

Exercise 7.2

Question # 1

Evaluate the following:

(i) ${}^{20}P_3$

(ii) ${}^{16}P_4$

Solution

(i)

$${}^{20}P_3 = \frac{20!}{(20-3)!} = \frac{20!}{17!} = \frac{20 \cdot 19 \cdot 18 \cdot 17!}{17!} = 20 \cdot 19 \cdot 18 = 6840$$

(ii)

$${}^{16}P_4 = \frac{16!}{(16-4)!} = \frac{16!}{12!} = \frac{16 \cdot 15 \cdot 14 \cdot 13 \cdot 12!}{12!} = 16 \cdot 15 \cdot 14 \cdot 13 = 43680$$

Others do yourself

Question # 2

Find the value of n when:

(i) ${}^nP_2 = 30$

(ii) ${}^{11}P_n = 11 \cdot 10 \cdot 9$

(iii) ${}^nP_4 : {}^{n-1}P_3 = 9:1$

Solution

(i)

$${}^nP_2 = 30 \Rightarrow \frac{n!}{(n-2)!} = 30 \Rightarrow \frac{n(n-1)(n-2)!}{(n-2)!} = 30$$

$$\Rightarrow n(n-1) = 30 \Rightarrow n(n-1) = 6 \cdot 5 \Rightarrow \boxed{n=6}$$

(ii)

$${}^{11}P_n = 11 \cdot 10 \cdot 9 \Rightarrow \frac{11 \cdot 10 \cdot 9 \cdot 8!}{(11-n)!} = 11 \cdot 10 \cdot 9$$

$$\Rightarrow \frac{8!}{(11-n)!} = 1 \Rightarrow 8! = (11-n)! \Rightarrow 8 = 11-n \Rightarrow n = 11-8$$

$$\Rightarrow \boxed{n=3}$$

(iii)

$${}^nP_4 : {}^{n-1}P_3 = 9:1 \Rightarrow \frac{{}^nP_4}{{}^{n-1}P_3} = \frac{9}{1} \Rightarrow {}^nP_4 = 9 \cdot {}^{n-1}P_3$$

$$\Rightarrow \frac{n!}{(n-4)!} = 9 \frac{(n-1)!}{(n-1-3)!} \Rightarrow \frac{n(n-1)!}{(n-4)!} = 9 \frac{(n-1)!}{(n-4)!} \Rightarrow \boxed{n=9}$$

Question # 3

Prove from the first principle that:

(i) ${}^nP_r = n \cdot {}^{n-1}P_{r-1}$

(ii) ${}^nP_r = {}^{n-1}P_r + r \cdot {}^{n-1}P_{r-1}$

Solution

(i)

$$\text{R.H.S} = n \cdot {}^{n-1}P_{r-1} = n \cdot \frac{(n-1)!}{(n-1-(r-1))!} = \frac{n(n-1)!}{(n-1-r+1)!} = \frac{n!}{(n-r)!} = {}^nP_r = \text{L.H.S}$$

(ii)

$$\begin{aligned} \text{R.H.S} &= {}^{n-1}P_r + r \cdot {}^{n-1}P_{r-1} = \frac{(n-1)!}{(n-1-r)!} + r \cdot \frac{(n-1)!}{(n-1-r+1)!} \\ &= \frac{(n-1)!}{(n-r-1)!} + r \cdot \frac{(n-1)!}{(n-r)!} = \frac{(n-1)!}{(n-r-1)!} + r \cdot \frac{(n-1)!}{(n-r)(n-r-1)!} \\ &= \frac{(n-1)!}{(n-r-1)!} \left(1 + r \cdot \frac{1}{(n-r)} \right) = \frac{(n-1)!}{(n-r-1)!} \left(\frac{n-r+r}{(n-r)} \right) \\ &= \frac{(n-1)!}{(n-r-1)!} \left(\frac{n}{(n-r)} \right) = \frac{n(n-1)!}{(n-r)(n-r-1)!} \\ &= \frac{n!}{(n-r)!} = {}^nP_r = \text{L.H.S} \end{aligned}$$

Question # 4

Do yourself as Question # 5

Question # 5

How many signals can be given by 6 flags of different colours, when any number of flags can be used at a time?

Solution

Total number of flags = $n = 6$

Number of signal using one flag = ${}^6P_1 = 6$

Number of signal using two flag = ${}^6P_2 = 30$

Number of signal using three flag = ${}^6P_3 = 120$

Number of signal using four flag = ${}^6P_4 = 360$

Number of signal using five flag = ${}^6P_5 = 720$

Number of signal using six flag = ${}^6P_6 = 720$

Total number of signals = $6 + 30 + 120 + 360 + 720 + 720 = 1956$

Question # 6

How many words can be formed from the letters of the following words using all letters when no letter is to be repeated:

(i) PLSNE

(ii) OBJECT

(iii) FASTING

Solution

(i)

Since number of letters in PLANE = $n = 5$

Therefore total words form = ${}^5P_5 = 120$

(ii)

Since number of letters in OBJECT = $n = 6$

Therefore total words forms = ${}^6P_6 = 720$

(iii)

Since number of letters in FASTING = $n = 7$

Therefore total words forms = ${}^7P_7 = 5040$

Question # 7

How many 3 – digit numbers can be formed by using each one of the digits 2,3,5,7,9 only once?

Solution

Number of digits = $n = 5$

So numbers forms taken 3 digits at a time = ${}^5P_3 = 60$

Question # 8

Find the numbers greater than 23000 that can be formed from the digits 1,2,3,5,6 without repeating any digit.

Solution

Number greater than 23000 can be formed as

Number of numbers of the form 23*** = ${}^3P_3 = 6$

Number of numbers of the form 25*** = ${}^3P_3 = 6$

Number of numbers of the form 26*** = ${}^3P_3 = 6$

Number of numbers of the form 3**** = ${}^4P_4 = 24$

Number of numbers of the form 5**** = ${}^4P_4 = 24$

Number of numbers of the form 6**** = ${}^4P_4 = 24$

Thus the total number formed = $6 + 6 + 6 + 24 + 24 + 24 = 90$

Alternative solution:

Permutation of 5 digits numbers = ${}^5P_5 = 120$

Numbers less than 23000 are of the form 1****

Then permutations = ${}^4P_4 = 24$

If number less than 23000 are of the form 21***

Then permutations = ${}^3P_3 = 6$

Thus number greater than 23000 formed = $120 - 24 - 6 = 90$

Question # 9

Find the number of 5 digits numbers that can be formed from the digits 1, 2, 4, 6, 8 (when no digit is repeated), but

(i) the digits 2 and 8 are next to each other.

(ii) the digits 2 and 8 are not next to each other.

Solution

Total number of digits = 5

(i) If we take 28 as a single digit then number of numbers = ${}^4P_4 = 24$

If we take 82 as a single digit then number of numbers = ${}^4P_4 = 24$

So the total numbers when 2 and 8 are next to each other = $24 + 24 = 48$

- (ii) Number of total permutation = ${}^5P_5 = 120$
thus number of numbers when 2 and 8 are not next to each other = $120 - 48 = 72$
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Question # 10

How many 6 – digit numbers can be formed, without repeating any digit from the digits 0,1,2,3,4,5? In how many of them will 0 be at the ten places?

Solution

Since number of permutation of 6 digits = ${}^6P_6 = 720$

But 0 at extreme left is meaning less

so number of permutation when 0 is at extreme left = ${}^5P_5 = 120$

Thus the number formed by 6 digits = $720 - 120 = 600$

Now if we fix 0 at ten place then number formed = ${}^5P_5 = 120$

Question # 11

How many 5 – digit multiples of 5 can be formed from the digits 2,3,5,7,9, when no digit is repeated.

Solution

Number of digits = 5

For multiple of 5 we must have 5 at extreme right so number formes =

$${}^4P_4 = 24$$

Question # 12

In how many ways can 8 books including 2 books on English be arranged on a shelf in such a way that the English books are never together.

Solution

Total numbers of books = 8

Total number of permutation = ${}^8P_8 = 40320$

Let E_1 and E_2 denotes two English books then

Number of permutation when E_1E_2 place together = ${}^7P_7 = 5040$

Number of permutation when E_2E_1 place together = ${}^7P_7 = 5040$

So total permutation when E_1 and E_2 together = $5040 + 5040 = 10080$

Required permutation when English books are not together = $40320 - 10080 = 30240$

Question # 13

Find the number of arrangement of 3 books on English and 5 books on Urdu for placing them on a shelf such that the books on the same subjects are together.

Solution

Let E_1, E_2, E_3 be the book on English and U_1, U_2, U_3, U_4, U_5 be the book on Urdu

Then the permutation when

books are arranged as $E_1, E_2, E_3, U_1, U_2, U_3, U_4, U_5 = {}^3P_3 \times {}^5P_5 = 6 \times 120 = 720$

books are arranged as $U_1, U_2, U_3, U_4, U_5, E_1, E_2, E_3 = {}^5P_5 \times {}^3P_3 = 120 \times 6 = 720$
 so total permutation when books of same subject are together = $720 + 720$
 $= 1440$

Question # 14

In how many ways can 5 boys and 4 girls be seated on bench so that the girls and the boys occupy alternative seats?

Solution

Let the five boys be B_1, B_2, B_3, B_4, B_5 and the four girls are G_1, G_2, G_3, G_4 then there seats plane is $B_1, G_1, B_2, G_2, B_3, G_3, B_4, G_4, B_5$

$$\begin{aligned} \text{Then the permutations} &= {}^5P_1 \times {}^4P_1 \times {}^4P_1 \times {}^3P_1 \times {}^3P_1 \times {}^2P_1 \times {}^2P_1 \times {}^1P_1 \times {}^1P_1 \\ &= 5 \times 4 \times 4 \times 3 \times 3 \times 2 \times 2 \times 1 \times 1 = 2880 \end{aligned}$$
