Exercise 6.8

Q.No 1 $1, \frac{1}{3}, \frac{1}{9}, \dots\dots$

$a_1 = 1 \quad , \quad r = \frac{a_2}{a_1} = \frac{1/3}{1} = \frac{1}{3} \quad , n = 15$

Now $S_n = \frac{a_1(1-r^n)}{1-r}$

$= \frac{1\left(1-\left(\frac{1}{3}\right)^{15}\right)}{1-\frac{1}{3}} = \frac{1-\frac{1}{3^{15}}}{2/3}$

$= \frac{3}{2}\left[\frac{3^{15}-1}{3^{15}}\right] = \frac{3}{2}\left(\frac{14348907-1}{14348907}\right)$

$= \frac{43046718}{28697814}$

$= \frac{7174453}{4782969}$ ÷ing by 6. Ans

Q.No 2 i)

$0.2 + 0.22 + 0.222 + \dots\dots$

$= 2\left[0.1 + 0.11 + 0.111 + \dots \text{ to } n \text{ terms}\right]$

$= \frac{2}{9}\left[0.9 + 0.99 + 0.999 + \dots \text{ to } n \text{ terms}\right]$

$= \frac{2}{9}\left[(1-0.1) + (1-0.01) + (1-0.001) + \dots\dots \text{ to } n \text{ terms}\right]$

$= \frac{2}{9}\left[(1+1+1+\dots\dots \text{ to } n \text{ terms}) - (0.1 + 0.01 + 0.001 + \dots \text{ to } n \text{ terms})\right]$

$= \frac{2}{9}\left(n - \left(\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \dots \text{ to } n \text{ terms}\right)\right)$

$a_1 = \frac{1}{10}, \; r = \frac{1}{10}, \; n = n$

$= \frac{2}{9}\left(n - \frac{\frac{1}{10}\left(1-\left(\frac{1}{10}\right)^n\right)}{1-\frac{1}{10}}\right)$

$= \frac{2}{9}\left(n - \frac{\frac{1}{10}\left(1-\frac{1}{10^n}\right)}{9/10}\right)$

$= \frac{2}{9}\left[n - \frac{1}{9}\left(1-\frac{1}{10^n}\right)\right]$ Answer

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ii) $3 + 33 + 333 + \dots\dots$ to n terms

$= \frac{1}{3}\left[9 + 99 + 999 + \dots \text{ to } n \text{ terms}\right]$

$= \frac{1}{3}\left[(10-1) + (100-1) + (1000-1) + \dots\dots \text{ to } n \text{ terms}\right]$

$= \frac{1}{3}\left[(10 + 100 + 1000 + \dots\dots \text{ to } n \text{ terms}) - (1+1+1+\dots\dots \text{ to } n \text{ terms})\right]$

$= \frac{1}{3}\left[(10 + 100 + 1000 + \dots \text{ to } n \text{ terms}) - n\right]$

$a_1 = 10, \; r = 10, \; n = n$

$= \frac{1}{3}\left[\frac{10(10^n-1)}{10-1} - n\right]$

$= \frac{1}{3}\left[\frac{10(10^n-1)}{9} - n\right]$ Answer

OR

$= \frac{1}{3}\left[\frac{10}{9}(10^n-1) - n\right]$ Answer

Q.No 3 i)

$1 + (a+b) + (a^2+ab+b^2) + \dots\dots$ to n terms

÷ing and ×ing by $a-b$.

$= \frac{a-b}{a-b}\left[1 + (a+b) + (a^2+ab+b^2) + \dots\dots \text{ to } n \text{ terms}\right]$

$= \frac{1}{a-b}\left[(a-b) + (a-b)(a+b) + (a-b)(a^2+ab+b^2) + \dots \text{ to } n \text{ terms}\right]$

$= \frac{1}{a-b}\left[(a-b) + (a^2-b^2) + (a^3-b^3) + \dots\dots \text{ to } n \text{ terms}\right]$

$= \frac{1}{a-b}\left[(a + a^2 + a^3 + \dots\dots \text{ to } n \text{ terms}) - (b + b^2 + b^3 + \dots\dots \text{ to } n \text{ terms})\right]$

$= \frac{1}{a-b}\left[\frac{a(a^n-1)}{a-1} - \frac{b(b^n-1)}{b-1}\right]$

$= \frac{1}{a-b}\left[\frac{a(a^n-1)(b-1) - b(b^n-1)(a-1)}{(a-1)(b-1)}\right]$

$= \frac{a(b-1)(a^n-1) - b(a-1)(b^n-1)}{(a-b)(a-1)(b-1)}$ Answer

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ii)

$r + (1+k)r^2 + (1+k+k^2)r^3 + \dots\dots$ to n terms

÷ing and ×ing by $1-k$

$= \frac{1-k}{1-k}\left(r + (1+k)r^2 + (1+k+k^2)r^3 + \dots\dots \text{ to } n \text{ terms}\right)$

$= \frac{1}{1-k}\left(r(1-k) + (1-k)(1+k)r^2 + (1-k)(1+k+k^2)r^3 + \dots\dots \text{ to } n \text{ terms}\right)$

$= \frac{1}{1-k}\left(r(1-k) + (1-k^2)r^2 + (1-k^3)r^3 + \dots\dots \text{ to } n \text{ terms}\right)$

$$= \frac{1}{1-K}\left[r - rK + r^2 - r^2K^2 + r^3 - r^3K^3 + \dots\dots \text{ to } n \text{ terms}\right]$$

$$= \frac{1}{1-K}\left\{(r + r^2 + r^3 + \dots \text{ to } n \text{ terms}) - (rK + r^2K^2 + r^3K^3 + \dots \text{ to } n \text{ terms})\right\}$$

$$= \frac{1}{1-K}\left(\frac{r(r^n-1)}{r-1} - \frac{rK((rK)^n-1)}{rK-1}\right)$$

$$= \frac{r}{1-K}\left(\frac{r^n-1}{r-1} - \frac{K(r^nK^n-1)}{rK-1}\right)$$

Answer

Q.No4 $2 + (1-i) + \frac{1}{i} + \dots$ to 8 terms

$a_1 = 2,\ r = \frac{1-i}{2};\ n = 8$

Now $S_n = \frac{a_1(r^n-1)}{r-1}$

$$= \frac{2\left(\left(\frac{1-i}{2}\right)^8 - 1\right)}{\frac{1-i}{2} - 1}$$

$$= \frac{2\left(\frac{(1-i)^8}{2^8} - 1\right)}{\frac{1-i-2}{2}}$$

$$= 4\left(\frac{\frac{[(1-i)^2]^4}{256} - 1}{-1-i}\right)$$

$$= 4\left[\frac{\frac{(1-2i+i^2)^4 - 256}{256}}{-1-i}\right]$$

$$= \frac{4\{(1-2i-1)^4 - 256\}}{256(-1-i)}$$

$$= \frac{\{(-2i)^4 - 256\}}{64(-1-i)}$$

$$= \frac{(-2)^4(i)^4 - 256}{64(-1-i)} \qquad \left|\ i^4 = (i^2)^2 = (-1)^2 = 1\right.$$

$$= \frac{16(1) - 256}{64(-1-i)} = \frac{-240}{64(-1-i)}$$

$$= \frac{-15}{-4(1+i)} = \frac{15}{4(1+i)} \cdot \frac{1-i}{1-i}$$

$$= \frac{15(1-i)}{4((1)^2-(i)^2)} = \frac{15(1-i)}{4(2)} = \frac{15(1-i)}{8}$$

Ans

Q.No.5 i) $\frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \dots\dots$

$a_1 = \frac{1}{5},\ r = \frac{1}{5}$

Now $S = \frac{a_1}{1-r} = \frac{1/5}{1-1/5}$

$$= \frac{1/5}{4/5} = \frac{1}{4}$$

Answer

ii) & iii) Do yourself

iv) $2 + 1 + 0.5 + \dots\dots$

$a_1 = 2,\ r = \frac{1}{2} = 0.5$

Now $S = \frac{a_1}{1-r} = \frac{2}{1-0.5}$

$$= \frac{2}{0.5} = 4$$

Answer

v) $4 + 2\sqrt{2} + 2 + \sqrt{2} + 1 + \dots\dots$

$a_1 = 4,\ r = \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

Now

$$S = \frac{a_1}{1-r} = \frac{4}{1-1/\sqrt{2}}$$

$$= \frac{4}{\frac{\sqrt{2}-1}{\sqrt{2}}} = \frac{4\sqrt{2}}{\sqrt{2}-1}$$

Answer OR

$$= \frac{4\sqrt{2}}{\sqrt{2}-1} \cdot \frac{\sqrt{2}+1}{\sqrt{2}+1}$$

$$= \frac{4((\sqrt{2})^2 + \sqrt{2})}{(\sqrt{2})^2 - (1)^2} = \frac{4(2+\sqrt{2})}{2-1}$$

$$= 4(2+\sqrt{2})$$

Answer

vi) Do yourself

Q.No6 i) $1.\dot{3}\dot{4}$

$= 1.343434\dots\dots$

$= 1 + 0.343434\dots\dots$

$= 1 + (0.34 + 0.0034 + 0.000034 + \dots)$

$a_1 = 0.34,\ r = \frac{0.0034}{0.34} = 0.01$

$$= 1 + \frac{0.34}{1-0.01} = 1 + \frac{0.34}{0.99} = 1 + \frac{34}{99}$$

$$= \frac{99+34}{99} = \frac{133}{99}$$

Answer

ii) $0.\dot{2}5\dot{9}$

$= 0.259259259\ldots\ldots$

$= 0.259 + 0.000259 + 0.000000259 + \ldots\ldots$

$a_1 = 0.259$, $r = \frac{0.000259}{0.259} = 0.001$

$= \frac{0.259}{1-0.001} = \frac{0.259}{0.999}$

$= \frac{259}{999}$ Answer

i) $1.1\dot{4}\dot{7}$

$= 1.1474747\ldots\ldots$

$= 1.1 + 0.0474747\ldots\ldots$

$= 1.1 + (0.047 + 0.00047 + 0.0000047 + \ldots\ldots)$

$a_1 = 0.047$, $r = \frac{0.00047}{0.047} = 0.01$

$= 1.1 + \frac{0.047}{1-0.01} = 1.1 + \frac{0.047}{0.99}$

$= \frac{11}{10} + \frac{47/1000}{99/100} = \frac{11}{10} + \frac{47}{1000}\cdot\frac{100}{99}$

$= \frac{11}{10} + \frac{47}{990} = \frac{1089+47}{990}$

$= \frac{1136}{990}$ Answer

QNo7 $r + (1+k)r^2 + (1+k+k^2)r^3 + \ldots\ldots$

÷ing & ×ing by $1-k$

$= \frac{1-k}{1-k}\left(r + (1+k)r^2 + (1+k+k^2)r^3 + \ldots\ldots\right)$

$= \frac{1}{1-k}\left[(1-k)r + (1-k)(1+k)r^2 + (1-k)(1+k+k^2)r^3 + \ldots\ldots\right]$

$= \frac{1}{1-k}\left((1-k)r + (1-k^2)r^2 + (1-k^3)r^3 + \ldots\ldots\right)$

$= \frac{1}{1-k}\left(r - rk + r^2 - r^2k^2 + r^3 - r^3k^3 + \ldots\ldots\right)$

$= \frac{1}{1-k}\left\{(r + r^2 + r^3 + \ldots\ldots) - (rk + r^2k^2 + r^3k^3 + \ldots\ldots)\right\}$

$= \frac{1}{1-k}\left(\frac{r}{1-r} - \frac{rk}{1-rk}\right)$ $\because S = \frac{a_1}{1-r}$

$= \frac{1}{1-k}\left\{\frac{r(1-rk) - rk(1-r)}{(1-r)(1-rk)}\right\}$

$= \frac{r - r^2k - rk + r^2k}{(1-k)(1-r)(1-rk)}$

$= \frac{r(1-k)}{(1-k)(1-r)(1-rk)}$

$= \frac{r}{(1-r)(1-rk)}$ Answer

QNo8 $y = \frac{x}{2} + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \ldots\ldots$

$a_1 = \frac{x}{2}$, $r = \frac{\frac{1}{4}x^2}{x/2} = \frac{\frac{1}{8}x^3}{\frac{1}{4}x^2} = \frac{x}{2}$

So This is an infinite Geometric series also $0 < x < 2 \Rightarrow 0 < \frac{x}{2} < 1$

i.e $r = \frac{x}{2} < 1$ so solution exists.

$\Rightarrow y = \frac{a_1}{1-r} = \frac{x/2}{1-x/2}$

$\Rightarrow y = \frac{x/2}{\frac{2-x}{2}} \Rightarrow y = \frac{x}{2-x}$

$\Rightarrow y(2-x) = x \Rightarrow 2y - xy = x$

$\Rightarrow 2y = x + xy \Rightarrow 2y = x(1+y)$

$\Rightarrow \frac{2y}{1+y} = x$ i.e $x = \frac{2y}{1+y}$

Answer

~~QNo9 Do yourself~~

Hint $0 < x < \frac{3}{2} \Rightarrow 0 < \frac{2}{3}x < 1$

QNo10

Distance travel in 1st fall = 27 m

" " " 2nd fall = $\frac{2}{3} \times 27 = 18$ m

" " " 3rd fall = $\frac{2}{3} \times 18 = 12$ m

So Sequence of fall is

$27 + 18 + 12 + \ldots$

This is infinite geometric sequence

if S_1 denotes distance travel by ball in fall then $a_1 = 27$, $r = \frac{18}{27} = \frac{2}{3} < 1$

$\Rightarrow S_1 = \frac{a_1}{1-r} = \frac{27}{1-2/3} = \frac{27}{1/3} = 81$ m

Now

Distance travel in 1st rebound $= \frac{2}{3} \times 27 = 18$

" " 2nd rebound $= \frac{2}{3} \times 18 = 12$

" " 3rd rebound $= \frac{2}{3} \times 12 = 8$

So sequence of rebound is

$18 + 12 + 8 + \cdots\cdots$

which is infinite Geometric series

If S_2 denotes distance travel by ball in rebound then $a_1 = 18$, $r = \frac{2}{3} < 1$

$$S_2 = \frac{a_1}{1-r} = \frac{18}{1-\frac{2}{3}} = \frac{18}{\frac{1}{3}} = 54$$

Now

total distance $= S_1 + S_2$

$= 81 + 54 = 135m$

Q No. 11 Same as Q No 10

Q No. 12 $y = 1 + 2x + 4x^2 + 8x^3 + \cdots\cdots$

i) $a_1 = 1$, $r = \frac{2x}{1} = \frac{4x^2}{2x} = 2x$

So $y = \frac{a_1}{1-r} = \frac{1}{1-2x}$

$\Rightarrow y = \frac{1}{1-2x} \Rightarrow y(1-2x) = 1$

$\Rightarrow y - 2xy = 1 \Rightarrow -2xy = 1 - y$

$\Rightarrow 2xy = y - 1 \Rightarrow x = \frac{1-y}{2x}$ proved

ii) Now series is convergent if

$|r| < 1$

$\Rightarrow |2x| < 1 \Rightarrow \pm 2x < 1$

$\Rightarrow \pm x < \frac{1}{2}$

$\Rightarrow x < \frac{1}{2}$ and $-x < \frac{1}{2}$

$\Rightarrow x < \frac{1}{2}$ and $x > -\frac{1}{2}$

$\Rightarrow -\frac{1}{2} < x < \frac{1}{2}$

hence series convergent if

$x \in \left(-\frac{1}{2}, \frac{1}{2}\right)$ or $-\frac{1}{2} < x < \frac{1}{2}$

Q No 13 Do yourself as Q No 12

Q No 14

Let the infinite geometric series

$a_1 + a_1 r + a_1 r^2 + \cdots\cdots$

then $S = \frac{a_1}{1-r}$

$\Rightarrow 9 = \frac{a_1}{1-r}$ $\because S = 9$ (given)

$\Rightarrow 9(1-r) = a_1$ ——— (i)

Now square of its terms.

$a_1^2 + a_1^2 r^2 + a_1^2 r^4 + \cdots\cdots$

then $S = \frac{a_1^2}{1-r^2}$ $\because a_1 = a_1^2,\ r = r^2$

$\Rightarrow \frac{81}{5} = \frac{a_1^2}{1-r^2}$ $\because S = \frac{81}{5}$ (given)

$\Rightarrow \frac{81}{5}(1-r^2) = a_1^2$

$\Rightarrow \frac{81}{5}(1-r)(1+r) = [9(1-r)]^2$ from (i)

$\Rightarrow \frac{\cancel{81}}{5}\cancel{(1-r)}(1+r) = \cancel{81}(1-r)^{\cancel{2}}$

$\Rightarrow \frac{1}{5}(1+r) = (1-r)$

$\Rightarrow (1+r) = 5 - 5r$

$\Rightarrow r + 5r = 5 - 1$

$\Rightarrow 6r = 4 \Rightarrow r = \frac{4}{6} = \frac{2}{3}$

putting in (i)

$9\left(1 - \frac{2}{3}\right) = a_1 \Rightarrow 9\left(\frac{1}{3}\right) = a_1$

$\Rightarrow a_1 = 3$

Now

$a_1 r = (3)\left(\frac{2}{3}\right) = 2$

$a_1 r^2 = (3)\left(\frac{2}{3}\right)^2 = (3)\left(\frac{4}{9}\right) = \frac{4}{3}$

$a_1 r^3 = (3)\left(\frac{2}{3}\right)^3 = (3)\left(\frac{8}{27}\right) = \frac{8}{9}$

Thus

$3 + 2 + \frac{4}{3} + \frac{8}{9} + \cdots\cdots$

is the required series

END