Exercise 13.2

Question # 1

Prove that:

(1) $\sin^{-1}\dfrac{5}{13}+\sin^{-1}\dfrac{7}{25}=\cos^{-1}\dfrac{253}{325}$

Solution L.H.S $=\sin^{-1}\dfrac{5}{13}+\sin^{-1}\dfrac{7}{25}$

$$\because \sin^{-1}A+\sin^{-1}B=\sin^{-1}\left(A\sqrt{1-B^2}+B\sqrt{1-A^2}\right)$$

$$=\sin^{-1}\left(\frac{5}{13}\sqrt{1-\left(\frac{7}{25}\right)^2}+\frac{7}{25}\sqrt{1-\left(\frac{5}{13}\right)^2}\right)$$

$$=\sin^{-1}\left(\frac{5}{13}\sqrt{1-\frac{49}{625}}+\frac{7}{25}\sqrt{1-\frac{25}{169}}\right)$$

$$=\sin^{-1}\left(\frac{5}{13}\sqrt{\frac{576}{625}}+\frac{7}{25}\sqrt{\frac{144}{169}}\right)=\sin^{-1}\left(\frac{5}{13}\left(\frac{24}{25}\right)+\frac{7}{25}\left(\frac{12}{13}\right)\right)$$

$$=\sin^{-1}\left(\frac{120}{325}+\frac{84}{325}\right)=\sin^{-1}\left(\frac{204}{325}\right)$$

$$=\frac{\pi}{2}-\cos^{-1}\left(\frac{204}{325}\right) \qquad \because\ \sin^{-1}\theta=\frac{\pi}{2}-\cos^{-1}\theta$$

$$=\cos^{-1}(0)-\cos\left(\frac{204}{325}\right) \qquad \because\ \frac{\pi}{2}=\cos^{-1}(0)$$

$$\because\ \cos^{-1}A-\cos^{-1}B=\cos^{-1}\left(AB+\sqrt{\left(1-A^2\right)\left(1-B^2\right)}\right)$$

$$=\cos^{-1}\left((0)\left(\frac{204}{325}\right)+\sqrt{\left(1-(0)^2\right)\left(1-\left(\frac{204}{325}\right)^2\right)}\right)$$

$$=\cos^{-1}\left(0+\sqrt{(1-0)\left(1-\frac{41616}{105625}\right)}\right)=\cos^{-1}\left(\sqrt{(1)\left(\frac{64009}{105625}\right)}\right)$$

$$=\cos^{-1}\left(\sqrt{\frac{64009}{105625}}\right)=\cos^{-1}\frac{253}{325}=\text{R.H.S}$$

Question # 2

Prove that: $\tan^{-1}\dfrac{1}{4}+\tan^{-1}\dfrac{1}{5}=\tan^{-1}\left(\dfrac{9}{19}\right)$

Solution L.H.S $=\tan^{-1}\dfrac{1}{4}+\tan^{-1}\dfrac{1}{5}$

$$= \tan^{-1}\left(\frac{\frac{1}{4}+\frac{1}{5}}{1-\left(\frac{1}{4}\right)\left(\frac{1}{5}\right)}\right) \quad \because \tan^{-1}A + \tan^{-1}B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$$

$$= \tan^{-1}\left(\frac{\frac{9}{20}}{1-\frac{1}{20}}\right) = \tan^{-1}\left(\frac{\frac{9}{20}}{\frac{19}{20}}\right)$$

$$= \tan^{-1}\left(\frac{9}{19}\right) = \text{R.H.S}$$

Question # 3
Prove that:

$$2\tan^{-1}\frac{2}{3} = \sin^{-1}\frac{12}{13}$$

Solution Suppose

$$\alpha = \sin^{-1}\frac{12}{13} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

$$\Rightarrow \sin\alpha = \frac{12}{13}$$

Now $\cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{1-\left(\frac{12}{13}\right)^2} = \sqrt{1-\frac{144}{169}} = \sqrt{\frac{25}{169}} = \frac{5}{13}$

Now $\tan\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} = \sqrt{\frac{1-\frac{5}{13}}{1+\frac{5}{13}}} = \sqrt{\frac{\frac{8}{13}}{\frac{18}{13}}} = \sqrt{\frac{8}{18}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$

$$\Rightarrow \frac{\alpha}{2} = \tan^{-1}\left(\frac{2}{3}\right) \quad \Rightarrow \alpha = 2\tan^{-1}\frac{2}{3} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

from (i) and (ii)

$$2\tan^{-1}\frac{2}{3} = \sin^{-1}\frac{12}{13} \qquad \textit{proved.}$$

Question # 4
Prove that:

$$\tan^{-1}\frac{120}{119} = 2\cos^{-1}\frac{12}{13}$$

Solution Suppose

$$\alpha = \tan^{-1}\frac{120}{119} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

$$\Rightarrow \tan\alpha = \frac{120}{119}$$

Now $\sec\alpha = \sqrt{1+\tan^2\alpha}$

$$=\sqrt{1+\left(\frac{120}{119}\right)^2}=\sqrt{1+\frac{14400}{14161}}$$

$$=\sqrt{\frac{28561}{14161}}=\frac{169}{119}$$

So $\qquad \cos\alpha=\frac{1}{\sec\alpha}=\frac{1}{169/119}=\frac{119}{169}$

Now $\qquad \cos\frac{\alpha}{2}=\sqrt{\frac{1+\cos\alpha}{2}}=\sqrt{\frac{1+119/169}{2}}=\sqrt{\frac{288/169}{2}}=\sqrt{\frac{288}{2\times169}}=\sqrt{\frac{144}{169}}=\frac{12}{13}$

$$\Rightarrow \frac{\alpha}{2}=\cos^{-1}\frac{12}{13} \qquad \Rightarrow \alpha=2\cos^{-1}\frac{12}{13} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

From (i) and (ii)

$$\tan^{-1}\frac{120}{119}=2\cos^{-1}\frac{12}{13} \qquad \textit{proved.}$$

Question # 5

Prove that:

$$\sin^{-1}\frac{1}{\sqrt{5}}+\cot^{-1}3=\frac{\pi}{4}$$

Solution Suppose

$$\alpha=\sin^{-1}\frac{1}{\sqrt{5}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

$$\Rightarrow \sin\alpha=\frac{1}{\sqrt{5}}$$

Now $\quad \cos\alpha=\sqrt{1-\sin^2\alpha}=\sqrt{1-\left(1/\sqrt{5}\right)^2}=\sqrt{1-1/5}=\sqrt{4/5}=2/\sqrt{5}$

So $\qquad \tan\alpha=\frac{\sin\alpha}{\cos\alpha}=\frac{1/\sqrt{5}}{2/\sqrt{5}}=\frac{1}{2}$

$$\Rightarrow \alpha=\tan^{-1}1/2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

From (i) and (ii)

$$\sin^{-1}\frac{1}{\sqrt{5}}=\tan^{-1}\frac{1}{2}$$

Now $\quad \cot^{-1}3=\tan^{-1}\frac{1}{3} \qquad\qquad \because \cot^{-1}x=\tan^{-1}\frac{1}{x}$

And $\qquad \text{L.H.S}=\sin^{-1}\frac{1}{\sqrt{5}}+\cot^{-1}3$

$$= \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3}$$

$$= \tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)}\right) = \tan^{-1}\left(\frac{\frac{5}{6}}{1-\frac{1}{6}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{5}{6}}{\frac{5}{6}}\right) = \tan^{-1}(1) = \frac{\pi}{4} = \text{R.H.S} \qquad \textit{proved.}$$

Question # 6

Prove that:

$$\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{8}{17} = \sin^{-1}\left(\frac{77}{85}\right)$$

Solution L.H.S $= \sin^{-1}\frac{3}{5} + \sin^{-1}\frac{8}{17}$

$$= \sin^{-1}\left(\frac{3}{5}\sqrt{1-\left(\frac{8}{17}\right)^2} + \frac{8}{17}\sqrt{1-\left(\frac{3}{5}\right)^2}\right)$$

$$= \sin^{-1}\left(\frac{3}{5}\sqrt{1-\frac{64}{289}} + \frac{8}{17}\sqrt{1-\frac{9}{25}}\right) = \sin^{-1}\left(\frac{3}{5}\sqrt{\frac{225}{289}} + \frac{8}{17}\sqrt{\frac{16}{25}}\right)$$

$$= \sin^{-1}\left(\frac{3}{5}\left(\frac{15}{17}\right) + \frac{8}{17}\left(\frac{4}{5}\right)\right) = \sin^{-1}\left(\frac{45}{85} + \frac{32}{85}\right)$$

$$= \sin^{-1}\left(\frac{77}{85}\right) = \text{R.H.S} \qquad \textit{proved.}$$

Question # 7

Prove that:

$$\sin^{-1}\frac{77}{85} - \sin^{-1}\frac{3}{5} = \cos^{-1}\left(\frac{15}{17}\right)$$

Solution L.H.S $= \sin^{-1}\frac{77}{85} - \sin^{-1}\frac{3}{5}$

$$= \left(\frac{\pi}{2} - \cos^{-1}\frac{77}{85}\right) - \left(\frac{\pi}{2} - \cos^{-1}\frac{3}{5}\right) \qquad \because \sin^{-1}x = \frac{\pi}{2} - \cos^{-1}x$$

$$= \frac{\pi}{2} - \cos^{-1}\frac{77}{85} - \frac{\pi}{2} + \cos^{-1}\frac{3}{5} = \cos^{-1}\frac{3}{5} - \cos^{-1}\frac{77}{85}$$

$$= \cos^{-1}\left(\left(\frac{3}{5}\right)\left(\frac{77}{85}\right) + \sqrt{\left(1-\left(\frac{3}{5}\right)^2\right)\left(1-\left(\frac{77}{85}\right)^2\right)}\right)$$

$$= \cos^{-1}\left(\frac{231}{425} + \sqrt{\left(1-\frac{9}{25}\right)\left(1-\frac{5929}{7225}\right)}\right)$$

$$= \cos^{-1}\left(\frac{231}{425} + \sqrt{\left(16/25\right)\left(1296/7225\right)}\right)$$

$$= \cos^{-1}\left(\frac{231}{425} + \sqrt{\frac{20736}{180625}}\right) = \cos^{-1}\left(\frac{231}{425} + \frac{144}{425}\right)$$

$$= \cos^{-1}\left(\frac{375}{425}\right) = \cos^{-1}\left(\frac{15}{17}\right) = \text{L.H.S}$$

Question # 8

Prove that:

$$\cos^{-1}\frac{63}{65} + 2\tan^{-1}\frac{1}{5} = \sin^{-1}\left(\frac{3}{5}\right)$$

Solution Suppose $\alpha = \cos^{-1}\frac{63}{65}$ ………………… (i)

$$\Rightarrow \cos\alpha = \frac{63}{65}$$

Now $\sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1-\left(63/65\right)^2} = \sqrt{1-\frac{3969}{4225}} = \sqrt{\frac{256}{4225}} = \frac{16}{65}$

$\Rightarrow \alpha = \sin^{-1}\left(\frac{16}{65}\right)$ ………………… (ii)

So from equation (i) and (ii)

$$\cos^{-1}\left(\frac{63}{65}\right) = \sin^{-1}\left(\frac{16}{65}\right)$$

Now suppose $\beta = \tan^{-1}\frac{1}{5}$ ………………… (iii)

$$\Rightarrow \tan\beta = 1/5$$

So $\sec\alpha = \sqrt{1+\tan^2\alpha} = \sqrt{1+\left(\frac{1}{5}\right)^2} = \sqrt{1+\frac{1}{25}} = \sqrt{\frac{26}{25}} = \frac{\sqrt{26}}{5}$

So $\cos\beta = \frac{1}{\sec\beta} = \frac{1}{\sqrt{26}/5} = \frac{5}{\sqrt{26}}$

As $\frac{\sin\beta}{\cos\beta} = \tan\beta \quad \Rightarrow \sin\beta = \tan\beta\cdot\cos\beta$

$$\Rightarrow \sin\beta = \left(\frac{1}{5}\right)\left(\frac{5}{\sqrt{26}}\right) = \frac{1}{\sqrt{26}}$$

$\Rightarrow \beta = \sin^{-1}\frac{1}{\sqrt{26}}$ ………………… (iv)

From (iii) and (iv)

$$\tan^{-1}\frac{1}{5}=\sin^{-1}\frac{1}{\sqrt{26}}$$

Now L.H.S = $\cos^{-1}\frac{63}{65}+2\tan^{-1}\frac{1}{5}$

$$=\sin^{-1}\frac{16}{65}+2\sin^{-1}\frac{1}{\sqrt{26}} \quad =\sin^{-1}\frac{16}{65}+\left(\sin^{-1}\frac{1}{\sqrt{26}}+\sin^{-1}\frac{1}{\sqrt{26}}\right)$$

$$=\sin^{-1}\frac{16}{65}+\sin^{-1}\left(\frac{1}{\sqrt{26}}\sqrt{1-\left(\frac{1}{\sqrt{26}}\right)^2}+\frac{1}{\sqrt{26}}\sqrt{1-\left(\frac{1}{\sqrt{26}}\right)^2}\right)$$

$$=\sin^{-1}\frac{16}{65}+\sin^{-1}\left(\frac{1}{\sqrt{26}}\sqrt{1-\frac{1}{26}}+\frac{1}{\sqrt{26}}\sqrt{1-\frac{1}{26}}\right)$$

$$=\sin^{-1}\frac{16}{65}+\sin^{-1}\left(\frac{1}{\sqrt{26}}\sqrt{\frac{25}{26}}+\frac{1}{\sqrt{26}}\sqrt{\frac{25}{26}}\right)$$

$$=\sin^{-1}\frac{16}{65}+\sin^{-1}\left(\frac{1}{\sqrt{26}}\cdot\frac{5}{\sqrt{26}}+\frac{1}{\sqrt{26}}\cdot\frac{5}{\sqrt{26}}\right)$$

$$=\sin^{-1}\frac{16}{65}+\sin^{-1}\left(\frac{5}{26}+\frac{5}{26}\right)=\sin^{-1}\frac{16}{65}+\sin^{-1}\left(\frac{5}{13}\right)$$

$$=\sin^{-1}\left(\frac{16}{65}\sqrt{1-\left(\frac{5}{13}\right)^2}+\frac{5}{13}\sqrt{1-\left(\frac{16}{65}\right)^2}\right)$$

$$=\sin^{-1}\left(\frac{16}{65}\sqrt{1-\frac{25}{169}}+\frac{5}{13}\sqrt{1-\frac{256}{4225}}\right)$$

$$=\sin^{-1}\left(\frac{16}{65}\sqrt{1-\frac{25}{169}}+\frac{5}{13}\sqrt{1-\frac{256}{4225}}\right) \quad =\sin^{-1}\left(\frac{16}{65}\sqrt{\frac{144}{169}}+\frac{5}{13}\sqrt{\frac{3969}{4225}}\right)$$

$$=\sin^{-1}\left(\frac{16}{65}\sqrt{\frac{144}{169}}+\frac{5}{13}\sqrt{\frac{3969}{4225}}\right)=\sin^{-1}\left(\frac{16}{65}\left(\frac{12}{13}\right)+\frac{5}{13}\left(\frac{63}{65}\right)\right)$$

$$=\sin^{-1}\left(\frac{192}{845}+\frac{315}{845}\right)=\sin^{-1}\left(\frac{3}{5}\right) \quad = \text{R.H.S}$$

Question # 9

Prove that:

$$\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{3}{5}-\tan^{-1}\frac{8}{19}=\frac{\pi}{4}$$

Solution L.H.S $=\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{3}{5}-\tan^{-1}\frac{8}{19}$

$$= \tan^{-1}\left(\frac{3/4 + 3/5}{1-(3/4)(3/5)}\right) - \tan^{-1}\frac{8}{19}$$

$$= \tan^{-1}\left(\frac{27/20}{1-9/20}\right) - \tan^{-1}\frac{8}{19} \quad = \tan^{-1}\left(\frac{27/20}{11/20}\right) - \tan^{-1}\frac{8}{19}$$

$$= \tan^{-1}\left(\frac{27}{11}\right) - \tan^{-1}\left(\frac{8}{19}\right) \quad = \tan^{-1}\left(\frac{27/11 - 8/19}{1+(27/11)(8/19)}\right)$$

$$= \tan^{-1}\left(\frac{425/209}{1+216/209}\right) = \tan^{-1}\left(\frac{425/209}{1+216/209}\right)$$

$$= \tan^{-1}\left(\frac{425/209}{425/209}\right) = \tan^{-1}(1) = \frac{\pi}{4} = \text{R.H.S} \qquad \textit{proved.}$$

Question # 10

Do Yourself

Question # 11

Prove that:

$$\tan^{-1}\frac{1}{11} + \tan^{-1}\frac{5}{6} = \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2}$$

Solution L.H.S $= \tan^{-1}\frac{1}{11} + \tan^{-1}\frac{5}{6}$

$=$ *Solve this* …………….. (i)

R.H.S $= \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2}$

$=$ *Solve this* ………………. (ii)

From (i) and (ii)

L.H.S = R.H.S

Question # 12

Prove that:

$$2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} = \frac{\pi}{4}$$

Solution L.H.S $= 2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7}$

$$= \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{3}}{1-\frac{1}{3}\frac{1}{3}}\right) + \tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\left(\frac{\frac{1+1}{3}}{1-\frac{1}{9}}\right) + \tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\left(\frac{\frac{2}{3}}{\frac{9-1}{9}}\right) + \tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\left(\frac{\frac{2}{3}}{\frac{8}{9}}\right) + \tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\left(\frac{2\times 9}{3\times 8}\right) + \tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\frac{1}{7} \Rightarrow \tan^{-1}\left(\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\times\frac{1}{7}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{21+4}{28}}{1-\frac{3}{28}}\right) \Rightarrow \tan^{-1}\left(\frac{\frac{25}{28}}{\frac{28-3}{28}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{25}{28}}{\frac{25}{28}}\right) \Rightarrow \tan^{-1}(1)$$

$$= \frac{\pi}{4} = R.H.S.$$

Question # 13

Show that:

$\cos\left(\sin^{-1} x\right) = \sqrt{1-x^2}$

Solution Suppose $y = \sin^{-1} x$

$\Rightarrow \sin y = x$

Since $\cos y = \sqrt{1-\sin^2 y} = \sqrt{1-x^2}$

$$\Rightarrow \quad \cos\left(\sin^{-1}x\right)=\sqrt{1-x^2} \qquad \textit{Proved}$$

Question # 14
Show that:

$$\sin\left(2\cos^{-1}x\right)=2x\sqrt{1-x^2}$$

Solution Suppose $y=\cos^{-1}x$

Then $\cos y = x$

Also $\sin y=\sqrt{1-\cos^2 y}=\sqrt{1-x^2}$

Now
$$\begin{aligned}\sin\left(2\cos^{-1}x\right)&=\sin(2y)\\&=2\sin y\cdot\cos y\\&=2\sqrt{1-x^2}\cdot x\\&=2x\sqrt{1-x^2}\end{aligned}$$

Question # 15
Show that:

$$\cos\left(2\sin^{-1}x\right)=1-2x^2$$

Solution Suppose $y=\sin^{-1}x \;\Rightarrow\; \sin y = x$

& $\cos y=\sqrt{1-\sin^2 y}=\sqrt{1-x^2}$

Now
$$\begin{aligned}\cos\left(2\sin^{-1}x\right)&=\cos 2y\\&=\cos^2 y-\sin^2 y\\&=\left(\sqrt{1-x^2}\right)^2-x^2=1-x^2-x^2\\&=1-2x^2 \qquad \textit{Proved}\end{aligned}$$

Question # 16
Show that:

$$\tan^{-1}(-x)=-\tan^{-1}x$$

Solution Suppose $y=\tan^{-1}(-x)$ …………….. (i)

$\Rightarrow \tan y=-x \quad \Rightarrow -\tan y = x$

$\Rightarrow \tan(-y)=x \qquad \because -\tan\theta=\tan(-\theta)$

$\Rightarrow -y=\tan^{-1}x$

$\Rightarrow y=-\tan^{-1}x$ ………………. (ii)

From equation (i) and (ii)

$$\tan^{-1}(-x)=-\tan^{-1}x \qquad \textit{Proved}$$

Question # 17

Do yourself as above

Question # 18
Show that:

$$\cos^{-1}(-x) = \pi - \cos^{-1} x$$

Solution Suppose $y = \pi - \cos^{-1} x$ ………………… (i)

$$\Rightarrow \pi - y = \cos^{-1} x \Rightarrow \cos(\pi - y) = x$$

$$\Rightarrow \cos\pi \cos y + \sin\pi \sin y) = x \Rightarrow (-1)\cos y + (0)\sin y) = x$$

$$\Rightarrow -\cos y + 0 = x \Rightarrow -\cos y = x$$

$$\Rightarrow \cos y = -x \qquad \Rightarrow y = \cos^{-1}(-x) \text{ ……….. (ii)}$$

From (i) and (ii)

$$\cos^{-1}(-x) = \pi - \cos^{-1} x \qquad \textit{Proved}$$

Question # 19

Show that:

$$\tan\left(\sin^{-1} x\right) = \frac{x}{\sqrt{1-x^2}}$$

Solution Suppose $y = \sin^{-1} x \Rightarrow \sin y = x$

Now $\cos y = \sqrt{1-\sin^2 y} = \sqrt{1-x^2}$

& $\tan y = \dfrac{\sin y}{\cos y} = \dfrac{x}{\sqrt{1-x^2}}$

Now $\tan\left(\sin^{-1} x\right) = \tan y = \dfrac{x}{\sqrt{1-x^2}}$ *proved*

Question # 20

Given that $x = \sin^{-1}\dfrac{1}{2}$, find the values of the following trigonometric functions:

$\sin x, \cos x, \tan x, \cot x, \sec x$ and $\csc x$.

Solution Since $x = \sin^{-1}\dfrac{1}{2} \Rightarrow \sin x = \dfrac{1}{2}$

Now $\cos x = \sqrt{1-\sin^2 x} = \sqrt{1-\left(\frac{1}{2}\right)^2} = \sqrt{1-\frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} , \qquad \cot x = \frac{1}{\tan x} = \frac{1}{\frac{1}{\sqrt{3}}} = \sqrt{3}$$

$$\sec x = \frac{1}{\cos x} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} , \qquad \operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\frac{1}{2}} = 2$$
