SOLVED EXERCISE 3.1

1. Express the following as a ratio a : b and as a fraction in its simplest (lowest) form.

(i) Rs.750, Rs. 1250

Solution:

$750:1250$

$=\frac{750}{10}:\frac{1250}{10}$ (Divided by 10)

$=75$ 125

$=\frac{75}{10}:\frac{125}{10}$ (Divided by 5)

$=15:25$

$=\frac{15}{5}:\frac{25}{5}$ (Divided by 5)

$=3:5$

$=\frac{3}{4}$

(ii) 450cm, 3 m

Solution:

450 cm : 3m = 450 cm : 300 cm 1m = 100 cm

$=450:300$

$=\frac{450}{10}=\frac{300}{10}$ (Divided by 10)

$=45:30$

$=\frac{45}{5}=\frac{30}{5}$ (Divided by 5)

$=9:6$

$=\frac{9}{3}=\frac{6}{3}$ (Divided by 3)

$=3:2$

$=\frac{3}{2}$

(iii) 4kg, 2kg 750gm

Solution:

4 kg : 2 kg 750 gm = 4000 g : 2750 g 1 kg = 1000g

$$= 4000 : 2750$$

$$= \frac{400}{10} : \frac{2750}{10} \qquad \text{(Divided by 10)}$$

$$= 400 : 275$$

$$= \frac{400}{5} = \frac{275}{5} \qquad \text{(Divided by 5)}$$

$$= 80 : 55$$

$$= \frac{80}{5} = \frac{55}{5} \qquad \text{(Divided by 5)}$$

$$= 16 : 11$$

$$= \frac{16}{11}$$

(iv) 27min 30 sec, 1 hour

Solution:

$$27 \text{ min. } 30 \text{ sec} : 1 \text{ hour} = (27 \times 60 + 30) \text{ sec} : (1 \times 60 \times 60) \text{ sec}$$

$$= 1650 : 3600$$

$$= \frac{1650}{10} = \frac{3600}{10} \qquad \text{(Divided by 10)}$$

$$= 165 : 360$$

$$= \frac{165}{5} = \frac{360}{5} \qquad \text{(Divided by 5)}$$

$$= 33 : 72$$

$$= \frac{33}{3} = \frac{72}{3} \qquad \text{(Divided by 3)}$$

$$= 11 : 24$$

$$= \frac{11}{24}$$

(v) 75°, 225°

Solution:

$$75^\circ : 225^\circ = \frac{75}{5} : \frac{225}{5} \qquad \text{(Divided by 5)}$$

$$= 15 : 45$$

$$= \frac{15}{5} = \frac{45}{5} \qquad \text{(Divided by 5)}$$

$$= 3 : 15$$

$$= \frac{3}{3} : \frac{15}{3} \qquad \text{(Divided by 3)}$$

$$= 1:5 \Rightarrow = \frac{1}{5}$$

2. In a class of 60 students, 25 students are girls and remaining students are boys. Compute the ratio of

(i) Boys to total students

Solution:

Total students = 60
Number of girls students = 25
Number of boys students = 60 – 25 = 35

$$25 : 60 = \frac{25}{5} : \frac{60}{5} \qquad \text{(Divided by 5)}$$
$$= 7:12$$

(ii) Boys to girls

Solution:

$$35:25 = \frac{35}{5} : \frac{25}{5} \qquad \text{(Divided by 5)}$$
$$= 7:5$$

3. If 3 (4x – 5y) = 2x – 7y, find the ratio x : y.

Solution:

$$3(4x - 5y) = 2x - 7y$$
$$12x - 15y = 2x - 7y$$
$$12x - 2x = 15y - 7y$$
$$10x = 8y$$
$$\frac{x}{y} = \frac{8}{10}$$
$$\frac{x}{y} = \frac{4}{5} \qquad \text{(Divided by 2)}$$
$$\Rightarrow \quad x : y = 4 : 5$$

4. Find the value of p, if the ratios 2p + 5 : 3p + 4 and 3 : 4 are equal.

Solution:

As the given ratios are equal, so

$$2p + 5 : 3p + 4 = 3 : 4$$
$$\frac{2p + 5}{3p + 4} = \frac{3}{4}$$
$$3(3p + 4) = 4(2p + 5)$$

$$9p + 12 = 8p + 20$$
$$9p - 8p = 20 - 12$$
$$p = 8$$

5. If the ratios 3x + 1 : 6 + 4x and 2 : 5 are equal. Find the value of x.

Solution:

As the given ratios are equal, so

$$3x + 1 : 6 + 4x = 2 : 5$$
$$\frac{3x+1}{6+4x} = \frac{2}{5}$$
$$5(3x+1) = 2(6+4x)$$
$$15x + 5 = 12 + 8x$$
$$15x - 8x = 12 - 5$$
$$7x = 7$$

Dividing throughout by '7', we get

$$x = 1$$

6. Two numbers are in the ratio 5 : 8. If 9 is added to each number, we get a new ratio 8 : 11. Find the numbers,

Solution:

Because the ratio of two numbers is 5 : 8.

Multiply each number of the ratio with x. then the numbers be 5x, 8x and the ratio becomes 5x : 8x.

Now according to the given condition, we have

$$\frac{5x+9}{8x+9} = \frac{8}{11}$$
$$8(8x+9) = 11(5x+9)$$
$$64x + 72 = 55x + 99$$
$$64x - 55x = 99 - 72$$
$$9x = 27$$

Dividing both sides by '9', we get

$$x = 3$$

The required numbers are

$$5x = 5(3) = 15$$
$$8x = 8(3) = 24$$

7. If 10 is added in each number of the ratio 4:13, we get a new ratio 1 : 2. What are the numbers?

Solution:

Because the ratio of two numbers is 4 : 13.

Multiply each number of the ratio with x. then the numbers be 4x, 13x and the ratio becomes 4x : 13x.

Now according to the given condition, we have

$$\frac{4x+10}{13x+10}=\frac{1}{2}$$

$$1(13x+10)=2(4x+10)$$

$$13x+10=8x+20$$

$$13x-8x=20-10$$

$$5x=10$$

Dividing both sides by '5', we get

$$x=2$$

The required numbers are

$$4x=4\ (2)=8$$

$$13x=13\ (2)=26$$

8. Find the cost of 8kg of mangoes, if 5kg of mangoes cost Rs. 250.

Solution:

Let the cost of 8 kg mangoes be x – rupees.

Then in proportion form, we have

8 kg : 5 kg :: Rs. x : Rs. 250

Product of means = Product of extremes

$$(5)(x) = (8)(250)$$

$$5x = 8\times 250$$

$$x=\frac{8\times 250}{5}$$

$$x=8\times 50$$

$$x = \text{Rs. } 400$$

9. If a : b = 7 : 6, find the value of 3a + 5b : 7b – 5a.

Solution:

Given that a : b = 7 : 6

or $\frac{a}{b}=\frac{7}{6}$

Now $3a + 5b : 7b - 5a = \dfrac{3a+5b}{7b-5a}$

$$=\frac{\frac{3a+5b}{b}}{\frac{7b-5a}{b}}$$ (Dividing numerator and denominator by 'b')

$$=\frac{3\left(\frac{a}{b}\right)+5\left(\frac{b}{b}\right)}{7\left(\frac{b}{b}\right)-5\left(\frac{a}{b}\right)}$$

$$=\frac{3\left(\frac{7}{6}\right)+5}{7-5\left(\frac{7}{6}\right)}$$

$$=\frac{\frac{21}{6}+5}{7-\frac{35}{6}}=\frac{\frac{21+5\times 6}{6}}{\frac{7\times 6-35}{6}}$$

$$=\frac{\frac{21+30}{6}}{\frac{42-35}{6}}=\frac{\frac{51}{6}}{\frac{7}{6}}$$

$$=\frac{51}{6}\times\frac{6}{7}=\frac{51}{7}$$

Hence,

$3a+5b : 7b-5a = 51 : 7$

10. Complete the following:

(i) If $\frac{24}{7}=\frac{6}{x}$, then $4x =$ ________

Solution:

$$\frac{24}{7}=\frac{6}{x}$$

$$(24)(x)=(6)(7)$$

$$(6\times 4)(x)=(6)(7)$$

$$4x=\frac{6\times 7}{6}$$

$$4x=7$$

(ii) If $\frac{5a}{3x}=\frac{15b}{y}$, then $ay =$ ________

Solution:

$$\frac{5a}{3x}=\frac{15b}{y}$$

$$(5a)(y) = (3x)(15b)$$
$$(5)(ay) = (3x)(15b)$$
$$ay = \frac{(3x)(15b)}{5}$$
$$ay = 9bx$$

(iii) If $\frac{9pq}{21m} = \frac{18p}{5m}$, then $5q$ = ________

Solution:

$$(5m)(9pq) = (21m)(18p)$$
$$(9mp)(5q) = (21m)(18p)$$
$$5q = (21m)(18p)$$
$$5q = (21)(2)$$
$$5q = 41$$

11. Find x in the following proportions.

(i) $3x - 2 : 4 :: 2x + 3 : 7$

Solution:

$$3x - 2 : 4 :: 2x + 3 : 7$$
$$3x - 2 : 4 = 2x + 3 : 7$$
$\therefore$ Product of extremes = Product of means
$$(3x - 2)(7) = (4)(2x + 3)$$
$$21x - 14 = 8x + 12$$
$$13x = 26$$
Dividing both sides by '13', we get
$$x = 2$$

(ii) $\frac{3x-1}{7} : \frac{3}{5} :: \frac{2x}{3} : \frac{7}{5}$

Solution:

$$\frac{3x-1}{7} : \frac{3}{5} :: \frac{2x}{3} : \frac{7}{5}$$
$$\frac{3x-1}{7} : \frac{3}{5} = \frac{2x}{3} : \frac{7}{5}$$
Multiplying throughout by '105' (L.C.M. of 3, 5, 7), we get

$$105\times\frac{3x-1}{7}:105\times\frac{3}{5}=105\times\frac{2x}{3}:105\times\frac{7}{5}$$

$$15\times(3x-1):21\times3=35\times2x:21\times7$$

$$(45x-15):\ 63=70x:147$$

$\therefore$ Product of extremes = Product of means

$$(45x-15)(147)=(63)(70x)$$

$$6615x-2205=4410x$$

$$6615x-4410x=2205$$

$$2205x=2205$$

Dividing both sides by '2205', we get

$$x=1$$

(iii) $\frac{x-3}{2}:\frac{5}{x-1}::\frac{x-1}{3}:\frac{4}{x+4}$

Solution:

$$\frac{x-3}{2}:\frac{5}{x-1}::\frac{x-1}{3}:\frac{4}{x+4}$$

$$\frac{x-3}{2}:\frac{5}{x-1}=\frac{x-1}{3}:\frac{4}{x+4}$$

$\therefore$ Product of extremes = Product of means

$$\left(\frac{x-3}{2}\right)\left(\frac{4}{x+4}\right)=\left(\frac{5}{x-1}\right)\left(\frac{x-1}{3}\right)$$

$$\frac{2(x-3)}{x+4}=\frac{5}{3}$$

$$\frac{2x-6}{x+4}=\frac{5}{3}$$

$$3(2x-6)=5(x+4)$$

$$6x-18=5x+20$$

$$6x-5x=18+20$$

$$x=38$$

(iv) $p^2+pq+q^2:x::\frac{p^3-q^3}{p+q}:(p-q)^2$

Solution:

$$p^2+pq+q^2:x::\frac{p^3-q^3}{p+q}:(p-q)^2$$

$$p^2 + pq + q^2 : x = \frac{(p^3 - q^3)}{p + q} : (p - q)^2$$

$\therefore$ Product of means = Product of extremes

$$x \times \frac{p^3 - q^3}{p + q} = (p - q)^2 (p^2 + pq + q^2)$$

$$x = \frac{(p - q)(p - q)(p^2 + pq + q^2)(p + q)}{(p - q)(p^2 + pq + q^2)}$$

$$x = p^2 - q^2$$

(v) $8 - x : 11 - x :: 16 - x : 25 - x$

Solution:

$8 - x : 11 - x :: 16 - x : 25 - x$
$8 - x : 11 - x = 16 - x : 25 - x$

$\therefore$ Product of extremes = Product of means

$$(8 - x)(25 - x) = (11 - x)(16 - x)$$

$$200 - 8x - 25x - x^2 = 176 - 11x - 16x - x^2$$

$$200 - 33x - x^2 = 176 - 27x - x^2$$

$$200 - 33x = 176 - 27x$$

$$-33x + 27x = 176 - 200$$

$$-6x = -24$$

$$\Rightarrow \quad x = 4$$

(c) Variation:

The word variation is frequently used in all sciences. There are two types of variations:
(i) Direct variation (ii) Inverse variation.

(i) Direct Variation

If two quantities are related in such a way that increase (decrease) in one quantity causes increase (decrease) in the other quantity, then this variation is called direct variation.

In other words, if a quantity varies directly with regard to a quantity x. We say that y is directly proportional to x and is written as $y \propto x$ or $y = kx$. i.e., $\frac{y}{x} = k, k \neq 0$.

The sign read as "varies as" is called the sign of proportionality or variation, while $k \neq 0$ is known as constant of variation. e.g.,

(i) Faster the speed of a car, longer the distance it covers.

(ii) The smaller the radius of the circle, smaller the circumference is.

(ii) Inverse Variation

If two quantities are related in such a way that when one quantity increases, the other decreases is called inverse variation.

In other words, if a quantity y varies inversely with regard to quantity x. We say that y is inversely proportional to x or y varies inversely as x and is written as $y \propto \frac{1}{x}$ or $y = \frac{k}{x}$.

i.e., $xy = k$, where $k \neq 0$ is the constant of variation.

SOLVED EXERCISE 3.2

1. If y varies directly as x, and y = 8 when x = 1, find

(i) y in terms of x

Solution:

Given that y varies directly as x.

Therefore $\quad y \propto x$

$\Rightarrow \quad y = kx$ ________ (i)

Where k is constant of variation.

Put x = 2 and y = 8 in eq. (i), we have

$$8 = k(2)$$

or $\quad 2k = 8$

$\Rightarrow \quad k = 4$

Put k = 4 in eq. (i), we get

$$y = 4x.$$

(ii) y when x = 5

Solution:

Given that y varies directly as x.

. Therefore $\quad y \propto x$

$\Rightarrow \quad y = kx$ ________ (i)

Where k is constant of variation.

Put x = 2 and y = 4 in eq. (i), we have

$$8 = k(2)$$

or $\quad 2k = 8$

$\Rightarrow \quad k = 4$

Put k = 4 and x = 5 in eq. (i), we get

$$y = (50 (4) = 20$$

(iii) x when y = 28

Solution:

Given that y varies directly as x.

Therefore $\quad y \propto x$

$\Rightarrow \quad y = kx$ ________ (i)

Where k is constant of variation.

Put x = 2 and y = 8 in eq. (i), we have