Exercise 2.6

Q1. Identify the following statements as true or false.

(i) $\sqrt{-3}\times\sqrt{-3}=3$ False

(ii) $i^{73}=-i$ False

(iii) $i^{10}=-1$ True

(iv) Complex conjugate of $\left(-6i+i^2\right)$ is $(-1+6i)$ True

(v) Difference of a complex number $z=a+bi$ and its conjugate is a real number. False

(vi) If $(a-1)-(b+3)i=5+8i$ then $a=6$ and $b=-11$. True

(vii) Product of a complex number and its conjugate is always a non-negative real number. True

Q2. Express each complex number in the standard form $a+bi$, where 'a' and 'b' are real numbers.

(i) $(2+3i)+(7-2i)$

$=2+3i+7-2i$

$=(2+7)+(3-2)i$

$=9+i$

(ii) $2(5+4i)-3(7+4i)$

$=10+8i-21-12i$

$=(10-21)+(8-12)i$

$=-11-4i$

(iii) $-1(-3+5i)-(4+9i)$

$=3-5i-4-9i$

$=(3-4)+(-5-9)i$

$=-1-14i$

(iv) $2i^2+6i^3+3i^{16}-6i^{19}+4i^{25}$

$=2(-1)+6i^2.i+3\left(i^2\right)^8-6i^{18}.i+4i^{24}.i$

$=-2+6(-1).i+3(-1)^8-6\left(i^2\right)^9.i+4\left(i^2\right)^{12}.i$

$=-2-6i+3(1)-6(-1)^9.i+4(-1)^{12}.i$

$=-2-6i+3-6(-1)i+4(1).i$

$=-2-\cancel{6i}+3+\cancel{6i}+4i$

$=1+4i$

Q3. Simplify and write your answer in the form $a+bi$

(i) $(-7+3i)(-3+2i)$

$=21-14i-9i+6i^2$

$=21-23i+6(-1)$

$=21-6-23i$

$=15-23i$

(ii) $\left(2-\sqrt{-4}\right)\left(3-\sqrt{-4}\right)$

$=\left(2-\sqrt{4}.\sqrt{-1}\right)\left(3-\sqrt{4}\sqrt{-1}\right)$

$=(2-2i)(3-2i)$

$=6-4i-6i+4i^2$

$=6-10i+4(-1)$

$=6-10i-4$

$=2-10i$

(iii) $\left(\sqrt{5}-3i\right)^2$

$=\left(\sqrt{5}\right)^2+(3i)^2-2\left(\sqrt{5}\right)(3i)$

$=5+9i^2-6\sqrt{5}i$

$=5+9(-1)-6\sqrt{5}i$

$=5-9-6\sqrt{5}i$

$=-4-6\sqrt{5}i$

(iv) $(2-3i)\left(\overline{3-2i}\right)$

$=(2-3i)(3+2i)$

$=6+4i-9i-6i^2$

$=6-5i-6(-1)$

$=6-5i+6$

$=12-5i$

Q4. Simplify and write your answer in the form of $a+bi$

(i) $\dfrac{-2}{1+i}$

$=\dfrac{-2}{1+i}\times\dfrac{1-i}{1-i}$

$=\dfrac{-2(1-i)}{(1)^2-(i)^2}$

$=\dfrac{-2(1-i)}{1-i^2}$

$=\dfrac{-2(1-i)}{1-(-1)}$

$=\dfrac{-2(1-i)}{1+1}$

$=\dfrac{-\cancel{2}(1-i)}{\cancel{2}}$

$=-(1-i)$

$=-1+i$

(ii) $\dfrac{2+3i}{4-i}$

$=\dfrac{2+3i}{4-i}\times\dfrac{4+i}{4+i}$

$=\dfrac{(2+3i)(4+i)}{(4)^2-(i)^2}$

$=\dfrac{8+2i+12i+3i^2}{16-i^2}$

$=\dfrac{8+14i+3(-1)}{16-(-1)}$

$=\dfrac{8+14i-3}{16+1}$

$=\dfrac{5+14i}{17}$

$=\dfrac{5}{17}+\dfrac{14}{17}i$

(iii) $\dfrac{9-7i}{3+i}$

$=\dfrac{9-7i}{3+i}\times\dfrac{3-i}{3-i}$

$=\dfrac{(9-7i)(3-i)}{(3)^2-(i)^2}$

$=\dfrac{27-9i-21i+7i^2}{9-i^2}=\dfrac{27-30i+7(-1)}{9-(-1)}$

$=\dfrac{27-7-30i}{9+1}$

$=\dfrac{20-30i}{10}$

$=\dfrac{20}{10}-\dfrac{30}{10}i$

$=2-3i$

(iv) $\dfrac{2-6i}{3+i}-\dfrac{4+i}{3+i}$

$=\dfrac{(2-6i)-(4+i)}{3+i}$

$=\dfrac{2-6i-4-i}{3+i}$

$=\dfrac{-2-7i}{3+i}$

$=\dfrac{-2-7i}{3+i}\times\dfrac{3-i}{3-i}$

$$=\frac{(-2-7i)(3-i)}{(3)^2-(i)^2}$$

$$=\frac{-6+2i-21i+7i^2}{9-i^2}$$

$$=\frac{-6-19i+7(-1)}{9-(-1)}$$

$$=\frac{-6-7-19i}{9+1}$$

$$=\frac{-13-19i}{10}$$

$$=\frac{-13}{10}-\frac{19}{10}i$$

) $\left(\frac{1+i}{1-i}\right)^2$

$$=\frac{(1)^2+(i)^2+2(1)(i)}{(1)^2+(i)^2-2(1)(i)}$$

$$=\frac{1+i^2+2i}{1+i^2-2i}$$

$$=\frac{\cancel{1}-\cancel{1}+2i}{\cancel{1}-\cancel{1}-2i}$$

$$=\frac{2i}{-2i}$$

$$=-1$$

$$=-1+0i$$

$$\frac{1}{(2+3i)(1-i)}$$

$$=\frac{1}{2-2i+3i-3i^2}$$

$$=\frac{1}{2+i-3(-1)}$$

$$=\frac{1}{2+i+3}$$

$$=\frac{1}{5+i}$$

$$=\frac{1}{5+i}\times\frac{5-i}{5-i}$$

$$=\frac{5-i}{(5)^2-(i)^2}$$

$$=\frac{5-i}{25-i^2}$$

$$=\frac{5-i}{25-(-1)}$$

$$=\frac{5-i}{25+1}$$

$$=\frac{5-i}{26}$$

$$=\frac{5}{26}-\frac{1}{26}i$$

Q5. **Calculate (a)** $\bar{z}$ **(b)** $z+\bar{z}$ **c)** $z-\bar{z}$ **(d)** $z.\bar{z}$ **for each of the following.**

(i) $z=0-i$

(a) $\bar{z}=0+i$

(b) $z+\bar{z}=0-i+0+i=0$

(c) $z-\bar{z}=0-i-(0+i)$

$=0-i-0-i$

$=-2i$

(d) $z.\bar{z}=(0-i)(0+i)$

$=(0)^2-(i)^2=0-(-1)$

$=1$

(ii) $z=2+i$

(a) $\bar{z}=2-i$

(b) $z+\bar{z}=2+\cancel{i}+2-\cancel{i}$

$=4$

(c) $z-\bar{z}=(2+i)-(2-i)$

$=\cancel{2}+i-\cancel{2}+i$

$=2i$

(d) $z.\bar{z} = (2+i)(2-i)$

$= (2)^2 - (i)^2$

$= 4 - i^2$

$= 4 - (-1)$

$= 4 + 1$

$= 5$

(iii) $z = \dfrac{1+i}{1-i}$

$= \dfrac{1+i}{1-i} \times \dfrac{1+i}{1+i}$

$= \dfrac{(1+i)^2}{(1)^2 - (i)^2}$

$= \dfrac{(1)^2 + (i)^2 + 2(1)(i)}{1-i^2}$

$= \dfrac{1+i^2+2i}{1-(-1)} = \dfrac{1-1+2i}{1+1}$

$= \dfrac{\cancel{2}i}{\cancel{2}} = i$

$z = 0 + i$

(a) $\bar{z} = 0 - i$

(b) $z + \bar{z} = 0 + i + 0 - i = 0$

(c) $z - \bar{z} = 0 + i - (0 - i)$

$= 0 + i - 0 + i$

$= 2\,i$

(d) $z.\bar{z} = (0+i)(0-i)$

$= (0)^2 - (i)^2 = 0 - (-1)$

$= 0 + 1 = 1$

(iv) $z = \dfrac{4-3i}{2+4i}$

$= \dfrac{4-3i}{2+4i} \times \dfrac{2-4i}{2-4i}$

$= \dfrac{(4-3i)(2-4i)}{(2)^2 - (4i)^2}$

$= \dfrac{8-16i-6i+12i^2}{4-16i^2}$

$= \dfrac{8-22i+12(-1)}{4-16(-1)}$

$= \dfrac{8-12-22i}{4+16}$

$= \dfrac{-4-22i}{20}$

$= -\dfrac{4}{20} - \dfrac{22}{20}i$

$z = -\dfrac{1}{5} - \dfrac{11}{10}i$

(a) $\bar{z} = -\dfrac{1}{5} + \dfrac{11}{10}i$

(b) $z + \bar{z} = -\dfrac{1}{5} + \cancel{\dfrac{11}{10}i} - \dfrac{1}{5} + \cancel{\dfrac{11}{10}i}$

$= -\dfrac{2}{5}$

(c) $z - \bar{z} = -\dfrac{1}{5} - \dfrac{11}{10}i - \left(-\dfrac{1}{5} + \dfrac{11}{10}i\right)$

$= -\cancel{\dfrac{1}{5}} - \dfrac{11}{10}i + \cancel{\dfrac{1}{5}} - \dfrac{11}{10}i$

$= -\dfrac{22}{10}i$

$= -\dfrac{11}{5}i$

(d) $z.\bar{z} = \left(-\dfrac{1}{5} - \dfrac{11}{10}i\right)\left(-\dfrac{1}{5} + \dfrac{11}{10}i\right)$

$= \left(-\dfrac{1}{5}\right)^2 \quad \left(\dfrac{11}{10}i\right)^2$

$= \dfrac{1}{25} - \dfrac{121}{100}i^2$

$= \frac{1}{25} - \frac{121}{100}(-1)$

$= \frac{1}{25} + \frac{121}{100}$

$= \frac{4+121}{100}$

$= \frac{125}{100}$

$= \frac{5}{4}$

Q6. If $z = 2+3i$ and $w = 5-4i$, show that:

(i) $\overline{z+w} = \overline{z} + \overline{w}$

Sol: L.H.S $= \overline{z+w}$

$z+w = 2+3i+5-4i$

$z+w = 7-i$

$\overline{z+w} = 7+i$

Now R.H.S $= \overline{z} + \overline{w}$

$\overline{z} = 2-3i$

$\overline{w} = 5+4i$

$\overline{z} + \overline{w} = 2-3i+5+4i$

$= 7+i$

Hence $\overline{z+w} = \overline{z} + \overline{w}$

(ii) $\overline{z-w} = \overline{z} - \overline{w}$

Sol: L.H.S $= \overline{z-w}$

$z-w = 2+3i-(5-4i)$

$= 2+3i-5+4i$

$= -3+7i$

$\overline{z-w} = -3-7i$

R.H.S $= \overline{z} - \overline{w}$

$\overline{z} = 2-3i$

$\overline{w} = 5+4i$

$\overline{z} - \overline{w} = (2-3i)-(5+4i)$

$= 2-3i-5-4i$

$= -3-7i$

Hence $\overline{z-w} = \overline{z} - \overline{w}$

(iii) $\overline{z.w} = \overline{z}.\overline{w}$

L.H.S $= \overline{z.w}$

$z.w = (2+3i)(5-4i)$

$= 10-8i+15i-12i^2$

$= 10+7i-12(-1)$

$= 10+7i+12$

$= 22+7i$

$\overline{z.w} = 22-7i$

R.H.S $= \overline{z}.\overline{w}$

$\overline{z} = 2-3i$

$\overline{w} = 5+4i$

$\overline{z}.\overline{w} = (2-3i)(5+4i)$

$= 10+8i-15i-12i^2$

$= 10-7i-12(-1)$

$= 10-7i+12$

$\overline{z}.\overline{w} = 22-7i$

Hence $\overline{z.w} = \overline{z}.\overline{w}$

(iv) $\overline{\left(\frac{z}{w}\right)} = \frac{\overline{z}}{\overline{w}}$, where $w \neq 0$

LHS $= \overline{\left(\frac{z}{w}\right)}$

$\frac{z}{w} = \frac{2+3i}{5-4i}$

$= \frac{2+3i}{5-4i} \times \frac{5+4i}{5+4i}$

$= \frac{(2+3i)(5+4i)}{(5)^2-(4i)^2} = \frac{10+8i+15i+12i^2}{25-16i^2}$

$$=\frac{10+23i+12(-1)}{25-16(-1)}$$

$$=\frac{10-12+23i}{25+16}$$

$$=\frac{-2+23i}{41}$$

$$=-\frac{2}{41}+\frac{23}{41}i$$

$$\overline{\left(\frac{z}{w}\right)}=-\frac{2}{41}-\frac{23}{41}i$$

R.H.S $=\frac{\overline{z}}{\overline{w}}$

$$\overline{z}=2-3i$$

$$\overline{w}=5+4i$$

$$\frac{\overline{z}}{\overline{w}}=\frac{2-3i}{5+4i}$$

$$=\frac{(2-3i)(5-4i)}{(5)^2-(4i)^2}$$

$$=\frac{10-8i-15i+12i^2}{25-16i^2}$$

$$=\frac{10-23i+12(-1)}{25-16(-1)}$$

$$=\frac{10-12-23i}{25+16}$$

$$=\frac{-2-23i}{41}$$

$$=-\frac{2}{41}-\frac{23}{41}i$$

Hence $\overline{\left(\frac{z}{w}\right)}=\frac{\overline{z}}{\overline{w}}$

(v) $\frac{1}{2}(z+\overline{z})$ is the real part of z

Sol: $z=2+3i$

Now $\overline{z}=2-3i$

$$\frac{1}{2}(z+\overline{z})=\frac{1}{2}(2+\cancel{3i}+2-\cancel{3i})$$

$$=\frac{1}{\cancel{2}}(\cancel{4})$$

$$\frac{1}{2}(z+\overline{z})=2$$

$$\frac{1}{2}(z+\overline{z})=\text{Re}(z)$$

Hence $\frac{1}{2}(z+\overline{z})$ is equal to the real part of z.

(vi) $\frac{1}{2i}(z-\overline{z})$ is the real part of z.

Sol. $z=2+3i$

Now $\overline{z}=2-3i$

$$\frac{1}{2i}(z-\overline{z})=\frac{1}{2i}[(2+3i)-(2-3i)]$$

$$=\frac{1}{2i}(\cancel{2}+3i-\cancel{2}+3i)$$

$$=\frac{6i}{3i}$$

$$=3$$

$$\frac{1}{2i}(z-\overline{z})=\text{R}(z)$$

Hence proved that $\frac{1}{2i}(z-\overline{z})$ is equal to the real part of z.

Q7. Solve the following equation for real x and y

(i) $(2-3i)(x+yi)=4+i$

$$(x+yi)=\frac{4+i}{2-3i}$$

$$=\frac{4+i}{2-3i}\times\frac{2+3i}{2+3i}$$

$$=\frac{(4+i)(2+3i)}{(2)^2-(3i)^2}$$

$$=\frac{8+12i+2i+3i^2}{4-9i^2}$$

$$=\frac{8+14i+3(-1)}{4-9(-1)}$$

$$=\frac{8-3+14i}{4+9}$$

$$=\frac{5+14i}{13}$$

$$(x+yi)=\frac{5}{13}+\frac{14}{13}i$$

$$\Rightarrow x=\frac{5}{13} \text{ and } y=\frac{14}{13}$$

(ii) $(3-2i)(x+yi)=2(x-2yi)+2i-1$

$$3x+3yi-2xi-2yi^2=2x-4yi+2i-1$$

$$3x+(3y-2x)i-2y(-1)=2x-1+(2-4y)i$$

$$(3x+2y)+(3y-2x)i=(2x-1)+(2-4y)i$$

$\Rightarrow$ $3x+2y=2x-1$ ....(i) and

$3y-2x=2-4y$ ......(ii)

From (i) $3x-2x+2y=-1$

$x+2y=-1$ ....(iii)

From (ii) $-2x+3y+4y=2$

$-2x+7y=2$ ....(iv)

Multiplying (iii) by 2 and adding in (iv)

$$\cancel{2x}+4y=\cancel{-2}$$

$$-\cancel{2x}+7y=\cancel{2}$$

$$11y=0$$

$$y=\frac{0}{11}$$

$$\boxed{y=0}$$

Putting value of y in (iii)

$$x+2y=-1$$

$$x+2(0)=-1$$

$$x+0=-1$$

$$\boxed{x=-1}$$

(iii) $(3+4i)^2-2(x-yi)=x+yi$

$$(3)^2+(4i)^2+2(3)(4i)-2x+2yi=x+yi$$

$$9+16i^2+24i-2x+2yi=x+yi$$

$$9+16(-1)+24i-2x+2yi=x+yi$$

$$9-16+24i-2x+2yi=x+yi$$

$$-7-2x+(24+2y)i=x+yi$$

$\Rightarrow$ $x=-7-2x$

$$x+2x=-7$$

$$3x=-7$$

$$\boxed{x=\frac{-7}{3}}$$

and $24+2y=y$

$$2y-y=-24$$

$$\boxed{y=-24}$$