حل مشق 6.5

1. حل کیجیے۔

(i) $\dfrac{\log 81}{\log 9}$

حل:

$$\frac{\log 81}{\log 9} = \frac{\log 3^4}{\log 3^2} = \frac{4\log 3}{2\log 3} = \frac{4}{2} = 2$$

(ii) $\dfrac{\log 36}{\log 6}$

حل:

$$\frac{\log 36}{\log 6} = \frac{\log 6^2}{\log 6} = \frac{2\log 6}{\log 6} = 2$$

(iii) $\dfrac{\log 243}{\log 9}$

حل:

$$\frac{\log 243}{\log 9} = \frac{\log 3^5}{\log 3^2} = \frac{5\log 3}{2\log 3} = \frac{5}{2}$$

حل کیجیے۔

(i) $\log 5 + \log 4 + \log 3 - \log 6$

حل:

$$\log 5 + \log 4 + \log 3 - \log 6$$
$$= \log(5 \times 4 \times 3) - \log 6$$
$$= \log \frac{(5 \times 4 \times 3)}{6}$$
$$= \log\left(\frac{60}{6}\right)$$
$$= \log(10) \quad = 1$$

(ii) $\log 5 + \log 20 + \log 24 + \log 25 - \log 60$

حل:

$$\log 5 + \log 20 + \log 24 + \log 25 - \log 60$$
$$= \log(5 \times 20 \times 24 \times 25) - \log 60$$
$$= \log \frac{(5 \times 20 \times 24 \times 25)}{60}$$
$$= \log \frac{(60000)}{60}$$
$$= \log(1000) = 3$$

(iii) $2\log 3 + 3\log 4 + 4\log 5 - 2\log 6$

حل:

$$2\log 3 + 3\log 4 + 4\log 5 - 2\log 6$$
$$= \log 3^2 + \log 4^3 + \log 5^4 - \log 6^2$$
$$= \log 9 + \log 64 + \log 625 - \log 36$$
$$= \log(9 \times 64 \times 625) - \log 36$$
$$= \log \frac{(9 \times 64 \times 625)}{36}$$
$$= \log\left(\frac{360000}{36}\right)$$
$$= \log(10000) = 4$$

(iv) $2\log 5 + \log 8 - \frac{1}{2}\log 4$

حل:

$$2\log 5 + \log 8 - \frac{1}{2}\log 4$$
$$= \log 5^2 + \log 8 - \log 4^{\frac{1}{2}}$$
$$= \log 25 + \log 8 - \log\left(2^2\right)^{\frac{1}{2}}$$

$= \log 25 + \log 8 - \log 2$

$= \log(25 \times 8) - \log 2$

$= \log\left(\frac{25 \times 8}{2}\right)$

$= \log(100) = 2$

(v) log 200 + log 5

حل:

$\log 200 + \log 5$

$= \log(200 \times 5)$

$= \log(1000) = 3$

3. لوگارتھم کا جدول استعمال کیے بغیر حل کیجیے۔

(i) log 1.3472 + log 22.79 − log 5

حل:

$\log 1.3472 + \log 22.79 - \log 5$

$= \log(1.3472 \times 22.79) - \log 5$

$= \log\left(\frac{1.3472 \times 22.79}{5}\right)$

(ii) log 22.13 + log 0.354 + log 7 − log 3

حل:

$\log 22.13 + \log 0.354 + \log 7 - \log 3$

$= \log(22.13 \times 0.354 \times 7) - \log 3$

$= \log\left(\frac{22.13 \times 0.354 \times 7}{3}\right)$

(iii) log 57.86 + log 4.385 − log 2.391 − log 3.072

حل:

$\log 57.86 + \log 4.385 - \log 2.391 - \log 3.072$

$= \log(57.86 \times 4.385) - \log 2.391 - \log 3.072$

$= \log\left(\frac{57.86 \times 4.385}{3.072 \times 2.391}\right)$

4. لوگارتھم کے جدول کی مدد سے حل کیجیے۔

(i) $\frac{2.38 \times 3.901}{4.83}$

حل:

Let $x = \frac{2.38 \times 3.901}{4.83}$

$\log x = \log\left(\frac{2.38 \times 3.901}{4.83}\right)$

$\log x = \log(2.38 \times 3.901) - \log(4.83)$

$\log x = \log 2.38 + \log 3.901 - \log 4.83$

$\log x = 0.3766 + 0.5912 - 0.6839$

$\log x = 0.2839$

$x = \text{Antilog}\,(0.2839)$

$x = 1.923$

(ii) $\dfrac{\mathbf{8.67 \times 3.94}}{\mathbf{1.78}}$

حل:

$$\text{Let } x = \frac{8.67 \times 3.94}{1.78}$$

$$\log x = \log\left(\frac{8.67 \times 3.94}{1.78}\right)$$

$$\log x = \log(8.67 \times 3.94) - \log(1.78)$$

$$\log x = \log 8.67 + \log 3.94 - \log 1.78$$

$$\log x = 0.9380 + 0.5955 - 0.2504$$

$$\log x = 1.2831$$

$x = \text{Antilog}\,(1.2831)$

$x = 19.19$

(iii) $\dfrac{\mathbf{25.36 \times 3.4569}}{\mathbf{9.87 \times 8.93}}$

حل:

$$\text{Let } x = \frac{25.36 \times 3.4569}{9.87 \times 8.93}$$

$$\log x = \log \frac{25.36 \times 3.4569}{9.87 \times 8.93}$$

$$\log x = \log(25.36 \times 3.4569) - \log(9.87 \times 8.93)$$

$$\log x = \log 25.36 + \log 3.4569 - \log 9.87 - \log 8.93$$

$$\log x = 1.4041 + 0.53887 - 0.9943 - 0.9509$$

$$\log x = -0.0024$$

$$x = Anti\log(-0.0024)$$

$$x = 0.9945$$

5۔ ثابت کیجیے۔

(i) $\mathbf{\log\left(\dfrac{a^2}{bc}\right) + \log\left(\dfrac{b^2}{ca}\right) + \log\left(\dfrac{c^2}{ab}\right) = 0}$

حل:

$$\text{L.H.S.} = \log\left(\frac{a^2}{bc}\right) + \log\left(\frac{b^2}{ca}\right) + \log\left(\frac{c^2}{ab}\right) = 0$$

$$= \log a^2 - \log(bc) + \log b^2 - \log(ca) + \log c^2 - \log(ab)$$

$$= 2\log a - (\log b + \log c) + 2\log b - (\log c + \log a) + 2\log c - (\log a + \log b)$$

$$= 2\log a - \log b - \log c + 2\log b - \log c - \log a + 2\log c - \log a - \log b$$

$$= 2\log a - \log a - \log a + 2\log b - \log b - \log b + 2\log c - \log c - \log c$$

$$= 2\log a - 2\log a + 2\log b - 2\log b + 2\log c - 2\log c$$

$= 0$

$\therefore$ R.H.S.

(ii) $\mathbf{3\log 2 + 2\log 3 + \log 5 = \log 360}$

حل:

L.H.S.$= 3\log 2 + 2\log 3 + \log 5 = \log 360$

$= \log 2^3 + \log 3^2 + \log 5$

$= \log 8 + \log 9 + \log 5$

$= \log(8 \times 9 \times 5)$

$= \log 360 \quad = \text{R.H.S.}$

(iii) $\mathbf{5\log 3 - \log 9 = \log 27}$

حل:

$L.H.S. = 5\log 3 - \log 9$

$= \log 3^5 - \log 9$

$= \log 243 - \log 9$

$= \log\left(\frac{243}{9}\right)$

$= \log 27 \quad = \text{R.H.S.}$

(iv) $\mathbf{\log\left(\frac{75}{16}\right) + \log\left(\frac{32}{243}\right) - 2\log\left(\frac{5}{9}\right) = \log 2}$

$L.H.S. = \log\left(\frac{75}{16}\right) + \log\left(\frac{32}{243}\right) - 2\log\left(\frac{5}{9}\right)$

$= \log\left(\frac{75}{16} \times \frac{32}{243}\right) - \log\left(\frac{5}{9}\right)^2$

$= \log\left(\frac{25}{1} \times \frac{2}{81}\right) - \log\left(\frac{5^2}{9^2}\right)$

$= \log\left(\frac{5^2 \times 2}{9^2}\right) - \log\left(\frac{5^2}{9^2}\right)$

$= \left(\log 5^2 + \log 2 - \log 9^2\right) - \left(\log 5^2 - \log 9^2\right)$

$= \log 5^2 + \log 2 - \log 9^2 - \log 5^2 + \log 9^2$

$= \log 5^2 - \log 5^2 + \log 9^2 - \log 9^2 + \log 2$

$= \log 2$

$\therefore$ R.H.S.

(v) $\mathbf{2\log\left(\frac{11}{13}\right) + \log\left(\frac{130}{77}\right) - \log\left(\frac{55}{91}\right) = \log 2}$

حل: رقم غلط ہے۔

L.H.S. $= 2\log\left(\frac{11}{13}\right) + \log\left(\frac{130}{77}\right) - \log\left(\frac{55}{91}\right)$

$$= \log\left(\frac{11}{13}\right)^2 + \log\left(\frac{130}{77}\right) - \log\left(\frac{55}{91}\right)$$

$$= \log\left(\frac{11^2}{13^2}\right) + \log\left(\frac{130}{77}\right) - \log\left(\frac{55}{91}\right)$$

$$= \log\left(\frac{121}{169}\right) + \log\left(\frac{130}{77}\right) - \log\left(\frac{55}{91}\right)$$

$$= \log\left(\frac{121}{169} \times \frac{130}{77}\right) - \log\left(\frac{55}{91}\right)$$

$$= \log\left(\frac{11}{169} \times \frac{130}{7}\right) - \log\left(\frac{55}{91}\right)$$

$$= \log\left(\frac{\frac{11}{169} \times \frac{130}{7}}{\frac{55}{91}}\right)$$

$$= \log\left(\frac{11}{169} \times \frac{130}{7} \times \frac{91}{55}\right)$$

$$= \log\left(\frac{1}{169} \times \frac{130}{1} \times \frac{13}{5}\right)$$

$$= \log\left(\frac{1}{169} \times \frac{26}{1} \times \frac{13}{1}\right)$$

$$= \log\left(\frac{26}{13}\right) = \log 2 = \text{R.H.S}$$

ثابت کیجیے۔

$3\log 4 + 2\log 5 - \frac{1}{3}\log 64 - \frac{1}{2}\log 16 = 2$

L.H.S. $= 3\log 4 + 2\log 5 - \frac{1}{3}\log 64 - \frac{1}{2}\log 16$

$$= 3\log 4 + \log 5^2 - \log 64^{\frac{1}{3}} - \log 16^{\frac{1}{2}}$$

$$= 3\log 4 + \log 25 - \log\left(4^3\right)^{\frac{1}{3}} - \log\left(4^2\right)^{\frac{1}{2}}$$

$$= 3\log 4 + \log 25 - \log 4 - \log 4$$

$$= 3\log 4 + \log 25 - 2\log 4$$

$$= \log 4 + \log 25$$

$$= \log(4 \times 25)$$

$$= \log(100) = 2$$

$$= \text{R. H. S.}$$

7. ثابت کیجیے۔

$\log(1\times2\times3) = \log 1 + \log 2 + \log 3$

حل:

$$\text{L.H.S.} = \log(1\times2\times3)$$
$$= \log 1 + \log 2 + \log 3 \quad \because \log mn = \log m + \log n$$
$$= \text{R.H.S.}$$

8. لوگارتھم کا جدول استعمال کیجیے اور درج ذیل کو حل کیجیے۔

(i) $69.13 \times 0.34 \times 0.014$

حل: فرض کیا:

$$x = 69.13\times0.34\times0.014$$
$$\log x = \log(69.13\times0.34\times0.014)$$
$$\log x = \log 69.13 + \log 0.34 + \log 0.014$$
$$\log x = 1.8397 + \bar{1}.5315 + \bar{2}.1461$$
$$\log x = 1.8399 - 1 + 0.5315 - 2 + 0.1461$$
$$\log x = \bar{0}.4825$$
$$\log x = -1 + 0.4825$$
$$\log x = -0.5175$$
$$x = \text{Antilog } (-0.5175)$$ (ضد لوگارتھم کے جدول سے)
$$x = 0.3292$$

(ii) $\dfrac{8.67\times3.94}{1.78}$

حل: فرض کیا:

$$x = \frac{8.67\times3.94}{1.78}$$
$$\log x = \log\frac{8.67\times3.94}{1.78}$$
$$\log x = \log 8.67 + \log 3.94 - \log 1.78$$
$$\log x = 0.9380 + 0.5955 - 0.2504$$
$$\log x = (1.2831)$$
$$x = 19.19$$

(iii) $\dfrac{4}{3}\times3.142\times(1.5)^3$

حل: فرض کیا:

$$x = \frac{4}{3}\times3.142\times(1.5)^3$$
$$\log x = \log\left[\frac{4}{3}\times3.142\times(1.5)^3\right]$$
$$\log x = \log\left(\frac{4}{3}\right) + \log(3.142) + \log(1.5)^3$$
$$\log x = \log(4) - \log(3) + \log 3.142 + 3\log(1.5)$$
$$\log x = 0.6021 - 0.4771 + 0.4972 + 3(0.1761)$$

$$\log x = 0.6021 - 0.4771 + 0.4972 + 0.5283$$
$$\log x = 1.1505$$
$$x = Anti\log(1.1505)$$
$$x = 14.15$$

(iv) $\dfrac{(25.36)^2 \times (0.4569)}{847.5}$

حل: فرض کیا:

$$x = \frac{(25.36)^2 \times (0.4569)}{847.5}$$
$$\log x = \log \frac{(25.36)^2 \times (0.4569)}{847.5}$$
$$\log x = \log(25.36)^2 + \log\{0.4569) - \log 847.5$$
$$\log x = 2\log(25.36) + \log(0.4569) - \log(847.5)$$
$$\log x = 2(1.4041) + \bar{1}.6599 - 2.9281$$
$$\log x = 2.8082 - 1 + 0.6599 - 2.9281$$
$$\log x = -0.46$$
$$\log x = -0.46 + 1 - 1$$
$$\log x = 0.54 + \bar{1}$$
$$\log x = \bar{1}.54$$
$$x = Anti\log(\bar{1}.54)$$ (ضدلوگارتھم کے جدول سے)
$$x = 0.3467$$

(v) $\dfrac{0.9876 \times (16.42)^2}{(4.567)^{\frac{1}{3}}}$

حل: فرض کیا:

$$x = \frac{0.9876 \times (16.42)^2}{(4.567)^{\frac{1}{3}}}$$
$$\log x = \log\left(\frac{0.9876 \times (16.42)^2}{(4.567)^{\frac{1}{3}}}\right)$$
$$\log x = \log(0.9876) + \log(16.42)^2 - \log(4.567)^{\frac{1}{3}}$$
$$\log x = \log(0.9876) + 2\log(1642) - \frac{1}{3}\log(4.567)$$
$$\log x = \bar{1}.9946 + 2(1.2154) - \frac{1}{3}(0.6596)$$
$$\log x = -1 + 0.9946 + 2.4308 - 0.2199$$
$$\log x = 2.2055$$

x = Antilog (2.055)

x = 160.5

(vi) $\sqrt{\dfrac{3\sqrt{\mathbf{0.0125}}\times\sqrt{\mathbf{31.15}}}{\mathbf{0.00081}}}$

حل:

$x=\sqrt{\dfrac{3\sqrt{0.0125}\times\sqrt{31.15}}{0.00081}}$ فرض کریں

$\log x=\log\sqrt{\dfrac{3\sqrt{0.0125}\times\sqrt{31.15}}{0.00081}}$ اطراف کا log لینے سے

$$\log x=\log\left[\frac{3(0.0125)^{1/2}\times(31.15)^{1/2}}{0.00081}\right]^{1/2}$$

$$=\frac{1}{2}\left[\log 3+\frac{1}{2}\log 0.0125+\frac{1}{2}\log(31.15)-\log(0.0008\right]$$

$$=\frac{1}{2}\left[0.4771+\frac{1}{2}(\bar{2}.0969)+\frac{1}{2}(1.4935)-\bar{4}.9085\right]$$

$$\log x=\frac{1}{2}\left[0.47771+\frac{1}{2}(-2+0.969)+0.7468-(-4+0.908)\right]$$

$$2\frac{1}{2}[0.4771-1+0.0485+0.7468+4-0.9085]$$

$$=\frac{1}{2}[3.3639]$$

$\log x$ = 1.68195

x = Antilog (1.68195)

x = 48.07 Ans

(vii) $\dfrac{(\mathbf{6.45})^3\times(\mathbf{0.00034})^{\frac{1}{3}}\times(\mathbf{981.9})}{(\mathbf{9.37})^2\times(\mathbf{8.93})^{\frac{1}{4}}\times(\mathbf{0.0617})}$

$x=\dfrac{(6.45)^3\times(0.00034)^{\frac{1}{3}}\times(981.9)}{(9.37)^2\times(8.93)^{\frac{1}{4}}\times(0.0617)}$ فرض کیا

$$\log x=\log\left[\frac{(6.45)^3\times(0.00034)^{\frac{1}{3}}\times(981.9)}{(9.37)^2\times(8.93)^{\frac{1}{4}}\times(0.0617)}\right]$$

$$\log x = \log(6.45)^3 + \log(0.00034)^{\frac{1}{3}} + \log(981.9) - \log(9.37)^2$$
$$-\log(8.93)^{\frac{1}{4}} - \log(0.0617)$$

$$\log x = 3\log(6.45) + \frac{1}{3}\log(0.00034) + \log(981.9)$$
$$-2\log(9.37) - \frac{1}{4}\log(8.93) - \log(0.0617)$$

$$\log x = 3(0.8096) + \frac{1}{3}(\bar{4}.5315) + 2.9921 - 2(0.9717) - \frac{1}{4}(0.9509) - \bar{2}.790$$

$$\log x = 2.4287 + \frac{1}{3}(-4 + 0.5315) + 2.9921 - 1.9435 - 0.2377 - (-2 + 0.7903)$$

$$\log x = 2.4287 + \frac{1}{3}(-3.4685) + 2.9921 - 1.9435 - 0.2377 - (-1.2097)$$

$$\log x = 2.4287 - 1.1562 + 2.9921 - 1.9435 - 0.2377 + 1.2097$$

$$\log x = 3.2931$$

x = Antilog (3.2931)

x = 1963

(viii) $\dfrac{(\mathbf{0.0437})^{\frac{2}{3}} \times (\mathbf{1.407})^2}{(\mathbf{0.0015})^{\frac{1}{3}} \times (\mathbf{1.235})^{\frac{1}{7}}}$

حل:

Let $x = \dfrac{(0.0437)^{\frac{2}{3}} \times (1.407)^2}{(0.0015)^{\frac{1}{3}} \times (1.235)^{\frac{1}{7}}}$

$$\log x = \log\left[\frac{(0.0437)^{\frac{2}{3}} \times (1.407)^2}{(0.0015)^{\frac{1}{3}} \times (1.235)^{\frac{1}{7}}}\right]$$

$$\log x = \log(0.0437)^{\frac{2}{3}} + \log(1.407)^2 - \log(0.0015)^{\frac{1}{3}} - \log(1.235)^{\frac{1}{7}}$$

$$\log x = \frac{2}{3}\log(0.0437) + 2\log(1.407) - \frac{1}{3}\log(0.0015) - \frac{1}{7}\log(1.235)$$

$$\log x = \frac{2}{3}(\bar{2}.6405) + 2(0.1483) - \frac{1}{3}(\bar{3}.1761) - \frac{1}{7}(0.0917)$$

$$\log x = \frac{2}{3}(-2 + 0.6405) + 0.2966 - \frac{1}{3}(-3 + 0.1761) - 0.0131$$

$$\log x = \frac{2}{3}(-1.3595) + 0.2966 - \frac{1}{3}(-2.8239) - 0.0131$$

$$\log x = -0.9063 + 0.2966 + 0.9413 - 0.0131$$

$$\log x = 0.3185$$

x = Antilog (0.3185)

x = 2.082

.9 اگر $v = \sqrt{\frac{g\ell}{2\pi}}$ ہو تو v معلوم کیجیے جبکہ $\ell = 150, g = 32.16, \pi = 3.142$

حل:

$$v = \sqrt{\frac{g\ell}{2\pi}} = \left[\frac{32.16 \times 150}{2 \times 3.142}\right]^{\frac{1}{2}}$$

$$\log v = \log\left[\frac{32.16 \times 150}{2 \times 3.142}\right]^{\frac{1}{2}}$$

$$\log v = \frac{1}{2}\log\left[\frac{32.16 \times 150}{2 \times 3.142}\right]$$

$$\log v = \frac{1}{2}[\log 32.16 + \log 150 - \log 2 - \log 3.142]$$

$$\log v = \frac{1}{2}[1.5073 + 2.1761 - 0.3010 - 0.4972]$$

$$\log v = \frac{1}{2}[2.8852]$$

$$\log v = 1.4426$$

v = Antilog (1.4426)

v = 27.71

.10 اگر $H = \frac{I^2Rt}{4.2}$ ہو تو H معلوم کیجیے جبکہ $I = 1.3, R = 6.7$ اور $t = 25$

$$H = \frac{I^2Rt}{4.2} = \frac{(1.3)^2 \times 6.7 \times 25}{4.2}$$

$$\log H = \log\left[\frac{(1.3)^2 \times 6.7 \times 25}{4.2}\right]$$

$$\log H = \log(1.3)^2 + \log(6.7) + \log(25) - \log(4.2)$$

$$\log H = 2\log(1.3) + \log(6.7) + \log 25 - \log(4.2)$$

$$\log H = 2(0.1139) + 0.8261 + 1.3979 - 0.6232$$

$$\log H = 0.2278 + 0.8261 + 1.3979 - 0.6232$$

$$\log H = 1.8286$$

H = 67.40

اگر $h = \dfrac{v}{\pi\left(R^2 - r^2\right)}$ ہو تو h معلوم کیجیے جبکہ $v = 1190, R = 83.6, r = 62.4$ اور $\pi = 3.14$

$$h = \frac{v}{\pi\left(R^2 - r^2\right)} = \frac{1190}{3.14\left[(83.6)^2 - (62.4)^2\right]}$$

$$= \frac{1190}{3.14[3095.2]} = \frac{1190}{9718.928}$$

$$\log h = \log\left(\frac{1190}{9718.928}\right)$$

$$= \log 1190 - \log(9718.928)$$

$$= 3.0755 - 3.9876$$

$$= -0.9121$$

h = Antilog (-0.9121)

h = 0.1224