

مشق 3.5

منفر کیجیے۔

$$\frac{1}{a} + \frac{2}{a+1} - \frac{3}{a+2} \quad -1$$

حل:

$$\begin{aligned} & \frac{1}{a} + \frac{2}{a+1} - \frac{3}{a+2} \\ &= \frac{(a+1)(a+2) + 2a(a+2) - 3a(a+1)}{a(a+1)(a+2)} \\ &= \frac{a^2 + 3a + 2 + 2a^2 + 4a - 3a^2 - 3a}{a(a+1)(a+2)} \end{aligned}$$

$$\begin{aligned}
&= \frac{3a^2 - 3a^2 + 3a + 4a - 3a + 2}{a(a+1)(a+2)} \\
&= \frac{4a + 2}{a(a+1)(a+2)} \\
&= \frac{2(2a+1)}{a(a+1)(a+2)}
\end{aligned}$$

پس

یا

$$\frac{2a}{(x-2a)} - \frac{x-a}{x^2-5ax+6a^2} + \frac{2}{x-3a} \quad -2$$

حل:

$$\begin{aligned}
&\frac{2a}{(x-2a)} - \frac{x-a}{x^2-5ax+6a^2} + \frac{2}{x-3a} \\
&= \frac{2a}{(x-2a)} - \frac{x-a}{x^2-3ax-2ax+6a^2} + \frac{2}{x-3a} \\
&= \frac{2a}{(x-2a)} - \frac{x-a}{x^2(x-3a)-2a(x-3a)} + \frac{2}{x-3a} \\
&= \frac{2a}{x-2a} - \frac{x-a}{(x-3a)(x-2a)} + \frac{2}{x-3a} \\
&= \frac{2a(x-3a)-x+a+2(x-2a)}{(x-3a)(x-2a)} \\
&= \frac{2ax-6a^2-x+a+2x-4a}{(x-3a)(x-2a)} \\
&= \frac{2ax+x-3a-6a^2}{(x-3a)(x-2a)} \\
&= \frac{x(2a+1)-3a(1+2a)}{(x-3a)(x-2a)} \\
&= \frac{(2a+1)(x-3a)}{(x-3a)(x-2a)} \\
&= \frac{2a+1}{x-2a}
\end{aligned}$$

پس

$$\frac{1}{a^2+1} - \frac{a^4}{a^2+1} + \frac{a^6}{a^2-1} - \frac{1}{a^2-1} \quad -3$$

حل:

$$\begin{aligned}
&\frac{1}{a^2+1} - \frac{a^4}{a^2+1} + \frac{a^6}{a^2-1} - \frac{1}{a^2-1} \\
&= \left[\frac{1}{a^2+1} - \frac{a^4}{a^2+1} \right] + \left[\frac{a^6}{a^2-1} - \frac{1}{a^2-1} \right]
\end{aligned}$$

$$\begin{aligned}
&= \frac{1-a^4}{a^2+1} + \frac{a^6-1}{a^2-1} \Rightarrow \frac{(1)-(a^2)^2}{a^2+1} + \frac{(a^2)^3-(1)^3}{a^2-1} \\
&= \frac{(1-a^2)(1+a^2)}{(a^2+1)} + \frac{(a^2-1)(a^4+a^2+1)}{(a^2-1)} \\
&= 1-a^2+a^4+a^2+1 \\
&= 2+a^4
\end{aligned}$$

$$\frac{1}{x^2+x+1} - \frac{1}{x^2-x+1} + \frac{2x+1}{x^4+x^2+1} \quad -4$$

$$\frac{1}{x^2+x+1} - \frac{1}{x^2-x+1} + \frac{2x+1}{x^4+x^2+1}$$

$$\begin{aligned}
x^4+x^2+1 &= x^4+x^3+x^2+1-x^3 \\
&= x^4+2x^3+1-x^3 \\
&= (x^3+1)^2-x^3 \\
&= (x^3+1+x)(x^3+1-x) \\
&= (x^3+x+1)(x^3-x+1)
\end{aligned}$$

چونکہ

لہذا

$$= \frac{1}{x^2+x+1} - \frac{1}{x^2-x+1} + \frac{2x+1}{(x^2+x+1)(x^2-x+1)}$$

$$= \frac{(x^2-x+1)-(x^2+x+1)+2x+1}{(x^2+x+1)(x^2-x+1)}$$

$$= \frac{x^2-x+1-x^2-x-1+2x+1}{(x^2+x+1)(x^2-x+1)}$$

$$= \frac{x^2-x^2-x-x+1-1+2x+1}{(x^2+x+1)(x^2-x+1)}$$

$$= \frac{-2x+2x+1}{(x^2+x+1)(x^2-x+1)}$$

$$= \frac{1}{(x^2+x+1)(x^2-x+1)}$$

$$= \frac{1}{x^4+x^2+1}$$

پس

یا

$$\frac{a^2(b-c)}{(a+b)(a+c)} - \frac{b^2(c-a)}{(b+c)(b+a)} + \frac{c^2(a-b)}{(c+a)(c+b)}$$

-5

$$\frac{a^2(b-c)}{(a+b)(a+c)} - \frac{b^2(c-a)}{(b+c)(b+a)} + \frac{c^2(a-b)}{(c+a)(c+b)}$$

حل:

$$\begin{aligned}
&= \frac{a^2(b-c)(b+c) - b^2(c-a)(a+c) + c^2(a-b)(a+b)}{(a+b)(a+c)(b+c)} \\
&= \frac{a^2(b^2 - c^2) - b^2(c^2 - a^2) + c^2(a^2 - b^2)}{(a+b)(a+c)(b+c)} \\
&= \frac{a^2b^2 - a^2c^2 - b^2c^2 + a^2b^2 + a^2c^2 - b^2c^2}{(a+b)(a+c)(b+c)} \\
&= \frac{2a^2b^2 - 2b^2c^2}{(a+b)(a+c)(b+c)} \\
&= \frac{2b^2(a^2 - c^2)}{(a+b)(a+c)(b+c)} \\
&= \frac{2b^2(a-c)(a+c)}{(a+b)(a+c)(b+c)} \\
&= \frac{2b^2(a-c)}{(a+b)(b+c)}
\end{aligned}$$

پس

$$\frac{1}{x-1} + \frac{1}{x+1} - \frac{x+2}{x^2+x+1} - \frac{x-2}{x^2-x+1} \quad -6$$

حل:

$$\begin{aligned}
&\frac{1}{x-1} + \frac{1}{x+1} - \frac{x+2}{x^2+x+1} - \frac{x-2}{x^2-x+1} \\
&= \frac{1}{x-1} - \frac{x+2}{x^2+x+1} - \frac{1}{x+1} - \frac{x-2}{x^2-x+1} \\
&= \frac{(x^2+x+1) - (x+2)(x-1)}{(x-1)(x^2+x+1)} + \frac{x^2-x+1 - (x-2)(x+1)}{(x+1)(x^2-x+1)} \\
&= \frac{x^2+x+1 - (x^2-x+2x-2)}{(x-1)(x^2+x+1)} + \frac{x^2-x+1 - (x^2+x-2x-2)}{(x+1)(x^2-x+1)} \\
&= \frac{x^2+x+1 - (x^2+x-2)}{(x-1)(x^2+x+1)} + \frac{x^2-x+1 - (x^2-x-2)}{(x+1)(x^2-x+1)} \\
&= \frac{x^2+x+1 - x^2 - x + 2}{(x-1)(x^2+x+1)} + \frac{x^2-x+1 - x^2 + x + 2}{(x+1)(x^2-x+1)} \\
&= \frac{3}{(x-1)(x^2+x+1)} + \frac{3}{(x+1)(x^2-x+1)} \\
&= \frac{3}{x^3-1} + \frac{3}{x^3+1}
\end{aligned}$$

$$\begin{aligned}
 &= \frac{3(x^3 + 1) + 3(x^3 - 1)}{(x^3 - 1)(x^3 + 1)} \\
 &= \frac{3x^3 + 3 + 3x^3 - 3}{x^6 - 1} \\
 &= \frac{6x^3}{x^6 - 1}
 \end{aligned}$$

پس

$$\frac{a^2 + ab + b^2}{a + b} + \frac{a^2 - ab + b^2}{a - b} \quad -7$$

حل:

$$\begin{aligned}
 &\frac{a^2 + ab + b^2}{a + b} + \frac{a^2 - ab + b^2}{a - b} \\
 &= \frac{(a - b)(a^2 + ab + b^2) + (a + b)(a^2 - ab + b^2)}{(a + b)(a - b)} \\
 &= \frac{a^3 - b^3 + a^3 + b^3}{a^2 - b^2} \\
 &= \frac{2a^3}{a^2 - b^2}
 \end{aligned}$$

پس

$$\frac{x^4 - y^4}{x^2 - 2xy + y^2} \times \frac{x - y}{x(x + y)} \div \frac{x^2 + y^2}{x} \quad -8$$

حل:

$$\begin{aligned}
 &\frac{x^4 - y^4}{x^2 - 2xy + y^2} \times \frac{x - y}{x(x + y)} \div \frac{x^2 + y^2}{x} \\
 &= \frac{(x^2 - y^2)(x^2 + y^2)}{(x - y)^2} \times \frac{x - y}{x(x + y)} \times \frac{x}{x^2 + y^2} \\
 &= \frac{(x - y)(x + y)(x^2 + y^2)}{(x - y)^2} \times \frac{(x - y)}{x(x + y)} \times \frac{x}{x^2 + y^2} \\
 &= \frac{\cancel{(x - y)^2} \times \cancel{(x + y)} \times \cancel{x} \times \cancel{(x^2 + y^2)}}{\cancel{(x - y)^2} \times \cancel{x} \times \cancel{(x + y)} \times \cancel{(x^2 + y^2)}} \\
 &= 1
 \end{aligned}$$

پس

$$\frac{x^2 - 1}{x^2 + x - 2} \times \frac{x^3 + 8}{x^4 + 4x^2 + 16} \div \frac{x^2 + x}{x^3 + 2x^2 + 4x} \quad -9$$

حل:

$$\frac{x^2 - 1}{x^2 + x - 2} \times \frac{x^3 + 8}{x^4 + 4x^2 + 16} \div \frac{x^2 + x}{x^3 + 2x^2 + 4x}$$

$$\begin{aligned}
x^2 + x - 2 &= x^2 + 2x - x - 2 \\
&= x(x + 2) - 1(x + 2) \\
&= (x + 2)(x - 1) \\
x^3 + 8 &= x^3 + 2^3 \\
&= (x + 2)(x^2 - 2x + 4) \\
x^2 + x &= x(x + 1) \\
x^2 - 1 &= (x - 1)(x + 1)
\end{aligned}$$

$$\begin{aligned}
x^3 + 2x^2 + 4x &= x(x^2 + 2x + 4) \\
x^4 + 4x^2 + 16 &= (x^2)^2 + 2(x^2)(4) + (4)^2 - 4x^2 \\
&= (x^2 + 4)^2 - (2x)^2 \\
&= (x^2 + 2x + 4)(x^2 - 2x + 4)
\end{aligned}$$

$$\begin{aligned}
&= \frac{\cancel{(x-1)}(x+1)}{(x+2)\cancel{(x-1)}} \times \frac{\cancel{(x+2)}\cancel{(x^2-2x+4)}}{(x^2+2x+4)\cancel{(x^2-2x+4)}} \div \frac{\cancel{x}(x+1)}{\cancel{x}(x^2+2x+4)} \\
&= \frac{\cancel{(x+1)} \times \frac{1}{x^2+2x+4} \times \frac{\cancel{(x^2+2x+4)}}{\cancel{(x+1)}}}{1} \\
&= 1
\end{aligned}$$

$$\frac{a^3 + 64b^3}{a^2 + 20ab + 64b^2} \div \frac{a^2 - 4ab + 16b^2}{a^2 + 4ab + 16b^2} \times \frac{a^2 + 12ab - 64b^2}{a^3 - 64b^3} \quad -10$$

حل: چونکہ

$$\begin{aligned}
a^3 + 64b^3 &= a^3 + (4b)^3 \\
&= (a + 4b)(a^2 - 4ab + 16b^2) \\
a^3 - 64b^3 &= a^3 - (4b)^3 \\
&= (a - 4b)(a^2 + 4ab + 16b^2) \\
a^2 + 20ab + 64b^2 &= a^2 + 16ab + 4ab + 64b^2 \\
&= a(a + 16b) + 4b(a + 16b) \\
&= (a + 16b)(a + 4b) \\
a^2 + 12ab - 64b^2 &= a^2 + 16ab - 4ab - 64b^2 \\
&= a(a + 16b) - 4b(a + 16b) \\
&= (a + 16b)(a - 4b)
\end{aligned}$$

لہذا دیا گیا اظہار یہ اس طرح لکھا جاسکتا ہے۔

$$\begin{aligned}
&= \frac{(a + 4b)(a^2 - 4ab + 16b^2)}{(a + 4b)(a + 16b)} \div \frac{a^2 - 4ab + 16b^2}{a^2 + 4ab + 16b^2} \times \frac{(a + 16b)(a - 4b)}{(a - 4b)(a^2 + 4ab + 16b^2)} \\
&= \frac{(a^2 - 4ab + 16b^2)}{(a + 16b)} \times \frac{a^2 + 4ab + 16b^2}{a^2 - 4ab + 16b^2} \times \frac{(a + 16b)}{a^2 + 4ab + 16b^2} \\
&= \frac{(a^2 - 4ab + 16b^2)}{(a^2 - 4ab + 16b^2)} \times \frac{(a^2 + 4ab + 16b^2)}{(a^2 + 4ab + 16b^2)} \times \frac{(a + 16b)}{(a + 16b)} \\
&= 1
\end{aligned}$$

پس

$$\frac{a}{(a+b)^2 - 2ab} \times \frac{a^4 - b^4}{(a+b)^3 - 3ab(a+b)} \div \frac{(a+b)^2 - 4ab}{(a+b)^2 - 3ab} \quad -11$$

چونکہ حل:

$$\begin{aligned} a^4 - b^4 &= (a^2 + b^2)(a^2 - b^2) \\ &= (a^2 + b^2)(a+b)(a-b) \\ (a+b)^2 - 2ab &= a^2 + b^2 + 2ab - 2ab \\ &= a^2 + b^2 \\ (a+b)^3 - 3ab(a+b) &= a^3 + b^3 + 3ab(a+b) - 3ab(a+b) \\ &= a^3 + b^3 \\ (a+b)^2 - 4ab &= a^2 + b^2 + 2ab - 4ab \\ &= a^2 + b^2 - 2ab \\ &= (a-b)^2 \\ (a+b)^3 - 3ab &= a^3 + b^3 + 2ab - 3ab \\ &= a^3 - ab + b^3 \end{aligned}$$

لہذا

$$\begin{aligned} &\frac{a}{a^2 + b^2} \times \frac{(a^2 + b^2)(a+b)(a-b)}{a^3 + b^3} \div \frac{(a-b)^2}{a^2 - ab + b^2} \\ &= \frac{\cancel{a(a^2 + b^2)}(a+b)\cancel{(a-b)}}{\cancel{(a^2 + b^2)}(a+b)\cancel{(a^2 - ab + b^2)}} \times \frac{\cancel{(a^2 - ab + b^2)}}{\cancel{(a-b)}} \\ &= \frac{a}{a-b} \quad \text{پس} \end{aligned}$$

$$\frac{a^2 - 1}{a^2 - a - 2} \div \frac{a^2 + 5a + 6}{a^2 - 5a + 6} \div \frac{a^2 - 4a + 3}{a^2 + 4a + 3} \quad -12$$

چونکہ حل:

$$\begin{aligned} &\frac{a^2 - 1}{a^2 - a - 2} \div \frac{a^2 + 5a + 6}{a^2 - 5a + 6} \div \frac{a^2 - 4a + 3}{a^2 + 4a + 3} \\ a^2 - a - 2 &= a^2 - 2a + a - 2 \\ &= a(a-2) + 1(a-2) \\ &= (a-2)(a+1) \\ a^2 + 5a + 6 &= a^2 + 3a + 2a + 6 \\ &= a(a+3) + 2(a+3) \\ &= (a+3)(a+2) \\ a^2 - 5a + 6 &= a^2 - 3a - 2a + 6 \\ &= a(a-3) - 2(a-3) \\ &= (a-3)(a-2) \end{aligned}$$

$$a^2 - 4a + 3$$

$$= a^2 - 3a - a + 3$$

$$= a(a - 3) - 1(a - 3)$$

$$= (a - 3)(a - 1)$$

$$a^2 + 4a + 3$$

$$= a^2 + 3a + a + 3$$

$$= a(a + 3) + 1(a + 3)$$

$$= (a + 3)(a + 1)$$

لہذا دیئے ہوئے اظہار دیئے کو اس طرح لکھنے سے

$$= \frac{(a-1)(a+1)}{(a-2)(a+1)} \div \frac{(a+3)(a+2)}{(a-3)(a-2)} \div \frac{(a-3)(a-1)}{(a+3)(a+1)}$$

$$= \frac{\cancel{(a-1)}}{\cancel{(a-2)}} \times \frac{\cancel{(a-3)} \cancel{(a-2)}}{\cancel{(a+3)} (a+2)} \times \frac{\cancel{(a+3)} (a+1)}{\cancel{(a-3)} \cancel{(a-1)}}$$

$$= \frac{a+1}{a+2}$$

پس