Exercise 4.4

1. Rationalize the denominator

(i) $$\frac{3}{4\sqrt{3}}=\frac{3}{4\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{3\sqrt{3}}{4\sqrt{3\text{x}3}}$$

$$=\frac{3\sqrt{3}}{4(3)}=\frac{\sqrt{3}}{4}$$

(ii) $$\frac{14}{\sqrt{98}}=\frac{14}{7\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$$

$$=\frac{\cancel{14}\sqrt{2}}{\cancel{14}}=\sqrt{2}$$

(iii) $$\frac{6}{\sqrt{8}.\sqrt{27}}=\frac{6}{2\sqrt{2}.3\sqrt{3}}$$

$$=\frac{\cancel{6}}{\cancel{6}\sqrt{6}}$$

$$=\frac{1}{\sqrt{6}}$$

$$=\frac{1}{\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}}$$

$$=\frac{\sqrt{6}}{6}$$

(iv) $$\frac{1}{3+2\sqrt{5}}=\frac{1}{3+2\sqrt{5}}\times\frac{3-2\sqrt{5}}{3-2\sqrt{5}}$$

$$=\frac{3-2\sqrt{5}}{(3)^2-(2\sqrt{5})^2}=\frac{3-2\sqrt{5}}{9-20}$$

$$=\frac{3-2\sqrt{5}}{-11}$$

(v) $$\frac{15}{\sqrt{31}-4}$$

$$=\frac{15}{\sqrt{31}-4}\times\frac{\sqrt{31}+4}{\sqrt{31}+4}$$

$$=\frac{15\left(\sqrt{31}+4\right)}{\left(\sqrt{31}\right)^2-(4)^2}$$

$$=\frac{15\left(\sqrt{31}+4\right)}{31-16}$$

$$=\frac{\cancel{15}\left(\sqrt{31}+4\right)}{\cancel{15}}$$

$$=\sqrt{31}+4$$

(vi) $$\frac{2}{\sqrt{5}-\sqrt{3}}=\frac{2}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$=\frac{2(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2}$$

$$=\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{5-3}$$

$$=\frac{\cancel{2}\left(\sqrt{5}+\sqrt{3}\right)}{\cancel{2}}$$

$$=\sqrt{5}+\sqrt{3}$$

(vii) $$\frac{\sqrt{3}-1}{\sqrt{3}+1}=\frac{\sqrt{3}-1}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$=\frac{\left(\sqrt{3}-1\right)\left(\sqrt{3}-1\right)}{\left(\sqrt{3}\right)^2-(1)^2}$$

$$=\frac{\left(\sqrt{3}-1\right)^2}{3-1}$$

$$=\frac{\left(\sqrt{3}\right)^2+1^2-2(1)\sqrt{3}}{2}$$

$$=\frac{3+1-2\sqrt{3}}{2}$$

$$= \frac{4-2\sqrt{3}}{2}$$
$$= \frac{2\left(2-\sqrt{3}\right)}{2}$$
$$= 2-\sqrt{3}$$

(viii) $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$$= \frac{\left(\sqrt{5}+\sqrt{3}\right)^2}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$$
$$\frac{\left(\sqrt{5}+\sqrt{3}\right)^2}{5-3}$$
$$= \frac{\left(\sqrt{5}\right)^2+\left(\sqrt{3}\right)^2+2\left(\sqrt{5}\right)\left(\sqrt{3}\right)}{2}$$
$$= \frac{5+3+2\sqrt{15}}{2}$$
$$= \frac{8+2\sqrt{15}}{2}$$
$$= \frac{2\left(4+\sqrt{15}\right)}{2}$$
$$= 4+\sqrt{15}$$

(2) Find conjugate of $x+\sqrt{y}$:

(i) $3+\sqrt{7}$

Conjugate of $3+\sqrt{7}$ is $3-\sqrt{7}$

(ii) $4-\sqrt{5}$

Conjugate of $4-\sqrt{5}$ is $4+\sqrt{5}$

(iii) $2+\sqrt{3}$

Conjugate of $2+\sqrt{3}$ is $2-\sqrt{3}$

(iv) $2+\sqrt{5}$

Conjugate of $2+\sqrt{5}$ is $2-\sqrt{5}$

(v) $5+\sqrt{7}$

Conjugate of $5+\sqrt{7}$ is $5-\sqrt{7}$

(vi) $4-\sqrt{15}$

Conjugate of $4-\sqrt{15}$ is $4+\sqrt{15}$

(vii) $7-\sqrt{6}$

Conjugate of $7-\sqrt{6}$ is $7+\sqrt{6}$

(viii) $9+\sqrt{2}$

Conjugate of $9+\sqrt{2}$ is $9-\sqrt{2}$

Q.3 If $x=2-\sqrt{3}$ find $\frac{1}{x}$

(i)
$$x = 2-\sqrt{3}$$
$$\frac{1}{x} = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$
$$\frac{1}{x} = \frac{2+\sqrt{3}}{(2)^2-\left(\sqrt{3}\right)^2}$$
$$\frac{1}{x} = \frac{2+\sqrt{3}}{4-3}$$
$$\frac{1}{x} = 2+\sqrt{3}$$

(ii) $x=4-\sqrt{17}$ **find** $\frac{1}{x}$

$$\frac{1}{x} = \frac{1}{4-\sqrt{17}} \times \frac{4+\sqrt{17}}{4+\sqrt{17}}$$
$$\frac{1}{x} = \frac{4+\sqrt{17}}{(4)^2-\left(\sqrt{17}\right)^2}$$
$$= \frac{4+\sqrt{17}}{16-17}$$
$$= \frac{4+\sqrt{17}}{-1}$$
$$= -(4+\sqrt{17})$$
$$= -4-\sqrt{17}$$

(iii) **If** $x=\sqrt{3}+2$**, find** $x+\frac{1}{x}$

$$x=\sqrt{3}+2$$

$$\frac{1}{x}=\frac{1}{\sqrt{3}+2}\times\frac{\sqrt{3}-2}{\sqrt{3}-2}$$

$$\frac{1}{x}=\frac{\sqrt{3}-2}{\left(\sqrt{3}\right)^2-(2)^2}$$

$$\frac{1}{x}=\frac{\sqrt{3}-2}{3-4}$$

$$\frac{1}{x}=\frac{\sqrt{3}-2}{-1}$$

$$\frac{1}{x}=-\sqrt{3}+2=2-\sqrt{3}$$

$$x+\frac{1}{x}=\cancel{\sqrt{3}}+2-\cancel{\sqrt{3}}+2$$

$$x+\frac{1}{x}=4$$

Q4. Simplify

(i) $\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}+\frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$

$$\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}\times\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$=\frac{\left(1+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}+\frac{\left(1-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$$

$$=\frac{\left(1+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{3}\right)}{5-3}+\frac{\left(1-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{3}\right)}{5-3}$$

$$=\frac{\sqrt{5}-\sqrt{3}+\sqrt{2}\sqrt{5}-\sqrt{2}\sqrt{3}}{2}+\frac{\sqrt{5}+\sqrt{3}-\sqrt{2}\sqrt{5}-\sqrt{2}\sqrt{3}}{2}$$

$$=\frac{\sqrt{5}-\sqrt{3}+\sqrt{10}-\sqrt{6}}{2}+\frac{\sqrt{5}+\sqrt{3}-\sqrt{10}-\sqrt{6}}{2}$$

$$=\frac{\sqrt{5}-\cancel{\sqrt{3}}+\cancel{\sqrt{10}}-\sqrt{6}+\sqrt{5}+\cancel{\sqrt{3}}-\cancel{\sqrt{10}}-\sqrt{6}}{2}$$

$$=\frac{2\sqrt{5}-2\sqrt{6}}{2}$$

$$=\frac{\cancel{2}\left(\sqrt{5}-\sqrt{6}\right)}{\cancel{2}}$$

$$=\sqrt{5}-\sqrt{6}$$

(ii) $\frac{1}{2+\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}+\frac{1}{2+\sqrt{5}}$

$$=\frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}$$

$$\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}+\frac{1}{2+\sqrt{5}}\times\frac{2-\sqrt{5}}{2-\sqrt{5}}$$

$$=\frac{2-\sqrt{3}}{(2)^2-\left(\sqrt{3}\right)^2}+\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}+\frac{2-\sqrt{5}}{(2)^2-\left(\sqrt{5}\right)^2}$$

$$=\frac{2-\sqrt{3}}{4-3}+\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{5-3}+\frac{2-\sqrt{5}}{4-5}$$

$$=2-\sqrt{3}+\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{2}+\frac{2-\sqrt{5}}{-1}$$

$$=\cancel{2}-\cancel{\sqrt{3}}+\sqrt{5}+\cancel{\sqrt{3}}-\cancel{2}+\sqrt{5}\ =2\sqrt{5}$$

(iii) $\frac{2}{\sqrt{5}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{2}}-\frac{3}{\sqrt{5}+\sqrt{2}}$

$$=\frac{2}{\sqrt{5}+\sqrt{3}}\times\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{2}}$$

$$\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}-\frac{3}{\sqrt{5}+\sqrt{2}}\times\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

$$=\frac{2\left(\sqrt{5}-\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{5}\right)^2}+\frac{\sqrt{3}-\sqrt{2}}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}-\frac{3\left(\sqrt{5}-\sqrt{2}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}$$

$$=\frac{2\left(\sqrt{5}-\sqrt{3}\right)}{5-3}+\frac{\sqrt{3}-\sqrt{2}}{3-2}-\frac{3\left(\sqrt{5}-\sqrt{2}\right)}{5-2}$$

$$=\frac{\cancel{2}\left(\sqrt{5}-\sqrt{3}\right)}{\cancel{2}}+\frac{\sqrt{3}-\sqrt{2}}{1}-\frac{\cancel{3}\left(\sqrt{5}-\sqrt{2}\right)}{\cancel{3}}$$

$$=\cancel{\sqrt{5}}-\cancel{\sqrt{3}}+\cancel{\sqrt{3}}-\cancel{\sqrt{2}}-\cancel{\sqrt{5}}+\cancel{\sqrt{2}}$$

$= 0$

Q5(i) If $x=2+\sqrt{3}$, find value of $x-\frac{1}{x}$ and $\left(x-\frac{1}{x}\right)^2$

$$x=2+\sqrt{3}$$

$$\frac{1}{x}=\frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$\frac{1}{x}=\frac{2-\sqrt{3}}{(2)^2-\left(\sqrt{3}\right)^2}$$

$$\frac{1}{x}=2-\sqrt{3}$$

$$x-\frac{1}{x}=2+\sqrt{3}-\left(2-\sqrt{3}\right)$$

$$=\cancel{2}+\sqrt{3}-\cancel{2}+\sqrt{3}$$

$$=2\sqrt{3}$$

$$\left(x-\frac{1}{x}\right)^2=\left(2\sqrt{3}\right)^2$$

$$\left(x-\frac{1}{x}\right)^2=12$$

(ii) If $x=\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$ find the value of $x+\frac{1}{x}$, $x^2+\frac{1}{x}$ and $x^3+\frac{1}{x^3}$

$$x=\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}\times\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

$$x=\frac{\left(\sqrt{5}-\sqrt{2}\right)^2}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}$$

$$x=\frac{\left(\sqrt{5}\right)^2+\left(\sqrt{2}\right)^2-2\left(\sqrt{5}\right)\left(\sqrt{2}\right)}{5-2}$$

$$x=\frac{5+2-2\sqrt{10}}{3}$$

$$x=\frac{7-2\sqrt{10}}{3}$$

$$\frac{1}{x}=\frac{3}{7-2\sqrt{10}}\times\frac{7+2\sqrt{10}}{7+2\sqrt{10}}$$

$$\frac{1}{x}=\frac{3\left(7+2\sqrt{10}\right)}{(7)^2-\left(2\sqrt{10}\right)^2}$$

$$\frac{1}{x}=\frac{3\left(7+2\sqrt{10}\right)}{49-40}$$

$$\frac{1}{x}=\frac{3\left(7+2\sqrt{10}\right)}{9}$$

$$\frac{1}{x}=\frac{7+2\sqrt{10}}{3}$$

$$x+\frac{1}{x}=\frac{7-2\sqrt{10}}{3}+\frac{7+2\sqrt{10}}{3}$$

$$=\frac{7-\cancel{2\sqrt{10}}+7+\cancel{2\sqrt{10}}}{3}=\frac{14}{3}$$

Now

$$x+\frac{1}{x}=\frac{14}{3}$$

Squaring

$$\left(x+\frac{1}{x}\right)^2=\left(\frac{14}{3}\right)^2$$

$$x^2+\frac{1}{x^2}+2=\frac{196}{9}$$

$$x^2+\frac{1}{x^2}=\frac{196}{9}-2$$

$$x^2+\frac{1}{x^2}=\frac{196-18}{9}=\frac{178}{9}$$

Also

$$x^3+\frac{1}{x^3}=?$$

$$x+\frac{1}{x}=\frac{14}{3}$$

$$\left(x+\frac{1}{x}\right)^3=\left(\frac{14}{3}\right)^3$$

$$x^3+\frac{1}{x^3}+3(x)\left(\frac{1}{x}\right)\left(x+\frac{1}{x}\right)=\frac{2744}{27}$$

$$x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)=\frac{2744}{27}$$

$$x^3+\frac{1}{x^3}+\cancel{3}\left(\frac{14}{\cancel{3}}\right)=\frac{2744}{27}$$

$$x^3+\frac{1}{x^3}=\frac{2744}{27}-14$$

$$=\frac{2366}{27}$$

Q6. Determine the rational numbers a and b. If

$$\frac{\sqrt{3}-1}{\sqrt{3}+1}+\frac{\sqrt{3}+1}{\sqrt{3}-1}=a+b\sqrt{3}$$

$$\frac{\sqrt{3}-1}{\sqrt{3}+1}+\frac{\sqrt{3}+1}{\sqrt{3}-1}=a+b\sqrt{3}$$

$$\frac{\sqrt{3}-1}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}+\frac{\sqrt{3}+1}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}=a+b\sqrt{3}$$

$$\frac{\left(\sqrt{3}-1\right)^2}{\left(\sqrt{3}\right)^2-(1)^2}+\frac{\left(\sqrt{3}+1\right)^2}{\left(\sqrt{3}\right)^2-(1)^2}=a+b\sqrt{3}$$

$$\frac{\left(\sqrt{3}\right)^2+(1)^2-2\left(\sqrt{3}\right)(1)}{3-1}+\frac{\left(\sqrt{3}\right)^2+(1)^2+2\sqrt{3}}{3-1}=a+b\sqrt{3}$$

$$\frac{3+1-2\sqrt{3}}{2}+\frac{3+1+2\sqrt{3}}{2}=a+b\sqrt{3}$$

$$\frac{4-2\sqrt{3}}{2}+\frac{4+2\sqrt{3}}{2}=a+b\sqrt{3}$$

$$\frac{\cancel{2}\left(2-\sqrt{3}\right)}{\cancel{2}}+\frac{\cancel{2}\left(2+\sqrt{3}\right)}{\cancel{2}}=a+b\sqrt{3}$$

$$2-\cancel{\sqrt{3}}+2+\cancel{\sqrt{3}}=a+b\sqrt{3}$$

$$4=a+b\sqrt{3}$$

$$\Rightarrow\ a+b\sqrt{3}=4$$

Hence on comparing the two sides, we get

$\Rightarrow\ a=4$ and $b=0$