Exercise 5.2

Q.1 **Factorize**

(i) $x^4+\frac{1}{x^4}-3$

$=x^4+\frac{1}{x^4}-2-1$

$=(x^2)^2+\left(\frac{1}{x^2}\right)^2-2(x^2)\left(\frac{1}{x^2}\right)-1$

$=\left(x^2-\frac{1}{x^2}\right)^2-(1)^2$

$=\left(x^2-\frac{1}{x^2}+1\right)\left(x^2-\frac{1}{x^2}-1\right)$

(ii) $3x^4+12y^4$

$=3(x^4+4y^4)$

$=3\left[(x^2)^2+(2y^2)^2+2(x^2)(2y^2)-4x^2y^2\right]$

$=3\left[(x^2+2y^2)^2-(2xy)^2\right]$

$=3(x^2+2y^2+2xy)(x^2+2y^2-2xy)$

$=3(x^2+2xy+2y^2)(x^2-2xy+2y^2)$

(iii) $a^4+3a^2b^2+4b^4$

$a^4+4a^2b^2+4b^4-a^2b^2$

$=(a^2)^2+2(a^2)(2b^2)+(2b^2)^2-a^2b^2$

$=(a^2+2b^2)^2-(ab)^2$

$=(a^2+2b^2+ab)(a^2+2b^2-ab)$

$=(a^2+ab+2b^2)(a^2-ab+2b^2)$

(iv) $4x^4+81$

$=(2x^2)^2+(9)^2+2(2x^2)(9)-36x^2$

$=(2x^2+9)^2-(6x)^2$

$=(2x^2+9+6x)(2x^2+9-6x)$

$=(2x^2+6x+9)(2x^2-6x+9)$

(v) x^4+x^2+25

$=(x^2)^2+2(x^2)(5)+(5)^2-9x^2$

$=(x^2+5)^2-(3x)^2$

$=(x^2+5+3x)(x^2+5-3x)$

$=(x^2+3x+5)(x^2-3x+5)$

(vi) x^4+4x^2+16

$=(x^2)^2+2(x^2)(4)+(4)^2-4x^2$

$=(x^2+4)^2-(2x)^2$

$=(x^2+4+2x)(x^2+4-2x)$

$=(x^2+2x+4)(x^2-2x+4)$

Q.2 (i) $x^2+14x+48$

$=x^2+6x+8x+48$

$=x(x+6)+8(x+6)$

$=(x+6)(x+8)$

(ii) $x^2-21x+108$

$=x^2-9x-12x+108$

$=x(x-9)-12(x-9)$

$=(x-9)(x-12)$

(iii) $x^2-11x-42$

$=x^2+3x-14x-42$

$=x(x+3)-14(x+3)$

$=(x+3)(x-14)$

(iv) $x^2+x-132$

$=x^2+12x-11x-132$

$=x(x+12)-11(x+12)$

$=(x+12)(x-11)$

Q.3 (i) $4x^2+12x+5$

$=4x^2+2x+10x+5$

$=2x(2x+1)+5(2x+1)$

$=(2x+1)(2x+5)$

(ii) $30x^2+7x-15$

$=30x^2+25x-18x-15$

$=5x(6x+5)-3(6x+5)$

$=(6x+5)(5x-3)$

(iii) $24x^2-65x+21$

$=24x^2-56x-9x+21$

$=8x(3x-7)-3(3x-7)$

$=(3x-7)(8x-3)$

(iv) $5x^2-16x-21$

$=5x^2+5x-21x-21$

$=5x(x+1)-21(x+1)$

$=(x+1)(5x-21)$

(v) $4x^2-17xy+4y^2$

$=4x^2-16xy-xy+4y^2$

$=4x(x-4y)-y(x-4y)$

$=(x-4y)(4x-y)$

(vi) $3x^2-38xy-13y^2$

$=3x^2-39xy+xy-13y^2$

$=3x(x-13y)+y(x-13y)$

$=(x-13y)(3x+y)$

(vii) $5x^2+33xy-14y^2$

$=5x^2+35xy-2xy-14y^2$

$=5x(x+7y)-2y(x+7y)$

$=(x+7y)(5x-2y)$

(viii) $\left(5x-\frac{1}{x}\right)^2+4\left(5x-\frac{1}{x}\right)+4$

$=\left(5x-\frac{1}{x}\right)^2+2\left(5x-\frac{1}{x}\right)(2)+(2)^2$

$=\left(5x-\frac{1}{x}+2\right)^2$

$=\left(5x-\frac{1}{x}+2\right)\left(5x-\frac{1}{x}+2\right)$

Q.4 (i) $(x^2+5x+4)(x^2+5x+6)-3$

Let $x^2+5x=y$

then

$(x^2+5x+4)(x^2+5x+6)-3$

$=(y+4)(y+6)-3$

$=y^2+4y+6y+24-3$

$=y^2+10y+21$

$=y^2+3y+7y+21$

$=y(y+3)+7(y+3)$

$=(y+3)(y+7)$

Putting value of y

$=(x^2+5x+3)(x^2+5x+7)$

(ii) $(x^2-4x)(x^2-4x-1)-20$

Let $x^2-4x=y$

then

$$(x^2-4x)(x^2-4x-1)-20$$
$$=y(y-1)-20$$
$$=y^2-y-20$$
$$=y^2+4y-5y-20$$
$$=y(y+4)-5(y+4)$$
$$=(y+4)(y-5)$$

Putting value of y

$$=(x^2-4x+4)(x^2-4x-5)$$
$$=\left[(x)^2-2(x)(2)+(2)^2\right]\left[x^2+x-5x-5\right]$$
$$=(x-2)^2\left[x(x+1)-5(x+1)\right]$$
$$=(x-2)^2(x+1)(x-5)$$

(iii) $(x+2)(x+3)(x+4)(x+5)-15$

$$=\left[(x+2)(x+5)\right]\left[(x+3)(x+4)\right]-15$$
$$=(x^2+2x+5x+10)(x^2+3x+4x+12)-15$$
$$=(x^2+7x+10)(x^2+7x+12)-15$$

Let $x^2+7x=y$

$$=(y+10)(y+12)-15$$
$$=y^2+10y+12y+120-15$$
$$=y^2+22y+105$$
$$=y^2+7y+15y+105$$
$$=y(y+7)+15(y+7)$$
$$=(y+7)(y+15)$$

Putting value of 'y'

$$(x^2+7x+7)(x^2+7x+15)$$

(iv) $(x+4)(x-5)(x+6)(x-7)-504$

$$=(x^2+4x-5x-20)(x^2+6x-7x-42)-504$$
$$=(x^2-x-20)(x^2-x-42)-504$$

Let $x^2-x=y$

$$=(y-20)(y-42)-504$$
$$=y^2-20y-42y+840-504$$
$$=y^2-62y+336$$
$$=y^2-6y-56y+336$$
$$=y(y-6)-56(y-6)$$
$$=(y-6)(y-56)$$

Putting value of 'y'

$$=(x^2-x-6)(x^2-x-56)$$
$$=(x^2+2x-3x-6)(x^2+7x-8x-56)$$
$$=\left[x(x+2)-3(x+2)\right]\left[x(x+7)-8(x+7)\right]$$
$$=(x+2)(x-3)(x+7)(x-8)$$

(v) $(x+1)(x+2)(x+3)(x+6)-3x^2$

$$=(x+1)(x+6)(x+2)(x+3)-3x^2$$
$$=(x^2+x+6x+6)(x^2+2x+3x+6)-3x^2$$
$$=(x^2+6+7x)(x^2+6+5x)-3x^2$$
$$=\frac{x^2}{x^2}\left[(x^2+6+7x)(x^2+6+5x)-3x^2\right]$$
$$=x^2\left[\frac{(x^2+6+7x)(x^2+6+5x)}{x^2}-\frac{3x^2}{x^2}\right]$$
$$=x^2\left[\left(x+\frac{6}{x}+7\right)\left(x+\frac{6}{x}+5\right)-3\right]$$

Let $x+\frac{6}{x}=y$

$$=x^2\left[(y+7)(y+5)-3\right]$$
$$=x^2(y^2+7y+5y+35-3)$$
$$=x^2(y^2+12y+32)$$
$$=x^2(y^2+4y+8y+32)$$
$$=x^2\left[y(y+4)+8(y+4)\right]$$
$$=x^2(y+4)(y+8)$$

Putting value of y

$$= x^2\left(x+\frac{6}{x}+4\right)\left(x+\frac{6}{x}+8\right)$$

$$= x^2\left(\frac{x^2+4x+6}{x}\right)\left(\frac{x^2+8x+6}{x}\right)$$

$$= (x^2+4x+6)(x^2+8x+6)$$

$$= (x^2+4x+6)(x^2+8x+6)$$

Q.5

(i) $x^3+48x-12x^2-64$

$= x^3-12x^2+48x-64$

$= (x)^3-3(x^2)(4)+3(x)(4)^2-(4)^3$

$= (x-4)^3$

$= (x-4)(x-4)(x-4)$

(ii) $8x^3+60x^2+150x+125$

$= (2x)^3+3(2x)^2(5)+3(2x)(5)^2+(5)^3$

$= (2x+5)^3$

$= (2x+5)(2x+5)(2x+5)$

(iii) $x^3-18x^2+108x-216$

$= (x)^3-3(x)^2(6)+3(x)(6)^2-(6)^3$

$= (x-6)^3$

$= (x-6)(x-6)(x-6)$

(iv) $8x^3-125y^3-60x^2y+150xy^2$

$= 8x^3-60x^2y+150xy^2-125y^3$

$= (2x)^3-3(2x)^2(5y)+3(2x)(5y)^2-(5y)^3$

$= (2x-5y)^3$

$= (2x-5y)(2x-5y)(2x-5y)$

Q.6 (i) $27+8x^3$

$= (3)^3+(2x)^3$

$= (3+2x)\left[(3)^2-(3)(2x)+(2x)^2\right]$

$= (3+2x)(9-6x+4x^2)$

or $= (2x+3)(4x^2-6x+9)$

(ii) $125x^3-216y^3$

$= (5x)^3-(6y)^3$

$= (5x-6y)\left[(5x)^2+(5x)(6y)+(6y)^2\right]$

$= (5x-6y)(25x^2+30xy+36y^2)$

(iii) $64x^3+27y^3$

$= (4x)^3+(3y)^3$

$= (4x+3y)\left[(4x)^2-(4x)(3y)+(3y)^2\right]$

$= (4x+3y)(16x^2-12xy+9y^2)$

(iv) $8x^3+125y^3$

$= (2x)^3+(5y)^3$

$= (2x+5y)\left[(2x)^2-(2x)(5y)+(5y)^2\right]$

$= (2x+5y)(4x^2-10xy+25y^2)$