

EXERCISE 7.2

Question # 1

Given $A(2,5)$, $B(-1,1)$ and $C(2,-6)$

(i) $\overrightarrow{AB} = (-1-2)\hat{i} + (1-5)\hat{j} = -3\hat{i} - 4\hat{j}$

(ii) From above $\overrightarrow{AB} = -3\hat{i} - 4\hat{j}$

Also $\overrightarrow{CB} = (2+1)\hat{i} + (-6-1)\hat{j} = 3\hat{i} - 7\hat{j}$

Now

$$\begin{aligned} 2\overrightarrow{AB} - \overrightarrow{CB} &= 2(-3\hat{i} - 4\hat{j}) - (3\hat{i} - 7\hat{j}) \\ &= -6\hat{i} - 8\hat{j} - 3\hat{i} + 7\hat{j} \\ &= -9\hat{i} - \hat{j} \end{aligned}$$

(iii) Do yourself as above

Question # 2

(i) $\underline{u} = \hat{i} + 2\hat{j} - \hat{k}$

$$\underline{v} = 3\hat{i} - 2\hat{j} + 2\hat{k}$$

$$\underline{w} = 5\hat{i} - \hat{j} + 3\hat{k}$$

$$\begin{aligned} \underline{u} + 2\underline{v} + \underline{w} &= \hat{i} + 2\hat{j} - \hat{k} + 2(3\hat{i} - 2\hat{j} + 2\hat{k}) \\ &\quad + (5\hat{i} - \hat{j} + 3\hat{k}) \\ &= \hat{i} + 2\hat{j} - \hat{k} + 6\hat{i} - 4\hat{j} + 4\hat{k} + 5\hat{i} - \hat{j} + 3\hat{k} \\ &= 12\hat{i} - 3\hat{j} - 6\hat{k} \end{aligned}$$

(ii) Do yourself

(iii)

$$\begin{aligned} 3\underline{v} + \underline{w} &= 3(3\hat{i} - 2\hat{j} + 2\hat{k}) + 5\hat{i} - \hat{j} + 3\hat{k} \\ &= 9\hat{i} - 6\hat{j} + 6\hat{k} + 5\hat{i} - \hat{j} + 3\hat{k} \\ &= 14\hat{i} - 7\hat{j} + 9\hat{k} \end{aligned}$$

Now $|3\underline{v} + \underline{w}| = \sqrt{(14)^2 + (-7)^2 + (9)^2}$
 $= \sqrt{196 + 49 + 81} = \sqrt{326}$

Question # 3

(i) $\underline{v} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$$\begin{aligned} \Rightarrow |\underline{v}| &= \sqrt{(2)^2 + (3)^2 + (4)^2} \\ &= \sqrt{4 + 9 + 16} = \sqrt{29} \end{aligned}$$

Unit vector of $\underline{v} = \hat{v} = \frac{\underline{v}}{|\underline{v}|} = \frac{2\hat{i} + 3\hat{j} + 4\hat{k}}{\sqrt{29}}$
 $= \frac{2}{\sqrt{29}}\hat{i} + \frac{3}{\sqrt{29}}\hat{j} + \frac{4}{\sqrt{29}}\hat{k}$

Hence direction cosines of \underline{v} are

$$\frac{2}{\sqrt{29}}, \frac{3}{\sqrt{29}}, \frac{4}{\sqrt{29}}.$$

(ii) Do yourself as above.

(iii) Do yourself as (i)

Question # 4

Since $|\alpha\hat{i} + (\alpha+1)\hat{j} + 2\hat{k}| = 3$
 $\Rightarrow \sqrt{\alpha^2 + (\alpha+1)^2 + (2)^2} = 3$
 $\Rightarrow \sqrt{\alpha^2 + \alpha^2 + 2\alpha + 1 + 4} = 3$

On squaring both sides

$$\begin{aligned} 2\alpha^2 + 2\alpha + 5 &= 9 \\ \Rightarrow 2\alpha^2 + 2\alpha + 5 - 9 &= 0 \\ \Rightarrow 2\alpha^2 + 2\alpha - 4 &= 0 \\ \Rightarrow \alpha^2 + \alpha - 2 &= 0 \\ \Rightarrow \alpha^2 + 2\alpha - \alpha - 2 &= 0 \\ \Rightarrow \alpha(\alpha+2) - 1(\alpha+2) &= 0 \\ \Rightarrow (\alpha+2)(\alpha-1) &= 0 \\ \Rightarrow \alpha+2 = 0 \quad \text{or} \quad \alpha-1 &= 0 \\ \Rightarrow \alpha = -2 \quad \text{or} \quad \alpha = 1 \end{aligned}$$

Question # 5

Given $\underline{v} = \hat{i} + 2\hat{j} - \hat{k}$

$$\begin{aligned} |\underline{v}| &= \sqrt{(1)^2 + (2)^2 + (-1)^2} \\ &= \sqrt{1 + 4 + 1} = \sqrt{6} \end{aligned}$$

Now

$$\begin{aligned} \hat{v} &= \frac{\underline{v}}{|\underline{v}|} = \frac{\hat{i} + 2\hat{j} - \hat{k}}{\sqrt{6}} \\ &= \frac{1}{\sqrt{6}}\hat{i} + \frac{2}{\sqrt{6}}\hat{j} - \frac{1}{\sqrt{6}}\hat{k} \end{aligned}$$

Question # 6

Given $\underline{a} = 3\hat{i} - \hat{j} - 4\hat{k}$

$$\underline{b} = -2\hat{i} - 4\hat{j} - 3\hat{k}$$

$$\underline{c} = \hat{i} + 2\hat{j} - \hat{k}$$

Suppose that

$$\begin{aligned} \underline{d} &= 3\underline{a} - 2\underline{b} + 4\underline{c} \\ \Rightarrow \underline{d} &= 3(3\hat{i} - \hat{j} - 4\hat{k}) \\ &\quad - 2(-2\hat{i} - 4\hat{j} - 3\hat{k}) \\ &\quad + 4(\hat{i} + 2\hat{j} - \hat{k}) \\ &= 9\hat{i} - 3\hat{j} - 12\hat{k} + 4\hat{i} + 8\hat{j} + 6\hat{k} + 4\hat{i} + 8\hat{j} - 4\hat{k} \\ &= 17\hat{i} - 13\hat{j} - 10\hat{k} \end{aligned}$$

Now

$$|\underline{d}| = \sqrt{(17)^2 + (-13)^2 + (-10)^2}$$

$$= \sqrt{289 + 169 + 100} = \sqrt{558} = 3\sqrt{62}$$

Now

$$\begin{aligned}\underline{\hat{d}} &= \frac{\underline{d}}{|\underline{d}|} = \frac{17\underline{\hat{i}} - 13\underline{\hat{j}} - 10\underline{\hat{k}}}{3\sqrt{62}} \\ &= \frac{17}{3\sqrt{62}}\underline{\hat{i}} - \frac{13}{3\sqrt{62}}\underline{\hat{j}} - \frac{10}{3\sqrt{62}}\underline{\hat{k}}\end{aligned}$$

Question # 7

Consider $\underline{a} = 2\underline{\hat{i}} - 3\underline{\hat{j}} + 6\underline{\hat{k}}$

$$\begin{aligned}|\underline{a}| &= \sqrt{(2)^2 + (-3)^2 + (6)^2} \\ &= \sqrt{4 + 9 + 36} = \sqrt{49} = 7\end{aligned}$$

Now

$$\begin{aligned}\underline{\hat{a}} &= \frac{\underline{a}}{|\underline{a}|} = \frac{2\underline{\hat{i}} - 3\underline{\hat{j}} + 6\underline{\hat{k}}}{7} \\ &= \frac{2}{7}\underline{\hat{i}} - \frac{3}{7}\underline{\hat{j}} + \frac{6}{7}\underline{\hat{k}}\end{aligned}$$

Let \underline{b} be a vector having magnitude 4
i.e. $|\underline{b}| = 4$

Since \underline{b} is parallel to \underline{a}

$$\text{therefore } \underline{\hat{b}} = \underline{\hat{a}} = \frac{2}{7}\underline{\hat{i}} - \frac{3}{7}\underline{\hat{j}} + \frac{6}{7}\underline{\hat{k}}$$

$$\begin{aligned}\text{Now } \underline{b} &= |\underline{b}| \underline{\hat{b}} = 4 \left(\frac{2}{7}\underline{\hat{i}} - \frac{3}{7}\underline{\hat{j}} + \frac{6}{7}\underline{\hat{k}} \right) \\ &= \frac{8}{7}\underline{\hat{i}} - \frac{12}{7}\underline{\hat{j}} + \frac{24}{7}\underline{\hat{k}}\end{aligned}$$

(ii) Do yourself.

Question # 8

Given $\underline{u} = 2\underline{\hat{i}} + 3\underline{\hat{j}} + 4\underline{\hat{k}}$

$$\underline{v} = -\underline{\hat{i}} + 3\underline{\hat{j}} - \underline{\hat{k}}$$

$$\underline{w} = \underline{\hat{i}} + 6\underline{\hat{j}} + z\underline{\hat{k}}$$

Since \underline{u} , \underline{v} and \underline{w} are sides of triangle
therefore

$$\underline{u} + \underline{v} = \underline{w}$$

$$\Rightarrow 2\underline{\hat{i}} + 3\underline{\hat{j}} + 4\underline{\hat{k}} - \underline{\hat{i}} + 3\underline{\hat{j}} - \underline{\hat{k}} = \underline{\hat{i}} + 6\underline{\hat{j}} + z\underline{\hat{k}}$$

$$\Rightarrow \underline{\hat{i}} + 6\underline{\hat{j}} + 3\underline{\hat{k}} = \underline{\hat{i}} + 6\underline{\hat{j}} + z\underline{\hat{k}}$$

Equating coefficient of $\underline{\hat{k}}$ only, we have

$$3 = z \text{ i.e. } \boxed{z = 3}$$

Question # 9

Position vector (p.v) of point $A = 2\underline{\hat{i}} - \underline{\hat{j}} + \underline{\hat{k}}$

p.v of point $B = 3\underline{\hat{i}} + \underline{\hat{j}}$

p.v. of point $C = 2\underline{\hat{i}} + 4\underline{\hat{j}} - 2\underline{\hat{k}}$

p.v. of point $D = -\underline{\hat{i}} - 2\underline{\hat{j}} + \underline{\hat{k}}$

\overrightarrow{AB} = p.v. of B - p.v. of A

$$= 3\underline{\hat{i}} + \underline{\hat{j}} - 2\underline{\hat{i}} - \underline{\hat{j}} - \underline{\hat{k}} = \underline{\hat{i}} + 2\underline{\hat{j}} - \underline{\hat{k}}$$

$$\overrightarrow{CD} = \text{p.v. of } D - \text{p.v. of } C$$

$$= -\underline{\hat{i}} - 2\underline{\hat{j}} + \underline{\hat{k}} - 2\underline{\hat{i}} - 4\underline{\hat{j}} + 2\underline{\hat{k}}$$

$$= -3\underline{\hat{i}} - 6\underline{\hat{j}} + 3\underline{\hat{k}}$$

$$= -3(\underline{\hat{i}} + 2\underline{\hat{j}} - \underline{\hat{k}}) = -3\overrightarrow{AB}$$

$$\text{i.e. } \overrightarrow{CD} = \lambda \overrightarrow{AB} \text{ where } \lambda = -3$$

Hence \overrightarrow{AB} and \overrightarrow{CD} are parallel.

Question # 10 (i)

$$\underline{v} = 2\underline{\hat{i}} - 4\underline{\hat{j}} + 4\underline{\hat{k}}$$

$$\begin{aligned}|\underline{v}| &= \sqrt{(2)^2 + (-4)^2 + (4)^2} \\ &= \sqrt{4 + 16 + 16} = \sqrt{36} = 6\end{aligned}$$

$$\begin{aligned}\text{Now } \underline{\hat{v}} &= \frac{\underline{v}}{|\underline{v}|} = \frac{2\underline{\hat{i}} - 4\underline{\hat{j}} + 4\underline{\hat{k}}}{6} \\ &= \frac{1}{3}\underline{\hat{i}} - \frac{2}{3}\underline{\hat{j}} + \frac{2}{3}\underline{\hat{k}}\end{aligned}$$

The two vectors of length 2 and parallel to \underline{v} are $2\underline{\hat{v}}$ and $-2\underline{\hat{v}}$.

$$2\underline{\hat{v}} = 2 \left(\frac{1}{3}\underline{\hat{i}} - \frac{2}{3}\underline{\hat{j}} + \frac{2}{3}\underline{\hat{k}} \right) = \frac{2}{3}\underline{\hat{i}} - \frac{4}{3}\underline{\hat{j}} + \frac{4}{3}\underline{\hat{k}}$$

$$-2\underline{\hat{v}} = -2 \left(\frac{1}{3}\underline{\hat{i}} - \frac{2}{3}\underline{\hat{j}} + \frac{2}{3}\underline{\hat{k}} \right) = -\frac{2}{3}\underline{\hat{i}} + \frac{4}{3}\underline{\hat{j}} - \frac{4}{3}\underline{\hat{k}}$$

Question # 10 (ii)

Given $\underline{v} = \underline{\hat{i}} - 3\underline{\hat{j}} + 4\underline{\hat{k}}$, $\underline{w} = a\underline{\hat{i}} + 9\underline{\hat{j}} - 12\underline{\hat{k}}$

Since \underline{v} and \underline{w} are parallel therefore there exists $\lambda \in \mathbb{R}$ such that

$$\underline{v} = \lambda \underline{w}$$

$$\Rightarrow \underline{\hat{i}} - 3\underline{\hat{j}} + 4\underline{\hat{k}} = \lambda (a\underline{\hat{i}} + 9\underline{\hat{j}} - 12\underline{\hat{k}})$$

$$\Rightarrow \underline{\hat{i}} - 3\underline{\hat{j}} + 4\underline{\hat{k}} = a\lambda \underline{\hat{i}} + 9\lambda \underline{\hat{j}} - 12\lambda \underline{\hat{k}}$$

Comparing coefficients of $\underline{\hat{i}}$, $\underline{\hat{j}}$ and $\underline{\hat{k}}$

$$1 = a\lambda \dots\dots\dots(i)$$

$$-3 = 9\lambda \dots\dots\dots(ii)$$

$$4 = -12\lambda \dots\dots\dots(iii)$$

$$\text{From (ii) } \lambda = -\frac{3}{9} \Rightarrow \lambda = -\frac{1}{3}$$

Putting in equation (i)

$$1 = a \left(-\frac{1}{3} \right) \Rightarrow -3 = a \text{ i.e. } \boxed{a = -3}$$

Question # 10 (c)

Consider $\underline{v} = \underline{\hat{i}} - 2\underline{\hat{j}} + 3\underline{\hat{k}}$

$$\begin{aligned}|\underline{v}| &= \sqrt{(1)^2 + (-2)^2 + (3)^2} \\ &= \sqrt{1 + 4 + 9} = \sqrt{14}\end{aligned}$$

Now

$$\hat{v} = \frac{\underline{v}}{|\underline{v}|} = \frac{\hat{i} - 2\hat{j} + 3\hat{k}}{\sqrt{14}}$$

$$= \frac{1}{\sqrt{14}}\hat{i} - \frac{2}{\sqrt{14}}\hat{j} + \frac{3}{\sqrt{14}}\hat{k}$$

Let \underline{a} be a vector having magnitude 5 i.e. $|\underline{a}| = 5$

Since \underline{a} is parallel to \underline{v} but opposite in direction, therefore

$$\hat{a} = -\hat{v} = -\frac{1}{\sqrt{14}}\hat{i} + \frac{2}{\sqrt{14}}\hat{j} - \frac{3}{\sqrt{14}}\hat{k}$$

Now

$$\underline{a} = |\underline{a}|\hat{a} = 5\left(-\frac{1}{\sqrt{14}}\hat{i} + \frac{2}{\sqrt{14}}\hat{j} - \frac{3}{\sqrt{14}}\hat{k}\right)$$

$$= -\frac{5}{\sqrt{14}}\hat{i} + \frac{5}{\sqrt{14}}\hat{j} - \frac{5}{\sqrt{14}}\hat{k}$$

Question # 10 (d)

Suppose that $\underline{v} = 3\hat{i} - \hat{j} + 4\hat{k}$ and

$$\underline{w} = a\hat{i} + b\hat{j} - 2\hat{k}$$

$\therefore \underline{v}$ and \underline{w} are parallel

\therefore there exists $\lambda \in \mathbb{R}$ such that

$$\underline{v} = \lambda \underline{w}$$

$$\Rightarrow 3\hat{i} - \hat{j} + 4\hat{k} = \lambda(a\hat{i} + b\hat{j} - 2\hat{k})$$

$$\Rightarrow 3\hat{i} - \hat{j} + 4\hat{k} = a\lambda\hat{i} + b\lambda\hat{j} - 2\lambda\hat{k}$$

Comparing coefficients of \hat{i} , \hat{j} and \hat{k}

$$3 = a\lambda \dots\dots\dots(i)$$

$$-1 = b\lambda \dots\dots\dots(ii)$$

$$4 = -2\lambda \dots\dots\dots(iii)$$

From equation (iii)

$$-\frac{4}{2} = \lambda \Rightarrow \lambda = -2$$

Putting value of λ in equation (i)

$$3 = a(-2) \Rightarrow \boxed{a = -\frac{3}{2}}$$

Putting value of λ in equation (ii)

$$-1 = b(-2) \Rightarrow \boxed{b = \frac{1}{2}}$$

Question # 11 (i)

$$\underline{v} = 3\hat{i} - \hat{j} + 2\hat{k}$$

$$|\underline{v}| = \sqrt{(3)^2 + (-1)^2 + (2)^2}$$

$$= \sqrt{9+1+4} = \sqrt{14}$$

Let \hat{v} be unit vector along \underline{v} . Then

$$\hat{v} = \frac{\underline{v}}{|\underline{v}|} = \frac{3\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{14}}$$

$$= \frac{3}{\sqrt{14}}\hat{i} - \frac{1}{\sqrt{14}}\hat{j} + \frac{2}{\sqrt{14}}\hat{k}$$

$$\hat{v} = \left[\frac{3}{\sqrt{14}}, -\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}} \right]$$

Hence the direction cosines of \underline{v} are

$$\frac{3}{\sqrt{14}}, -\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}.$$

Question # 11 (ii)

$$\underline{v} = 6\hat{i} - 2\hat{j} + \hat{k}$$

$$|\underline{v}| = \sqrt{(6)^2 + (-2)^2 + (1)^2}$$

$$= \sqrt{36+4+1} = \sqrt{41}$$

Let \hat{v} be unit vector along \underline{v} . Then

$$\hat{v} = \frac{\underline{v}}{|\underline{v}|} = \frac{6\hat{i} - 2\hat{j} + \hat{k}}{\sqrt{41}}$$

$$= \frac{6}{\sqrt{41}}\hat{i} - \frac{2}{\sqrt{41}}\hat{j} + \frac{1}{\sqrt{41}}\hat{k}$$

$$\hat{v} = \left[\frac{6}{\sqrt{41}}, -\frac{2}{\sqrt{41}}, \frac{1}{\sqrt{41}} \right]$$

Hence the direction cosines of \underline{v} are

$$\frac{6}{\sqrt{41}}, -\frac{2}{\sqrt{41}}, \frac{1}{\sqrt{41}}.$$

Question # 11 (iii)

$$P = (2, 1, 5), Q = (1, 3, 1)$$

$$\overrightarrow{PQ} = (1-2)\hat{i} + (3-1)\hat{j} + (1-5)\hat{k}$$

$$= -\hat{i} + 2\hat{j} - 4\hat{k}$$

$$|\overrightarrow{PQ}| = \sqrt{(-1)^2 + (2)^2 + (-4)^2}$$

$$= \sqrt{1+4+16} = \sqrt{21}$$

Let \hat{v} be unit vector along \overrightarrow{PQ} . Then

$$\hat{v} = \frac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|} = \frac{-\hat{i} + 2\hat{j} - 4\hat{k}}{\sqrt{21}}$$

$$= \frac{-1}{\sqrt{21}}\hat{i} + \frac{2}{\sqrt{21}}\hat{j} - \frac{4}{\sqrt{21}}\hat{k}$$

$$\hat{v} = \left[\frac{-1}{\sqrt{21}}, \frac{2}{\sqrt{21}}, -\frac{4}{\sqrt{21}} \right]$$

Hence the direction cosines of \overrightarrow{PQ} are

$$\frac{-1}{\sqrt{21}}, \frac{2}{\sqrt{21}}, -\frac{4}{\sqrt{21}}.$$

Question # 12(i)

$45^\circ, 45^\circ, 60^\circ$ will be direction angles of the vectors if

$$\cos^2 45^\circ + \cos^2 45^\circ + \cos^2 60^\circ = 1$$

$$\text{L.H.S} = \cos^2 45^\circ + \cos^2 45^\circ + \cos^2 60^\circ$$

$$= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{1}{4} = \frac{5}{4} \neq \text{R.H.S}$$

Therefore given angles are not direction angles.

Question # 12(ii)

$30^\circ, 45^\circ, 60^\circ$ will be direction angles of the vectors if

$$\cos^2 30^\circ + \cos^2 45^\circ + \cos^2 60^\circ = 1$$

$$\text{L.H.S} = \cos^2 30^\circ + \cos^2 45^\circ + \cos^2 60^\circ$$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$= \frac{3}{4} + \frac{1}{2} + \frac{1}{4} = \frac{3}{2} \neq \text{R.H.S}$$

Therefore given angles are not direction angles.

Question # 12 (iii)

$30^\circ, 60^\circ, 60^\circ$ will be direction angles of the vectors if

$$\cos^2 45^\circ + \cos^2 60^\circ + \cos^2 60^\circ = 1$$

$$\text{L.H.S} = \cos^2 45^\circ + \cos^2 60^\circ + \cos^2 60^\circ$$

$$= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$= \frac{1}{2} + \frac{1}{4} + \frac{1}{4}$$

$$= 1 = \text{R.H.S}$$

Therefore given angles are direction angles.
