EXERCISE 3.4

1. $A=\begin{bmatrix}1&-2&5\\-2&3&-1\\5&-1&0\end{bmatrix}$ $\quad B=\begin{bmatrix}-3&1&-2\\1&0&-1\\-2&-1&2\end{bmatrix}$

Show That $(A+B)$ is Symmetric

Sol.

$$A+B=\begin{bmatrix}1&-2&5\\-2&3&-1\\5&-1&0\end{bmatrix}+\begin{bmatrix}-3&1&-2\\1&0&-1\\-2&-1&2\end{bmatrix}$$

$$=\begin{bmatrix}-2&-1&3\\-1&3&-2\\3&-2&2\end{bmatrix}$$

$$(A+B)^t=\begin{bmatrix}-2&-1&3\\-1&3&-2\\3&-2&2\end{bmatrix}=A+B$$

Hence $(A+B)$ is symmetric Matrix

2. If $A=\begin{bmatrix}1&2&0\\3&2&-1\\-1&3&2\end{bmatrix}$

Then show That

i) $A+A^t$ is symmetric

Sol.

$$A+A^t=\begin{bmatrix}1&2&0\\3&2&-1\\-1&3&2\end{bmatrix}+\begin{bmatrix}1&3&-1\\2&2&3\\0&-1&2\end{bmatrix}$$

$$=\begin{bmatrix}2&5&-1\\5&4&2\\-1&2&4\end{bmatrix}$$

$$(A+A^t)^t=\begin{bmatrix}2&5&-1\\5&4&2\\-1&2&4\end{bmatrix}=A+A^t$$

$\Rightarrow$ $(A+A^t)$ is symmetric

ii) $$A-A^t=\begin{bmatrix}1&2&0\\3&2&-1\\-1&3&2\end{bmatrix}-\begin{bmatrix}1&3&-1\\2&2&3\\0&-1&2\end{bmatrix}$$

$$=\begin{bmatrix}0&-1&1\\1&0&-4\\-1&4&0\end{bmatrix}$$

$$(A-A^t)^t=\begin{bmatrix}0&1&-1\\-1&0&4\\1&-4&0\end{bmatrix}=-\begin{bmatrix}0&-1&1\\1&0&-4\\-1&4&0\end{bmatrix}$$

$$=-(A-A^t)$$

$\Rightarrow$ $A-A^t$ is skew Symmetric

3 i) If A is square matrix of order 3 Show That $A+A^t$ is symmetric

Sol. $(A+A^t)^t=A^t+(A^t)^t$

$=A^t+A=A+A^t$

$\Rightarrow$ $A+A^t$ is symmetric

ii) $A-A^t$ is skew Symmetric

$(A-A^t)^t=A^t-(A^t)^t=A^t-A$

$=-(A-A^t)$

$\Rightarrow$ $A-A^t$ is Skew Symmetric

4. If the matrices A and B are symmetric matrix $AB=BA$ Show that AB is symmetric

Sol $A^t=A \quad B^t=B$

To prove $(AB)^t=AB$

L.H.S $(AB)^t=B^tA^t=BA=AB=$ R.H.S

Hence AB is symmetric

5. Show that AA^t and A^tA are Symmetric for any matrix of order 2×3

Sol. let $A=\begin{bmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\end{bmatrix}$

$$AA^t=\begin{bmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\end{bmatrix}\begin{bmatrix}a_{11}&a_{21}\\a_{12}&a_{22}\\a_{13}&a_{23}\end{bmatrix}$$

$$=\begin{bmatrix}a_{11}^2+a_{12}^2+a_{13}^2 & a_{11}a_{21}+a_{12}a_{22}+a_{13}a_{23}\\ a_{21}a_{11}+a_{22}a_{12}+a_{23}a_{13} & a_{21}^2+a_{22}^2+a_{23}^2\end{bmatrix}$$

$$(AA^t)^t=\begin{bmatrix}a_{11}^2+a_{12}^2+a_{13}^2 & a_{21}a_{11}+a_{22}a_{12}+a_{23}a_{13}\\ a_{11}a_{12}+a_{12}a_{22}+a_{13}a_{23} & a_{21}^2+a_{22}^2+a_{23}^2\end{bmatrix}$$

$=AA^t$

Hence AA^t is symmetric

$$A^tA=\begin{bmatrix}a_{11}&a_{21}\\a_{12}&a_{22}\\a_{13}&a_{23}\end{bmatrix}\begin{bmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\end{bmatrix}$$

$$=\begin{bmatrix}a_{11}^2+a_{21}^2 & a_{11}a_{12}+a_{21}a_{22} & a_{11}a_{13}+a_{21}a_{23}\\ a_{21}a_{11}+a_{22}a_{21} & a_{12}^2+a_{22}^2 & a_{12}a_{13}+a_{22}a_{23}\\ a_{11}a_{13}+a_{21}a_{23} & a_{12}a_{13}+a_{22}a_{23} & a_{13}^2+a_{23}^2\end{bmatrix}$$

$$(A^tA)^t=\begin{bmatrix}a_{11}^2+a_{21}^2 & a_{21}a_{11}+a_{22}a_{21} & a_{13}a_{11}+a_{23}a_{21}\\ a_{11}a_{12}+a_{21}a_{22} & a_{12}^2+a_{22}^2 & a_{13}a_{12}+a_{23}a_{22}\\ a_{11}a_{13}+a_{21}a_{23} & a_{12}a_{13}+a_{22}a_{23} & a_{13}^2+a_{23}^2\end{bmatrix}$$

$=A^tA$

Thus A^tA is symmetric

6. i) If $A = \begin{pmatrix} i & 1+i \\ 1 & -i \end{pmatrix}$

Show that $A+(\bar{A})^t$ is Hermitian

To prove $\left(\overline{A+(\bar{A})^t}\right)^t = A+(\bar{A})^t$

sol. $\bar{A} = \begin{bmatrix} -i & 1-i \\ 1 & i \end{bmatrix}$ $\quad (\bar{A})^t = \begin{bmatrix} -i & 1 \\ 1-i & i \end{bmatrix}$

$A+(\bar{A})^t = \begin{bmatrix} i & 1+i \\ 1 & -i \end{bmatrix} + \begin{bmatrix} -i & 1 \\ 1-i & i \end{bmatrix} = \begin{bmatrix} 0 & 2+i \\ 2-i & 0 \end{bmatrix}$

$\overline{A+(\bar{A})^t} = \begin{pmatrix} 0 & 2-i \\ 2+i & 0 \end{pmatrix}$

$\left(\overline{A+(\bar{A})^t}\right)^t = \begin{pmatrix} 0 & 2+i \\ 2-i & 0 \end{pmatrix}$

$= A+(\bar{A})^t$

Thus $A+(\bar{A})^t$ is Hermitian

ii) $A-(\bar{A})^t$ is skew Hermitian

To prove $\left(\overline{A-(\bar{A})^t}\right)^t = -\left(A-(\bar{A})^t\right)$

$A-(\bar{A})^t = \begin{bmatrix} i & 1+i \\ 1 & -i \end{bmatrix} - \begin{bmatrix} -i & 1 \\ 1-i & i \end{bmatrix} = \begin{bmatrix} 2i & i \\ i & -2i \end{bmatrix}$

$\overline{A-(\bar{A})^t} = \begin{bmatrix} -2i & -i \\ -i & 2i \end{bmatrix}$

$\left(\overline{A-(\bar{A})^t}\right)^t = \begin{bmatrix} -2i & -i \\ -i & 2i \end{bmatrix}$

$= -\begin{bmatrix} 2i & i \\ i & -2i \end{bmatrix} = -\left(A-(\bar{A})^t\right)$

Thus $A-(\bar{A})^t$ is skew Hermitian.

7. If A is symmetric or skew symmetric show that A^2 symmetric

sol. If A is symmetric $A^t = A$

To prove $(A^2)^t = A^2$

$(A^2)^t = (A\cdot A)^t = A^t A^t = A A = A^2$

Thus A^2 is symmetric

If A is skew symmetric $A^t = -A$

$(A^2)^t = (A\cdot A)^t = A^t A^t = (-A)(-A) = A^2$

Thus A^2 is symmetric

8. If $A = \begin{bmatrix} 1 \\ 1-i \\ -i \end{bmatrix}$ $\quad A(\bar{A})^t = ?$

Sol $\bar{A} = \begin{pmatrix} 1 \\ 1+i \\ i \end{pmatrix}$ $\quad (\bar{A})^t = \begin{pmatrix} 1 & 1+i & i \end{pmatrix}$

$A(\bar{A})^t = \begin{pmatrix} 1 \\ 1-i \\ -i \end{pmatrix} \begin{pmatrix} 1 & 1+i & i \end{pmatrix}$

$A(\bar{A})^t = \begin{pmatrix} 1 & 1-i & -i \\ 1+i & 1-i^2 & -i-i^2 \\ i & i-i^2 & -i^2 \end{pmatrix}$

$= \begin{pmatrix} 1 & 1-i & -i \\ 1+i & 2 & -i+1 \\ i & 1+i & 1 \end{pmatrix}$

9. Find the inverse by matrices, Row, Column

i) $A = \begin{pmatrix} 1 & 2 & -3 \\ 0 & -2 & 0 \\ -2 & -2 & 2 \end{pmatrix}$

Cofactors of $A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$

$A_{11} = (-1)^{1+1} \begin{vmatrix} -2 & 0 \\ 2 & 2 \end{vmatrix} = (-1)^2(-4-0) = -4$

$A_{12} = (-1)^{1+2} \begin{vmatrix} 0 & 0 \\ -2 & 2 \end{vmatrix} = (-1)^3(0+0) = 0$

$A_{13} = (-1)^{1+3} \begin{vmatrix} 0 & -2 \\ -2 & -2 \end{vmatrix} = (-1)^4(0-4) = -4$

$A_{21} = (-1)^{2+1} \begin{vmatrix} 2 & -3 \\ -2 & 2 \end{vmatrix} = (-1)^3(4-6) = 2$

$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & -3 \\ -2 & 2 \end{vmatrix} = (-1)^4(2-6) = -4$

$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 2 \\ -2 & -2 \end{vmatrix} = (-1)^5(-2+4) = -2$

$A_{31} = (-1)^{3+1} \begin{vmatrix} 2 & -3 \\ -2 & 0 \end{vmatrix} = (-1)^4(0-6) = -6$

$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & -3 \\ 0 & 0 \end{vmatrix} = (-1)^5(0+0) = 0$

$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 2 \\ 0 & -2 \end{vmatrix} = (-1)^6(-2-0) = -2$

Cofactors of $A = \begin{pmatrix} -4 & 0 & -4 \\ 2 & -4 & -2 \\ -6 & 0 & -2 \end{pmatrix}$

$\text{Adj}A = (\text{Cofactors of }A)^t = \begin{bmatrix} -4 & 2 & -6 \\ 0 & -4 & 0 \\ -4 & -2 & -2 \end{bmatrix}$

$|A| = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$

$= 1(-4) + 2(0) + (-3)(-4)$

$= -4 + 0 + 12 = 8$

$A^{-1} = \frac{1}{|A|}\text{Adj}A = \frac{1}{8}\begin{bmatrix} -4 & 2 & -6 \\ 0 & -4 & 0 \\ -4 & -2 & -2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{4} & -\frac{3}{4} \\ 0 & -\frac{1}{2} & 0 \\ -\frac{1}{2} & -\frac{1}{4} & -\frac{1}{4} \end{bmatrix}$

Find Inverse of A by Column operation

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & -2 & 0 \\ -2 & -2 & 2 \\ \hline 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \underset{\sim}{C} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ -2 & 2 & -4 \\ \hline 1 & -2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ By $C_2 - 2C_1 \to C_2'$, $C_3 + 3C_1 \to C_3'$

$$\underset{\sim}{C} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & -1 & -4 \\ \hline 1 & 1 & 3 \\ 0 & -1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ By $-\frac{1}{2}C_2 \to C_2'$

$$\underset{\sim}{C} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & -1 & 1 \\ \hline 1 & 1 & -3/4 \\ 0 & -1/2 & 0 \\ 0 & 0 & -1/4 \end{bmatrix}$$ By $-\frac{1}{4}C_1 \to C_1'$

$$\underset{\sim}{C} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \hline -1/2 & 1/4 & -3/4 \\ 0 & -1/2 & 0 \\ -1/2 & -1/4 & -1/4 \end{bmatrix}$$ By $C_1 + 2C_3 \to C_1'$, $C_2 + C_3 \to C_2'$

Hence the inverse of matrix A is $\begin{bmatrix} -1/2 & 1/4 & -3/4 \\ 0 & -1/2 & 0 \\ -1/2 & -1/4 & -1/4 \end{bmatrix}$

10. Find the rank of the matrices

(i) $\begin{bmatrix} 1 & -1 & 2 & 1 \\ 2 & -6 & 5 & 1 \\ 3 & 5 & 4 & -3 \end{bmatrix}$

Sol. $\underset{\sim}{R} \begin{bmatrix} 1 & -1 & 2 & 1 \\ 0 & -4 & 1 & -1 \\ 0 & 8 & -2 & -6 \end{bmatrix}$ By $R_2 - 2R_1 \to R_2'$, $R_3 - 3R_1 \to R_3'$

$\underset{\sim}{R} \begin{bmatrix} 1 & -1 & 2 & 1 \\ 0 & 1 & -1/4 & 1/4 \\ 0 & 4 & -1 & -3 \end{bmatrix}$ By $-\frac{1}{4}R_2 \to R_2'$, $\frac{1}{2}R_3 \to R_3'$

$\underset{\sim}{R} \begin{bmatrix} 1 & 0 & 7/4 & 5/4 \\ 0 & 1 & -1/4 & 1/4 \\ 0 & 0 & 0 & -4 \end{bmatrix}$ By $R_1 + R_2 \to R_1'$, $R_3 - 4R_2 \to R_3'$

$\underset{\sim}{R} \begin{bmatrix} 1 & 0 & 7/4 & 5/4 \\ 0 & 1 & -1/4 & 1/4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ By $-\frac{1}{4}R_3 \to R_3'$

Hence the rank of given matrix is 3

(ii) $\begin{bmatrix} 1 & -4 & 0 \\ 2 & -5 & 1 \\ 1 & -2 & 3 \\ 3 & -7 & 4 \end{bmatrix}$

$$R \sim \begin{pmatrix} 1 & -4 & -7 \\ 0 & 3 & 15 \\ 0 & 2 & 10 \\ 0 & 5 & 25 \end{pmatrix} \begin{matrix} R_2 - 2R_1 \to R_2' \\ R_3 - R_1 \to R_3' \\ R_4 - 3R_1 \to R_4' \end{matrix} \quad R \sim \begin{pmatrix} 1 & -4 & -7 \\ 0 & 1 & 5 \\ 0 & 2 & 10 \\ 0 & 1 & 5 \end{pmatrix} \begin{matrix} R_2 - R_3 \to R_2' \\ \frac{1}{5}R_3 \to R_3' \end{matrix}$$

$$R \sim \begin{pmatrix} 1 & 0 & 13 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{ By } \begin{matrix} R_1 + 4R_2 \to R_1', \quad R_1 - R_2 \to R_4' \\ R_3 - 2R_2 \to R_3 \end{matrix}$$

Hence The rank of given matrix is 2

(iii) $\begin{pmatrix} 3 & -1 & 3 & 0 & -1 \\ 1 & 2 & -1 & -3 & -2 \\ 2 & 3 & 4 & 2 & 5 \\ 2 & 5 & -2 & -3 & 3 \end{pmatrix}$

Sol. $\begin{pmatrix} 1 & -1 & 3 & 0 & -1 \\ 3 & 2 & -1 & -3 & -2 \\ 2 & 3 & 4 & 2 & 5 \\ 2 & 5 & -2 & -3 & 3 \end{pmatrix}$ By $R_1 \leftrightarrow R_2$

$$R \sim \begin{pmatrix} 1 & 2 & -1 & -3 & -2 \\ 0 & -7 & 6 & 9 & 5 \\ 0 & -1 & 6 & 8 & 9 \\ 0 & 1 & 0 & 3 & 7 \end{pmatrix} \text{ By } \begin{matrix} R_2 - 3R_1 \to R_2' \\ R_3 - 2R_1 \to R_3' \\ R_4 - 2R_1 \to R_4' \end{matrix}$$

$$R \sim \begin{pmatrix} 1 & 2 & -1 & -3 & -2 \\ 0 & 1 & 0 & 3 & 7 \\ 0 & -1 & 6 & 8 & 9 \\ 0 & -7 & 6 & 9 & 5 \end{pmatrix} \text{ By } R_2 \leftrightarrow R_4$$

$$R \sim \begin{pmatrix} 1 & 0 & -1 & -9 & -16 \\ 0 & 1 & 0 & 3 & 7 \\ 0 & 0 & 6 & 11 & 16 \\ 0 & 0 & 6 & 30 & 54 \end{pmatrix} \text{ By } \begin{matrix} R_1 - 2R_2 \to R_1' \\ R_3 + R_2 \to R_3' \\ R_4 + 7R_2 \to R_4' \end{matrix}$$

$$R \sim \begin{pmatrix} 1 & 0 & -1 & -9 & -16 \\ 0 & 1 & 0 & 3 & 7 \\ 0 & 0 & 6 & 11 & 16 \\ 0 & 0 & 1 & 5 & 9 \end{pmatrix} \quad \frac{1}{6}R_4 \to R_4'$$

$$R \sim \begin{pmatrix} 1 & 0 & -1 & -9 & -16 \\ 0 & 1 & 0 & 3 & 7 \\ 0 & 0 & 1 & 5 & 9 \\ 0 & 0 & 6 & 11 & 16 \end{pmatrix} \quad R_3 \leftrightarrow R_4$$

$$R \sim \begin{pmatrix} 1 & 0 & 0 & -4 & -7 \\ 0 & 1 & 0 & 3 & 7 \\ 0 & 0 & 1 & 5 & 9 \\ 0 & 0 & 0 & -19 & -38 \end{pmatrix} \quad \begin{matrix} R_1 + R_3 \to R_1' \\ R_4 - 6R_3 \to R_4' \end{matrix}$$

$$R \sim \begin{pmatrix} 1 & 0 & 0 & -4 & -7 \\ 0 & 1 & 0 & 3 & 7 \\ 0 & 0 & 1 & 5 & 9 \\ 0 & 0 & 0 & 1 & 2 \end{pmatrix} \quad -\frac{1}{19}R_4 \to R_4'$$

$$R \sim \begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 2 \end{pmatrix} \quad \begin{matrix} R_1 + 4R_4 \to R_1' \\ R_2 - 3R_4 \to R_2' \\ R_3 - 5R_4 \to R_3' \end{matrix}$$

Hence the rank of given matrix is 4.

9 (ii) Let $B = \begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 3 \\ 1 & 0 & 2 \end{bmatrix}$

Sol. Cofactors of $B = \begin{pmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{pmatrix}$

$B_{11} = (-1)^{1+1} \begin{vmatrix} -1 & 3 \\ 0 & 2 \end{vmatrix} = (-1)^2(-2-0) = 1(-2) = -2$

$B_{12} = (-1)^{1+2} \begin{vmatrix} 0 & 3 \\ 1 & 2 \end{vmatrix} = (-1)^3(0-3) = -1(-3) = 3$

$B_{13} = (-1)^{1+3} \begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix} = (-1)^4(0+1) = 1(1) = 1$

$B_{21} = (-1)^{2+1} \begin{vmatrix} 2 & -1 \\ 0 & 2 \end{vmatrix} = (-1)^3(4+0) = -1(4) = -4$

$B_{22} = (-1)^{2+2} \begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix} = (-1)^4(2+1) = 1(3) = 3$

$B_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix} = (-1)^5(0-2) = -1(-2) = 2$

$B_{31} = (-1)^{3+1} \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix} = (-1)^4(6-1) = 1(5) = 5$

$B_{32} = (-1)^{3+2} \begin{vmatrix} 1 & -1 \\ 0 & 3 \end{vmatrix} = (-1)^5(3+0) = -1(3) = -3$

$B_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 2 \\ 0 & -1 \end{vmatrix} = (-1)^6(-1-0) = 1.(-1) = -1$

Cofactors of $B = \begin{pmatrix} -2 & 3 & 1 \\ -4 & 3 & 2 \\ 5 & -3 & -1 \end{pmatrix}$

$\text{Adj } B = (\text{Cofactors of } B)^t = \begin{pmatrix} -2 & -4 & 5 \\ 3 & 3 & -3 \\ 1 & 2 & -1 \end{pmatrix}$

$|B| = \begin{vmatrix} 1 & 2 & -1 \\ 0 & -1 & 3 \\ 1 & 0 & 2 \end{vmatrix} = 1(-2-0) - 2(0-3) - 1(0+1)$

$= 1(-2) - 2(-3) - 1(1) = -2 + 6 - 1 = 3$

$B^{-1} = \frac{1}{|B|} \text{Adj } B = \frac{1}{3}\begin{pmatrix} -2 & -4 & 5 \\ 3 & 3 & -3 \\ 1 & 2 & -1 \end{pmatrix} = \begin{pmatrix} -2/3 & -4/3 & 5/3 \\ 1 & 1 & -1 \\ 1/3 & 2/3 & -1/3 \end{pmatrix}$

Now B^{-1} By Row operations

$B = \left(\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & -1 & 3 & 0 & 1 & 0 \\ 1 & 0 & 2 & 0 & 0 & 1 \end{array}\right)$

$\sim \left(\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & -1 & 3 & 0 & 1 & 0 \\ 0 & -2 & 3 & -1 & 0 & 1 \end{array}\right) \quad R_3 - R_1 \to R_3'$

$\sim \left(\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -3 & 0 & -1 & 0 \\ 0 & -2 & 3 & -1 & 0 & 1 \end{array}\right) \quad -R_2 \to R_2'$

$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 5 & 1 & 2 & 0 \\ 0 & 1 & -3 & 0 & -1 & 0 \\ 0 & 0 & -3 & -1 & 2 & 1 \end{array}\right) \quad R_1 - 2R_2 \to R_1', \; R_3 + 2R_2 \to R_3'$

$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 5 & 1 & 2 & 0 \\ 0 & 1 & -3 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1/3 & 2/3 & -1/3 \end{array}\right) \quad -\frac{1}{3}R_3 \to R_3'$