EXERCISE 3.1

Question # 1(i)

$$y = x^2 - 1 \quad \ldots\ldots (i)$$

$x = 3$ & $\delta x = 3.02 - 3 = 0.02$

$$y + \delta y = (x + \delta x)^2 - 1$$

$$\Rightarrow \delta y = (x + \delta x)^2 - 1 - x^2 + 1$$

$$= (x + \delta x)^2 - x^2$$

Put $x = 3$ & $\delta x = 0.02$

$$\delta y = (3 + 0.02)^2 - (3)^2$$

$$\Rightarrow \boxed{\delta y = 0.1204}$$

Taking differential of (i)

$$dy = d(x^2 - 1)$$

$$\Rightarrow dy = 2x\, dx$$

Put $x = 3$ & $dx = \delta x = 0.02$

$$dy = 2(3)(0.02) \quad \Rightarrow \quad \boxed{dy = 0.12}$$

Question # 1(ii)

Do yourself as above.

Question # 1(iii)

$$y = \sqrt{x} = x^{\frac{1}{2}} \quad \ldots\ldots. (i)$$

$x = 4$ & $\delta x = 4.41 - 4 = 0.41$

$$y + \delta y = (x + \delta x)^{\frac{1}{2}}$$

$$\Rightarrow \delta y = (x + \delta x)^{\frac{1}{2}} - x^{\frac{1}{2}}$$

Put $x = 4$ & $\delta x = 0.41$

$$\delta y = (4 + 0.41)^{\frac{1}{2}} - (4)^{\frac{1}{2}}$$

$$= 2.1 - 2 \quad \Rightarrow \quad \boxed{\delta y = 0.1}$$

Taking differential of (i)

$$dy = \frac{d}{dx}\left(x^{\frac{1}{2}}\right) dx$$

$$= \frac{1}{2} x^{-\frac{1}{2}}\, dx = \frac{1}{2x^{\frac{1}{2}}}\, dx$$

Put $x = 4$ & $dx = \delta x = 0.41$

$$dy = \frac{1}{2(4)^{\frac{1}{2}}}(0.41) = \frac{0.41}{4}$$

$$\Rightarrow \boxed{dy = 0.1025}$$

Question # 2(i)

$$xy + x = 4$$

Taking differential on both sides

$$d(xy) + dx = d(4)$$

$$\Rightarrow xdy + ydx + dx = 0$$

$$\Rightarrow xdy + (y+1)dx = 0$$

$$\Rightarrow xdy = -(y+1)dx$$

$$\Rightarrow \frac{dy}{dx} = -\frac{y+1}{x}$$

$$\& \quad \frac{dx}{dy} = -\frac{x}{y+1}$$

Question # 2(ii)

Do yourself as above

Question # 2(iii)

$$x^4 + y^2 = xy^2$$

Taking differential

$$d(x^4) + d(y^2) = d(xy^2)$$

$$\Rightarrow 4x^3 dx + 2ydy = x \cdot 2ydy + y^2 dx$$

$$\Rightarrow 2ydy - 2xydy = y^2 dx - 4x^3 dx$$

$$\Rightarrow 2y(1-x)dy = (y^2 - 4x^3)dx$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^2 - 4x^3}{2y(1-x)}$$

$$\& \quad \frac{dx}{dy} = \frac{2y(1-x)}{y^2 - 4x^3}$$

Question # 2(iv)

$$xy - \ln x = c$$

Taking differential

$$d(xy) - d(\ln x) = d(c)$$

$$\Rightarrow xdy + ydx - \frac{1}{x}dx = 0$$

$$\Rightarrow xdy = \frac{1}{x}dx - ydx$$

$$= \left(\frac{1}{x} - y\right)dx$$

$$\Rightarrow xdy = \left(\frac{1-xy}{x}\right)dx$$

$$\Rightarrow \frac{dy}{dx} = \frac{1-xy}{x^2}$$

$$\& \quad \frac{dx}{dy} = \frac{x^2}{1-xy}$$

Question # 3(i)

Let $y = f(x) = \sqrt[4]{x}$

where $x = 16$ and $\delta x = dx = 1$

Taking differential of above

$$dy = d\left(\sqrt[4]{x}\right)$$

$$= d(x)^{\frac{1}{4}}$$

$$= \frac{1}{4} x^{\frac{1}{4}-1} dx$$

$$= \frac{1}{4} x^{-\frac{3}{4}} dx$$

$$= \frac{1}{4x^{\frac{3}{4}}} dx$$

Put $x = 16$ and $dx = 1$

$$dy = \frac{1}{4(16)^{\frac{3}{4}}}(1)$$

$$= \frac{1}{4\left(2^4\right)^{\frac{3}{4}}} = \frac{1}{4(8)} = 0.03125$$

Now $f(x+dx) \approx y+dy$

$= f(x)+dy \quad \because \quad y=f(x)$

$\Rightarrow \sqrt[4]{16+1} \approx \sqrt[4]{16}+0.03125$

$\Rightarrow \sqrt[4]{17} \approx \left(2^4\right)^{\frac{1}{4}}+0.03125$

$= 2+0.03125 = 2.03125$

Question # 3(ii)

Let $y=f(x)=(x)^{\frac{1}{3}}$

Where $x=8$ & $\delta x=dx=0.2$

Taking differential of above

$dy = d(x)^{\frac{1}{3}}$

$= \frac{1}{3}(x)^{-\frac{2}{3}}dx = \frac{1}{3x^{\frac{2}{3}}}dx$

Put $x=8$ and $dx=0.2$

$dy = \frac{1}{3(8)^{\frac{2}{3}}}(0.2)$

$= \frac{1}{3\left(2^3\right)^{\frac{2}{3}}}(0.2) = \frac{1}{3(4)}(0.2)$

$= 0.01667$

Now $f(x+\delta x) \approx y+dy$

$= f(x)+dy \quad \because \quad y=f(x)$

$\Rightarrow (8+0.2)^{\frac{1}{3}} = (8)^{\frac{1}{3}}+0.01667$

$\Rightarrow (8.02)^3 = 2+0.01667$

$= 2.01667$

Question # 3(iii)

Let $y = f(x) = x^{\frac{1}{5}}$

Where $x=32$ & $\delta x=dx=-1$

Try yourself as above.

Question # 3(iv)

Let $y=f(x)=\cos x$

Where $x=30^\circ$ & $\delta x=-1^\circ=-\frac{\pi}{180}\text{ rad}$

$=-0.01745\text{ rad}$

Now $dy = d(\cos x)$

$= -\sin x\,dx$

Put $x=30^\circ$ and $dx=\delta x=-0.01745$

$dy = -\sin 30^\circ(-0.01745)$

$= -(0.5)(-0.01745) = 0.008725$

Now $f(x+\delta x) \approx y+dy$

$= f(x)+dy$

$\Rightarrow \cos(30-1) = \cos 30^\circ+0.008725$

$\Rightarrow \cos 29^\circ = 0.866+0.008725$

$= 0.8747$

Question # 3(v)

Let $y=f(x)=\sin x$

Where $x=60^\circ$ & $\delta x=1^\circ=\frac{\pi}{180}\text{ rad}$

$=0.01745\text{ rad}$

Now $dy = d(\sin x)$

$= \cos x\,dx$

Put $x=60^\circ$ and $dx=\delta x=0.01745$

$dy = \cos 60^\circ(0.01745)$

$= (0.5)(0.01745) = 0.008725$

Now $f(x+\delta x) \approx y+dy$

$= f(x)+dy$

$\Rightarrow \sin(60+1) = \sin 60^\circ+0.008725$

$\Rightarrow \sin 61^\circ = 0.866+0.008725$

$= 0.8747$

Question # 4

Let x be the length of side of cube where

$x=5$ & $\delta x=5.02-5=0.02$

Assume V denotes the volume of the cube.

Then $V = x\cdot x\cdot x$

$= x^3$

Taking differential

$dV = 3x^2dx$

Put $x=5$ & $dx=\delta x=0.02$

$dV = 3(5)^2(0.02)$

$= 1.5$

Hence increase in volume is 1.5 cubic unit.

Question # 5

Let x denotes diameter of a disc

Where $x=44\text{ cm}$ & $\delta x=44.4-44=0.4$

Then radius $= \frac{x}{2}$

Let A denotes the area of the disc

Then $A = \pi(\text{radius})^2$

$= \pi\left(\frac{x}{2}\right)^2 = \frac{\pi}{4}x^2$

Taking differential

$dA = d\left(\frac{\pi}{4}x^2\right)$

$= \frac{\pi}{4}\cdot 2x\cdot dx = \frac{\pi}{2}x\,dx$

Put $x=44$ and $dx=\delta x=0.4$

$dA = \frac{\pi}{2}(44)(0.4)$

$= (3.14)(22)(0.4)$

$= 27.65$

Hence change in area is 27.65 cm^2
