حل مشق 6.1

1. درج ذیل میں جذر اور مجذور بتایئے:

(i) $\sqrt{3}$

حل: $3 = \text{مجذور}$ $\sqrt{3} = \text{جذر}$

(ii) $4+3\sqrt{a}$

حل: $a = \text{مجذور}$ $\sqrt{a} = \text{جذر}$

(iii) $\sqrt{11}$

حل: $11 = \text{مجذور}$ $\sqrt{11} = \text{جذر}$

(iv) $8-2\sqrt{6}$

حل: $6 = \text{مجذور}$ $\sqrt{6} = \text{جذر}$

(v) $\frac{\sqrt{5}}{7}$

حل: $5 = \text{مجذور}$ $\sqrt{5} = \text{جذر}$

(vi) $\frac{9}{\sqrt{13}}$

حل: $13 = \text{مجذور}$ $\sqrt{13} = \text{جذر}$

2. درج ذیل کو قوت نما کی شکل میں لکھیے:

(i) $\sqrt{a^3}$

حل: $\sqrt{a^3} = \left(a^3\right)^{\frac{1}{2}} = a^{\frac{3}{2}}$

(ii) $\sqrt[5]{a^3}$

حل: $\sqrt[5]{a^3} = \left(a^3\right)^{\frac{1}{5}} = a^{\frac{3}{5}}$

(iii) $\frac{1}{\sqrt[p]{a^k}}$

حل: $\frac{1}{\sqrt[p]{a^k}} = \frac{1}{\left(a^k\right)^{\frac{1}{p}}} = \frac{1}{a^{\frac{k}{p}}} = a^{-\frac{k}{p}}$

(iv) $\frac{1}{\sqrt[b]{a^k}}$

حل: $\frac{1}{\sqrt[b]{a^k}} = \frac{1}{\left(a^k\right)^{\frac{1}{b}}} = \frac{1}{a^{\frac{k}{b}}} = a^{-\frac{k}{b}}$

3. جذر کی شکل میں لکھ کر حل کیجیے۔

(i) $(25)^{\frac{1}{2}}$

حل: $(25)^{\frac{1}{2}} = \sqrt{25} = \sqrt{5^2} = \left(5^2\right)^{\frac{1}{2}} = 5$

(ii) $(64)^{\frac{1}{3}}$

حل: $(64)^{\frac{1}{3}} = \sqrt[3]{64} = \sqrt[3]{4^3} = \left(4^3\right)^{\frac{1}{3}} = 4$

(iii) $(81)^{\frac{1}{4}}$

حل: $(81)^{\frac{1}{4}} = \sqrt[4]{81} = \sqrt[4]{3^4} = \left(3^4\right)^{\frac{1}{4}} = 3$

(iv) $(27)^{\frac{1}{3}}$

حل: $(27)^{\frac{1}{3}} = \sqrt[3]{27} = \sqrt[3]{3^3} = \left(3^3\right)^{\frac{1}{3}} = 3$

(v) $(27)^{\frac{2}{3}}$

حل: $27)^{\frac{2}{3}} = \left(27^2\right)^{\frac{1}{3}} = \sqrt[3]{(27)^2} = \sqrt[3]{\left(3^3\right)^2} = 3^2 = 9$

(vi) $8^{-\frac{1}{3}}$

حل: $8^{-\frac{1}{3}} = \frac{1}{8^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{8}} = \frac{1}{\sqrt[3]{2^3}} = \frac{1}{2}$

(vii) $(1000)^{\frac{2}{3}}$

حل: $(1000)^{\frac{2}{3}} = \left[\left((1000)^2\right)\right]^{\frac{1}{3}} = \sqrt[3]{(1000)^2} = \sqrt[3]{\left(10^3\right)^2} = 10^2 = 100$

(viii) $(64)^{\frac{1}{2}}$

حل:

$$(64)^{\frac{1}{2}} = \sqrt{64} = \sqrt{8^2} = \left(8^2\right)^{\frac{1}{2}} = 8$$

مختصر کیجیے اور جواب قوت نما کی شکل میں لکھیے۔

(i) $\sqrt{a^{16}}$ حل:

$$\sqrt{a^{16}} = \left(a^{16}\right)^{\frac{1}{2}} = a^8$$

(ii) $\sqrt[3]{a^{15}}$ حل:

$$\sqrt[3]{a^{15}} = \left(a^{15}\right)^{\frac{1}{3}} = a^5$$

(iii) $\sqrt[3]{27a^9}$ حل:

$$\sqrt[3]{27a^9} = \left(27a^9\right)^{\frac{1}{3}} = \left(3^3\right)^{\frac{1}{3}}\left(a^9\right)^{\frac{1}{3}} = 3a^3$$

(iv) $\sqrt[3]{8a^9}$ حل:

$$\sqrt[3]{8a^9} = \left(8a^9\right)^{\frac{1}{3}} = \left(2^3\right)^{\frac{1}{3}}\left(a^9\right)^{\frac{1}{3}} = 2a^3$$

(v) $\sqrt[4]{x^{32}}$ حل:

$$\sqrt[4]{x^{32}} = \left(x^{32}\right)^{\frac{1}{4}} = x^8$$

(vi) $\sqrt[4]{81x^{20}}$ حل:

$$\sqrt[4]{81x^{20}} = \left(81x^{20}\right)^{\frac{1}{4}} = \left(3^4\right)^{\frac{1}{4}}\left(x^{20}\right)^{\frac{1}{4}} = 3x^5$$

(vii) $\sqrt[3]{125x^9y^{15}}$ حل:

$$\sqrt[3]{125x^9y^{15}} = \left(125x^9y^{15}\right)^{\frac{1}{3}} = \left(5^3\right)^{\frac{1}{3}}\left(x^9\right)^{\frac{1}{3}}\left(y^{15}\right)^{\frac{1}{3}} = 5x^3y^5$$

(viii) $\sqrt{(8+y)^7}$ حل:

$$\sqrt{(8+y)^7} = \left[(8+y)^7\right]^{\frac{1}{2}} = (8+y)^{\frac{7}{2}}$$

(ix) $\sqrt[4]{16x^2y^6}$ حل:

$$\sqrt[4]{16x^2y^6} = \left(16x^2y^6\right)^{\frac{1}{4}} = \left(2^4\right)^{\frac{1}{4}}\left(x^2\right)^{\frac{1}{4}}\left(y^6\right)^{\frac{1}{4}} = 2x^{\frac{1}{2}}y^{\frac{3}{2}}$$

(x) $\sqrt[4]{\frac{x^5y^6}{z^2}}$ حل:

$$\sqrt[4]{\frac{x^5y^6}{z^2}} = \left(\frac{x^5y^6}{z^2}\right)^{\frac{1}{4}} = \frac{\left(x^5\right)^{\frac{1}{4}}\left(y^6\right)^{\frac{1}{4}}}{\left(z^2\right)^{\frac{1}{4}}} = \frac{x^{\frac{5}{4}}y^{\frac{3}{2}}}{z^{\frac{1}{2}}}$$

(xi) $\sqrt[3]{\frac{8x}{x+y}}$ حل:

$$\sqrt[3]{\frac{8x}{x+y}} = \left(\frac{8x}{x+y}\right)^{\frac{1}{3}} = \frac{\left(2^3\right)^{\frac{1}{3}}x^{\frac{1}{3}}}{(x+y)^{\frac{1}{3}}} = \frac{2x^{\frac{1}{3}}}{(x+y)^{\frac{1}{3}}}$$

(xii) $\sqrt[p]{\frac{y^n}{a^m}}$ حل:

$$\sqrt[p]{\frac{y^n}{a^m}} = \left(\frac{y^n}{a^m}\right)^{\frac{1}{p}} = \frac{\left(y^n\right)^{\frac{1}{p}}}{\left(a^m\right)^{\frac{1}{p}}} = \frac{y^{\frac{n}{p}}}{a^{\frac{m}{p}}}$$

5. مختصر کیجیے:

(i) $\sqrt{3} \times \sqrt{7}$ حل:

$$\sqrt{3} \times \sqrt{7} = \sqrt{3 \times 7} = \sqrt{21}$$

(ii) $\sqrt[5]{4} \times \sqrt[5]{128}$ حل:

$$\sqrt[5]{4} \times \sqrt[5]{128} = \sqrt[5]{4 \times 128} = \sqrt[5]{512}$$

(iii) $\sqrt[5]{81} \times \sqrt[5]{27}$ حل:

$$\sqrt[5]{81} \times \sqrt[5]{27} = \sqrt[5]{81 \times 27} = \sqrt[5]{2187}$$

(iv) $\sqrt{2} \div \sqrt[9]{32}$

$$\sqrt{2} \div \sqrt[9]{32} = \frac{\sqrt{2}}{\sqrt[9]{32}} = \frac{2^{\frac{1}{2}}}{32^{\frac{1}{9}}} = \frac{(2)^{\frac{1}{2}}}{\left(2^5\right)^{\frac{1}{9}}} = \frac{2^{\frac{1}{2}}}{2^{\frac{5}{9}}} = \frac{1}{2^{\frac{5}{9}} \times 2^{-\frac{1}{2}}}$$

$$= \frac{1}{2^{\frac{10-9}{18}}} = \frac{1}{2^{\frac{1}{18}}} = \frac{1}{\sqrt[18]{2}}$$

(v) $\sqrt[5]{118} \div \sqrt[5]{2}$

$$\sqrt[5]{118} \div \sqrt[5]{2} = \frac{\sqrt[5]{118}}{\sqrt[5]{2}} = \sqrt[5]{59}$$ حل:

(vi) $\sqrt{27} \div \sqrt{81}$

$$\sqrt{27} \div \sqrt{81} = \frac{\sqrt{27}}{\sqrt{81}} = \sqrt{\frac{27}{81}} = \sqrt{\frac{1}{3}}$$ حل:

(vii) $a^{\frac{1}{4}} \times a^{\frac{2}{3}}$

$$a^{\frac{1}{4}} \times a^{\frac{2}{3}} = a^{\frac{1}{4}+\frac{2}{3}} = a^{\frac{3+8}{12}} = a^{\frac{11}{12}} = \left(a^{11}\right)^{\frac{1}{12}} = \sqrt[12]{a^{11}}$$ حل:

(viii) $x^{\frac{6}{7}} \times y^{\frac{1}{4}}$

$$x^{\frac{6}{7}} \times y^{\frac{1}{4}} = x^{\frac{6}{7}} \times y^{\frac{1}{4}} = x^{\frac{6}{7} \times \frac{4}{4}} \times y^{\frac{1}{4}} = x^{\frac{24}{7} \times \frac{1}{4}} \times y^{\frac{1}{4}} = \sqrt[4]{x^{\frac{24}{7}} y}$$ حل:

(ix) $\left(x^{\frac{3}{4}}\ y^{\frac{1}{6}}\right)^6$

$$\left(x^{\frac{3}{4}}\ y^{\frac{1}{6}}\right)^6 = \left(x^{\frac{3}{4}}\right)^6 \left(y^{\frac{1}{6}}\right)^6 = x^{\frac{9}{2}} y = y\sqrt{x^9}$$ حل:

(x) $\left(x^3y^2\right)^{\frac{1}{2}} \times \left(y^3x^4\right)^{-\frac{1}{3}}$

حل:

$$\left(x^3y^2\right)^{\frac{1}{2}} \times \left(y^3x^4\right)^{-\frac{1}{3}} = \left(x^3\right)^{\frac{1}{2}}\left(y^2\right)^{\frac{1}{2}} \times \left(y^3\right)^{-\frac{1}{3}}\left(x^4\right)^{-\frac{1}{3}}$$

$$= x^{\frac{3}{2}} \times y \times y^{-1}x^{-\frac{4}{3}} = x^{\frac{3}{2}-\frac{4}{3}} \times y^{1-1} = x^{\frac{9-8}{6}} \times y^{o}$$

$$= x^{\frac{1}{6}} \times 1 = x^{\frac{1}{6}} = \sqrt[6]{x}$$

(xi) $\left(x^3y^2\right)^{\frac{1}{4}} \times \left(x^{\frac{1}{3}}y\right)^{\frac{3}{4}}$

حل:

$$\left(x^3y^2\right)^{\frac{1}{4}} \times \left(x^{\frac{1}{3}}y\right)^{\frac{3}{4}} = \left(x^3\right)^{\frac{1}{4}}\left(y^2\right)^{\frac{1}{4}} \times \left(x^{\frac{1}{3}}\right)^{\frac{3}{4}} y^{\frac{3}{4}}$$

$$= x^{\frac{3}{4}}y^{\frac{2}{4}} \times x^{\frac{1}{4}}y^{\frac{3}{4}} = x^{\frac{3}{4}+\frac{1}{4}} \times y^{\frac{2}{4}+\frac{3}{4}} = x^{\frac{4}{4}} \times y^{\frac{5}{4}} = \left(x^4 \times y^5\right)^{\frac{1}{4}} = \sqrt[4]{x^4y^5}$$

(xii) $\left(a^{\frac{1}{4}}b^{\frac{1}{3}}\right)^{-\frac{1}{2}} \div \left(a^{\frac{1}{3}}b^{\frac{1}{4}}\right)^{-5}$

حل:

$$\left(a^{\frac{1}{4}}b^{\frac{1}{3}}\right)^{-\frac{1}{2}} \div \left(a^{\frac{1}{3}}b^{\frac{1}{4}}\right)^{-5} = \left(a^{\frac{1}{4}}\right)^{-\frac{1}{2}}\left(b^{\frac{1}{3}}\right)^{-\frac{1}{2}} \div \left(a^{\frac{1}{3}}\right)^{-5}\left(b^{\frac{1}{4}}\right)^{-5}$$

$$= a^{-\frac{1}{8}}b^{-\frac{1}{6}} \div a^{-\frac{5}{3}}b^{-\frac{5}{4}} = a^{-\frac{1}{8}}b^{-\frac{1}{6}} \times a^{\frac{5}{3}}b^{\frac{5}{4}} = a^{-\frac{1}{8}+\frac{5}{3}} \times b^{-\frac{1}{6}+\frac{5}{4}}$$

$$= a^{\frac{37}{24}} \times a^{\frac{13}{12}} = a^{\frac{37}{24}} \times b^{\frac{13\times2}{12\times2}} = a^{\frac{37}{24}} \times b^{\frac{26}{24}} = \sqrt[24]{a^{37}b^{26}}$$

(xiii) $\left(x^2y^3\right)^{\frac{1}{5}} \times \left(x^{\frac{1}{3}}y^2\right)^{\frac{1}{4}}$

حل:

$$\left(x^2y^3\right)^{\frac{1}{5}} \times \left(x^{\frac{1}{3}}y^2\right)^{\frac{1}{4}} = \left(x^2\right)^{\frac{1}{5}}\left(y^3\right)^{\frac{1}{5}} \times \left(x^{\frac{1}{3}}\right)^{\frac{1}{4}}\left(y^2\right)^{\frac{1}{4}}$$

$$= x^{\frac{2}{5}}y^{\frac{3}{5}} \times x^{\frac{1}{12}}y^{\frac{1}{2}} = x^{\frac{2}{5}+\frac{1}{12}} \times y^{\frac{3}{5}+\frac{1}{2}} = x^{\frac{29}{60}}y^{\frac{11}{10}}$$