EXERCISE 2.1

1. Write the following sets in set-builder notation :

Solution.

(i) $\{1, 2, 3, \ldots, 1000\} = \{x \mid x \in \mathrm{N} \wedge x \leq 1000\}$

(ii) $\{0, 1, 2, \ldots, 100\} = \{x \mid x \in \mathrm{W} \wedge x \leq 100\}$

(iii) $\{0, \pm 1, \pm 2, \ldots, \pm 1000\} = \{x \mid x \in \mathrm{Z} \wedge -1000 \leq x \leq 1000\}$

(iv) $\{0, -1, -2, \ldots, -500\} = \{x \mid x \in \mathrm{Z} \wedge -500 \leq x \leq 0\}$

(v) $\{100, 101, 102, \ldots, 400\} = \{x \mid x \in \mathrm{Z} \wedge 100 \leq x \leq 400\}$

$= \{x \mid x \in \mathrm{N} \wedge 100 \leq x \leq 400\}$

(vi) $\{-100, -101, -102, \ldots, -500\} = \{x \mid x \in \mathrm{Z} \wedge -500 \leq x \leq -100\}$

(vii) {Peshawar, Lahore, Karachi, Quetta}

$= \{x \mid x$ is a capital of a province of Pakistan $\}$

(viii) {January, June, July}

$= \{x \mid x$ is a month of the Calender year beginning with letterJ $\}$

(ix) **The set of all odd natural numbers**

$= \{x \mid x$ is an odd natural number$\}$

(x) **The set of all rational numbers.** $= \{x \mid x \in Q\}$

(xi) **The set of all real numbers between 1 and 2** $= \{x \mid x \in R \wedge 1 < x < 2\}$

(xii) **The set of all integers between –100 and 1000**

$= \{x \mid x \in Z \wedge -100 < x < 1000\}$

2. Write each of the following sets in descriptive and tabular form:

Solution.

(i) $\{x \mid x \in N \wedge x \leq 10\}$

Descriptive Form ↓	*Tabular Form* ↓
The set of first ten natural numbers	{1, 2, 3, .. , 10}
(ii) $\{x \mid x \in N \wedge 4 < x < 12\}$	
The set of natural numbers between 4 and 12	{5, 6, 7, ... , 11}
(iii) $\{x \mid x \in Z \wedge -5 < x < 5\}$	
The set of integers between –5 and 5	{–4, –3, –2, ... , 4}
(iv) $\{x \mid x \in E \wedge 2 < x \leq 4\}$	
The set of even integers between 2 and 5	{4}
(v) $\{x \mid x \in P \wedge x < 12\}$	
The set of prime numbers less than 12	{2, 3, 5, 7, 11}
(vi) $\{x \mid x \in O \wedge 3 < x < 12\}$	
The set of odd integers between 3 and 12	{5, 7, 9, 11}
(vii) $\{x \mid x \in E \wedge 4 \leq x \leq 10\}$	
The set of even integers between 2 and 12	{4, 6, 8, 10}
(viii) $\{x \mid x \in E \wedge 4 < x < 6\}$	
The set of even integers between 4 and 6	{ }
(ix) $\{x \mid x \in O \wedge 5 \leq x \leq 7\}$	
The set of odd integers from 5 upto 7	{5, 7}
(x) $\{x \mid x \in O \wedge 5 < x < 7\}$	
The set of odd integers greater or equal 5 and less than 7	{ }
(xi) $\{x \mid x \in N \wedge x + 4 = 0\}$	

The set of natural numbers x satisfying $x + 4 = 0$

Tabular form : as $x + 4 = 0 \Rightarrow x = -4$ which $\notin N \Rightarrow \{\ \}$

(xii) $\{x \mid x \in Q \wedge x^2 = 2\}$

The set of rational numbers x satisfying $x^2 = 2$

Tabular form : as $x^2 = 2 \Rightarrow x = \sqrt{2}$ which $\notin Q \Rightarrow \{\ \}$

(xiii) $\{x \mid x \in R \wedge x = x\}$

The set of real numbers x satisfying $x = x$

Tabular form : $x = x$ is satisfied by all reals. $\Rightarrow$ not possible

(xiv) $\{x \mid x \in Q \wedge x = -x\}$

The set of rational numbers x satisfying $x = -x$

Tabular form : $x = -x \Rightarrow x + x = 0 \Rightarrow 2x = 0 \Rightarrow x = 0 \Rightarrow \{0\}$

(xv) $\{x \mid x \in R \wedge x \neq x\}$

The set of real numbers x satisfying $x \neq x$

Tabular form : $x \neq x$ as there is no real number which is not equal to itself $\Rightarrow \{\ \}$

(xvi) $\{x \mid x \in R \wedge x \notin Q\}$

The set of real numbers which are not rational. not possible

Tabular form : set of reals is the union of rational & irrational numbers, so irrational $\Rightarrow Q'$

3. Which of the following sets are finite and which of these are infinite ?

Solution.

(i)	**The set of students of your class.**	[Finite]
(ii)	**The set of all schools in Pakistan.**	[Finite]
(iii)	**The set of natural numbers between 3 and 10.**	[Finite]
(iv)	**The set of rational numbers between 3 and 10.**	[Infinite]
(v)	**The set of real numbers between 0 and 1.**	[Infinite]
(vi)	**The set of rationals between 0 and 1.**	[Infinite]
(vii)	**The set of whole between 0 and 1.**	[Finite]
(viii)	**The set of all leaves of trees in Pakistan.**	[Finite]
(ix)	$P(N)$	[Infinite]
(x)	$P\{a, b, c\}$	[Finite]

(xi) $\{1, 2, 3, 4, \ldots\}$ [Infinite]

(xii) $\{1, 2, 3, \ldots, 100000000\}$ [Finite]

(xiii) $\{x \mid x \in R \wedge x \neq x\}$ [Finite]

(xiv) $\{x \mid x \in R \wedge x^2 = -16\}$ [Finite]

(xv) $\{x \mid x \in Q \wedge x^2 = 5\}$ [Finite]

(xvi) $\{x \mid x \in Q \wedge 0 \leq x \leq 1\}$ [Infinite]

4. Write two proper subsets of each of the following sets :

(i) $\{a, b, c\}$ (ii) $\{0, 1\}$ (iii) N (iv) Z (v) Q

(vi) R (vii) W (viii) $\{x \mid x \in Q \wedge 0 < x \leq 2\}$

Solution.

(i) Two proper subsets of $\{a, b, c\}$ are : $\{a\}, \{a, b\}$

(ii) Two proper subsets of $\{0, 1\}$ are : $\{0\}, \{1\}$

(iii) Two proper subsets of N are : $\{1\}, \{1, 2\}$

(iv) Two proper subsets of Z are : $\{1\}, \{1, 2\}$

(v) Two proper subsets of Q are : $\{1\}, \{1, 2\}$

(vi) Two proper subsets of R are : $\{1\}, \{1, 2\}$

(vii) Two proper subsets of W are : $\{1\}, \{1, 2\}$

(viii) Two proper subsets of $\{x \mid x \in Q \wedge 0 < x \leq 2\}$ are : $\{1\}, \{1, 2\}$

5. Is there any set which has no proper subset ? If so, name the set.

Solution. Yes, empty set or $\{\}$ or $\varnothing$ is the set which has no proper subset.

6. What is the difference between $\{a, b\}$ and $\{\{a, b\}\}$?

Solution.

$\{a, b\}$ is a set which contains two elements a and b.

and $\{\{a, b\}\}$ is a set which contains only one element $\{a, b\}$.

7. Which of the following sentences are true & which of them are false ?

(i) $\{1, 2\} = \{2, 1\}$ (ii) $\varnothing \subseteq \{\{a\}\}$ (iii) $\{a\} \subseteq \{\{a\}\}$

(iv) $\{a\} \in \{\{a\}\}$ (v) $a \in \{\{a\}\}$ (vi) $\varnothing \in \{\{a\}\}$

Solution.

(i) $\{1, 2\} = \{2, 1\}$ [True]

(ii) $\varnothing \subseteq \{\{a\}\}$ [True]

(iii) $\{a\} \subseteq \{\{a\}\}$ [False]

(iv) $\{a\} \in \{\{a\}\}$ [True]

(v) $a \in \{\{a\}\}$ [False]

(vi) $\varnothing \in \{\{a\}\}$ [False]

8. What is the number of elements of the power set of each of the following sets ?

(i) $\{\ \}$ (ii) $\{0, 1\}$ (iii) $\{1, 2, 3, 4, 5, 6, 7\}$

(iv) $\{0, 1, 2, 3, 4, 5, 6, 7\}$ (v) $\{a, \{b, c\}\}$ (vi) $\{\{a, b\}, (b, c\}, \{d, e\}\}$

Solution.

Note that formula to find the number of elements in power set is 2^n.

Number of elements in the

(i) Power set of $\{\ \}$ is $2^0 = 1$

(ii) Power set of $\{0, 1\}$ is $2^2 = 4$

(iii) Power set of $\{1, 2, 3, 4, 5, 6, 7\}$ is $2^7 = 128$

(iv) Power set of $\{0, 1, 2, 3, 4, 5, 6, 7\}$ is $2^8 = 256$

(v) Power set of $\{a, \{b, c\}\}$ is $2^2 = 4$

(v) Power set of $\{\{a, b\}, (b, c\}, \{d, e\}\}$ is $2^3 = 8$

9. Write down the power set of each of the following sets :

(i) $\{9, 11\}$ (ii) $\{+, -, \times, \div\}$ (iii) $\{\varnothing\}$ (iv) $\{a, \{b, c\}\}$

Solution.

(i) Power set of $\{9, 11\}$ is : $\{\varnothing, \{9\}, \{11\}, \{9, 11\}\}$

(ii) Power set of $\{+, -, \times, \div\}$ is $\{\varnothing, \{+\}, \{-\}, \{\times\}, \{+\}, \{+, -\}, \{+, \times\}, \{+, \div\}, \{-, \times\}, \{-, \div\}, \{\times, \div\}, \{+, -, \times\}, \{+, -, \div\}, \{+, \times, \div\}, \{-, \times, \div\}, \{+, -, \times, \div\}\}$

(iii) Power set of $\{\varnothing\}$ is : $\{\varnothing, \{\varnothing\}\}$

(iv) Power set of $\{a, \{b, c\}\}$ is : $\{\varnothing, \{a\}, \{\{b, c\}\}, \{a, \{b, c\}\}\}$

10. Which of the pairs of sets are equivalent ? Which of them are also equal ?

Solution.

(i) $\{a, b, c\}$, $\{1, 2, 3\}$

are equivalent sets (since, each has three elements)

(ii) **The set of first 10 whole numbers ;** $\{0, 1, 2, \ldots, 9\}$

are equal sets (since, each has same ten elements)

(iii) **The set of angles of a quadrilateral *ABCD*;**
set of the sides of the same quadrilateral.
are equivalent sets (since, each has four elements)

(iv) **Set of the sides of a hexagon *ABCDEF*;**
set of the angles of the same hexagon.
are equivalent sets (since, each has six elements)

(v) **{ 1, 2, 3, 4, ... } ; { 2, 4, 6, 8, ... }**
are equivalent sets (since, 1–1 correspondence can be established)

(vi) **{ 1, 2, 3, 4, ... } ;** $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\right\}$
are equivalent sets (since, 1–1 correspondence can be established)

(vii) **{ 5, 10, 15, 20, ..., 55555 } ; { 5, 10, 15, 20 , ... }**
are not equivalent sets
(since, first set has finite and second infinite number of elements).