Exercise 2.5

Convert the following theorems to logical form and prove them by constructing truth table:-

Question # 1

$(A \cap B)' = A' \cup B'$

Solution

The corresponding formula of logic is

$$\sim (p \wedge q) = \sim p \vee \sim q$$

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim (p \wedge q)$	$\sim p \vee \sim q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

The last two columns of the above table shows that $\sim (p \wedge q) = \sim p \vee \sim q$ and hence $(A \cap B)' = A' \cup B'$.

Question # 2

$(A \cup B) \cup C = A \cup (B \cup C)$

Solution

The corresponding formula of logic is

$$(p \vee q) \vee r = p \vee (q \vee r)$$

p	q	r	$p \vee q$	$q \vee r$	$(p \vee q) \vee r$	$p \vee (q \vee r)$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T
T	F	F	T	F	T	T
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	F	T	T	T
F	F	F	F	F	F	F

The last two columns of the above table shows that $(p \vee q) \vee r = p \vee (q \vee r)$ and hence $(A \cup B) \cup C = A \cup (B \cup C)$.

Question # 3

$(A \cap B) \cap C = A \cap (B \cap C)$

Solution

The corresponding formula of logic is

$$(p \wedge q) \wedge r = p \wedge (q \wedge r)$$

p	q	r	$p\wedge q$	$q\wedge r$	$(p\wedge q)\wedge r$	$p\wedge(q\wedge r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	F	T	F	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

The last two columns of the above table shows that $(p\wedge q)\wedge r = p\wedge(q\wedge r)$ and hence $(A\cap B)\cap C = A\cap(B\cap C)$.

Question # 4

$A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

Solution

$A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

The corresponding formula of logic is

$p\vee(q\wedge r)=(p\vee q)\wedge(p\vee r)$

p	q	r	$q\wedge r$	$p\vee q$	$p\vee r$	$p\vee(q\wedge r)$	$(p\vee q)\wedge(p\vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	T	F	F	F
F	F	T	F	F	T	F	F
F	F	F	F	F	F	F	F

The last two columns of the above table shows that $p\vee(q\wedge r)=(p\vee q)\wedge(p\vee r)$ and hence $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$.
