Since 1 and 3 are the root of the equation

$x^4 + 2x^3 - 13x^2 - 14x + 24 = 0$

Then by synthetic division, we get

$$
\begin{array}{r|rrrrr}
 & 1 & 2 & -13 & -14 & 24 \\
3 & \downarrow & 3 & 15 & 6 & -24 \\
\hline
 & 1 & 5 & 2 & -8 & |\,0 \\
-4 & \downarrow & -4 & -4 & 8 & \\
\hline
 & 1 & 1 & -2 & |\,0 &
\end{array}
$$

The depressed equation is

$$x^2 + x - 2 = 0$$
$$x^2 - 2x + x - 2 = 0$$
$$x(x - 2) + 1(x - 2) = 0$$
$$(x + 1)(x - 2) = 0$$

Either $x + 1 = 0$ or $x - 2 = 0$

$x = -1$ $\quad$ $x = 2$

Thus, − 4, −1, 2 and 3 are the roots of given equation.

Simultaneous equations:

A system of equations having a common solution is called a system of simultaneous equations.

The set of all the ordered pairs (x, y), .which satisfies the system of equations is called the solution set of the system.

SOLVED EXERCISE 2.7

Solve the following simultaneous equations.

1. $\quad \mathbf{x + y = 5 \quad ; \quad x^2 - 2y - 14 = 0}$

Solution:

$x + y = 5$ ________ (i)

$x^2 - 2y - 14 = 0$ ________ (ii)

From (i), we have

$y = 5 - x$ ________ (iii)

Put value of y in eq. (ii), we get

$$x^2 - 2(5 - x) - 14 = 0$$
$$x^2 - 10 + 2x - 14 = 0$$
$$x^2 + 2x - 24 = 0$$
$$x^2 + 6x - 4x - 24 = 0$$
$$x(x + 6) - 4(x + 6) = 0$$
$$(x - 4)(x + 6) = 0$$

Either $x - 4 = 0$ or $x + 6 = 0$

$x = 4$ $\quad$ $x = -6$

Put x = 4 in eq. (iii), we get $\quad$ Put x = − 6 in eq (iii), we get

$y = 5 - 4$
$= 1$

$y = 5 - (-6)$
$= 5 + 6$
$= 11$

The ordered pairs are (4, 1) are (– 6, 11).
Thus, solution set = {(4, 1), (– 6, 11)}

2. $\mathbf{3x - 2y = 1}$; $\mathbf{x^2 + xy - y^2 = 1}$

Solution:

$3x - 2y = 1$ ________ (i)
$x^2 + xy - y^2 = 1$ ________ (ii)
From (i), we hae
$2y = 3x - 1$

$$y = \frac{1}{2}(3x - 1)$$

$$y = \frac{3}{2}x - \frac{1}{2} \quad \text{\_\_\_\_\_\_\_\_ (iii)}$$

Put value of y in eq. (ii), we get

$$x^2 + x\left(\frac{3}{2}x - \frac{1}{2}\right) - \left(\frac{3}{2}x - \frac{1}{2}\right)^2 = 1$$

$$x^2 + \frac{3}{2}x^2 - \frac{1}{2}x - \frac{9}{4}x^2 + \frac{3}{2}x - \frac{1}{4} = 1$$

Multiplying both sides by '4', we get
$4x^2 + 6x^2 - 2x - 9x^2 + 6x - 1 = 4$
$4x^2 + 6x^2 - 9x^2 - 2x + 6x - 1 - 4 = 0$
$x^2 + 4x - 5 = 0$
$x^2 + 5x - x - 5 = 0$
$x(x + 5) - 1(x + 5) = 0$
$(x - 1)(x + 5) = 0$

Either $x - 1 = 0$ or $x + 5 = 0$
$x = 1$ $x = -5$

Put $x = 1$ in eq. (iii), we get

$$y = \frac{3}{2}(1) - \frac{1}{2}$$
$$= \frac{3}{2} - \frac{1}{2}$$
$$= \frac{2}{2}$$
$$= 1$$

Put $x = -5$ in eq (iii), we get

$$y = \frac{3}{2}(-5) - \frac{1}{2}$$
$$= -\frac{15}{2} - \frac{1}{2}$$
$$= -\frac{16}{2}$$
$$= -8$$

The ordered pairs are (1, 1) are (–5, –8).
Thus, solution set = {(1, 1), (– 5, –8)}

3. $x - y = 7$; $\frac{2}{x} - \frac{5}{y} = 2$

Solution:

$$x - y = 7 \quad \text{\_\_\_\_\_\_\_\_\_\_ (i)}$$

$$\frac{2}{x} - \frac{5}{y} = 2$$

$$\frac{2y - 5x}{xy} = 2$$

$$2y - 5x = 2xy \quad \text{\_\_\_\_\_\_\_\_\_\_ (ii)}$$

From (i), we hae

$$y = x - 7 \quad \text{\_\_\_\_\_\_\_\_\_\_ (iii)}$$

Put value of y in eq. (ii), we get

$$2(x-7) - 5x = 2x(x-7)$$

$$2x - 14 - 5x = 2x^2 - 14x$$

$$-3x - 14 = 2x^2 - 14x$$

or $2x^2 - 14x + 3x + 14 = 0$

$$2x^2 - 11x + 14 = 0$$

$$2x^2 - 7x - 4x + 14 = 0$$

$$x(2x-7) - 2(2x-7) = 0$$

$$(x-2)(2x-7) = 0$$

Either $x - 2 = 0$, $x = 2$ or $2x - 7 = 0$, $2x = 7$, $x = \frac{7}{2}$

Put $x = 2$ in eq. (iii), we get

$$y = 2 - 7$$

$$= -5$$

Put $x = \frac{7}{2}$ in eq (iii), we get

$$y = \frac{7}{2} - 7$$

$$y = -\frac{7}{2}$$

The ordered pairs are $(2,-5), \left(\frac{7}{2}, -\frac{7}{2}\right)$

Thus, solution set = $\left\{(2,-5), \left(\frac{7}{2}, -\frac{7}{2}\right)\right\}$

4. $x + y = a - b$; $\frac{a}{x} - \frac{b}{y} = 2$

Solution:

$$x + y = a - b \quad \text{\_\_\_\_\_\_\_\_\_\_ (i)}$$

$$\frac{a}{x}-\frac{b}{y}=2$$

$$\frac{ay-bx}{xy}=2 \qquad \_\_\_\_\_\_\_\_\_\_ \text{(ii)}$$

From eq. (i), we have

$y = a - b - x$ __________ (iii)

Put value of y in eq. (ii),, we get

$a(a-b-x) - bx - 2x(a-b-x)$

$2x^2 - 2ax - ax + 2bx - bx - a^2 - ab = 0$

$2x^2 - 3ax + bx + a^2 - ab = 0$

$2x^2 - (3a-b)x + (a^2 - ab) = 0$

Using quadratic formula

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-\left[-(3a-b) \pm \sqrt{\left[-(3a-b)\right]^2 - 4(2)\left(a^2-ab\right)}\right]}{4(2)}$$

$$= \frac{(3a-b) \pm \sqrt{9a^2 - 6ab + b^2 - 8a^2 + 8ab}}{4}$$

$$= \frac{(3a-b) \pm \sqrt{a^2 + 2ab + b^2}}{4}$$

$$= \frac{(3a-b) \pm \sqrt{(a+b)^2}}{4} = \frac{(3a-b) \pm (a+b)}{4}$$

Either $x = \dfrac{(3a-b) \pm (a+b)}{4}$ or $x = \dfrac{(3a-b) \pm (a+b)}{4}$

$$= \frac{3a - b + a + b}{4} \qquad\qquad = \frac{3a - b - a - b}{4}$$

$$= \frac{4a}{4} \qquad\qquad = \frac{2a-2b}{4}$$

$$= a \qquad\qquad = \frac{2(a-b)}{4} = \frac{a-b}{2}$$

Put $x = a$ in eq. (iii), we get

$$y = a - b - a$$

$$= -b$$

Put $x = \dfrac{a-b}{2}$ in eq. (iii), we get

$$y = (a-b) - \frac{a-b}{2}$$

$$= \frac{2a - 2b - a + b}{2}$$

$$= \frac{a-b}{2}$$

$\therefore$ The ordered pairs are $(a, -b), \left(\frac{a-b}{2}, \frac{a-b}{2}\right)$

Thus, solution set = $\left\{(a,-b), \left(\frac{a-b}{2}, \frac{a-b}{2}\right)\right\}$

5. $\mathbf{x^2 + (y-1)^2 = 10}$; $\mathbf{x^2 + y^2 + 4x = 1}$

Solution:

$x^2 + (y-1)^2 = 10$
$x^2 + y^2 - 2y + 1 = 10$
$x^2 + y^2 - 2y = 10 - 1$
$x^2 + y^2 - 2y = 9$ ________ (i)
$x^2 + y^2 + 4x = 1$ ________ (ii)

Subtract eq. (ii) from eq. (i), we have

$$\begin{array}{r} x^2 + y^2 - 2y = 9 \\ \underline{\pm x^2 \pm y^2 \pm 4x = \pm 1} \\ -4x - 2y = 8 \end{array}$$

$-2(2x + y) = 8$

$\Rightarrow$ $2x + y = -4$
$y = -2x - 4$ ________ (iii)

Put the value of y in eq. (ii). We have

$x^2 + (-2x - 4)^2 + 4x = 1$
$x^2 + \{-(2x + 4)\}^2 + 4x = 1$
$x^2 + 4x^2 + 16x + 16 + 4x - 1 = 0$
$5x^2 + 20x + 15 = 0$
$5(x^2 + 4x + 3) = 0$

$\Rightarrow$ $x^2 + 4x + 3 = 0$
$x^2 + 3x + x + 3 = 0$
$x(x + 3) + 1(x + 3) = 0$
$(x + 1)(x + 3) = 0$

Either $x + 3 = 0$ or $x + 1 = 0$
$x = -3$ $x = -1$

Put $x = -3$ in eq (iii) we get,
$y = -2(-3) - 4$
$= -6 - 4$
$= -10$

Put $x = -1$ in eq (ii)
$y = -2(-1) - 4$
$= -2 - 4$
$= -6$

$\therefore$ The ordered pairs are $(-3, -10), (-1, -6)$

Thus, solution set = $\{(-3, -10), (-1, -6)\}$

6. $\mathbf{(x + 1)^2 + (y + 1)^2 = 5}$; $\mathbf{(x + 2)^2 + y^2 = 5}$

Solution:

$(x + 1)^2 + (y + 1)^2 = 5$

$x^2 + 2x + 1 + y^2 + 2y + 1 = 5$
$x^2 + y^2 + 2x + 2y = 5 - 1 - 1$
$x^2 + y^2 + 2x + 2y = 3$ ______ (i)
$(x + 2)^2 + y^2 = 5$
$x^2 + 4x + 4 + y^2 = 5$
$x^2 + y^2 + 4x = 5 - 4$
$x^2 + y^2 + 4x = 1$ ______ (ii)
Subtract eq. (ii) from eq. (i), we have

$$\begin{array}{l} x^2 + y^2 + 2x + 2y = 3 \\ \underline{\pm x^2 \pm y^2 \pm 4 \ = \pm 1} \\ \quad -2x + 2y = 2 \end{array}$$

$2(-x + y) = 2$

$\Rightarrow$ $-x + y = 1$

$y = x + 1$ ______ (iii)

Put the value of y in eq. (ii), we have

$x^2 + (x+1)^2 + 4x = 1$

$x^2 + x^2 + 2x + 1 + 4x = 1$

$2x^2 + 6x + 1 - 1 = 0$

$2x^2 + 6x + 0 = 0$

$2x^2 + 6x = 0$

$2x(x+3) = 0$

Either $2x = 0$ or $x + 3 = 0$

Put $x = 0$ in eq. (iii), we get
$y = 0 + 1$
$= 0$

Put $x = -3$ in eq. (iii), we get
$y = -3 + 1$
$= -2$

$\therefore$ The ordered pairs are $(0, 1)$, $(-3, -2)$

Thus, solution set = $\{(0,1),(-3,-2)\}$

7. $\mathbf{x^2 + 2y^2 = 22}$; $\mathbf{5x^2 + y^2 = 29}$

Solution:

$x^2 + 2y^2 = 22$ ______ (i)
$5x^2 + y^2 = 29$ ______ (ii)

Multiply eq. (ii) by '2' then subtract eq. (ii) from eq. (i) we get

$$\begin{array}{l} \underline{\pm 10x^2 \pm 2y^2 = \pm 58} \\ \quad -9x^2 = -36 \end{array}$$

$\Rightarrow$ $x^2 = 4$

Put the value of $x^2 = 4$ in eq. (i), we get

$4 + 2y^2 = 22$

$$2y^2 = 22 - 4$$
$$2y^2 = 18$$
$$\Rightarrow \quad y^2 = 9$$
$$y = \pm 3$$
Thus, solution set = $\{(\pm 2, \pm 3)\}$

8. $4x^2 - 5y^2 = 6$; $3x^2 + y^2 = 14$

Solution:

$4x^2 - 5y^2 = 6$ ________ (i)
$3x^2 + y^2 = 14$ ________ (ii)

Multiply eq. (ii) by 5 then add eq. (i) and (ii) we get.

$$3x^2 - 5y^2 = 6$$
$$\underline{15x^2 + 5y^2 = 70}$$
$$19x^2 = 76$$
$$\Rightarrow \quad x^2 = 4$$

Put the value of $x^2 = 4$ in eq. (ii), we get

$$3(4) + y^2 = 14$$
$$12 + y^2 = 14$$
$$y2 = 14 - 12$$
$$y2 = 2$$
$$y = \pm \sqrt{2}$$

Thus, solution set = $\{(\pm 2, \pm\sqrt{2})\}$

9. $7x^2 - 3y^2 = 4$; $2x^2 + 5y^2 = 7$

Solution:

$7x^2 - 3y^2 = 4$ ________ (i)
$2x^2 + 5y^2 = 7$ ________ (ii)

Multiply eq. (i) by '5' and eq. (ii) by add eq. (i) and (ii), we get

$$35x^2 - 15y^2 = 20$$
$$\underline{6x^2 + 15y = 21}$$
$$41x^2 = 41$$
$$\Rightarrow \quad x^2 = 1$$
$$x = \pm 1$$

Put the value of $x^2 = 1$ in eq. (ii), we get

$$2(1) + 5y^2 = 7$$
$$2 + 5y^2 = 7$$
$$5y^2 = 7 - 2$$

$\Rightarrow \quad y^2 = 1$

$y = \pm 1$

Thus, solution set $= \{(\pm 1, \pm 1)\}$

10. $\mathbf{x^2 + 2y^2 = 3 \quad ; \quad x^2 + 4xy - 5y^2 = 0}$

Solution:

$$x^2 + 2y^2 = 3 \qquad \text{\_\_\_\_\_\_ (i)}$$

$$x^2 + 4xy - 5y^2 = 0$$

$$x^2 + 5xy - xy - 5y^2 = 0$$

$$x(x + 5y) - y(x + 5y) = 0$$

$$(x - y)(x + 5y) = 0$$

$x - y = 0$ or $x + 5y = 0$

or $y = x$ $\qquad$ $x = -5y$

Put $y = x$ in eq. (i), we get

$$x^2 + 2x^2 = 3$$

$$3x^2 = 3$$

$$\Rightarrow \quad x^2 = 1$$

$$x = \pm 1$$

Put $x = -5y$ in eq. (i), we get

$$(-5y)^2 + 2y^2 = 3$$

$$25y^2 + 2y^2 = 3$$

$$27y^2 = 3$$

$$\Rightarrow \quad y^2 = \frac{1}{9}$$

$$y = \pm \frac{1}{3}$$

Put $x = \pm 1$ in $y = x$, we get

$y = \pm 1$

Put $y = \pm \frac{1}{3}$ in $x = -5y$, we get

$$x = -5\left(\pm\frac{1}{3}\right)$$

$$x = \mp\frac{5}{3}$$

$\therefore$ The ordered pairs are $(1, 1)$, $(-1, 1)$ $\qquad$ $\therefore$ The ordered pairs are $\left(\frac{5}{3}, -\frac{1}{3}\right), \left(-\frac{5}{3}, \frac{1}{3}\right)$

Thus, solution set $= \left\{(-1, 1), (-1, 1), \left(\frac{5}{3}, -\frac{1}{3}\right), \left(-\frac{5}{3}, \frac{1}{3}\right)\right\}$

11. $\mathbf{3x^2 - y^2 = 26 \quad ; \quad 3x^2 - 5xy - 2y^2 = 0}$

Solution:

$$3x^2 - y^2 = 26 \qquad \text{\_\_\_\_\_\_ (i)}$$

$$3x^2 - 5xy - 2y^2 = 0$$

$$3x^2-9xy-4xy-12y^2=0$$
$$3x(x-3y)+4y(x-3y)=0$$
$$(3x+4y)(x-3y)=0$$

Either $3x+4y=0$ or $x-3y=0$

$3x=-4y$ $\quad$ $x=3y$

$$x=-\frac{4}{3}y$$

Put $x=-\frac{4}{3}y$ in eq. (i), we get

Put $x=3y$ in eq. (i), we get

$$3(3y)^2-y^2=26$$
$$3(9x)^2-y^2=26$$
$$27y2-y^2=26$$
$$26y^2=26$$
$$\Rightarrow \quad y^2=1$$
$$y=\pm 1$$

Put $y=\pm 1$ in $x=2y$, we get

$$x=3(\pm 1)$$
$$x=\pm 3$$

$\therefore$ The ordered pairs are (3, 1), (−3 , 1)

$$3\left(-\frac{4}{3}y\right)^2-y^2=26$$
$$3\left(\frac{16}{9}y^2\right)-y^2=26$$
$$\frac{16}{9}y^2-y^2=26$$
$$\frac{16y^2-3y^2}{3}=26$$
$$13y^2=\frac{26\times 3}{13}$$
$$y^2=6$$
$$y=\pm\sqrt{6}$$

Put $y=\pm\sqrt{6}$ in $x=-\frac{4}{3}y$, we get

$$x=-\frac{4}{3}\left(\pm\sqrt{6}\right)$$
$$x=\mp\frac{4\sqrt{3}}{3}$$

$\therefore$ The ordered pairs are $\left(\frac{-4\sqrt{3}}{3},\sqrt{6}\right),\left(\frac{4\sqrt{3}}{3},-\sqrt{6}\right)$

Thus, solution set =

12. $x^2 + xy = 5$; $y^2 + xy = 3$

Solution:

$x^2 + xy = 5$ ________ (i)

$y^2 + xy = 3$ ________ (ii)

Multiply eq. (i) by '3' and eq. (ii) by '5' then subtract eq. (ii) from eq. (i), we get.

$$3x^2 + 3xy = 15$$

$$\underline{\pm 5y^2 \pm 5xy = \pm 15}$$

$$3x^2 - 2xy - 5y^2 = 0$$

$$3x^2 - 5xy + 3xy - 5y^2 = 0$$

$$x(3x - 5y) + y(3x - 5y) = 0$$

$$(x + y)(3x - 5y) = 0$$

Either $x + y = 0$ or $3x - 5y = 0$

$y = -x$

$3x = 5y$

$x = \frac{5}{3}y$

Put $x = -x$ in eq. (i), we get

$$x^2 + x(-x) = 5$$

$$x^2 - x^2 = 5$$

$$0 = 5$$

which is not possible

Put $x = \frac{5}{3}y$ in eq. (i), we get

$$\left(\frac{5}{3}y\right)^2 + \left(\frac{5}{3}y\right)y = 5$$

$$\frac{25}{9}y^2 + \frac{5}{3}y^2 = 5$$

$$\frac{25y^2 + 15y^2 = 5}{9}$$

$$\frac{40y^2}{9} = 5$$

$$40y^2 = 45$$

$$y^2 = \frac{45}{40}$$

$$y^2 = \frac{9}{8}$$

$$y = \pm\frac{3}{2\sqrt{2}}$$

$$\text{Put } y = \pm \frac{3}{2\sqrt{2}} \text{ in } x = \frac{5}{3}y, \text{ we get}$$

$$x = \frac{5}{3}\left(\pm\frac{3}{2\sqrt{2}}\right)$$

$$x \pm \frac{5}{2\sqrt{2}}$$

$\therefore$ The ordered pairs are $\left(\frac{5}{2\sqrt{2}}, -\frac{3}{2\sqrt{2}}\right), \left(-\frac{5}{2\sqrt{2}}, -\frac{3}{2\sqrt{2}}\right)$

Thus, solution set = $\left\{\left(\frac{5}{2\sqrt{2}}, -\frac{3}{2\sqrt{2}}\right), \left(-\frac{5}{2\sqrt{2}}, -\frac{3}{2\sqrt{2}}\right)\right\}$

13. $\mathbf{x^2 - 2xy = 7}$; $\mathbf{xy + 3y^2 = 2}$

Solution:

$x^2 - 2xy = 7$ ________ (i)

$xy + 3y^2 = 2$ ________ (ii)

Multiply eq (i) by '2' and eq. (ii) by '7' then subtract eq. (ii) from eq (i), we get

$$\begin{array}{l} 2x^2 - 4xy = 14 \\ \underline{\pm 21y^2 \pm 7xy = \pm 14} \\ 2x^2 - 11xy - 21y^2 = 0 \end{array}$$

$$2x^2 - 14xy + 3xy - 21y^2 = 0$$

$$2x(x - 7y) + 3y(x - 7y) = 0$$

$$(2x + 3y)(x - 7y) = 0$$

Either $2x + 3y = 0$ or $x - 7y = 0$

$2x = -3y$ $\quad$ $x = 7t$

Put $x = -\frac{3}{2}y$ in eq. (ii), we get $\quad$ Put $x = 7y$ in eq. (ii), we get

$$x = -\frac{3}{2}\left(\pm\frac{2}{\sqrt{3}}\right) \qquad = \pm\frac{7}{\sqrt{5}}$$

$$= \mp\sqrt{3}$$

$\therefore$ The ordered pairs are $\left(-\sqrt{3}, \frac{2}{\sqrt{3}}\right), \left(\sqrt{3}, -\frac{2}{\sqrt{3}}\right)$

$\therefore$ The ordered pairs are $\left(\frac{7}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right), \left(\frac{7}{\sqrt{5}}, -\frac{1}{\sqrt{5}}\right)$

Thus, solution set = $\left\{\left(\frac{7}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right), \left(\frac{7}{\sqrt{5}}, -\frac{1}{\sqrt{5}}\right), \left(-\sqrt{3} \frac{2}{\sqrt{3}}\right), \left(\sqrt{3} - \frac{2}{\sqrt{3}}\right)\right\}$