

Exercise 12.6

Ques 1

$$a = 7, \quad b = 7, \quad c = 9.$$

$$s = \frac{a+b+c}{2} = \frac{7+7+9}{2} = \frac{23}{2} = 11.5$$

Now

$$\begin{aligned} \cos \frac{\alpha}{2} &= \sqrt{\frac{s(s-a)}{bc}} = \sqrt{\frac{(11.5)(11.5-7)}{(7)(9)}} \\ &= \sqrt{\frac{(11.5)(4.5)}{63}} = \sqrt{0.821} = 0.906 \end{aligned}$$

$$\Rightarrow \frac{\alpha}{2} = \cos^{-1}(0.906) = 24.99 \approx 25$$

$$\Rightarrow \alpha = 2(25) \Rightarrow \boxed{\alpha = 50^\circ}$$

Now

$$\begin{aligned} \cos \frac{\beta}{2} &= \sqrt{\frac{s(s-b)}{ca}} = \sqrt{\frac{11.5(11.5-7)}{(9)(7)}} \\ &= \sqrt{0.821} = 0.906 \end{aligned}$$

$$\Rightarrow \frac{\beta}{2} = \cos^{-1}(0.906) = 24.99 \approx 25^\circ$$

$$\Rightarrow \beta = 2(25) \Rightarrow \boxed{\beta = 50^\circ}$$

Now

$$\alpha + \beta + \gamma = 180^\circ$$

$$\begin{aligned} \Rightarrow \gamma &= 180 - \alpha - \beta \\ &= 180 - 50 - 50 \end{aligned}$$

$$\Rightarrow \boxed{\gamma = 80^\circ}$$

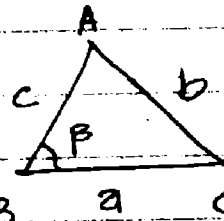
Notes: You may use any formula in above Ques 1

Ques 2 to 5

Do yourself

QNo6 $a = 37.34$, $b = 3.24$, $c = 35.06$

Since angle opposite to smallest side is smallest
therefore β is the smallest angle



&

$$\cos \beta = \frac{c^2 + a^2 - b^2}{2ca}$$

$$= \frac{(35.06)^2 + (37.34)^2 - (3.24)^2}{2(35.06)(37.34)}$$

$$= \frac{2612.98}{2618.28} = 0.9979$$

$$\Rightarrow \beta = \cos^{-1}(0.9979)$$

$$= 3.82$$

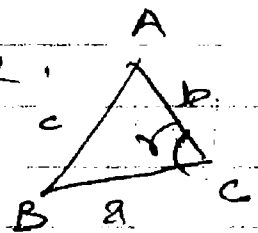
\Rightarrow

$$\boxed{\beta = 3^{\circ}50'}$$

QNo7 Let $a = 16$, $b = 20$, $c = 33$

Since angle opposite to the largest side is largest, here $c = 33$ is largest side
therefore γ is the largest angle.

$$\cos \gamma = \frac{a^2 + b^2 - c^2}{2ab}$$



$$x^2 + x + 1$$

Do yourself

$$\gamma = ?$$

QNo8 Let $a = x^2 + x + 1$, $b = 2x + 1$, $c = x^2 - 1$

Since $a = x^2 + x + 1$ is the greatest side
Therefore α is the greatest angle.

Now

$$\cos \alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{(2x+1)^2 + (x^2-1)^2 - (x^2+x+1)^2}{2(2x+1)(x^2-1)}$$

$$= \frac{4x^2+4x+1 + x^4-2x^2+1 - (x^4+x^2+1+2x^3+2x+2x^2)}{2(2x^3+x^2-2x-1)}$$

$$= \frac{x^4 + \cancel{2x^2} + 4x + 2 - x^4 - x^2 - 1 - 2x^3 - 2x - \cancel{2x^2}}{2(2x^3+x^2-2x-1)}$$

$$= \frac{-2x^3 - x^2 + 2x + 1}{2(2x^3+x^2-2x-1)} = \frac{-(2x^3+x^2-2x-1)}{2(2x^3+x^2-2x-1)}$$

$$\Rightarrow \cos \alpha = -\frac{1}{2} \Rightarrow \alpha = \cos^{-1}\left(-\frac{1}{2}\right)$$

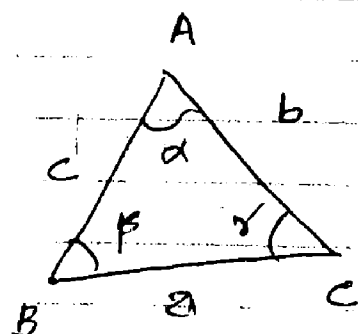
$$\Rightarrow \boxed{\alpha = 120^\circ}$$

Q No 9 Let $a=413$, $b=214$, $c=375$

Since

$$\cos \alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

Do yourself.



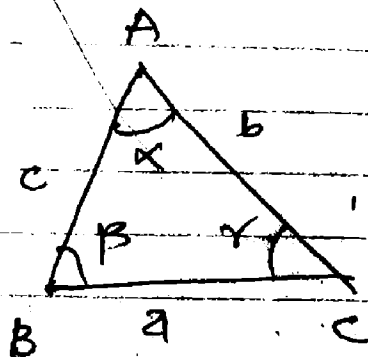
Find all angle in this Question.

Q No 10 Let $a=6$, $b=9$, $c=13$.

$$\text{Now } \cos \alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{(9)^2 + (13)^2 - (6)^2}{2(9)(13)}$$

$$= \frac{214}{234} = 0.9145$$



$$\Rightarrow \alpha = \cos^{-1}(0.9145) = 23.86^\circ$$

$$\Rightarrow \boxed{\alpha = 23^\circ 52'}$$

Now

$$\cos \beta = \frac{c^2 + a^2 - b^2}{2ca}$$

$$= \frac{(13)^2 + (6)^2 - (9)^2}{2(13)(6)} = \frac{124}{156} = 0.7948$$

$$\Rightarrow \beta = \cos^{-1}(0.7948) = 37^\circ 21'$$

$$\Rightarrow \boxed{\beta = 37^\circ 21'}$$

Now

$$\alpha + \beta + \gamma = 180$$

$$\Rightarrow \gamma = 180 - \alpha - \beta$$

$$= 180 - 23^\circ 52' - 37^\circ 21'$$

$$\Rightarrow \boxed{\gamma = 118^\circ 47'}$$

Thus the road of villages make angles $23^\circ 52'$, $37^\circ 21'$ and $118^\circ 47'$ to each other.