Exercise 6.9

Question # 1

A man deposits in a bank Rs. 8 in the first year, Rs. 24 in the second year, Rs. 72 in the third year and so on. Find the amount he will have deposited in the bank by the fifth year.

Solution

The sequence of deposit is 8, 24, 72, …………

Here $a_1 = 8 \;\; r = \frac{24}{8} = \frac{72}{24} = 3, \; n = 5$

Since $S_n = \frac{a_1(r^n - 1)}{r - 1} \qquad \Rightarrow \; S_5 = \frac{8(3^5 - 1)}{3 - 1} = \frac{8(243 - 1)}{2} = 4(242) = 968$

Thus he has to deposited Rs. 968 up to the fifth year.

Question # 2

A man borrow Rs. 32760 with out interest and agrees to repay the loan in installment being twice the preceding one. Find the amount of the last installment, if the amount of the first installment is Rs. 8.

Solution

Here $a_1 = 8, \quad r = 2, \quad S_n = 32760, \quad n = ?, \quad a_n = ?$

Since $S_n = \frac{a_1(r^n - 1)}{r - 1}$

$\Rightarrow \; 32760 = \frac{8(2^n - 1)}{2 - 1} \qquad \Rightarrow \; 32760 = \frac{8(2^n - 1)}{1} \qquad \Rightarrow \; 32760 = 8(2^n - 1)$

$\Rightarrow \; 4095 = (2^n - 1) \qquad \Rightarrow \; 4095 + 1 = 2^n \qquad \Rightarrow \; 4096 = 2^n$

$\Rightarrow \; (2)^{12} = 2^n \qquad \Rightarrow \; 12 = n$

Now $a_{12} = a_1 r^{11} \quad \Rightarrow \; a_{12} = (8)(2)^{11} = (8)(2048) = 16384$

Hence the last instalment is Rs. 16384.

Question # 3

The population of a certain village is 62500. What will be its population after 3 years if it increase geometrically at the rate of 4% annually?

Solution

Here $a_1 = 62500, \;\; n = 4, \;\; r = 1 + \frac{4}{100} = 1 + 0.04 = 1.04$

Since $a_n = a_1 r^{n-1} \;\; \Rightarrow \;\; a_4 = (62500)(1.04)^{4-1} = (62500)(1.04)^3$

$= (62500)(1.1249) = 70304$

Thus the population after 3 years is 70304.

Question # 4

The enrollment of a famous school doubled after every eight years from 1970 to 1994. If the enrollment was 6000 in 1994, what was its enrollment in 1970.

Solution

Let the enrolment in 1970 is a_1

also $a_n = 6000$, $r = 2$, $n = 4$

Since $a_n = a_1 r^{n-1}$

$\Rightarrow\ 6000 = a_1(2)^{4-1} \quad \Rightarrow\ 6000 = a_1(2)^3 \quad \Rightarrow\ 6000 = a_1(8)$

$\Rightarrow\ \dfrac{6000}{4} = a_1 \Rightarrow\ a_1 = 750$

Thus the enrolment was 750.

Question # 5

A singular cholera bacteria produces two complete bacteria in ½ hours. If we starts with a colony of A bacteria, how many bacteria will we have in n hours.

Solution

The colony of bacteria in the start = $a_1 = A$

Then $r = 2$, $n = 2n + 1$

Since $a_n = a_1 r^{n-1} \quad \Rightarrow\ a_{2n+1} = (A)(2)^{2n+1-1} = A(2)^{2n}$

Thus bacteria after n hours will be $A(2)^n$.

Question # 6

Joining the mid points of the sides of an equilateral triangle, an equilateral triangle having half the perimeter of the original triangle is obtained. We form a sequence of nested equilateral triangles in the manner described above with the original triangle having parameter $\dfrac{3}{2}$. What will be the total perimeter of all the triangles formed in this way?

Solution

Here $a_1 = \dfrac{3}{2}$, $r = \dfrac{1}{2}$

So the series is $\dfrac{3}{2} + \dfrac{3}{4} + \dfrac{3}{8} + \ldots\ldots\ldots\ldots\ldots$

Which is infinite geometric series

Now $S = \dfrac{a_1}{1-r} = \dfrac{3/2}{1-1/2} = \dfrac{3/2}{1/2} = 3$ Answer

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