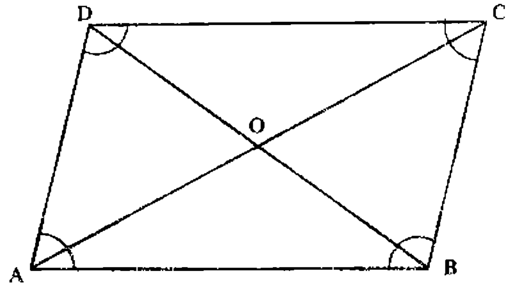
EXERCISE 11.2

(1) Prove that a quadrilateral is a parallelogram if its

(a) Opposite angles are congruent.

(b) Diagonals bisect each other.



Given Given ABCD is a quadrilateral.

$m\angle A = m\angle C$,

$m\angle B = m\angle D$

To prove ABCD is a parallelogram.

Proof

Statements	Reasons
$m\angle A = m\angle C$ (i)	Given
$m\angle B = m\angle D$ (ii)	Given
Now	
$m\angle A + m\angle B + m\angle C + m\angle D = 360^\circ$	Angles of a quad.
$m\angle A + m\angle B + m\angle A + m\angle B = 360^\circ$	From (i), (ii)
$m\angle A + m\angle A + m\angle B + m\angle B = 360^\circ$	Rearranging
$2m\angle A + 2m\angle B = 360^\circ$	
$(m\angle A + m\angle B) = 360^\circ/2 = 180^\circ$	Dividing by 2
$\therefore \overline{AD} \parallel \overline{BC}$	As $m\angle A + m\angle B = 180^\circ$ (sum of interior angles)
Similarly it can be Proved that $\overline{AB} \parallel \overline{CD}$	
Hence ABCD is a parallelogram.	

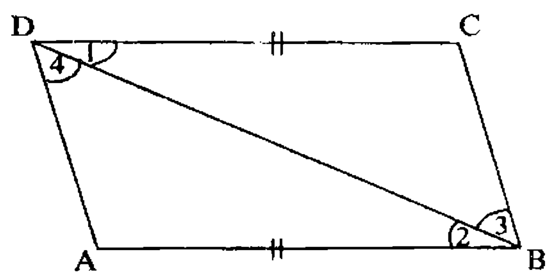
(2) prove that a quadrilateral is a parallelogram if its opposite sides are congruent.

Given

In quadrilateral

ABCD, $\overline{AB} \cong \overline{DC}$,

$\overline{AD} \cong \overline{BC}$



Required

ABCD is a $\parallel$ gm

$\overline{AB} \parallel \overline{CD}, \overline{AD} \parallel \overline{BC}$

Construction

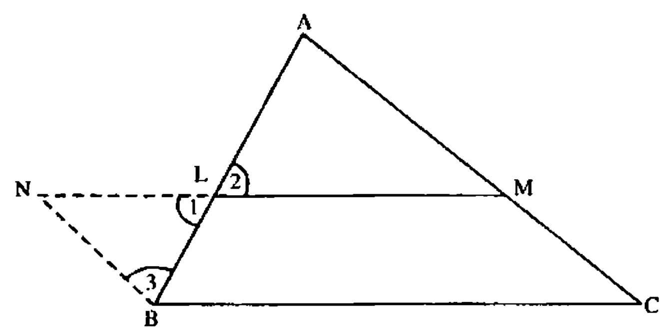
Join point B to D and name the angles $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$

Proof

Statements			Reasons
	$\Delta ABD \leftrightarrow \Delta CDB$		
	$\overline{AD} \cong \overline{CB}$		Given
	$\overline{AB} \cong \overline{CD}$		Given
	$\overline{BD} \cong \overline{BD}$		Common
$\therefore$	$\Delta ABD \cong \Delta CDB$		S.S.S $\cong$ S.S.S
So	$\angle 2 \cong \angle 1$	(i)	Corresponding angles of Congruent triangles
	$\angle 4 \cong \angle 3$	(ii)	Alternate angles
Hence	$\overline{AB} \parallel \overline{CD}$	(iii)	$\angle 2$ and $\angle 1$ are congruent
Similarly	$\overline{BC} \parallel \overline{AD}$	(iv)	Alternate angles $\angle 3$, $\angle 4$ congruent
$\therefore$	ABCD is a parallelogram.		From iii, iv

Theorem

The line segment, joining the mid-points of two sides of a triangle, is parallel to the third side and is equal to one half if its length.



Given In ΔABC, the mid-points of $\overline{AB}$ and $\overline{AC}$ are L and M respectively.

To Prove

$$\overline{LM} \parallel \overline{BC} \text{ and } m\overline{LM} = \frac{1}{2} m\overline{BC}$$

Construction

Join M to L and produce $\overline{ML}$ to N such that $\overline{ML} \cong \overline{LN}$ .Join N to B. and in the figures name the angles $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$ as shown.

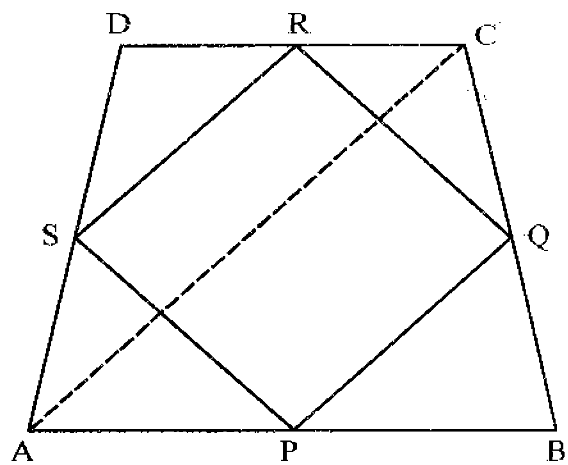
Proof

Statements		Reasons
In	$\Delta BLN \leftrightarrow \Delta ALM$	
	$\overline{BL} \cong \overline{AL}$,	Given
	$\angle 1 \cong \angle 2$	Vertical angles
	$\overline{NL} \cong \overline{ML}$	Construction

$\therefore$ $\Delta BLN \cong \Delta ALM$	S.A.S. postulate
$\angle A \cong \angle 3$ .....(i)	(corresponding angles of congruent triangles)
and $\overline{NB} \cong \overline{AM}$ .....(ii)	(corresponding sides of congruent triangles)
But $\overline{NB} \parallel \overline{AM}$	From (i), alternate $\angle$s
Thus $\overline{NB} \parallel \overline{MC}$ ....(iii)	(M is a point of $\overline{AC}$)
$\overline{MC} \cong \overline{AM}$ ....(iv)	Given
$\overline{NB} \cong \overline{MC}$ ... (v)	{from (ii) and (iv)}
$\therefore$ BCMN is a parallelogram	From (iii) and (v)
$\therefore$ $\overline{BC} \parallel \overline{LM}$ or $\overline{BC} \parallel \overline{NL}$	(Opposite sides of a parallelogram BCMN)
$\overline{BC} \cong \overline{NM}$ .....(vi)	(Opposite sides of parallelogram)
$m\overline{LM} = \frac{1}{2} m\overline{NM}$ .....(vii)	Construction
Hence $m\overline{LM} = \frac{1}{2} m\overline{BC}$	{from (vi) and (vii)}

Example

The line segments, joining the mid-points of the sides of a quadrilateral, taken in order, form a parallelogram.



Given

A quadrilateral ABCD, in which P is the mid-point of $\overline{AB}$, Q is the mid-point of $\overline{BC}$, R is the mid-point of $\overline{CD}$, S is the mid-point of $\overline{DA}$.

P is joined to Q, Q is joined to R. R is joined to S and S is joined to P.

To prove

PQRS is a parallelogram.

Construction

Join A to C.

Proof

Statements	Reasons
In ΔDAC,	
$\left.\begin{array}{l}\overline{SR} \parallel \overline{AC} \\ m\overline{SR} = \frac{1}{2} m\overline{AC}\end{array}\right\}$	S is the mid-point of $\overline{DA}$ R is the mid-point of $\overline{CD}$
In ΔBAC,	
$\left.\begin{array}{l}\overline{PQ} \parallel \overline{AC} \\ m\overline{PQ} = \frac{1}{2} m\overline{AC}\end{array}\right\}$	P is the mid-point of $\overline{AB}$ Q is the mid-point of $\overline{BC}$
$\overline{SR} \parallel \overline{PQ}$	Each $\parallel$ $\overline{AC}$
$m\overline{SR} = m\overline{PQ}$	Each $= \frac{1}{2} m\overline{AC}$
Thus PQRS is a parallelogram	$\overline{SR} \parallel \overline{PQ}, m\overline{SR} = m\overline{PQ}$ (proved)