Exercise 2.4

Question # 1

Write the converse, inverse and contrapositive of the following conditions:

(i) $\sim p \to q$ (ii) $q \to p$ (iii) $\sim p \to \sim q$

Solution

(i) Conditional: $\sim p \to q$

Converse: $q \to \sim p$

Inverse: $p \to \sim q$

Contrapositive: $\sim q \to p$

(ii) Conditional: $q \to p$

Converse: $p \to q$

Inverse: $\sim q \to \sim p$

Contrapositive: $\sim p \to \sim q$

(iii) Conditional: $\sim p \to \sim q$

Converse: $\sim q \to \sim p$

Inverse: $p \to q$

Contrapositive: $q \to p$

(iv) *Do yourself as above*

Question # 2

Construct truth tables for the following statements:

(i) $(p \to \sim p) \vee (p \to q)$ (ii) $(p \wedge \sim p) \to q$ (iii) $\sim (p \to q) \leftrightarrow (p \wedge \sim q)$

Solution

(i) Statement: $(p \to \sim p) \vee (p \to q)$

p	Q	$\sim p$	$p \to \sim p$	$p \to q$	$(p \to \sim p) \vee (p \to q)$
T	T	F	F	T	T
T	F	F	F	F	F
F	T	T	T	T	T
F	F	T	T	T	T

(ii) Statement: $(p \wedge \sim p) \to q$

p	Q	$\sim p$	$p \wedge \sim p$	$(p \wedge \sim p) \to q$
T	T	F	F	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

(iii) Statement: $\sim (p \to q) \leftrightarrow (p \wedge \sim q)$

p	Q	$\sim q$	$p \to q$	$\sim (p \to q)$	$p \wedge \sim q$	$(p \wedge \sim q) \leftrightarrow \sim (p \to q)$
T	T	F	T	F	F	T
T	F	T	F	T	T	T
F	T	F	T	F	F	T
F	F	T	T	F	F	T

Tautology*:*

The statement, which is true for all possible values of the variables in it, is called *tautology*.

Contingency:

The statement, which is true or false depending upon the truth values of the variables involved in it, is called a *contingency*.

Absurdity or Contradiction:

The statement, which is false for all the possible values of the variables involved in it, is called an *absurdity* or *contradiction*.

Question # 3

Show that each of the following statements is a tautology:

(i) $(p \wedge q) \to p$ (ii) $p \to (p \vee q)$

(iii) $\sim (p \to q) \to p$ (iv) $\sim q \wedge (p \to q) \to \sim p$

Solution

(i) Statement: $(p \wedge q) \to p$

P	q	$p \wedge q$	$p \wedge q \to p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

The last column of the above table shows that the statement is true for all values of p and q thus given statement is tautology.

(ii) Statement: $p \to (p \vee q)$

p	q	$p \vee q$	$p \to (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

The last column of the above table shows that the statement is true for all values of p and q thus given statement is tautology

(iii) Statement: $\sim (p \to q) \to p$

p	Q	$p \to q$	$\sim (p \to q)$	$\sim (p \to q) \to p$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

The last column of the above table shows that the statement is true for all values of p and q thus given statement is tautology.

(iv) Statement: $\sim q \wedge (p \to q) \to \sim p$

p	Q	$\sim p$	$\sim q$	$p \to q$	$\sim q \wedge (p \to q)$	$\sim q \wedge (p \to q) \to \sim p$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

The last column of the above table shows that the statement is true for all values of p and q thus given statement is tautology.

Question # 4

Determined whether each of the following is a tautology, a contingency or an absurdity:

(i) $p \wedge \sim p$ (ii) $p \to (q \to p)$ (iii) $q \vee (\sim q \vee p)$

Solution

(i) Statement: $p \wedge \sim p$

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

The last column of the above table shows that the statement is false for all values of p and q thus given statement is absurdity.

(ii) Statement: $p \to (q \to p)$

p	q	$q \to p$	$p \to (q \to p)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

The last column of the above table shows that the statement is true for all values of p and q thus given statement is tautology.

(iii) Statement: $q \vee (\sim q \vee p)$

P	q	$\sim q$	$\sim q \vee p$	$q \vee (\sim q \vee p)$
T	T	F	T	T
T	F	T	T	T
F	T	F	F	T
F	F	T	T	T

The last column of the above table shows that the statement is true for all values of p and q thus given statement is tautology.

Question # 5

Prove that

$p \vee (\sim p \wedge \sim q) \vee (p \wedge q) = p \vee (\sim p \wedge \sim q)$

Solution Consider the truth table

P	Q	$\sim p$	$\sim q$	$p \wedge q$	$\sim p \wedge \sim q$	$p \vee (\sim p \wedge \sim q) \vee (p \wedge q)$	$p \vee (\sim p \wedge \sim q)$
T	T	F	F	T	F	T	T
T	F	F	T	F	F	T	T
F	T	T	F	F	F	F	F
F	F	T	T	F	T	T	T

The last two column of the above table are identical this shows that the statement $p \vee (\sim p \wedge \sim q) \vee (p \wedge q)$ and $p \vee (\sim p \wedge \sim q)$ are equal

i.e. $p \vee (\sim p \wedge \sim q) \vee (p \wedge q) = p \vee (\sim p \wedge \sim q)$
