EXERCISE 4.2

Q.1

$$x^4 - 6x^2 + 8 = 0$$

$$(x^2)^2 - 6x^2 + 8 = 0$$

put $x^2 = y$ then

$$y^2 - 6y + 8 = 0$$

$8y^2$: $-2y - 4y$

$$y^2 - 2y - 4y + 8 = 0$$

$$y(y-2) - 4(y-2) = 0$$

$$(y-2)(y-4) = 0$$

$$y - 2 = 0, \quad y - 4 = 0$$

$$y = 2, \quad y = 4$$

If $y = 2$ then

$$x^2 = 2$$

$$x = \pm\sqrt{2}$$

If $y = 4$ then

$$x^2 = 4$$

$$x = \pm\sqrt{4}$$

$$x = \pm 2$$

$$\{\pm\sqrt{2}, \pm 2\}$$

Q.2

$$x^{-2} - 10 = 3x^{-1}$$

$$x^{-2} - 3x^{-1} - 10 = 0$$

$$(x^{-1})^2 - 3x^{-1} - 10 = 0$$

put $x^{-1} = y$ then

$$y^2 - 3y - 10 = 0$$

$-10y^2$: $2y - 5y$

$$y^2 + 2y - 5y - 10 = 0$$

$$y(y+2) - 5(y+2) = 0$$

$$(y-5)(y+2) = 0$$

$$y - 5 = 0, \quad y + 2 = 0$$

$$y = 5, \quad y = -2$$

If $y = 5$ then

$$x^{-1} = 5$$

$$\frac{1}{x} = 5$$

$$\Rightarrow x = 1/5$$

If $y = -2$ then

$$x^{-1} = -2$$

$$\frac{1}{x} = -2$$

$$x = -1/2$$

$$\{1/5, -1/2\}$$

Q.3 $x^6 - 9x^3 + 8 = 0$

$(x^3)^2 - 9x^3 + 8 = 0$

put $x^3 = y$ then

$y^2 - 9y + 8 = 0$

$y^2 - y - 8y + 8 = 0$ ($8y^2$ → $-y$, $-8y$)

$y(y-1) - 8(y-1) = 0$

$(y-1)(y-8) = 0$

$y - 1 = 0$, $y - 8 = 0$

$y = 1$, $y = 8$

If $y = 1$ then

$x^3 = 1$

$x^3 - 1 = 0$

$x^3 - (1)^3 = 0$

If $y = 8$ then

$x^3 = 8$

$x^3 - 8 = 0$

$x^3 - (2)^3 = 0$

$\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

So

$(x-1)(x^2+x+1) = 0$

$x - 1 = 0$, $x^2 + x + 1 = 0$

$x = 1$, $x^2 + x + 1 = 0$

$(x-2)(x^2+2x+4) = 0$

$x - 2 = 0$, $x^2 + 2x + 4 = 0$

$x = 2$, $x^2 + 2x + 4 = 0$

Using $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$

$x = \frac{-1 \pm \sqrt{1-4}}{2}$

$x = \frac{-1 \pm \sqrt{-3}}{2}$

$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(4)}}{2(1)}$

$x = \frac{-2 \pm \sqrt{4-16}}{2}$

$x = \frac{-2 \pm \sqrt{-12}}{2}$

$x = \frac{-2 \pm \sqrt{4}\sqrt{-3}}{2}$

$x = \frac{-2 \pm 2\sqrt{-3}}{2}$

$x = 2\left(\frac{-1 \pm \sqrt{-3}}{2}\right)$

$x = -1 \pm \sqrt{-3}$

$\left\{1, 2, \frac{-1+\sqrt{-3}}{2}, \frac{-1-\sqrt{-3}}{2}, -1+\sqrt{-3}, -1-\sqrt{-3}\right\}$

Q.4 $8x^6 - 19x^3 - 27 = 0$

$8(x^3)^2 - 19x^3 - 27 = 0$

put $x^3 = y$ Then

$8y^2 - 19y - 27 = 0$

$8y^2 + 8y - 27y - 27 = 0$ ($-216y^2$ → $8y$, $-27y$)

$8y(y+1) - 27(y+1) = 0$

$(y+1)(8y-27) = 0$

$y + 1 = 0$, $8y - 27 = 0$

$y = -1$, $y = 27/8$

If $y = -1$ then

$x^3 = -1$

$x^3 + 1 = 0$

$x^3 + (1)^3 = 0$

$(x)^3 + (1)^3 = 0$

If $y = 27/8$ then

$x^3 = 27/8$

$8x^3 = 27$

$8x^3 - 27 = 0$

$(2x)^3 - (3)^3 = 0$

$\because a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

& $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$(x+1)(x^2 - x + 1) = 0$

Either $x + 1 = 0$

$\Rightarrow x = -1$

OR $x^2 - x + 1 = 0$

$(2x-3)(4x^2 + 6x + 9) = 0$

Either $2x - 3 = 0$

$\Rightarrow x = 3/2$

OR $4x^2 + 6x + 9 = 0$

Using $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$

$x = \frac{1 \pm \sqrt{1-4}}{2}$

$x = \frac{1 \pm \sqrt{-3}}{2}$

$x = \frac{-6 \pm \sqrt{(6)^2 - 4(4)(9)}}{2(4)}$

$x = \frac{-6 \pm \sqrt{36-144}}{8}$

$x = \frac{-6 \pm \sqrt{-108}}{8}$

$x = \frac{-6 \pm \sqrt{36 \times (-3)}}{8}$

$x = \frac{-6 \pm 6\sqrt{-3}}{8}$

$x = 6\left(\frac{-1 \pm \sqrt{-3}}{8}\right)$ → $x = 3\left(\frac{-1 \pm \sqrt{-3}}{4}\right)$

$\left\{-1, 3/2, \frac{1 \pm \sqrt{-3}}{2}, 3\left(\frac{-1 \pm \sqrt{-3}}{4}\right)\right\}$

Q.5 $x^{-1/5} + 8 = 6x^{1/5}$

$x^{1/5} - 6x^{1/5} + 8 = 0$

$(x^{1/5})^2 - 6x^{1/5} + 8 = 0$

put $x^{1/5} = y$ Then

$y^2 - 6y + 8 = 0$

$y^2 - 2y - 4y + 8 = 0$ ($8y^2$ → $-2y$, $-4y$)

$y(y-2) - 4(y-2) = 0$

$(y-2)(y-4) = 0$

$y-2=0$, $y-4=0$

If $y = 2$ Then

$x^{1/5} = 2$

$(x^{1/5})^5 = 2^5$

$x = 32$

If $y = 4$ Then

$x^{1/5} = 4$

$(x^{1/5})^5 = 4^5$

$x = 1024$

$\{32, 1024\}$

Q6 $(x+1)(x+2)(x+3)(x+4) = 24$

Re-arranging it

$(x+1)(x+4) \cdot (x+2)(x+3) = 24$

$(x^2+4x+x+4) \cdot (x^2+3x+2x+6) = 24$

$(x^2+5x+4) \cdot (x^2+5x+6) = 24$

put $x^2+5x = y$ Then

$(y+4)(y+6) = 24$

$y^2 + 6y + 4y + 24 = 24$

$y^2 + 10y + 24 - 24 = 0$

$y^2 + 10y = 0$

$y(y+10) = 0$

$y = 0$, $y + 10 = 0 \Rightarrow y = -10$

If $y = 0$ Then

$x^2 + 5x = 0$

$x(x+5) = 0$

$x = 0$, $x+5 = 0$

$x = -5$

If $y = -10$ Then

$x^2 + 5x = -10$

$x^2 + 5x + 10 = 0$

Using $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(10)}}{2(1)}$

$x = \frac{-5 \pm \sqrt{25-40}}{2} \Rightarrow x = \frac{-5 \pm \sqrt{-15}}{2}$

$\left\{0, -5, \frac{-5 \pm \sqrt{-15}}{2}\right\}$

Q.7 $(x-1)(x+5)(x+8)(x+2) - 880 = 0$

$(x-1)(x+5)(x+8)(x+2) = 880$

Re-arranging it

$(x-1)(x+8) \cdot (x+2)(x+5) = 880$

$(x^2+8x-x-8) \cdot (x^2+5x+2x+10) = 880$

$(x^2+7x-8) \cdot (x^2+7x+10) = 880$

put $x^2+7x = y$ Then

$(y-8)(y+10) = 880$

$y^2 + 10y - 8y - 80 = 880$

$y^2 + 2y - 80 - 880 = 0$

$y^2 + 2y - 960 = 0$

$y^2 + 32y - 30y - 960 = 0$ ($-960y^2$ → $32y$, $-30y$)

$y(y+32) - 30(y+32) = 0$

$(y-30)(y+32) = 0$

$y - 30 = 0$, $y + 32 = 0$

$y = 30$, $y = -32$

If $y = 30$ Then

$x^2 + 7x = 30$

$x^2 + 7x - 30 = 0$

$x^2 - 3x + 10x - 30 = 0$,

$x(x-3) + 10(x-3) = 0$,

If $y = -32$ Then

$x^2 + 7x = -32$

$x^2 + 7x + 32 = 0$

Using $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$(x-3)(x+10)=0$

$x-3=0,\ x+10=0$

$x=3,\ x=-10$

$x=\frac{-7\pm\sqrt{(7)^2-4(1)(32)}}{2(1)}$

$x=\frac{-7\pm\sqrt{49-128}}{2}$

$x=\frac{-7\pm\sqrt{-79}}{2}$

$\left\{3,\ -10,\ \frac{-7\pm\sqrt{-79}}{2}\right\}$

Q.8 $(x-5)(x-7)(x+6)(x+4)-504=0$

$(x-5)(x-7)(x+6)(x+4)=504$

Re-arranging it

$(x-5)(x+4)\cdot(x-7)(x+6)=504$

$(x^2+4x-5x-20)\cdot(x^2+6x-7x-42)=504$

$(x^2-x-20)\cdot(x^2-x-42)=504$

put $x^2-x=y$ Then

$(y-20)(y-42)=504$

$y^2-42y-20y+840-504=0$

$y^2-62y+336=0$

$y^2-6y-56y+336=0$

$336y^2$ → $-6y$, $-56y$

$y(y-6)-56(y-6)=0$

$(y-6)(y-56)=0$

$y-6=0,\ y-56=0$

$y=6,\ y=56$

If $y=6$ Then

$x^2-x=6$

$x^2-x-6=0$

$x^2+2x-3x-6=0$

$x(x+2)-3(x+2)=0$

$(x-3)(x+2)=0$

$x-3=0,\ x+2=0$

$x=3,\ x=-2$

If $y=56$ Then

$x^2-x=56$

$x^2-x-56=0$

$x^2+7x-8x-56=0$

$x(x+7)-8(x+7)=0$

$(x-8)(x+7)=0$

$x-8=0,\ x+7=0$

$x=8,\ x=-7$

$\{3,-2,8,-7\}$

Q.9 $(x-1)(x-2)(x-8)(x+5)+360=0$

$(x-1)(x-2)(x-8)(x+5)=-360$

Re-arranging it

$(x-1)(x-2)\cdot(x-8)(x+5)=-360$

$(x^2-2x-x+2)\cdot(x^2+5x-8x-40)=-360$

$(x^2-3x+2)\cdot(x^2-3x-40)=-360$

put $x^2-3x=y$ Then

$(y+2)(y-40)=-360$

$y^2-40y+2y-80+360=0$

$y^2-38y+280=0$

$280y^2$ → $-10y$, $-28y$

$y^2-10y-28y+280=0$

$y(y-10)-28(y-10)=0$

$(y-10)(y-28)=0$

$y-10=0,\ y-28=0$

$y=10,\ y=28$

If $y=10$ Then

$x^2-3x=10$

$x^2-3x-10=0$

$x^2+2x-5x-10=0$

$x(x+2)-5(x+2)=0$

$(x-5)(x+2)=0$

$x-5=0,\ x+2=0$

$x=5,\ x=-2$

If $y=28$ Then

$x^2-3x=28$

$x^2-3x-28=0$

$x^2+4x-7x-28=0$

$x(x+4)-7(x+4)=0$

$(x-7)(x+4)=0$

$x-7=0,\ x+4=0$

$x=7,\ x=-4$

$\{5,-2,7,-4\}$.

Q.10 $(x+1)(2x+3)(2x+5)(x+3)=945$

Re-arranging it

$(x+1)(x+3)\cdot(2x+3)(2x+5)=945$

$(x^2+3x+x+3)\cdot(4x^2+10x+6x+15)=945$

$(x^2+4x+3)\cdot(4x^2+16x+15)=945$

$(x^2+4x+3)\cdot(4(x^2+4x)+15)=945$

put $x^2+4x = y$ Then

$(y+3)(4y+15) = 945$

$4y^2+15y+12y+45-945 = 0$

$4y^2 + 27y - 900 = 0$

$4y^2 - 48y + 75y - 900 = 0$ ($-3600y^2$: $-48y$, $75y$)

$4y(y-12)+75(y-12) = 0$

$(y-12)(4y+75) = 0$

$y-12=0$, $4y+75=0$

$y = 12$, $y = -75/4$

If $y = 12$ Then

$x^2+4x = 12$

$x^2+4x-12=0$

$x^2-2x+6x-12=0$

$x(x-2)+6(x-2)=0$

$(x-2)(x+6)=0$

$x-2=0$, $x+6=0$

$x=2$, $x=-6$

If $y = -75/4$ Then

$x^2+4x = -75/4$

$4x^2+16x = -75$

$4x^2+16x+75=0$

using $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x = \frac{-16\pm\sqrt{(16)^2-4(4)(75)}}{2(4)}$

$x = \frac{-16\pm\sqrt{256-1200}}{8}$

$x = \frac{-16\pm\sqrt{-944}}{8} \Rightarrow x = \frac{-16\pm\sqrt{16\times(-59)}}{8}$

$x = \frac{-16\pm 4\sqrt{-59}}{8} \Rightarrow x = \frac{4(-4\pm\sqrt{-59})}{8}$

$x = \frac{-4\pm\sqrt{-59}}{2} \Rightarrow x = \frac{-4\pm\sqrt{59}\sqrt{-1}}{2}$

$x = \frac{-4\pm\sqrt{59}\,i}{2}$ $\because \sqrt{-1} = i$ (iota)

$\left\{-6, 2, \frac{-4\pm\sqrt{59}\,i}{2}\right\}$

Q.11 $(2x-7)(x^2-9)(2x+5)-91=0$

$(2x-7)(x-3)(x+3)\cdot(2x+5) = 91$

Re-arranging it

$(2x-7)(x+3)\cdot\ (x-3)(2x+5) = 91$

$(2x^2+6x-7x-21)\cdot(2x^2+5x-6x-15)=91$

$(2x^2-x-21)\cdot(2x^2-x-15) = 91$

put $2x^2-x = y$ Then

$(y-21)(y-15) = 91$

$y^2-15y-21y+315-91 = 0$

$y^2-36y+224=0$

$y^2-8y-28y+224=0$ ($224y^2$: $-8y$, $-28y$)

$y(y-8)-28(y-8)=0$

$(y-8)(y-28)=0$

$y-8=0$, $y-28=0$

$y = 8$, $y = 28$

If $y=8$ then,

$2x^2-x=8$

$2x^2-x-8=0$

using $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x = \frac{-(-1)\pm\sqrt{(-1)^2-4(2)(-8)}}{2(2)}$

$x = \frac{1\pm\sqrt{1+64}}{4}$

$x = \frac{1\pm\sqrt{65}}{4}$

If $y=28$ then,

$2x^2-x = 28$

$2x^2-x-28=0$

$2x^2-8x+7x-28=0$

$2x(x-4)+7(x-4)=0$

$(x-4)(2x+7)=0$

$x-4=0$, $2x+7=0$

$x=4$, $x=-7/2$

$\left\{4, -7/2, \frac{1\pm\sqrt{65}}{4}\right\}$

Q.12 $(x^2+6x+8)(x^2+14x+48) = 105$

Factorizing it

$(x^2+2x+4x+8)\cdot(x^2+6x+8x+48)=105$

$[x(x+2)+4(x+2)][x(x+6)+8(x+6)]=105$

$(x+2)(x+4)\cdot(x+6)(x+8) = 105$

Re-arranging it

$(x+2)(x+8)\cdot\ (x+4)(x+6) = 105$

$(x^2+8x+2x+16)\cdot(x^2+6x+4x+24)=105$

$(x^2+10x+16)\cdot(x^2+10x+24)=105$

put $x^2+10x = y$ Then

$(y+16)(y+24) = 105$

$y^2+24y+16y+384-105=0$

$y^2+40y+279=0$

$y^2+9y+31y+279=0$

$279y^2$ → $9y$, $31y$

$y(y+9)+31(y+9)=0$

$(y+9)(y+31)=0$

$y+9=0$, $y+31=0$

$y=-9$, $y=-31$

If $y=-9$ then,

$x^2+10x=-9$

$x^2+10x+9=0$

$x^2+x+9x+9=0$

$x(x+1)+9(x+1)=0$

$(x+1)(x+9)=0$

$x+1=0$, $x+9=0$

$x=-1$, $x=-9$

If $y=-31$ Then

$x^2+10x=-31$

$x^2+10x+31=0$

Using $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-10\pm\sqrt{(10)^2-4(1)(31)}}{2(1)}$

$x=\frac{-10\pm\sqrt{100-124}}{2}$

$x=\frac{-10\pm\sqrt{-24}}{2}$ $\Rightarrow x=\frac{-10\pm\sqrt{4(-6)}}{2}$

$x=\frac{-10\pm2\sqrt{-6}}{2}$ $\Rightarrow x=\frac{2(-5\pm\sqrt{-6})}{2}$

$x=-5\pm\sqrt{-6}$ $\Rightarrow x=-5\pm\sqrt{6}\sqrt{-1}$

$x=-5\pm\sqrt{6}\,i$ $\because \sqrt{-1}=i$

$\{-1, -9, -5\pm\sqrt{-6}\}$

OR $\{-1, -9, -5\pm\sqrt{6}\,i\}$

Q.13 $(x^2+6x-27)(x^2-2x-35)=385$

Factorizing it

$(x^2-3x+9x-27)\cdot(x^2+5x-7x-35)=385$

$[x(x-3)+9(x-3)]\,[x(x+5)-7(x+5)]=385$

$(x-3)(x+9)\cdot(x-7)(x+5)=385$

Re-arranging it

$(x-3)(x+5)\cdot(x-7)(x+9)=385$

$(x^2+5x-3x-15)\cdot(x^2+9x-7x-63)=385$

$(x^2+2x-15)\cdot(x^2+2x-63)=385$

put $x^2+2x=y$ Then

$(y-15)(y-63)=385$

$y^2-63y-15y+945-385=0$

$y^2-78y+560=0$

$y^2-8y-70y+560=0$

$560y^2$ → $-8y$, $-70y$

$y(y-8)-70(y-8)=0$

$(y-8)(y-70)=0$

$y-8=0$, $y-70=0$

$y=8$, $y=70$

If $y=8$ Then

$x^2+2x=8$,

$x^2+2x-8=0$,

$x^2-2x+4x-8=0$

$x(x-2)+4(x-2)=0$

$(x-2)(x+4)=0$

$x-2=0$, $x+4=0$

$x=2$, $x=-4$

If $y=70$ Then

$x^2+2x=70$

$x^2+2x-70=0$

Using $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-2\pm\sqrt{(2)^2-4(1)(-70)}}{2(1)}$

$x=\frac{-2\pm\sqrt{4+280}}{2}$

$x=\frac{-2\pm\sqrt{284}}{2}$ $\rightarrow x=\frac{-2\pm\sqrt{4\times71}}{2}$

$x=\frac{-2\pm2\sqrt{71}}{2}$ $\rightarrow x=\frac{2(-1\pm\sqrt{71})}{2}$

$x=-1\pm\sqrt{71}$ $\{2, -4, -1\pm\sqrt{71}\}$

Q.14 $4\cdot2^{2x+1}-9\cdot2^{x}+1=0$

$4\cdot2^{2x}\cdot2^{1}-9\cdot2^{x}+1=0$

$4\cdot(2^{x})^2\cdot2-9\cdot2^{x}+1=0$

$8\cdot(2^{x})^2-9\cdot2^{x}+1=0$

put $2^x = y$ Then

$8y^2 - 9y + 1 = 0$

$8y^2 - 8y - y + 1 = 0$

$8y(y-1) - 1(y-1) = 0$

$(y-1)(8y-1) = 0$

$y-1=0, \quad 8y-1=0$

$y = 1, \quad y = 1/8$

If $y=1$ Then, If $y = 1/8$ Then

$2^x = 1, \quad 2^x = 1/8$

$2^x = 2^0, \quad 2^x = \frac{1}{2^3}$

$2^x = 2^0, \quad 2^x = 2^{-3}$

$\Rightarrow x = 0, \quad x = -3$

$\{0, -3\}$

Q.15 $2^x + 2^{-x+6} - 20 = 0$

$2^x + 2^{-x} \cdot 2^6 - 20 = 0$

$2^x + 2^{-x} \cdot 64 - 20 = 0$

$2^x + \frac{64}{2^x} - 20 = 0$

put $2^x = y$ Then

$y + \frac{64}{y} - 20 = 0$

$y^2 + 64 - 20y = 0$

$y^2 - 20y + 64 = 0$

$y^2 - 4y - 16y + 64 = 0$

$y(y-4) - 16(y-4) = 0$

$(y-4)(y-16) = 0$

$y-4=0, \quad y-16=0$

$y = 4, \quad y = 16$

If $y=4$ Then If $y=16$ Then

$2^x = 4, \quad 2^x = 16$

$2^x = 2^2 \qquad 2^x = 2^4$

$\Rightarrow x = 2 \qquad x = 4$

$\{2, 4\}$

Q.16 $4^x - 3 \cdot 2^{x+3} + 128 = 0$

$(2^2)^x - 3 \cdot 2^x \cdot 2^3 + 128 = 0$

$(2^x)^2 - 3 \cdot 2^x \cdot 8 + 128 = 0$

$(2^x)^2 - 24 \cdot 2^x + 128 = 0$

put $2^x = y$ Then

$y^2 - 24y + 128 = 0$

$y^2 - 8y - 16y + 128 = 0$

$y(y-8) - 16(y-8) = 0$

$(y-8)(y-16) = 0$

$y-8=0, \quad y-16=0$

$y = 8, \quad y = 16$

If $y=8$ Then, If $y=16$ Then

$2^x = 8, \quad 2^x = 16$

$2^x = 2^3, \quad 2^x = 2^4$

$\rightarrow x = 3, \quad x = 4$

$\{3, 4\}$

Q.17 $3^{2x-1} - 12 \cdot 3^x + 81 = 0$

$3^{2x} \cdot 3^{-1} - 12 \cdot 3^x + 81 = 0$

$(3^x)^2 \cdot \frac{1}{3} - 12 \cdot 3^x + 81 = 0$

Multiplying by 3

$(3^x)^2 - 36 \cdot 3^x + 243 = 0$

put $3^x = y$ Then

$y^2 - 36y + 243 = 0$

$y^2 - 9y - 27y + 243 = 0$

$y(y-9) - 27(y-9) = 0$

$(y-9)(y-27) = 0$

$y-9=0$, $y-27=0$

$y=9$, $y=27$

If $y=9$ Then, If $y=27$ Then

$3^x=9$, $3^x=27$

$3^x=3^2$, $3^x=3^3$

$\Rightarrow x=2$, $x=3$

$\{2,3\}$

Q.18 $\left(x+\frac{1}{x}\right)^2-3\left(x+\frac{1}{x}\right)-4=0$

put $x+\frac{1}{x}=y$ then

$y^2-3y-4=0$

$y^2+y-4y-4=0$

$y(y+1)-4(y+1)=0$

$(y+1)(y-4)=0$

$y+1=0$, $y-4=0$

$y=-1$, $y=4$

If $y=-1$ Then, If $y=4$ Then

$x+\frac{1}{x}=-1$ | $x+\frac{1}{x}=4$

$x^2+1=-x$ | $x^2+1=4x$

$x^2+x+1=0$ | $x^2-4x+1=0$

Using $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-1\pm\sqrt{(1)^2-4(1)(1)}}{2(1)}$ | $x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(1)}}{2(1)}$

$x=\frac{-1\pm\sqrt{1-4}}{2}$ | $x=\frac{4\pm\sqrt{16-4}}{2}$

$x=\frac{-1\pm\sqrt{-3}}{2}$ | $x=\frac{4\pm\sqrt{12}}{2}$

$x=\frac{-1\pm\sqrt{3}\sqrt{-1}}{2}$ | $x=\frac{4\pm\sqrt{4\times3}}{2}$

$x=\frac{-1\pm\sqrt{3}i}{2}$ | $x=\frac{4\pm2\sqrt{3}}{2}=2\pm\sqrt{3}$

$\left\{\frac{-1\pm\sqrt{3}i}{2},\ 2\pm\sqrt{3}\right\}$

Q.19 $x^2+x-4+\frac{1}{x}+\frac{1}{x^2}=0$

$x^2+\frac{1}{x^2}+x+\frac{1}{x}-4=0$

put $x+\frac{1}{x}=y$ then

$\left(x+\frac{1}{x}\right)^2=y^2$

$x^2+\frac{1}{x^2}+2=y^2 \Rightarrow x^2+\frac{1}{x^2}=y^2-2$

Given equation takes form

$y^2-2+y-4=0$

$y^2+y-6=0$

$y^2-2y+3y-6=0$

$y(y-2)+3(y-2)=0$

$(y-2)(y+3)=0$

$y-2=0$, $y+3=0$

$y=2$, $y=-3$

If $y=2$ then, If $y=-3$ Then

$x+\frac{1}{x}=2$, $x+\frac{1}{x}=-3$

$x^2+1=2x$, $x^2+1=-3x$

$x^2-2x+1=0$, $x^2+3x+1=0$

$(x-1)^2=0$ Using $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x-1=0$

$\Rightarrow x=1$ $x=\frac{-3\pm\sqrt{(3)^2-4(1)(1)}}{2(1)}$

$x=\frac{-3\pm\sqrt{9-4}}{2} \Rightarrow x=\frac{-3\pm\sqrt{5}}{2}$

$\left\{1,\ \frac{-3\pm\sqrt{5}}{2}\right\}$

Q.20 $\left(x-\frac{1}{x}\right)^2+3\left(x+\frac{1}{x}\right)=0$

$x^2+\frac{1}{x^2}-2+3\left(x+\frac{1}{x}\right)=0$

put $x+\frac{1}{x}=y$

$\Rightarrow\left(x+\frac{1}{x}\right)^2=y^2$

$x^2+\frac{1}{x^2}+2=y^2 \Rightarrow x^2+\frac{1}{x^2}=y^2-2$

Then given equation takes form

$$y^2-2-2+3y=0$$
$$y^2+3y-4=0$$
$$y^2-y+4y-4=0$$
$$y(y-1)+4(y-1)=0$$
$$(y-1)(y+4)=0$$
$$y-1=0,\quad y+4=0$$
$$y=1,\quad y=-4$$

If $y=1$ then, If $y=-4$ then

$$x+\frac{1}{x}=1,\quad x+\frac{1}{x}=-4$$
$$x^2+1=x,\quad x^2+1=-4x$$
$$x^2-x+1=0,\quad x^2+4x+1=0$$

Using $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(1)}}{2(1)}$$
$$x=\frac{1\pm\sqrt{1-4}}{2}$$
$$x=\frac{1\pm\sqrt{-3}}{2}$$
$$x=\frac{1\pm\sqrt{3}\sqrt{-1}}{2}$$
$$x=\frac{1\pm\sqrt{3}\,i}{2}$$

$$x=\frac{-4\pm\sqrt{(4)^2-4(1)(1)}}{2(1)}$$
$$x=\frac{-4\pm\sqrt{16-4}}{2}$$
$$x=\frac{-4\pm\sqrt{12}}{2}$$
$$x=\frac{-4\pm\sqrt{4\times3}}{2}$$
$$x=\frac{-4\pm2\sqrt{3}}{2}$$
$$x=\frac{2(-2\pm\sqrt{3})}{2}$$
$$x=-2\pm\sqrt{3}$$

OR $\left\{\frac{1\pm\sqrt{-3}}{2},\ -2\pm\sqrt{3}\right\}$

OR $\left\{\frac{1\pm\sqrt{3}\,i}{2},\ -2\pm\sqrt{3}\right\}$

Q.21 $2x^4-3x^3-x^2-3x+2=0$

Dividing by x^2

$$\frac{2x^4}{x^2}-\frac{3x^3}{x^2}-\frac{x^2}{x^2}-\frac{3x}{x^2}+\frac{2}{x^2}=\frac{0}{x^2}$$
$$2x^2-3x-1-\frac{3}{x}+\frac{2}{x^2}=0$$
$$2x^2+\frac{2}{x^2}-3x-\frac{3}{x}-1=0$$
$$2\left(x^2+\frac{1}{x^2}\right)-3\left(x+\frac{1}{x}\right)-1=0$$

put $x+\frac{1}{x}=y$

$$\Rightarrow \left(x+\frac{1}{x}\right)^2=y^2$$
$$x^2+\frac{1}{x^2}+2=y^2 \Rightarrow x^2+\frac{1}{x^2}=y^2-2$$

The given equation takes form

$$2(y^2-2)-3y-1=0$$
$$2y^2-4-3y-1=0$$
$$2y^2-3y-5=0$$
$$2y^2+2y-5y-5=0$$
$$2y(y+1)-5(y+1)=0$$
$$(y+1)(2y-5)=0$$
$$y+1=0,\quad 2y-5=0\Rightarrow y=5/2$$
$$y=-1,\quad y=5/2$$

If $y=-1$ Then

$$x+\frac{1}{x}=-1$$
$$x^2+1=-x$$
$$x^2+x+1=0$$

Using $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-1\pm\sqrt{(1)^2-4(1)(1)}}{2(1)}$$
$$x=\frac{-1\pm\sqrt{1-4}}{2}$$
$$x=\frac{-1\pm\sqrt{-3}}{2}\Rightarrow x=\frac{-1\pm\sqrt{3}\,i}{2}$$

If $y=5/2$ then

$$x+\frac{1}{x}=5/2$$
$$2x^2+2=5x$$
$$2x^2-5x+2=0$$
$$2x^2-x-4x+2=0$$
$$x(2x-1)-2(2x-1)=0$$
$$(x-2)(2x-1)=0$$
$$x-2=0,\quad 2x-1=0$$
$$x=2,\quad x=1/2$$

$\left\{2,1/2,\frac{-1\pm\sqrt{-3}}{2}\right\}$ OR. $\left\{2,1/2,\frac{-1\pm\sqrt{3}\,i}{2}\right\}$

Q.22 $2x^4+3x^3-4x^2-3x+2=0$

Dividing by x^2

$$\frac{2x^4}{x^2}+\frac{3x^3}{x^2}-\frac{4x^2}{x^2}-\frac{3x}{x^2}+\frac{2}{x^2}=\frac{0}{x^2}$$

$$2x^2+3x-4-\frac{3}{x}+\frac{2}{x^2}=0$$

$$2x^2+\frac{2}{x^2}+3x-\frac{3}{x}-4=0$$

$$2\left(x^2+\frac{1}{x^2}\right)+3\left(x-\frac{1}{x}\right)-4=0$$

put $x-\frac{1}{x}=y$

$$\rightarrow \left(x-\frac{1}{x}\right)^2=y^2 \Rightarrow x^2+\frac{1}{x^2}-2=y^2$$

$$x^2+\frac{1}{x^2}=y^2+2$$

Given equation takes form

$$2(y^2+2)+3y-4=0$$

$$2y^2+4+3y-4=0$$

$$2y^2+3y=0$$

$$y(2y+3)=0$$

$y=0$, $2y+3=0$

$y=0$, $y=-3/2$

If $y=0$ then

$x-\frac{1}{x}=0$

$x^2-1=0$

$(x-1)(x+1)=0$

$x-1=0, x+1=0$

$x=1, x=-1$

If $y=-3/2$ then

$x-\frac{1}{x}=-3/2$

$2x^2-2=-3x$

$2x^2+3x-2=0$

$2x^2-x+4x-2=0$

$x(2x-1)+2(2x-1)=0$

$(2x-1)(x+2)=0$

$2x-1=0, \quad x+2=0$

$x=1/2, \quad x=-2$

$\{1, -1, 1/2, -2\}$

Q.23 $6x^4-35x^3+62x^2-35x+6=0$

Dividing by x^2

$$\frac{6x^4}{x^2}-\frac{35x^3}{x^2}+\frac{62x^2}{x^2}-\frac{35x}{x^2}+\frac{6}{x^2}=\frac{0}{x^2}$$

$$6x^2-35x+62-\frac{35}{x}+\frac{6}{x^2}=0$$

$$6x^2+\frac{6}{x^2}-35x-\frac{35}{x}+62=0$$

$$6\left(x^2+\frac{1}{x^2}\right)-35\left(x+\frac{1}{x}\right)+62=0$$

put $x+\frac{1}{x}=y$

$$\rightarrow \left(x+\frac{1}{x}\right)^2=y^2$$

$$x^2+\frac{1}{x^2}+2=y^2 \rightarrow x^2+\frac{1}{x^2}=y^2-2$$

Given equation takes form

$$6(y^2-2)-35y+62=0$$

$$6y^2-12-35y+62=0$$

$$6y^2-35y+50=0$$

$$6y^2-15y-20y+50=0$$

$$3y(2y-5)-10(2y-5)=0$$

$$(2y-5)(3y-10)=0$$

$2y-5=0$, $3y-10=0$

$y=5/2$, $y=10/3$

If $y=5/2$ then

$x+\frac{1}{x}=5/2$

$2x^2+2=5x$

$2x^2-5x+2=0$

$2x^2-x-4x+2=0$

$x(2x-1)-2(2x-1)=0$

$(2x-1)(x-2)=0$

$2x-1=0, x-2=0$

$x=1/2, x=2$

If $y=10/3$ then

$x+\frac{1}{x}=10/3$

$3x^2+3=10x$

$3x^2-10x+3=0$

$3x^2-x-9x+3=0$

$x(3x-1)-3(3x-1)=0$

$(3x-1)(x-3)=0$

$3x-1=0, x-3=0$

$x=1/3, x=3$

$\{1/2, 2, 1/3, 3\}$

Q.24 $x^4 - 6x^2 + 10 - \frac{6}{x^2} + \frac{1}{x^4} = 0$

$$x^4 + \frac{1}{x^4} - 6x^2 - \frac{6}{x^2} + 10 = 0$$

$$x^4 + \frac{1}{x^4} - 6\left(x^2 + \frac{1}{x^2}\right) + 10 = 0$$

put $x^2 + \frac{1}{x^2} = y$

$$\rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = y^2$$

$$x^4 + \frac{1}{x^4} + 2 = y^2 \rightarrow x^4 + \frac{1}{x^4} = y^2 - 2$$

Given equation takes form.

$$y^2 - 2 - 6y + 10 = 0$$

$$y^2 - 6y + 8 = 0$$

$$y^2 - 2y - 4y + 8 = 0$$

$$y(y-2) - 4(y-2) = 0$$

$$(y-2)(y-4) = 0$$

$$y - 2 = 0, \quad y - 4 = 0$$

$$y = 2, \quad y = 4$$

If $y = 2$ then

$$x^2 + \frac{1}{x^2} = 2$$

$$x^4 + 1 = 2x^2$$

$$x^4 - 2x^2 + 1 = 0$$

$$(x^2 - 1)^2 = 0$$

$$x^2 - 1 = 0$$

$$(x-1)(x+1) = 0$$

$$x - 1 = 0, \; x + 1 = 0$$

$$x = 1, \; x = -1$$

If $y = 4$ then

$$x^2 + \frac{1}{x^2} = 4$$

$$x^4 + 1 = 4x^2$$

$$x^4 - 4x^2 + 1 = 0$$

$$(x^2)^2 - 4x^2 + 1 = 0$$

Let $x^2 = z$ then

$$z^2 - 4z + 1 = 0$$

$$z = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)}$$

$$z = \frac{4 \pm \sqrt{16 - 4}}{2} \rightarrow z = \frac{4 \pm \sqrt{12}}{2}$$

$$z = \frac{4 \pm \sqrt{4 \times 3}}{2} \rightarrow z = \frac{4 \pm 2\sqrt{3}}{2} = 2\left(\frac{2 \pm \sqrt{3}}{2}\right)$$

$$z = 2 \pm \sqrt{3} \quad \text{so} \quad x^2 = 2 \pm \sqrt{3}$$

$$\rightarrow x = \pm\sqrt{2 \pm \sqrt{3}}$$

$$\left\{1, -1, \pm\sqrt{2 \pm \sqrt{3}}\right\}$$