Exercise 6.7

Q.No 1 i) −2 and 8

$a = -2$ and $b = 8$

Since $G.M = \pm\sqrt{ab}$

$= \pm\sqrt{(-2)(8)}$

$= \pm\sqrt{-16} = \pm 4i$

ii) $-2i$ and $8i$

$a = -2i$, $b = 8i$

Since $G.M = \pm\sqrt{ab}$

$= \pm\sqrt{(-2i)(8i)}$

$= \pm\sqrt{-16i^2}$

$= \pm\sqrt{16}$ $\quad \because i^2 = -1$

$= \pm 4$

Q.No 2 i) Let G_1, G_2 are two G.Ms between 1 and 8.

then 1, G_1, G_2, 8 are in G.P.

here $a_1 = 1$, $a_4 = 8$

$\Rightarrow a_1 r^3 = 8$

$\Rightarrow (1) r^3 = 8$

$\Rightarrow r^3 = (2)^3$

$\Rightarrow r = 2$

Now

$G_1 = a_2 = a_1 r = (1)(2) = 2$

$G_2 = a_3 = a_1 r^2 = (1)(2)^2 = (1)(4) = 4$

hence 2, 4 are two G.Ms between 1 and 8

ii) Do yourself as (i)

Q.No 3 i) Let G_1, G_2 and G_3 are three G.Ms between 1 and 16.

then 1, G_1, G_2, G_3, 16 are in G.P.

here $a_1 = 1$, $a_5 = 16$

$\Rightarrow a_1 r^4 = 16$

$\Rightarrow 1 \cdot r^4 = 16$

$\Rightarrow r^4 = (2)^4$

$\Rightarrow r = 2$

Now $G_1 = a_2 = a_1 r = (1)(2) = 2$

$G_2 = a_3 = a_1 r^2 = (1)(2)^2 = 4$

$G_3 = a_4 = a_1 r^3 = (1)(2)^3 = 8$

Hence 2, 4, 8 are three G.Ms between 1 and 16.

Q.No.3 (ii) Do yourself

Q.No 4 Let G_1, G_2, G_3 and G_4 are four G.Ms between 3 and 96.

then 3, G_1, G_2, G_3, G_4, 96 are in G.P.

here $a_1 = 3$, $a_6 = 96$

$\Rightarrow a_1 r^5 = 96$

$\Rightarrow 3 r^5 = 96$

$\Rightarrow r^5 = 96/3 = 32$

$\Rightarrow r^5 = (2)^5 \Rightarrow \boxed{r = 2}$

Now

$G_1 = a_2 = a_1 r = (3)(2) = 6$

$G_2 = a_3 = a_1 r^2 = (3)(2)^2 = 12$

$G_3 = a_4 = a_1 r^3 = (3)(2)^3 = 24$

$G_4 = a_5 = a_1 r^4 = (3)(2)^4 = 48$

Hence 6, 12, 24, 48 are four G.Ms between 3 and 96.

Q.No.5

Suppose $A > G$

then $\frac{x+y}{2} > \pm\sqrt{xy}$

$\Rightarrow x + y > \pm 2\sqrt{xy}$

$\Rightarrow x + y \mp 2\sqrt{xy} > 0$

$\Rightarrow (\sqrt{x})^2 + (\sqrt{y})^2 \mp \sqrt{xy} > 0$

$\Rightarrow (\sqrt{x} \mp \sqrt{y})^2 > 0$

which is true as square is always +ive. Hence $A > G$.

Q.No6 We know that

$$G.M = \sqrt{ab} \quad \text{——— (i)}$$

but we have given

$$G.M = \frac{a^n + b^n}{a^{n-1} + b^{n-1}} \quad \text{——— (ii)}$$

Comparing (i) and (ii)

$$\frac{a^n + b^n}{a^{n-1} + b^{n-1}} = \sqrt{ab}$$

$$\Rightarrow a^n + b^n = a^{1/2} b^{1/2} (a^{n-1} + b^{n-1})$$

$$\Rightarrow a^n + b^n = a^{n-1/2} b^{1/2} + a^{1/2} b^{n-1/2}$$

$$\Rightarrow a^n - a^{n-\frac{1}{2}} b^{1/2} = a^{1/2} b^{n-1/2} - b^n$$

$$\Rightarrow a^{n-\frac{1}{2}} (a^{1/2} - b^{1/2}) = b^{n-1/2} (a^{1/2} - b^{1/2})$$

$$\Rightarrow a^{n-1/2} = b^{n-1/2}$$

$$\Rightarrow \frac{a^{n-1/2}}{b^{n-1/2}} = 1$$

$$\Rightarrow \left(\frac{a}{b}\right)^{n-1/2} = \left(\frac{a}{b}\right)^0 \quad \because \left(\frac{a}{b}\right)^0 = 1$$

$$\Rightarrow n - \frac{1}{2} = 0 \Rightarrow \boxed{n = 1/2}$$

Q.No7 Let a and b be two +ive integers then by given condition.

$$A.M = G.M + 2$$

$$\Rightarrow \frac{a+b}{2} = \sqrt{ab} + 2$$

$$\Rightarrow a + b = 2(\sqrt{ab} + 2) \quad \text{——— (i)}$$

Also we have given

$$a + b = 20 \quad \text{——— (ii)}$$

Comparing (i) & (ii)

$$2(\sqrt{ab} + 2) = 20 \Rightarrow 2\sqrt{ab} + 4 = 20$$

$$\Rightarrow 2\sqrt{ab} = 20 - 4$$

$$= 16$$

$$\Rightarrow \sqrt{ab} = \frac{16}{2} = 8$$

$$\Rightarrow ab = 64 \quad \text{by squaring}$$

$$\Rightarrow b = \frac{64}{a} \quad \text{——— (iii)}$$

putting in (ii)

$$a + \frac{64}{a} = 20$$

$$\Rightarrow a^2 + 64 = 20a$$

$$\Rightarrow a^2 - 20a + 64 = 0$$

$$\Rightarrow a^2 - 16a - 4a + 64 = 0$$

$$\Rightarrow a(a-16) - 4(a-16) = 0$$

$$\Rightarrow (a-16)(a-4) = 0$$

$$a - 16 = 0 \quad , \quad a - 4 = 0$$

$$a = 16 \quad , \quad a = 4$$

putting in (iii)

$$b = \frac{64}{16} = 4 \quad , \quad b = \frac{64}{4} = 16$$

hence 16, 4 OR 4, 16 are required numbers.

Q.No.8 Let a, and b be two required number then

$$A.M = 5$$

$$\Rightarrow \frac{a+b}{2} = 5$$

$$\Rightarrow a + b = 10 \quad \text{——— (i)}$$

Also

$$G.M = 4$$

$$\Rightarrow \sqrt{ab} = 4$$

$$\Rightarrow ab = 16 \quad \text{on squaring}$$

$$\Rightarrow b = \frac{16}{a} \quad \text{——— (ii)}$$

putting in (i)

$$a + \frac{16}{a} = 10$$

$$\Rightarrow a^2 + 16 = 10a$$

$$\Rightarrow a^2 - 10a + 16 = 0$$

$$\Rightarrow a^2 - 8a - 2a + 16 = 0$$

$$\Rightarrow a(a-8) - 2(a-8) = 0$$

$$\Rightarrow (a-8)(a-2) = 0$$

$$a - 8 = 0 \quad , \quad a - 2 = 0$$

$$a = 8 \quad , \quad a = 2$$

putting in (ii)

$$b = \frac{16}{8} = 2 \quad , \quad b = \frac{16}{2} = 8$$

hence 8, 2 OR 2, 8 are required numbers.

END