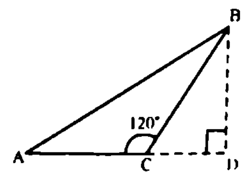
SOLVED EXERCISE 8.1

Q1. Given $m\overline{AC} = 1cm$, $m\overline{BC} = 2cm$, $m\angle C = 120°$. Compute the length AB and the area of ΔABC.

Hint: $(\overline{AB})^2 = (\overline{AC})^2 + (\overline{BC})^2 + 2\ mAC.\ m\overline{CD}$

Where $(m\overline{CD}) = (m\overline{BC})\cos(180° - C)$ *(Use theorem 1).*

Solution:



Given

$$m\overline{AC} = 1cm;\ m\overline{BC} = 2cm;\ m\angle C = 120°$$

Required: $m\overline{AB} = ?$

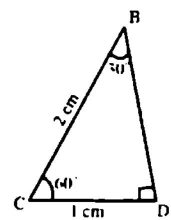
and Area of ABC = ?

$$m\overline{AB}^2 = m\overline{AC}^2 + m\overline{BC}^2 + 2m\overline{AC}\,m\overline{CD}$$

$$= (1)^2 + (2)^2 + 2(1)(\overline{CD})$$

$$= 1 + 4 + 2\overline{CD}$$

$$= 1 + 4 + 2\overline{CD} \quad \_\_\_\_\_\_\_\_\_ (i)$$



In Δ BCD

$m\angle BCD = 60°$

and $m\angle CBD = 30°$

The side opposite to $\angle 30°$ is $\overline{CD}$ which is $\frac{1}{2}\overline{CB}$, the hypotenuse of right Δ CDB.

$$\overline{CD} = 1cm$$

By putting the value of $\overline{CD}$ in eq. (i)

$$m\overline{AB}^2 = 5 + 2(1)(1) = 5 + 2 = 7,$$

$$m\overline{AB}^2 = 7 \Rightarrow m\overline{AB} = \sqrt{7}cm = 2.646cm$$

And $\quad m\overline{CB}^2 = m\overline{CD}^2 + \overline{BD}^2$

$$2^2 = 1^2 + m\overline{BD}^2$$

$$4 = 1 + m\overline{BD}^2$$

$$mBD^2 = 4 - 1 = 3$$

$$h = m\overline{BD} = \sqrt{3}$$

Area of ABC.

$$= \frac{1}{2}\text{ base} \times \text{height}$$

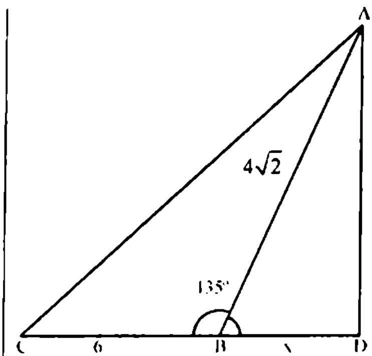
$$= \frac{1}{2} \text{ m } \overline{AC} \times \text{m } \overline{BD}$$

$$= \frac{1}{2} \times 1 \times \sqrt{3}$$

$$\text{Area of ABC} = \frac{\sqrt{3}}{2} \text{cm}$$

Q2. Find m $\overline{AC}$ if m$\overline{CB}$ = 6 cm, $\overline{CB}$ = 6 cm, m $\overline{AB}$ = $4\sqrt{2}$ cm and m$\angle$ABC = 135°.

Solution:



Let m $\overline{BD}$ = x

In Δ ABD, we have

$$\text{Cos } 45° = \frac{\overline{BD}}{\overline{AB}}$$

$$\frac{1}{\sqrt{2}} = \frac{x}{4\sqrt{2}}$$

$$\sqrt{2}\, x = 4\sqrt{2}$$

$$x = 4 \text{ cm}$$

we know that

To Prove:

$$\left(m\overline{AC}\right)^2 = \left(m\overline{CB}\right)^2 + \left(m\overline{AB}\right)^2 + 2 \times m\overline{CB} \times m\overline{BD}$$

$$= (6)^2 + \left(4\sqrt{2}\right)^2 + 2 \times 6 \times 4$$

$$= 36 + 32 + 48$$

$$= 116$$

$$\Rightarrow \quad m\overline{AC} = \sqrt{116} = \sqrt{4 \times 29} = 2\sqrt{29} \text{ cm}$$

THEOREM 2

8.1 (ii) In any triangle, the square on the side opposite to acute angle is equal to sum of the squares on the sides containing that acute angle diminished by twice the rectangle contained by one of those sides and the projection on it of the other.

Given:

ΔABC with an acute angle CAB at A.

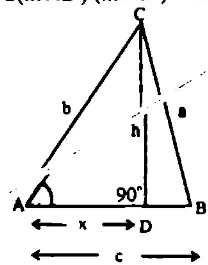
Take m $\overline{BC}$ a, m $\overline{CA}$ = b and mAB = c $\overline{AB}$

Draw $\overline{CD} = \overline{AB}$ so that $\overline{AD}$ is projection of $\overline{AC}$ on $\overline{AB}$

Also, $m\overline{AD} = x$ and $m\overline{CD} = h$

To prove:

$$(\overline{BC})^2 = (\overline{AC})^2 + (\overline{AB})^2 - 2(m\overline{AB})(m\overline{AD}) \quad \text{i.e.,} \quad a^2 = b^2 + c^2 - 2cx$$



Proof:

Statements	Reasons
In $\angle$ rt ΔCDA	
$m\angle CDA = 90°$	Given
$(\overline{AC})^2 = (\overline{AD})^2 + (\overline{CD})^2$	Pythagoras Theorem
i.e., $b^2 = x^2 + h^2$ (i)	
In $\angle$rt ΔCDS,	
$m\ CDB = 90°$	Given
$(\overline{BC})^2 = (\overline{AD})^2 + (\overline{CD})^2$	Pythagoras Theorem
$a^2 = (c - x)^2 + h^2$	From the figure
or $a^2 = c^2 - 2cx + x^2 + h^2$ (ii)	
$a^2 = c^2 - 2cx + b^2$	Using (i) and (ii)
Hence, $a^2 = b^2 + c^2 - 2cx$	
i,e., $(\overline{BC})^2 = (\overline{AC})^2 + (\overline{AB})^2 - 2(mAB)(m\overline{AD})$	

THEOREM 3

(Apollonius theorem)

**8.1 (iii) In any triangle, the sum of the squares on ,
the square on half the third side together
median which bisects the third side.**

Given

In a ΔABC, the median $\overline{AD}$ bisects $\overline{BC}$ i.e., $m\overline{BD} =$,

To prove

$(\overline{AB})^2 + (\overline{AC})^2 = 2(\overline{BD})^2 + 2(\overline{AD})^2$

Construction:

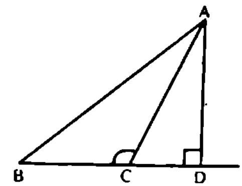
Draw $\overline{AF} \perp \overline{BC}$

Proof:

Statements	Reasons
In ΔADB Since $\angle ADB$ is acute at D	
$\therefore\ (\overline{AB})^2 = (\overline{BD})^2 + (\overline{AD})^2 - 2\, m\overline{BD} \,.\, m\overline{FD}$ (i)	Using Theorem 2
Now in ΔADC Since $\angle ADC$ is obtuse at D	
$(\overline{AC})^2 = (\overline{CD})^2 + (\overline{AD})^2 + 2\, m\overline{CD} \,.\, m\overline{FD}$	Using Theorem 1
$= (\overline{BD})^2 + (\overline{AD})^2 + 2\, m\overline{BD} \,.\, m\overline{FD}$ (ii)	
Thus $(\overline{AB})^2 + (\overline{AC})^2 = 2(\overline{BD})^2 + 2(\overline{AD})^2$	Adding (i) and (ii)

Example 1

In ΔABC, $\angle C$ is obtuse, $\overline{AD} \perp \overline{BC}$ produced, whereas $\overline{BD}$ is projection of $\overline{AB}$ on $\overline{BC}$. Prove that $(\overline{AC})^2 = (\overline{AB})^2 + 2(\overline{BC})^2 - 2\, m\overline{BC} \,.\, m\overline{BD}$



Given:

In a ΔABC, $\angle BCA$ is obtuse so that $\angle B$ is acute, $\overline{AD} \perp \overline{BC}$ produced, whereas $\overline{BD}$ is projection of $\overline{AB}$ on $\overline{BC}$ produced.

To prove:

$(\overline{AC})^2 = (\overline{AB})^2 + (\overline{BC})^2 - 2\, m\overline{BC} \,.\, m\overline{BD}$

Proof:

Statements	Reasons
In $\angle$ rt ΔAAD	
$(\overline{AB})^2 = (\overline{AD})^2 + (\overline{BD})^2$ (i)	Pythagoras Theorem
In $\angle$ rt ΔACD $(\overline{AC})^2 = (\overline{AD})^2 + (\overline{CD})^2$ (ii)	Pythagoras Theorem
or $(\overline{AC})^2 = (\overline{AD})^2 + (\overline{BD} - \overline{BC})^2$	$mBC + mCD = BD$
$(\overline{AC})^2 = (\overline{AD})^2 + (\overline{BD})^2 + (\overline{BC})^2 - 2\, m\overline{BC}\ m\overline{BD}$ (iii)	
$(\overline{AC})^2 = (\overline{AB})^2 + (\overline{BC})^2 - 2\, m\overline{BC}\ m\overline{BD}$	Using (i) and (iii)

Example 2

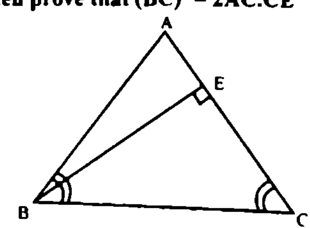
In an Isosceles ΔABC, if $mAB = mAC$ and $BE \perp AC$, then prove that $(BC)^2 = 2AC.CE$

Given

In an Isosceles ΔABC

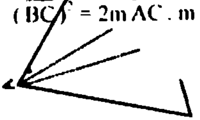
$m\overline{AB} = m\overline{AC}$ and $\overline{BE} \perp \overline{AC}$

whereas $\overline{CE}$ is the projection of $\overline{BC}$ upon on $\overline{AC}$.



To prove

$(\overline{BC})^2 = 2m\overline{AC} \,.\, m\overline{CE}$



Proof: .

Statements	Reasons
In an isosceles ΔABC with $m\overline{AB} = m\overline{AC}$. If $\angle C$ is acute,	
then $(\overline{AB})^2 = (\overline{AC})^2 + (\overline{BC})^2 - 2m\overline{AC}.m\overline{CE}$,	By Theorem 2
$(\overline{AC})^2 = (\overline{AC})^2 + (\overline{BC})^2 - 2\,m\overline{AC}.m\overline{CE}$	Given mAB = mAC
$\Rightarrow$ $(\overline{BC})^2 - 2m\overline{AC}.m\overline{CE} = 0$	Cancel $(AC)^2$ on bath sides
or $(\overline{BC})^2 = 2m\overline{AC}\ , m\overline{CE}$	

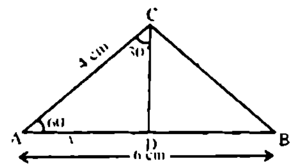
SOLVED EXERCISE 8.2

Q1. In a ΔABC calculate $m\overline{BC}$ when $m\overline{AB} = 6cm$, $m\overline{AC} = 4cm$ and $m\angle A = 60°$.

Solution:

Given: mAB = 6cm; mAC = 4cm; m CA = 60°.

Required: $m\overline{CB} = ?$



In Δ ABC, we have

$$(\overline{BC})^2 = (\overline{AB}) + (\overline{AC})^2 - 2(\overline{AB}).(\overline{AD})$$

$$= (6)^2 + (4)^2 \times 2(6)(x) \qquad \because \cos 60° = \frac{x}{4}$$

$$= 36 + 16 - 2(6)(2) \qquad \frac{1}{2} = \frac{x}{4}$$

$$= 52 - 24 \qquad 2x = 4$$

$$= 28 \qquad \Rightarrow \quad x = 2$$

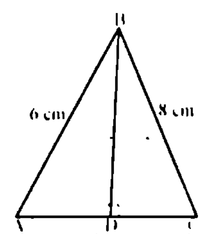
$$m\overline{BC} = \sqrt{28}$$

$$= 2\sqrt{7}\text{ cm} \Rightarrow = 5.29\text{ cm}$$

Q2. In ΔABC, $\overline{AB} = 6$ cm, $\overline{BC} = 8$ cm, $\overline{AC} = 9$ cm and D is the mid point of side $\overline{AC}$. Find length of the median $\overline{BD}$.

Solution:

According to the figure, we have



$$m\overline{AD} = \overline{DC}$$

and $$m\overline{AC} = m\overline{AD} + m\overline{DC}$$

$$m\overline{AC} = m\overline{AD} + m\overline{AD}$$

$$9 = 2m\overline{AD}$$

Or $$2m\overline{AD} = 9$$

$$m\overline{AD} = \frac{9}{2} = 4.5\text{ cm}$$