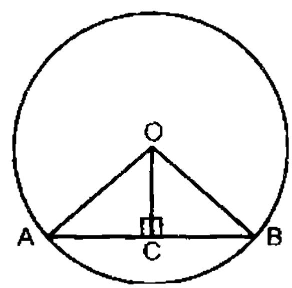
Exercise 12.1

1. Prove that the centre of a circle is on the right bisectors of each of its chords.



Given Circle with centre O

To Prove Centre of the circle is on right bisectors of each of its chords

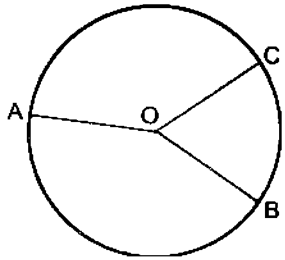
Construction

Draw any chord $\overline{AB}$. Draw $\overline{OC} \perp \overline{AB}$ join O with A and B.

Proof:

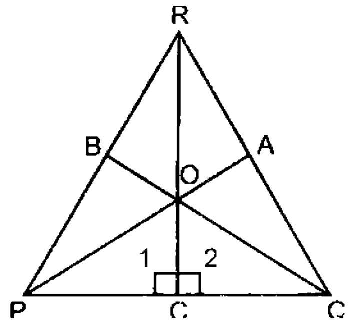
Statements	Reasons
In $\Delta OAC \leftrightarrow \Delta OBC$	
$\overline{OA} \cong \overline{OB}$	Radii of same circle
$\overline{OC} \cong \overline{OC}$	Common
$\angle ACO \cong \angle BCO$	Each of 90°
$\therefore \Delta ACO \cong \Delta BCO$	H.S $\cong$ H.S
$\therefore \overline{AC} \cong \overline{BC}$	Corresponding sides of the congruent triangles.
$\therefore \overline{OC}$ is the right bisector of $\overline{AB}$	

2. Where will be the centre of a circle passing through three non-collinear points and why?



Circle is the locus of a point which moves so that its distance from a fixed point O remains same. Otherwise no circle will be formed.

3. Three villages P, Q and R are not on the same line. The people of these villages want to make a Children Park at such a place which is equidistant from these three villages. After fixing the place, of Children park, prove that the Park is equidistant from the three villages.



Given

Three villages P, Q, R not on the same line.

To Prove

Park is equidistant from P, Q and R.

Construction

Complete the triangle PQR, draw the right bisectors of the sides $\overline{PQ}$ and $\overline{QR}$ cutting each other at O. Join O with P, Q and R. let O be the park.

Proof:

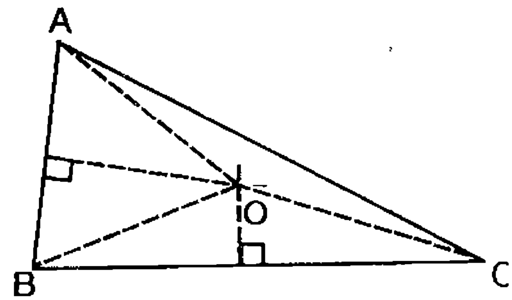
Statements	Reasons
In $\quad \Delta OPC \leftrightarrow \Delta OQC$	
$\overline{CP} \cong \overline{CQ}$	Construction
$\overline{OC} \cong \overline{OC}$	Common
$\angle 1 \cong \angle 2$	Each of 90°
$\therefore \quad \Delta OCP \cong \Delta OCQ$	$S.A.S \cong S.A.S$
$\therefore \quad \overline{OP} \cong \overline{OQ}$ .... (i)	Corresponding sides of congruent triangles
Similarly	
$\overline{OQ} \cong \overline{OR}$ .... (ii)	
$\therefore \quad \overline{OP} \cong \overline{OQ} \cong \overline{OR}$	

Theorem

The right bisectors of the sides of a triangle are concurrent.

Given

ΔABC



To Prove

The right bisectors of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ are concurrent.

Construction Draw the right bisectors of $\overline{AB}$ and $\overline{BC}$ which meet each other at the point O. Join O to A, B and C.

Proof:

Statements	Reasons
$\overline{OA} \cong \overline{OB}$ ......(i)	(Each point on right bisector of a segment is equidistant from its end points)
$\overline{OB} \cong \overline{OC}$ ......(ii)	as in (i)
$\overline{OA} \cong \overline{OC}$ ......(iii)	From (i) and (ii)
∴ Point O is on the right bisector of $\overline{CA}$. ......(iv)	(O is equidistant from A and C)
But point O is on the right bisector of $\overline{AB}$ and of $\overline{BC}$. ......(v)	construction
Hence the right bisectors of the three sides of a triangle are concurrent at O.	{from (iv) and (v)}

Note:

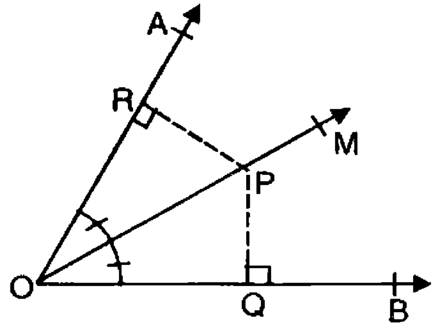
(a) The right bisectors of the sides of an acute triangle intersect each other inside the triangle.

(b) The right bisectors of the sides of a right triangle intersect each other on the hypotenuse.

(c) The right bisectors of the sides of an obtuse triangle intersect each other outside the triangle.

Theorem

Any point on the bisector of an angle is equidistant from its arms.



Given

A point P is on $\overrightarrow{OM}$, the bisectors of $\angle AOB$.

To Prove

$\overline{PQ} \cong \overline{PR}$ i.e., P is equidistant from $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

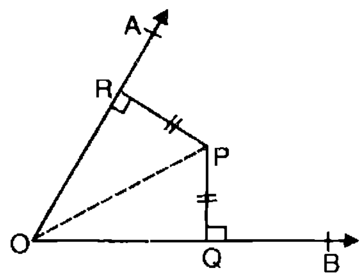
Construction

Draw $\overline{PR} \perp \overrightarrow{OA}$ and $\overline{PQ} \perp \overrightarrow{OB}$.

Proof:

Statements	Reasons
In $\Delta POQ \longleftrightarrow \Delta POR$	
$\overline{OP} \cong \overline{OP}$	Common
$\angle PQO \cong \angle PRO$	Construction
$\angle POQ \cong \angle POR$	Given
$\therefore$ $\Delta POQ \cong \Delta POR$	S.A.A. $\cong$ S.A.A.
Hence $\overline{PQ} \cong \overline{PR}$	(corresponding sides of congruent triangles)

Theorem Any point inside an angle, equidistant from its arms, is on the bisector of it.



Given

Any point P lies inside $\angle AOB$ such that $\overline{PQ} \cong \overline{PR}$, where $\overline{PQ} \perp \overrightarrow{OB}$ and $\overline{PR} \perp \overrightarrow{OA}$.

To Prove Point P is on the bisector of $\angle AOB$.

Construction Join P to O.

Proof:

Statements	Reasons
In $\Delta POQ \longleftrightarrow \Delta POR$	
$\angle PQO \cong \angle PRO$	Given (right angles)
$\overline{PO} \cong \overline{PO}$	Common
$\overline{PQ} \cong \overline{PR}$	Given
$\therefore$ $\Delta POQ \cong \Delta POR$	H.S. $\cong$ H.S.
Hence $\angle POQ \cong \angle POR$	(corresponding angles of congruent triangles)
i.e., P is on the bisector of $\angle AOB$.	