Exercise 6.1

Question 1:

Find the H.C.F of the following expressions.
(i) $39x^7y^3z$ and $91x^5y^6z^7$
(ii) $102xy^2z$, $85x^2yz$ and $187xyz^2$

Solution:

(i) $39x^7y^3z = 13 \times 3 \times x^7y^3z$
$91x^5y^6z^7 = 13 \times 7 \times x^5y^6z^7$
H.C.F = $13x^5y^3z$

(ii) $102xy^2z = 2 \times 3 \times 17xy^2z$
$85x^2yz = 3 \times 17x^2yz$
$187xyz^2 = 11 \times 17xyz^2$
H.C.F= $17xyz$

Question 2

Find the H.C.F of the following expressions by factorization.
(i) $x^2 + 5x + 6$, $x^2 - 4x - 12$
(ii) $x^3 - 27$, $x^2 + 6x - 27$, $2x^2 - 18$
(iii) $x^3 - 2x^2 + x$, $x^2 + 2x - 3$, $X^2 + 3x - 4$
(iv) $18(x^3 - 9x^2 + 8x)$, $24(x^2 - 3x + 2)$
(v) $36(3x^4 + 5x^3 - 2x^2)$, $54(27x^4 - x)$

Solution:

(i)

$$x^2 + 5x + 6 = x^2 + 3x + 2x + 6,$$
$$= x(x + 3) + 2(x + 3)$$
$$= (x + 3)(x + 2)$$

$$x^2 - 4x - 12 = x^2 - 6x + 2x - 12,$$
$$= x(x - 6) + 2(x - 6)$$
$$= (x - 6)(x + 2)$$

H.C.F= $x + 2$

(ii)

$$x^3 - 27 = x^3 - 3^3,$$
$$= (x - 3)(x^2 + 3x + 9)$$

$x^2+6x-27=x^2-3x+9x-27$

$=x(x-3)+9(x-3)$

$=(x-3)(x+9)$ ....(ii)

$2x^2-18=2(x^2-9)$

$=2\left[(x)^2-(3)^2\right]$

$=2(x+3)(x-3)$ ....(iii)

From (i), (ii) and (iii)

Common factors $=(x-3)$

$HCF=x-3$

iii) $x^3-2x^2+x,\ x^2+2x-3,\ x^2+3x-4$

Sol: By factorization

$x^3-2x^2-x=x(x^2-2x+1)$

$=x(x^2-x-x+1)$

$=x[x(x-1)-1(x-1)]$

$=x(x-1)(x-1)$ ....(i)

$x^2+2x-3=x^2-x+3x-3$

$=x(x-1)+3(x-1)$

$=(x-1)(x+3)$ ....(ii)

$x^2+3x-4=x^2-x+4x-4$

$=x(x-1)+4(x-1)$

$=(x-1)(x+4)$ ....(iii)

From (i), (ii) and (iii)

Common factors: $x-1$

HCF $=x-1$

iv) $18(x^3+9x^2+8x),\ 24(x^2-3x+2)$

Sol: By factorization

$18(x^3-9x^2+8x)=18x(x^2-9x+8)$

$=18x(x^2-x-8x+8)$

$=18x[x(x-1)-8(x-1)]$

$=2\times3\times3\ x(x-1)(x-8)$ ....(i)

$24(x^2-3x+2)=$

$24(x^2-x-2x+2)$

$=2\times2\times2\times3[x(x-1)-2(x-1)]$

$=2\times2\times2\times3(x-1)(x-2)$....(ii)

From (i) and (ii)

HCF $=2\times3(x-1)$

$=6(x-1)$

v) $36(3x^4+5x^3-2x^2),\ 54(27x^4-x)$

Sol: By factorization

$36(3x^4+5x^3-2x^2)=36x^2(3x^2+5x-2)$

$=36x^2(3x^2+6x-x-2)$

$=36x^2[3x(x+2)-1(x+2)]$

$=2\times2\times3\times3x.x(x+2)(3x-1)$ ....(i)

$54(27x^4-x)=54x(27x^3-1)$

$=54x\left[(3x)^3-(1)^3\right]$

$=54x(3x-1)\left[(3x)^2+(3x)(1)+(1)^2\right]$

$=2\times3\times3\times3x(3x-1)(9x^2+3x+1)$ .....(ii)

From (i) and (ii)

Common factors = $2,3,3,x,(3x-1)$

HCF = $2\times3\times3x(3x-1)$

$=18x(3x-1)$

Q3. Find the H.C.F of the following by division methal.

i) $p(x)=x^3+3x^2-16x+12,\ q(x)=x^3+x^2-10x+8$

Sol:

$$\begin{array}{r|l} & \quad\quad 1 \\ x^3+x^2-10x+8 & x^3+3x^2-16x+12 \\ & \underline{-x^3\pm x^2\mp10x\pm8} \\ & \quad 2x^2-6x+4 \end{array}$$

Dividing remainder by 2

x^2-3x+2

$$\begin{array}{r|l} & \quad x+4 \\ \hline x^2-3x+2 & \cancel{x^3}+x^2-10x+8 \\ & -\cancel{x^3}\mp 3x^2\pm 2x \\ \cline{2-2} & \cancel{4x^2}-\cancel{12x}+\cancel{8} \\ & -\cancel{4x^2}-\cancel{12x}\pm\cancel{8} \\ \cline{2-2} & \quad 0 \end{array}$$

Hence HCF $= x^2-3x+2$

ii) $P(x)=x^4+x^3-2x^2+x-3,$

$q(x)=5x^3+3x^2-17x+6$

$$\begin{array}{r|ll} & \quad x+2 & \\ \hline 5x^3+3x^2-17x+6 & x^4+x^3-2x^2+x-3 & \\ & \times 5 & \text{(Multiplying by 5)} \\ \cline{2-2} & \cancel{5x^4}+5x^3-10x^2+5x-15 & \\ & -\cancel{5x^4}\pm 3x^3\mp 17x^2\pm 6x & \\ \cline{2-2} & 2x^3+7x^2-x-15 & \\ & \times 5 & \text{(Multiplying by 5)} \\ \cline{2-2} & \cancel{10x^3}+35x^2-5x-75 & \\ & -\cancel{10x^3}\pm 6x^2\mp 34x\pm 12 & \\ \cline{2-2} & 29x^2+29x-87 & \end{array}$$

Divided by 29

x^2+x-3

$$\begin{array}{r|l} & \quad 5x-2 \\ \hline x^2+x-3 & \cancel{5x^3}+3x^2-17x+6 \\ & -\cancel{5x^3}\pm 5x^2\mp 15x \\ \cline{2-2} & -\cancel{2x^2}-\cancel{2x}+\cancel{6} \\ & \mp\cancel{2x^2}\mp\cancel{2x}\pm\cancel{6} \\ \cline{2-2} & \quad 0 \end{array}$$

Hence H.C.F $= x^2+x-3$

iii) $p(x)=2x^5-4x^4-6x,$

$q(x)=x^5+x^4-3x^3-3x^2$

$$\begin{array}{r|l} & \quad 2 \\ \hline x^5+x^4-3x^3-3x^2 & \cancel{2x^5}-4x^4-6x \\ & -\cancel{2x^5}\pm 2x^4\mp 6x^3\mp 6x^2 \\ \cline{2-2} & -6x^4+6x^3+6x^2-6x \end{array}$$

Dividing by -6

$x^4-x^3-x^2+x$

$$\begin{array}{r|l} & \quad x+2 \\ \hline x^4-x^3-x^2+x & \cancel{x^5}+x^4-3x^3-3x^2 \\ & -\cancel{x^5}\mp x^4\mp x^3\pm x^2 \\ \cline{2-2} & \cancel{2x^4}-\cancel{2x^3}-4x^2 \\ & -\cancel{2x^4}\mp\cancel{2x^3}\mp 2x^2\pm 2x \\ \cline{2-2} & -2x^2-2x \end{array}$$

Dividing by -2

x^2+x

$$\begin{array}{r|l} & \quad x^2-2x+1 \\ \hline x^2+x & \cancel{x^4}-x^3-x^2+x \\ & -\cancel{x^4}\pm x^3 \\ \cline{2-2} & -\cancel{2x^3}-x^2+x \\ & \mp\cancel{2x^3}\mp 2x^2 \\ \cline{2-2} & \cancel{x^2}+\cancel{x} \\ & \pm\cancel{x^2}\pm\cancel{x} \\ \cline{2-2} & \quad 0 \end{array}$$

Hence H.C.F $= x^2+x=x(x+1)$

Q4. Find the L.C.M of the following expressions:

i) $39x^7y^3z$ and $91x^5y^6z^7$

Sol: By factorization

$39x^7y^3z = 13\times 3\, x.x.x.x.x.x.x.y.y.y.z$

$91x^5y^6z^7 = 13\times 7\, x.x.x.x.x.y.y.y.y.y.y.z.z.z.z.z.z.z$

Hence L.C.M =

$13\times 3\times 7.\ x.x.x.x.x.x.x.y.y.y.y.y.y.z.z.z.z.z.z.z$

$= 273x^7y^6z^7$

ii) $102xy^2z$, $85x^2yz$ and $187xyz^2$

Sol: By factorization

$102xy^2z$	=	$2\times 3\times 17x.y.y.z$
$85x^2yz$	=	$5\times 17x.x.y.z$
$187xyz^2$	=	$11\times 17x.y.z.z$

Hence L.C.M $= 17\times11\times5\times3\times2.x.x.y.y.z.z$

$= 5610x^2y^2z^2$

Q5. Find the L.C.M of the following expressions by factorization:

i) $x^2-25x+100$ and x^2-x-20

Sol: By factorization

$x^2-25x+100 = x^2-5x-20x+100$

$= x(x-5)-20(x-5)$

$= (x-5)(x-20)$.............(i)

$x^2-x-20 = x^2-5x+4x-20$

$= x(x-5)+4(x-5)$

$= (x-5)(x+4)$ ....(ii)

From (i) and (ii)

L.C.M $= (x-5)(x-20)(x+4)$

ii) $x^2+4x+4,\ x^2-4,\ 2x^2+x-6$

Sol: By factorization

$x^2+4x+4 = x^2+2x+2x+4$

$= x(x+2)+2(x+2)$

$= (x+2)(x+2)$ ....(i)

$x^2-4 = (x)^2-(2)^2$

$= (x+2)(x-2)$ ....(ii)

$2x^2+x-6 = 2x^2+4x-3x-6$

$= 2x(x+2)-3(x+2)$

$= (x+2)(2x-3)$ ....(iii)

From (i), (ii) and (iii)

LCM $= (x+2)(x+2)(x-2)(2x-3)$

$= (x+2)^2(x-2)(2x-3)$

iii) $2(x^4-y^4),\ 3(x^3+2x^2y-xy^2-2y^3)$

Sol: By factorization

$2(x^4-y^4) = 2\left[(x^2)^2-(y^2)^2\right]$

$= 2(x^2+y^2)(x^2-y^2)$

$= 2(x^2+y^2)(x+y)(x-y)$ ....(i)

$3(x^3+2x^2y-xy^2-2y^3) = 3\left[x^2(x+2y)-y^2(x+2y)\right]$

$= 3(x+2y)(x^2-y^2)$

$= 3(x+2y)(x+y)(x-y)$ ....(ii)

From (i) & (ii)

L.C.M =

$2\times3(x+y)(x-y)(x^2+y^2)(x+2y)$

$= 6(x^4-y^4)(x+2y)$

iv) $4(x^4-1),6(x^3-x^2-x+1)$

Sol: By factorization

$4(x^4-1) = 4\left[(x^2)^2-(1)^2\right]$

$= 4(x^2+1)(x^2-1)$

$= 2\times2(x^2+1)\left[(x)^2-(1)^2\right]$

$= 2\times2(x^2+1)(x+1)(x-1)$ ....(i)

$6(x^3-x^2-x+1) = 6\left[x^2(x-1)-1(x-1)\right]$

$= 6(x-1)(x^2-1) = 2\times3(x-1)\left[(x)^2-(1)^2\right]$

$= 2\times3(x-1)(x-1)(x+1)$ ....(ii)

From (i) & (ii)

LCM= $2\times2\times3(x+1)(x-1)(x^2+1)(x-1)$

$= 12(x^4-1)(x-1)$

Q6. For what value of k is $(x+4)$, the H.C.F of $x^2+x-(2k+2)$ and $2x^2+kx-12$?

Sol: $k = ?$

$p(x) = x^2+x-(2k+2)$ and

$q(x) = 2x^2+kx-12$

As given that $x+4$ is HCF, so $p(x)$ and $q(x)$ will be exactly divisible by $(x+4)$

$$x+4\overline{\smash{\big)}\cancel{x^2}+x-(2k+2)}\quad x-3$$

$$\cancel{x^2}\pm 4x$$

$$\cancel{-3x}-(2k+2)$$

$$\cancel{\mp 3x}\mp 12$$

$$12-(2k+2)$$

$$=12-2k-2$$

$$=10-2k$$

As $p(x)$ is exactly divisible by $x+4$, so,

$$10-2k=0$$

$$10=2k$$

$$\frac{10}{2}=k$$

$$k=5$$

Q7. If $(x+3)(x-2)$ is the H.C.F of $p(x)=(x+3)(2x^2-3x+k)$ and $q(x)=(x-2)(3x^2+7x-l)$, find k and l.

Sol: $k=?$ and $l=?$

As $(x+3)(x-2)$ is the H.C.F, so $p(x)$ and $q(x)$ will be exactly divisible by $(x+3)(x-2)$ i.e., $\frac{p(x)}{HCF}$ has remainder zero.

$$\frac{\cancel{(x+3)}(2x^2-3x+k)}{\cancel{(x+3)}(x-2)}=\frac{2x^2-3x+k}{x-2}$$

i.e

$$x-2\overline{\smash{\big)}2x^2-3x+k}\quad 2x+1$$

$$\pm 2x^2\mp 4x$$

$$x+k$$

$$\pm x\mp 2$$

$$k+2$$

As remainder = 0, then

$$k+2=0$$

$$\boxed{k=-2}$$

and $\frac{q(x)}{HCF}$ has zero remainder

$$\frac{\cancel{(x-2)}(3x^2+7x-l)}{(x+3)\cancel{(x-2)}}=\frac{3x^2+7x-l}{x+3}$$

$$x+3\overline{\smash{\big)}\cancel{3x^2}+7x-l}\quad 3x-2$$

$$\cancel{\pm 3x^2}\pm 9x$$

$$\cancel{-2x}-l$$

$$\cancel{\mp 2x}\mp 6$$

$$-l+6$$

As remainder = 0

$$-l+6=0$$

$$-l=-6$$

$$\Rightarrow \boxed{l=6}$$

Q8. The LCM and HCF of two polynomials $p(x)$ and $q(x)$ are $2(x^4-1)$ and $(x+1)$ (x^2+1) respectively. If $p(x)=x^3+x+1$, find $q(x)$.

Sol: LCM $=2(x^4-1)$,

HCF $=(x+1)(x^2+1)$

$$p(x)=x^3+x^2+x+1,\ q(x)=?$$

As $\quad p(x)\times q(x)=(LCM)\times(HCF)$

$$q(x)=\frac{(LCM)\times(HCF)}{p(x)}$$

$$=\frac{2(x^4-1)\times(x+1)(x^2+1)}{x^3+x^2+x+1}$$

$$=\frac{2(x^4-1)\cancel{(x^3+x^2+x+1)}}{\cancel{x^3+x^2+x+1}}$$

$$q(x)=2(x^4-1)$$

Q9. Let $p(x)=10(x^2-9)(x^2-3x+2)$ and $q(x)=10x(x+3)(x-1)^2$. If the H.C.F. of $p(x), q(x)$ is $10(x+3)(x-1)$, find their L.C.M.

Sol: $p(x)=10(x^2-9)(x^2-3x+2)$,

$q(x)=10x(x+3)(x-1)^2$

H.C.F. $=10(x+3)(x-1)$, L.C.M = ?

As $(L.C.M)\times(H.C.F)=p(x)\times q(x)$

$$\text{L.C.M.} = \frac{p(x)\times q(x)}{H.C.F}$$

$$=\frac{\cancel{10}(x^2-9)(x^2-3x+2)\times 10x(x+3)(x-1)^2}{\cancel{10}(x+3)(x-1)}$$

$$=\frac{(x^2-9)(x^2-3x+2)\times 10x\cancel{(x+3)}\cancel{(x-1)}(x-1)}{\cancel{(x+3)}\cancel{(x-1)}}$$

$$=10x(x-1)(x^2-9)(x^2-3x+2)$$

$$=10x(x-1)(x^2-9)(x^2-x-2x+2)$$

$$=10x(x-1)(x^2-9)[x(x-1)-2(x-1)]$$

$$=10x(x-1)(x^2-9)(x-1)(x-2)$$

$$=10x(x-1)^2(x^2-9)(x-2)$$

Q10. Let the product of L.C.M and H.C.F of two polynomials be $(x+3)^2(x-2)(x+5)$. If one polynomial is $(x+3)(x-2)$ and the second polynomial is $x^2+kx+15$, find the value of k.

Sol: $k=?$

Product of L.C.M. & H.C.F is

$LCM\times HCF=(x+3)^2(x-2)(x+5)$

$p(x)=(x+3)(x-2)$

$q(x)=x^2+kx+15$

As $p(x)\times q(x)=LCM\times HCF$

$$(x+3)(x-2)(x^2+kx+15)$$

$$=(x+3)^2(x-2)(x+5)$$

$$x^2+kx+15=\frac{(x+3)\cancel{(x+3)}\cancel{(x-2)}(x+5)}{\cancel{(x+3)}\cancel{(x-2)}}$$

$$x^2+kx+15=(x+3)(x+5)$$

$$x^2+kx+15=x^2+3x+5x+15$$

$$x^2+kx+15=x^2+8x+15$$

Comparing co-efficient of 'x'

$\Rightarrow kx=8x$

$\boxed{k=8}$

Q11. Waqas wishes to distribute 128 bananas and also 176 apples equally among a certain number of children. Find the highest number of the Children. Who can get the fruit in this way?

Sol: No. of bananas = 128

No. of apples = 176

Highest no. of children who get the fruit in this way is H.C.F.

So No. of bananas =

$2\times2\times2\times2\times2\times2\times2$

No. of apples =

$2\times2\times2\times2\times11$

Hence required no. of children =

$2\times2\times2\times2=16$