left[x^2+xy+y^2-1\right]$$

(ii) $8x^3-\frac{1}{27y^3}$

$$=(2x)^3-\left(\frac{1}{3y}\right)^3$$

$$=\left(2x-\frac{1}{3y}\right)\left((2x)^2+(2x)\left(\frac{1}{3y}\right)+\left(\frac{1}{3y}\right)^2\right)$$

$$=\left(2x-\frac{1}{3y}\right)\left(4x^2+\frac{2x}{3y}+\frac{1}{9y^2}\right)$$

15. Find products, using formulae

(i) $(x^2+y^2)(x^4-x^2y^2+y^4)$

$$=(x^2)^3+(y^2)^3$$

$$\text{Ref}=(a+b)(a^2-ab+b^2)=a^3+b^3$$

$$=x^6+y^6$$

(ii) $(x^3-y^3)(x^6+x^3y^3+y^6)$

$$=(x^3)^3-(y^3)^3$$

$$\text{Ref.}(a-b)(a^2+ab+b^2)=a^3-b^3$$

$$=x^9-y^9$$

(iii) $(x-y)(x+y)(x^2+y^2)(x^2+xy+y^2)$

$$(x^2-xy+y^2)(x^4-x^2y^2+y^4)$$

$$=(x-y)(x^2+xy+y^2)(x+y)(x^2-xy+y^2)$$

$$(x^2+y^2)(x^4-x^2y^2+y^4)$$

$$=(x^3-y^3)(x^3+y^3)\left[(x^2)^3+(y^2)^3\right]$$

$$=\left[(x^3)^2-(y^3)^2\right]\left(x^6+y^6\right)$$

$$=(x^6-y^6)(x^6+y^6)$$

$$=(x^6)^2-(y^6)^2$$

$$=x^{12}-y^{12}$$

16. $(2x^2-1)(2x^2+1)(4x^4+2x^2+1)$

$(4x^4-2x^2+1)$

$=(2x^2-1)(4x^4+2x^2+1)(2x^2+1)$

$(4x^4-2x^2+1)$

$=((2x^2)^3-(1)^3)((2x^2)^3+(1)^3)$

$$= (8x^6 - 1)(8x^6 + 1)$$

$$= (8x^6)^2 - (1)^2$$

$$= 64x^{12} - 1$$