

Exercise 7.3

Q1. Solve the following in equalities.

i) $3x + 1 < 5x - 4$

$$1 + 4 < 5x - 3x$$

$$5 < 2x$$

$$\frac{5}{2} < x$$

or $x > \frac{5}{2}$

$$\text{Solution Set} = \left\{ x \mid x > \frac{5}{2} \right\}$$

ii) $4x - 10.3 \leq 21x - 1.8$

$$4x - 21x \leq 10.3 - 1.8$$

$$-17x \leq 8.5$$

$$17x \geq -8.5$$

$$x \geq -\frac{8.5}{17}$$

$$x \geq -0.5$$

$$\text{Solution Set} = \left\{ x \mid x \geq -0.5 \right\}$$

iii) $4 - \frac{1}{2}x \geq -7 + \frac{1}{4}x$

$$4 + 7 \geq \frac{1}{4}x + \frac{1}{2}x$$

$$11 \geq \frac{x + 2x}{4}$$

$$11 \geq \frac{3}{4}x$$

$$\frac{11 \times 4}{3} \geq x$$

$$\frac{44}{3} \geq x$$

or $x \leq \frac{44}{3}$

$$\text{Solution Set} = \left\{ x \mid x \leq \frac{44}{3} \right\}$$

iv) $x - 2(5 - 2x) \geq 6x - 3\frac{1}{2}$

$$x - 2(5 - 2x) \geq 6x - \frac{7}{2}$$

Multiplying both sides by 2

$$2x - 4(5 - 2x) \geq 12x - 7$$

$$2x - 20 + 8x \geq 12x - 7$$

$$2x + 8x - 12x \geq 20 - 7$$

$$-2x \geq 13$$

$$2x \leq -13$$

$$x \leq -\frac{13}{2}$$

$$\text{Solution Set} = \left\{ x \mid x \leq -\frac{13}{2} \right\}$$

v) $\frac{3x+2}{9} - \frac{2x+1}{3} > -1$

Multiplying both sides by 9

$$3x + 2 - 3(2x + 1) > -9$$

$$3x + 2 - 6x - 3 > -9$$

$$-3x - 1 > -9$$

$$-3x > 1 - 9$$

$$-3x > -8$$

$$x < \frac{-8}{-3}$$

$$x < \frac{8}{3}$$

$$\text{Solution Set} = \left\{ x \mid x < \frac{8}{3} \right\}$$

$$\text{vi) } 3(2x+1) - 2(2x+5) < 5(3x-2)$$

$$6x+3-4x-10 < 15x-10$$

$$2x-7 < 15x-10$$

$$10-7 < 15x-2x$$

$$3 < 13x$$

$$\frac{3}{13} < x$$

$$\text{or } x > \frac{3}{13}$$

$$\text{Solution Set} = \left\{ x \mid x > \frac{3}{13} \right\}$$

$$\text{vii) } 3(x-1) - (x-2) > -2(x+4)$$

$$3x-3-x+2 > -2x-8$$

$$2x-1 > -2x-8$$

$$2x+2x > 1-8$$

$$4x > -7$$

$$x > -\frac{7}{4}$$

$$\text{Solution Set} = \left\{ x \mid x > -\frac{7}{4} \right\}$$

$$\text{viii) } 2\frac{2}{3}x + \frac{2}{3}(5x-4) > -\frac{1}{3}(8x+7)$$

$$\frac{8}{3}x + \frac{2}{3}(5x-4) > -\frac{1}{3}(8x+7)$$

Multiplying both sides by 3

$$8x+2(5x-4) > -(8x+7)$$

$$8x+10x-8 > -8x-7$$

$$18x-8 > -8x-7$$

$$18x+8x > 8-7$$

$$26x > 1$$

$$x > \frac{1}{26}$$

$$\text{Solution Set} = \left\{ x \mid x > \frac{1}{26} \right\}$$

Q2. Solve the following inequalities.

$$\text{i) } -4 < 3x+5 < 8$$

$$-4 < 3x+5 \quad \text{and} \quad 3x+5 < 8$$

$$-4-5 < 3x \quad \text{and} \quad 3x < 8-5$$

$$-9 < 3x \quad \text{and} \quad 3x < 3$$

$$-\frac{9}{3} < x \quad \text{and} \quad x < \frac{3}{3}$$

$$-3 < x \quad \text{and} \quad x < 1$$

$$-3 < x < 1$$

$$\text{Solution Set} = \{ x \mid -3 < x < 1 \}$$

$$\text{ii) } -5 \leq \frac{4-3x}{2} < 1$$

$$-5 \leq \frac{4-3x}{2} \quad \text{and} \quad \frac{4-3x}{2} < 1$$

$$-10x \leq 4-3x \quad \text{and} \quad 4-3x < 2$$

$$-10-4 \leq -3x \quad \text{and} \quad -3x < 2-4$$

$$-14 \leq -3x \quad \text{and} \quad -3x < -2$$

$$14 \geq 3x \quad \text{and} \quad 3x > 2$$

$$\frac{14}{3} \geq x \quad \text{and} \quad x > \frac{2}{3}$$

$$\frac{14}{3} \geq x > \frac{2}{3}$$

$$\text{Solution Set} = \left\{ x \mid \frac{14}{3} \geq x > \frac{2}{3} \right\}$$

$$\text{iii) } -6 < \frac{x-2}{4} < 6$$

$$-6 < \frac{x-2}{4} \quad \text{and} \quad \frac{x-2}{4} < 6$$

$$-24 < x-2 \quad \text{and} \quad x-2 < 24$$

$$-24+2 < x \quad \text{and} \quad x < 24+2$$

$$-22 < x \quad \text{and} \quad x < 26$$

$$\text{Solution Set} = \{ x \mid -22 < x < 26 \}$$

$$\text{iv) } 3 \geq \frac{7-x}{2} \geq 1$$

$$\begin{array}{lll}
 3 \geq \frac{7-x}{2} & \text{and} & \frac{7-x}{2} \geq 1 \\
 6 \geq 7-x & \text{and} & 7-x \geq 2 \\
 6-7 \geq -x & \text{and} & -x \geq 2-7 \\
 -1 \geq -x & \text{and} & -x \geq -5 \\
 1 \leq x & \text{and} & x \leq 5 \\
 1 \leq x \leq 5
 \end{array}$$

$$\text{Solution Set} = \{x | 1 \leq x \leq 5\}$$

$$\begin{array}{lll}
 \text{v)} & 3x-10 \leq 5 < x+3 \\
 & 3x-10 \leq 5 & \text{and} & 5 < x+3 \\
 & -5-10 \leq -3x & \text{and} & -x < 3-5 \\
 & -15 \leq -3x & \text{and} & -x < -2 \\
 & 15 \geq 3x & \text{and} & x > 2 \\
 & 5 \geq x & \text{and} & x > 2 \\
 & & & 5 \geq x > 2
 \end{array}$$

$$\text{Solution Set} = \{x | 5 \geq x > 2\}$$

$$\begin{array}{lll}
 \text{vi)} & -3 \leq \frac{x-4}{-5} < 4 \\
 & -3 \leq \frac{x-4}{-5} & \text{and} & \frac{x-4}{-5} < 4
 \end{array}$$

$$\begin{array}{lll}
 \Rightarrow & 3 \geq \frac{x-4}{5} & \text{and} & \frac{x-4}{5} \\
 & 15 \geq x-4 & \text{and} & x-4 \\
 & 15+4 \geq x & \text{and} & x > 4 \\
 & 19 \geq x & \text{and} & x > - \\
 & 19 \geq x \geq -16
 \end{array}$$

$$\text{Solution Set} = \{x | 19 \geq x > -16\}$$

$$\begin{array}{lll}
 \text{vii)} & 1-2x < 5-x \leq 25-6x \\
 & 1-2x < 5-x & \text{and} & 5-x \leq 2 \\
 & 1-5 \leq 2x-x & \text{and} & 6x-x \leq \\
 & -4 < x & \text{and} & 5x \leq 20 \\
 & -4 < x & \text{and} & x \leq 4 \\
 & -4 < x \leq 4
 \end{array}$$

$$\text{Solution Set} = \{x | -4 < x \leq 4\}$$

$$\begin{array}{lll}
 \text{viii)} & 3x-2 < 2x+1 < 4x+17 \\
 & 3x-2 < 2x+1 & \text{and} & 2x+1 < 4x+17 \\
 & -2-1 < 2x-3x & \text{and} & 2x-4x < 17-1 \\
 & -3 < -x & \text{and} & -2x < 16 \\
 & 3 > x & \text{and} & 2x > -16 \\
 & 3 > x & \text{and} & x > -8 & 3 > x > -8
 \end{array}$$

$$\text{Solution Set} = \{x | 3 > x > -8\}$$