Exercise 4.8

$x^2 + \left[-(2x+4)\right]^2 + 4x = 1$

$x^2 + (2x+4)^2 + 4x - 1 = 0$

$x^2 + 4x^2 + 16x + 16 + 4x - 1 = 0$

$5x^2 + 20x + 15 = 0$

Dividing by 5 we get.

$\qquad x^2 + 4x + 3 = 0$

$\qquad x^2 + x + 3x + 3 = 0$

$\qquad x(x+1) + 3(x+1) = 0$

$\qquad (x+1)(x+3) = 0$

$x+1 = 0, \ x+3 = 0 \Rightarrow x = -1, \ x = -3$

If $x = -1$ then from eq③ | If $x = -3$ then from③

$\quad y = -(2(-1)+4)$ | $y = -(2(-3)+4)$

$\quad y = -(-2+4)$ | $y = -(-6+4)$

$\quad y = -2$ | $y = 2$

Hence $\left\{(-1,-2),(-3,2)\right\}$

EXERCISE.4.8

Q.1 $2x - y = 4 \longrightarrow ①$

$\qquad 2x^2 - 4xy - y^2 = 6 \longrightarrow ②$

From ① $\quad y = 2x - 4 \longrightarrow ③$

Putting value of y in equation ②

$\quad 2x^2 - 4x(2x-4) - (2x-4)^2 = 6$

$\quad 2x^2 - 8x^2 + 16x - (4x^2 - 16x + 16) - 6 = 0$

$\quad -6x^2 + 16x - 4x^2 + 16x - 16 - 6 = 0$

$\quad -10x^2 + 32x - 22 = 0$

Dividing by -2 we get

$\qquad 5x^2 - 16x - 11 = 0$

$\quad 5x^2 - 5x - 11x + 11 = 0$

$\quad 5x(x-1) - 11(x-1) = 0$

$\quad (x-1)(5x-11) = 0$

$x - 1 = 0, \ 5x - 11 = 0 \Rightarrow x = 1, \ x = 11/5$

If $x = 1$ then from ③ | If $x = 11/5$ then from③

$\quad y = 2(1) - 4$ | $y = 2(11/5) - 4$

$\quad y = 2 - 4 = -2$ | $y = 22/5 - 4$

| $y = \dfrac{22-20}{5} = 2/5$

$\left\{(1,-2),(11/5,2/5)\right\}$

Q.2 $x + y = 5 \longrightarrow ①$

$\qquad x^2 + 2y^2 = 17 \longrightarrow ②$

From ① $\quad y = 5 - x \longrightarrow ③$

Putting value of y in equation ②

$\quad x^2 + 2(5-x)^2 = 17$

$\quad x^2 + 2(25 - 10x + x^2) - 17 = 0$

$\quad x^2 + 50 - 20x + 2x^2 - 17 = 0$

$\quad 3x^2 - 20x + 33 = 0$

$\quad 3x^2 - 9x - 11x + 33 = 0$

$\quad 3x(x-3) - 11(x-3) = 0$

$\quad (x-3)(3x-11) = 0$

$\quad x - 3 = 0 \ ; \ 3x - 11 = 0$

$\quad x = 3 \ , \ x = 11/3$

If $x = 3$ then from③ | If $x = 11/3$ then from③

$\quad y = 5 - 3$ | $y = 5 - 11/3$

$\quad y = 2$ | $y = \dfrac{15-11}{3}$

| $y = 4/3$

$\left\{(3,2),(11/3,4/3)\right\}$

Q.3 $3x + 2y = 7 \longrightarrow ①$

$\qquad 3x^2 = 25 + 2y^2$

or $\quad 3x^2 - 2y^2 = 25 \longrightarrow ②$

From① $\quad 2y = 7 - 3x \Rightarrow y = \dfrac{7-3x}{2} \to ③$

Putting value of y in eq ②

$\quad 3x^2 - 2\left(\dfrac{7-3x}{2}\right)^2 = 25$

$\quad 3x^2 - 2\left(\dfrac{49 - 42x + 9x^2}{4}\right) - 25 = 0$

$\quad 3x^2 - \left(\dfrac{9x^2 - 42x + 49}{2}\right) - 25 = 0$

$\quad 6x^2 - (9x^2 - 42x + 49) - 50 = 0$

$\quad 6x^2 - 9x^2 + 42x - 49 - 50 = 0$

$\quad -3x^2 + 42x - 99 = 0$

Dividing by -3 we get

$$x^2 - 14x + 33 = 0$$
$$x^2 - 3x - 11x + 33 = 0$$
$$x(x-3) - 11(x-3) = 0$$
$$(x-3)(x-11) = 0$$
$$x - 3 = 0, \ x - 11 = 0 \Rightarrow x = 3, x = 11$$

If $x=3$ then from ③

$$y = \frac{7 - 3(3)}{2}$$
$$y = \frac{7-9}{2} = \frac{-2}{2} = -1$$

If $x=11$ then from ③

$$y = \frac{7 - 3(11)}{2}$$
$$y = \frac{7 - 33}{2} = \frac{-26}{2}$$
$$y = -13$$

$$\{(3,-1),(11,-13)\}$$

Q.4

$$x + y = 5 \longrightarrow ①$$
$$\frac{2}{x} + \frac{3}{y} = 2$$

Multiplying by xy we get

or $2y + 3x = 2xy \longrightarrow ②$

From ① $y = 5 - x \longrightarrow ③$

Putting value of y in equation ②

$$2(5-x) + 3x = 2x(5-x)$$
$$10 - 2x + 3x = 10x - 2x^2$$
$$10 + x = 10x - 2x^2$$
$$2x^2 - 10x + x + 10 = 0$$
$$2x^2 - 9x + 10 = 0$$
$$2x^2 - 4x - 5x + 10 = 0$$
$$2x(x-2) - 5(x-2) = 0$$
$$(x-2)(2x-5) = 0$$
$$x - 2 = 0, \ 2x - 5 = 0$$
$$x = 2, \quad x = \frac{5}{2}$$

If $x=2$ then from ③

$$y = 5 - 2$$
$$y = 3$$

If $x = \frac{5}{2}$ then from ③

$$y = 5 - \frac{5}{2}$$
$$y = \frac{10 - 5}{2}$$
$$y = \frac{5}{2}$$

$$\left\{(2,3),\left(\frac{5}{2},\frac{5}{2}\right)\right\}$$

Q.5

$$x + y = a + b \longrightarrow ①$$
$$\frac{a}{x} + \frac{b}{y} = 2$$

or $ay + bx = 2xy \longrightarrow ②$

From ① $y = a + b - x \longrightarrow ③$

Putting value of y in eq ②

$$a(a+b-x) + bx = 2x(a+b-x)$$
$$a^2 + ab - ax + bx = 2ax + 2bx - 2x^2$$
$$2x^2 - 2ax - 2bx - ax + bx + a^2 + ab = 0$$
$$2x^2 - 3ax - bx + a^2 + ab = 0$$

Using $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-[-(3a+b)] \pm \sqrt{(-(3a+b))^2 - 4(2)(a^2+ab)}}{2(2)}$$

$$x = \frac{(3a+b) \pm \sqrt{(3a+b)^2 - 8(a^2+ab)}}{4}$$

$$x = \frac{(3a+b) \pm \sqrt{9a^2 + b^2 + 6ab - 8a^2 - 8ab}}{4}$$

$$x = \frac{(3a+b) \pm \sqrt{a^2 + b^2 - 2ab}}{4}$$

$$x = \frac{(3a+b) \pm \sqrt{(a-b)^2}}{4}$$

$$x = \frac{(3a+b) \pm (a-b)}{4}$$

$$x = \frac{3a+b+a-b}{4}, \quad x = \frac{3a+b-(a-b)}{4}$$

$$x = \frac{4a}{4}, \quad x = \frac{3a+b-a+b}{4}$$

$$x = a, \quad x = \frac{2a+2b}{4} \Rightarrow x = \frac{a+b}{2}$$

If $x=a$ then from ③

$$y = a + b - a$$
$$y = b$$

If $x = \frac{a+b}{2}$ then from ③

$$y = a + b - \frac{a+b}{2}$$
$$y = \frac{2a + 2b - a - b}{2}$$
$$y = \frac{a+b}{2}$$

$\{(a,b), (\frac{a+b}{2}, \frac{a+b}{2})\}$

Q.6

$3x + 4y = 25 \longrightarrow ①$

$\frac{3}{x} + \frac{4}{y} = 2$

or $3y + 4x = 2xy \longrightarrow ②$

From ① $4y = 25 - 3x \Rightarrow y = \frac{25-3x}{4} \longrightarrow ③$

Putting value of y in equation ②

$3(\frac{25-3x}{4}) + 4x = 2x(\frac{25-3x}{4})$

Multiplying by 4 we get

$3(25-3x) + 4x = 2x(25-3x)$

$75 - 9x + 16x = 50x - 6x^2$

$75 + 7x = 50x - 6x^2$

$6x^2 - 50x + 7x + 75 = 0$

$6x^2 - 43x + 75 = 0$

$6x^2 - 18x - 25x + 75 = 0$

$6x(x-3) - 25(x-3) = 0$

$(x-3)(6x-25) = 0$

$x - 3 = 0 \quad , \quad 6x - 25 = 0$

$x = 3 \quad , \quad x = 25/6$

If x=3 then from ③	If x=25/6 than from ③
$y = \frac{25-3(3)}{4}$	$y = \frac{25-3(25/6)}{4}$
$y = \frac{25-9}{4}$	$y = \frac{1}{4}(25 - \frac{25}{2})$
$y = \frac{16}{4} = 4$	$y = \frac{1}{4}(\frac{50-25}{2})$
	$y = \frac{1}{4} \cdot \frac{25}{2} = \frac{25}{8}$

$\{(3,4), (25/6, 25/8)\}$

Q.7

$(x-3)^2 + y^2 = 5$

$x^2 - 6x + 9 + y^2 = 5$

$\rightarrow x^2 + y^2 - 6x + 4 = 0 \longrightarrow ①$

$2x = y + 6 \longrightarrow ②$

From ② $y = 2x - 6 \longrightarrow ③$

Putting value of y in equation (1)

$x^2 + (2x-6)^2 - 6x + 4 = 0$

$x^2 + 4x^2 - 24x + 36 - 6x + 4 = 0$

$5x^2 - 30x + 40 = 0$

Dividing by 5 we get

$x^2 - 6x + 8 = 0$

$x^2 - 2x - 4x + 8 = 0$

$x(x-2) - 4(x-2) = 0$

$(x-2)(x-4) = 0$

$x - 2 = 0 \quad , \quad x - 4 = 0$

$x = 2 \quad , \quad x = 4$

If x=2 then from ③	If x=4 then from ③
$y = 2(2) - 6$	$y = 2(4)$
$y = 4-6 = -2$	$y = 8-6 = 2$

$\{(2,-2),(4,2)\}$

Q.8

$(x+3)^2 + (y-1)^2 = 5$

or $x^2 + 6x + 9 + y^2 - 2y + 1 - 5 = 0$

$x^2 + y^2 + 6x - 2y + 5 = 0 \longrightarrow ①$

$x^2 + y^2 + 2x = 9$

or $x^2 + y^2 + 2x - 9 = 0$

Subtracting eq ② from eq ①

$x^2 + y^2 + 6x - 2y + 5 = 0$

$\underline{x^2 + y^2 + 2x \qquad - 9 = 0}$

$4x - 2y + 14 = 0$

Dividing by 2 we get

$2x - y + 7 = 0$

$\Rightarrow y = 2x + 7 \longrightarrow ③$

Putting value of y in equation ②

$x^2 + (2x+7)^2 + 2x - 9 = 0$

$x^2 + 4x^2 + 28x + 49 + 2x - 9 = 0$

$5x^2 + 30x + 40 = 0$

Dividing by 5 we get

$x^2 + 6x + 8 = 0$

$x^2 + 2x + 4x + 8 = 0$

$x(x+2) + 4(x+2) = 0$

$(x+2)(x+4) = 0$

$x + 2 = 0 \qquad , \qquad x + 4 = 0$

$x = -2 \quad , \quad x = -4$

If $x = -2$ then from ③ | If $x = -4$ then from ③

$y = 2(-2) + 7 \qquad | \qquad y = 2(-4) + 7$

$y = -4 + 7 \qquad | \qquad y = -8 + 7$

$y = 3 \qquad\qquad | \qquad y = -1$

$\{(-2, 3), (-4, -1)\}$

Q.9

$x^2 + (y+1)^2 = 18$

$x^2 + y^2 + 2y + 1 - 18 = 0$

$x^2 + y^2 + 2y - 17 = 0 \longrightarrow ①$

$(x+2)^2 + y^2 = 21$

$x^2 + 4x + 4 + y^2 - 21 = 0$

$x^2 + y^2 + 4x - 17 = 0 \longrightarrow ②$

Subtracting eq ② from eq ①

$\begin{array}{r} x^2 + y^2 + 2y - 17 = 0 \\ x^2 + y^2 + 4x - 17 = 0 \\ \hline 2y - 4x = 0 \end{array}$

$2y - 4x = 0 \rightarrow 2x = 4x \rightarrow y = 2x$

Putting value of y in equation ②

$x^2 + (2x)^2 + 4x - 17 = 0$

$x^2 + 4x^2 + 4x - 17 = 0$

$5x^2 + 4x - 17 = 0$

Using $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-4 \pm \sqrt{(4)^2 - 4(5)(-17)}}{2(5)}$

$x = \dfrac{-4 \pm \sqrt{16 + 340}}{10}$

$x = \dfrac{-4 \pm \sqrt{356}}{10} \rightarrow x = \dfrac{-4 \pm \sqrt{4 \times 89}}{10}$

$x = \dfrac{-4 \pm 2\sqrt{89}}{10} \rightarrow x = \dfrac{2(-2 \pm \sqrt{89})}{10}$

$x = \dfrac{-2 \pm \sqrt{89}}{5}$

If $x = \dfrac{-2 + \sqrt{89}}{5}$ then from ③ | If $x = \dfrac{-2 - \sqrt{89}}{5}$ Then from ③

$y = 2\left(\dfrac{-2 + \sqrt{89}}{5}\right) \qquad | \qquad y = 2\left(\dfrac{-2 - \sqrt{89}}{5}\right)$

$y = \dfrac{-4 + 2\sqrt{89}}{5} \qquad | \qquad y = \dfrac{-4 - 2\sqrt{89}}{5}$

$\left\{\left(\dfrac{-2 + \sqrt{89}}{5}, \dfrac{-4 + 2\sqrt{89}}{5}\right), \left(\dfrac{-2 - \sqrt{89}}{5}, \dfrac{-4 - 2x}{5}\right)\right.$

Q.10

$x^2 + y^2 + 6x = 1 \longrightarrow ①$

$x^2 + y^2 + 2(x+y) = 3$

or $x^2 + y^2 + 2x + 2y = 3 \longrightarrow ②$

Subtracting equation ② from ①

$\begin{array}{r} x^2 + y^2 + 6x = 1 \\ x^2 + y^2 + 2x + 2y = 3 \\ \hline 4x - 2y = -2 \end{array}$

Dividing by 2 we get

$2x - y = -1$

$\implies y = 2x + 1$

Putting values of y in equation ①

$x^2 + (2x+1)^2 + 6x = 1$

$x^2 + 4x^2 + 4x + 1 + 6x - 1 = 0$

$5x^2 + 10x = 0$

$5x(x + 2) = 0$

$5x = 0 \quad , \quad x + 2 = 0$

$x = 0 \quad , \quad x = -2$

If $x = 0$ then from ⑤ | If $x = -2$ then from ①

$y = 2(0) + 1 \qquad | \qquad y = 2(-2) + 1$

$y = 1 \qquad\qquad | \qquad y = -4 + 1 = -3$

$\{(0, 1), (-2, -3)\}$