Rewrite eq. (i) as $$x^2+\frac{b}{a}x+\frac{c}{a}=0$$

or $$x^2-\left(-\frac{b}{a}\right)x+\frac{c}{a}=0$$

$$x^2-(\alpha+\beta)x+\alpha\beta=0$$

or x^2 – (sum of roots) x + product of roots = 0 , that is,

$x^2-Sx+P=0$ where $S=\alpha+\beta$ and $P=\alpha\beta$

SOLVED EXERCISE 2.5

1. Write the quadratic equations having following roots.

(a) 1, 5

Solution:

Since 1 and 5 are the roots of the required quadratic equation, therefore,

S = Sum of roots = 1 + 5 = 6

P = Product of roots = (1) (5) = 5

Thus, the required quadratic equation is

$x^2-Sx+P=0$

$x^2-6x+5=0$

(b) 4, 9

Solution:

Since 4 and 9 are the roots of the required quadratic equation, therefore,

S = Sum of roots = 4 + 9 = 13

P = Product of roots = (4) (9) = 36

Thus, the required quadratic equation is

$x^2-Sx+P=0$

$x^2-13x+36=0$

(c) - 2, 3

Solution:

Since –2 and 3 are the roots of the required quadratic equation, therefore,

S = Sum of roots = – 2 + 3 = 1

P = Product of roots = (–2) (3) = – 6

Thus, the required quadratic equation is

$x^2-Sx+P=0$

$x^2-(1)x+(-6)=0$

$x^2-x-6=0$

(d) 0, – 3

Solution:

Since 0 and –3 are the roots of the required quadratic equation, therefore,

S = Sum of roots = 0 + (–3) = – 3

P = Product of roots = $(0)(-3) = 0$

Thus, the required quadratic equation is

$$x^2 - Sx + P = 0$$
$$x^2 - (3)x + 0 = 0$$
$$x^2 + 3x = 0$$

(e) 2, – 6

Solution:

Since 2 and -6 are the roots of the required quadratic equation, therefore,

S = Sum of roots = $2 + (-6) = -4$

P = Product of roots = $(2)(-6) = -4$

Thus, the required quadratic equation is

$$x^2 - Sx + P = 0$$
$$x^2 - (-4)\,x + (-12) = 0$$
$$x^2 + 4x - 12 = 0$$

(f) –1, –7

Solution:

Since -1 and -7 are the roots of the required quadratic equation, therefore,

S = Sum of roots = $(-1) + (-7) = -1 - 7 = -8$

P = Product of roots = $(-1)(-7) = 7$

Thus, the required quadratic equation is

$$x^2 - sx + p = 0$$
$$x^2 - (-8)\,x + 7 = 0$$
$$x^2 + 8x + 7 = 0$$

(g) $1 + i, 1 - i$

Solution:

Since $1 + i$ and $1 - i$ are the roots of the required quadratic equation, therefore.

S = Sum of roots = $1 + i, 1 - i = 2$

P = Product of roots = $(1 + i)(1 - i) = 1 - i^2 = 1 - (-1) = 2$

Thus, the required quadratic equation is

$$x^2 - Sx + P = 0$$
$$x^2 - 2x + 2 = 0$$
$$x^2 + 3x = 0$$

(h) $3 + \sqrt{2}, 3 - \sqrt{2}$

Solution:

Since $3 - \sqrt{2}$ and $3 - \sqrt{2}$ are the roots of the required quadratic equation, therefore,

S = Sum of roots = $3 + \sqrt{2} + 3 - \sqrt{2} = 6$

P = Product of roots = $\left(3 + \sqrt{2}\right)\left(3 - \sqrt{2}\right) = (3)^2 - \left(\sqrt{2}\right)^2 = 9 - 2 = 7$

Thus, the required quadratic equation is

$$x^2 - Sx + P = 0$$
$$x^2 - 6x + 7 = 0$$

2. If α, β are the roots of the equation $x^2 - 3x + 6 = 0$.

Form equations whose roots are

(a) $2\alpha + 1, 2\beta + 1$

Solution:

$$x^2 - 3x + 6 = 0$$

Here $a = 1, b = -3, c = 6$

As α, β be the roots of given equation.

Then $\alpha + \beta = -\frac{b}{a} = -\frac{(-3)}{1} = 3$ and $\alpha\beta = \frac{c}{a} = \frac{6}{1} = 6$

S = Sum of roots
$= (2\alpha + 1) + (2\beta + 1)$
$= 2\alpha + 1 + 2\beta + 1$
$= 2\alpha + 2\beta + 2$
$= 2(\alpha + \beta) + 2$
$= 2(3) + 2$
$= 8$

and P = Product of roots
$= (2\alpha + 1)(2\beta + 1)$
$= (2\alpha + 1)(2\beta + 1)$
$= 4\alpha\beta + 2\alpha + 2\beta + 1$
$= 4(6) + 2(3) + 1$
$= 24 + 6 + 1$
$= 31$

Thus, the required quadratic equation is
$$x^2 - Sx + P = 0$$
$$x^2 - 8x + 31 = 0$$

(b) α^2, β^2

Solution:

$$x^2 - 3x + 6 = 0$$

Here $a = 1, b = -3, c = 6$

As α, β be the roots of given equation.

Then $\alpha + \beta = -\frac{b}{a} = -\frac{(-3)}{1} = 3$ and $\alpha\beta = \frac{c}{a} = \frac{6}{1} = 6$

S = Sum of roots
$= \alpha^2 + \beta^2$
$= (\alpha + \beta)^2 - 2\alpha\beta$
$= (3)^2 - 2(6)$
$= 9 - 12$

and P = Product of roots
$= (\alpha^2 + 1)(\beta^2)$
$= (\alpha^2)(\beta +)$
$= (\alpha\beta)^2$
$= (6)^2$

$= -3$ $\qquad\qquad\qquad\qquad = 36$

Thus, the required quadratic equation is

$$x^2 - Sx + P = 0$$
$$x^2 - (-3)x + 36 = 0$$
$$x^2 + 3x + 36 = 0$$

(c) $\dfrac{1}{\alpha}, \dfrac{1}{\beta}$

Solution:

$$x^2 - 3x + 6 = 0$$

Here $a = 1, b = -3, c = 6$

As $\propto, \beta$ be the roots of given equation.

Then $\propto + \beta = -\dfrac{b}{a}$ and $\propto\beta = \dfrac{c}{a}$

$= -\dfrac{(-3)}{1}$ $\qquad = \dfrac{6}{1}$

$= 3$ $\qquad = 6$

S = Sum of roots and $\qquad$ P = Product of roots

$= \dfrac{1}{\propto} + \dfrac{1}{\beta}$ $\qquad = \left(\dfrac{1}{\propto}\right)\left(\dfrac{1}{\beta}\right)$

$= \dfrac{\propto + \beta}{\propto\beta}$ $\qquad = \dfrac{1}{\propto\beta}$

$= \dfrac{3}{6} = \dfrac{1}{2}$ $\qquad = \dfrac{1}{6}$

Thus, the required quadratic equation is

$$x^2 - Sx + P = 0$$
$$x^2 - \frac{1}{2}x + \frac{1}{6} = 0$$

$\Rightarrow \quad 6x^2 - 3x + 1 = 0$

(d) $\dfrac{\alpha}{\beta}, \dfrac{\beta}{\alpha}$

Solution:

$$x^2 - 3x + 6 = 0$$

Here $a = 1, b = -3, c = 6$

As $\propto, \beta$ be the roots of given equation.

Then $\propto + \beta = -\dfrac{b}{a}$ and $\propto\beta = \dfrac{c}{a}$

$= -\dfrac{(-3)}{1}$ $\qquad = \dfrac{6}{1}$

$= 3$ $\qquad = 6$

S = Sum of roots and P = Product of roots

$$= \frac{\alpha}{\beta} + \frac{\beta}{\alpha} \qquad = \left(\frac{\alpha}{\beta}\right)\left(\frac{\beta}{\alpha}\right)$$

$$= \frac{\alpha^2 + \beta^2}{\alpha\beta} \qquad = 1$$

$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$= \frac{(3)^2 - 2(6)}{6}$$

$$= \frac{9 - 12}{6}$$

$$= -\frac{3}{6}$$

$$= -\frac{1}{2}$$

Thus, the required quadratic equation is

$$x^2 - Sx + P = 0$$

$$x^2 - \left(-\frac{1}{2}\right)x + 1 = 0$$

$$\Rightarrow \quad 2x^2 + x + 2 = 0$$

(e) $\alpha + \beta, \frac{1}{\alpha}, \frac{1}{\beta}$

Solution:

$$x^2 - 3x + 6 = 0$$

Here $a = 1, b = -3, c = 6$

As α, β be the roots of given equation.

Then $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

$$= -\frac{(-3)}{1} \qquad = \frac{6}{1}$$

$$= 3 \qquad = 6$$

S = Sum of roots and P = Product of roots

$$= \alpha + \beta + \frac{1}{\alpha} + \frac{1}{\beta} \qquad = (\alpha + \beta)\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$$

$$= (\alpha + \beta) + \frac{\alpha + \beta}{\alpha\beta} \qquad = \frac{\alpha}{\alpha} + \frac{\alpha}{\beta} + \frac{\beta}{\alpha} + \frac{\beta}{\beta}$$

$$= 3 + \frac{3}{6}$$

$$= 3 + \frac{1}{2}$$

$$= \frac{7}{2}$$

$$= 1 + \frac{\alpha^2 + \beta^2}{\alpha\beta} + 1$$

$$= 2 + \frac{2 + (\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$= 2 + \frac{(3)^2 - 2(6)}{6}$$

$$= 2 + \frac{9 - 12}{6}$$

$$= 2 - \frac{3}{6}$$

$$= 2 - \frac{1}{2}$$

$$= \frac{3}{2}$$

Thus, the required quadratic equation is

$$x^2 - Sx + P = 0$$

$$x^2 - \frac{7}{2}x + \frac{3}{2} = 0$$

$$\Rightarrow \quad 2x^2 - 7x + 3 = 0$$

3. If α, β are the roots of the equation $x^2 + px + q = 0$.

Form equations whose roots are

(a) α^2, β^2

Solution:

$$x^2 + Px + q = 0$$

Here $a = 1, b = P, c = q$

As α, β be the roots of given equation.

Then $\alpha + \beta - \frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

$= -\frac{p}{1}$ $\qquad$ $= \frac{q}{1}$

$= -p$ $\qquad$ $= q$

S = Sum of roots and P = Product of roots

$= \alpha^2 + \beta^2$ $\qquad$ $= (\alpha)^2 (\beta^2)$

$= (\alpha + \beta)^2 - 2\alpha\beta$ $\qquad$ $= (\alpha\beta)^2$

$= (-p)^2 - 2q$ $\qquad$ $= q^2$

$= p^2 - 2q$

Thus, the required quadratic equation is

$$x^2 - Sx + P = 0$$

$$x^2 - (p^2 - 2q)\,x + q^2 = 0$$

(b) $\dfrac{\alpha}{\beta}, \dfrac{\beta}{\alpha}$

Solution:

$$x^2 + Px + q = 0$$

Here $a = 1, b = P, c = q$

As $\propto, \beta$ be the roots of given equation.

Then $\propto + \beta = -\dfrac{b}{a}$ and $\propto\beta = \dfrac{c}{a}$

$= -\dfrac{p}{1}$ $\qquad = \dfrac{q}{1}$

$= -p$ $\qquad = q$

S = Sum of roots and P = Product of roots

$$= \frac{\propto}{\beta} + \frac{\beta}{\propto} \qquad = \left(\frac{\propto}{\beta}\right)\left(\frac{\beta}{\propto}\right)$$

$$= \frac{\propto^2 + \beta^2}{\propto\beta} \qquad = 1$$

$$= \frac{(\propto + \beta)^2 - 2\propto\beta}{\propto\beta}$$

$$= \frac{(-p)^2 - 2q}{q}$$

$$= \frac{p^2 - 2q}{q}$$

Thus, the required quadratic equation is

$$x^2 - Sx + P = 0$$

$$x^2 - \left(\frac{p^2 - 2q}{q}\right) + 1 = 0$$

$$\Rightarrow \quad qx^2 - (p^2 - 2q) + q = 0$$

Synthetic Division:

Synthetic division is the process of finding the quotient and remainder, when a polynomial is divided by a linear polynomial. In Fact synthetic division is simply a shortcut of long division method.

SOLVED EXERCISE 2.6

1. Use synthetic division to find the quotient and the remainder, when

(i) $(x^2 + 7x - 1) + (x + 1)$