$\Rightarrow \quad x \in A$ and $\quad x \in B \cap C$

$\Rightarrow \quad x \in A$ and $\quad [x \in B$ or $x \in C]$

$\Rightarrow \quad [x \in A$ and $x \in B]$ or $[x \in A$ and $x \in C]$

$\Rightarrow \quad [x \in A \cap B] \quad$ or $[x \in A \cap C]$

$\Rightarrow \quad x \in (A \cap B) \cup (A \cup C)$

Hence by def. of subsets

$A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C) \qquad$ (i)

Similarly $\quad (A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C) \qquad$ (ii)

From (i) and (ii), we have , $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(g) De-Morgan's laws

For any two sets A and B, prove that

(i) $(A \cup B)' = A' \cap B'$

Proof: Let $x \in (A \cup B)'$

$\Rightarrow \quad x \notin A \cup B \qquad$ (by definition of complement of set)

$\Rightarrow \quad x \notin A$ and $\quad x \notin B$

$\Rightarrow \quad x \in A'$ and $\quad x \in B'$

$\Rightarrow \quad x \in A' \cap B' \qquad$ (by definition of intersection of sets)

$\Rightarrow \quad (A \cup B)' \subseteq (A \cup B)' \qquad$ (i)

Similarly $A' \cap B' \subseteq (A \cup B) \qquad$ (ii)

Using (i) and (ii), we have $(A \cup B)' = A' \cap B'$

(ii) Let $\quad x \in (A \cap B)'$

$\Rightarrow \quad x \notin A \cap B$

$\Rightarrow \quad x \notin A$ or $x \notin B$

$\Rightarrow \quad x \in A'$ or $x \in B'$

$\Rightarrow \quad x\ A' \cup B'$

$\Rightarrow \quad (A \cap B)' \subseteq A' \cup B' \qquad$ (i)

Let $\quad y \in A' \cap B'$

$\Rightarrow \quad y \in A \cap B$

$\Rightarrow \quad y \notin A$ or $x \notin B$

$\Rightarrow \quad y \notin A \cap B$

$\Rightarrow \quad y \in (A \cap B)'$

$\Rightarrow \quad (A' \cap B)' \subseteq A' \cap B' \qquad$ (ii)

From (i) and (ii) we have proved that

$(A \cap B)' = A' \cup B'$

SOLVED EXERCISE 5.2

1. If $X = \{1,3. 5, 7, \ldots,19\}$, $\quad Y = \{0,2,4, 6, 8, \ldots,20\}$
$Z = \{2, 3,5,7,11,13,17,19,23\}$, then find the following.

(i) $X \cup (Y \cup Z)$

Solution:

$$Y \cup Z = \{0, 2, 4, 6, 8, \ldots, 20\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$$
$$= \{0, 2, 3, 4, \ldots, 17, 19, 20, 23\}$$
$$X \cup (Y \cup Z) = \{1, 3, 5, 7, .., 19\} \cup \{0, 2, 3, 4, \ldots, 17, 19, 20, 23\}$$
$$= \{0, 1, 2, 3, \ldots, 30, 33\}$$

(ii) $(X \cup Y) \cup Z$

Solution:

$$X \cup Y = \{1, 3, 5, 7, .., 19\} \cup \{0, 2, 4, 6, 8, \ldots, 20\}$$
$$= \{0, 1, 2, 3, \ldots, 19, 20\}$$
$$(X \cup Y \cup Z = \{0, 1, 2, 3, .., 19, 20\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$$
$$= \{0, 1, 2, 3, .., 20, 23\}$$

(iii) $X \cap (Y \cap Z)$

Solution:

$$Y \cap Z = \{0, 2, 4, 6, 8, \ldots, 20\} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$$
$$= \phi$$
$$X \cap (Y \cap Z)$$
$$X \cap Y = \{1, 3, 5, 7, \ldots, 19\} \cap \phi$$
$$= \phi$$

(iv) $(X \cap Y) \cap Z$

Solution:

$$X \cap Y = \{1, 3, 5, 7, \ldots, 19\} \cap \{0, 2, 4, 6, 8, .., 20\}$$
$$= \phi$$
$$(X \cap Y) \cap Z = \phi \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$$
$$= \phi$$

(v) $X \cup (Y \cap Z)$

Solution:

$$Y \cap Z = \{0, 2, 4, 6, 8, \ldots, 20\} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$$
$$= \{2\}$$
$$X \cup (Y \cap Z) = \{1, 3, 5, 7, \ldots, 19\} \cup \{2\}$$
$$= \{1, 2, 3, 5, 7, .., 19\}$$

(vi) $(X \cup Y) \cap (X \cup Z)$

Solution:

$$X \cup Y = \{1, 3, 5, 7, \ldots, 19\} \cup \{0, 2, 4, 6, 8, \ldots, 20\}$$
$$= \{0, 1, 2, 3, .., 19, 20\}$$
$$X \cup Z = \{1, 3, 5, 7, .., 19\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$$
$$= \{1, 2, 3, 5, 7, .., 17, 19, 23\}$$
$$(X \cup Y) \cap (X \cup Z) = \{0, 1, 2, 3, .., 19, 20\} \cap \{1, 2, 3, 5, 7, \ldots, 17, 19, 23\}$$
$$= \{1, 2, 3, 5, 7, \ldots, 19\}$$

(vii) $X \cap (Y \cup Z)$

Solution:

$Y \cup Z = \{0, 2, 4, 6, 8, ..., 20\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$
$= \{0, 2, 3, 4, 5, 6, ..., 19, 20, 23\}$

$X \cap (Y \cup Z) = \{1, 3, 5, 7, ..., 19\} \cap \{0, 2, 3, 4, 5, 6, ..., 19, 20\}$
$= \{3, 5, 7, ..., 19\} \cap \{0, 2, 4, 6, 8, ..., 20\}$
$= \phi$

$X \cap Z = \{1, 3, 5, 7, ..., 19\} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$
$= \{3, 5, 7, 11, 13, 17, 19\}$

(viii) $(X \cap Y) \cup (X \cap Z)$

Solution:

$X \cap Y = \{1, 3, 5, 7, ..., 19\} \cap \{0, 2, 4, 6, 8, ..., 20\}$
$= \phi$

$X \cap Z = \{1, 3, 5, 7, ..., 19\} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$
$= \{3, 5, 7, 11, 13, 17, 19\}$

2. If 4 = {1, 2, 3, 4. 5, 6}, B = {2, 4, 6, 8}, C- {1, 4, 8}. Prove the following identities:

(i) $A \cap B = B \cap A$

Solution:

L.H.S. $= A \cap B$
$= \{1, 2, 3, 4, 5, 6\} \cap \{2, 4, 6, 8\}$
$= \{2, 4, 6\}$ ________ (i)

R.HS. $= B \cap A$
$= \{2, 4, 6, 8\} \cap \{1, 2, 3, 4, 5, 6\}$
$= \{2, 4, 6\}$ ________ (ii)

From (i) and (ii), we have

L.H.S = R.H.S.

Hence proved.

(ii) $A \cup B = B \cup A$

Solution:

L.H.S. $= A \cup B$
$= \{1, 2, 3, 4, 5, 6\} \cup \{2, 4, 6, 8\}$
$= \{1, 2, 3, 4, 5, 6, 8\}$ ________ (i)

R.HS. $= B \cup A$
$= \{2, 4, 6, 8\} \cup \{1, 2, 3, 4, 5, 6\}$
$= \{1, 2, 3, 4, 5, 6, 8\}$ ________ (ii)

From (i) and (ii), we have

L.H.S = R.H.S.

Hence proved.

(iii) $A \cup (B \cup C) = (A \cap B) \cup (A \cap C)$

Solution:

L.H.S. $= A \cap (B \cup C)$

$= \{1, 2, 3, 4, 5, 6\} \cap (\{2, 4, 6, 8\} \cap \{1, 4, 8\})$

$= \{1, 2, 3, 4, 5, 6,\} \cap \{1, 2, 4, 6, 8\}$

$= \{1, 2, 3, 4, 5, 6\}$ ________ (i)

R.HS. $= (A \cap B) \cup (A \cup C)$

$= (\{1, 2, 3, 4, 5, 6\} \cap \{2, 4, 6, 8\}) \cup (\{1, 2, 3, 4, 5, 6\} \cap \{1, 4, 8\})$

$= \{2, 4, 6\} \cup \{1, 4\}$

$= \{1, 2, 3, 4, 5, 6\}$ ________ (ii)

From (i) and (ii), we have

L.H.S = R.H.S.

Hence Proved.

(iv) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Solution:

L.H.S. $= A \cup (B \cap C)$

$= \{1, 2, 3, 4, 5, 6\} \cup (\{2, 4, 6, 8\} \cap \{1, 4, 8\})$

$= \{1, 2, 3, 4, 5, 6,\} \cup \{4, 8\}$

$= \{1, 2, 3, 4, 5, 6, 8\}$ ________ (i)

R.HS. $= (A \cup B) \cap (A \cup C)$

$= (\{1, 2, 3, 4, 5, 6\} \cup \{2, 4, 6, 8\}) \cap (\{1, 2, 3, 4, 5, 6\} \cap \{1, 4, 8\})$

$= \{1, 2, 3, 4, 6, 8\} \cap \{1, 2, 3, 4, 5, 6, 8\}$

$= \{1, 2, 3, 4, 5, 6, 8\}$ ________ (ii)

From (i) and (ii), we have

L.H.S = R.H.S.

Hence Proved.

3. If $U = \{1,2, 3, 4, 5, 6, 7, 8, 9, 10\}$ $A = \{1,3, 5, 7,. 9\}$, $B = \{2, 3,5, 7\}$, then verify the De-Morgan's Laws

i.e., $(A \cap B) = A' \cup B'$ and $(A \cup B)' = A' \cap B'$

Solution:

L.H.S. $= A' \cup B'$

$= \cup - (A \cap B)$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - (\{1, 3, 5, 7, 9\} \cap \{2, 3, 5, 7\})$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{3, 5, 7\}$

$= \{1, 2, 4, 6, 8, 9, 10\}$ ________ (i)

R.H.S. $= A' \cup B'$

$= [\cup - A] \cup [\cup - B]$

$= (\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{1, 3, 5, 7, 9\})$

$\cup (\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{2, 3, 5, 7\})$

$= \{2, 4, 6, 8, 10\} \cup \{1, 4, 6, 8, 9, 10\}$

$= \{1, 2, 4, 6, 8, 9, 10\}$ ________ (ii)

From (i) and (ii), we have

L.H.S = R.H.S.

(ii) $(A \cup B)' = A' \cap B'$

L.H.S. $= A' \cup B'$

$= \cup - (A \cup B)$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - (\{1, 3, 5, 7, 9\} \cap \{2, 3, 5, 7\})$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{1, 2, 3, 5, 7, 9\}$

$= \{4, 6, 8, 9, 10\}$ ________ (i)

R.H.S. $= A' \cap B'$

$= [\cup - A] \cap [\cup - B]$

$= (\{1, 2, 3, \ldots\ldots 10\} - \{1, 3, 5, 7, 9\}) \cap (\{1, 2, 3, \ldots\ldots, 10\} - \{2, 3, 5, 7\})$

$= \{2, 4, 6, 8, 10\} \cap \{1, 4, 6, 8, 9, 10\}$

$= \{4, 6, 8, 9, 10\}$ ________ (ii)

From (i) and (ii), we have

L.H.S = R.H.S.

4. If $U = \{1, 2, 3, \ldots, 20)$, $X = \{1, 3, 7, 9, 15, 18, 20\}$ and $Y = \{1, 3, 5, \ldots, 17\}$, then show that

(i) $X - Y = X \cap Y'$

Solution:

L.H.S. $= X \cap Y'$

$= \{1, 3, 5, 7, 9, 15, 18, 20\} \cap (\cup - Y)$

$= \{1, 2, 5, 7, 9, 15, 18, 20\} \cap (\{1, 2, 3, \ldots, 20\} - \{1, 3, 5, \ldots\ldots 17\})$

$= \{1, 3, 5, 7, 9, 15, 18, 20\} \cap \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$

$= \{18, 20\}$ ________ (ii)

From (i) and (ii), we have

L.H.S = R.H.S.

Hence Proved.

(ii) $Y - X = Y \cap X'$

Solution:

L.H.S. $= Y - X$

$= \{1, 3, 5, \ldots\ldots 17\} - \{1, 2, 5, 7, 9, 15, 18, 20\}$

$= \{5, 11, 13, 17\}$ ________ (i)

R.H.S. $= Y \cap X'$

$= Y \cap (\cup - X)$

$= \{1, 3, 5, \ldots\ldots, 17\} \cap (\{1, 3, \ldots\ldots, 20\} - \{1, 3, 5, 7, 9, 15, 18, 20\}$

$= \{1, 3, 5, \ldots\ldots, 17\} \cap \{2, 4, 5, 6, 8, 10, 11, 12, 13, 14, 16, 19\}$

$= \{5, 11, 13, 17\}$ ________ (ii)

From (i) and (ii), we have

L.H.S = R.H.S.

Hence Proved.

Verify the fundamental properties for given sets:

(a) A and B are any two subsets of U, then A u B = B u A (commutative law).

For example

$A = \{1,3,5, 7\}$ and $B = \{2,3,, 5,7\}$

then $A \cup B = \{1, 3, 5, 7\} \cup \{2, 3, 5, 7\} = \{1, 2, 3, 5, 7\}$

and $B \cup A = \{2, 3, 5, 7\} \cup \{1, 3, 5, 7\} = \{1, 2, 3, 5, 7\}$

Hence, verified that $A \cup B = B \cup A$.

(b) Commutative property of intersection

For example $A = \{1, 3, 5, 7\}$ and $B = [2, 3, 5, 7\}$

Then $A \cap B = \{1,3,5,7\} \cap \{2,3,5,7\}=\{3,5,7\}$

and $B \cap A = \{2, 3,5, 7\} \cap \{1,3, 5, 7\} = \{3, 5, 7\}$

Hence, verified that $A \cap B = B \cap A$.

(c) If A, B and C are the subsets of U, then $(A \cup B) \cup C = A \cup (B \cup C)$.

(Associative law)

Suppose $A = \{1,2,4,8\}$; $B = \{2,4,6\}$

And $C = \{3,4,5,6\}$

Then L.H.S. $= (A \cup B) \cup C$

$= (\{1,2,4,8\} \cup \{2,4,6\}) \cup \{3.4,5,6\}$

$= \{1,2,4,6,8\} \cup \{.3,4,5.6\}$

$= \{1,2,3,4.5,.6,8\}$

and R.H.S. $= A \cup (B \cup C)$

$= \{42. 4. 8\} \cup (\{2, 4, 6\} \cup (3, 4, 5, 6\})$

$= \{1,2, 4,8\} \cup \{2.3,4, 5, 6\}$

$= \{42,3, 4, 5, 6,8\}$

L.H.S. = R.H.S.

Hence, union of Sets is associative.

(d) If A, B and C are the subsets of U, then $(A \cap B) \cap C = A \cap (B \cap C)$

(Associative Law).

Suppose ., $A = \{1, 2,4, 8\}$; $5 = \{2, 4, 6\}$ and $C = \{3,4. 5, 6\}$

then L.H.S, $= (A \cap B) \cap C$

$= (\{1, 2,4, 8\} \cap \{2, 4, 6\}) \cap \{3, 4, 5, 6\}$

$= \{2, 4\} \cap \{3. 4, 5, 6\} = \{4\}$

and R.H.S. $= A \cap (B \cap C)$

$= \{1, 2, 4, 8\} \cap (\{2, 4, 6\} \cap \{3, 4, 5, 6\})$

$= (1,2, 4, 8\}; = \{4, 6\} - \{4\}$

L.H.S. = R.H.S.

Hence, intersection of sets is associative.

Distributive laws

(e) Union is distributive over intersection of sets

If A, B and C are the subsets of universal set U, then $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Solution: Suppose $A = \{1, 2, 4, 8\}$, $B = \{2, 4, 6\}$ and $C \{3, 4, 5, 6\}$

then L.H.S $= A \cup (B \cap C)$

$= \{1,2,4.8\} \cup (\{2,4,6) \cap \{3,4,5.6\})$

$= \{1,2,4,8) \cup \{4,6\} - \{1,2,4,6,8\}$

and R.H.S $= (A \cup B) \cap (A \cup C)$

$= (\{1, 2, 4, 8\} \cup \{2, 4, 6\}) \cap (\{1, 2, 4, 8\} \cup \{3, 4, .5, 6\})$

$= (1,2,4,6,8\} \cap \{1,2,3,4,5,6,8\}$

$= \{1,2,4,6,8\}$

L.H.S = R.H.S

(f) Intersection is distributive over union of sets

To proved $\cap (B \cup C) = (A \cap B) \cup (A \cap B)$

Suppose $A = \{1, 2, 3, 4. 5, \ldots, 20\}$

$B = \{5,10,15.20,25,30\}$

$C = \{3,9,15,21,27,33\}$

L.H.S. $= A \cap (B \cup C)$

$= \{1,2,3,4,5, \ldots,20\} \cap (\{.5, 10, 15, 20, 25, 30\} \cup \{3,9, 15,21,27,33\})$

$= \{1,2,3,4,5, \ldots, 20\} \cap \{3, 5, 9, 10, 15, 20, 21, 25, 27, 30, 33\}$

$= \{3,5,9,10,15,20\}$

R.H.S. $= (A \cap B) \cup (A \cap C)$

$= (\{1,2,3,4, \ldots,20\}$ n $\{5, 10,15, 20,25, 30\})$

$\cup (\{1, 2, 3, 4, 5, \ldots, 20\} \cap \{3, 9,15,21, 27, 33\})$

$= \{5, 10, 15, 20\} \cup \{3, 9, 15\} = \{3, 5, 9, 10, 15,20\}$

L.H.S. = R.H.S.

(g) De Morgan's Laws $(A \cap B)' = A' \cup B'$ and $(A \cup B)' = A' \cap B'$

Suppose $U = \{1,2,3,4, \ldots, 10\}$

$A = \{2,4,6,8.10\} \Rightarrow \{1,3,5,7,9\}$

$B = \{1. 2, 3, 4, 5, 6\} \Rightarrow B' = \{7, 8. 9, 10\}$

Now consider $A \cap B = \{2, 4, 6, 8, 10\} \cap \{1, 2, 3, 4, 5, 6\}$

$= \{2. 4, .6\}$

Then L.H.S. $= (A \cap B)' = U - (A \cap B)$

$= \{1,2, 3, 4,\ldots,10) - \{2, 4, 6\}$

$= \{1,3,5,7,8,9,10\}$

and R.H.S. $= A \cup B'$

$= \{1,3,5, 7, 9\} \cup \{7,8,9,10\}$

$= \{1,3,5,7,8,9,10\}$

L.H.S. = R.H.S.

$(A \cup B)' = A' \cap B'$

Suppose $U = \{1. 2, 3, 4, \ldots, 10\}$

$A = \{2, 4, 6, 8, 10\} \Rightarrow A' = \{-1, 3, 5, 7, 9\}$

$B = \{1, 2, 3, 4, 5, 6\} \Rightarrow B' = \{7, 8, 9, 10\}$

Now consider $A \cup B = \{2, 4, 6, 8, 10\} \cup \{1,2, 3, 4, 5, 6\}$

$= \{1,2,3,4,5,6,8,10\}$

L.H.S. $= (A \cup B)' = U - (A \cup B)$

$= \{1,2,3,4,\ldots, 10\} - \{1,2, 3, 4, 5, 6, 8, 10\}$

$= \{7,9\}$

and R.H.S $A' = B' = \{1,3,5,7,9\} \cap \{7,8,9, 10\}$

$= \{7,9\}$

L.H.S.= R.H.S.

Venn Diagram:

British mathematician John Venn (1834 – 1923) introduced rectangle for a universal set U and its subsets A and B as closed figures inside this rectangle.

Use Venn diagrams to represent:

(a) Union and intersection of sets

Disjoint sets — Overlapping sets — $A \subseteq B$

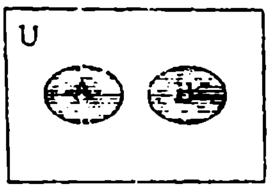


Fig. 1

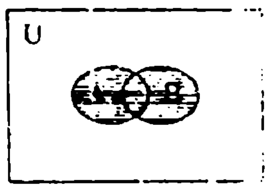


Fig. 2

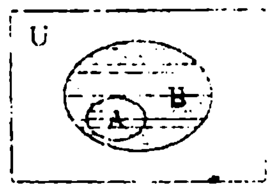


Fig. 3

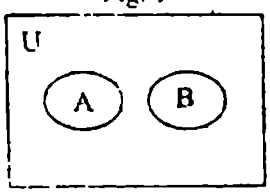


Fig. 4

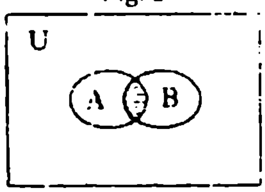


Fig. 5

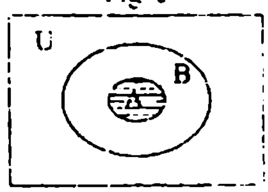
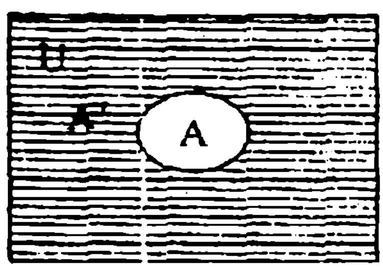


Fig. 6

(Regions shown by horizontal line segments in figures 1 to 6.)

(b) Complement of a set

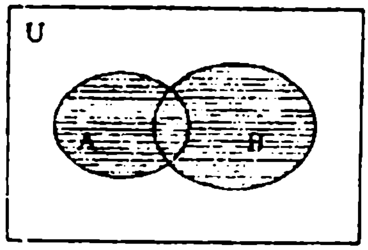


U – A =A' is shown by horizontal line segments.

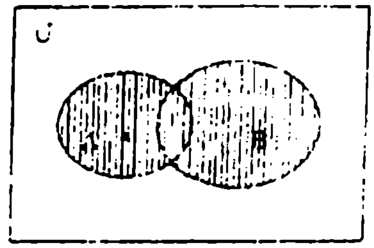
Use Venn diagram to verify:

(a) Commutative law for union and intersection of sets.

A∪B



B ∪ A



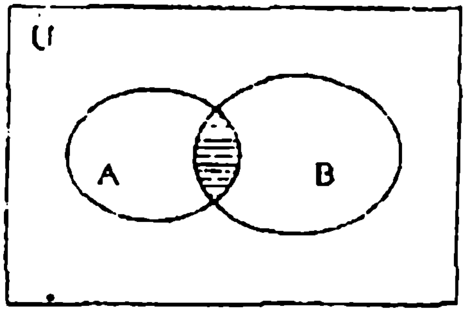
A ∪ B is shown by horizontal line segments,

B ∪ A is shown by vertical line segments.

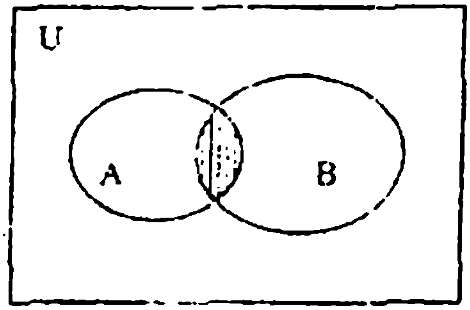
The regions shown in both cases are equal.

Thus A ∪ B = B ∪ A.,

A ∩ B



B ∩ A



A ∩ B is shown by horizontal line segments.

B ∩ A is shown by vertical line segments.

The regions shown in both cases are equal.

Thus A ∩ B = B ∩ A.

(b) De Morgan's laws

(i) (A ∪ B)' = A' ∩ B'

(ii) (A ∩ B)' = A' ∪ B'

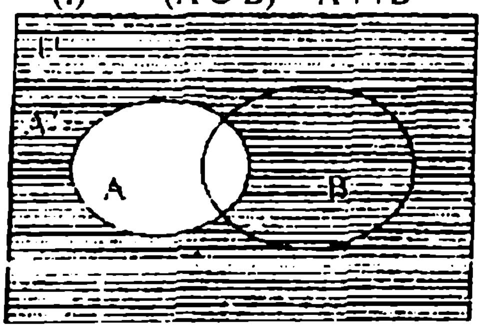


Fig. 1: A' is shown by horizontal line segments

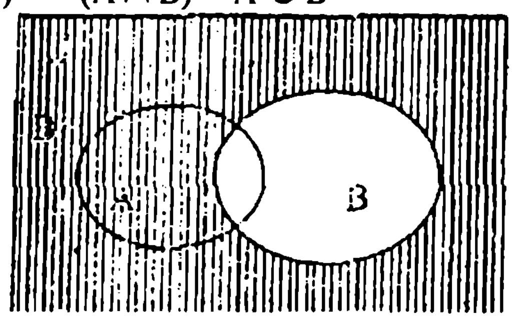


Fig. 2: B' is shown by vertical line segments

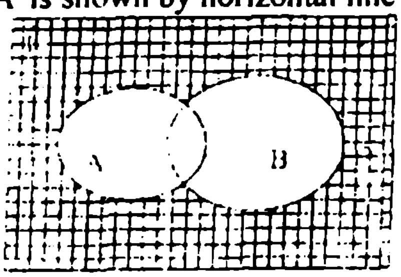


Fig. 3: A' ∩ B' is shown by squares

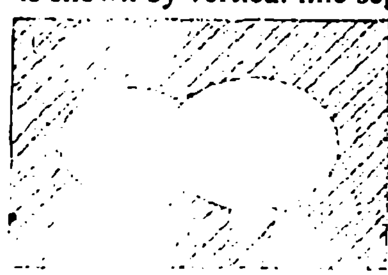


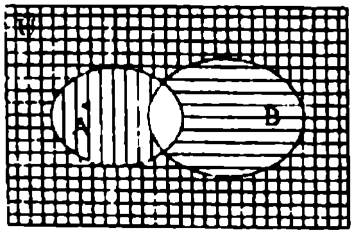
Fig. 4: (A ∪ B)' is shown by slanting line segments

Regions shown in Fig. 3 and Fig. 4 are equal.

Thus (A ∪ B)' = A' ∩ B'

(A ∩ B)' = A' ∪ B'

A' ∪ B'



(A ∩ B)'

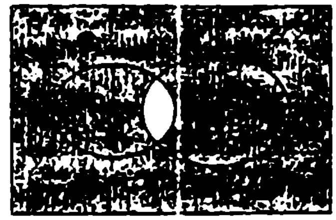


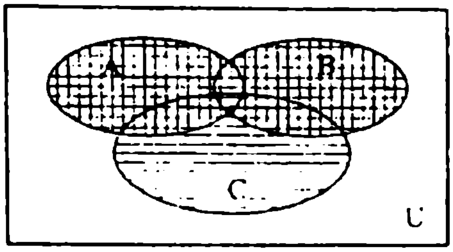
Fig. 5 A' ∪ B' is shown by squares, horizontal and vertical line segments.

Fig.6 U − (A ∩ B) = (A ∩ B) is shown by squares, horizontal .

Regions shown in Fig. 5 and Fig. 6 are equal.

Thus (A ∩ B)' = A' ∪ B'

(c) Associative law:



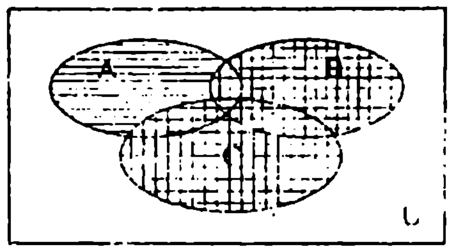


Fig. 1

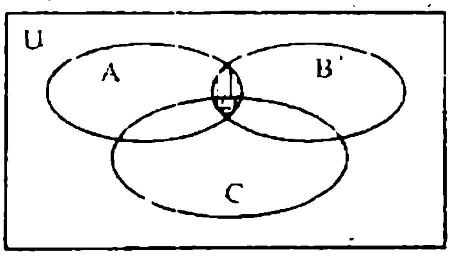
(A ∪ B) ∪ C is shown in the above figure,

Fig. 2

A ∪ (5 ∪ C) is shown in the above figure.

Regions shown in fig. 1 and fig. 2 by different ways are equal.

Thus (A ∪ B) ∪ C = A ∪ (B ∪ C)



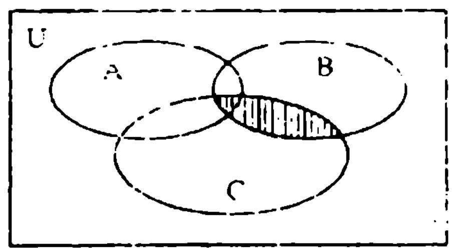


Fig. 3

Fig. 4

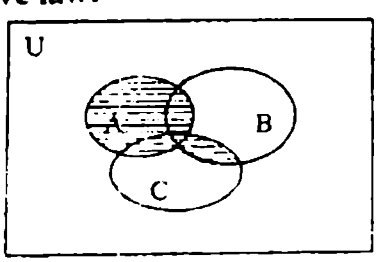
(A ∩ B) ∩ C is shown in figure 3 by double

A ∩ (B ∩ C) is shown in figure 4 by double crossing line segments

Regions shown in Fig. 3 and fig. 4 are equal.

Thus (A ∩ B)' ∩ C' = A ∩ (B ∩ C)

(d) Distributive law:



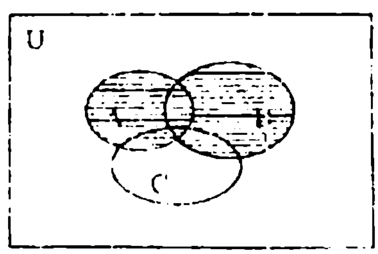
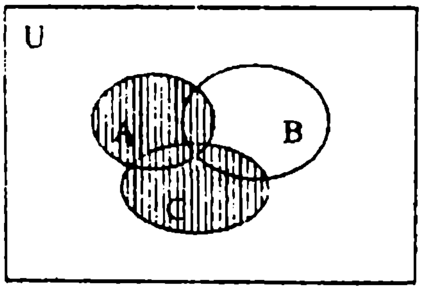


Fig. 1: $A \cup (B \cap C)$ is shown by horizontal line segments in the above figure.

Fig. 2: A u B is shown by horizontal line segments in the above figure.



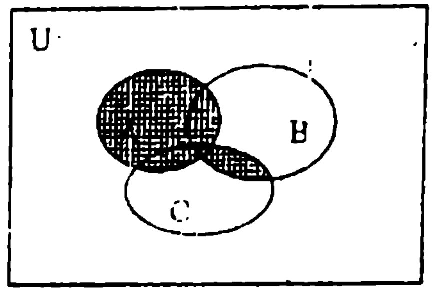
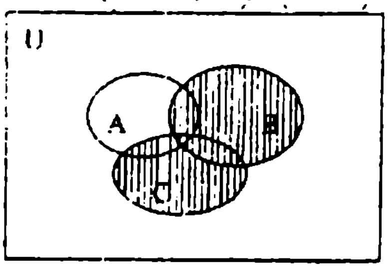


Fig. 3: $A \cup C$ is shown by vertical line segments in Fig. 3,

Fig. 4: $(A \cup B) \cap (A \cup C)$ is shown by double crossing line segments in Fig. 4.

Regions shown in Fig. 1 and Fig. 4 are equal.

Thus $A \cup (B \cap C) = (A \cup B') \cap (A \cup C)$



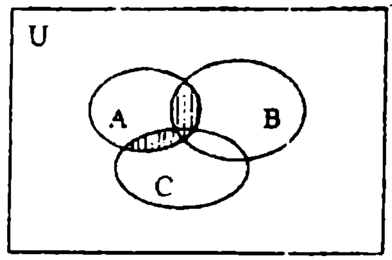


Fig. 5: $B \cup C$ is shown by vertical line segments in Fig. 5.

Fig. 6: $A \cap (B \cup C)$ is shown in Fig. 6 by vertical line segments.

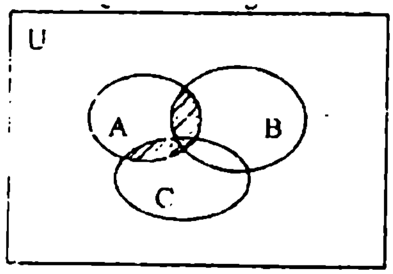


Fig. 7: $(A \cap B) \cup (A \cap C)$ is shown in Fig. 7 by slanting line segments.

Regions displayed in Fig. 6 and Fig, 7 are equal.

Thus $A \cap (A \cup C) = (A \cap B) \cup (A \cap C)$

SOLVED EXERCISE 5.3

1. If $U = \{1, 2, 3, 4, \ldots, 10\}$
$A = \{1, 3, 5, 7, 9\}$
$B = \{1, 4, 7, 10\}$ then verify the following questions,

(i) $A - B = A \cap B'$

L.H.S. $= A - B$

$= \{1, 3, 5, 7, 9\} - \{1, 4, 7, 10\}$

$= \{3, 5, 9\}$ ________(i)

R.H.S. $= A \cap B'$