$y = \frac{1+\sqrt{33}}{2}, y = \frac{1-\sqrt{33}}{2}$

If $y = \frac{1+\sqrt{33}}{2}$ then $x = \frac{1+\sqrt{33}}{2}$

If $y = \frac{1-\sqrt{33}}{2}$ then $x = \frac{1-\sqrt{33}}{2}$

$x = 4$

If $y = -4/3$ then $x = 2(-4/3)$

$x = -8/3$

$\left\{\left(\frac{1+\sqrt{33}}{2}, \frac{1+\sqrt{33}}{2}\right), \left(\frac{1-\sqrt{33}}{2}, \frac{1-\sqrt{33}}{2}\right), (4,2), (-8/3, -4/3)\right\}$

(C) When both equations are quadratic but one of them involving xy in product form.

Solution Method

In this type we balance constant terms of both equations, then subtracting each other we get Homogeneous Quadratic Equation and solve as homogeneous case.

Questions of such type are Exp ③, Q. 7, 8, 9, 10

Example 3

$$x^2 - y^2 = 5 \longrightarrow ①$$
$$4x^2 - 3xy = 18 \longrightarrow ②$$

To balance constant terms multiplying eq. ① by 18 and eq. ② by 5 we get

$$18x^2 - 18y^2 = 90 \longrightarrow ③$$
$$20x^2 - 15xy = 90 \longrightarrow ④$$

Subtracting eq ③ from eq ④

$$\begin{array}{r} 20x^2 - 15xy = 90 \\ \mp 18x^2 \mp 18y^2 = 90 \\ \hline 2x^2 + 18y^2 - 15xy = 0 \end{array}$$

$$2x^2 - 15xy + 18y^2 = 0$$

Factorizing it we get

$$2x^2 - 12xy + 3xy - 18y^2 = 0$$
$$2x(x-6y) - 3y(x-6y) = 0$$
$$(x-6y)(2x-3y) = 0$$
$$x - 6y = 0 \quad , \quad 2x - 3y = 0$$
$$x = 6y \quad , \quad x = \frac{3}{2}y$$

If $x = 6y$ then from ①

$(6y)^2 - y^2 = 5$

$36y^2 - y^2 = 5$

$35y^2 = 5$

$y^2 = \frac{5}{35} \rightarrow y^2 = \frac{1}{7}$

$y = \pm\frac{1}{\sqrt{7}}$

$y = \frac{1}{\sqrt{7}}, y = \frac{-1}{\sqrt{7}}$

If $y = \frac{1}{\sqrt{7}}$ then $x = 6\left(\frac{1}{\sqrt{7}}\right) = \frac{6}{\sqrt{7}}$

If $y = \frac{-1}{\sqrt{7}}$ then $x = 6\left(\frac{-1}{\sqrt{7}}\right) = \frac{-6}{\sqrt{7}}$

If $x = \frac{3}{2}y$ then from ①

$\left(\frac{3}{2}y\right)^2 - y^2 = 5$

$\frac{9}{4}y^2 - y^2 = 5$

$9y^2 - 4y^2 = 20$

$5y^2 = 20$

$y^2 = 4 \rightarrow y = \pm 2$

$y = 2, y = -2$

If $y = 2$ then $x = \frac{3}{2}(2) \rightarrow x = 3$

If $y = -2$ then $x = \frac{3}{2}(-2) = -3$

$\left\{\left(\frac{6}{\sqrt{7}}, \frac{1}{\sqrt{7}}\right), \left(\frac{-6}{\sqrt{7}}, \frac{-1}{\sqrt{7}}\right), (3,2), (-3,-2)\right\}$

EXERCISE 4.3

Q.1 $2x^2 = 6 + 3y^2$

or $2x^2 - 3y^2 = 6$

and $3x^2 - 5y^2 = 7$

Put $x^2 = u$, $y^2 = v$ thus we get

$$2u - 3v = 6 \longrightarrow ①$$
$$3u - 5v = 7 \longrightarrow ②$$

To balance coefficient of u multiplying eq ① by 3 and eq ② by 2 we get

$$6u - 9v = 18 \longrightarrow ③$$
$$6u - 10v = 14 \longrightarrow ④$$

Subtracting eq ③ from eq ④

$$\begin{array}{r} 6u - 10v = 14 \\ \mp 6u \pm 9v = \mp 18 \\ \hline -v = -4 \end{array} \rightarrow v = 4$$

Putting value of v in ① we get

$$2u-3(4)=6$$

$$2u-12=6 \Rightarrow 2u=6+12$$

$$2u=18 \Rightarrow u=9$$

If $u=9$ then | If $v=4$ then

$x^2=9 \Rightarrow x=\pm 3$ | $y^2=4 \Rightarrow y=\pm 2$

$\{(\pm 3, \pm 2)\}$

or $\{(3,\pm 2),(-3,\pm 2)\}$

or $\{(3,2),(3,-2),(-3,2),(-3,-2)\}$

Q.2 $8x^2=y^2$

or $8x^2-y^2=0$

and $x^2+2y^2=19$

Put $x^2=u$ and $y^2=v$ we get

$$8u-v=0 \longrightarrow ①$$

$$u+2v=19 \longrightarrow ②$$

To balance coefficient of v multiply eq ① by 2 we get

$$16u-2v=0 \longrightarrow ③$$

Adding ② and ③

$$\begin{array}{r} u+2v=19 \\ 16u-2v=0 \\ \hline 17u=19 \end{array}$$

$\Rightarrow u=\frac{19}{17}$, Putting value of u in ① we get

$$8\left(\frac{19}{17}\right)-v=0 \Rightarrow v=\frac{152}{17}$$

If $u=19/17$ then | If $v=\frac{152}{17}$ then

$x^2=\frac{19}{17}$ | $y^2=\frac{152}{17}$

$x=\pm\sqrt{\frac{19}{17}}$ | $y=\pm\sqrt{\frac{152}{17}}$

$y=\pm\sqrt{\frac{4\times 38}{17}}$

$y=\pm 2\sqrt{\frac{38}{17}}$

$\left\{\left(\pm\sqrt{\frac{19}{17}}, \pm 2\sqrt{\frac{38}{17}}\right)\right\}$

Q.3 $2x^2-8=5y^2$ or $2x^2-5y^2=8$

and $x^2-13=-2y^2$ or $x^2-2y^2=13$

Put $x^2=u$ and $y^2=v$ we get

$$2u-5v=8 \longrightarrow ①$$

$$u+2v=13 \longrightarrow ②$$

To balance coefficient of u multiply eq ② by 2 we get

$$2u+4v=26 \longrightarrow ③$$

Subtracting eq ① from eq ③

$$\begin{array}{r} 2u+4v=26 \\ 2u-5v=8 \\ \hline 9v=18 \end{array}$$

$\Rightarrow v=2$

put $v=2$ in eq ② we get

$$u+2(2)=13 \Rightarrow u=13-4 \Rightarrow u=9$$

If $u=9$ then | If $v=2$ then

$x^2=9$ | $y^2=2$

$x=\pm 3$ | $y=\pm\sqrt{2}$

$\{(\pm 3, \pm\sqrt{2})\}$

Q.4 $x^2-5xy+6y^2=0 \longrightarrow ①$

$x^2+y^2=45 \longrightarrow ②$

Factorizing homogeneous equation we get

$$x^2-2xy-3xy+6y^2=0$$

$$x(x-2y)-3y(x-2y)=0$$

$$(x-2y)(x-3y)=0$$

$$x-2y=0 \quad , \quad x-3y=0$$

$$x=2y \quad , \quad x=3y$$

If $x=2y$ then from ② | If $x=3y$ then from ②

$(2y)^2+y^2=45$ | $(3y)^2+y^2=45$

$4y^2+y^2=45$ | $9y^2+y^2=45$

$5y^2=45$ | $10y^2=45$

$y^2=9 \Rightarrow y=\pm 3$ | $y^2=\frac{45}{10} \Rightarrow y^2=\frac{9}{2}$

$y=3$, $y=-3$ | $y=\pm\frac{3}{\sqrt{2}}$

If $y=3$ then | $y=\frac{3}{\sqrt{2}}$, $y=\frac{-3}{\sqrt{2}}$

$x=2(3)=6$ | If $y=\frac{3}{\sqrt{2}}$ then

If $y=-3$ then | $x=3\left(\frac{3}{\sqrt{2}}\right)=\frac{9}{\sqrt{2}}$

$x = 2(-3)$

$x = -6$

$\{(6,3),(-6,-3),$

If $y = \frac{-3}{\sqrt{2}}$ then

$x = 3\left(\frac{-3}{\sqrt{2}}\right) = \frac{-9}{\sqrt{2}}$

$\left(\frac{9}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right), \left(\frac{-9}{\sqrt{2}}, \frac{-3}{\sqrt{2}}\right)\}$

Q.5 $12x^2 - 25xy + 12y^2 = 0 \longrightarrow ①$

$4x^2 + 7y^2 = 148 \longrightarrow ②$

Factorizing homogeneous equation we get

$12x^2 - 16xy - 9xy + 12y^2 = 0$

$4x(3x - 4y) - 3y(3x - 4y) = 0$

$(3x - 4y)(4x - 3y) = 0$

$3x - 4y = 0 \quad , \quad 4x - 3y = 0$

$3x = 4y \quad , \quad 4x = 3y$

$x = \frac{4}{3}y \quad , \quad x = \frac{3}{4}y$

If $x = \frac{4}{3}y$ then from ②

$4\left(\frac{4}{3}y\right)^2 + 7y^2 = 148$

$\frac{64}{9}y^2 + 7y^2 = 148$

$64y^2 + 63y^2 = 1332$

$127y^2 = 1332$

$y^2 = \frac{1332}{127}$

$y = \pm\sqrt{\frac{1332}{127}}$

$y = \pm\sqrt{\frac{4 \times 333}{127}}$

$y = \pm 2\sqrt{\frac{333}{127}}$

$y = 2\sqrt{\frac{333}{127}}, y = -2\sqrt{\frac{333}{127}}$

If $y = 2\sqrt{\frac{333}{127}}$ then

$x = \frac{4}{3}\left(2\sqrt{\frac{333}{127}}\right)$

$= \frac{8}{3}\sqrt{\frac{333}{127}}$

If $y = -2\sqrt{\frac{333}{127}}$ then

$x = \frac{4}{3}\left(-2\sqrt{\frac{333}{127}}\right)$

$x = \frac{-8}{3}\sqrt{\frac{333}{127}}$

If $x = \frac{3}{4}y$ then from ②

$4\left(\frac{3}{4}y\right)^2 + 7y^2 = 148$

$\frac{9}{4}y^2 + 7y^2 = 148$

$9y^2 + 28y^2 = 592$

$37y^2 = 592$

$y^2 = \frac{592}{37} = 16$

$y = \pm\sqrt{16}$

$y = \pm 4$

$y = 4, y = -4$

If $y = 4$ then

$x = \frac{3}{4}(4)$

$x = 3$

If $y = -4$ then

$x = \frac{3}{4}(-4)$

$x = -3$

$\left\{\left(\frac{8}{3}\sqrt{\frac{333}{127}}, 2\sqrt{\frac{333}{127}}\right), \left(\frac{-8}{3}\sqrt{\frac{333}{127}}, -2\sqrt{\frac{333}{127}}\right)\right.$

$\left.(3,4), (-3,-4)\right\}$

Q.6 $12x^2 - 11xy + 2y^2 = 0 \longrightarrow ①$

$2x^2 + 7xy = 60 \longrightarrow ②$

Factorizing homogeneous equation we get

$12x^2 - 8xy - 3xy + 2y^2 = 0$

$4x(3x - 2y) - y(3x - 2y) = 0$

$(3x - 2y)(4x - y) = 0$

$3x - 2y = 0 \quad , \quad 4x - y = 0$

$3x = 2y, 4x = y \Rightarrow x = \frac{2}{3}y, x = \frac{1}{4}y$

If $x = \frac{2}{3}y$ then from ②

$2\left(\frac{2}{3}y\right)^2 + 7\left(\frac{2}{3}y\right)y = 60$

$\frac{8}{9}y^2 + \frac{14}{3}y^2 = 60$

$8y^2 + 42y^2 = 540$

$50y^2 = 540$

$y^2 = \frac{540}{50} \Rightarrow y = \frac{54}{5}$

$y = \pm\sqrt{\frac{54}{5}}$

$y = \pm\sqrt{\frac{9 \times 6}{5}}$

$y = \pm 3\sqrt{\frac{6}{5}}$

$y = 3\sqrt{\frac{6}{5}}, y = -3\sqrt{\frac{6}{5}}$

If $y = 3\sqrt{\frac{6}{5}}$ then

$x = \frac{2}{3}\left(3\sqrt{\frac{6}{5}}\right) = 2\sqrt{\frac{6}{5}}$

If $y = -3\sqrt{\frac{6}{5}}$ then

$x = \frac{2}{3}\left(-3\sqrt{\frac{6}{5}}\right)$

$x = -2\sqrt{\frac{6}{5}}$

If $x = \frac{1}{4}y$ then from ②

$2\left(\frac{1}{4}y\right)^2 + 7\left(\frac{1}{4}y\right)y = 60$

$\frac{1}{8}y^2 + \frac{7}{4}y^2 = 60$

$y^2 + 14y^2 = 480$

$15y^2 = 480$

$y^2 = \frac{480}{15} \Rightarrow y^2 = 32$

$y = \pm\sqrt{32}$

$y = \pm\sqrt{16 \times 2}$

$y = \pm 4\sqrt{2}$

$y = 4\sqrt{2}, y = -4\sqrt{2}$

If $y = 4\sqrt{2}$ then

$x = \frac{1}{4}(4\sqrt{2}) = \sqrt{2}$

If $y = -4\sqrt{2}$ then

$x = \frac{1}{4}(-4\sqrt{2})$

$x = -\sqrt{2}$

$\left\{\left(2\sqrt{\frac{6}{5}}, 3\sqrt{\frac{6}{5}}\right), \left(-2\sqrt{\frac{6}{5}}, -3\sqrt{\frac{6}{5}}\right)\right.$

$\left.(\sqrt{2}, 4\sqrt{2}), (-\sqrt{2}, -4\sqrt{2})\right\}$

Q.7 $x^2 - y^2 = 16 \longrightarrow ①$

$xy = 15 \longrightarrow ②$

To balance constant terms multiply eq ① by 15 and eq ② by 16 we get

$15x^2 - 15y^2 = 240 \longrightarrow ③$

$16xy = 240 \longrightarrow ④$

Subtracting eq ④ from eq ③

$15x^2 - 15y^2 - 16xy = 0$

$15x^2 - 16xy - 15y^2 = 0$

Factorizing

$15x^2 - 25xy + 9xy - 15y^2 = 0$

$5x(3x - 5y) + 3y(3x - 5y) = 0$

$(3x - 5y)(5x + 3y) = 0$

$3x - 5y = 0 \quad , \quad 5x + 3y = 0$

$3x = 5y \quad , \quad 5x = -3y$

$x = \frac{5}{3}y \quad , \quad x = -\frac{3}{5}y$

If $x = \frac{5}{3}y$ then from ②

$(\frac{5}{3}y)y = 15$

$\frac{5}{3}y^2 = 15$

$y^2 = 15 \times \frac{3}{5}$

$y^2 = 9 \Rightarrow y = \pm 3$

$y = 3, y = -3$

If $y = 3$ then

$x = \frac{5}{3}(3) = 5$

If $y = -3$ then

$x = \frac{5}{3}(-3)$

$x = -5$

If $x = -\frac{3}{5}y$ then from ②

$(-\frac{3}{5}y)y = 15$

$-\frac{3}{5}y^2 = 15$

$y^2 = 15(-\frac{5}{3})$

$y^2 = -25$

$y = \pm\sqrt{-25} \Rightarrow y = \pm 5i$

$y = 5i, y = -5i$

If $y = 5i$ then

$x = \frac{-3}{5}(5i) = -3i$

If $y = -5i$ then

$x = \frac{-3}{5}(-5i)$

$x = 3i$

$\{(5,3), (-5,-3), (-3i, 5i), (3i, -5i)\}$

Q.8 $x^2 + xy = 9 \longrightarrow ①$

$x^2 - y^2 = 2 \longrightarrow ②$

To balance constant term multiply eq ① by 2 and eq ② by 9 we get

$2x^2 + 2xy = 18 \longrightarrow ③$

$9x^2 - 9y^2 = 18 \longrightarrow ④$

Subtracting eq ③ from ④

$9x^2 - 9y^2 = 18$

$\underline{2x^2 + 2xy = 18}$

$7x^2 - 9y^2 - 2xy = 0$

$7x^2 - 2xy - 9y^2 = 0$

$7x^2 + 7xy - 9xy - 9y^2 = 0$

$7x(x+y) - 9y(x+y) = 0$

$(x+y)(7x - 9y) = 0$

$x + y = 0 \quad , \quad 7x - 9y = 0$

$x = -y \quad , \quad x = \frac{9}{7}y$

If $x = -y$ then from ②

$y^2 - y^2 = 2$

$0 = 2$

which is impossible

If $x = \frac{9}{7}y$ then from ②

$(\frac{9}{7}y)^2 - y^2 = 2$

$\frac{81}{49}y^2 - y^2 = 2$

$81y^2 - 49y^2 = 98$

$32y^2 = 98$

$y^2 = \frac{98}{32} = \frac{49}{16}$

$y = \pm\frac{7}{4}$

$y = \frac{7}{4}, y = -\frac{7}{4}$

If $y = \frac{7}{4}$ then $x = \frac{9}{7}(\frac{7}{4}) = \frac{9}{4}$

If $y = -\frac{7}{4}$ then $x = \frac{9}{7}(\frac{-7}{4}) = -\frac{9}{4}$

$\{(-\frac{9}{4}, -\frac{7}{4}), (\frac{9}{4}, \frac{7}{4})\}$

Q.9

$y^2 - 7 = 2xy$ or $y^2 - 2xy = 7 \longrightarrow ①$

$2x^2 + 3 = xy$ or $2x^2 - xy = -3 \longrightarrow ②$

To balance coefficients multiply eq ① by 3 and eq ② by 7 we get

$3y^2 - 6xy = 21 \longrightarrow ③$

$14x^2 - 7xy = -21 \longrightarrow ④$

Adding eq ③ and eq ④

$$3y^2 - 6xy = 21$$
$$14x^2 - 7xy = -21$$
$$14x^2 + 3y^2 - 13xy = 0$$

$$14x^2 - 13xy + 3y^2 = 0$$
$$14x^2 - 7xy - 6xy + 3y^2 = 0$$
$$7x(2x-y) - 3y(2x-y) = 0$$
$$(2x-y)(7x-3y) = 0$$
$$2x - y = 0 \quad , \quad 7x - 3y = 0$$
$$x = \frac{1}{2}y \quad , \quad x = \frac{3}{7}y$$

If $x = \frac{1}{2}$ then from ①

$$y^2 - 2\left(\frac{1}{2}y\right)y = 7$$
$$y^2 - y^2 = 7$$
$$0 = 7$$

which is impossible

If $x = \frac{3}{7}y$ then from ①

$$y^2 - 2\left(\frac{3}{7}y\right)y = 7$$
$$y^2 - \frac{6}{7}y^2 = 7$$
$$7y^2 - 6y^2 = 49$$
$$y^2 = 49$$
$$y = \pm 7$$
$$y = 7, \; y = -7$$

If $y = 7$ then $x = \frac{3}{7}(7) \rightarrow x = 3$

If $y = -7$ then $x = \frac{+3}{7}(-7) \rightarrow x = -3$

$$\{(3,7), (-3,-7)\}$$

Q.10

$$x^2 + y^2 = 5 \longrightarrow ①$$
$$xy = 2 \longrightarrow ②$$

To balance coefficients multiply eq ① by 2 and ② by 5 we get

$$2x^2 + 2y^2 = 10 \longrightarrow ③$$
$$5xy = 10 \longrightarrow ④$$

Subtracting eq ④ from eq ③

$$2x^2 + 2y^2 - 5xy = 0$$
$$2x^2 - 5xy + 2y^2 = 0$$
$$2x^2 - xy - 4xy + 2y^2 = 0$$
$$x(2x-y) - 2y(2x-y) = 0$$
$$(x-2y)(2x-y) = 0$$
$$x - 2y = 0 \quad , \quad 2x - y = 0$$
$$x = 2y \quad , \quad x = \frac{1}{2}y$$

If $x = 2y$ then from ②

$$(2y)y = 2$$
$$2y^2 = 2$$
$$y^2 = 1 \rightarrow y = \pm 1$$
$$y = 1, \; y = -1$$

If $y = 1$ then $x = 2(1) = 2$

If $y = -1$ then $x = 2(-1) = -2$

If $x = \frac{1}{2}y$ then from ②

$$\left(\frac{1}{2}y\right)y = 2$$
$$y^2 = 4$$
$$y = \pm 2$$
$$y = 2, \; y = -2$$

If $y = 2$ then $x = \frac{1}{2}(+2) = +1$

If $y = +2$ then $x = \frac{1}{2}(-2) = -1$

$$\{(2,1), (-2,-1), (1,2), (-1,-2)\}$$

Theoretical Problems on Quadratic Equation:

To solve theoretical problems we should keep following steps in mind.

(i) Read the problem carefully.

(ii) Suppose the unknown quantities as x, y, z c.l.c

(iii) Translate the problem in to symbols. For example if we are given

(a) 5 is greater then x, we write it as $5 - x$

(b) x is greater then y, we write it as $x - y$

(c) x is greater then z by 5, we write it as $x - z = 5$

(d) 5 is less than x, we write it as $x - 5$

Example.1 Let x be the one part then $12 - x$ will be another part.