

## EXERCISE 2.4

### Question # 1 (i)

$$y = \sqrt{\frac{1-x}{1+x}}$$

Put  $u = \frac{1-x}{1+x}$

So  $y = \sqrt{u} \Rightarrow y = u^{\frac{1}{2}}$

Now diff.  $u$  w.r.t.  $x$

$$\begin{aligned} \frac{du}{dx} &= \frac{d}{dx} \left( \frac{1-x}{1+x} \right) \\ &= \frac{(1+x) \frac{d}{dx}(1-x) - (1-x) \frac{d}{dx}(1+x)}{(1+x)^2} \\ &= \frac{(1+x)(-1) - (1-x)(1)}{(1+x)^2} \\ &= \frac{-1-x-1+x}{(1+x)^2} \\ \Rightarrow \frac{du}{dx} &= \frac{-2}{(1+x)^2} \end{aligned}$$

Now diff.  $y$  w.r.t.  $u$

$$\begin{aligned} \frac{dy}{du} &= \frac{d}{du} u^{\frac{1}{2}} \\ &= \frac{1}{2} u^{-\frac{1}{2}} = \frac{1}{2} \left( \frac{1-x}{1+x} \right)^{-\frac{1}{2}} \\ \Rightarrow \frac{dy}{du} &= \frac{1}{2} \left( \frac{1+x}{1-x} \right)^{\frac{1}{2}} \end{aligned}$$

Now by chain rule

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} \\ &= \frac{1}{2} \left( \frac{1+x}{1-x} \right)^{\frac{1}{2}} \cdot \frac{-2}{(1+x)^2} \\ &= \frac{(1+x)^{\frac{1}{2}}}{(1-x)^{\frac{1}{2}}} \cdot \frac{-1}{(1+x)^2} \\ &= \frac{-1}{(1-x)^{\frac{1}{2}} (1+x)^{\frac{3}{2}}} \\ &= \frac{-1}{\sqrt{1-x} (1+x)^{\frac{3}{2}}} \quad \text{Answer} \end{aligned}$$

### Question # 1(ii)

$$y = \sqrt{x+\sqrt{x}}$$

Let  $u = x + \sqrt{x} = x + x^{\frac{1}{2}}$   
 $\Rightarrow y = \sqrt{u} = u^{\frac{1}{2}}$

Diff.  $u$  w.r.t.  $x$

$$\begin{aligned} \frac{du}{dx} &= \frac{d}{dx} \left( x + x^{\frac{1}{2}} \right) \\ &= 1 + \frac{1}{2} x^{-\frac{1}{2}} = 1 + \frac{1}{2\sqrt{x}} \\ &= \frac{2\sqrt{x} + 1}{2\sqrt{x}} \end{aligned}$$

Now diff.  $y$  w.r.t.  $u$

$$\begin{aligned} \frac{dy}{du} &= \frac{d}{du} u^{\frac{1}{2}} \\ &= \frac{1}{2} u^{-\frac{1}{2}} = \frac{1}{2u^{\frac{1}{2}}} = \frac{1}{2(x+\sqrt{x})^{\frac{1}{2}}} \\ \Rightarrow \frac{dy}{du} &= \frac{1}{2\sqrt{x+\sqrt{x}}} \end{aligned}$$

Now by chain rule

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} \\ &= \frac{1}{2\sqrt{x+\sqrt{x}}} \cdot \frac{2\sqrt{x}+1}{2\sqrt{x}} \\ &= \frac{2\sqrt{x}+1}{4\sqrt{x} \cdot \sqrt{x+\sqrt{x}}} \quad \text{Answer} \end{aligned}$$

### Question # 1 (iii)

$$y = x \sqrt{\frac{a+x}{a-x}}$$

Put  $u = \frac{a+x}{a-x}$

So  $y = x\sqrt{u} = x(u)^{\frac{1}{2}}$

Diff. w.r.t.  $x$

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} x(u)^{\frac{1}{2}} \\ &= x \frac{d}{dx} (u)^{\frac{1}{2}} + (u)^{\frac{1}{2}} \frac{d}{dx} x \\ &= x \frac{1}{2} (u)^{-\frac{1}{2}} \frac{du}{dx} + (u)^{\frac{1}{2}} (1) \end{aligned}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x}{2} (u)^{-\frac{1}{2}} \frac{du}{dx} + (u)^{\frac{1}{2}} \dots\dots\dots (i)$$

Now diff.  $u$  w.r.t.  $x$

$$\begin{aligned}\frac{du}{dx} &= \frac{d}{dx}\left(\frac{a+x}{a-x}\right) \\ &= \frac{(a-x)\frac{d}{dx}(a+x) - (a+x)\frac{d}{dx}(a-x)}{(a-x)^2} \\ &= \frac{(a-x)(0+1) - (a+x)(0-1)}{(a-x)^2} \\ &= \frac{(a-x)(1) - (a+x)(-1)}{(a-x)^2} \\ &= \frac{a-x+a+x}{(a-x)^2} = \frac{2a}{(a-x)^2}\end{aligned}$$

Using value of  $u$  and  $\frac{du}{dx}$  in eq. (i)

$$\begin{aligned}\frac{dy}{dx} &= \frac{x}{2}\left(\frac{a+x}{a-x}\right)^{-\frac{1}{2}} \cdot \frac{2a}{(a-x)^2} + \left(\frac{a+x}{a-x}\right)^{\frac{1}{2}} \\ &= \frac{(a+x)^{-\frac{1}{2}}}{(a-x)^{-\frac{1}{2}}} \cdot \frac{ax}{(a-x)^2} + \frac{(a+x)^{\frac{1}{2}}}{(a-x)^{\frac{1}{2}}} \\ &= \frac{ax}{(a+x)^{\frac{1}{2}}(a-x)^{2-\frac{1}{2}}} + \frac{(a+x)^{\frac{1}{2}}}{(a-x)^{\frac{1}{2}}} \\ &= \frac{ax}{(a+x)^{\frac{1}{2}}(a-x)^{\frac{3}{2}}} + \frac{(a+x)^{\frac{1}{2}}}{(a-x)^{\frac{1}{2}}} \\ &= \frac{ax + (a+x)(a-x)}{(a+x)^{\frac{1}{2}}(a-x)^{\frac{3}{2}}} \\ \Rightarrow \boxed{\frac{dy}{dx} = \frac{ax + a^2 - x^2}{(a+x)^{\frac{1}{2}}(a-x)^{\frac{3}{2}}}}\end{aligned}$$

### Question # 1 (iv) & (v)

*Do yourself as above*

### Question # 2 (i)

$$3x + 4y + 7 = 0$$

Diff. w.r.t.  $x$ .

$$\begin{aligned}\frac{d}{dx}(3x + 4y + 7) &= \frac{d}{dx}(0) \\ \Rightarrow 3(1) + 4\frac{dy}{dx} + 0 &= 0 \\ \Rightarrow 4\frac{dy}{dx} &= -3 \Rightarrow \boxed{\frac{dy}{dx} = -\frac{3}{4}}\end{aligned}$$

### Question # 2 (ii)

$$xy + y^2 = 2$$

Differentiating w.r.t.  $x$

$$\begin{aligned}\frac{d}{dx}(xy + y^2) &= \frac{d}{dx}(2) \\ \Rightarrow \frac{d}{dx}(xy) + \frac{d}{dx}y^2 &= 0 \\ \Rightarrow x\frac{dy}{dx} + y\frac{dx}{dx} + 2y\frac{dy}{dx} &= 0 \\ \Rightarrow (x + 2y)\frac{dy}{dx} + y(1) &= 0 \\ \Rightarrow (x + 2y)\frac{dy}{dx} &= -y \\ \Rightarrow (x + 2y)\frac{dy}{dx} &= -y \\ \Rightarrow \boxed{\frac{dy}{dx} = \frac{-y}{x + 2y}}\end{aligned}$$

### Question # 2 (iii)

*Do yourself*

### Question # 2(iv)

$$4x^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

Differentiating w.r.t.  $x$

$$\begin{aligned}\frac{d}{dx}(4x^2 + 2hxy + by^2 + 2gx + 2fy + c) &= \frac{d}{dx}(0) \\ \Rightarrow 4\frac{d}{dx}(x^2) + 2h\frac{d}{dx}(xy) + b\frac{d}{dx}(y^2) \\ &\quad + 2g\frac{d}{dx}(x) + 2f\frac{d}{dx}(y) + \frac{d}{dx}(c) = 0 \\ \Rightarrow 4(2x) + 2h\left(x\frac{dy}{dx} + y(1)\right) + b \cdot 2y\frac{dy}{dx} \\ &\quad + 2g(1) + 2f\frac{dy}{dx} + 0 = 0 \\ \Rightarrow 8x + 2hx\frac{dy}{dx} + 2hy + 2by\frac{dy}{dx} \\ &\quad + 2g + 2f\frac{dy}{dx} = 0\end{aligned}$$

$$\begin{aligned}\Rightarrow 2(hx + by + f)\frac{dy}{dx} + 2(4x + hy + g) &= 0 \\ \Rightarrow 2(hx + by + f)\frac{dy}{dx} &= -2(4x + hy + g) \\ \Rightarrow (hx + by + f)\frac{dy}{dx} &= -(4x + hy + g) \\ \Rightarrow \boxed{\frac{dy}{dx} = -\frac{4x + hy + g}{hx + by + f}}\end{aligned}$$

**Question # 2 (v)**

$$x\sqrt{1+y} + y\sqrt{1+x} = 0 \Rightarrow x(1+y)^{\frac{1}{2}} + y(1+x)^{\frac{1}{2}} = 0$$

Differentiating w.r.t.  $x$

$$\Rightarrow \frac{d}{dx} \left[ x(1+y)^{\frac{1}{2}} \right] + \frac{d}{dx} \left[ y(1+x)^{\frac{1}{2}} \right] = \frac{d}{dx}(0)$$

$$\Rightarrow x \frac{d}{dx} (1+y)^{\frac{1}{2}} + (1+y)^{\frac{1}{2}} \frac{dx}{dx} + y \frac{d}{dx} (1+x)^{\frac{1}{2}} + (1+x)^{\frac{1}{2}} \frac{dy}{dx} = 0$$

$$\Rightarrow x \cdot \frac{1}{2} (1+y)^{-\frac{1}{2}} \frac{dy}{dx} + (1+y)^{\frac{1}{2}} (1) + y \cdot \frac{1}{2} (1+x)^{-\frac{1}{2}} (1) + (1+x)^{\frac{1}{2}} \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{x}{2(1+y)^{\frac{1}{2}}} \frac{dy}{dx} + (1+y)^{\frac{1}{2}} + \frac{y}{2(1+x)^{\frac{1}{2}}} + (1+x)^{\frac{1}{2}} \frac{dy}{dx} = 0$$

$$\Rightarrow \left[ \frac{x}{2(1+y)^{\frac{1}{2}}} + (1+x)^{\frac{1}{2}} \right] \frac{dy}{dx} = - \left[ (1+y)^{\frac{1}{2}} + \frac{y}{2(1+x)^{\frac{1}{2}}} \right]$$

$$\Rightarrow \left[ \frac{x + 2(1+x)^{\frac{1}{2}}(1+y)^{\frac{1}{2}}}{2(1+y)^{\frac{1}{2}}} \right] \frac{dy}{dx} = - \left[ \frac{2(1+x)^{\frac{1}{2}}(1+y)^{\frac{1}{2}} + y}{2(1+x)^{\frac{1}{2}}} \right]$$

$$\Rightarrow \left[ \frac{x + 2\sqrt{(1+x)(1+y)}}{2\sqrt{1+y}} \right] \frac{dy}{dx} = - \left[ \frac{2\sqrt{(1+x)(1+y)} + y}{2\sqrt{1+x}} \right]$$

$$\Rightarrow \frac{dy}{dx} = - \frac{2\sqrt{(1+x)(1+y)} + y}{2\sqrt{1+x}} \cdot \frac{2\sqrt{1+y}}{x + 2\sqrt{(1+x)(1+y)}}$$

$$\Rightarrow \frac{dy}{dx} = - \frac{\sqrt{1+y} (2\sqrt{(1+x)(1+y)} + y)}{\sqrt{1+x} (x + 2\sqrt{(1+x)(1+y)})} \quad \text{Answer}$$

**Question # 2 (vi)**

$$y(x^2 - 1) = x\sqrt{x^2 + 4}$$

Differentiating w.r.t  $x$

$$\frac{d}{dx} y(x^2 - 1) = \frac{d}{dx} x(x^2 + 4)^{\frac{1}{2}}$$

$$\Rightarrow y \frac{d}{dx} (x^2 - 1) + (x^2 - 1) \frac{dy}{dx} = x \frac{d}{dx} (x^2 + 4)^{\frac{1}{2}} + (x^2 + 4)^{\frac{1}{2}} \frac{dx}{dx}$$

$$\Rightarrow y(2x) + (x^2 - 1) \frac{dy}{dx} = x \frac{1}{2} (x^2 + 4)^{-\frac{1}{2}} (2x) + (x^2 + 4)^{\frac{1}{2}} (1)$$

$$\Rightarrow 2xy + (x^2 - 1) \frac{dy}{dx} = \frac{x^2}{(x^2 + 4)^{\frac{1}{2}}} + (x^2 + 4)^{\frac{1}{2}}$$

$$\Rightarrow (x^2 - 1) \frac{dy}{dx} = \frac{x^2}{(x^2 + 4)^{\frac{1}{2}}} + (x^2 + 4)^{\frac{1}{2}} - 2xy$$

$$\Rightarrow (x^2 - 1) \frac{dy}{dx} = \frac{x^2}{(x^2 + 4)^{\frac{1}{2}}} + (x^2 + 4)^{\frac{1}{2}} - 2xy$$

$$\Rightarrow (x^2 - 1) \frac{dy}{dx} = \frac{x^2 + x^2 + 4 - 2xy(x^2 + 4)}{(x^2 + 4)^{\frac{1}{2}}} \Rightarrow \boxed{\frac{dy}{dx} = \frac{2x^2 + 4 - 2xy(x^2 + 4)}{(x^2 - 1)\sqrt{x^2 + 4}}}$$

**Question # 3 (i)**

Since  $x = \theta + \frac{1}{\theta}$

$$\Rightarrow x = \theta + \theta^{-1}$$

Differentiating  $x$  w.r.t.  $\theta$

$$\frac{dx}{d\theta} = \frac{d}{d\theta}(\theta + \theta^{-1})$$

$$= 1 - \theta^{-2} = 1 - \frac{1}{\theta^2} = \frac{\theta^2 - 1}{\theta^2}$$

$$\Rightarrow \frac{d\theta}{dx} = \frac{\theta^2}{\theta^2 - 1}$$

Now  $y = \theta + 1$

Diff. w.r.t.  $\theta$

$$\frac{dy}{d\theta} = \frac{d}{d\theta}(\theta + 1) \Rightarrow \frac{dy}{d\theta} = 1$$

Now by chain rule

$$\frac{dy}{dx} = \frac{dy}{d\theta} \cdot \frac{d\theta}{dx}$$

$$= \frac{dy}{d\theta} \cdot \frac{d\theta}{dx} = 1 \cdot \frac{\theta^2}{\theta^2 - 1}$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{\theta^2}{\theta^2 - 1}}$$

**Question # 3 (ii)**

Since  $x = \frac{a(1-t^2)}{1+t^2}$  \*Correction

Diff. w.r.t.  $t$

$$\frac{dx}{dt} = a \frac{d}{dt} \left( \frac{1-t^2}{1+t^2} \right)$$

$$= a \frac{(1+t^2) \frac{d}{dt}(1-t^2) - (1-t^2) \frac{d}{dt}(1+t^2)}{(1+t^2)^2}$$

$$= a \frac{(1+t^2)(-2t) - (1-t^2)(2t)}{(1+t^2)^2}$$

$$= a \frac{-2t - 2t^3 - 2t + 2t^3}{(1+t^2)^2}$$

$$\Rightarrow \frac{dx}{dt} = \frac{-4at}{(1+t^2)^2} \Rightarrow \frac{dt}{dx} = \frac{(1+t^2)^2}{-4at}$$

Now  $y = \frac{2bt}{1+t^2}$

Diff. w.r.t.  $t$

$$\frac{dy}{dt} = \frac{d}{dt} \left( \frac{2bt}{1+t^2} \right)$$

$$= \frac{(1+t^2) \frac{d}{dt} 2bt - 2bt \frac{d}{dt} (1+t^2)}{(1+t^2)^2}$$

$$= \frac{(1+t^2) 2b(1) - 2bt(2t)}{(1+t^2)^2}$$

$$= \frac{2b + 2bt^2 - 4bt^2}{(1+t^2)^2} = \frac{2b - 2bt^2}{(1+t^2)^2}$$

$$= \frac{2b(1-t^2)}{(1+t^2)^2}$$

Now by chain rule

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$= \frac{2b(1-t^2)}{(1+t^2)^2} \cdot \frac{(1+t^2)^2}{-4at}$$

$$\Rightarrow \boxed{\frac{dy}{dx} = -\frac{b(1-t^2)}{2at}}$$

**Question # 4**

Since  $x = \frac{1-t^2}{1+t^2}$

Differentiating w.r.t.  $t$ , we get (solve yourself as above)

$$\frac{dx}{dt} = \frac{-4t}{1+t^2} \Rightarrow \frac{dt}{dx} = \frac{1+t^2}{-4t}$$

Now  $y = \frac{2bt}{1+t^2}$

Differentiating w.r.t.  $t$ , we get (solve yourself as above)

$$\frac{dy}{dt} = \frac{2(1-t^2)}{(1+t^2)^2}$$

Now by chain rule

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$= \frac{2(1-t^2)}{(1+t^2)^2} \cdot \frac{(1+t^2)^2}{-4t}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1-t^2}{2t}$$

Multiplying both sides by  $y$

$$\begin{aligned} \Rightarrow y \frac{dy}{dx} &= -y \cdot \frac{1-t^2}{2t} \\ &= -\frac{2t}{1+t^2} \cdot \frac{1-t^2}{2t} \end{aligned}$$

$$\Rightarrow y \frac{dy}{dx} = -\frac{1-t^2}{1+t^2}$$

$$\Rightarrow y \frac{dy}{dx} = -x \quad \because x = \frac{1-t^2}{1+t^2}$$

$$\Rightarrow y \frac{dy}{dx} + x = 0 \quad \text{Proved.}$$

### Question # 5(i)

Suppose  $y = x^2 - \frac{1}{x^2}$  and  $u = x^4$

Diff.  $y$  w.r.t  $x$

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} \left( x^2 - \frac{1}{x^2} \right) \\ &= \frac{d}{dx} (x^2 - x^{-2}) = 2x + 2x^{-3} \\ &= 2 \left( x + \frac{1}{x^3} \right) \\ \Rightarrow \frac{dy}{dx} &= 2 \left( \frac{x^4 + 1}{x^3} \right) \end{aligned}$$

Now diff.  $u$  w.r.t  $x$

$$\begin{aligned} \frac{du}{dx} &= \frac{d}{dx} (x^4) \\ \Rightarrow \frac{du}{dx} &= 4x^3 \end{aligned}$$

Now by chain rule

$$\begin{aligned} \frac{dy}{du} &= \frac{dy}{dx} \cdot \frac{dx}{du} \\ &= \frac{dy}{dx} \cdot \frac{1}{\frac{du}{dx}} \\ \Rightarrow \frac{dy}{du} &= 2 \left( \frac{x^4 + 1}{x^3} \right) \cdot \frac{1}{4x^3} \\ \Rightarrow \boxed{\frac{dy}{du} = \frac{x^4 + 1}{2x^6}} \end{aligned}$$

### Question # 4(ii)

Let  $y = (1+x^2)^n$  and  $u = x^2$

Differentiation  $y$  w.r.t  $x$

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} (1+x^2)^n \\ &= n(1+x^2)^{n-1} \frac{d}{dx} (1+x^2) \\ &= n(1+x^2)^{n-1} (2x) \\ &= 2nx(1+x^2)^{n-1} \end{aligned}$$

Now differentiating  $u$  w.r.t  $x$

$$\frac{du}{dx} = \frac{d}{dx} x^2$$

$$= 2x \quad \Rightarrow \frac{dx}{du} = \frac{1}{2x}$$

Now by chain rule

$$\begin{aligned} \frac{dy}{du} &= \frac{dy}{dx} \cdot \frac{dx}{du} \\ \Rightarrow \frac{dy}{du} &= 2nx(1+x^2)^{n-1} \cdot \frac{1}{2x} \\ \Rightarrow \boxed{\frac{dy}{du} = n(1+x^2)^{n-1}} \end{aligned}$$

### Question # 5 (iii)

Let  $y = \frac{x^2+1}{x^2-1}$  and  $u = \frac{x-1}{x+1}$

Diff.  $y$  w.r.t  $x$

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} \left( \frac{x^2+1}{x^2-1} \right) \\ &= \text{Solve yourself} = \frac{-4x}{(x^2-1)} \end{aligned}$$

Now diff.  $u$  w.r.t  $x$

$$\begin{aligned} \frac{du}{dx} &= \frac{d}{dx} \left( \frac{x-1}{x+1} \right) \\ &= \text{Solve yourself} = \frac{2}{(x+1)^2} \end{aligned}$$

$$\Rightarrow \frac{du}{dx} = \frac{(x+1)^2}{2}$$

Now by chain rule

$$\begin{aligned} \frac{dy}{du} &= \frac{dy}{dx} \cdot \frac{dx}{du} \\ &= \frac{-4x}{(x^2-1)} \cdot \frac{(x+1)^2}{2} \\ &= \frac{-2x}{(x-1)(x+1)} \cdot (x+1)^2 \\ \Rightarrow \boxed{\frac{dy}{dx} = \frac{-2x(x+1)}{(x-1)}} \end{aligned}$$

### Question # 5(iv)

Let  $y = \frac{ax+b}{cx+d}$  and  $u = \frac{ax^2+b}{ax^2+d}$

Diff.  $y$  w.r.t.  $x$

$$\frac{dy}{dx} = \frac{d}{dx} \left( \frac{ax+b}{cx+d} \right)$$

$$\begin{aligned}
&= \frac{(cx+d)\frac{d}{dx}(ax+b) - (ax+b)\frac{d}{dx}(cx+d)}{(cx+d)^2} \\
&= \frac{(cx+d)(a) - (ax+b)(c)}{(cx+d)^2} \\
&= \frac{acx+ad-acx-bc}{(cx+d)^2} \\
\Rightarrow \frac{dy}{dx} &= \frac{ad-bc}{(cx+d)^2}
\end{aligned}$$

Now diff.  $u$  w.r.t  $x$

$$\begin{aligned}
\frac{du}{dx} &= \frac{d}{dx} \left( \frac{ax^2+b}{ax^2+d} \right) \\
&= \frac{(ax^2+d)\frac{d}{dx}(ax^2+b) - (ax^2+b)\frac{d}{dx}(ax^2+d)}{(ax^2+d)^2} \\
&= \frac{(ax^2+d)(2ax) - (ax^2+b)(2ax)}{(ax^2+d)^2} \\
&= \frac{2ax(ax^2+d-ax^2-b)}{(ax^2+d)^2} \\
&= \frac{2ax(d-b)}{(ax^2+d)^2} \\
\Rightarrow \frac{dx}{du} &= \frac{(ax^2+d)^2}{2ax(d-b)}
\end{aligned}$$

Now by chain rule

$$\begin{aligned}
\frac{dy}{du} &= \frac{dy}{dx} \cdot \frac{dx}{du} \\
&= \frac{ad-bc}{(cx+d)^2} \cdot \frac{(ax^2+d)^2}{2ax(d-b)} \\
\Rightarrow \boxed{\frac{dy}{dx} &= \frac{(ad-bc)(ax^2+d)^2}{2ax(cx+d)^2(d-b)}}
\end{aligned}$$

### Question # 5 (v)

Let  $y = \frac{x^2+1}{x^2-1}$  and  $u = x^3$

Diff.  $y$  w.r.t  $x$

$$\begin{aligned}
\frac{dy}{dx} &= \frac{d}{dx} \left( \frac{x^2+1}{x^2-1} \right) \\
&= \text{Solve yourself} \\
&= \frac{-4x}{(x^2-1)^2}
\end{aligned}$$

Now diff.  $u$  w.r.t  $x$

$$\begin{aligned}
\frac{du}{dx} &= \frac{d}{dx} x^3 \\
&= 3x^2 \\
\Rightarrow \frac{dx}{du} &= \frac{1}{3x^2} \\
\text{Now by chain rule} \\
\frac{dy}{du} &= \frac{dy}{dx} \cdot \frac{dx}{du} \\
&= \frac{-4x}{(x^2-1)^2} \cdot \frac{1}{3x^2} \\
\Rightarrow \boxed{\frac{dy}{dx} &= \frac{-4}{3x(x^2-1)^2}}
\end{aligned}$$