Exercise 15

1. Verify that the Δs having the following measures of sides are right-angled.

(i) $a = 5$ cm, $b = 12$ cm, $c = 13$ cm

Ans. $(Hyp)^2 = (Perp.)^2 + (Base)^2$

$(13)^2 = (12)^2 + (5)^2$

$169 = 144 + 25$

$169 = 169$

$\therefore$ The triangle is right angled.

(ii) $a = 1.5$ cm, $b = 2$ cm, $c = 2.5$ cm

Ans. $(Hyp)^2 = (Perp.)^2 + (Base)^2$

$(2.5)^2 = (1.5)^2 + (2)^2$

$625 = 2.25 + 4$

$6.25 = 6.25$

$\therefore$ The triangle is right angled.

(iii) $a = 9$ cm, $b = 12$ cm, $c = 15$ cm

Ans. $(Hyp)^2 = (Perp.)^2 + (Base)^2$

$(15)^2 = (12)^2 + (9)^2$

$225 = 144 + 81$

$225 = 225$

$\therefore$ The triangle is right angled.

(iv) $a = 16$ cm, $b = 30$ cm, $c = 34$ cm

Ans. $(Hyp)^2 = (Perp.)^2 + (Base)^2$

$(34)^2 = (30)^2 + (16)^2$

$1156 = 900 + 256$

$1156 = 1156$

$\therefore$ The triangle is right angled.

2. Verify that $a^2 + b^2$, $a^2 - b^2$ and $2ab$ are the measures of the sides of a right angled triangle where a and b are any two real numbers ($a > b$).

Ans. In right angle triangle.

$H^2 = P^2 + B^2$

$(a^2 + b^2)^2 = a^4 + b^4 + 2a^2b^2$ ....(i)

$(a^2 - b^2)^2 = a^4 + b^4 - 2a^2b^2$ ....(ii)

$(2ab)^2 = 4a^2b^2$ .... (iii)

Adding (ii) and (iii) we get

$(a^2 - b^2)^2 + (2ab)^2 = a^4 + b^4 - 2a^2b^2 + 4a^2b^2$

$= a^4 + b^4 + 2a^2b^2$ ....(iv)

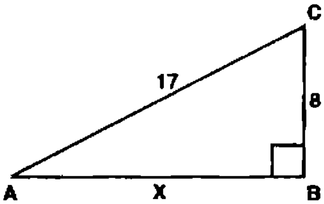
Comparing (i) and (iv), we get

$(a^2 - b^2)^2 + (2ab)^2 = (a^2 + b^2)^2$

Hence $a^2 + b^2$, $a^2 - b^2$ and $2ab$ are measures of the sides of a right angled triangle where $a^2 + b^2$ is Hypotenuse.

3. The three sides of a triangle are of measure 8, x and 17 respectively. For what value of x will it become base of a right angled triangle?

Ans:



Consider a right angled triangle

With $\overline{AB} = x$

$\overline{BC} = 8$

and $\overline{AC} = 17$

If x is the base of right angled ΔABC then we know by Pythagoras theorem that

$$(\text{hyp})^2 = (\text{Base})^2 + (\text{perp})^2$$
$$(17)^2 = x^2 + (8)^2$$
$$289 = x^2 + 64$$
$$x^2 + 64 = 289$$
$$x^2 = 289 - 64$$
$$x^2 = 225$$
$$x = \sqrt{225}$$

As x is measure of side

So x = 15 units

4. In an isosceles Δ, the base $\overline{BC} = 28$ cm, and $\overline{AB} = \overline{AC} = 50$cm.

If $\overline{AD} \perp \overline{BC}$, then find:

(i) Length of $\overline{AD}$

(ii) Area of ΔABC

Given

$m\overline{AC} = m\overline{AB} = 50\text{ cm}$

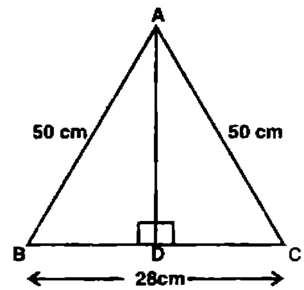
$m\overline{BC} = 28\text{cm}$

$\overline{AD} \perp \overline{BC}$

To Prove

$m\overline{AD} = ?$

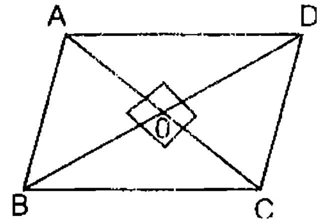
Area of $\Delta ABC = ?$



Proof

Statements	Reasons
In right angled triangle	
$m\overline{CD} = 14\text{cm}$	$\overline{CD} = \frac{1}{2} m\overline{BC}$
$m\overline{AC} = 50\text{cm}$	Given
$(mAD)^2 = (mAC)^2 - (mCD)^2$	
$(mAD)^2 = (50)^2 - (14)^2$ $= 2500 - 196$ $= 2304$	$(mAC)^2 = (mAD)^2 - (mCD)^2$ (by Pythagoras theorem)
$\sqrt{(mAD)^2} = \sqrt{2304}$	
$m\overline{AD} = 18$ cm	Taking square root of both sides
(ii) Area of $\Delta ABC = \dfrac{\text{Base} \times \text{Altitude}}{2}$ $= \dfrac{28 \times 48}{2}$ $= 14 \times 28$ $= 672$ sq.cm	

In a quadrilateral ABCD, the diagonals $\overline{AC}$ and $\overline{BD}$ are perpendicular to each other. Prove that:



$$m\overline{AB}^2 + m\overline{CD}^2 = m\overline{AD}^2 + m\overline{BC}^2.$$

Given: Quadrilateral ABCD diagonal $\overline{AC}$ and $\overline{BD}$ are perpendicular to each other.

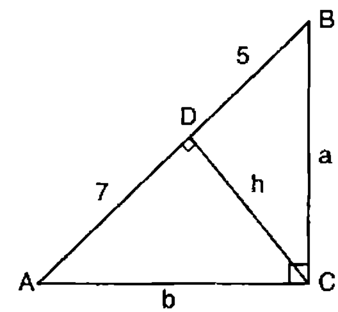
To Prove:

$$m\left(\overline{AB}\right)^2 + m\left(\overline{CD}\right)^2 = m\left(\overline{AD}\right)^2 + m\left(\overline{BC}\right)^2$$

Proof

Statements	Reasons
In right triangle AOB	
$m\left(\overline{AB}\right)^2 = m\left(\overline{AO}\right)^2 + m\left(\overline{OB}\right)^2$ ....,(i)	By Pythagoras theorem
In right triangle COD	
$m\left(\overline{CD}\right)^2 = m\left(\overline{OC}\right)^2 + m\left(\overline{OD}\right)^2$ ....,(ii)	By Pythagoras theorem
In right triangle AOD	
$m\left(\overline{AD}\right)^2 = m\left(\overline{AO}\right)^2 + m\left(\overline{OD}\right)^2$ ....,(iii)	By Pythagoras theorem
In right triangle BOC	
$m\left(\overline{BC}\right)^2 = m\left(\overline{OB}\right)^2 + m\left(\overline{OC}\right)^2$ ....,(iv)	By Pythagoras theorem
$m\left(\overline{AB}\right)^2 + m\left(\overline{CD}\right)^2 = m\left(\overline{AO}\right)^2 + m\left(\overline{OB}\right)^2 + m\left(\overline{OC}\right)^2 + m\left(\overline{OD}\right)^2$ ....,(v)	By adding (i) and (ii)
$m\left(\overline{AD}\right)^2 + m\left(\overline{BC}\right)^2 = m\left(\overline{AO}\right)^2 + m\left(\overline{OD}\right)^2 + m\left(\overline{OB}\right)^2 + m\left(\overline{OC}\right)^2$ ....(vi)	By adding (ii) and (iv)
$\left(m\overline{AB}\right)^2 + \left(m\overline{CD}\right)^2 = \left(m\overline{BC}\right)^2 + \left(m\overline{AD}\right)^2$	By adding (v) and (vi)

6. (i) In the ΔABC as shown in the figure, $m\angle ACB = 90^0$ and $\overline{CD} \perp \overline{AB}$. Find the lengths a, h and b if $m\overline{BD} = 5$ units and $m\overline{AD} = 7$ units.



Given: A ΔABC as shown

$m\ \angle ACB = 90^0$

and $\overline{CD} \perp \overline{AB}$

To prove : a, h and b.

In right angled ΔBDC

$a^2 = 25 + h^2$ .... (i)

in right angled ΔADC

$b^2 = 49 + h^2$ .... (ii)

in right angled ΔABC

$a^2+b^2 = 144$ .... (iii)

adding (i) and (ii)

$a^2+b^2 = 74+2h^2$ .... (iv)

from (iii) and (iv)

$$74 + 2h^2 = 144$$
$$2h^2 = 144-74$$
$$2h^2 = 70$$
$$h^2 = 35$$
$$h = \sqrt{35}$$

Now we will find a and b

Put $h^2 = 35$ (in Eq. 1)

$$a^2 = 25+35$$
$$a^2 = 60$$
$$a = \sqrt{60}$$
$$= \sqrt{4\times15}$$
$$a = 2\sqrt{15}$$

now put $h^2 = 35$ (in Eq. 2)

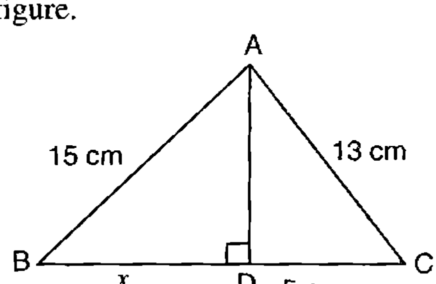
$$b^2 = 49+35$$
$$b^2 = 48$$
$$b = \sqrt{84}$$
$$b = \sqrt{4\times21}$$
$$b = 2\sqrt{21}$$

SO $a = 2\sqrt{15}$

$h = \sqrt{35}$

$b = 2\sqrt{21}$

(ii) Find the value of x in the shown in the figure.



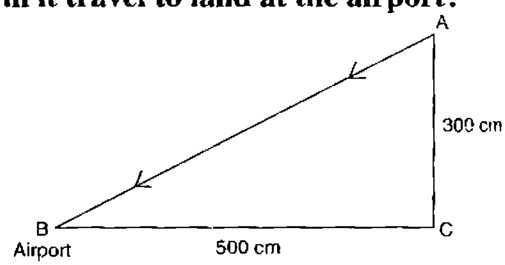
In right angled triangle ADC

$$m(\overline{AC})^2 = m(\overline{AD})^2 + m(\overline{DC})^2$$
$$(13)^2 = (AD)^2 + (5)^2$$
$$169 = (AD)^2 + 25$$
$$(AD)^2 = 169 - 25$$
$$(AD)^2 = 144$$
$$AD = \sqrt{144}$$
$$AD = 12cm$$

In right angled triangle ABD

$$(AB)^2 = (AD)^2 + (BD)^2$$
$$(15)^2 = (12)^2 + x^2$$
$$225 = 144 + x^2$$
$$x^2 = 225 - 144$$
$$x^2 = 81$$
$$x = 9 \text{ cm}$$

7. A plane is at a height of 300 m and is 500 m away from the airport as shown in the figure. How much distance will it travel to land at the airport?



Here A be the position of plane and B be the position of airport.

$$m\overline{AC} = 500m$$
$$m\overline{BC} = 300m$$
$$m\overline{AB} = ?$$

Applying Pythagoras theorem on right angled triangle ABC

$$|\overline{AB}|^2 = |\overline{AC}|^2 + |\overline{BC}|^2$$
$$= (500)^2 + (300)^2$$
$$= 250000 + 90000$$
$$= 34000$$
$$|\overline{AB}|^2 = 34\times10000$$

so $|\overline{AB}| = \sqrt{34\times10000}$

$$= \sqrt{34\times100\times100}$$

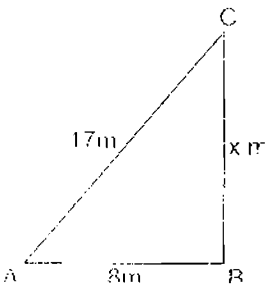
$= 100\sqrt{34}m$

So required distance is $100\sqrt{34}m$

8. A ladder 17 m long rests against a vertical wall. The foot of the ladder is 8m away from the base of the wall. How high up the wall will the ladder reach?

Ans. Let the height of ladder = x m

in right angled triangle



$(Hyp)^2 = (Perp.)^2 + (Base)^2$

$(17)^2 = (x)^2 + (8)^2$

$289 = x^2 + 64$

$x^2 = 289 - 64$

$x^2 = 225$

$x = \sqrt{225} = 15m$

9. A student travels to his school by the route as shown in the figure. Find $m\overline{AD}$, the direct distance from his house to school.

According to figure, $m\overline{AB} = 2km$

$m\overline{BC} = 6km$

$m\overline{CD} = 3km$

Here $m\overline{AB}$ and $m\overline{CD}$ are perpendicular

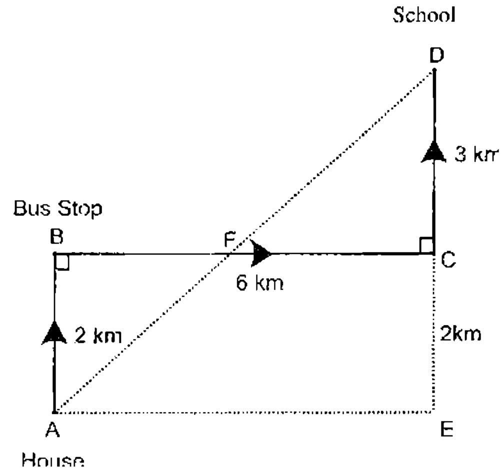
Perpendicular = $\overline{AB} + \overline{CD}$

$= 2 + 3$

$= 5km$

According to Pythagoras theorem

$(H)^2 = P^2 + B^2$



$(m\overline{AD})^2 = (5)^2 + (6)^2 = 25 + 36$

$(m\overline{AD})^2 = 61$

$m\overline{AD} = \sqrt{61}$ Km

10. Which of the following are true and which are false?

(i) In a right angled triangle greater angle is $90°$. (T)

(ii) In a right angled triangle right angle is $60°$. (F)

(iii) In a right triangle hypotenuse is a side opposite to right angle. (T)

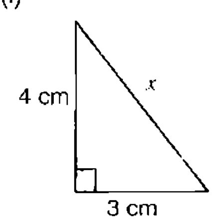
(iv) If a, b, c are sides of right angled triangle with c as longer side then $c^2 = a^2 + b^2$. (T)

(v) If 3 cm and 4 cm are two sides of a right angled triangle, then hypotenuse is 5 cm. (T)

(vi) If hypotenuse of an isosceles right triangle is $\sqrt{2}$ cm then each of other side is of length 2 cm.(F)

11. Find the unknown value in each of the following figures.

(i)



By Pythagoras theorem

$(Hyp)^2 = (Perp.)^2 + (Base)^2$

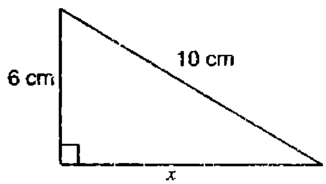
$x^2 = (4)^2 + (3)^2$

$x^2 = 16 + 9$

$x^2 = 25 \Rightarrow x = \sqrt{25}$

$x = 5cm$

(ii)



By Pythagoras theorem

$(Hyp)^2 = (Perp.)^2 + (Base)^2$

$(10)^2 = (6)^2 + (x)^2$

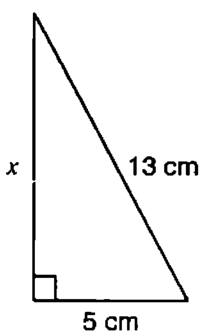
$100 = 36 + x^2$

$x^2 = 64$

$x = \sqrt{64}$

$X = 8cm$

(iii)



By Pythagoras theorem

$(Hyp)^{\cdot} = (Perp.)^2 + (Base)^2$

$(13)^2 = (x)^2 + (2)^2$

$169 = x^2 + 25$

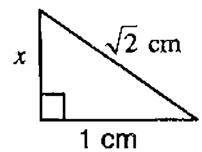
$x^2 = 169 - 25$

$x^2 = 144$

$x = \sqrt{144}$

$x = 12cm$

(iv)



By Pythagoras theorem

$(Hyp.)^2 = (Perp.)^2 + (Base)^2$

$(\sqrt{2})^2 = (x)^2 + (1)^2$

$\left(\sqrt{2}\right)^2 = (x)^2 + (1)^2$

$2 = x^2 + 1$

$x^2 = 2 - 1$

$x^2 = 1$

$x = \sqrt{1} = 1cm$