$A' = U - A$
$= \{1, 2, 3, \ldots, 10\} - \{2, 4, 6, 8\}$
$= \{1, 3, 5, 7, 9. 10\}$

Perform operations on sets:

Example: If $U = \{1, 2, 3, \ldots, 10\}$, $A = \{2, 3, 5, 7\}$, $B = \{3, 5, 8\}$ then

Find (i) $A \cup B$ (ii) $A \cap B$ (iii) $A - B$

Solution: (i) $A \cup B = \{2, 3, 5, 7\} \cup \{3, 5, 8\}$
$= \{2, 3, 5, 7, 8\}$

(ii) $A \cap B = \{2, 3, 5, 7\} \cap \{3, 5, 8\}$
$= \{3, 5\}$

(iii) $A \backslash B = \{2, 3, 5, 7\} \backslash \{3, 5, 8\}$
$= \{2. 7\}$

(iv) $A' = U - A = \{1, 2, 3, \ldots, 10\} - \{2, 3. 5, 7\}$
$= \{1, 4, 6, 8, 9{:}10\}$

$B' = U - B = \{1, 2, 3, \ldots, 10\} - \{3, 5, 8\}$
$= \{1, 2, 4, 6, 7, 9, 10\}$

SOLVED EXERCISE 5.1

1. If $X = \{1, 4, 7, 9\}$ and $Y = \{2, 4, 5, 9\}$
Then find:

(i) $X \cup Y$ **(ii)** $X \cap Y$
(iii) $Y \cup X$ **(iv)** $Y \cap X$

Solution:

(i) $X \cup Y = \{1, 4, 7, 9\} \cup \{2, 4, 5, 9\}$
$= \{1, 2, 4, 7, 9\}$

(ii) $X \cap Y = \{1, 4, 7, 9\} \cap \{2, 4, 5, 9\}$
$= \{4, 9\}$

(iii) $Y \cup X = \{2, 4, 5. 9\}\{1, 4, 7. 9\}$
$= \{1, 2, 4, 5, 7, 9\}$

(iv) $Y \cup X = \{2, 4, 5, 9\} \cap \{1, 4, 7, 9\}$
$= \{4. 9\}$

2. If X Set of prime numbers less than or equal to 17 and Set of first 12 natural numbers, then find the following.

(i) $X \cup Y$ **(ii)** $Y \cup X$ **(iii)** $Z \cap Y$ **(iv)** $Y \cap X$

Solution:

$X = \{2, 3, 5, 7, 11, 13, 17\}, Y = \{1, 2, 3, 4, \ldots\ldots, 12\}$

(i) $X \cup Y = \{2,3,5,7,11,13,17\}, \cup = \{1,2,3,4,\ldots\ldots,12\}$

$= \{1,2,3,4,\ldots,12,13,17\}$

$= \{1,2,3,4,\ldots,12\} \cup \{13,17\}$

$= Y \cup \{13,17\}$

(ii) $Y \cup X = \{1,2,3,4,..,12\} \cup \{2,3,5,7,11,13,17\}$

$= \{1,2,3,4,\ldots,12,13,17\}$

$= \{1,2,3,4,..,12\} \cup \{13,17\}$

$Y \cup \{13,17\}$

(iii) $X \cap Y = \{2,3,5,7,11,13,17\} \cap \{1,2,3,..,12\}$

$= \{2,3,5,7,11\}$

(iv) $Y \cap X = \{1,2,3,4,\ldots,12\} \cap \{2,3,5,7,11,13,17\}$

$= \{2,3,5,7,11\}$

3. If $X = \phi$, $Y = Z^+$, $F = O^+$, then
find: (i) $X \cup Y$ (ii) $X \cup T$ (iii) $Y \cup T$ (iv) $X \cap Y$
(v) $X \cap T$ (vi) $Y \cap T$

Solution:

$X = \phi, Y = Z^+, T = O^+$

(i) $X \cup Y = \phi \cup Z^+$

$= Z^+ = Y$

(ii) $X \cup T = \phi \cup O^+$

$= O^+ = T$

(iii) $Y \cup T = Z^+ \cup O^+$

$= Z^+ = Y$

(iv) $X \cap Y = \phi \cap Z^+$

$= \phi$

(v) $X \cap T = \phi \cap O^+$

$= \phi$

(vi) $Y \cap T = Z^+ \cap O^+$

$= O^+ = T$

4. If $U = \{x \mid x \in N \wedge 3 < x \leq 25\}$, $X = \{x \mid x \text{ is prime} \wedge 8 < x < 25\}$
and $Y = \{x \mid x \in W \wedge 4 \leq x \leq 17\}$. Find the value of:
i) $(X \cup Y)'$ (ii) $X' \cap Y'$ (iii) $(X \cap Y)'$ (iv) $X' \cup Y'$

Solution:

$U = \{4, 5, 6, 7, ..., 24, 25\}$

$X = \{11, 13, 17, 19, 23\}$

$Y = \{4, 5, 6, 7, ..., 16, 17\}$

(i) $(X \cup Y)' = U - (X \cup Y)$

Now

$X \cup Y = \{11, 13, 17, 19, 23\}\ \{4, 5, 6, 7, \ldots, 16, 17\}$
$= \{4. 5, 6, 7, \ldots, 16, 17, 19, 23\}$

$(X \cup Y)' = \{4, 5, 6, 7, \ldots, 24, 25\} - \{4, 5, 6, 7, \ldots, 16, 17, 19, 23\}$
$= \{18, 20, 21, 22, 24, 25\}$

(ii) $X' \cap Y'$

Now $X' = \cup - X$
$= \{4, 5, 6, 7, \ldots, 24, 25\} - \{11, 13, 17, 19, 23\}$
$= \{4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25\}$

$Y' = \cup - Y$
$= \{4, 5, 6, 7, \ldots, 24, 25\} - \{4, 5, 6, 7, \ldots, 16, 17\}$
$= \{18, 19, 20, \ldots, 24, 25\}$

$X' \cap Y' = \{4, 5, 6, 7, 8, 9, 10, 12, 14, 16, 18, 20, 21, 22, 24, 25\}$
$\cap \{18, 19, 20, \ldots, 24, 25\}$
$= \{18, 20, 21, 22, 24, 25\}$

(iii) $(X \cap Y)' = U - (X \cap Y)$

Now

$X \cap Y = \{11, 13, 17, 19, 23\} \cap \{4, 5, 6, 7, .., 16, 17\}$
$= \{11, 13, 17\}$

$(X \cap Y)' = \{4, 5, 6, 7, \ldots, 24, 25\} - \{11, 13, 17\}$
$= \{4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 18, 20, 21, 22. 24, 25\}$

$Y' = \cup - Y$
$= \{4, 5, 6, 7, ..., 24, 25\} - \{4, 5, 6, 7, \ldots, 16, 17\}$
$\{18. 19, 20, \ldots, 24, 25\}$

$Y' = \cup - Y$
$= \{4, 5, 6, \ldots, 10, 12, 14, 15, 16, 18, ..., 25\} \cup\ = \{18, 19, 20, \ldots, 24, 25\}$
$= \{4, 5, ..., 10, 12, 14, 16, 18, \ldots, 25\}$

5. If $X = \{2, 4, 6, \ldots, 20\}$ and $Y = \{4, \&, 12, ..., 24\}$, then find the following:
(i) $X - Y$ **(ii) $Y - X$**

Solution:

$X = \{2, 4, 6, \ldots, 20\}$, $Y = \{4. 8. 12, \ldots, 24\}$

i) $X - Y = \{2, 4, 6, .., 20\} - \{4, 8, 12, .., 24\}$
$= \{2, 6, 10, 14, 18\}$

(ii) $Y - X = \{4, 8, 12, \ldots, 24\} - \{2, 4, 6, ..., 20\}$
$= \{24\}$

6. If A = N and B = W, then find the value of

(i) A – B **(ii) B – A**

Solution:

A = N and B = W

(i) $A - B = N - W$

$= \phi$

(ii) $B - A = W - N$

$= \{0, 1, 2, \ldots\} - \{1, 2, \ldots\}$

$= \{0\}$

Properties of Union and Intersection:

(a) Commutative property of union.

For any two sets A and B, prove that $A \cup B = B \cup A$.

Proof:

Let $x \in A \cup B$

$\Rightarrow \quad x \in A \quad$ or $\quad x \in B$ (by definition of union of sets)

$\Rightarrow \quad x \in B \quad$ or $\quad x \in A$

$\Rightarrow \quad x \in B \cup A$

$\Rightarrow \quad A \cup B \subseteq B \cup A$ (i)

Now let $y \in B \cup A$

$\Rightarrow \quad y \in B \quad$ or $\quad y \in A$ (by definition of union of sets)

$\Rightarrow \quad y \in A \quad$ or $\quad y \in B$

$\Rightarrow \quad y \in A \cup B$

$\Rightarrow \quad B \cup A \subseteq A \cup B$ (ii)

From (i) and (ii), we have $A \cup B = B \cup A$. (by definition of equal sets)

(b) Commutative property of intersection

For any two sets A and B, prove that $A \cup B$

Proof: Let $x \in A \cap B$

$\Rightarrow \quad x \in A \quad$ and $\quad x \in B$ (by definition of intersection of sets)

$\Rightarrow \quad x \in B \quad$ and $\quad x \in A$

$\Rightarrow \quad x \in B \cap A$

$\Rightarrow \quad A \cap B \subseteq B \cap A$ (i)

Now let $y \in B \cap A$

$\Rightarrow \quad y \in B \quad$ and $\quad y \in A$ (by definition of intersection of sets)

$\Rightarrow \quad y \in A \quad$ and $\quad y \in B$

$\Rightarrow \quad y \in A \cap B$

Therefore, $B \cap A \subseteq A \cap B$ (ii)

From (i) and (ii), we have $A \cap B = B \cap A$ (by definition of equal sets)

(c) Associative property of union

For any three sets A, B and C, prove that $(A \cup B) \cup C = A \cup (B \cup C)$

Proof: Let $x \in (A \cup B) \cup C$

$\Rightarrow \quad x \in (A \cup B) \quad \text{or} \quad x \in C$

$\Rightarrow \quad x \in A \text{ or } x \in B \text{ or } x \in C$

$\Rightarrow \quad x \in A \text{ or } x \in B \text{ or } A \in C$

$\Rightarrow \quad x \in A \text{ or } x \in B \cup C$

$\Rightarrow \quad x \in A \cup (B \cup C)$

$(A \cup B) \cup C \subseteq A \cup (B \cup C) \qquad$ (i)

Similarly $\quad A \cup (B \cup C) \subseteq (A \cup B) \cup C \qquad$ (ii)

From (i) and (ii), we have

$$(A \cup B) \cup C = A \cup (B \cup C)$$

(d) Associative property of intersection

For any three sets A, B and C, prove that $(A \cap B) \cap C = A \cap (B \cap C\}$

Proof: Let $\in (A \cap B) \cap C$

$\Rightarrow \quad x \in (A \cap B) \text{ and } x \in C$

$\Rightarrow \quad (x \in A \text{ and } x \in 5) \text{ and } x \in C$

$\Rightarrow \quad x \in A \text{ and } (x \in B) \text{ and } \in C)$

$\Rightarrow \quad x \in A \text{ and } x \in B \cap C$

$\Rightarrow \quad x \in A \cap (B \cap C)$

$\therefore \quad (A \cap B) \cap C \quad \subseteq \quad A \cap (B \cap C) \qquad$ (i)

Similarly $\quad A \cap (B \cap C) \quad \subseteq \quad (A \cap B) \cap C \qquad$ (ii)

From (i) and (ii), we have

$$(A \cap B) \cap C = A \cap (B \cap C)$$

(e) Distributive property of union over intersection

For any three sets A, B and C, prove that $A \cup (B \cup C) = (A \cup B) \cap (A \cup C)$

Proof: Let $x \in A \cup (B \cup C)$

$\Rightarrow \quad x \in A \text{ or } x \in B \cap C$

$\Rightarrow \quad x \in A \text{ or } (x \in B \text{ and } x \in C)$

$\Rightarrow \quad (x \in A \text{ or } x \in B) \text{ and } (x \in A \text{ or } x \in C)$

$\Rightarrow \quad x \in A \text{ U } B \text{ and } x \in A \cup C$

$\Rightarrow \quad x \in (A \cup B) \cap (A \cup C)$

Therefore $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$

Similarly, now let $y \in (A \cup B) \cap (A \cup C)$

$\Rightarrow \quad y \in (A \cup B) \text{ and } y \in (A \cup C)$

$\Rightarrow \quad (y \in A \text{ or } y \in B) \text{ and } (y \in A \text{ or } y \in C)$

$\Rightarrow \quad y \in A \text{ or } (y \in B \text{ and } y \in C)$

$\Rightarrow \quad y \in A \text{ or } y \in B \cap C$

$\Rightarrow \quad y \in y \in \cup \; (B \cap C)$

$\Rightarrow \quad (A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C) \qquad$ (ii)

From (i) and (ii), we have $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(f) Distributive property of intersection over union

For any three sets A, B and C, prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Proof: Let $x \in A \cap (B \cup C)$

$\Rightarrow \quad x \in A$ and $\quad x \in B \cap C$

$\Rightarrow \quad x \in A$ and $\quad [x \in B$ or $x \in C]$

$\Rightarrow \quad [x \in A$ and $x \in B]$ or $[x \in A$ and $x \in C]$

$\Rightarrow \quad [x \in A \cap B] \quad$ or $[x \in A \cap C]$

$\Rightarrow \quad x \in (A \cap B) \cup (A \cup C)$

Hence by def. of subsets

$A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C) \qquad$ (i)

Similarly $\quad (A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C) \qquad$ (ii)

From (i) and (ii), we have , $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(g) De-Morgan's laws

For any two sets A and B, prove that

(i) $(A \cup B)' = A' \cap B'$

Proof: Let $x \in (A \cup B)'$

$\Rightarrow \quad x \notin A \cup B \qquad$ (by definition of complement of set)

$\Rightarrow \quad x \notin A$ and $\quad x \notin B$

$\Rightarrow \quad x \in A'$ and $\quad x \in B'$

$\Rightarrow \quad x \in A' \cap B' \qquad$ (by definition of intersection of sets)

$\Rightarrow \quad (A \cup B)' \subseteq (A \cup B)' \qquad$ (i)

Similarly $A' \cap B' \subseteq (A \cup B) \qquad$ (ii)

Using (i) and (ii), we have $(A \cup B)' = A' \cap B'$

(ii) Let $\quad x \in (A \cap B)'$

$\Rightarrow \quad x \notin A \cap B$

$\Rightarrow \quad x \notin A$ or $x \notin B$

$\Rightarrow \quad x \in A'$ or $x \in B'$

$\Rightarrow \quad x\ A' \cup B'$

$\Rightarrow \quad (A \cap B)' \subseteq A' \cup B' \qquad$ (i)

Let $\quad y \in A' \cap B'$

$\Rightarrow \quad y \in A \cap B$

$\Rightarrow \quad y \notin A$ or $x \notin B$

$\Rightarrow \quad y \notin A \cap B$

$\Rightarrow \quad y \in (A \cap B)'$

$\Rightarrow \quad (A' \cap B)' \subseteq A' \cap B' \qquad$ (ii)

From (i) and (ii) we have proved that

$(A \cap B)' = A' \cup B'$

SOLVED EXERCISE 5.2

1. If $X = \{1,3. 5, 7, \ldots,19\}$, $\quad Y = \{0,2,4, 6, 8, \ldots,20\}$
$Z = \{2, 3,5,7,11,13,17,19,23\}$, then find the following.

(i) $X \cup (Y \cup Z)$

Solution: