It is given by the formula:
Range = $X_{max} - X_{min} = X_m - X_0$
where $X_{max} = X_m$ = the maximum, highest or largest observation.
$X_{min} = X_o$ = the minimum lowest or smallest observation.
The formula to find range for grouped continuous data is given below:
Range = (Upper class boundary of last group) – (lower class boundary of first group).

(ii) Variance:
Variance is defined as the mean of the squared deviations of x_i (i = , 2, ... n) observations from their arithmetic mean. In symbols,

$$\text{Variance of X} = \text{Var (X)} = S^2 = \frac{\Sigma\left(X - \bar{X}\right)^2}{n}$$

(iii) Standard Deviation:
Standard deviation is defined as the positive square root of mean of the squared deviations of X_i (i = 1, 2, ... n) observations from their arithmetic mean. In symbols we write.

$$\text{Standard Deviation of X} = \text{S.D (X)} = S = \sqrt{\frac{\Sigma\left(X - \bar{X}\right)^2}{n}}$$

Computation of Variance and Standard Deviation:
We use the following formulae to compute Variance and Standard Deviation for Ungrouped and Grouped Data.

Ungrouped Data
The formula of Variance is given by:

$$\text{Var (X)} = S^2 = \frac{\Sigma X^2}{n} - \left(\frac{\Sigma X}{n}\right)^2$$

And Standard deviation is given by:

$$\text{S.D}(X) = S = \sqrt{\left[\frac{\Sigma X^2}{n} - \left(\frac{\Sigma X}{n}\right)^2\right]}$$

SOLVED EXERCISE 6.3

1. What do you understand by Dispersion?

Solution:

Dispersion:
Dispersion means the spread or scatterness of observations in a data set. By dispersion we mean the extent to which the observations in a sample or in a population are spread out.
The main measures of dispersion are range, variance and standard deviation.

2. How do you define measure of dispersion?

Solution:

The measures that are used to determine the degree or extent of variation in a data set are called measure of dispersion.

3. Define Range, Standard deviation and Variance.

Solution:

Range:

Range measures the extent of variation between two extreme observations of a data set. It is given by the formula Range = $X_{max} - X_{min}$

Range = (upper C. B of the last group) – (lower C. B of first group)

Variance:

The mean of the squared deviations of x_i (i = 1, 2, ..., n) observations from their arithmetic mean.

$$\text{Variance} = S^2 = \frac{\Sigma(X-\overline{X})^2}{n}$$

$$= S^2 = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2$$

Standard Deviation:

The positive square root of the squared deviations of x_i (i = 1, 2, 3, ..., n) observations from their mean.

$$\text{Standard Deviation} = S = \sqrt{\frac{\Sigma(X-\overline{X})^2}{n}}$$

$$= S = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$$

4. The salaries of five teachers in Rupees are as follows.
11500, 12400, 15000, 14500, 14800.
Find Range and standard deviation.

Solution:

$$X = 11500, 12400, 15000, 14500, 14800$$

Here, $X_{max} = 15000$, $X_{max} = 115000$

$$\text{Range} = X_{max} - X_{min}$$

$$= 15000 - 11500 = 3500$$

$$\overline{X} = \frac{\Sigma x}{n}$$

$$= \frac{11500 + 12400 + 15000 + 14500 + 14800}{5}$$

$$= \frac{68200}{5} = 13640$$

X	$X - \overline{X}$	$(X - \overline{X})^2$
11500	–2140	4579600
12400	–1240	1537600
15000	1360	1849600
14500	860	739600
14800	1160	1345600

$\Sigma (X - \overline{X})^2 = 10052000, \; n = 5$

$$S.D = S = \sqrt{\frac{\Sigma(X-\overline{X})^2}{n}} = \sqrt{\frac{10052000}{5}}$$

$$= \sqrt{2010400} = 1417.88$$

5. a. Find the standard deviation "S" of each set of numbers:

(i) 12, 6, 7, 3, 15, 10, 18, 5

(ii) 9, 3, 8, 8, 9, 8, 9, 18.

b. Calculate variance for the data: 10, 8, 9, 7, 5, 12, 8 6, 8, 2.

Solution:

(i)

X	$X - \overline{X}$	$(X - \overline{X})^2$
12	2.5	6.25
6	–3.5	12.25
7	–2.5	6.25
3	–6.5	42.25
15	5.5	30.25
10	0.5	0.25
18	8.5	72.25
5	–4.5	20.25

$\Sigma x = 76$ $\qquad\qquad \Sigma (X - \overline{X})^2 = 190, \; n = 8$

$$\overline{X} = \frac{76}{8} = 9.5$$

$$S.D = S = \sqrt{\frac{\Sigma(X-\overline{X})^2}{n}} = \sqrt{\frac{190}{8}}$$

$$= \sqrt{23.75} = 4.87$$

(ii)

X	$X - \overline{X}$	$(X - \overline{X})^2$
9	0	0
3	–6	36
8	–1	1

8	−1	1
9	0	0
8	−1	1
9	0	0
18	9	81

$\Sigma x = 72$ $\qquad\qquad$ $\Sigma (X - X)^2 = 120$

$n = 8$

$$\overline{X} = \frac{\Sigma n}{n} = \frac{72}{8} = 9$$

$$S.D = S = \sqrt{\frac{\Sigma\left(X - \overline{X}\right)^2}{n}} = \sqrt{\frac{120}{8}}$$

$$= \sqrt{15} = 3.87$$

b. Calculate variance for the data: 10, 8, 9, 7, 5, 12, 8 6, 8, 2.

Solution:

(i)

X	$X - \overline{X}$	$(X - \overline{X})^2$
10	2.5	6.25
8	0.5	25
9	1.5	2.25
7	−0.5	.25
5	−2.5	6.25
12	4.5	20.25
8	0.5	.25
6	−1.5	2.25
8	0.5	.25
2	−5.5	30.25

$\Sigma x = 75$ $\qquad\qquad$ $\Sigma (X - \overline{X})^2 = 68.5$

$n = 10$

$$\overline{X} = \frac{\Sigma x}{n} = \frac{75}{10} = 7.5$$

$$\text{Variance} = S^2 = \frac{\Sigma\left(X - \overline{X}\right)^2}{n}$$

$$= \frac{68.5}{10} = 6.85$$

6. The length of 32 items are given below. Find the mean length and standard deviation of the distribution.

Length	20-22	23-25	26-28	29-31	32-34
Frequency	3	6	12	9	2

Solution:

C.I	f	Mid point (x)	fx	$X-\overline{X}$	$(X-\overline{X})^2$	$f(X-\overline{X})^2$
20 – 22	3	21	63	–6	36	108
23 – 25	6	24	144	–3	9	54
26 – 28	12	27	324	0	0	0
29 – 31	9	30	270	3	9	81
32 – 34	2	33	66	6	36	72
	32	$\sum fx = 867$			90	315

$$\overline{X} = \frac{\sum fx}{n} = \frac{667}{32} = 27.093 = 27 \text{ approx.}$$

$$S.D = S^2 = \sqrt{\frac{\Sigma\left(X-\overline{X}\right)^2}{n}} = \sqrt{\frac{315}{32}}$$

$$= \sqrt{9.84375} = 3.137$$

7. **For the following distribution of marks calculate Range.**

Marks in, percentage	Frequency/ (No
33 — 40	28
41— 50	31
51 — 60	12
61 — 70	9
71 — 75	5

Solution:

C.I	Class Boundaries	f
33 – 40	32.5 – 40.5	28
41 – 50	40.5 – 50.5	32
51 – 60	50.5 – 60.5	12
61 – 70	60.5 – 70.5	9
71 – 75	70.5 – 75.5	5

Here, $X_{max} = 75.5$

$X_{min} = 32.5$

$$\text{Range} = X_{max} - X_{min}$$
$$= 75.5 - 32.5$$
$$= 43$$

SOLVED MISCELLANEOUS EXERCISE - 6

1. **Multiple Choice Questions**

Three possible answers are given for the following question. Tick (✓) the correct answer.

(i) **A grouped frequency table is also called**