SOLVED EXERCISE 1.4

Solve the following equations.

(1) $2x + 5 = \sqrt{7x + 16}$

Solution:

$2x + 5 = \sqrt{7x+16}$ _____ (i)

Squaring both sides, we get

$$(2x+5)^2 = \left(\sqrt{7x+16}\right)^2$$

$$4x^2 + 20x + 25 = 7x + 16$$

$$4x^2 + 20x - 7x + 25 - 16 = 0$$

$$4x^2 + 13x + 9 = 0$$

$$4x^2 + 9x + 4x + 9 = 0$$

$$x(4x+9) + 1(4x+9) = 0$$

$$(x+1)(4x+9) = 0$$

Either $x + 1 = 0$ or $4x + 9 = 0$

$x = -1$ $\quad$ $4x = -9$

$$x = -\frac{9}{4}$$

Check:

Put $x = -1$ in eq. (i), we get

$2(-1) + 5 = \sqrt{7(-1)+16} \quad \Rightarrow \quad -2 + 5 = \sqrt{-7+16}$

$3 = \sqrt{9} \quad \Rightarrow \quad 3 = 3$ (which is true)

Put $x = -\frac{9}{4}$ in eq. (i), we get

$2\left(-\frac{9}{4}\right) + 5 = \sqrt{7\left(-\frac{9}{4}\right)+16} \quad \Rightarrow \quad -\frac{9}{2} + 5 = \sqrt{-\frac{63}{4}+16}$

$\frac{1}{2} = \sqrt{\frac{1}{4}} \quad \Rightarrow \quad \frac{1}{2} = \frac{1}{2}$ (Which is true)

Thus, solution set $= \left\{-1, -\frac{9}{4}\right\}$

2) $\sqrt{x+3} = 3x - 1$

Solution:

$\sqrt{x+3} = 3x - 1$ _____ (i)

Squaring both sides, we get

$$\left(\sqrt{x+3}\right)^2 = (3x-1)^2$$
$$x+3 = 9x^2 - 6x + 1$$
$$9x^2 - 6x + 1 - x - 3 = 0$$
$$9x^2 - 7x - 2 = 0$$
$$9x^2 - 9x + 2x - 2 = 0$$
$$9x(x-1) + 2(x-1) = 0$$
$$(9x+2)(x-1) = 0$$

Either $9x + 2 = 0$ or $x - 1 = 0$

$9x = -2$ $\quad$ $x = 1$

$x = -\dfrac{2}{9}$

Check:

Put $x = -\dfrac{2}{9}$ in eq (i), we get

$\sqrt{-\dfrac{2}{9}+3} = 3\left(-\dfrac{2}{9}\right) - 1 \quad \Rightarrow \quad \sqrt{\dfrac{25}{9}} = -\dfrac{2}{3} - 1$

$\sqrt{\dfrac{25}{9}} = -\dfrac{5}{3} \quad \Rightarrow \quad \dfrac{5}{3} \neq -\dfrac{5}{3}$ (which is not true)

Put $x = 1$ in eq. (i), we get

$\sqrt{1+3} = 3(1) - 1 \quad \Rightarrow \quad \sqrt{4} = 3 - 1$

$2 = 2$ (Which is true).

Thus, solution set = $\{1\}$

(3) $4x = \sqrt{13x+14} - 3$

Solution:

$4x = \sqrt{13x+14} - 3$ ______ (i)

$4x + 3 = \sqrt{13x+14}$

Squering both sides, we get

$$(4x+3)^2 = \left(\sqrt{13x+14}\right)^2$$
$$16x^2 + 24x + 9 = 13x + 14$$
$$16x^2 + 24x - 13x + 9 - 14 = 0$$
$$16x^2 + 11x - 5 = 0$$
$$16x^2 + 16x - 5x - 5 = 0$$
$$16x(x+1) - 5(x+1) = 0$$
$$(16x-5)(x+1) = 0$$

Either $16x - 5 = 0$ or $x + 1 = 0$

$$16x = 5 \qquad\qquad x = -1$$

$$x = \frac{5}{16}$$

Check:

Put $x = \frac{5}{16}$ in eq. (i), we get

$$4\left(\frac{5}{16}\right) = \sqrt{13\left(\frac{5}{16}\right) + 14} - 3 \qquad \Rightarrow \qquad \frac{5}{4} = \sqrt{\frac{65}{16} + 14} - 3$$

$$\frac{5}{4} = \sqrt{\frac{289}{16}} - 3 \qquad \Rightarrow \qquad \frac{5}{4} = \frac{17}{4} - 3$$

$$\frac{5}{4} = \frac{5}{4} \qquad \text{(Which is true)}$$

Put $x = -1$ in eq, (i), we get

$$4(-1) = \sqrt{13(-1) + 14} - 3 \qquad \Rightarrow \qquad -4 = \sqrt{-13 + 14} - 3$$

$$-4 = \sqrt{1} - 3 \qquad \Rightarrow \qquad -4 = 1 - 3$$

$$-4 \neq -2 \qquad \text{(Which is not true)}$$

Thus, solution set = $\left\{\frac{5}{16}\right\}$

4. $\sqrt{3x + 100} - x = 4$

Solution:

$$\sqrt{3x + 100} - x = 4 \_\_\_\_\_\_\_ \text{(i)}$$

$$\sqrt{3x + 100} = x + 4$$

Squaring both sides

$$\left(\sqrt{3x + 100}\right)^2 = (x + 4)^2$$

$$3x + 100 = x^2 + 8x + 16$$

$$x^2 + 8x + 16 - 3x - 100 = 0$$

$$x^2 + 5x - 84 = 0$$

$$x^2 + 12x - 7x - 84 = 0$$

$$x(x + 12) - 7(x + 12) = 0$$

Either $x - 7 = 0$ or $x + 12 = 0$

$$x = 7 \qquad\qquad x = -12$$

Check:

Put $x = 7$ in eq. (i), we get

$$\sqrt{3(7) + 100} - 7 = 4 \qquad \Rightarrow \qquad \sqrt{21 + 100} - 7 = 4$$

$$\sqrt{121} - 7 = 4 \qquad \Rightarrow \qquad 11 - 7 = 4$$

$$4 = 4 \qquad \text{(Whih is true)}$$

Put $x = -12$ in eq. (i), we get

$$\sqrt{3(-12)+100}-(-12)=4 \quad \Rightarrow \quad \sqrt{-36+100}+12=4$$

$$\sqrt{64}=12=4 \quad \Rightarrow \quad 8+12=4$$

$$20 \neq 4 \quad \text{(Which is nt true)}$$

Thus, Solution set = {7}

(5) $\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60}$

Solution:

$$\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60} \quad \_\_\_\_\_\_ \text{(i)}$$

Squaring both sides, we get

$$\left(\sqrt{x+5}+\sqrt{x+21}\right)^2=\left(\sqrt{x+60}\right)^2$$

$$(x+5)+(x+21)+2\sqrt{(x+5)(x+21)}=x+60$$

$$x+5+x+21+2\sqrt{x^2+26x+105}=x+60$$

$$2x+26+2\sqrt{x^2+26x+105}=x+60$$

$$2\sqrt{x^2+26x+105}=x+60-2x-26$$

$$2\sqrt{x^2+26x+105}=-x+34$$

$$2\sqrt{x^2+26+105}=-(x-34)$$

Squaring both sides, we get

$$\left(2\sqrt{x^2+26+105}\right)^2=\left[-(x-34)\right]^2$$

$$4\left(x^2+26x+105\right)=x^2-68x+1156$$

$$4x^2+104x+420=x^2-68x+1156$$

$$4x^2-x^2+104x+68x+420-1156=0$$

$$3x^2+172x-736=0$$

Here a = 3, b = 172, c = – 736

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-172\pm\sqrt{(172)^2-4(3)(-736)}}{2(3)}$$

$$x=\frac{-172\pm\sqrt{29584+8832}}{6}$$

$$x=\frac{-172\pm\sqrt{38416}}{6}$$

$$x=\frac{-172-196}{6} \quad \text{or} \quad x=\frac{-172+196}{6}$$

$$x=-\frac{368}{6} \qquad x=\frac{24}{6}$$

$$x-\frac{184}{3} \qquad x=4$$

Check:

$x=-\dfrac{184}{3}$ in eq. (i), we get

$$\sqrt{-\frac{184}{3}+5}+\sqrt{\frac{-184}{3}+21}=\sqrt{-\frac{184}{3}+60}$$

$$\sqrt{-\frac{169}{3}}+\sqrt{-\frac{121}{3}}=\sqrt{-\frac{4}{3}} \qquad \text{(Which is not true)}$$

Put $x = 4$ in eq. (i), we get

$$\sqrt{4+5}+\sqrt{4+21}=\sqrt{4+60}$$

$$\sqrt{9}+\sqrt{25}=\sqrt{64}$$

$$3+5=8$$

$$8=8 \qquad \text{(Which is true)}$$

Thus, solution set = $\{8\}$

(6) $\sqrt{x-1}+\sqrt{x-2}+\sqrt{x+6}$

Solution:

$\sqrt{x-1}+\sqrt{x-2}+\sqrt{x+6}$ ________ (i)

Squaring both sides, we get

$$\left(\sqrt{x+1}+\sqrt{x-2}\right)^2=\left(\sqrt{x+6}\right)^2$$

$$(x+1)+(x-2)+2\sqrt{(x+1)(x-2)}=x+6$$

$$x+1+x-2+2\sqrt{x^2-x-2}=x+6$$

$$2x-1+2\sqrt{x^2-x-2}=x+6$$
$$2\sqrt{x^2-x-2}=x+6-2x+1$$
$$2\sqrt{x^2-x-2}=-x+7$$
$$2\sqrt{x^2-x-2}=-(x-7)$$
Squaring both sides, we get
$$\left(2\sqrt{x^2-x-2}\right)^2=\left[-(x-7)\right]^2$$
$$4\left(x^2-x-2\right)=x^2-14x+49$$
$$4x^2-4x-8=x^2-14x+49$$
$$4x^2-x^2-4x+14x-8-49=0$$
$$3x^2+10x-57=0$$
Here a = 3, b = 10, c = – 57
$$x=\frac{-10\pm\sqrt{(10)^2-4(3)(-57)}}{2(3)}$$
$$x=\frac{-10\pm\sqrt{100+684}}{6}$$
$$x=\frac{-10\pm\sqrt{784}}{6}$$
$$x=\frac{-10\pm 28}{6}$$
$$x=\frac{-10-28}{6} \quad \text{or} \quad x=\frac{-10+28}{6}$$
$$x=\frac{-38}{6} \qquad x=\frac{18}{6}$$
$$x=-\frac{19}{3} \qquad x=3$$

Check:

Put $x=-\dfrac{19}{3}$ in eq. (i), we get
$$\sqrt{-\frac{19}{3}+1}+\sqrt{\frac{-19}{3}-2}=\sqrt{-\frac{19}{3}+6}$$
$$\sqrt{\frac{-16}{3}}+\sqrt{\frac{-25}{3}}=\sqrt{-\frac{1}{3}} \quad \text{(Which is not true)}$$
Put x = 3 in eg. (i), we get

$$\sqrt{3+1}+\sqrt{3-2}=\sqrt{3+6}$$
$$\sqrt{4}+\sqrt{1}=\sqrt{9}$$
$$2+1=3$$
$$3=3 \text{ (Which is true)}$$

Thus, solution set = $\{3\}$

(7) $\sqrt{11-x}+\sqrt{6-x}=\sqrt{27-x}$

Solution:

$$\sqrt{11-x}+\sqrt{6-x}=\sqrt{27-x} \qquad \_\_\_\_\_\_\_\_ \text{(i)}$$

Squaring both sides, we get

$$\left(\sqrt{11-x}+\sqrt{6-x}\right)^2=\left(\sqrt{27-x}\right)^2$$
$$(11-x)+(6-x)+2\sqrt{(11-x)(6-x)}=27-x$$
$$11-x+6-x+2\sqrt{(11-x)(6-x)}=27-x$$
$$17-2x+2\sqrt{x^2-17x+66}=27-x$$
$$2\sqrt{x^2-17x+66}=27-x-17+2x$$
$$2\sqrt{x^2-17x+66}=10+x$$

Squaring both sides, we get

$$\left(2\sqrt{x^2-17x+66}\right)^2=(10+x)^2$$
$$4\left(x^1-17x+66\right)=100+20x+x^2$$
$$4x^2-68x+264=x^2+20x+100$$
$$4x^2-x^2-68x+20x+264-100=0$$
$$3x^2-88x+164=0$$

Here $a=3$, $b=-88$, $c=164$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-88)\pm\sqrt{(-88)^2-4(3)(164)}}{2(3)}$$

$$x=\frac{88\pm\sqrt{7744-1968}}{6}$$

$$x=\frac{88\pm\sqrt{5776}}{6}$$

$$x=\frac{88\pm 76}{6}$$

$$x=\frac{88-76}{6}, \qquad x=\frac{88+76}{6}$$

$$x=\frac{12}{6}, \qquad x=\frac{164}{6}$$

$$x=2 \qquad x=\frac{82}{3}$$

Check:

Put $x = 2$ in eq. (i), we get

$$\sqrt{11-2}+\sqrt{6-2}=\sqrt{27-2}$$

$$\sqrt{9}+\sqrt{4}=\sqrt{25}$$

$$3+2=5 \quad \Rightarrow \quad 5=5 \text{ (Which is true)}$$

Put $x=\frac{82}{3}$ in eq. (i), we get

$$\sqrt{11-\frac{82}{3}}+\sqrt{6-\frac{82}{3}}=\sqrt{27-\frac{82}{3}}$$

$$\sqrt{-\frac{49}{3}}+\sqrt{-\frac{64}{3}}=\sqrt{-\frac{1}{3}} \text{ (Which is not true)}$$

Thus, Solution set = {2}

(8) $\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$

Solution:

$$\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$$

Squaring both sides, we get

$$\left(\sqrt{4a+x}-\sqrt{a-x}\right)^2=\left(\sqrt{a}\right)^2$$

$$(4a+x)-(a-x)-2\sqrt{(4a+x)(a-x)}=a$$

$$4a+x-a+x-2\sqrt{4a^2-3ax-x^2}=a$$
$$3a+2x-2\sqrt{4a^2-3ax-x^2}=a$$
$$-2\sqrt{4a^2-3ax-x^2}=a-3a-2x$$
$$-2\sqrt{4a^2-3ax-x^2}=-2a-2x$$
$$-2\sqrt{4a^2-3ax-x^2}=-2(a+x)$$
$$\Rightarrow \quad \sqrt{4a^2-3ax-x^2}=(a+x)$$

Squaring both sides, we get

$$\left(\sqrt{4a^2-3ax-x^2}\right)^2=(a+x)^2$$
$$4a^2-3ax-x^2=a^2+x^2+2ax$$
$$-x^2-x^2-3ax-2ax+4a^2-a^2=0$$
$$-2x^2-5ax+3a^2=0$$
$$-\left(2x^2+5ax-3a^2\right)=0$$
$$\Rightarrow \quad 2x^2+5ax-3a^2=0$$
$$2x^2+6ax-ax-3a^2=0$$
$$2x(x+3a)-a(x+3a)=0$$
$$(2x-a)(x+3a)=0$$

Either $2x-a=0$ or $x+3a=0$

$2x=a$ $\qquad$ $x=-3a$

$x=\dfrac{a}{2}$

Thus, Solution set = $\left\{-3a, \dfrac{a}{2}\right\}$

(9) $\sqrt{x^2+x+1}-\sqrt{x^2+x-1}=1$

Solution:

$$\sqrt{x^2+x+1}-\sqrt{x^2+x-1}=1 \qquad \text{\_\_\_\_\_\_ (i)}$$

Let $x^2+x=y$

So eq. (i) becomes

$$\sqrt{y+1}-\sqrt{y-1}=1$$

Squaring both sides, we get

$$\left(\sqrt{y+1}-\sqrt{y-1}\right)^2=1$$
$$(y+1)+(y-1)-2\sqrt{(y+1)(y-1)}=1$$

$$y+1+y-1-2\sqrt{y^2-1}=1$$

$$2y-2\sqrt{y^2-1}=1$$

$$-2\sqrt{y^2-1}=1-2y$$

Squaring both sides, we get

$$\left(-2\sqrt{y^2-1}\right)^2=(1-2y)^2$$

$$4\left(y^2-1\right)=1-4y+4y+4y^2$$

$$4y^2-4=1-4y+4y^2$$

$$4y^2-4-1+4y-4y^2=0$$

$$4y-5=0$$

$$y=\frac{5}{4}$$

Put $y=\frac{5}{4}$ in $x^2+x=y$, we get

$$x^2+x=\frac{5}{4}$$

$\Rightarrow$ $4x^2+4x=5$

$$4x^2+4x-5=0$$

Here $a=4,\ b=4,\ c=-5$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-4\pm\sqrt{(4)^2-4(4)(-5)}}{2(4)}$$

$$x=\frac{-4\pm\sqrt{16+80}}{8}$$

$$x=\frac{-4\pm\sqrt{96}}{8}$$

$$x=\frac{88\pm 76}{6}$$

$$x=\frac{-4\pm 4\sqrt{6}}{8}=\frac{4\left(-1\pm\sqrt{6}\right)}{8}=\frac{-1\pm\sqrt{6}}{2}$$

(10) $\sqrt{x^2+3x+8}+\sqrt{x^2+3x+2}=3$

Solution:

$$\sqrt{x^2+3x+8}+\sqrt{x^2+3x+2}=3 \quad \_\_\_\_\_\_\_\_(i)$$

Let $x^2 + 3x = y$

So eq. (i) becomes

$$\sqrt{y+8} - \sqrt{y+2} = 3$$

Squaring both sides, we get

$$\left(\sqrt{y+8} + \sqrt{y+2}\right)^2 = 9$$

$$(y+8) + (y+2) + 2\sqrt{(y+8)(y+2)} = 9$$

$$y + 8 + y + 2 + 2\sqrt{y^2 + 10y + 16} = 9$$

$$2y + 10 + 2\sqrt{y^2 + 10y + 16} = 9$$

$$2\sqrt{y^2 + 10y + 16} = 9 - 2y - 10$$

$$2\sqrt{y^2 + 10y + 16} = -2y - 1$$

$$2\sqrt{y^2 + 10y + 16} = -(2y + 1)$$

Squaring both sides, we get

$$\left(2\sqrt{y^2 + 10y + 16}\right)^2 = \left[-(2y+1)\right]^2$$

$$4\left(y^2 + 10y + 16\right) = 4y^2 + 4y + 1$$

$$4y^2 + 40y + 64 = 4y^2 + 4y + 1$$

$$4y^2 - 4y^2 + 40y - 4y + 64 - 1 = 0$$

$$36y + 63 = 0$$

$$36y = -63$$

$$y = -\frac{63}{36}$$

Put $y = -\frac{63}{36}$ in $x^2 + 3x = y$, we get

$$x^2 + 3x = -\frac{53}{36}$$

$\Rightarrow$ $36x^2 + 108x = -63$

$$36x^2 + 108x + 63 = 0$$

Here $a = 36$, $b = 108$, $c = 63$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x=\frac{108\pm\sqrt{(108)^2-4(36)(63)}}{2(36)}$$

$$x=\frac{-108\pm\sqrt{11664+9072}}{72}$$

$$x=\frac{-108\pm\sqrt{2592}}{72}$$

$$x=\frac{108\pm36\sqrt{2}}{72}$$

$$x=\frac{36\left(-3\pm\sqrt{2}\right)}{72}$$

$$x=\frac{-3\pm\sqrt{2}}{2}$$

Thus, Solution set = $\left\{\frac{-3\pm\sqrt{2}}{2}\right\}$

11) $\sqrt{x^2+3x+9}+\sqrt{x^2+3x+4}=5$

Solution:

$$\sqrt{x^2+3x+9}+\sqrt{x^2+3x+4}=5 \quad \text{\_\_\_\_\_ (i)}$$

Let $x^2+3x=y$

So eq. (i) becomes

$$\sqrt{y+9}+\sqrt{y+1}=5$$

Squaring both sides, we get

$$\left(\sqrt{y+9}+\sqrt{y+4}\right)^2=25$$

$$(y+9)+(y+4)+2\sqrt{(y+9)(y+4)}=25$$

$$y+9+y+4+2\sqrt{y^2+13+36}=25$$

$$2y+13+2\sqrt{y^2+13y+36}=25$$

$$2\sqrt{y^2+13y+36}=25-2y-13$$

$$2\sqrt{y^2+13y+36}=25-2y+12$$

$$2\sqrt{y^2+13y+36}=25-2(y-6)$$

$\Rightarrow$ $\sqrt{y^2+13y+36}=-(y-6)$

Squaring both sides, we get

$$\left(\sqrt{y^2+13y+36}\right)^2\left[-(y-6)\right]^2$$

$$y^2 + 13y + 36 = y^2 - 12y + 36$$
$$y^2 - y^2 + 13y + 12y + 36 - 36 = 0$$
$$25y = 0$$
$$\Rightarrow \quad y = 0$$

Put $y = 0$ in $x^2 + x^2 + y$, we get

$$x^2 + 3x = y$$
$$x^2 + 3x = 0$$
$$x^2 + 3x = 0$$
$$x(x + 3) = 0$$

Either $x = 0$ or $x + 3 = 0$

Thus, solution set = $\{-3, 0\}$

SOLVED MISCELLANEOUS EXERCISE - 1

Q1. Multiple Choice Questions:
Four possible answers are given for the following questions. Tick (✓) the correct answer.

(i) Standard form of quadratic equation is:
(a) $bx + c = 0$, $b \neq 0$ (b) $ax^2 + bx + c = 0, a \neq 0$
(c) $ax^2 = bx, a \neq 0$ (d) $ax^2 = 0, a \neq 0$

(ii) The number of terms in a standard quadratic equation $ax^2 + bx + c = 0$ is
(a) 1 (b) 2 (c) 3 (d) 4

(iii) The number of methods to solve a quadrate equation is:
(a) 1 (b) 2 (c) 3 (d) 4

(iv) The quadratic formula is:
(a) $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (b) $\dfrac{b \pm \sqrt{b^2 - 4ac}}{2a}$
(c) $\dfrac{-b \pm \sqrt{b^2 + 4ac}}{2a}$ (d) $\dfrac{b \pm \sqrt{b^2 + 4ac}}{2a}$

(v) Two linear factors of $x^2 - 15x + 56$ are:
(a) $(x - 7)$ and $(x + 8)$ (b) $(x + 7)$ and $(x - 8)$
(c) $(x - 7)$ and $(x - 8)$ (d) $(x + 7)$ and $(x + 8)$

(vi) An equation, which remains unchanged when x is replaced by $\frac{1}{x}$ is called a/an
(a) Exponential equation (b) Reciprocal equation
(c) Radical equation (d) None of these

(vii) An equation 6fthetype $3^x + 3^{2x} + 6 = 0$ is a/an:
(a) Exponential equation (b) Radical equation