EXERCISE 3.8

Question # 1(i)

$$x\frac{dy}{dx}=1+y$$

$$\Rightarrow\ x\,dy=(1+y)dx\quad\Rightarrow\ \frac{dy}{1+y}=\frac{dx}{x}$$

Integrating both sides

$$\int\frac{dy}{1+y}=\int\frac{dx}{x}$$

$$\Rightarrow\ \ln(1+y)=\ln x+\ln c$$

$$=\ln cx$$

$$\Rightarrow\ 1+y=cx$$

$$\Rightarrow\ y=cx-1\qquad \textit{Proved}$$

Question # 1(ii)

$$x^2(2y+1)\frac{dy}{dx}-1=0$$

$$\Rightarrow\ x^2(2y+1)\frac{dy}{dx}=1\quad\Rightarrow\ x^2(2y+1)dy=dx$$

$$\Rightarrow\ (2y+1)dy=\frac{1}{x^2}dx$$

On integrating

$$\int(2y+1)dy=\int\frac{1}{x^2}dx$$

$$\Rightarrow\ 2\int ydy+\int dy=\int x^{-2}dx$$

$$\Rightarrow\ 2\cdot\frac{y^2}{2}+y=\frac{x^{-2+1}}{-2+1}+c$$

$$\Rightarrow\ y^2+y=\frac{x^{-1}}{-1}+c$$

$$\Rightarrow\ y^2+y=c-\frac{1}{x}\qquad \textit{Proved}$$

Question # 1(iii)

$$y\frac{dy}{dx}-e^{2x}=1$$

$$\Rightarrow\ y\frac{dy}{dx}=1+e^{2x}\quad\Rightarrow\ ydy=\left(1+e^{2x}\right)dx$$

On integrating

$$\int ydy=\int\left(1+e^{2x}\right)dx$$

$$\Rightarrow\ \frac{y^2}{2}=x+\frac{e^{2x}}{2}+\frac{c}{2}\quad\Rightarrow\ y^2=2x+e^{2x}+c$$

$$\Rightarrow\ y^2=2x+e^{2x}+c$$

Question # 1(iv)

$$\frac{1}{x}\frac{dy}{dx}-2y=0$$

$$\Rightarrow\ \frac{1}{x}\frac{dy}{dx}=2y\quad\Rightarrow\ \frac{dy}{dx}=2xy$$

$$\Rightarrow\ \frac{dy}{y}=2xdx$$

On integrating

$$\int\frac{dy}{y}=2\int xdx$$

$$\Rightarrow\ \ln y=2\cdot\frac{x^2}{2}+\ln c$$

$$=x^2+\ln c$$

$$=x^2\ln e+\ln c\qquad \because\ \ln e=1$$

$$=\ln e^{x^2}+\ln c$$

$$\Rightarrow\ \ln y=\ln ce^{x^2}$$

$$\Rightarrow\ y=ce^{x^2}\qquad \textit{Proved}$$

Question # 1(v)

$$\frac{dy}{dx}=\frac{y^2+1}{e^{-x}}\quad\Rightarrow\ \frac{dy}{y^2+1}=e^xdx$$

Integrating both sides

$$\Rightarrow\ \int\frac{dy}{y^2+1}=\int e^xdx$$

$$\Rightarrow\ Tan^{-1}y=e^x+c$$

$$\Rightarrow\ y=Tan\left(e^x+c\right)$$

Question # 2

$$\frac{dy}{dx}=-y\quad\Rightarrow\ \frac{dy}{y}=-dx$$

On integrating

$$\int\frac{dy}{y}=-\int dx$$

$$\ln y=-x+\ln c$$

$$=-x\ln e+\ln c\qquad \because\ \ln e=1$$

$$=\ln e^{-x}+\ln c$$

$$\Rightarrow\ \ln y=\ln ce^{-x}\quad\Rightarrow\ y=ce^{-x}$$

Question # 3

$$ydx+xdy=0\quad\Rightarrow\ ydx=-xdy$$

$$\Rightarrow\ \frac{dx}{x}=-\frac{dy}{y}$$

On integrating

$$\ln x=-\ln y+\ln c$$

$$\Rightarrow\ \ln x=\ln\frac{c}{y}$$

$$\Rightarrow\ x=\frac{c}{y}\quad\Rightarrow\ xy=c$$

Question # 4

Do yourself

Question # 5

$$\frac{dy}{dx}=\frac{y}{x^2}\quad\Rightarrow\ \frac{dy}{y}=x^{-2}dx$$

Integrating

$$\int\frac{dy}{y}=\int x^{-2}dx$$

$$\Rightarrow\ \ln y=\frac{x^{-2+1}}{-2+1}+\ln c\quad\Rightarrow\ \ln y=\frac{x^{-1}}{-1}+\ln c$$

$$\Rightarrow\ \ln y=-\frac{1}{x}+\ln c$$

$$\Rightarrow \ln y = -\frac{1}{x}\ln e + \ln c$$
$$= \ln e^{-\frac{1}{x}} + \ln c$$
$$\Rightarrow \ln y = \ln ce^{-\frac{1}{x}} \quad \Rightarrow y = ce^{-\frac{1}{x}}$$

Question # 6

$$\sin y \operatorname{cosec} y\frac{dy}{dx} = 1$$
$$\Rightarrow \sin y\ dy = \frac{dx}{\operatorname{cosec} x} \quad \Rightarrow \sin y\ dy = \sin x\, dx$$

Integrating

$$\int \sin y\ dy = \int \sin x\, dx$$
$$\Rightarrow -\cos y = -\cos x - c$$
$$\Rightarrow \cos y = \cos x + c$$

Question # 7

$$xdy + y(x-1)dx = 0$$
$$\Rightarrow xdy = -y(x-1)dx$$
$$\Rightarrow \frac{dy}{y} = -\frac{x-1}{x}dx \quad \Rightarrow \frac{dy}{y} = -\left(\frac{x}{x} - \frac{1}{x}\right)dx$$
$$\Rightarrow \frac{dy}{y} = -\left(1 - \frac{1}{x}\right)dx$$

On integrating

$$\int \frac{dy}{y} = -\int \left(1 - \frac{1}{x}\right)dx$$
$$\Rightarrow \ln y = -x - \ln x + \ln c$$
$$= -x\ln e - \ln x + \ln c$$
$$= \ln e^{-x} - \ln x + \ln c$$
$$\Rightarrow \ln y = \ln cxe^{-x} \quad \Rightarrow y = cxe^{-x}$$

Question # 8

$$\frac{x^2+1}{y+1} = \frac{x}{y}\frac{dy}{dx}$$
$$\Rightarrow \frac{x^2+1}{x}\,dx = \frac{y+1}{y}dy$$

On integrating

$$\int \frac{x^2+1}{x}\,dx = \int \frac{y+1}{y}dy$$
$$\Rightarrow \int\left(\frac{x^2}{x} + \frac{1}{x}\right)dx = \int\left(\frac{y}{y} + \frac{1}{y}\right)dy$$
$$\Rightarrow \int\left(x + \frac{1}{x}\right)dx = \int\left(1 + \frac{1}{y}\right)dy$$
$$\Rightarrow \int x\ dx + \int \frac{1}{x}dx = \int dy + \int \frac{1}{y}dy$$
$$\Rightarrow \frac{x^2}{2} + \ln x = y + \ln y - \ln c$$
$$\Rightarrow \frac{x^2}{2}\ln e + \ln x + \ln c = y\ln e + \ln y$$
$$\Rightarrow \ln e^{\frac{x^2}{2}} + \ln x + \ln c = \ln e^y + \ln y$$
$$\Rightarrow \ln cxe^{\frac{x^2}{2}} = \ln ye^y$$
$$\Rightarrow cxe^{\frac{x^2}{2}} = ye^y \quad \text{i.e. } ye^y = cxe^{\frac{x^2}{2}}$$

Question # 9

Do yourself

Question # 10

Do yourself

Question # 11

$$\frac{dy}{dx} + \frac{2xy}{2y+1} = x$$
$$\Rightarrow \frac{dy}{dx} = x - \frac{2xy}{2y+1}$$
$$= x\left(1 - \frac{2y}{2y+1}\right)$$
$$= x\left(\frac{2y+1-2y}{2y+1}\right)$$
$$\Rightarrow \frac{dy}{dx} = x\left(\frac{1}{2y+1}\right) \quad \Rightarrow (2y+1)dy = x\,dx$$

Now do yourself

Question # 12

$$\left(x^2 - yx^2\right)\frac{dy}{dx} + y^2 + xy^2 = 0$$
$$\Rightarrow \left(x^2 - yx^2\right)\frac{dy}{dx} = -y^2 - xy^2$$
$$\Rightarrow x^2(1-y)\frac{dy}{dx} = -y^2(1+x)$$
$$\Rightarrow \frac{1-y}{y^2}dy = -\frac{1+x}{x^2}dx$$

Now do yourself

Question # 13

$$\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$$
$$\Rightarrow \sec^2 x \tan y\, dx = -\sec^2 y \tan x\, dy$$
$$\Rightarrow \frac{\sec^2 x}{\tan x}dx = -\frac{\sec^2 y}{\tan y}dy$$

On integrating

$$\int \frac{\sec^2 x}{\tan x}dx = -\int \frac{\sec^2 y}{\tan y}dy$$
$$\Rightarrow \int \frac{\frac{d}{dx}(\tan x)}{\tan x}dx = -\int \frac{\frac{d}{dy}(\tan y)}{\tan y}dy$$
$$\Rightarrow \ln\tan x = -\ln\tan y + \ln c$$
$$\Rightarrow \ln\tan x + \ln\tan y = \ln c$$
$$\Rightarrow \ln(\tan x \tan y) = \ln c$$
$$\Rightarrow \tan x \tan y = c$$

Question # 14

$$\left(y - x\frac{dy}{dx}\right) = 2\left(y^2 + \frac{dy}{dx}\right)$$
$$\Rightarrow y - x\frac{dy}{dx} = 2y^2 + 2\frac{dy}{dx}$$
$$\Rightarrow y - 2y^2 = 2\frac{dy}{dx} + x\frac{dy}{dx}$$
$$\Rightarrow y(1-2y) = (2+x)\frac{dy}{dx}$$
$$\Rightarrow \frac{dx}{2+x} = \frac{dy}{y(1-2y)}$$

On integrating

$$\int \frac{dx}{2+x} = \int \frac{dy}{y(1-2y)} \text{ ……….. (i)}$$

Now consider

$$\frac{1}{y(1-2y)} = \frac{A}{y}+\frac{B}{1-2y}$$

$$\Rightarrow 1 = A(1-2y)+By \text{ ………. (ii)}$$

Put $y=0$ in (ii)

$$1 = A(1-2(0))+0 \Rightarrow A=1$$

Put $1-2y=0 \Rightarrow 2y=1 \Rightarrow y=\frac{1}{2}$ in (ii)

$$1 = 0+B\left(\frac{1}{2}\right) \Rightarrow B=2$$

So $\dfrac{1}{y(1-2y)} = \dfrac{1}{y}+\dfrac{2}{1-2y}$

Using in (i) $\leftarrow *$

$$\int \frac{dx}{2+x} = \int\left(\frac{1}{y}+\frac{2}{1-2y}\right)dy$$

$$= \int\frac{1}{y}dy+\int\frac{2}{1-2y}dy$$

$$= \int\frac{1}{y}dy-\int\frac{-2}{1-2y}dy$$

$$\Rightarrow \int \frac{dx}{2+x} = \int\frac{1}{y}dy-\int\frac{\frac{d}{dx}(1-2y)}{1-2y}dy$$

$$\Rightarrow \ln(2+x) = \ln y-\ln(1-2y)-\ln c$$

$$\Rightarrow \ln(2+x)+\ln c = \ln y-\ln(1-2y)$$

$$\Rightarrow \ln c(2+x) = \ln\frac{y}{(1-2y)}$$

$$\Rightarrow c(2+x) = \frac{y}{(1-2y)}$$

$$\Rightarrow y = c(2+x)(1-2y)$$

Alternative $(\leftarrow *)$

$$\int \frac{dx}{2+x} = \int\left(\frac{1}{y}+\frac{2}{1-2y}\right)dx$$

$$= \int\frac{1}{y}dy+\int\frac{2}{1-2y}dy$$

$$= \int\frac{1}{y}dy-\int\frac{2}{2y-1}dy$$

$$\Rightarrow \int \frac{dx}{2+x} = \int\frac{1}{y}dy-\int\frac{\frac{d}{dx}(2y-1)}{2y-1}dy$$

$$\Rightarrow \ln(2+x) = \ln y-\ln(2y-1)-\ln c$$

$$\Rightarrow \ln(2+x)+\ln c = \ln y-\ln(2y-1)$$

$$\Rightarrow \ln c(2+x) = \ln\frac{y}{(2y-1)}$$

$$\Rightarrow c(2+x) = \frac{y}{(2y-1)}$$

i.e. $\dfrac{y}{(2y-1)} = c(2+x)$

Review

- $\int \tan x\,dx = \ln|\sec x| = -\ln|\cos x|$
- $\int \cot x\,dx = \ln|\sin x| = -\ln|\csc x|$
- $\int \sec x\,dx = \ln|\sec x+\tan x|$
- $\int \csc x\,dx = \ln|\csc x-\cot x|$

Question # 15

$$1+\cos x\tan y\frac{dy}{dx} = 0$$

$$\Rightarrow \cos x\tan y\frac{dy}{dx} = -1$$

$$\Rightarrow \tan y\,dy = -\frac{1}{\cos x}dx$$

$$\Rightarrow \tan y\,dy = -\sec x\,dx$$

On integrating

$$\int\tan y\,dy = -\int\sec x\,dx$$

$$\Rightarrow -\ln|\cos y| = -\ln|\sec x+\tan x|-\ln c$$

$$\Rightarrow \ln|\cos y| = +\ln|\sec x+\tan x|+\ln c$$

$$\Rightarrow \ln|\cos y| = \ln|c(\sec x+\tan x)|$$

$$\Rightarrow \cos y = c(\sec x+\tan x)$$

Question # 16

$$y-x\frac{dy}{dx} = 3\left(1+x\frac{dy}{dx}\right)$$

$$\Rightarrow y-x\frac{dy}{dx} = 3+3x\frac{dy}{dx}$$

$$\Rightarrow y-3 = 3x\frac{dy}{dx}+x\frac{dy}{dx}$$

$$= (3x+x)\frac{dy}{dx}$$

$$\Rightarrow y-3 = 4x\frac{dy}{dx} \quad \Rightarrow \frac{dx}{x} = 4\frac{dy}{y-3}$$

Now do yourself

Question # 17

$$\sec x+\tan y\frac{dy}{dx} = 0$$

$$\Rightarrow \tan y\frac{dy}{dx} = -\sec x$$

$$\Rightarrow \tan y\,dy = -\sec x\,dx$$

Now do yourself as Question # 15

Question # 18

$$\left(e^x+e^{-x}\right)\frac{dy}{dx} = e^x-e^{-x}$$

$$\Rightarrow dy = \frac{e^x-e^{-x}}{e^x+e^{-x}}dx$$

On integrating

$$\int dy = \int\frac{e^x-e^{-x}}{e^x+e^{-x}}dx$$

$$\Rightarrow y = \int\frac{\frac{d}{dx}\left(e^x+e^{-x}\right)}{e^x+e^{-x}}dx$$

$$\Rightarrow y = \ln\left(e^x+e^{-x}\right)+c$$

Question # 19

$$\frac{dy}{dx}-x = xy^2 \Rightarrow \frac{dy}{dx} = x+xy^2$$
$$\Rightarrow \frac{dy}{dx} = x\left(1+y^2\right) \Rightarrow \frac{dy}{1+y^2} = x\,dx$$
$$\Rightarrow \int\frac{dy}{1+y^2} = \int x\,dx$$
$$\Rightarrow Tan^{-1}y = \frac{x^2}{2}+c$$
$$\Rightarrow y = Tan\left(\frac{x^2}{2}+c\right)$$

Question # 20

$$\frac{dx}{dt} = 2x \Rightarrow \frac{dx}{x} = 2dt$$
$$\Rightarrow \int\frac{dx}{x} = 2\int dt$$
$$\Rightarrow \ln x = 2t+\ln c$$
$$= \ln e^{2t}+\ln c \qquad \because \ln e^x = x$$
$$\Rightarrow \ln x = \ln ce^{2t}$$
$$\Rightarrow x = ce^{2t} \;\ldots\ldots\text{ (i)}$$

When $t=0$ then $x=4$, putting in (i)

$$4 = ce^{2(0)} \Rightarrow 4 = c\,e^0$$
$$\Rightarrow 4 = c(1) \Rightarrow c = 4$$

Putting in (i)

$$\Rightarrow x = 4e^{2t}$$

Question # 21

$$\frac{ds}{dt}+2st = 0$$
$$\Rightarrow \frac{ds}{dt} = -2st \Rightarrow \frac{ds}{s} = -2t\,dt$$

On integrating

$$\int\frac{ds}{s} = -2\int t\,dt$$
$$\Rightarrow \ln s = -2\frac{t^2}{2}+\ln c$$
$$= -t^2+\ln c$$
$$= \ln e^{-t^2}+\ln c \qquad \because \ln e^x = x$$
$$\Rightarrow \ln s = \ln ce^{-t^2}$$
$$\Rightarrow s = ce^{-t^2} \;\ldots\ldots\text{ (i)}$$

When $t=0$ then $s=4e$, using in (i)

$$4e = ce^{-(0)^2} \Rightarrow 4e = c(1)$$
$$\Rightarrow c=4e$$

Putting in (i)

$$s = 4e\cdot e^{-t^2}$$
$$\Rightarrow s = 4e^{1-t^2}$$

Question # 22

Number of bacteria initially	= 200
No. of bacteria after two hours	= 2(200)
	= 400
No. of bacteria after four hours	= 2(400)
	= 800 *Ans.*

Question # 23

i) When a body is projected upward its acceleration is $-g$. (where $g=980$ cm/sec^2)

i.e. acceleration = $\dfrac{dv}{dt} = -g$,

where v is velocity of ball.

$$\Rightarrow \frac{dv}{dt} = -980$$
$$\Rightarrow dv = -980\,dt$$

On integrating

$$\int dv = -980\int dt$$
$$\Rightarrow v = -980t+c_1 \;\ldots\ldots\text{ }(i)$$

Initially, when $t=0$ then $v=2450$ cm/sec

$$2450 = -980(0)+c_1$$
$$\Rightarrow c_1 = 2450$$

Putting in (i)

$$\boxed{v = -980t+2450}$$

ii) Since velocity $= v = \dfrac{dx}{dt}$

where x is height of ball.

$$\Rightarrow \frac{dx}{dt} = -980t+2450$$
$$\Rightarrow dx = \left(-980t+2450\right)dt$$

Integrating

$$\int dx = \int\left(-980t+2450\right)dt$$
$$\Rightarrow x = -980\frac{t^2}{2}+2450t+c_2$$
$$\Rightarrow x = -490t^2+2450t+c_2 \;\ldots\ldots\text{ }(ii)$$

Initially, when $t=0$ then $x=0$

$$0 = -490\,(0)+2450(0)+c_2$$
$$\Rightarrow c_2 = 0$$

Putting value of c_2 in (ii)

$$\Rightarrow x = -490t^2+2450t+0$$
$$\Rightarrow \boxed{x = 2450t-490t^2}$$

iii)

$$\because v = -980t+2450$$

When body is at max. height then $v=0$

$$\Rightarrow -980t+2450 = 0$$
$$\Rightarrow 980t = 2450 \Rightarrow t = \frac{2450}{980}$$
$$\Rightarrow t = 2.5\,\text{sec}$$

Since $x = 2450t-4980t^2$

When $t=2.5$ sec

$$x = 2450\left(2.5\right)-490\left(2.5\right)^2$$
$$= 6125-3062.5$$
$$= 3062.5$$

Hence ball attains max. height of 3062.5 cm.