Exercise 3.3

Q1. Write the following into sum or difference.

i) $\log(A\times B)$

Sol: $\log(A\times B) = \log A + \log B$

ii) $\log\dfrac{15.2}{30.5}$

Sol: $\log\dfrac{15.2}{30.5} = \log 15.2 - \log 30.5$

iii) $\log\dfrac{21\times5}{8}$

Sol: $\log\dfrac{21\times5}{8} = \log 21 + \log 5 - \log 8$

iv) $\log\sqrt[3]{\dfrac{7}{15}}$

Sol: $\log\sqrt[3]{\dfrac{7}{15}} = \log\left(\dfrac{7}{15}\right)^{\frac{1}{3}} = \dfrac{1}{3}\log\left(\dfrac{7}{15}\right)$

$= \dfrac{1}{3}(\log 7 - \log 15)$

v) $\log\dfrac{(22)^{\frac{1}{3}}}{5^3}$

Sol: $\log\dfrac{(22)^{\frac{1}{3}}}{5^3} = \log(22)^{\frac{1}{3}} - \log 5^3$

$= \dfrac{1}{3}\log 22 - 3\log 5$

vi) $\log\dfrac{25\times47}{29}$

$= \log 25 + \log 47 - \log 29$

Q2. Express

$\log x - 2\log x + 3\log(x+1) - \log(x^2-1)$

as a single logarithm

Sol:

$\log x - 2\log x + 3\log(x+1) - \log(x^2-1)$

$= \log x - \log x^2 + \log(x+1)^3 - \log(x^2-1)$

$= \log x + \log(x+1)^3 - \log x^2 - \log(x^2-1)$

$= \log\dfrac{x(x+1)^3}{x^2(x^2-1)}$

$= \log\dfrac{(x+1)^3}{x(x-1)(x+1)}$

$= \log\dfrac{(x+1)^2}{x(x-1)}$

Q3. Write the following in the form of a single logarithm.

i) $\log 21 + \log 5$

Sol: $\log 21 + \log 5$

$= \log 21\times5$

ii) $\log 25 - 2\log 3$

$= \log 25 - \log 3^2$

$= \log\dfrac{25}{3^2} = \log\dfrac{25}{9}$

iii) $2\log x - 3\log y$

Sol: $2\log x - 3\log y$

$= \log x^2 - \log y^3$

$= \log\dfrac{x^2}{y^3}$

iv) $\log 5 + \log 6 - \log 2$

Sol: $\log 5 + \log 6 - \log 2$

$= \log\dfrac{5\times6}{2}$

Q4. Calculate the following:

i) $\log_3 2 \times \log_2 81$

Sol: As we know that $\log_a n = \frac{\log_b n}{\log_b a}$

$$\therefore \log_3 2 \times \log_2 81 = \frac{\log 2}{\log 3} \times \frac{\log 81}{\log 2}$$

$$= \frac{\log 81}{\log 3}$$

$$= \frac{\log 3^4}{\log 3}$$

$$= \frac{4 \log 3}{\log 3}$$

$$= 4$$

ii) $\log_5 3 \times \log_3 25$

Sol: As we know that

$$\log_a n = \frac{\log_b n}{\log_b a}$$

$$\log_5 3 \times \log_3 25 = \frac{\cancel{\log 3}}{\log 5} \times \frac{\log 25}{\log 3}$$

$$= \frac{\log 25}{\log 5}$$

$$= \frac{\log 5^2}{\log 5}$$

$$= \frac{2 \log 5}{\log 5}$$

$$= 2$$

Q5. If $\log 2 = 0.3010$, $\log 3 = 0.4771$, $\log 5 = 0.6990$, **then find the values of the following.**

i) $\log 32$

Sol: $\log 32$

$$= \log 2^5$$

$$= 5 \log 2$$

$$= 5(0.3010)$$

$$= 1.5050$$

ii) $\log 24$

$$= \log 8 \times 3$$

$$= \log 2^3 \times 3$$

$$= \log 2^3 + \log 3$$

$$= 3 \log 2 + \log 3$$

$$= 3(0.3010) + 0.4771$$

$$= 0.9030 + 0.4771$$

$$= 1.3801$$

iii) $\log \sqrt{3\frac{1}{3}}$

$$= \log \sqrt{\frac{10}{3}}$$

$$= \log \left(\frac{2 \times 5}{3}\right)^{\frac{1}{2}}$$

$$= \frac{1}{2} \log \left(\frac{2 \times 5}{3}\right) = \frac{1}{2}(\log 2 + \log 5 - \log 3)$$

$$= \frac{1}{2}(0.3010 + 0.6990 - 0.4771)$$

$$= \frac{1}{2}(0.5229)$$

$$= 0.2615$$

iv) $\log \frac{8}{3}$

$$= \log \frac{2^3}{3}$$

$$= \log 2^3 - \log 3$$

$$= 3 \log 2 - \log 3$$

$$= 3(0.3010) - 0.4771$$

$$= 0.4259$$

v) $\log 30$

$$= \log 2 \times 3 \times 5$$

$$= \log 2 + \log 3 + \log 5$$

$$= 0.3010 + 0.4771 + 0.6990$$

$$= 1.4771$$