حل مشق 8.1

1. اگر $A=\{1,4,7,8\}, B=\{4,6,8,9\}$ اور $C=\{3,4,5,7\}$ ہوتو درج ذیل معلوم کیجیے۔

(i) $A\cup B$ (ii) $B\cup C$ (iii) $A\cap C$

(iv) $A\cap(B\cap C)$ (v) $(A\cup B)\cup C$ (vi) $(A\cap B)\cap C$

حل:

$A=\{1,4,7,8\}, B=\{4,6,8,9\}, C=\{3,4,5,7\}$

$A\cup B=\{1,4,7,8\}\cup\{4,6,8,9\}$

$=\{1,4,6,7,8,9\}$

$B\cup C=\{4,6,8,9\}\cup\{3,4,5,7\}$

$=\{3,4,5,6,7,8,9\}$

$A\cap C=\{1,4,7,8\}\cap\{3,4,5,7\}$

$=\{4,7\}$

$A\cap(B\cap C)=\{1,4,7,8\}\cap(\{4,6,8,9\}\cap\{3,4,5,7\})$

$=\{1,4,7,8\}\cap\{4\}$

$=\{4\}$

$(A\cup B)\cup C=(\{1,4,7,8\}\cup\{4,6,8,9\})\cup\{3,4,5,7\}$

$=\{1,4,6,7,8,9\}\cup\{3,4,5,7\}$

$=\{1,3,4,5,6,7,8,9\}$

$(A\cap B)\cap C=(\{1,4,7,8\}\cap\{4,6,8,9\})\cap\{3,4,5,7\}$

$=\{4,8\}\cap\{3,4,5,7\}$

$=\{4\}$

2. اگر $A=\{1,7,11,15,17,21\}, B=\{11,17,19,23\}$ اور $C=\{2,3,5\}$ ہوتو

ثابت کیجیے کہ $(A\cap B)\cap C=A\cap(B\cap C)$

حل:

$A=\{1,7,11,15,17,21\}, B=\{11,17,19,23\}, C=\{2,3,5\}$

$$L.H.S = (A \cap B) \cap C$$
$$= (\{1,7,11,15,17,21\} \cap \{11,17,19,23\}) \cap \{2,3,5\}$$
$$= \{11,17\} \cap \{2,3,5\}$$
$$= \{\ \} \_\_\_\_\_ (i)$$
$$R.H.S = A \cap (B \cap C)$$
$$= \{1,7,11,15,17,21\} \cap (\{11,17,19,23\} \cap \{2,3,5\})$$
$$= \{1,7,11,15,17,21\} \cap \{\ \}$$
$$= \{\ \} \_\_\_\_\_ (ii)$$

وات(i)اور(ii)سے ثابت ہوا کہ

$$(A \cap B) \cap C = A \cap (B \cap C)$$

اگر $A = \{2,4,6\}, B = \{3,6,9,12\}$ اور $C = \{4,6,8,10\}$ ہوتو

ثابت کیجیے کہ $A \cup (B \cup C) = (A \cup B) \cup C$

حل: $A = \{2,4,6\}, B = \{3,6,9,12\}, C = \{4,6,8,10\}$

$$L.H.S = A \cup (B \cup C)$$
$$= \{2,4,6\} \cup (\{3,6,9,12\} \cup \{4,6,8,10\})$$
$$= \{2,4,6\} \cup \{3,4,6,8,9,10,12\}$$
$$= \{2,3,4,6,8,9,10,12\} \quad \_\_\_\_\_ (i)$$
$$R.H.S = (A \cup B) \cup C$$
$$= (\{2,4,6\} \cup \{3,6,9,12\}) \cup \{4,6,8,10\}$$
$$= \{2,3,4,6,9,12\} \cup \{4,6,8,10\}$$
$$= \{2,3,4,6,8,9,10,12\} \quad \_\_\_\_\_ (ii)$$

ات(i)اور(ii)سے ثابت ہوا کہ

$$A \cup (B \cup C) = (A \cup B) \cup C$$

اگر $A = \{2,3,5,7,9\}, B = \{1,3,5,7\}$ اور $C = \{2,3,4,5,6\}$ ہوتو

ثابت کیجیے کہ $(A \cap B) \cap C = A \cap (B \cap C)$

ل: $A = \{2,3,5,7,9\}, B = \{1,3,5,7\}, C = \{2,3,4,5,6\}$

$$L.H.S = (A \cap B) \cap C$$
$$= (\{2,3,5,7,9\} \cap \{1,3,5,7\}) \cap \{2,3,4,5,6\}$$
$$= \{3,5,7\} \cap \{2,3,4,5,6\}$$
$$= \{3,5\} \quad \_\_\_\_\_ (i)$$

$$R.H.S = A \cap (B \cap C)$$
$$= \{2,3,5,7,9\} \cap (\{1,3,5,7\} \cap \{2,3,4,5,6\})$$
$$= \{2,3,5,7,9\} \cap \{3,5\}$$
$$= \{3,5\} \quad \text{_____ (ii)}$$

مساوات (i) اور (ii) سے ثابت ہوا کہ

$$(A \cap B) \cap C = A \cap (B \cap C)$$

5. اگر $U = \{7,8,9,10,11,12,13,14\}$، $A = \{7,10,13,14\}$ اور $B = \{7,8,11,12\}$ ہو تو ثابت کیجیے کہ $(A \cap B)' = A^c \cup B^c$

حل: $U = \{7,8,9,10,11,12,13,14\}, A = \{7,10,13,14\}, B = \{7,8,11,12\}$

$$L.H.S. = (A \cap B)^C$$
$$A \cap B = \{7,10,13,14\} \cap \{7,8,11,12\}$$
$$= \{7\}$$
$$(A \cap B)^C = U - (A \cap B)$$
$$= \{7,8,9,10,11,12,13,14\} - \{7\}$$
$$= \{8,9,10,11,12,13,14\} \quad \text{_____ (i)}$$
$$R.H.S. = A^c \cup B^c$$
$$A^C = U - A$$
$$= \{7,8,9,10,11,12,13,14\} - \{7,10,13,14\}$$
$$= \{8,9,11,12\}$$
$$B^C = U - B$$
$$= \{7,8,9,11,12,13,14\} - \{7,8,11,12\}$$
$$= \{9,10,13,14\}$$
$$A^c \cup B^c = \{8,9,11,12\} \cup \{9,10,13,14\}$$
$$R.H.S. = \{8,9,10,11,12,13,14\} \quad \text{_____ (ii)}$$

مساوات (i) اور (ii) سے ثابت ہوا کہ

$$(A \cap B)^c = A^c \cup B^c$$

6. ڈی مارگن کے قوانین ثابت کیجیے اگر $U = \{4,6,8,9,10\}$ اور $A = \{4,6\}, B = \{6,8,9\}$

حل: $\cup = \{4,6,8,9,10\}, A = \{4,6\}, B = \{6,8,9\}$

ڈی مارگن کے قوانین:

(i) $(A \cap B)^c = A^c \cup B^c$ (ii) $(A \cup B)^c = A^c \cap B^c$

$$L.H.S. = (A \cap B)^c$$

$A \cap B = \{4,6\} \cap \{6,8,9\}$

$= \{6\}$

$(A \cap B)^c = U - (A \cap B)$

$= \{4,6,8,9,10\} - \{6\}$

$= \{4,8,9,10\}$ _____ (i)

R.H.S. $= A^c \cup B^c$

$A^c = U - A$

$= \{4,6,8,9,10\} - \{4,6\}$

$= \{8,9,10\}$

$B^c = U - B$

$= \{4,6,8,9,10\} - \{6,8,9\}$

$= \{4.10\}$

$A^c \cup B^c = \{8,9,10\} \cup \{4,10\}$

$= \{4,8,9,10\}$ _____ (ii)

مساوات(i)اور(ii)سے ثابت ہوا کہ

$(A \cap B)^c = A^c \cup B^c$

(ii) $(A \cup B)^c = A^c \cap B^c$

L.H.S. $= (A \cup B)^c$

$A \cup B = \{4,6\} \cup \{6,8,9\}$

$= \{4,6,8,9\}$

$(A \cup B)^c = U - (A \cup B)$

$= \{4,6,8,9,10\} - \{4,6,8,9\}$

$= \{10\}$ _____ (i)

R.H.S. $= A^c \cap B^c$

$A^c = U - A$

$= \{4,6,8;9,10\} - \{4,6\}$

$= \{8,9,10\}$

$B^c = U - B$

$= \{4,6,8,9,10\} - \{6,8,9\}$

$= \{4,10\}$

$A^c \cap B^c = \{8,9,10\} \cap \{4,10\}$

$= \{10\}$ _____(ii)

مساوات(i)اور(ii) سے ثابت ہوا کہ

$(A \cup B)^c = A^c \cap B^c$

7. اگر $U = \{1,2,3,4,5,6,7,8,9\}$, $A = \{2,3,6,9\}$ اور $B = \{1,3,6,7,8\}$ ہو تو
ثابت کیجیے کہ $(A \cup B)^c = A^c \cap B^c$

$U = \{1,2,3,4,5,6,7,8,9\}, A = \{2,3,6,9\}, B = \{1,3,6,7,8\}$

L.H.S $= (A \cup B)^c$

$A \cup B = \{2,3,6,9\} \cup \{1,3,6,7,8\}$

$= \{1,2,3,6,7,8,9\}$

$(A \cup B)^c = U - (A \cup B)$

$= \{1,2,3,4,5,6,7,8,9\} - \{1,2,3,6,7,8,9\}$

$= \{4,5\}$ _____ (i)

R.H.S $= A^c \cap B^c$

$A^c = U - A$

$= \{1,2,3,4,5,6,7,8,9\} - \{2,3,6,9\}$

$= \{1,4,5,7,8\}$

$B^c = U - B$

$= \{1,2,3,4,5,6,7,8,9\} - \{1,3,6,7,8\}$

$= \{2,4,5,9\}$

$A^c \cap B^c = \{1,4,5,7,8\} \cap \{2,4,5,9\}$

$= \{4,5\}$ _____ (ii)

مساوات(i)اور(ii) سے ثابت ہوا کہ

$(A \cup B)^c = A^c \cap B^c$

8. خالی جگہ پُر کیجیے۔

i) $A \cup A =$ _____ (ii) $A \cap A =$ _____

iii) $A \cup \phi =$ _____ (iv) $A \cap \phi =$ _____

v) $\phi \cup \phi =$ _____ (vi) $(A \cap B)^c =$ _____

vii) $(A \cup B)^c =$ _____ (viii) $(A^c)^c =$ _____

ix) $\phi \cap \phi^c =$ _____ (x) $A \cap A^c =$ _____

حل:

(i) A **(ii)** A **(iii)** A **(iv)** ϕ **(v)** ϕ

(vi) $A^c \cup B^c$ **(vii)** $A^c \cap B^c$ **(viii)** A **(ix)** ϕ **(x)** ϕ