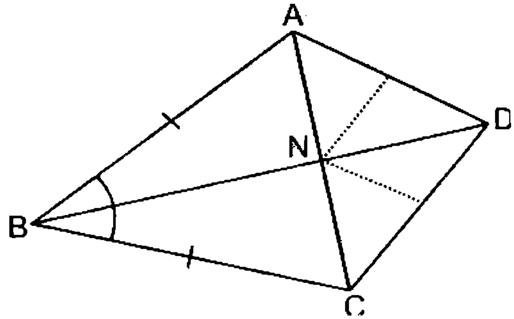
Exercise 12.2

1. In a quadrilateral ABCD, $\overline{AB} \cong \overline{BC}$ and the right bisectors of $\overline{AD}, \overline{CD}$ meet each other at point N. prove that $\overline{BN}$ is a bisector of $\angle ABC$.



Given Quadrilateral ABCD in which $\overline{AB} \cong \overline{BC}$. Right bisectors of $\overline{AD}$ and $\overline{CD}$ meet each other at point N.

To prove $\overline{BN}$ is a bisector of $\angle ABC$

Construction Join N with A, B, C, D

Proof:

Statements	Reasons
$\overline{NC} \cong \overline{ND}$ .... (i)	N is on the right bisector of $\overline{CD}$
$\overline{NA} \cong \overline{ND}$ .... (ii)	N is on the right bisector of $\overline{AD}$
$\overline{NA} \cong \overline{NC}$ .... (iii)	By (i) and (ii)
In $\Delta ABN \leftrightarrow \Delta CBN$	
$\overline{AB} \cong \overline{BC}$	Given
$\overline{BN} \cong \overline{BN}$	Common
$\overline{NA} \cong \overline{NC}$	Proved
$\therefore$ $\Delta ABN \cong \Delta CBN$	S.S.S $\cong$ S.S.S
$\angle ABN \cong \angle CBN$	Corresponding angles of congruent
$\therefore$ BN is a bisector of $\angle ABC$.	triangles.

2. The bisectors of $\angle A$, $\angle B$ and $\angle C$ of a quadrilateral ABCP meet each other at point O. Prove that the bisectors of $\angle P$ will also pass through the point O.

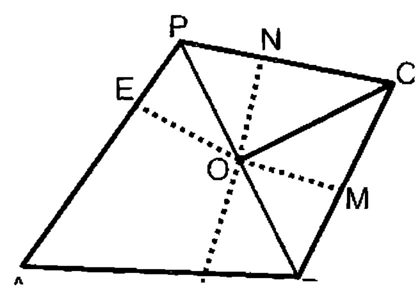
Given Bisector of the angles A, B, C meet at O.

To Prove

Bisector of $\angle P$ will also pass through O.

Construction

From O draw $\perp$ on the sides of quadrilateral BCP.

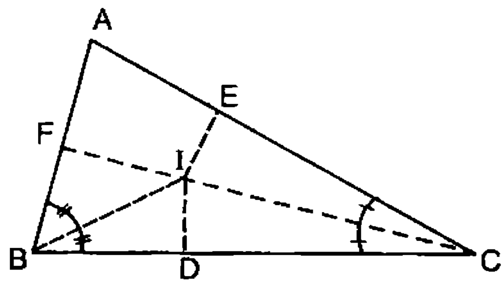


Proof:

Statements	Reasons
$\overline{OE} \cong \overline{OL}$ .... (i)	O is on the bisector of $\angle A$
$\overline{OL} \cong \overline{OM}$ .... (ii)	O is on the bisector of $\angle B$
$\overline{OM} \cong \overline{ON}$ .... (iii)	O is on the bisector of $\angle C$
$\therefore$ $\overline{OE} \cong \overline{ON}$	By (i) and (ii), (iii)
$\therefore$ O is on the bisector of $\angle P$.	$\overline{OE} \cong \overline{ON}$

Theorem

The bisectors of the angles of a triangle are concurrent.



Given

ΔABC

To Prove

The bisectors of $\angle A$, $\angle B$ and $\angle C$ are concurrent.

Construction

Draw the bisectors of $\angle B$ and $\angle C$ which intersect at point I. From I, draw $\overline{IF} \perp \overline{AB}$, $\overline{ID} \perp \overline{BC}$ and $\overline{IE} \perp \overline{CA}$.

Proof:

Statements	Reasons
$\overline{ID} \cong \overline{IF}$	(Any point on bisector of an angle is equidistant from its arms)
Similarly,	
$\overline{ID} \cong \overline{IE}$	
$\therefore$ $\overline{IE} \cong \overline{IF}$	Each $\cong$ ID, proved.
So, the point I is on the bisector of $\angle A$ ....(i)	
Also the point I is on the bisectors of $\angle ABC$ and $\angle BCA$. ....(ii)	Construction
Thus the bisectors of $\angle A$, $\angle B$ and $\angle C$ are concurrent at I.	{from (i) and (ii)}