SOLVED EXERCISE 3.2

1. If y varies directly as x, and y = 8 when x = 1, find

(i) y in terms of x

Solution:

Given that y varies directly as x.

Therefore $y \propto x$

$\Rightarrow$ $y = kx$ _______ (i)

Where k is constant of variation.

Put x = 2 and y = 8 in eq. (i), we have

$8 = k(2)$

or $2k = 8$

$\Rightarrow$ $k = 4$

Put k = 4 in eq. (i), we get

$y = 4x.$

(ii) y when x = 5

Solution:

Given that y varies directly as x.

Therefore $y \propto x$

$\Rightarrow$ $y = kx$ _______ (i)

Where k is constant of variation.

Put x = 2 and y = 4 in eq. (i), we have

$8 = k(2)$

or $2k = 8$

$\Rightarrow$ $k = 4$

Put k = 4 and x = 5 in eq. (i), we get

$y = (50(4) = 20$

(iii) x when y = 28

Solution:

Given that y varies directly as x.

Therefore $y \propto x$

$\Rightarrow$ $y = kx$ _______ (i)

Where k is constant of variation.

Put x = 2 and y = 8 in eq. (i), we have

$$8 = k(2)$$

or $\quad 2k = 8$

$\Rightarrow \quad k = 4$

Put k = 4 and y = 28 in eq. (i), we get

$$28 = 4x$$

or $\quad 4x = 28$

$\Rightarrow \quad x = 7$

2. If $y \propto x$, and y = 7 when x = 3 find

(i) y in terms of x

Solution:

Given that y varies directly as x.

Therefore $\quad y \propto x$

$\Rightarrow \quad y = kx$ ________ (i)

Where k is constant of variation.

Put x = 3 and y = 7 in eq. (i), we have

$$7 = k(3)$$

or $\quad 3k = 7$

$\Rightarrow \quad k = \frac{7}{3}$

Put $k = \frac{7}{3}$ in eq. (i), we get

$$y = \frac{7}{3}x.$$

(ii) x when y = 35 and y when x = 18

Solution:

Given that y varies directly as x.

Therefore $\quad y \propto x$

$\Rightarrow \quad y = kx$ ________ (i)

Where k is constant of variation.

Put x = 3 and y = 7 in eq. (i), we have

$$7 = k(3)$$

or $\quad 3k = 7$

$\Rightarrow \quad k = \frac{7}{3}$

Put $k = \frac{7}{3}$ and y = 35 in eq. (i), we get

$$35 = \frac{7}{3}x$$

$$x = 35 \times \frac{7}{3}$$

$$x = 5 \times 3 = 15$$

Put $k = \frac{7}{3}$ and $x = 18$ in eq. (i), we get

$$y = \left(\frac{7}{3}\right)(18)$$

$$y = 7 \times 6 = 42$$

3. If R $\propto$ T and R = 5 when r = 8, find the equation connecting R and T. Also find R when T= 64 and F when R = 20.

Solution:

Given that $R \propto T$

$\Rightarrow$ $R = KT$ ________ (i)

Put R = 5 and T = 8 in eq. (i), we get

$$5 = k(8)$$

or $8k = 5$

$\Rightarrow$ $k = \frac{5}{8}$

Put $k = \frac{5}{8}$ in eq. (i), we get

$$R = \frac{5}{8}T \quad \text{\_\_\_\_\_\_\_\_ (ii)}$$

Put T = 64 in eq. (ii), we get

$$R = \frac{5}{8}(64)$$

$$= 5 \times 8 = 40$$

Put R = 20 in eq. (ii), we get

$$20 = \frac{5}{8}T$$

or $\frac{5}{8}T = 20$

$$T = 20 \times \frac{5}{8}$$

$$T = 4 \times 8 = 32$$

4. If $\propto T^2$ and R = 8 when T = 3, find R when T = 6.

Solution:

Given that $R \propto T^2$

$\Rightarrow$ $R = KT^2$ ________ (i)

Put R = 8 and T = 3 in eq. (i), we get

$$8 = k(3)^2$$

$$8 = 9k$$

or $\quad 9k = 8$

$\Rightarrow \quad k = \frac{8}{9}$

Put $k = \frac{8}{9}$ $T = 6$ in eq. (i), we get

$$R = \left(\frac{8}{9}\right)(6)^2$$

$$R = \frac{8}{9} \times 36 = 8 \times 4 = 32$$

5. If $V \propto R^3$ and $V = 5$ when $R = 3$, find R, when $V = 625$.

Solution:

Given that $V \propto R^3$

$\Rightarrow \quad V = KR^3 \quad$ ________ (i)

Put $V = 5$ and $k = 3$ in eq. (i), we get

$5 = k(3)^3$

$5 = 27k$

or $\quad 27 = k5$

$\Rightarrow \quad k = \frac{5}{27}$

Put $k = \frac{5}{27}$ and $V = 625$ in eq. (i), we get

$$625 = \frac{5}{27} R^3$$

or $\quad \frac{5}{27} R^3 = 625$

$$R^3 = 625 \times \frac{27}{5}$$

$$R^3 = 125 \times 27$$

$$R^3 = 5^3 \times 3^3$$

$$R^3 = (5 \times 3)^3$$

$\Rightarrow \quad R = 5 \times 3 = 15$

6. If w varies directly as u^3 and $w = 81$ when $u = 3$. Find w, when $u = 5$.

Solution:

Given that $\quad w \propto u^3$

$\Rightarrow \quad w = ku^3$ ________ (i)

Put $w = 81$ and $u = 3$ in eq. (i), we get

$81 = k(3)^3$

$81 = 27k$

$$\text{or} \quad 27k = 81$$
$$\Rightarrow \quad k = 3$$

Put $k = 3$ and $u = 5$ in eq (i), we get

$$W = (3)(5)^3$$
$$W = 3 \times 125 = 375$$

7. If y varies inversely as x and y = 7 when x = 2, find y when x = 126.

Solution:

Given that $y \propto \frac{1}{x} \Rightarrow y = \frac{k}{x}$ ________________(i)

Put $y = 7$ and $x = 2$ in eq. (i), we get

$$7 = \frac{k}{2}$$
$$k = 14$$

Put $k = 14$ and $x = 126$ in eq. (i), we get.

$$y = \frac{14}{126} \Rightarrow y = \frac{1}{9}$$

8. If $y \propto \frac{1}{x}$ and y = 4 when x = 3, find x when y = 24.

Solution:

Given that $y \propto \frac{1}{x}$

$\Rightarrow \quad y = \frac{k}{x}$ __________ (i)

Put $y = 4$ and $x = 3$ in eq. (i), we get

$$4 = \frac{k}{3}$$
$$k = 12$$

Put $k = 12$ and $y = 24$ in eq. (i), we get

$$24 = \frac{12}{x}$$
$$x = \frac{12}{24} = \frac{1}{2}$$

9. If $y \propto \frac{1}{z}$ and w = 5 when z = 7, find w when $z = \frac{175}{4}$.

Solution:

Given that $w \propto \frac{1}{z}$

$$\Rightarrow \qquad w = \frac{k}{z} \quad \text{———— (i)}$$

Put w = 5 and z = 7 in eq. (i), we get

$$5 = \frac{k}{7}$$

$$k = 35$$

Put k = 35 and $z = \frac{175}{4}$ in eq. (i), we get

$$w = \frac{35}{175/4}$$

$$w = 35 \times \frac{4}{175}$$

$$w = \frac{4}{5}$$

10. $A \propto \frac{1}{r^2}$ and A = 2 when r = 3, find r when A = 72.

Solution:

Given that $\qquad A \propto \frac{1}{r^2}$

$$\Rightarrow \qquad A = \frac{1}{r^2}$$

Put A = 2 and r = 3 in eq. (i), we get

$$2 = \frac{k}{(3)^2}$$

$$K = 18$$

Put k = 18 and A = 72 in eq. (i), we get

$$72 = \frac{18}{r^2}$$

$$r^2 = \frac{18}{72}$$

$$r^2 = \frac{1}{4}$$

$$r = \pm\frac{1}{2}$$

11. $a \propto \frac{1}{b^2}$ and a = 3 when b = 4, find a, when b = 8.

Solution:

Given that $a \propto \frac{1}{b^2}$

$$\Rightarrow \qquad a = \frac{k}{b^2} \text{ \_\_\_\_\_\_\_\_ (i)}$$

Put $a = 3$ and. $b = 4$ in eq. (1), we get

$$3 = \frac{k}{(4)^2}$$

$$3 = \frac{k}{16}$$

$$k = 48$$

Put $k = 48$ and $b = 8$ in eq. (i), we get

$$a = \frac{48}{(8)^2} = \frac{48}{64}$$

$$a = \frac{3}{4}$$

12. $V \propto \frac{1}{r^2}$ and $V = 5$ when $r = 3$, find V when $r = 6$ and r when $V = 320$.

Solution:

Given that $V \propto \frac{1}{r^3}$

$$\Rightarrow \qquad V = \frac{k}{r^3} \text{ \_\_\_\_\_\_\_\_ (i)}$$

Put $V = 5$ and $r = 3$ in eq. (i), we get

$$5 = \frac{k}{(3)^3}$$

$$5 = \frac{k}{27}$$

$$k = 135$$

Put $k = 135$ and $r = 6$ in eq. (i), we get

$$V = \frac{135}{(6)^3}$$

$$V = \frac{135}{216} = \frac{5}{8}$$

Put $K = 135$ and $V = 320$ in eq. (i), we get

$$320 = \frac{135}{320}$$

$$r^3 = \frac{135}{320}$$

$$r^3 = \frac{27}{64}$$

$$r^3 = \left(\frac{3}{4}\right)^3$$

$$\Rightarrow \qquad r = \frac{3}{4}$$

13. $m \propto \frac{1}{n^3}$ and $m = 2$ when $n = 4$, find m when $n = 6$ and n when $m = 432$.

Solution:

Given that $\qquad m \propto \frac{1}{n^3}$

$$\Rightarrow \qquad m = \frac{k}{n^3} \quad \_\_\_\_\_\_\_\_ \text{(i)}$$

Put $m = 2$ and $n = 4$ in eq. (i), we get

$$2 = \frac{k}{(4)^3}$$

$$2 = \frac{k}{64}$$

$$k = 128$$

Put $k = 128$ and $n = 6$ in eq. (i), we get

$$m = \frac{128}{(6)^3}$$

$$m = \frac{128}{216} = \frac{16}{27}$$

Put $k = 128$ and $m = 432$ in eq. (i), we get

$$432 = \frac{128}{n^3}$$

$$n^3 = \frac{128}{432}$$

$$n^3 = \frac{8}{27}$$

$$n^3 = \left(\frac{2}{3}\right)^3$$

$$\Rightarrow \qquad n = \frac{2}{3}$$

Find 3^{rd}, 4^{th}, mean and continued proportion:

We are already familiar with proportions that if quantities a, b, c and d are in proportion, then a : b :: c : d

i.e., product of extremes = product of means

Third Proportional

If three quantities a, b and c are related as a : b :: b : c, then c is called the third proportion.

Fourth Proportional

If four quantities a, b, c and d are related as

a : b :: c : d

Then d is called the fourth proportional.

Mean Proportional

If three quantities a, b and c are related as a : b :: b : c,
then b is called the mean proportional.

Continued Proportion

If three quantities a, b and c are related as

a : b :: b : c

where a is first, b is the mean and c is the third proportional, then a, b and c are in continued proportion.

SOLVED EXERCISE 3.3

1. Find a third proportional to

(i) 6, 12

Solution:

Let C be the third proportional, then

$$6 : 12 :: 12 : C$$

$\therefore$ Product of extremes = Product of means

$$6C = 12 \times 12$$

$$6C = 144$$

$$C = \frac{144}{6}$$

$$C = 24$$

(ii) $a^2 - b^2, a - b$

Solution:

Let C be the third proportional, then

$$a^3 : 3a^2 :: 3a^2 : C$$

Product of extremes = Product of means