Put W = 800, d = 120 and T = 40 in eq. (ii), we get

$$40 = \frac{800 \times 120}{200P}$$

$$P = \frac{800 \times 120}{200 \times 40}$$

$$P = 12 \text{ hp}$$

9. **The kinetic energy (K.E.) of a body varies jointly as the mass "m" of the body and the square of its velocity "v". If the kinetic energy is 4320 ft/lb when the mass is 45 lb and the velocity is 24 ft/sec. Determine the kinetic energy of a 3000 lb automobile travelling 44 ft/sec.**

Given that $K.E \propto MV^2$

$\Rightarrow$ $K.E = KmV^2$ ______(i)

Put K.E = 4320, m = 45 and V = 24 in eq. (i), we get

$$4320 = k(45)(24)^2$$

$$K = \frac{4320}{45 \times 576}$$

$$K = \frac{1}{6}$$

Put $K = \frac{1}{6}$ in eq. (i), we get

$$K.E = \frac{1}{6} mV^2 \text{ ______(ii)}$$

Put m = 3000 and V = 44 in eq. (ii), we get

$$K.E = \frac{1}{6} (3000)(44)^2$$

$$= 968000$$

SOLVED MISCELLANEOUS EXERCISE - 3

1. **Multiple Choice Questions**

Four possible answers are given for the following questions. Tick (✓) the correct answer.

(i) In a ratio a : b, a is called
(a) relation (b) antecedent (c) consequent (d) None of these

(ii) In a ratio x : y, y is called
(a) relation (b) antecedent (c) consequent (d) None of these

(iii) In a proportion a : b :: c : d, a and d are called,
(a) means (b) extremes
(c) third proportional (d) None of these

(iv) In a proportion a : b :: c : d, b and c are called

(a) means (b) extremes

(c) fourth proportional (d) None of these

(v) In continued proportion a : b = b : c, ac = b^2, b is said to be ________ proportional between a and c.

(a) third (b) fourth (c) means (d) None of these

(vi) In continued proportion a : b = b : c, c is said to be ______ proportional to a and b.

(a) third (b) fourth (c) means (d) None of these

(vii) Find x in proportion 4 : x :: 5 : 15

(a) $\frac{75}{4}$ (b) $\frac{4}{3}$ (c) $\frac{3}{4}$ (d) 12

(viii) If $u \propto v^2$, then

(a) $u = v^2$ (b) $u = kv^2$ (c) $uv^2 = k$ (d) $uv^2 = 1$

(ix) If $y^2 \propto \frac{1}{x^3}$, then

(a) $y^2 = \frac{k}{x^3}$ (b) $y^2 = \frac{1}{x^3}$ (c) $y^2 = x^2$ (d) $y^2 = kx^3$

(x) If $\frac{u}{v} = \frac{v}{w} = k$, then

(a) $u = wk^2$ (b) $u = vk^2$ (c) $u = w^2k$ (d) $u = v^2k$

(xi) The third proportional of x^2 wdy^2 is:

(a) $\frac{y^2}{x^2}$ (b) x^2y^2 (c) $\frac{y^4}{x^2}$ (d) $\frac{y^2}{x^4}$

(xii) The fourth proportional w of x : y :: v : w is

(a) $\frac{xy}{v}$ (b) $\frac{vy}{x}$ (c) xyv (d) $\frac{x}{vy}$

(xiii) If a : b = x : y, then alternant property is

(a) $\frac{a}{x} = \frac{b}{y}$ (b) $\frac{a}{b} = \frac{x}{y}$

(c) $\frac{a+b}{b} = \frac{x+y}{y}$ (d) $\frac{a-b}{x} = \frac{x-y}{y}$

(xiv) If a : b = x : y, then inverted property is

(a) $\frac{a}{x} = \frac{b}{y}$ (b) $\frac{a}{a-b} = \frac{x}{x-y}$

(c) $\frac{a+b}{b}=\frac{x+y}{y}$ (d) $\frac{b}{a}=\frac{y}{x}$

(xv) If, $\frac{a}{b}=\frac{c}{d}$ then component property is:

(a) $\frac{a}{a+b}=\frac{c}{a+d}$ (b) $\frac{a}{a-b}=\frac{c}{a-d}$

(c) $\frac{ad}{bc}$ (d) $\frac{a-b}{b}=\frac{c-d}{d}$

Answers:

(i)	b	(ii)	c	(iii)	b	(iv)	a	(v)	c
(vi)	a	(vii)	d	(viii)	b	(ix)	a	(x)	a
(xi)	c	(xii)	b	(xiii)	a	(xiv)	d	(xv)	a

2. Write short answers of the following questions.

(i) Define ratio and give one example.

Ans: Ratio:

A relation between two quantities of the same kind is called ratio.

(ii) Define proportion.

Ans: Proportion:

A proportion is a statement, which is expressed as equivalence of two ratios.

(iii) Define direct variation.

Ans: Direct variation:

If two quantities are related in such a way that increase (decrees) in one quantity causes increase (decrease) in the other quantity is called direct variation.

(iv) Define inverse variation.

Ans: Inverse Variation:

If two quantities are related in such a way that when one quantity increases, the other decreases is called inverse variation.

(v) State theorem of componendo-dividendo.

Ans: The theorem of componendo-dividendo is

If $a : b = c : d$, then

$a + b : a - b = c + d : c - d$

(vi) Find x, if 6 : x : : 3 : 5,

Ans: $6 : x :: 3 : 5$

Product of means = Product of extremes

$$(x)(3) = (6)(5)$$
$$3x = 30$$
$$\Rightarrow \quad x = 10$$

(vii) If x and y^2 varies directly, and x = 27 when y = 4. Find the value of y when x = 3.

Ans: Given that $x \propto y^2$

$\Rightarrow$ $x = ky^2$ ______ (i)

Put x = 27 and y = 4 in eq. (i), we get

$$27 = K(4)^2$$

$$K = \frac{27}{16}$$

Put $K = \frac{27}{16}$ in eq. (i), we get

$$x = \frac{27}{16}y^2 \text{ ______ (ii)}$$

Put x = 3 in eq. (ii), we get

$$3 = \frac{27}{16}y^2$$

$$y^2 = 3 \times \frac{16}{27}$$

$$y^2 = \frac{16}{9} \quad \Rightarrow \quad y = \pm\frac{4}{3}$$

(viii) If u and v varies inversely, and u = 8, when v =3. Find v when u = 12.

Ans: Given that $u \propto \frac{1}{V}$

$\Rightarrow$ $u = \frac{K}{V}$ ______ (i)

Put u = 8 and V = 3 in eq. (i), we get

$$8 = \frac{K}{3}$$

$$K = 24$$

Put K = 24in eq. (i), we get

$$u = \frac{24}{V} \text{ ______ (ii)}$$

Put u = 12 in eq. (ii), we get

$$12 = \frac{24}{V} \quad \Rightarrow \quad V = \frac{24}{12} = 2$$

(ix) Find the fourth proportional to 8, 7, 6.

Ans: Let x be the fourth proportional, then

$$8 : 7 :: 6 : x$$

Product of extremes = Product of means

$$(8)(x) = (7)(6)$$

$$8x = 42$$

$$x = \frac{42}{8} \quad \Rightarrow \quad x = \frac{21}{4}$$

(x) Find a mean proportional to 16 and 49.

Ans: Let x be the fourth proportional, then

$$16 : m :: m : 49$$

Product of extremes = Product of means

$$m.\,m = 16 \times 49$$

$$m = 784$$

$$\Rightarrow \quad m = \sqrt{784} = \pm 28$$

(xi) Find a third proportional to 28 and 4.

Ans: Let x be the fourth proportional, then

$$28 : 4 :: 4 : C$$

Product of extremes = Product of means

$$(28)(C) = (4)(4)$$

$$28\,C = 16$$

$$C = \frac{16}{28}$$

$$C = \frac{4}{7}$$

(xii) If $y \propto \frac{x^2}{z}$ and y = 28 when x = 7, z = 2, then find y.

Ans: Given $y \propto \frac{x^2}{Z}$

$$y = \frac{x^2}{Z} \text{ ------- (i)}$$

Put x = 7 and y = 28 in eq. (i), we get

$$28 = K\frac{(7)^2}{2}$$

$$28 = K\frac{49}{2}$$

or $\quad K = \frac{56}{49}$

$$K = \frac{8}{7}$$

Put $K = \frac{8}{7}$ in eq. (i), we get

$$y = \frac{8x^2}{72} \text{ ------- (ii)}$$

Put x = 7 and Z = 2 in eq. (ii), we get

$$y = \frac{8(7)^2}{7(2)} = 28$$

(xiii) If $z \propto xy$ and $z = 36$ when $x = 2, y = 3$, then find z.

Ans: Given $Z \propto xy$

$$z = Kxy \text{ \_\_\_\_\_\_\_ (i)}$$

Put $z = 36$ $x = 2$ and $y = 3$ in eq. (i), we get

$$36 = K(2)(3)$$

or $36 = 6k$

$\Rightarrow$ $K = 6$

Put $K = 6$ in eq. (i), we get

$$Z = 6\,xy$$

(xiv) If $w \propto \dfrac{1}{v^2}$ and $w = 2$ when $v = 3$, then find w.

Ans: Given $W \propto \dfrac{1}{V^2}$

$\Rightarrow$ $W = \dfrac{K}{q^2}$ _______ (i)

Put $W = 2$ and $y = 3$ in eq. (i), we get

$$2 = \frac{K}{(3)^2}$$

$$2 = \frac{K}{9}$$

$K = 18$

Put $K = 18$ in eq. (i), we get

$$W = \frac{18}{V^2}$$

Q3. Fill in the blanks:

(i) The simplest form of the ratio $\dfrac{(x+y)(x^2+xy+y^2)}{x^3-y^3}$ is __________

(ii) In a ratio x : y ; x is called __________.

(iii) In a ratio a : b; b is called__________.

(iv) In a proportion a : b :: x : y, a and y are called __________.

(v) In a proportion p : q :: m : n; q and m are called__________.

(vi) In proportion 7 : 4 :: p : 8, p = __________.

(vii) If 6: m :: 9 : 12, then m = __________.

(viii) If x and y varies directly, then x = __________.

(ix) If v varies directly as u^3, then u^3 = __________.

(x) If w varies inversely as p^2, then k = __________.

(xi) A third proportional of 12 and 4, is __________.

(xii) The fourth proportional of 15, 6, 5 is ————.

(xiii) The mean proportional of $4m^2n^4$ and p^6 is ————.

(xiv) The continued proportion of 4, m and 9 is ————.

Answer:

(i)	$\frac{x+y}{x-y}$	(ii)	Antecedent	(iii)	Consequent
(iv)	Extremes	(v)	Means	(vi)	P = 14
(vii)	m = 8	(viii)	Ky	(ix)	$\frac{V}{K}$
(x)	P^2w	(xi)	$\frac{4}{3}$	(xii)	2
xiii)	$\pm mn^2p^2$	(xiv)	$m = \pm 6$		

SUMMARY

- A relation between two quantities of the same kind is called ratio.
- A proportion is a statement, which is expressed as equivalence of two ratios.
- If two ratios a: b and c: d are equal, then we can write a : b = c : d
- If two quantities are related in such a way that increase (decrease) in one quantity causes increase (decrease) in the other quantity is called direct variation.
- If two quantities are related in such a way that when one quantity increases, the other decreases is called inverse variation.
- Theorem on proportions:

  (1) Theorem of Invertendo
  If c : b = c : d, then b : a = d : c

  (2) Theorem of Alternando
  If a : b = c : d, then a : c = b : d

  (3) Theorem of Componendo.
  If a : b = c : d, then
  (i) a + b : b = c : d : d
  and (ii) a : a + b = c : c + d

  (4) Theorem of Dividendo
  If a : h = c : d, then
  (i) a – b : b = c – d : d
  (ii) a : a – b = c : c – d

  (5) Theorem of Componendo-dividendo
  If a : b = c : d, then
  a + b : a – b = c + d : c – d

- A combination of direct and inverse variations of one or more than one variable forms joint variation.