EXERCISE 5.3

Question # 1

Given $f(x,y)=2x+5y$

$2y-x\le 8$.............(i)

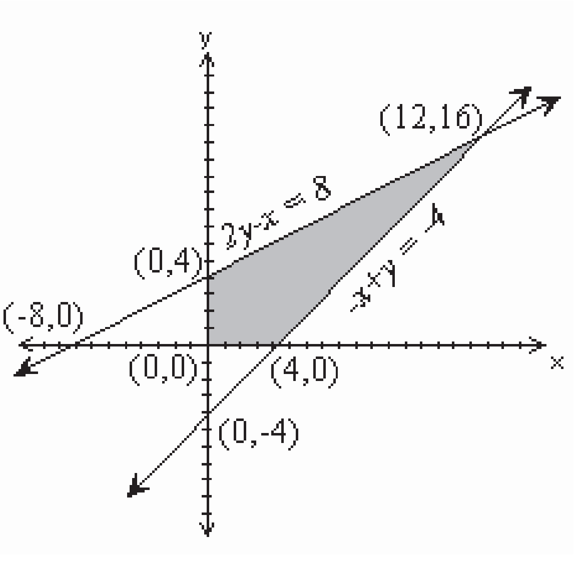
$x-y\le 4$

$\Rightarrow\ -x+y\ge -4$ .... (ii)

The associated equations of (i) and (ii) are

$2y-x=8$.............(iii)

$-x+y=-4$ .... (iv)



Put $x=0$ in (iii)

$2y-0=8 \quad\Rightarrow\quad 2y=8 \quad\Rightarrow\quad y=4$

Put $y=0$ in (iii)

$2(0)-x=8 \quad\Rightarrow\quad 0-x=8 \quad\Rightarrow\quad x=-8$

$\Rightarrow$ $(0,4)$ and $(-8,0)$ lies on (iii).

Put $x=0$ in (iv)

$-0+y=-4 \quad\Rightarrow\quad y=-4$

Put $y=0$ in (iv)

$-x+0=-4 \quad\Rightarrow\quad x=4$

$\Rightarrow$ $(0,-4)$ and $(4,0)$ lies on (iv).

For intersection, subtracting (iii) and (iv)

$$\begin{array}{rcl} -x+2y &=& 8 \\ \underset{+}{-}x\underset{-}{+}\ y &=& \underset{+}{-}\ 4 \\ \hline y &=& 12 \end{array}$$

Putting values of y in (iv)

$-x+12=-4 \quad\Rightarrow\quad x=4+12=16$

$\Rightarrow$ $(16,12)$ is point of intersection of (iii) and (iv)

Form the graph we see that the corner points of feasible region are $(4,0)$, $(0,0)$, $(0,4)$ and $(16,12)$.

Now we find value of $f(x,y)$ at corner points

$f(4,0) \;=\; '2(4)+5(0) \;=\; 8+0 \;=\; 8$

$f(0,0) \;=\; 2(0)+5(0) \;=\; 0+0 \;=\; 0$

$f(0,4) \;=\; 2(0)+5(4) \;=\; 0+20 \;=\; 20$

$f(16,12) \;=\; 2(16)+5(12) \;=\; 32+60 \;=\; 92$

Hence f is maximum at the corner point $(16,12)$.

Question # 2

Given $f(x,y)=x+3y$

$2x+5y\le 30$.............(i)

$5x+4y\le 20$.............(ii)

The associated equations of (i) and (ii) are

$2x+5y=30$.............(iii)

$5x+4y=20$.............(iv)

Put $x=0$ in (iii)

$2(0)+5y=30 \quad\Rightarrow\quad 5y=30 \quad\Rightarrow\quad y=6$

Put $y=0$ in (iii)

$2x+5(0)=30 \quad \Rightarrow \; 2x=30 \quad \Rightarrow \; x=15$

$\Rightarrow \; (0,6)$ and $(15,0)$ lies on (iii).

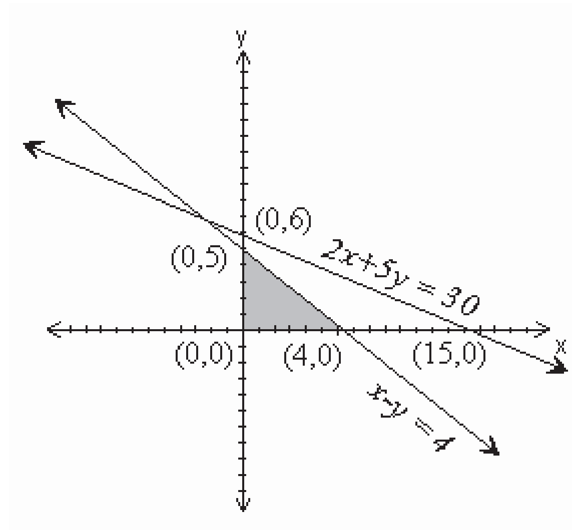
Put $x=0$ in (iv)

$$5(0)+4y=20 \quad \Rightarrow \; 4y=20 \quad \Rightarrow \; y=5$$

Put $y=0$ in (iv)

$$5x+4(0)=20 \quad \Rightarrow \; 5x=20 \quad \Rightarrow \; x=4$$

$\Rightarrow \; (0,5)$ and $(4,0)$ lies on (iv).



Form the graph we see that the corner points of feasible region are $(0,0)$, $(4,0)$ and $(0,5)$.

Now we find value of $f(x,y)$ at corner points

$$f(0,0)=0+3(0)=0$$
$$f(4,0)=4+3(0)=4$$
$$f(0,5)=0+3(5)=15$$

Hence f is maximum at $(0,5)$.

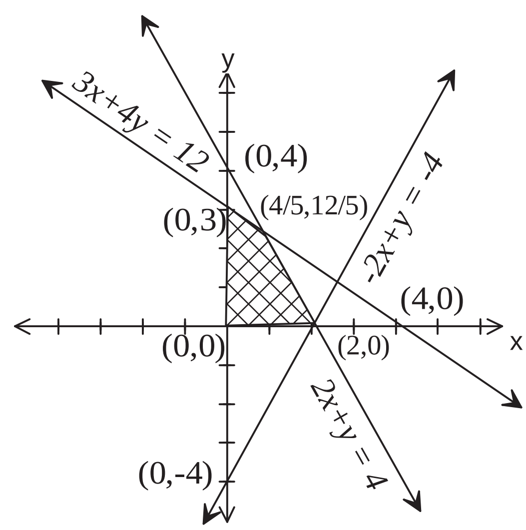
Question # 3

Given $f(x,y)=2x+3y$

$$3x+4y\le 12 ....(i)$$
$$2x+y\le 4 ....(ii)$$
$$2x-y\le 4$$
$$\Rightarrow \; -2x+y\ge -4 ....(iii)$$

The associated equations of (i), (ii) and (iii) are

$$3x+4y=12 ....(iv)$$
$$2x+y=4 ....(v)$$
$$-2x+y=-4 .... (vi)$$



Put $x=0$ in (iv)

$3(0)+4y=12 \quad \Rightarrow \; 4y=12 \quad \Rightarrow \; y=3$

Put $y=0$ in (iv)

$3x+4(0)=12 \quad \Rightarrow 3x=12 \quad \Rightarrow \; x=4$

$\Rightarrow \; (0,3)$ and $(4,0)$ lies on (iv).

Put $x=0$ in (v)

$2(0)+y=4 \quad \Rightarrow \; y=4$

Put $y=0$ in (v)

$2x+(0)=4 \quad \Rightarrow \; 2x=4 \quad \Rightarrow \; x=2$

$\Rightarrow \; (0,4)$ and $(2,0)$ lies on (v).

Put $x=0$ in (vi)

$-2(0)+y=-4 \Rightarrow \; y=-4$

Put $y=0$ in (vi)

$-2x+0=-4 \Rightarrow \; y=2$

$\Rightarrow \; \left(0,-4\right)$ and $\left(2,0\right)$ lies on line (vi).

For intersection of $\left(iv\right)$ and (v), $\times$ing (v) by 4 and subtracting from (iv).

$$\begin{array}{rcl} 3x+\;\;4y &=& 12 \\ \underline{-}8x\underline{+}\;\;4y &=& \underline{-}16 \\ \hline -5x \qquad &=& -4 \end{array}$$

$$\Rightarrow \; x=\frac{4}{5}$$

Putting values of x in (iv)

$$2\left(\frac{4}{5}\right)+y=4 \quad\Rightarrow\quad y=4-\frac{8}{5} \quad\Rightarrow\quad x=\frac{12}{5}$$

$\Rightarrow \left(\frac{4}{5},\frac{12}{5}\right)$ is point of intersection of (iii) and (iv)

Form the graph we see that the corner points of feasible region are $(0,0)$, $(2,0)$ $(0,3)$ and $\left(\frac{4}{5},\frac{12}{5}\right)$.

Now we find value of $f(x,y)$ at corner points

$$f(0,0)=2(0)+3(0)=0$$
$$f(2,0)=2(2)+3(0)=4$$
$$f(0,3)=2(0)+3(3)=9$$
$$f\left(\frac{4}{5},\frac{12}{5}\right)=2\left(\frac{4}{5}\right)+3\left(\frac{12}{5}\right)=\frac{8}{5}+\frac{36}{5}=\frac{44}{5}=8\frac{4}{5}$$

Hence f is maximum at $(0,3)$.

Question # 4

Given: $f(x,y)=2x+y$ **Correction*

$$x+y\geq 3 \text{ ....}(i)$$
$$7x+5y\leq 35 \text{ ....}(ii)$$

The associated equations of (i) and (ii) are

$$x+y=3 \text{ ....}(iii)$$
$$7x+5y=35 \text{ ....}(iv)$$

Put $x=0$ in (iii)

$$0+y=3 \quad\Rightarrow\quad y=3$$

Put $y=0$ in (iii)

$$x+0=3 \quad\Rightarrow\quad x=3$$

$\Rightarrow$ $(0,3)$ and $(3,0)$ lies on (iii).

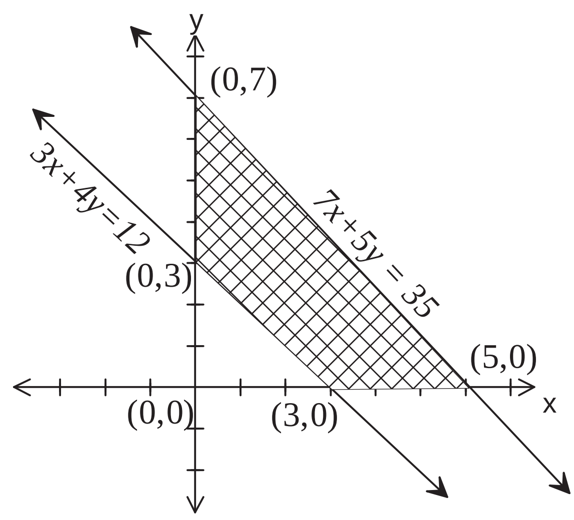
Put $x=0$ in (iv)

$$7(0)+5y=35 \quad\Rightarrow\quad y=7$$

Put $y=0$ in (iv)

$$7x+5(0)=35 \quad\Rightarrow\quad x=5$$

$\Rightarrow$ $(0,7)$ and $(5,0)$ lies on (iv).



From graph we see that the corner points of feasible region are $(3,0)$ $(0,3)$, $(5,0)$ and $(0,7)$.

Now we find value of $f(x,y)$ at corner points

$$f(3,0) = 2(3)+0 = 6$$
$$f(0,3) = 2(0)+3 = 3$$
$$f(5,0) = 2(5)+0 = 10$$
$$f(0,7) = 2(0)+7 = 7$$

Hence f is minimum at corner point $(0,3)$.

Question # 5

Do yourself

Question # 6

Do yourself

Question # 7

Let x and y denotes units of food X and food Y respectively.

Let $f(x,y)$ denotes the cost function, then we have to minimize

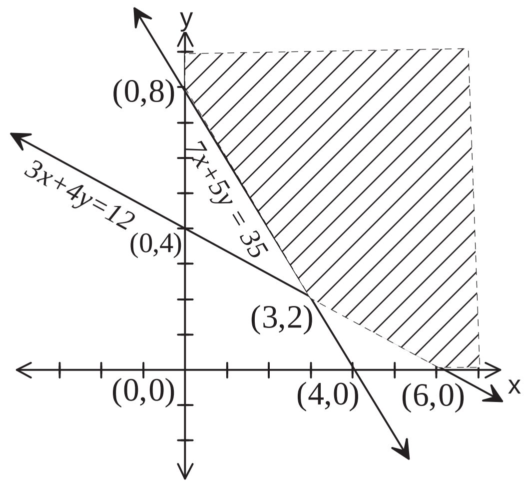
$$f(x,y) = 25x+30y$$

subject to the constraints

$$2x+3y \geq 12 \;\ldots\ldots\ldots\; (i)$$

$$4x+2y \geq 16$$

$$\Rightarrow \; 2x+y \geq 8 \;\ldots\ldots\ldots\ldots\ldots\; (ii)$$



The associated equations of (i) and (ii) are

$$2x+3y = 12 \;\ldots\ldots\ldots\ldots\; (iii)$$

$$2x+y = 8 \;\ldots\ldots\ldots\ldots\; (iv)$$

Put $x=0$ in (iii) $\Rightarrow$ $3y=12$ $\Rightarrow$ $y=4$

Put $y=0$ in (iii) $\Rightarrow$ $2x=12$ $\Rightarrow$ $x=6$

$\Rightarrow$ $(0,4)$ & $(6,0)$ lies on (iii)

Put $x=0$ in (iv) $\Rightarrow$ $y=8$

Put $y=0$ in (iv) $\Rightarrow$ $2x=8$ $\Rightarrow$ $x=4$

$\Rightarrow$ $(0,8)$ & $(4,0)$ lies on (iv)

For intersection of (iii) & (iv), –ing (iii) & (iv)

$$\begin{array}{rcl} 2x + 3y & = & 12 \\ \underline{\underset{-}{}2x \underset{-}{+} \;\; y} & \underline{=} & \underline{\underset{-}{}8} \\ 2y & = & 4 \end{array} \qquad \Rightarrow \; y=2$$

Put $y=2$ in (iv)

$$2x+2=8 \;\Rightarrow\; 2x=6 \;\Rightarrow\; x=3$$

$\Rightarrow$ $(3,2)$ is intersection of (iii) & (iv)

From graph, we see that corner points are $(6,0)$, $(3,2)$ and $(0,8)$.

Now

$$f(6,0) = 25(6)+30(0) = 150$$

$$f(3,2) = 25(3)+30(2) = 75+60 = 135$$

$$f(0,8) = 25(0)+30(8) = 240$$

Since $f(x,y)$ is minimum at $(3,2)$ therefore

Hence 3 unit of food X and 2 unit of food Y are used to minimize the cost.

Question # 8

Let x denotes number of fans and y denotes number of sewing machines.

Then profit function

$$f(x,y) = 22x+18y$$

Subject to the constraints

$$x+y \leq 20 \;\ldots\ldots\ldots\ldots\; (i)$$

$$360x+240y \leq 5760$$

$$\Rightarrow \; 9x+6y \leq 144 \;\ldots\ldots\ldots\ldots\; (ii) \qquad (\div\text{ing by } 40)$$

We have to maximize $f(x,y)$

The associated equation of (i) and (ii) are

$$x+y = 20 \;\ldots\ldots\ldots\; (iii)$$

$$9x+6y = 144 \;\ldots\ldots\ldots\ldots\; (iv)$$

Put $x=0$ in (iii) $\Rightarrow$ $y=20$

Put $y=0$ in (iii) $\Rightarrow$ $x=20$

$\Rightarrow$ $(0,20)$ & $(20,0)$ lies of (iii)

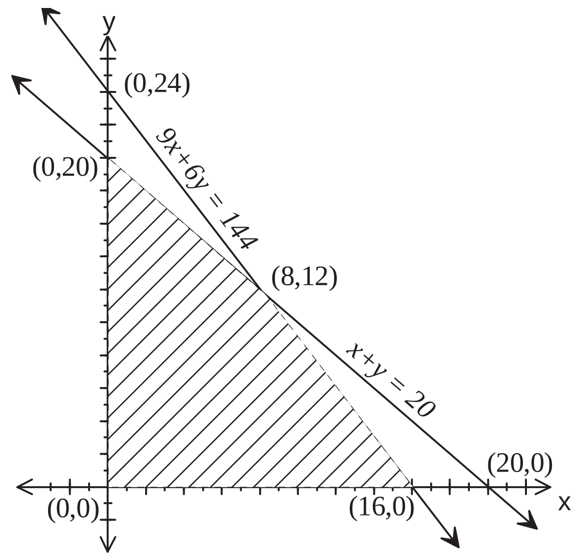
Now put $x=0$ in (iv) $\Rightarrow$ $6y=144$ $\Rightarrow$ $y=24$

Put $y=0$ in (iv) $\Rightarrow$ $9x=144$ $\Rightarrow$ $x=16$

$\Rightarrow$ $(0,24)$ & $(16,0)$ lies on (iv)

For point of intersection of (iii) and (iv)

Multiplying eq. (iii) by 6 and subtracting from (iv)



$$\begin{array}{rcl} 9x+6y & = & 144 \\ \underline{-6x \pm 6y} & = & \underline{-120} \\ 3x & = & 24 \end{array} \quad \Rightarrow \quad x=8$$

Putting value of x in (iii)

$$8+y = 20 \Rightarrow y=20-8$$
$$\Rightarrow y=12$$

$\Rightarrow$ $(8,12)$ is the intersection of (iii) and (iv)

Now from graph we see that $(0,0)$, $(16,0)$, $(8,12)$ and $(0,20)$ are corner points of feasible region.

Now

$$f(0,0) = 0+0 = 0$$
$$f(16,0) = 22(16)+18(0) = 352$$
$$f(8,12) = 22(16)+18(12) = 392$$
$$f(0,20) = 22(0)+18(20) = 360$$

Since $f(x,y)$ is maximum at $(8,12)$. Thus 8 fans and 12 sewing machine to maximize the profile.

Question # 9

Let x denotes the unit of product A and y denotes the unit of product B

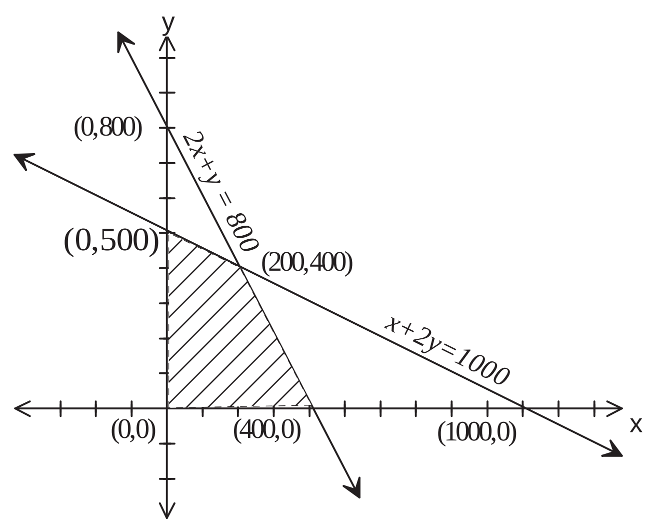
Then profit function is

$$f(x,y) = 30x+20y$$

subject to the constraints

$$2x+y \leq 800 \;\ldots\ldots\ldots\; (i)$$
$$x+2y \leq 1000 \;\ldots\ldots\ldots\ldots\; (ii)$$

The corresponding equations of (i) and (ii) are



$$2x+y = 800 \;\ldots\ldots\ldots\; (iii)$$
$$x+2y = 1000 \;\ldots\ldots\ldots\; (iv)$$

Put $x=0$ in (iii) $\Rightarrow$ $y=800$

Put $y=0$ in (iii) $\Rightarrow$ $2x=800$

$\Rightarrow$ $x=400$

$\Rightarrow$ $(0,800)$ & $(400,0)$ lies on (iii)

Now put $x=0$ in (iv) $\Rightarrow$ $2y=1000$ $\Rightarrow$ $y=500$

Put $y=0$ in (iv) $\Rightarrow$ $x=1000$

Hence $(0,500)$ & $(1000,0)$ lies on (iv)

For point of intersection, $\times$ing eq. (iii) by 2 and $-$ing from (iv)

$$\begin{array}{rcl} 4x+2y & = & 1600 \\ \underline{-x \pm 2y} & = & \underline{-1000} \\ 3x & = & 600 \end{array} \quad \Rightarrow \quad x=200$$

Putting value of x in (iii)

$$2(200)+y = 800$$

$$\Rightarrow \quad y=800-400 \quad \Rightarrow \quad y=400$$

So $(200,400)$ is point of intersection of line (iii) & (iv)

From graph, we see that corner points of feasible region are

$$(0,500),\ (0,0),\ (400,0)\ \&\ (200,400).$$

Now

$$f(0,500) = 30(0)+20(500) = 10000$$

$$f(0,0) = 30(0)+20(0) = 0$$

$$f(400,0) = 30(400)+20(0) = 12000$$

$$f(200,400) = 30(200)+20(400) = 14000$$

Since $f(x,y)$ is maximum at $(200,400)$.

Thus 200 unit of product A and 400 unit of product B must be used to maximize the profit.