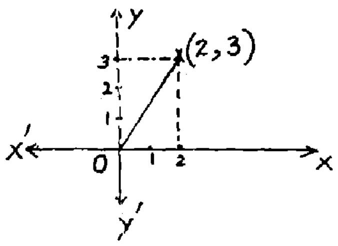
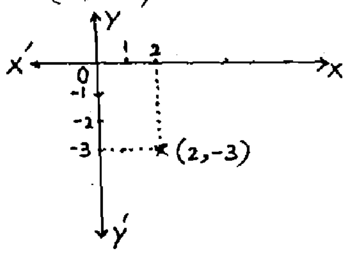
Exercise 1.3

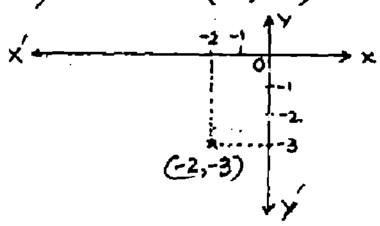
① i) $2+3i = (2,3)$



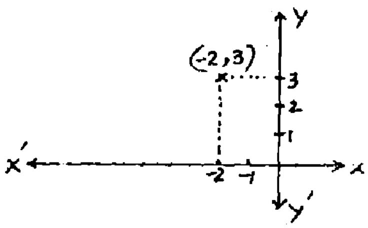
ii) $2-3i = (2,-3)$



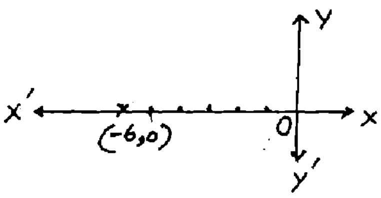
iii) $-2-3i = (-2,-3)$



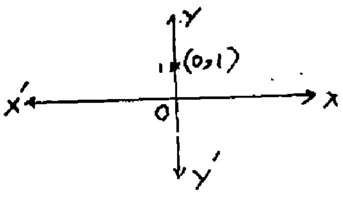
iv) $-2+3i = (-2,3)$



v) $-6 = -6+i.0 = (-6,0)$

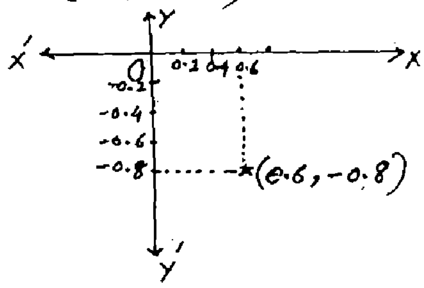


vi) $i = 0+i = (0,1)$

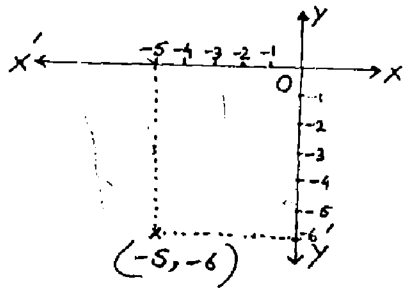


vii) $\frac{3}{5} - \frac{4}{5}i = 0.6 - 0.8i$

$= (0.6, -0.8)$



viii) $-5-6i = (-5,-6)$



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② i) Let $z = -3i$

Multiplicative inverse of z.

$= z^{-1} = \frac{1}{z} = \frac{1}{-3i} = \frac{1}{-3i} \times \frac{i}{i} = \frac{i}{-3i^2}$

$= \frac{i}{-3(-1)} = \frac{i}{3}$ Ans.

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ii) Let $z = 1-2i$

Multiplicative inverse of $z = \frac{1}{z}$

$= \frac{1}{1-2i} = \frac{1}{1-2i} \times \frac{1+2i}{1+2i}$

$= \frac{1+2i}{(1)^2-(2i)^2} = \frac{1+2i}{1-4i^2} = \frac{1+2i}{1-4(-1)}$

$= \frac{1+2i}{1+4} = \frac{1+2i}{5} = \frac{1}{5} + \frac{2}{5}i$ Ans

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iii) Let $z = -3-5i$

Multiplicative inverse of $z = \frac{1}{z}$

$= \frac{1}{-3-5i} = \frac{1}{-3-5i} \times \frac{-3+5i}{-3+5i}$

$= \frac{-3+5i}{(-3)^2-(5i)^2}$

$$\frac{\ }{9-25i^2} = \frac{\ }{9+25}$$

$$= \frac{-3+5i}{34} = \frac{-3}{34} + \frac{5}{34} i \text{ Ans.}$$

iv) Let $z = (1,2) = 1+2i$

Multiplicative inverse of $z = \frac{1}{z}$

$$= \frac{1}{1+2i} = \frac{1}{1+2i} \times \frac{1-2i}{1-2i} = \frac{1-2i}{(1)^2-(2i)^2}$$

$$= \frac{1-2i}{1-4i^2} = \frac{1-2i}{1-4(-1)} = \frac{1-2i}{1+4} = \frac{1-2i}{5}$$

$$= \frac{1}{5} - \frac{2}{5} i = \left(\frac{1}{5}, -\frac{2}{5}\right) \text{ Ans}$$

③ i) $i^{101} = (i^2)^{50} \cdot i = (-1)^{50} \cdot i = 1 \cdot i$

$= i$ Ans.

ii) $(-a i)^4 = (-a)^4 i^4 = a^4 (i^2)^2$

$= a^4 (-1)^2 = a^4 \cdot 1 = a^4$ Ans

iii) $i^{-3} = \frac{1}{i^3} = \frac{1}{i^2 \cdot i} = \frac{1}{(-1)(i)} = \frac{1}{-i}$

$= \frac{1}{-i} \times \frac{i}{i} = \frac{i}{-i^2} = \frac{i}{-(-1)} = \frac{i}{1}$

$= i$ Ans.

iv) $i^{-10} = \frac{1}{i^{10}} = \frac{1}{(i^2)^5} = \frac{1}{(-1)^5} = \frac{1}{-1}$

$= -1$ Ans.

④ Suppose that Let $z = a+bi$, $a, b \in \mathbb{R}$ ⟶ ①

$\bar{z} = z$

We shall prove that z is real

$\because \bar{z} = z$

$\Rightarrow \overline{a+ib} = a+ib$

$\Rightarrow a-ib = a+ib$

$\Rightarrow a-ib-a-ib = 0$

$\Rightarrow -2ib = 0$

$\Rightarrow b = 0 \quad \because -2i \neq 0$

$\therefore$ ① becomes

$z = a + i(0) = a+0 = a$

$\Rightarrow z$ is real.

Conversely suppose that z is real.

Then we shall prove that

$\bar{z} = z$.

$\because z$ is real

$\Rightarrow$ Imaginary part of $z = 0$

$\Rightarrow b = 0$

$\therefore$ ① becomes

$z = a + i(0) = a+0 = a$

$\Rightarrow z = a$ —— ②

Taking conjugate on both sides

$\bar{z} = \bar{a} \Rightarrow \bar{z} = a$ —— ③ $\because a \in \mathbb{R}$

$\therefore$ From ② and ③, we get

$\bar{z} = z$ (proved)

⑤ i) $5 + 2\sqrt{-4} = 5 + 2\sqrt{(-1)(4)}$

$= 5 + 2\sqrt{-1}\sqrt{4} = 5 + 2(i)(2) = 5+4i$ Ans.

ii) $(2+\sqrt{-3})(3+\sqrt{-3})$

$= (2+\sqrt{(-1)(3)})(3+\sqrt{(-1)(3)})$

$= (2+\sqrt{-1}\sqrt{3})(3+\sqrt{-1}\sqrt{3})$

$= (2+i\sqrt{3})(3+i\sqrt{3})$

$= 6 + i\,2\sqrt{3} + i\,3\sqrt{3} + i^2(\sqrt{3})^2$

$= 6 + 5\sqrt{3}\,i + (-1)(3)$

$= 6 + 5\sqrt{3}\,i - 3 = 3 + 5\sqrt{3}\,i$ Ans.

iii) $\frac{2}{\sqrt{5}+\sqrt{-8}} = \frac{2}{\sqrt{5}+\sqrt{(-1)(8)}} = \frac{2}{\sqrt{5}+\sqrt{-1}\sqrt{8}}$

$= \frac{2}{\sqrt{5} + i(2\sqrt{2})}$ $\left\{ \sqrt{8} = \sqrt{2\times2\times2} = \sqrt{2^2\times2} = 2\sqrt{2} \right.$

$$= \frac{2}{\sqrt{5} + 2\sqrt{2}\, i}$$

$$= \frac{2}{\sqrt{5} + 2\sqrt{2}\, i} \times \frac{\sqrt{5} - 2\sqrt{2}\, i}{\sqrt{5} - 2\sqrt{2}\, i}$$

$$= \frac{2(\sqrt{5} - 2\sqrt{2}\, i)}{(\sqrt{5})^2 - (2\sqrt{2}i)^2} = \frac{2(\sqrt{5} - 2\sqrt{2}\, i)}{5 - 4(2)\, i^2}$$

$$= \frac{2(\sqrt{5} - 2\sqrt{2}\, i)}{5 + 8} = \frac{2\sqrt{5} - 4\sqrt{2}\, i}{13}$$

$$= \frac{2\sqrt{5}}{13} - \frac{4\sqrt{2}}{13} i$$ Ans.

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iv) $$\frac{3}{\sqrt{6} - \sqrt{-12}} = \frac{3}{\sqrt{6} - \sqrt{(-1)(12)}} = \frac{3}{\sqrt{6} - \sqrt{-1}\sqrt{12}}$$

$$= \frac{3}{\sqrt{6} - i\sqrt{12}} = \frac{3}{\sqrt{6} - i(2\sqrt{3})}$$

$$\left\{ \sqrt{12} = \sqrt{2\times 2\times 3} = \sqrt{2^2 \times 3} = 2\sqrt{3} \right.$$

$$= \frac{3}{\sqrt{6} - 2\sqrt{3}\, i} \times \frac{\sqrt{6} + 2\sqrt{3}\, i}{\sqrt{6} + 2\sqrt{3}\, i}$$

$$= \frac{3(\sqrt{6} + 2\sqrt{3}\, i)}{(\sqrt{6})^2 - (2\sqrt{3}\, i)^2} = \frac{3(\sqrt{6} + 2\sqrt{3} i)}{6 - 4(3) i^2}$$

$$= \frac{3(\sqrt{6} + 2\sqrt{3}\, i)}{6 + 12} = \frac{3(\sqrt{6} + 2\sqrt{3}\, i)}{18}$$

$$= \frac{\sqrt{6} + 2\sqrt{3} i}{6} = \frac{\sqrt{6}}{6} + \frac{2\sqrt{3}}{6} i$$

$$= \frac{\sqrt{6}}{6} + \frac{\sqrt{3}}{3} i = \frac{\sqrt{6}}{\sqrt{6}\times\sqrt{6}} + \frac{\sqrt{3}}{\sqrt{3}\times\sqrt{3}} i$$

$$= \frac{1}{\sqrt{6}} + \frac{1}{\sqrt{3}} i$$ Ans

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⑥ i) $z^2 + \bar{z}^2$ is a real number.

Let $z = a + ib$, $a, b \in R$

then $\bar{z} = a - ib$

$$z^2 + \bar{z}^2 = (a+ib)^2 + (a-ib)^2$$

$$= a^2 + i^2 b^2 + 2abi + a^2 + i^2 b^2 - 2abi$$

$$= 2a^2 + 2i^2 b^2$$

$$= 2a^2 + 2(-1)b^2 = 2a^2 - 2b^2$$

which is real.

ii) $z^2 - \bar{z}^2$ is Imaginary.

Ans Let $z = a + ib$, $a, b \in R$

then $\bar{z} = a - ib$

Now $$z^2 - \bar{z}^2 = (a+ib)^2 - (a-ib)^2$$

$$= (a^2 + i^2 b^2 + 2abi) - (a^2 + i^2 b^2 - 2abi)$$

$$= a^2 + i^2 b^2 + 2abi - a^2 - i^2 b^2 + 2abi$$

$= 4abi$ which is imaginary number.

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⑦ Simplify

i) $$\left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)^3$$

$$= \left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)^2 \left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)$$

$$= \left[\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2} i\right)^2 + 2\left(-\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2} i\right)\right]\left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)$$

$$= \left(\frac{1}{4} + \frac{3}{4} i^2 - \frac{\sqrt{3}}{2} i\right)\left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)$$

$$= \left(\frac{1}{4} - \frac{3}{4} - \frac{\sqrt{3}}{2} i\right)\left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)$$

$$\left\{ \frac{1}{4} - \frac{3}{4} = \frac{1-3}{4} = -\frac{2}{4} = -\frac{1}{2} \right.$$

$$= \left(-\frac{1}{2} - \frac{\sqrt{3}}{2} i\right)\left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)$$

$$= \left(-\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2} i\right)^2 = \frac{1}{4} - \frac{3}{4} i^2$$

$$= \frac{1}{4} + \frac{3}{4} = \frac{1+3}{4} = \frac{4}{4} = 1$$ Ans.

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ii) $$\left(-\frac{1}{2} - \frac{\sqrt{3}}{2} i\right)^3$$

$$= \left(-\frac{1}{2} - \frac{\sqrt{3}}{2} i\right)^2 \left(-\frac{1}{2} - \frac{\sqrt{3}}{2} i\right)$$

$$= \left[\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2} i\right)^2 - 2\left(-\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2} i\right)\right]\left(-\frac{1}{2} - \frac{\sqrt{3}}{2} i\right)$$

$$= \left(\frac{1}{4} - \frac{3}{4} + \frac{\sqrt{3}}{2} i\right)\left(-\frac{1}{2} - \frac{\sqrt{3}}{2} i\right)$$

$$= \left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)\left(-\frac{1}{2} - \frac{\sqrt{3}}{2} i\right)$$

$$= \left(-\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2} i\right)^2$$

$$= \frac{1}{4} - \frac{3}{4} i^2 = \frac{1}{4} + \frac{3}{4} = \frac{1+3}{4} = \frac{4}{4} = 1$$

Ans

iii) $\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^{-2}\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)$

$=\dfrac{-\frac{1}{2}-\frac{\sqrt{3}}{2}i}{\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^2}$

$=\dfrac{-\frac{1}{2}-\frac{\sqrt{3}}{2}i}{\left(-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}i\right)^2+2\left(-\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}i\right)}$

$=\dfrac{-\frac{1}{2}-\frac{\sqrt{3}}{2}i}{\frac{1}{4}-\frac{3}{4}-\frac{\sqrt{3}}{2}i}=\dfrac{-\frac{1}{2}-\frac{\sqrt{3}}{2}i}{-\frac{1}{2}-\frac{\sqrt{3}}{2}i}=1$ Ans

(iii) $\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)^{-2}\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)^{1}$ (according to book)

$=\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)^{-1}=\dfrac{1}{-\frac{1}{2}-\frac{\sqrt{3}}{2}i}$

$=\dfrac{1}{-\frac{1}{2}-\frac{\sqrt{3}}{2}i}\times\dfrac{-\frac{1}{2}+\frac{\sqrt{3}}{2}i}{-\frac{1}{2}+\frac{\sqrt{3}}{2}i}$

$=\dfrac{-\frac{1}{2}+\frac{\sqrt{3}}{2}i}{\left(-\frac{1}{2}\right)^2-\left(\frac{\sqrt{3}}{2}i\right)^2}=\dfrac{-\frac{1}{2}+\frac{\sqrt{3}}{2}i}{\frac{1}{4}+\frac{3}{4}}$

$=\dfrac{-\frac{1}{2}+\frac{\sqrt{3}}{2}i}{\frac{4}{4}}=-\frac{1}{2}+\frac{\sqrt{3}}{2}i$

$=\dfrac{-1+\sqrt{3}i}{2}$ Ans

iv) $(a+bi)^2=a^2+b^2i^2+2abi$

$=a^2+b^2(-1)+2abi$

$=a^2-b^2+2abi$ Ans

v) $(a+bi)^{-2}=\dfrac{1}{(a+bi)^2}=\dfrac{1}{a^2+b^2i^2+2abi}$

$=\dfrac{1}{a^2-b^2+2abi}$

$=\dfrac{1}{(a^2-b^2)+2abi}\times\dfrac{(a^2-b^2)-2abi}{(a^2-b^2)-2abi}$

$=\dfrac{(a^2-b^2)-2abi}{(a^2-b^2)^2-(2abi)^2}$

$=\dfrac{(a^2-b^2)-2abi}{a^4+b^4-2a^2b^2-4a^2b^2i^2}$

$=\dfrac{(a^2-b^2)-2abi}{a^4+b^4-2a^2b^2+4a^2b^2}$

$=\dfrac{(a^2-b^2)-2abi}{a^4+b^4+2a^2b^2}=\dfrac{(a^2-b^2)-2abi}{(a^2+b^2)^2}$

$=\dfrac{a^2-b^2}{(a^2+b^2)^2}-\dfrac{2ab}{(a^2+b^2)^2}i$ Ans

vi) $(a+bi)^3=a^3+(bi)^3+3(a)(bi)(a+bi)$

$=a^3+b^3i^3+3abi(a+bi)$ $\boxed{\begin{aligned}i^3&=i^2\cdot i\\&=-1\cdot i\\&=-i\end{aligned}}$

$=a^3+b^3(-i)+3a^2bi+3ab^2i^2$

$=a^3-b^3i+3a^2bi-3ab^2$

$=a^3-3ab^2+3a^2bi-b^3i$

$=a^3-3ab^2+i(3a^2b-b^3)$ Ans

vii) $(a-bi)^3$

$=(a)^3-(bi)^3-3(a)(bi)(a-bi)$

$=a^3-b^3i^3-3abi(a-bi)$

$=a^3-b^3(-i)-3a^2bi+3ab^2i^2$

$=a^3+b^3i-3a^2bi-3ab^2$

$=a^3-3ab^2-3a^2bi+b^3i$

$=a^3-3ab^2-i(3a^2b-b^3)$ Ans

viii) $(3-\sqrt{-4})^{-3}=\dfrac{1}{[3-\sqrt{-4}]^3}=\dfrac{1}{[3-\sqrt{(-1)(4)}]^3}$

$=\dfrac{1}{(3-\sqrt{-1}\sqrt{4})^3}=\dfrac{1}{[3-i(2)]^3}=\dfrac{1}{(3-2i)^3}$

$=\dfrac{1}{(3)^3-(2i)^3-3(3)(2i)(3-2i)}$

$=\dfrac{1}{27-8i^3-18i(3-2i)}$

$$= \frac{1}{27-8(-i)-54i+36i^2}$$

$$= \frac{1}{27+8i-54i-36}$$

$$= \frac{1}{-9-46i}$$

$$= \frac{1}{-9-46i} \times \frac{-9+46i}{-9+46i}$$

$$= \frac{-9+46i}{(-9)^2-(46i)^2} = \frac{-9+46i}{81-2116i^2}$$

$$= \frac{-9+46i}{81+2116} = \frac{-9+46i}{2197}$$

$$= \frac{-9}{2197} + \frac{46}{2197}i$$ Ans.

End of Chapter No. 1.