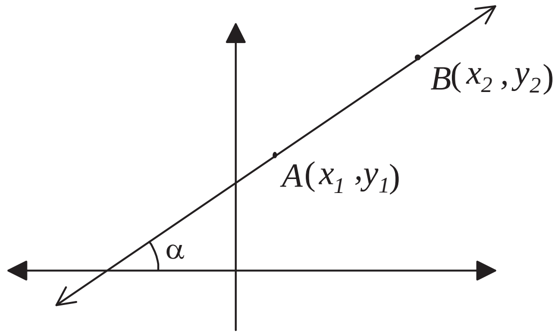
EXERCISE 4.3

◈ *Inclination of a Line:*

The angle α $(0^\circ \le \alpha < 180^\circ)$ measure anti-clockwise from positive $x-$axis to the straight line l is called *inclination* of a line l.



◈ *Slope or Gradient of Line*

The slope m of the line l is defined by:

$$m = \tan\alpha$$

If $A(x_1, y_1)$ and $B(x_2, y_2)$ be any two distinct points on the line l then

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

See proof on book at page: 191

◈ *Note:*

l is horizontal, iff $m = 0$ $(\because \alpha = 0^\circ)$

l is vertical, iff $m = \infty$ i.e. m is not defined. $(\because \alpha = 90^\circ)$

If slope of AB = slope of BC, then the points A, B and C are collinear i.e. lie on the same line.

◈ *Theorem*

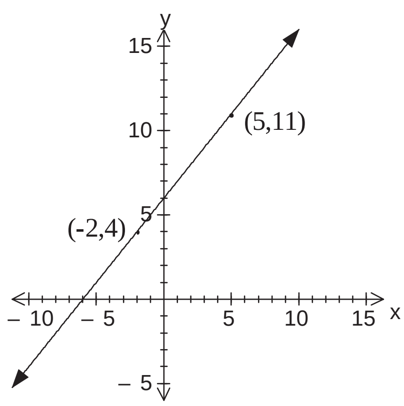
The two lines l_1 and l_2 with respective slopes m_1 and m_2 are

(i) Parallel iff $m_1 = m_2$

(ii) Perpendicular iff $m_1 m_2 = -1$ or $m_1 = -\dfrac{1}{m_2}$

◈ *Question # 1*

(i) $(-2,4)$; $(5,11)$

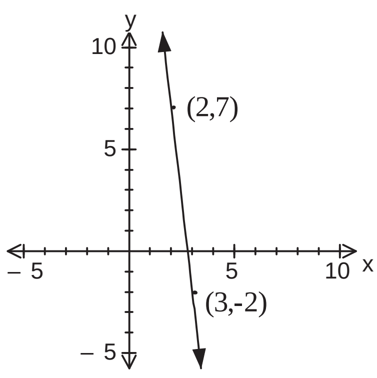


Slope $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{11-4}{5+2} = \dfrac{7}{7} = 1$

Since $\tan\alpha = m = 1$

$\Rightarrow \alpha = \tan^{-1}(1) = 45^\circ$

(ii) $(3,-2)$; $(2,7)$



Slope $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{7+2}{2-3} = \dfrac{9}{-1} = -9$

Since $\tan\alpha = m = -9$

$\Rightarrow -\tan\alpha = 9 \Rightarrow \tan(180 - \alpha) = 9$

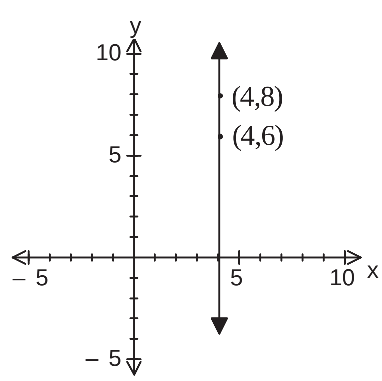
$\Rightarrow 180 - \alpha = \tan^{-1}(9)$

$\Rightarrow 180 - \alpha = 83^\circ 40'$

$\Rightarrow \alpha = 180 - 83^\circ 40'$

$\Rightarrow \alpha = 96^\circ 20'$

(ii) $(4,6)$; $(4,8)$



Slope $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{8-6}{4-4} = \dfrac{2}{0} = \infty$

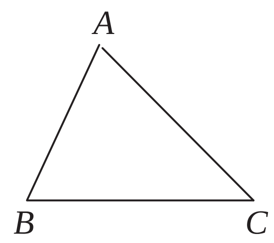
Since $\tan\alpha = m = \infty$

$\Rightarrow \alpha = \tan^{-1}(\infty) = 90^\circ$

◈ *Question # 2*

Since $A(8,6)$, $B(-4,2)$ and $C(-2,-6)$ are vertices of triangle therefore

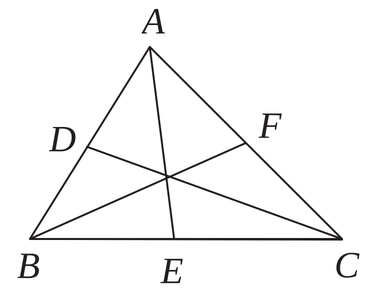
(i) Slope of side $AB = \dfrac{2-6}{-4-8} = \dfrac{-4}{-12} = \dfrac{1}{3}$



Slope of side $BC = \dfrac{-6-2}{-2+4} = \dfrac{-8}{2} = -4$

Slope of side $CA = \dfrac{6+6}{8+2} = \dfrac{12}{10} = \dfrac{6}{5}$

(ii) Let D, E and F are midpoints of sides AB, BC and CA respectively.
Then



Coordinate of $D = \left(\dfrac{8-4}{2}, \dfrac{6+2}{2}\right) = \left(\dfrac{4}{2}, \dfrac{8}{2}\right) = (2,4)$

Coordinate of $E = \left(\dfrac{-4-2}{2}, \dfrac{2-6}{2}\right) = \left(\dfrac{-6}{2}, \dfrac{-4}{2}\right) = (-3,-2)$

Coordinate of $F = \left(\dfrac{-2+8}{2}, \dfrac{-6+6}{2}\right) = \left(\dfrac{6}{2}, \dfrac{0}{2}\right) = (3,0)$

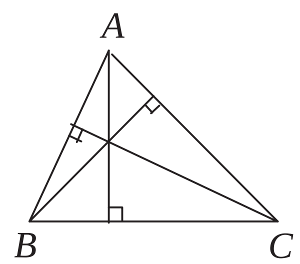
Hence Slope of median $AE = \dfrac{-2-6}{-3-8} = \dfrac{-8}{-11} = \dfrac{8}{11}$

Slope of median $BF = \dfrac{0-2}{3+4} = \dfrac{-2}{7}$

Slope of median $CD = \dfrac{4+6}{2+2} = \dfrac{10}{4} = \dfrac{5}{2}$

(iii) Since altitudes are perpendicular to the sides of a triangle therefore

Slope of altitude from vertex $A = \dfrac{-1}{\text{slope of side } BC} = \dfrac{-1}{-4} = \dfrac{1}{4}$



Slope of altitude from vertex $B = \dfrac{-1}{\text{slope of side } AC} = \dfrac{-1}{6/5} = -\dfrac{5}{6}$

Slope of altitude from vertex $C = \dfrac{-1}{\text{slope of side } AB} = \dfrac{-1}{1/3} = -3$

◈ *Question # 3*

(a) Let $A(-1,-3)$, $B(1,5)$ and $C(2,9)$ be given points

Slope of $AB = \dfrac{5+3}{1+1} = \dfrac{8}{2} = 4$

Slope of $BC = \dfrac{9-5}{2-1} = \dfrac{4}{1} = 4$

Since slope of $AB =$ slope of BC
Therefore A, B and C lie on the same line.

(b) & (c) *Do yourself as above*

(d) Let $A(a,2b), B(c,a+b)$ and $C(2c-a,2a)$ be given points.

Slope of $AB = \dfrac{(a+b)-2b}{c-a} = \dfrac{a-b}{c-a}$

Slope of $BC = \dfrac{2a-(a+b)}{(2c-a)-c} = \dfrac{2a-a-b}{2c-a-c} = \dfrac{a-b}{c-a}$

Since slope of $AB =$ slope of BC
Therefore A, B and C lie on the same line.

◈ Question # 4

Since $A(7,3)$, $B(k,-6)$, $C(-4,5)$ and $D(-6,4)$

Therefore slope of $AB = m_1 = \dfrac{-6-3}{k-7} = \dfrac{-9}{k-7}$

Slope of $CD = m_2 = \dfrac{4-5}{-6+4} = \dfrac{-1}{-2} = \dfrac{1}{2}$

(i) If AB and CD are parallel then $m_1 = m_2$

$$\Rightarrow \frac{-9}{k-7} = \frac{1}{2} \qquad \Rightarrow -18 = k-7$$

$$\Rightarrow k = -18+7 \qquad \Rightarrow \boxed{k=-11}$$

(ii) If AB and CD are perpendicular then $m_1 m_2 = -1$

$$\Rightarrow \left(\frac{-9}{k-7}\right)\left(\frac{1}{2}\right) = -1 \qquad \Rightarrow -9 = -2(k-7)$$

$$\Rightarrow 9 = 2k-14 \qquad \Rightarrow 2k = 9+14 = 23$$

$$\Rightarrow \boxed{k = \frac{23}{2}}$$

◈ Question # 5

Since $A(6,1), B(2,7)$ and $C(-6,-7)$ are vertices of triangle therefore

Slope of $\overline{AB} = m_1 = \dfrac{7-1}{2-6} = \dfrac{6}{-4} = -\dfrac{3}{2}$

Slope of $\overline{BC} = m_2 = \dfrac{-7-7}{-6-2} = \dfrac{-12}{-8} = \dfrac{7}{4}$

Slope of $\overline{CA} = m_3 = \dfrac{1+7}{6+6} = \dfrac{8}{12} = \dfrac{2}{3}$

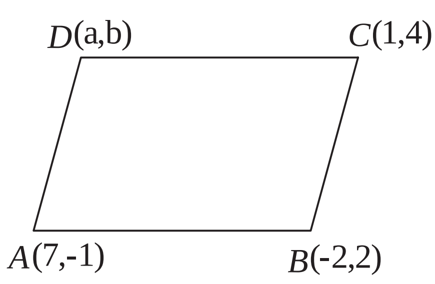
Since $m_1 m_3 = \left(-\dfrac{3}{2}\right)\left(\dfrac{2}{3}\right) = -1$

$\Rightarrow$ The triangle ABC is a right triangle with $m\angle A = 90^\circ$

> **REMEMBER**
> The symbols
> (i) $\parallel$ stands for 'parallel"
> (ii) $\nparallel$ stands for "not parallel"
> (iii) $\perp$ stands for "perpendicular"

◈ Question # 6

Let $D(a,b)$ be a fourth vertex of the parallelogram.



Slope of $\overline{AB} = \dfrac{2+1}{-2-7} = \dfrac{3}{-9} = -\dfrac{1}{3}$

Slope of $\overline{BC} = \dfrac{4-2}{1+2} = \dfrac{2}{3}$

Slope of $\overline{CD} = \dfrac{b-4}{a-1}$

Slope of $\overline{DA} = \dfrac{-1-b}{7-a}$

Since $ABCD$ is a parallelogram therefore

Slope of $\overline{AB} =$ Slope of $\overline{CD}$

$$\Rightarrow -\frac{1}{3} = \frac{b-4}{a-1} \qquad \Rightarrow -(a-1) = 3(b-4)$$

$$\Rightarrow -a+1-3b+12 = 0 \qquad \Rightarrow -a-3b+13 = 0 \text{ ....}(i)$$

Also slope of $\overline{BC} =$ slope of $\overline{DA}$

$$\Rightarrow \frac{2}{3} = \frac{-1-b}{7-a} \qquad \Rightarrow 2(7-a) = 3(-1-b) \qquad \Rightarrow 14-2a = -3-3b$$

$$\Rightarrow 14-2a+3+3b = 0 \qquad \Rightarrow -2a+3b+17 = 0 \text{.............}(ii)$$

Adding (i) and (ii)

$$\begin{array}{l} -a \;\; -3b+13=0 \\ \underline{-2a+3b+17=0} \\ -3a \qquad\quad +30=0 \end{array} \quad \Rightarrow \; 3a=30 \quad \Rightarrow \; \boxed{a=10}$$

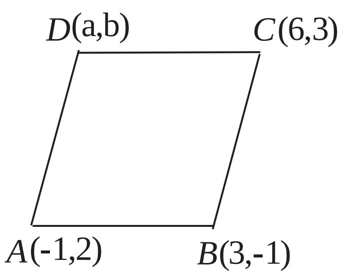
Putting value of a in (i)

$$-10-3b+13=0 \quad \Rightarrow \; -3b+3=0 \quad \Rightarrow \; 3b=3 \;\Rightarrow\; \boxed{b=1}$$

Hence $D(10,1)$ is the fourth vertex of parallelogram.

◈ *Question # 7*

Let $D(a,b)$ be a fourth vertex of rhombus.



Slope of $\overline{AB} = \dfrac{-1-2}{3+1} = \dfrac{-3}{4}$

Slope of $\overline{BC} = \dfrac{3+1}{6-3} = \dfrac{4}{3}$

Slope of $\overline{CD} = \dfrac{b-3}{a-6}$

Slope of $\overline{DA} = \dfrac{2-b}{-1-a}$

Since $ABCD$ is a rhombus therefore

Slope of $\overline{AB} =$ Slope of $\overline{CD}$

$$\Rightarrow \; -\frac{3}{4} = \frac{b-3}{a-6} \qquad \Rightarrow \; -3(a-6)=4(b-3)$$

$$\Rightarrow \; -3a+18=4b-12 \qquad \Rightarrow \; -3a+18-4b+12=0$$

$$\Rightarrow \; -3a-4b+30=0 \text{.............}(i)$$

Also slope of $\overline{BC} =$ slope of $\overline{DA}$

$$\Rightarrow \; \frac{4}{3} = \frac{2-b}{-1-a} \qquad \Rightarrow \; 4(-1-a)=3(2-b)$$

$$\Rightarrow \; -4-4a=6-3b \qquad \Rightarrow \; -4-4a-6+3b=0$$

$$\Rightarrow \; -4a+3b-10=0 \text{ ....}(ii)$$

$\times$ing eq. (i) by 3 and (ii) by 4 and adding.

$$\begin{array}{l} -9a \;\; -12b+90=0 \\ \underline{-16a+12b-40=0} \\ -25a \qquad\quad +50=0 \end{array} \quad \Rightarrow \; 25a=50 \quad \Rightarrow \; \boxed{a=2}$$

Putting value of a in (ii)

$$-4(2)+3b-10=0 \;\Rightarrow\; 3b-18=0 \quad \Rightarrow \; 3b=18 \;\Rightarrow\; \boxed{b=6}$$

Hence $D(2,6)$ is the fourth vertex of rhombus.

Now slope of diagonal $\overline{AC} = \dfrac{3-2}{6+1} = \dfrac{1}{7}$

Slope of diagonal $\overline{BD} = \dfrac{b-(-1)}{a-3} = \dfrac{6+1}{2-3} = \dfrac{7}{-1} = -7$

Since

$$(\text{Slope of } \overline{AC})(\text{Slope of } \overline{BD}) = \left(\frac{1}{7}\right)(-7) = -1$$

$\Rightarrow$ Diagonals of a rhombus are $\perp$ to each other.

◈ *Question # 8*

(a) Slope of line joining $(1,-2)$ and $(2,4) = m_1 = \dfrac{4+2}{2-1} = \dfrac{6}{1} = 6$

Slope of line joining $(4,1)$ and $(-8,2) = m_2 = \dfrac{2-1}{-8-4} = \dfrac{1}{-12}$

Since $m_1 \neq m_2$

Also $m_1 m_2 = 6 \cdot \dfrac{1}{-12} = -\dfrac{1}{2} \neq -1$

$\Rightarrow$ lines are neither parallel nor perpendicular.

(b) *Do yourself as above.*

◈ *Equation of Straight Line:*

(i) Slope-intercept form

Equation of straight line with slope m and $y-$intercept c is given by:

$$\boxed{y = mx + c}$$

See proof on book at page 194

(ii) Point-slope form

Let m be a slope of line and $A(x_1, y_1)$ be a point lies on a line then equation of line is given by:

$$\boxed{y - y_1 = m(x - x_1)}$$

See proof on book at page 195

(iii) Symmetric form

Let α be an inclination of line and $A(x_1, y_1)$ be a point lies on a line then equation of line is given by:

$$\boxed{\frac{y - y_1}{\cos\alpha} = \frac{x - x_1}{\sin\alpha}}$$

See proof on book at page 195

(iv) Two-points form

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be points lie on a line then it's equation is given by:

$$\boxed{y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)} \quad \text{or} \quad \boxed{y - y_2 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_2)} \quad \text{or} \quad \boxed{\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0}$$

See proof on book at page 196

(v) Two-intercept form

When a line intersect $x-$axis at $x = a$ and $y-$axis at $y = b$ i.e. $x-$intercept $= a$ and $y-$intercept $= b$, then equation of line is given by:

$$\boxed{\frac{x}{a} + \frac{y}{b} = 1}$$

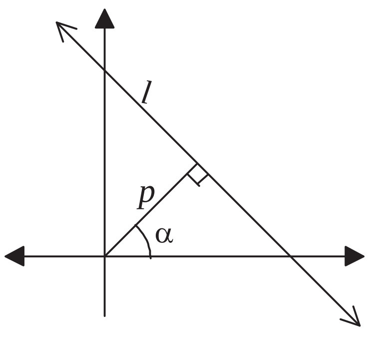
See proof on book at page 197

(vi) Normal form

Let p denoted length of perpendicular from the origin to the line and α is the inclination of the perpendicular then equation of line is given by:

$$\boxed{x\cos\alpha + y\sin\alpha = p}$$

See proof on book at page 198



◈ Question # 9

(a) Since slope of horizontal line $= m = 0$

$$\& \ (x_1, y_1) = (7, -9)$$

therefore equation of line:

$$y - (-9) = 0(x - 7)$$

$$\Rightarrow \ x + 9 = 0 \quad \textit{Answer}$$

(b) Since slope of vertical line $m = \infty = \dfrac{1}{0}$

$$\& \ (x_1, y_1) = (-5, 3)$$

therefore required equation of line

$$y - 3 = \infty\left(x - (-5)\right)$$

$$\Rightarrow \ y - 3 = \frac{1}{0}(x + 5) \qquad \Rightarrow \ 0(y - 3) = 1(x + 5)$$

$$\Rightarrow \ x + 5 = 0 \quad \textit{Answer}$$

(c) The line bisecting the first and third quadrant makes an angle of 45° with the $x-$axis therefore slope of line $= m = \tan 45^\circ = 1$

Also it passes through origin $(0,0)$, so its equation

$$y - 0 = 1(x - 0) \quad \Rightarrow \ y = x$$

$$\Rightarrow \ x - y = 0 \quad \textit{Answer}$$

(d) The line bisecting the second and fourth quadrant makes an angle of 135° with $x-$axis therefore slope of line $= m = \tan 135^\circ = -1$

Also it passes through origin $(0,0)$, so its equation

$$y - 0 = -1(x - 0) \quad \Rightarrow \ y = -x$$

$$\Rightarrow \ x + y = 0 \quad \textit{Answer}$$

◈ Question # 10

(a) $\because \ (x_1, y_1) = (-6, 5)$

and slope of line $= m = 7$

so required equation

$$y - 5 = 7\left(x - (-6)\right)$$

$$\Rightarrow \ y - 5 = 7(x + 6) \qquad \Rightarrow \ y - 5 = 7x + 42$$

$$\Rightarrow \ 7x + 42 - y + 5 = 0 \qquad \Rightarrow \ 7x - y + 47 = 0 \quad \textit{Answer}$$

(b) *Do yourself as above.*

(c) $\because \ (x_1, y_1) = (-8, 5)$

and slope of line $= m = \infty$

So required equation

$$y - 5 = \infty\left(x - (-8)\right)$$

$$\Rightarrow \ y - 5 = \frac{1}{0}(x + 8) \qquad \Rightarrow \ 0(y - 5) = 1(x + 8)$$

$$\Rightarrow \ x + 8 = 0 \quad \textit{Answer}$$

(d) The line through $(-5, -3)$ and $(9, -1)$ is

$$y - (-3) = \frac{-1 - (-3)}{9 - (-5)}\left(x - (-5)\right) \qquad \Rightarrow \ y + 3 = \frac{2}{14}(x + 5)$$

$$\Rightarrow \ y + 3 = \frac{1}{7}(x + 5) \qquad \Rightarrow \ 7y + 21 = x + 5$$

$$\Rightarrow \ x + 5 - 7y - 21 = 0 \qquad \Rightarrow \ x - 7y - 16 = 0 \quad \textit{Answer}$$

(e) $\because\ y-\text{intercept} = -7$

$\Rightarrow\ (0,-7)$ lies on a required line

Also slope $= m = -5$

So required equation

$$y-(-7) = -5(x-0)$$

$$\Rightarrow\ y+7 = -5x \quad \Rightarrow\ 5x+y+7=0 \qquad \textit{Answer}$$

(f) $\because\ x-\text{intercept} = -9$

$\Rightarrow\ (-9,0)$ lies on a required line

Also slope $= m = 4$

Therefore required line

$$y-0 = 4(x+9)$$

$$\Rightarrow\ y = 4x+9 \quad \Rightarrow\ 4x-y+9=0 \qquad \textit{Answer}$$

(e) $x-\text{intercept} = a = -3$

$y-\text{intercept} = b = 4$

Using two-intercept form of equation line

$$\frac{x}{a}+\frac{y}{b}=1 \quad \Rightarrow\ \frac{x}{-3}+\frac{y}{4}=0$$

$$\Rightarrow\ 4x-3y=-12 \qquad \times\text{ing by } -12$$

$$\Rightarrow\ 4x-3y+12=0 \qquad \textit{Answer}$$

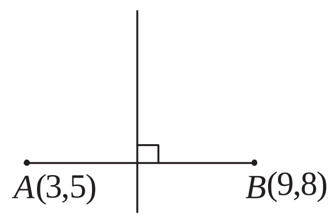
◈ *Question # 11*

Given points $A(3,5)$ and $B(9,8)$

Midpoint of $\overline{AB} = \left(\frac{3+9}{2},\frac{5+8}{2}\right) = \left(\frac{12}{2},\frac{13}{2}\right) = \left(6,\frac{13}{2}\right)$

Slope of $\overline{AB} = m = \frac{8-5}{9-3} = \frac{3}{6} = \frac{1}{2}$

Slope of line $\perp$ to $\overline{AB} = -\frac{1}{m} = -\frac{1}{\cancel{1}/2} = --2$



Now equation of $\perp$ bisector having slope -2 through $\left(6,\frac{13}{2}\right)$

$$\Rightarrow\ y-\frac{13}{2} = -2(x-6)$$

$$\Rightarrow\ y-\frac{13}{2} = -2x+12 \quad \Rightarrow\ y-\frac{13}{2}+2x-12=0$$

$$\Rightarrow\ 2x+y-\frac{37}{2}=0 \quad \Rightarrow\ 4x+2y-37=0 \qquad \textit{Answer}$$

◈ *Question # 12*

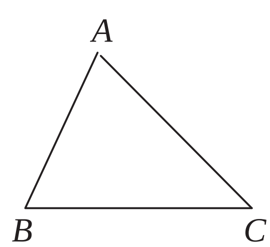
Given vertices of triangle are $A(-3,2)$, $B(5,4)$ and $C(3,-8)$.

Equation of sides:

Slope of $\overline{AB} = m_1 = \frac{4-2}{5-(-3)} = \frac{2}{8} = \frac{1}{4}$

Slope of $\overline{BC} = m_2 = \frac{-8-4}{3-5} = \frac{-12}{-2} = 6$

Slope of $\overline{CA} = m_3 = \frac{2-(-8)}{-3-3} = \frac{10}{-6} = -\frac{5}{3}$



Now equation of side $\overline{AB}$ having slope $\frac{1}{4}$ passing through $A(-3,2)$

[You may take $B(5,4)$ instead of $A(-3,2)$]

$$y-2=\frac{1}{4}\left(x-(-3)\right) \quad \Rightarrow \quad 4y-8=x+3$$

$$\Rightarrow \quad x+3-4y+8=0 \quad \Rightarrow \quad \boxed{x-4y+11=0}$$

Equation of side $\overline{BC}$ having slope 6 passing through $B(5,4)$.

$$y-4=6(x-5) \quad \Rightarrow \quad y-4=6x-30$$

$$\Rightarrow \quad 6x-30-y+4=0 \quad \Rightarrow \quad \boxed{6x-y-26=0}$$

Equation of side $\overline{CA}$ having slope $-\frac{5}{3}$ passing through $C(3,-8)$

$$y-(-8)=-\frac{5}{3}(x-3) \quad \Rightarrow \quad 3(y+8)=-5(x-3)$$

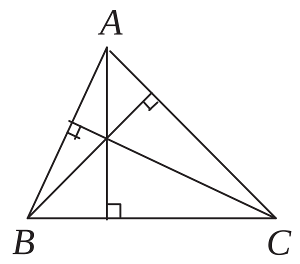
$$\Rightarrow \quad 3y+24=-5x+15 \quad \Rightarrow \quad 5x-15+3y+24=0$$

$$\Rightarrow \quad \boxed{5x+3y+9=0}$$

Equation of altitudes:

Since altitudes are perpendicular to the sides of triangle therefore

Slope of altitude on $\overline{AB}=-\frac{1}{m_1}=-\frac{1}{\cancel{1}/4}=-4$



Equation of altitude from $C(3,-8)$ having slope -4

$$y+8=-4(x-3) \quad \Rightarrow \quad y+8=-4x+12$$

$$\Rightarrow \quad 4x-12+y+8=0 \quad \Rightarrow \quad \boxed{4x+y-4=0}$$

Slope of altitude on $\overline{BC}=-\frac{1}{m_2}=-\frac{1}{6}$

Equation of altitude from $A(-3,2)$ having slope $-\frac{1}{6}$

$$y-2=-\frac{1}{6}(x+3) \quad \Rightarrow \quad 6y-12=-x-3$$

$$\Rightarrow \quad x+3+6y-12=0 \quad \Rightarrow \quad \boxed{x+6y-9=0}$$

Slope of altitude on $\overline{CA}=-\frac{1}{m_3}=-\frac{1}{-5\cancel{/3}}=\frac{3}{5}$

Equation of altitude from $B(5,4)$ having slope $\frac{3}{5}$

$$y-4=\frac{3}{5}(x-5) \quad \Rightarrow \quad 5y-20=3x-15$$

$$\Rightarrow \quad 3x-15-5y+20=0 \quad \Rightarrow \quad \boxed{3x-5y+5=0}$$

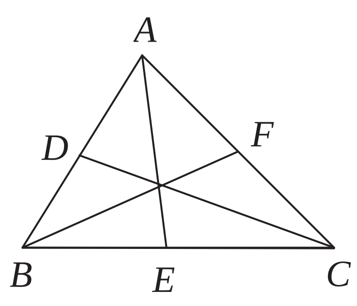
Equation of Medians:

Suppose D,E and F are midpoints of sides $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively.

Then coordinate of $D=\left(\frac{-3+5}{2},\frac{2+4}{2}\right)=\left(\frac{2}{2},\frac{6}{2}\right)=(1,3)$

Coordinate of $E=\left(\frac{5+3}{2},\frac{4-8}{2}\right)=\left(\frac{8}{2},\frac{-4}{2}\right)=(4,-2)$

Coordinate of $F=\left(\frac{3-3}{2},\frac{-8+2}{2}\right)=\left(\frac{0}{2},\frac{-6}{2}\right)=(0,-3)$



Equation of median $\overline{AE}$ by two-point form

$$y-2=\frac{-2-2}{4-(-3)}\left(x-(-3)\right)$$

$$\Rightarrow\ y-2=\frac{-4}{7}(x+3) \qquad \Rightarrow\ 7y-14=-4x-12$$

$$\Rightarrow\ 7y-14+4x+12=0 \qquad \Rightarrow\ \boxed{4x+7y-2=0}$$

Equation of median $\overline{BF}$ by two-point form

$$y-4=\frac{-3-4}{0-5}(x-5)$$

$$\Rightarrow\ y-4=\frac{-7}{-5}(x-5) \qquad \Rightarrow\ -5y+20=-7x+35$$

$$\Rightarrow\ -5y+20+7x-35=0 \qquad \Rightarrow\ \boxed{7x-5y-15=0}$$

Equation of median $\overline{CD}$ by two-point form

$$y-(-8)=\frac{3-(-8)}{1-3}(x-3)$$

$$\Rightarrow\ y+8=\frac{11}{-2}(x-3) \qquad \Rightarrow\ -2y-16=11x-33$$

$$\Rightarrow\ 11x-33+2y+16=0 \qquad \Rightarrow\ \boxed{11x+2y-17=0}$$

◈ *Question # 13*

Here $(x_1,y_1)=(-4,-6)$

Slope of given line $=m=\dfrac{-3}{2}$

$\because$ required line is $\perp$ to given line

$\therefore$ slope of required line $=-\dfrac{1}{m}=-\dfrac{1}{-3/2}=\dfrac{2}{3}$

Now equation of line having slope $\dfrac{2}{3}$ passing through $(-4,-6)$

$$y-(-6)=\frac{2}{3}(x-(-4))$$

$$\Rightarrow\ 3(y+6)=2(x+4) \qquad \Rightarrow\ 3y+18=2x+8$$

$$\Rightarrow\ 2x+8-3y-18=0 \qquad \Rightarrow\ 2x-3y-10=0$$ *Answer*

◈ *Question # 14*

Here $(x_1,y_1)=(11,-5)$

Slope of given line $=m=-24$

$\because$ required line is $\parallel$ to given line

$\therefore$ slope of required line $=m=-24$

Now equation of line having slope -24 passing through $(11,-5)$

$$y-(-5)=-24(x-11)$$

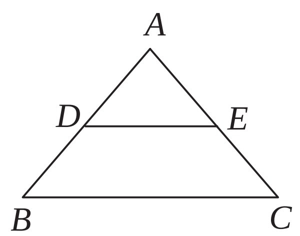
$$\Rightarrow\ y+5=-24x+264 \qquad \Rightarrow\ 24x-264+y+5=0$$

$$\Rightarrow\ 24x+y-259=0$$ *Answer*

◈ *Question # 15*

Given vertices $A(-1,2)$, $B(6,3)$ and $C(2,-4)$

Since D and E are midpoints of sides $\overline{AB}$ and $\overline{AC}$ respectively.



Therefore coordinate of $D=\left(\dfrac{-1+6}{2},\dfrac{2+3}{2}\right)=\left(\dfrac{5}{2},\dfrac{5}{2}\right)$

Coordinate of $E = \left(\dfrac{-1+2}{2}, \dfrac{2-4}{2}\right) = \left(\dfrac{1}{2}, \dfrac{-2}{2}\right) = \left(\dfrac{1}{2}, -1\right)$

Now slope of $\overline{DE} = \dfrac{-1-\frac{5}{2}}{\frac{1}{2}-\frac{5}{2}} = \dfrac{-\frac{7}{2}}{-\frac{4}{2}} = \dfrac{7}{4}$

slope of $\overline{BC} = \dfrac{-4-3}{2-6} = \dfrac{-7}{-4} = \dfrac{7}{4}$

Since slope of $\overline{DE} =$ slope of $\overline{BC}$

Therefore $\overline{DE}$ is parallel to $\overline{BC}$.

Now $\left|\overline{DE}\right| = \sqrt{\left(\dfrac{1}{2}-\dfrac{5}{2}\right)^2 + \left(-1-\dfrac{5}{2}\right)^2} = \sqrt{\left(-\dfrac{4}{2}\right)^2 + \left(-\dfrac{7}{2}\right)^2}$

$= \sqrt{4+\dfrac{49}{4}} = \sqrt{\dfrac{65}{4}} = \dfrac{\sqrt{65}}{2}$ ………... (i)

$\left|\overline{BC}\right| = \sqrt{(2-6)^2 + (-4-3)^2} = \sqrt{(-4)^2 + (-7)^2}$

$= \sqrt{16+49} = \sqrt{65}$ ………… (ii)

From (i) and (ii)

$$\left|\overline{DE}\right| = \frac{1}{2}\left|\overline{BC}\right| \qquad \text{Proved.}$$

◈ *Question # 16*

Let l denotes the number of litres of milk and p denotes the price of milk,

Then $(l_1, p_1) = (560, 12.50)$ & $(l_2, p_2) = (700, 12.00)$

Since graph of sale price and milk sold is a straight line

Therefore, from two point form, it's equation

$$p - p_1 = \frac{p_2 - p_1}{l_2 - l_1}(l - l_1)$$

$$\Rightarrow\ p - 12.50 = \frac{12.00-12.50}{700-560}(l-560)$$

$$\Rightarrow\ p - 12.50 = \frac{-0.50}{140}(l-560)$$

$$\Rightarrow\ 140p - 1750 = -0.50l + 280$$

$$\Rightarrow\ 140p - 1750 + 0.50l - 280 = 0$$

$$\Rightarrow\ 0.50l + 140p - 2030 = 0$$

ALTERNATIVE

You may use determinant form of two-point form to find an equation of line.

$$\begin{vmatrix} l & p & 1 \\ l_1 & p_1 & 1 \\ l_2 & p_2 & 1 \end{vmatrix} = 0$$

If $p = 12.25$

$$\Rightarrow\ 0.50l + 140(12.25) - 2030 = 0$$

$$\Rightarrow\ 0.50l + 1715 - 2030 = 0 \qquad \Rightarrow\ 0.50l - 315 = 0$$

$$\Rightarrow\ 0.50l = 315 \qquad \Rightarrow\ l = \frac{315}{0.50} = 630$$

Hence milkman can sell 630 litres milk at Rs. 12.25 per litre.

◈ *Question # 17*

Let p denotes population of Pakistan in million and t denotes year after 1961,

Then $(p_1, t_1) = (60, 1961)$ and $(p_2, t_2) = (95, 1981)$

Equation of line by two point form:

$$t - t_1 = \frac{t_2 - t_1}{p_2 - p_1}(p - p_1)$$

$$\Rightarrow\ t - 1961 = \frac{1981-1961}{95-60}(p-60)$$

$$\Rightarrow\ t-1961=\frac{20}{35}(p-60) \quad \Rightarrow\ t-1961=\frac{4}{7}(p-60)$$
$$\Rightarrow\ 7t-13727=4p-240 \quad \Rightarrow\ 7t-13727+240=4p$$
$$\Rightarrow\ 4p=7t-13487 \quad \Rightarrow\ p=\frac{7}{4}t-\frac{13487}{4} \quad \ldots\ldots\ldots\ldots (i)$$

This is the required equation which gives population in term of t.

(*a*) Put $t=1947$ in eq. (i)

$$p=\frac{7}{4}(1947)-\frac{13487}{4} \quad =3407.25-3371.75 \quad =35.5$$

Hence population in 1947 is 35.5 millions.

(*b*) Put $t=1997$ in eq. (i)

$$p=\frac{7}{4}(1997)-\frac{13487}{4}=3494.75-3371.75=123$$

Hence population in 1997 is 123 millions.

◈ *Question # 18*

Let p denotes purchase price of house in millions and t denotes year then

$$(p_1,t_1)=(1,1980) \quad \text{and} \quad (p_2,t_2)=(4,1996)$$

Equation of line by two point form:

$$t-t_1=\frac{t_2-t_1}{p_2-p_1}(p-p_1)$$
$$\Rightarrow\ t-1980=\frac{1996-1980}{4-1}(p-1)$$
$$\Rightarrow\ t-1980=\frac{16}{3}(p-1)$$
$$\Rightarrow\ 3t-5940=16p-16$$
$$\Rightarrow\ 3t-5940+16=16p \quad \Rightarrow\ 16p=3t-5924$$
$$\Rightarrow\ p=\frac{3}{16}t-\frac{5924}{16} \quad \Rightarrow\ p=\frac{3}{16}t-\frac{1481}{4} \quad \ldots\ldots\ldots\ldots (i)$$

> **ALTERNATIVE**
> You may use determinant form of two-point form to find an equation of line.
> $$\begin{vmatrix} p & t & 1 \\ p_1 & t_1 & 1 \\ p_2 & t_2 & 1 \end{vmatrix}=0$$

This is the required equation which gives value of house in term of t.

Put $t=1990$ in eq. (i)

$$p=\frac{3}{16}(1990)-\frac{1481}{4} \quad =373.125-370.25 \quad =2.875$$

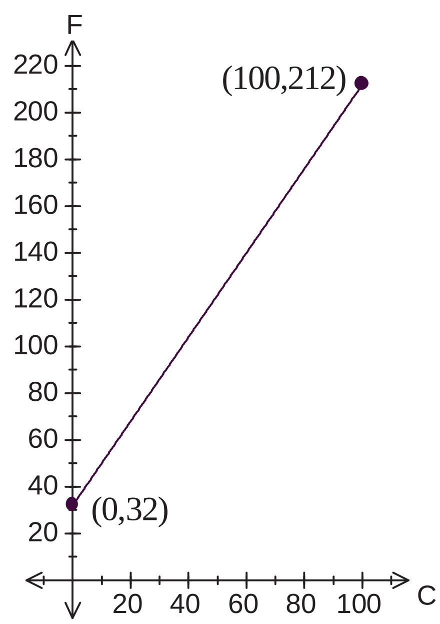
Hence value of house in 1990 is 2.875 millions.

◈ *Question # 19*

Since freezing point of water $=0^\circ C\ =\ 32^\circ F$
and boiling point of water $=100^\circ C\ =\ 212^\circ F$
therefore we have points $(C_1,F_1)=(0,32)$ and $(C_2,F_2)=(100,212)$

Equation of line by two point form

$$F-F_1=\frac{F_2-F_1}{C_2-C_1}(C-C_1)$$
$$\Rightarrow\ F-32=\frac{212-32}{100-0}(C-0)$$
$$\Rightarrow\ F-32=\frac{180}{100}C$$
$$\Rightarrow\ F=\frac{9}{5}C+32$$



Take scale 10ss = 20C and 10ss = 20F on x-axis and y-axis respectively to draw graph.

◈ Question # 20

Let s denotes entry test score and y denotes year.

Then we have $(s_1, y_1) = (592, 1998)$ and $(s_2, y_2) = (564, 2002)$

By two point form of equation of line

$$y - y_1 = \frac{y_2 - y_1}{s_2 - s_1}(s - s_1)$$

$$\Rightarrow\ y - 1998 = \frac{2002 - 1998}{564 - 592}(s - 592) \quad \Rightarrow\ y - 1998 = \frac{4}{-28}(s - 592)$$

$$\Rightarrow\ y - 1998 = -\frac{1}{7}(s - 592) \quad \Rightarrow\ 7y - 13986 = -s + 592$$

$$\Rightarrow\ 7y - 13986 + s - 592 = 0 \quad \Rightarrow\ s + 7y - 14578 = 0$$

Put $y = 2006$ in (i)

$$s + 7(2006) - 14578 = 0 \quad \Rightarrow\ s + 14042 - 14578 = 0$$

$$\Rightarrow\ s - 536 = 0 \quad \Rightarrow\ s = 536$$

Hence in 2006 the average score will be 536.

◈ Question # 21 (a)

(i) - Slope-intercept form

$$\because\ 2x - 4y + 11 = 0$$

$$\Rightarrow\ 4y = 2x + 11 \quad \Rightarrow\ y = \frac{2x + 11}{4}$$

$$\Rightarrow\ y = \frac{1}{2}x + \frac{11}{4}$$

is the intercept form of equation of line with $m = \frac{1}{2}$ and $c = \frac{11}{4}$

(ii) - Two-intercept form

$$\because\ 2x - 4y + 11 = 0 \quad \Rightarrow\ 2x - 4y = -11$$

$$\Rightarrow\ \frac{2}{-11}x - \frac{4}{-11}y = 1 \quad \Rightarrow\ \frac{x}{-11/2} + \frac{y}{11/4} = 1$$

is the two-point form of equation of line with $a = -\frac{11}{2}$ and $b = \frac{11}{4}$.

(iii) - Normal form

$$\because\ 2x - 4y + 11 = 0 \quad \Rightarrow\ 2x - 4y = -11$$

Dividing above equation by $\sqrt{(2)^2 + (-4)^2} = \sqrt{20} = 2\sqrt{5}$

$$\frac{2x}{2\sqrt{5}} - \frac{4y}{2\sqrt{5}} = \frac{-11}{2\sqrt{5}} \quad \Rightarrow\ \frac{x}{\sqrt{5}} - \frac{2y}{\sqrt{5}} = \frac{-11}{2\sqrt{5}}$$

$$\Rightarrow\ -\frac{x}{\sqrt{5}} + \frac{2y}{\sqrt{5}} = \frac{11}{2\sqrt{5}} \qquad \times\text{ing by } -1.$$

Suppose $\cos\alpha = -\frac{1}{\sqrt{5}} < 0$ and $\sin\alpha = \frac{2}{\sqrt{5}} > 0$

$\Rightarrow$ α lies in 2nd quadrant and $\alpha = \cos^{-1}\left(-\frac{1}{\sqrt{5}}\right) = 116.57^\circ$

Hence the normal form is

$$x\cos\left(116.57^\circ\right) + y\sin\left(116.57^\circ\right) = \frac{11}{2\sqrt{5}}$$

And length of perpendicular from $(0,0)$ to line $= p = \frac{11}{2\sqrt{5}}$

◈ Question # 21 (b)

(i) - Slope-intercept form

$$\because\ 4x+7y-2=0$$

$$\Rightarrow\ 7y=-4x+2 \qquad \Rightarrow\ y=\frac{-4x+2}{7}$$

$$\Rightarrow\ y=-\frac{4}{7}x+\frac{2}{7}$$

is the intercept form of equation of line with $m=-\frac{4}{7}$ and $c=\frac{2}{7}$

(ii) - Two-intercept form

$$\because\ 4x+7y-2=0 \qquad \Rightarrow\ 4x+7y=2$$

$$\Rightarrow\ 2x+\frac{7}{2}y=1 \qquad \div\text{ing by } 2$$

$$\Rightarrow\ \frac{x}{1/2}+\frac{y}{2/7}=1$$

is the two-point form of equation of line with $a=\frac{1}{2}$ and $b=\frac{2}{7}$.

(iii) - Normal form

$$\because\ 4x+7y-2=0$$

$$\Rightarrow\ 4x+7y=2$$

Dividing above equation by $\sqrt{(4)^2+(7)^2}=\sqrt{16+49}=\sqrt{65}$

$$\Rightarrow\ \frac{4}{\sqrt{65}}x+\frac{7}{\sqrt{65}}y=\frac{2}{\sqrt{65}} \quad .$$

Suppose $\cos\alpha=\frac{4}{\sqrt{65}}>0$ and $\sin\alpha=\frac{7}{\sqrt{65}}>0$

$\Rightarrow$ α lies in first quadrant and $\alpha=\cos^{-1}\left(\frac{4}{\sqrt{65}}\right)=60.26^\circ$

Hence the normal form is

$$x\cos\left(60.26^\circ\right)+y\sin\left(60.26^\circ\right)=\frac{2}{\sqrt{65}}$$

And length of perpendicular from $(0,0)$ to line $=p=\frac{2}{\sqrt{65}}$

◈ Question # 21 (c)

(i) - Slope-intercept form

$$\because\ 15y-8x+3=0$$

$$\Rightarrow\ 15y=8x-3 \qquad \Rightarrow\ y=\frac{8x-3}{15}$$

$$\Rightarrow\ y=\frac{8}{15}x-\frac{3}{15} \qquad \Rightarrow\ y=\frac{8}{15}x-\frac{1}{5}$$

is the intercept form of equation of line with $m=\frac{8}{15}$ and $c=-\frac{1}{5}$

(ii) - Two-intercept form

$$\because\ 15y-8x+3=0 \qquad \Rightarrow\ -8x+15y=-3$$

$$\Rightarrow\ \frac{8x}{3}-5y=1 \qquad \Rightarrow\ \frac{x}{3/8}+\frac{y}{-1/5}=1$$

is the two-point form of equation of line with $a=\frac{3}{8}$ and $b=-\frac{1}{5}$.

(iii) - Normal form

$$\because\ 15y-8x+3=0$$
$$\Rightarrow\ 8x-15y=3$$

Dividing above equation by $\sqrt{(8)^2+(-15)^2}=\sqrt{64+225}=\sqrt{289}=17$

$$\Rightarrow\ \frac{8}{17}x-\frac{15}{17}y=\frac{3}{17}\quad .$$

Suppose $\cos\alpha=\frac{8}{17}>0$ and $\sin\alpha=-\frac{15}{17}<0$

$\Rightarrow$ α lies in 4^{th} quadrant and $\alpha=\cos^{-1}\left(\frac{8}{17}\right)=298.07^\circ$

$\alpha=\cos^{-1}\left(\frac{8}{17}\right)$
$=61.93^\circ,\ 298.07^\circ$
Taking value that lies in 4^{th} quadrant.

Hence the normal form is

$$x\cos\left(298.07^\circ\right)+y\sin\left(298.07^\circ\right)=\frac{3}{17}$$

And length of perpendicular from $(0,0)$ to line $=p=\frac{3}{17}$

◈ *General equation of the straight line*

A general equation of straight line (General linear equation) in two variable x and y is given by:

$$ax+by+c=0$$

where a, b and c are constants and a and b are not simultaneously zero.

See proof on book at page: 199.

Note: Since $ax+by+c=0\ \Rightarrow\ by=-ax-c\ \Rightarrow\ y=-\frac{a}{b}x-\frac{c}{b}$

Which is an intercept form of equation of line with slope $m=-\frac{a}{b}$ and $c=-\frac{c}{b}$

◈ *Question # 22*

(a) Let $l_1:\ 2x+y-3=0$
$l_2:\ 4x+2y+5=0$

Slope of $l_1=m_1=-\frac{2}{1}=-2$

Slope of $l_2=m_2=-\frac{4}{2}=-2$

Since $m_1=m_2$ therefore l_1 and l_2 are parallel.

(b) Let $l_1:\ 3y=2x+5\ \Rightarrow\ 2x-3y+5=0$
$l_2:\ 3x+2y-8=0$

Slope of $l_1=m_1=-\frac{2}{-3}=\frac{2}{3}$

Slope of $l_2=m_2=-\frac{3}{2}=$

Since $m_1m_2=\left(\frac{2}{3}\right)\left(-\frac{3}{2}\right)=-1\ \Rightarrow\ l_1$ and l_2 are perpendicular.

(c) Let $l_1:\ 4y+2x-1=0\ \Rightarrow\ 2x+4y-1=0$
$l_2:\ x-2y-7=0$

Slope of $l_1=m_1=-\frac{2}{4}=-\frac{1}{2}$

Slope of $l_2 = m_2 = -\frac{1}{-2} = \frac{1}{2}$

Since $m_1 \neq m_2$ and $m_1 m_2 = \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right) = -\frac{1}{4} \neq -1$

$\Rightarrow$ l_1 and l_2 are neither parallel nor perpendicular.

(d) & (e) *Do yourself as above.*

◈ *Question # 23 (a)*

$$l_1:\ 3x - 4y + 3 = 0 \ldots\ldots\ldots\ldots(i)$$
$$l_2:\ 3x - 4y + 7 = 0 \ldots\ldots\ldots\ldots(ii)$$

We first convert l_1 and l_2 in normal form

$$(i) \Rightarrow -3x + 4y = 3$$

Dividing by $\sqrt{(-3)^2 + (4)^2} = \sqrt{25} = 5$

$$-\frac{3}{5}x + \frac{4}{5}y = \frac{3}{5}$$

Let $\cos\alpha = -\frac{3}{5} < 0$ and $\sin\alpha = \frac{4}{5} > 0$

$\Rightarrow$ α lies in 2nd quadrant and $\alpha = \cos^{-1}\left(-\frac{3}{5}\right) = 126.87^\circ$

$$\Rightarrow x\cos(126.87) + y\sin(126.87) = \frac{3}{5}$$

Hence distance of l_1 form origin $= \frac{3}{5}$

Now $(ii) \Rightarrow -3x + 4y = 7$

Dividing by $\sqrt{(-3)^2 + (4)^2} = \sqrt{25} = 5$

$$-\frac{3}{5}x + \frac{4}{5}y = \frac{7}{5}$$

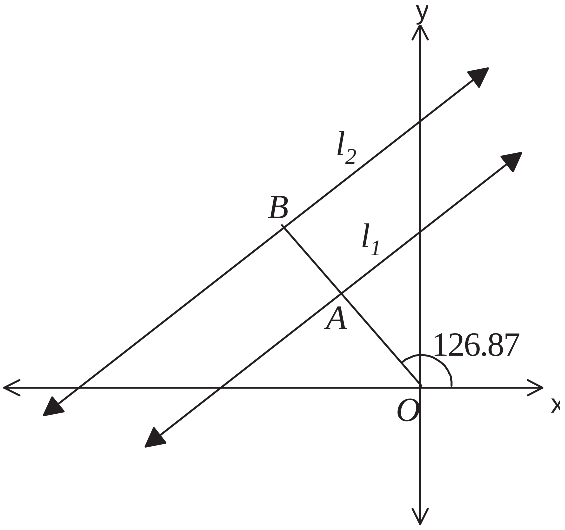
Let $\cos\alpha = -\frac{3}{5} < 0$ and $\sin\alpha = \frac{4}{5} > 0$

$\Rightarrow$ α lies in 1st quadrant

and $\alpha = \cos^{-1}\left(-\frac{3}{5}\right) = 126.87^\circ$

$$\Rightarrow x\cos(126.87) + y\sin(126.87) = 7/5$$

Hence distance of l_2 form origin $= 7/5$



From graph we see that both lines lie on the same side of origin therefore

Distance between lines $= |\overline{AB}| = |\overline{OB}| - |\overline{OA}| = \frac{7}{5} - \frac{3}{5} = \frac{4}{5}$

Let l_3 be a line parallel to l_1 and l_2, and lying midway between them. Then

Distance of l_3 from origin $= |\overline{OA}| + \frac{|\overrightarrow{AB}|}{2} = \frac{3}{5} + \frac{4/5}{2} = \frac{3}{5} + \frac{4}{10} = 1$

Hence equation of l_3

$$x\cos(126.87) + y\sin(126.87) = 1$$

$$\Rightarrow x\left(-\frac{3}{5}\right) + y\left(\frac{4}{5}\right) = 1 \quad \Rightarrow -3x + 4y = 5$$

$$\Rightarrow 3x - 4y + 5 = 0$$

◈ Question # 23 (b)

$$l_1:\ 12x+5y-6=0 \ldots\ldots\ldots\ldots(i)$$
$$l_2:\ 12x+5y+13=0 \ldots\ldots\ldots\ldots(ii)$$

We first convert l_1 and l_2 in normal form

$$(i) \Rightarrow 12x+5y=6$$

Dividing by $\sqrt{(12)^2+(5)^2}=\sqrt{169}=13$

$$\frac{12}{13}x+\frac{5}{13}y=\frac{6}{13}$$

Let $\cos\alpha=\frac{12}{13}>0$ and $\sin\alpha=\frac{5}{13}>0$

$\Rightarrow$ α lies in 1st quadrant and $\alpha=\cos^{-1}\left(\frac{12}{13}\right)=22.62^\circ$

$$\Rightarrow x\cos(22.62)+y\sin(22.62)=\frac{6}{13}$$

Hence distance of l_1 form origin $=\frac{6}{13}$

Now $(ii) \Rightarrow -12x-5y=13$

Dividing by $\sqrt{(12)^2+(5)^2}=\sqrt{169}=13$

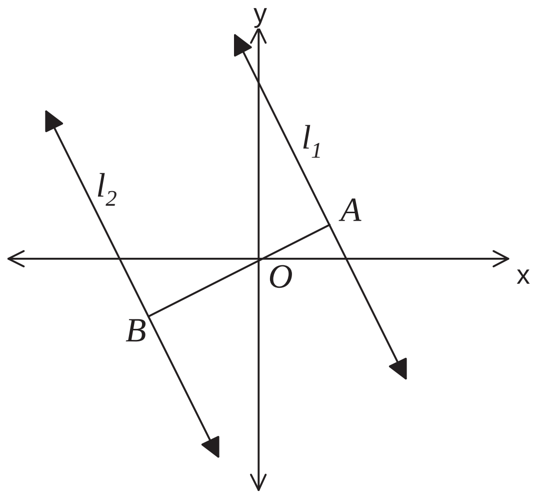
$$-\frac{12}{13}x-\frac{5}{13}y=1$$

Let $\cos\alpha=-\frac{12}{13}<0$ and $\sin\alpha=-\frac{5}{13}<0$

$\Rightarrow$ α lies in 3rd quadrant

and $\alpha=\cos^{-1}\left(-\frac{12}{13}\right)=202.62^\circ$

$$\Rightarrow x\cos(202.62)+y\sin(202.62)=1$$



Hence distance of l_2 form origin $=1$

From graph we see that lines lie on the opposite side of origin therefore

Distance between lines $=\left|\overline{AB}\right|=\left|\overline{OA}\right|+\left|\overline{OB}\right|=\frac{6}{13}+1=\frac{19}{13}$

Let l_3 be a line parallel to l_1 and l_2, and lying midway between them. Then

Distance of l_3 from origin $=\left|\overline{OB}\right|-\frac{\left|\overrightarrow{AB}\right|}{2}=1-\frac{19/13}{2}=1-\frac{19}{26}=\frac{7}{26}$

Hence equation of l_3

$$x\cos(202.62)+y\sin(202.62)=\frac{7}{26}$$

$$\Rightarrow x\left(-\frac{12}{13}\right)+y\left(-\frac{5}{13}\right)=\frac{7}{26} \qquad \Rightarrow -24x-10y=7$$

$$\Rightarrow 24x+10y+7=0$$

◈ Question # 23 (c)

Do yourself as Question # 23 (a)

◈ Question # 24

Let $l:\ 2x-7y+4=0$

REMEMBER
If $l:\ ax+by+c=0$ then slope of $l=-\dfrac{a}{b}$

Slope of $l=m=-\dfrac{2}{-7}=\dfrac{2}{7}$

Since required line is parallel to l

Therefore slope of required line $=m=\dfrac{2}{7}$

Now equation of line having slope $\dfrac{2}{7}$ passing through $(-4,7)$

$$y-7=\frac{2}{7}(x-(-4))$$
$$\Rightarrow\ 7(y-7)=2(x+4)$$
$$\Rightarrow\ 7y-49=2x+8\quad\Rightarrow\ 2x+8-7y+49=0$$
$$\Rightarrow\ 2x-7y+57=0$$

Question # 25

Given: $A(-15,-18)$, $B(10,7)$ and $(5,8)$

Slope of $\overline{AB}\ =\ m\ =\ \dfrac{7-(-8)}{10-(-15)}$

$$=\frac{7+8}{10+15}\ =\ \frac{15}{25}\ =\ \frac{3}{5}$$

Since required line is perpendicular to $\overline{AB}$

Therefore slope of required line $=-\dfrac{1}{m}=-\dfrac{1}{3/5}=-\dfrac{5}{3}$

Now equation of line having slope $-\dfrac{5}{3}$ through $(5,-8)$

$$y-(-8)=-\frac{5}{3}(x-5)$$
$$\Rightarrow\ 3(y+8)=-5(x-5)\quad\Rightarrow\ 3y+24=-5x+25$$
$$\Rightarrow\ 5x-25+3y+24=0\quad\Rightarrow\ 5x+3y-1=0\qquad Ans.$$

Question # 26

Let $l:\ 2x-y+3=0$

Slope of $l\ =\ m\ =\ -\dfrac{2}{-1}\ =\ 2$

Since required line is perpendicular to l

Therefore slope of required line $=\ -\dfrac{1}{m}\ =\ -\dfrac{1}{2}$

Let $y-$intercept of req. line $=c$

Then equation of req. line with slope $-\dfrac{1}{2}$ and $y-$intercept c

$$y=-\frac{1}{2}x+c\ \ \ldots\ldots\ldots\ldots\ (i)$$
$$\Rightarrow\ \frac{1}{2}x+y\ =\ c$$
$$\Rightarrow\ \frac{x}{2c}+\frac{y}{c}\ =\ 1$$

This is two intercept form of equation of line with

$x-$intercept $=2c$ and $y-$intercept $=\ c$

Since product of intercepts $=3$

$$\Rightarrow\ (c)(2c)=3 \quad \Rightarrow\ 2c^2=3 \quad \Rightarrow\ c^2=\frac{3}{2} \quad \Rightarrow\ c=\pm\sqrt{\frac{3}{2}}$$

Putting in (i)

$$\Rightarrow\ y = -\frac{1}{2}x\pm\sqrt{\frac{3}{2}}$$

$$\Rightarrow\ \frac{1}{2}x+y \mp \sqrt{\frac{3}{2}} = 0 \quad \Rightarrow\ \frac{1}{2}x+y \mp \sqrt{\frac{3\times 2}{2\times 2}} = 0$$

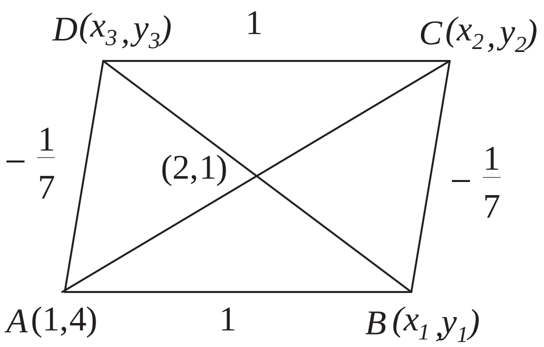
$$\Rightarrow\ \frac{1}{2}x+y \mp \frac{\sqrt{6}}{2} = 0$$

$\Rightarrow\ x+2y \mp \sqrt{6} = 0$ are the required equations.

Question # 27

Let $A(1,4)$ be a given vertex and $B(x_1,y_1), C(x_2,y_2)$ and $D(x_3,y_3)$ are remaining vertices of parallelogram.

Since diagonals of parallelogram bisect at $(2,1)$ therefore



$$(2,1)=\left(\frac{1+x_2}{2},\frac{4+y_2}{2}\right)$$

$$\Rightarrow\ 2=\frac{1+x_2}{2} \quad \text{and} \quad 1=\frac{4+y_2}{2}$$

$$\Rightarrow\ 4=1+x_2 \quad , \quad 2=4+y_2$$

$$\Rightarrow\ x_2=4-1 \quad , \quad y_2=-4+2$$

$$\Rightarrow\ x_2=3 \quad , \quad y_2=-2$$

Hence $C(x_2,y_2) = C(3,-2)$

Now slope of $\overline{AB}=1$

$$\Rightarrow\ \frac{y_1-4}{x_1-1}=1 \quad \Rightarrow\ y_1-4=x_1-1$$

$$\Rightarrow\ x_1-y_1-1+4=0 \quad \Rightarrow\ x_1-y_1+3=0 \ \ldots\ldots\ldots\ldots\ldots\ (i)$$

Also slope of $\overline{BC}=-\frac{1}{7}$

$$\Rightarrow\ \frac{y_2-y_1}{x_2-x_1}=-\frac{1}{7} \quad \Rightarrow\ \frac{-2-y_1}{3-x_1}=-\frac{1}{7}$$

$$\Rightarrow\ -14-7y_1=-3+x_1 \quad \Rightarrow\ -3-x_1+14+7y_1=0$$

$$\Rightarrow\ x_1+7y_1+11=0 \ \ldots\ldots\ldots\ldots(ii)$$

Subtracting (i) and (ii)

$$\begin{array}{r} x_1 - y_1 + 3 = 0 \\ \underset{-}{x_1} \underset{-}{+} 7y_1 \underset{-}{+} 11 = 0 \\ \hline -8y_1-8 = 0 \end{array}$$

$$\Rightarrow\ y_1+1=0 \quad \Rightarrow\ y_1=-1$$

Putting in (i)

$$x_1-(-1)+3 = 0 \quad \Rightarrow\ x_1+4=0 \quad \Rightarrow\ x_1=-4$$

$$\Rightarrow\ B(x_2,y_2) = B(-4,-1)$$

Now E is midpoint of BD

$$\Rightarrow\ (2,1) = \left(\frac{x_1+x_3}{2},\frac{y_1+y_3}{2}\right)$$

$$= \left(\frac{-4+x_3}{2},\frac{-1+y_3}{2}\right)$$

$$\Rightarrow \; 2 = \frac{-4+x_3}{2} \quad , \quad 1 = \frac{-1+y_3}{2}$$
$$\Rightarrow \; 4 = -4+x_3 \quad , \quad 2 = -1+y_3$$
$$\Rightarrow \; x_3 = 8 \quad , \quad y_3 = 3$$
$$\Rightarrow \; D(x_3, y_3) = D(8,3)$$

Hence $(-4,-1)$, $(3,-2)$ and $D(8,3)$ are remaining vertex of $\|_{gram}$.

Position of the point with respect to line (Page 204)

Consider $l: \; ax+by+c = 0$ with $b>0$

Then point $P(x_1, y_1)$ lies

i) above the line l if $ax_1+by_1+c > 0$

ii) below the line l if $ax_1+by_1+c < 0$

Corollary 1 (Page 205)

The point $P(x_1, y_1)$ lies above the line if ax_1+by_1+c and b have the same sign and the point $P(x_1, y_1)$ lies below the line if ax_1+by_1+c and b have opposite signs.

Question # 28

(a) $\quad 2x-3y+6 = 0$

To make coefficient of y positive we multiply above eq. with -1.

$$-2x+3y-6 = 0$$

Putting $(5,8)$ on L.H.S of above

$$-2(5)+3(8)+6 = -10+24-6 = 8 > 0$$

Hence $(5,8)$ lies above the line.

(b) Alternative Method

$$4x+3y-9 = 0$$ **Correction*

Putting $(-7,6)$ in L.H.S of given eq.

$$4(-7)+3(6)-9 = -28+18-9 = -19 \; \ldots\ldots\ldots (i)$$

Since coefficient of y and expression (i) have opposite signs therefore $(-7,6)$ lies below the line.

Question # 29

(a) $\quad 2x-3y+6 = 0$

To make coefficient of y positive we multiply above eq. with -1.

$$-2x+3y-6 = 0 \; \ldots\ldots\ldots (i)$$

Putting $(0,0)$ on L.H.S of (i)

$$-2(0)+3(0)-6 = -6 < 0$$

$\Rightarrow$ $(0,0)$ lies below the line.

Putting $(-4,7)$ on L.H.S of (i)

$$-2(-4)+3(7)-6 = 8+21-6$$
$$= 23 > 0$$

$\Rightarrow$ $(-4,7)$ lies above the line.

Hence $(0,0)$ and $(-4,7)$ lies on the opposite side of line.

(b) $\quad 3x-5y+8 = 0$

To make coefficient of y positive we multiply above eq. with -1.

$$-3x+5y-8 = 0 \; \ldots\ldots\ldots (i)$$

Putting $(2,3)$ on L.H.S of (i)

$$-3(2)+5(3)-8 = -6+15-8$$
$$= 1 > 0$$

$\Rightarrow$ $(2,3)$ lies above the line.

Putting $(-2,3)$ on L.H.S of (i)

$$-3(-2)+5(3)-8 = 6+15-8$$
$$= 13 > 0$$

$\Rightarrow$ $(-2,3)$ lies above the line

Hence $(2,3)$ and $(-2,3)$ lies on the same side of line.

Perpendicular distance of P(x₁,y₁) from line (Page 212)

The distance d from the point $P(x_1,y_1)$ to the line l, where $l:\ ax+by+c = 0$,

is given by: $$d = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

Question # 30

$$l:\ 6x-4y+9 = 0$$

Let d denotes distance of $P(6,-1)$ from line l then

$$d = \frac{|6(6)-4(-1)+9|}{\sqrt{(6)^2+(-4)^2}}$$
$$= \frac{|36+4+9|}{\sqrt{36+16}} = \frac{|49|}{\sqrt{52}} = \frac{49}{2\sqrt{13}}$$

Area of Triangular Region

Let $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ are vertices of triangle then

$$\text{Area of triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

If A, B and C are collinear then $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

### *Question # 31* 		*Do yourself as below (Just find the area)*

Question # 32

Given: $A(2,3)$, $B(-1,1)$, $C(4,-5)$

$$\text{Area of } \Delta ABC = \begin{vmatrix} 2 & 3 & 1 \\ -1 & 1 & 1 \\ 4 & -5 & 1 \end{vmatrix}$$
$$= \frac{1}{2}\big(2(1+5)-3(2-4)+1(5-4)\big)$$
$$= \frac{1}{2}(12+6+1) = \frac{1}{2}(19) = \frac{19}{2} \text{ sq. unit}$$

$\because$ Area of triangle $\neq 0$

$\Rightarrow$ A, B and C are not collinear.
