Exercise 6.2

Simplify each of the following as a rational expression.

Q1. $\dfrac{x^2-x-6}{x^2-9}+\dfrac{x^2+2x-24}{x^2-x-12}$

$=\dfrac{x^2-3x+2x-6}{(x)^2-(3)^2}+\dfrac{x^2+6x-4x-24}{x^2+3x-4x-12}$

$=\dfrac{x(x-3)+2(x-3)}{(x+3)(x-3)}+\dfrac{x(x+6)-4(x+6)}{x(x+3)-4(x+3)}$

$=\dfrac{(x-3)(x+2)}{(x+3)(x-3)}+\dfrac{(x+6)(x-4)}{(x+3)(x-4)}$

$=\dfrac{x+2}{x+3}+\dfrac{x+6}{x+3}=\dfrac{x+2+x+6}{x+3}$

$$=\frac{2x+8}{x+3}$$

$$=\frac{2(x+4)}{x+3}$$

Q2. $$\left[\frac{x+1}{x-1}-\frac{x-1}{x+1}-\frac{4x}{x^2+1}\right]+\frac{4x}{x^4-1}$$

$$=\left[\frac{(x+1)^2-(x-1)^2}{(x-1)(x+1)}-\frac{4x}{x^2+1}\right]+\frac{4x}{x^4-1}$$

$$=\left[\frac{(x^2+2x+1)-(x^2-2x+1)}{(x)^2-(1)^2}-\frac{4x}{x^2+1}\right]+\frac{4x}{x^4-1}$$

$$=\left[\frac{x^2+2x+1-x^2+2x-1}{x^2-1}-\frac{4x}{x^2+1}\right]+\frac{4x}{x^4-1}$$

$$=\left[\frac{4x}{x^2-1}-\frac{4x}{x^2+1}\right]+\frac{4x}{x^4-1}$$

$$=\left[\frac{4x(x^2+1)-4x(x^2-1)}{(x^2-1)(x^2+1)}\right]+\frac{4x}{x^4-1}$$

$$=\frac{4x^3+4x-4x^3+4x}{(x^2)^2-(1)^2}+\frac{4x}{x^4-1}$$

$$=\frac{8x}{x^4-1}+\frac{4x}{x^4-1}$$

$$=\frac{8x+4x}{x^4-1}$$

$$=\frac{12x}{x^4-1}$$

Q3. $$\frac{1}{x^2-8x+15}+\frac{1}{x^2-4x+3}-\frac{2}{x^2-6x+5}$$

$$=\frac{1}{x^2-3x-5x+15}+\frac{1}{x^2-3x-x+3}-\frac{2}{x^2-5x-x+5}$$

$$=\frac{1}{x(x-3)-5(x-3)}+\frac{1}{x(x-3)-1(x-3)}-\frac{2}{x(x-5)-1(x-5)}$$

$$=\frac{1}{(x-3)(x-5)}+\frac{1}{(x-3)(x-1)}-\frac{2}{(x-5)(x-1)}$$

$$=\frac{x-1+x-5-2(x-3)}{(x-1)(x-3)(x-5)}$$

$$=\frac{x-1+x-5-2x+6}{(x-1)(x-3)(x-5)}$$

$$=\frac{2x-6-2x+6}{(x-1)(x-3)(x-5)}$$

$$=\frac{0}{(x-1)(x-3)(x-5)}$$

$$=0$$

Q4. $$\frac{(x+2)(x+3)}{x^2-9}+\frac{(x+2)(2x^2-32)}{(x-4)(x^2-x-6)}$$

$$=\frac{(x+2)(x+3)}{(x)^2-(3)^2}+\frac{(x+2).2(x^2-16)}{(x-4)(x^2+2x-3x-6)}$$

$$=\frac{(x+2)(x+3)}{(x-3)(x+3)}+\frac{2(x+2)\left[(x)^2-(4)^2\right]}{(x-4)(x^2+2x-3x-6)}$$

$$=\frac{(x+2)}{x-3}+\frac{2(x+2)(x+4)(x-4)}{(x-4)(x+2)(x-3)}$$

$$=\frac{x+2}{x-3}+\frac{2x+8}{x-3}$$

$$=\frac{x+2+2x+8}{x-3}$$

$$=\frac{3x+10}{x-3}$$

Q5. $$\frac{x+3}{2x^2+9x+9}+\frac{1}{2(2x-3)}-\frac{4x}{4x^2-9}$$

$$=\frac{x+3}{2x^2+6x+3x+9}+\frac{1}{2(2x-3)}-\frac{4x}{(2x)^2-(3)^2}$$

$$=\frac{x+3}{2x(x+3)+3(x+3)}+\frac{1}{2(2x-3)}-\frac{4x}{(2x+3)(2x-3)}$$

$$= \frac{\cancel{x+3}}{\cancel{(x+3)}(2x+3)} + \frac{1}{2(2x-3)} - \frac{4x}{(2x+3)(2x-3)}$$

$$= \frac{1}{2x+3} + \frac{1}{2(2x-3)} - \frac{4x}{(2x+3)(2x-3)}$$

$$= \frac{2(2x-3)+2x+3-2(4x)}{2(2x+3)(2x-3)}$$

$$= \frac{4x-6+2x+3-8x}{2(2x+3)(2x-3)}$$

$$= \frac{-2x-3}{2(2x+3)(2x-3)}$$

$$= \frac{-1\cancel{(2x+3)}}{2\cancel{(2x+3)}(2x-3)}$$

$$= \frac{-1}{2(2x-3)}$$

$$= \frac{1}{2(3-2x)}$$

Q6. $A - \frac{1}{A}$, **where** $A = \frac{a+1}{a-1}$

so $\frac{1}{A} = \frac{a-1}{a+1}$

Now $A - \frac{1}{A} = \frac{a+1}{a-1} - \frac{a-1}{a+1}$

$$= \frac{(a+1)^2-(a-1)^2}{(a-1)(a+1)}$$

$$= \frac{(a^2+2a+1)-(a^2-2a+1)}{(a)^2-(1)^2}$$

$$= \frac{\cancel{a^2}+2a+\cancel{1}-\cancel{a^2}+2a-\cancel{1}}{a^2-1}$$

$$= \frac{4a}{a^2-1}$$

Q7. $\left[\frac{x-1}{x-2} + \frac{2}{2-x}\right] - \left[\frac{x+1}{x+2} + \frac{4}{4-x^2}\right]$

$$= \left[\frac{-(x-1)}{2-x} + \frac{2}{2-x}\right] - \left[\frac{x+1}{x+2} + \frac{4}{(2)^2-(x)^2}\right]$$

$$= \left[-\frac{(x-1)}{2-x} + \frac{2}{2-x}\right] - \left[\frac{x+1}{x+2} + \frac{4}{(2+x)(2-x)}\right]$$

$$= \left[\frac{-x+1+2}{2-x}\right] - \left[\frac{(x+1)(2-x)+4}{(2+x)(2-x)}\right]$$

$$= \frac{3-x}{2-x} - \left[\frac{2x-x^2+2-x+4}{(2+x)(2-x)}\right]$$

$$= \frac{3-x}{2-x} - \left[\frac{6+x-x^2}{(2+x)(2-x)}\right]$$

$$= \frac{3-x}{2-x} - \left[\frac{6+3x-2x-x^2}{(2+x)(2-x)}\right]$$

$$= \frac{3-x}{2-x} - \left[\frac{3(2+x)-x(2+x)}{(2+x)(2-x)}\right]$$

$$= \frac{3-x}{2-x} - \left[\frac{\cancel{(2+x)}(3-x)}{\cancel{(2+x)}(2-x)}\right]$$

$$= \frac{3-x}{2-x} - \frac{3-x}{2-x}$$

$$= \frac{3-x-3+x}{2-x}$$

$$= \frac{0}{2-x}$$

$$= 0$$

Q8. **What rational expression should be subtracted from** $\frac{2x^2+2x-7}{x^2+x-6}$ **to get** $\frac{x-1}{x-2} = ?$

Sol: Let the required expression be A,

then $\dfrac{2x^2+2x-7}{x^2+x-6}-A=\dfrac{x-1}{x-2}$

or $\dfrac{2x^2+2x-7}{x^2+x-6}-\dfrac{x-1}{x-2}=A$

So $A=\dfrac{2x^2+2x-7}{x^2+3x-2x-6}-\dfrac{x-1}{x-2}$

$=\dfrac{2x^2+2x-7}{x(x+3)-2(x+3)}-\dfrac{x-1}{x-2}$

$=\dfrac{2x^2+2x-7}{(x+3)(x-2)}-\dfrac{x-1}{x-2}$

$=\dfrac{2x^2+2x-7-(x-1)(x+3)}{(x+3)(x-2)}$

$=\dfrac{2x^2+2x-7-(x^2-x+3x-3)}{(x+3)(x-2)}$

$=\dfrac{(2x^2+2x-7)-(x^2+2x-3)}{(x+3)(x-2)}$

$=\dfrac{2x^2+2x-7-x^2-2x+3}{(x+3)(x-2)}$

$=\dfrac{x^2-4}{(x+3)(x-2)}$

$=\dfrac{(x)^2-(2)^2}{(x+3)(x-2)}$

$=\dfrac{(x+2)\cancel{(x-2)}}{(x+3)\cancel{(x-2)}}$

$=\dfrac{x+2}{x+3}$

Perform the indicated operations and simplify to the lowest forms.

Q9. $\dfrac{x^2+x-6}{x^2-x-6}\times\dfrac{x^2-4}{x^2-9}$

$=\dfrac{x^2+3x-2x-6}{x^2-3x+2x-6}\times\dfrac{(x)^2-(2)^2}{(x)^2-(3)^2}$

$=\dfrac{x(x+3)-2(x+3)}{x(x-3)+2(x-3)}\times\dfrac{(x+2)(x-2)}{(x+3)(x-3)}$

$=\dfrac{\cancel{(x+3)}(x-2)}{(x-3)\cancel{(x+2)}}\times\dfrac{\cancel{(x+2)}(x-2)}{\cancel{(x+3)}(x-3)}$

$=\dfrac{(x-2)^2}{(x-3)^2}$

Q10. $\dfrac{x^3-8}{x^2-4}\times\dfrac{x^2+6x+8}{x^2-2x+1}$

$=\dfrac{(x)^3-(2)^3}{(x)^2-(2)^2}\times\dfrac{x^2+2x+4x+8}{x^2-x-x+1}$

$=\dfrac{\cancel{(x-2)}\left[(x)^2+(x)(2)+(2)^2\right]}{\cancel{(x-2)}(x+2)}\times\dfrac{x(x+2)+4(x+2)}{x(x-1)-1(x-1)}$

$=\dfrac{x^2+2x+4}{\cancel{x+2}}\times\dfrac{\cancel{(x+2)}(x+4)}{(x-1)(x-1)}$

$=\dfrac{(x^2+2x+4)(x+4)}{(x-1)^2}$

Q11. $\dfrac{x^4-8x}{2x^2+5x-3}\times\dfrac{2x-1}{x^2+2x+4}\times\dfrac{x+3}{x^2-2x}$

$=\dfrac{x(x^3-8)}{2x^2+6x-x-3}\times\dfrac{2x-1}{x^2+2x+4}\times\dfrac{x+3}{x(x-2)}$

$=\dfrac{x\left[(x)^3-(2)^3\right]}{2x(x+3)-1(x+3)}\times\dfrac{2x-1}{x^2+2x+4}\times\dfrac{x+3}{x(x-2)}$

$=\dfrac{\cancel{x}\cancel{(x-2)}\cancel{(x^2+2x+4)}}{\cancel{(x+3)}\cancel{(2x-1)}}\times\dfrac{\cancel{2x-1}}{\cancel{x^2+2x+4}}\times\dfrac{\cancel{x+3}}{\cancel{x}\cancel{(x-2)}}$

$=1$

Q12. $\dfrac{2y^2+7y-4}{3y^2-13y+4}\div\dfrac{4y^2-1}{6y^2+y-1}$

$$=\frac{2y^2+8y-y-4}{3y^2-y-12y+4}\div\frac{(2y)^2-(1)^2}{6y^2+3y-2y-1}$$

$$=\frac{2y(y+4)-1(y+4)}{y(3y-1)-4(3y-1)}\div\frac{(2y+1)(2y-1)}{3y(2y+1)-1(2y+1)}$$

$$=\frac{(y+4)\cancel{(2y-1)}}{\cancel{(3y-1)}(y-4)}\div\frac{\cancel{(2y+1)}(2y-1)}{\cancel{(2y+1)}\cancel{(3y-1)}}$$

$$=\frac{(y+4)\cancel{(2y-1)}}{\cancel{(3y-1)}(y-4)}\times\frac{\cancel{(2y+1)}(3y-1)}{\cancel{(2y+1)}\cancel{(2y-1)}}$$

$$=\frac{y+4}{y-4}$$

Q13. $\left[\frac{x^2+y^2}{x^2-y^2}-\frac{x^2-y^2}{x^2+y^2}\right]\div\left[\frac{x+y}{x-y}-\frac{x-y}{x+y}\right]$

$$=\left[\frac{(x^2+y^2)^2-(x^2-y^2)^2}{(x^2-y^2)(x^2+y^2)}\right]\div\left[\frac{(x+y)^2-(x-y)^2}{(x-y)(x+y)}\right]$$

$$=\frac{x^4+y^4+2x^2y^2-(x^4+y^4-2x^2y^2)^2}{(x^2-y^2)(x^2+y^2)}$$

$$\div\frac{x^2+y^2+2xy-x^2-y^2+2xy}{x^2-y^2}$$

$$=\frac{\cancel{x^4}+\cancel{y^4}+2x^2y^2-\cancel{x^4}-\cancel{y^4}+2x^2y^2}{(x^2-y^2)(x^2+y^2)}$$

$$\div\frac{\cancel{x^2}+\cancel{y^2}+2xy-\cancel{x^2}-\cancel{y^2}+2xy}{x^2-y^2}$$

$$=\frac{4x^2y^2}{(x^2-y^2)(x^2+y^2)}\div\frac{4xy}{x^2-y^2}$$

$$=\frac{\cancel{4}\,\cancel{x}^2\,\cancel{y}^2}{\cancel{(x^2-y^2)}(x^2+y^2)}\times\frac{\cancel{x^2-y^2}}{\cancel{4xy}}$$

$$=\frac{xy}{x^2+y^2}$$