Hence, $\dfrac{x^5}{(x^2+1)^2} = x - \left[\dfrac{2x}{x^2+1} + \dfrac{-x}{(x^2+1)^2}\right]$

$$= x - \left[\frac{2x}{x^2+1} - \frac{x}{(x^2+1)^2}\right]$$

$$= x - \left[\frac{2x}{x^2+1} + \frac{x}{(x^2+1)^2}\right]$$

SOLVED MISCELLANEOUS EXERCISE - 4

Q1. Multiple Choice Questions

Four possible answers are given for the following questions. Tick (✓) the correct answer.

(i) The identity $(5x + 4)^2 = 25x^2 + 40x + 16$ is true for

(a) one value of x (b) two values of x

(c) all values of x (d) none of these

(ii) A function of the form $(x) = \dfrac{N(x)}{D(x)}$, with $D(x) \neq 0$, where N(x) and D(x) are polynomials in x is called

(a) an identity (b) an equation

(c) a fraction (c) none of these

(iii) A fraction in which the degree of the numerator is greater or equal the degree of denominator is called:

(a) a proper fraction (b) an improper fraction

(c) an equation (d) algebraic relation

(iv) A fraction in which the degree of numerator is less than the degree of the denominator is called

(a) an equation (b) an improper fraction

(c) an identity (d) a proper fraction

(v) $\dfrac{2x+1}{(x+1)(x-1)}$ is:

(a) an improper fraction (b) an equation

(c) a proper fraction (d) none of these

(vi) $(x + 3)^2 = x^2 + 6x + 9$ is:

(a) a linear equation (b) an equation

(c) an identity (d) none of these

(vii) $\frac{x^3+1}{(x-1)(x+2)}$ is

(a) a proper fraction
(b) an improper fraction
(c) an identity
(d) a constant term

(viii) Partial fractions of $\frac{x-2}{(x-1)(x+2)}$ are of the form

(a) $\frac{A}{x-1}+\frac{B}{x+2}$
(b) $\frac{Ax}{x-1}+\frac{B}{x+2}$
(c) $\frac{Ax}{x-1}+\frac{Bx+C}{x+2}$
(d) $\frac{Ax}{x-1}+\frac{C}{x+2}$

(ix) Partial fractions of $\frac{x+2}{(x+1)(x^2+2)}$ are of the form

(a) $\frac{A}{x+1}+\frac{B}{x^2+2}$
(b) $\frac{A}{x+1}+\frac{Bx+C}{x^2+2}$
(c) $\frac{Ax+B}{x+1}+\frac{C}{x^2+2}$
(d) $\frac{A}{x+1}+\frac{Bx}{x^2+2}$

(x) Partial fractions $\frac{x^2+1}{(x+1)(x-1)}$ of are of the form

(a) $\frac{A}{x+1}+\frac{B}{x-1}$
(b) $1+\frac{A}{x+1}+\frac{Bx+C}{x-1}$
(c) $\frac{Ax+B}{x+1}+\frac{C}{x^2+2}$
(d) $\frac{Ax+B}{(x+1)}+\frac{C}{x-1}$

Answer:

i)	c	ii)	c	iii)	b	iv)	d	v)	c
vi)	c	vii)	b	viii)	a	ix)	b	x)	c

Q2. Write short answers of the following questions.

(i) Define a rational fraction.

Ans: National Fraction

An expression of the form $\frac{N(X)}{D(x)}$ with $D(x) \neq 0$ is called a rational fraction.

(ii) What is a proper fraction?

Ans: Proper Fraction

A rational fraction $\frac{N(X)}{D(x)}$, with $D(x) \neq 0$ is called a proper fraction if degree of the polynomial $N(x) <$ degree of the polynomial $D(x)$.

(iii) What is an improper fraction?

Ans: Improper Fraction

A rational fraction $\frac{N(X)}{D(x)}$, with $D(x) \neq 0$ is called an improper fraction if degree of the polynomial $N(x)$ is greater than degree of $D(x)$.

(iv) What are partial fractions?

Ans: Partial Fraction

A single fraction written in the forms of its components is said to be resolved into partial fraction.

(v) How can we make partial fractions of. $\frac{x-2}{(x+2)(x+3)}$?

Ans: It is written as: $\frac{x-2}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3}$, then values of A and B as found.

(vi) Resolve $\frac{1}{x^2-1}$ into partial fractions.

Ans: $\frac{1}{x^2-1}$

$= \frac{1}{(x+1)(x-1)}$

Let $\frac{1}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$ ....(i)

Multiplying both sides by $(x+1)(x-1)$, we get

$1 = A(x-1) + B(x+1)$ ....(ii)

Put $x+1=0$ i.e; $x=-1$ in (ii)

$1 = A(-1-1) + 0$

$1 = 2A$

$\boxed{A = -\frac{1}{2}}$

Put $x-1=0$ i.e; $x=1$ in (ii)

$1 = 0 + B(1+1)$

$1 = 2B$

$\boxed{B = -\frac{1}{2}}$

Putting values of A, B in (i)

$\frac{1}{(x+1)(x-1)} = \frac{-1}{2(x+1)} + \frac{B}{2(x-1)}$ (Partial Fractions)

(vii) Find partial fractions of $\frac{3}{(x+1)(x-1)}$.

Ans: Let $\frac{3}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$ .....(i)

Multiplying by (x + 1) (x − 1), we get

$3 = A(x-1) + B(x+1)$ ....(ii)

Put $x - 1 = 0$ i.e; $x = -1$ in (ii)

$3 = B(1+1)$

$2B = 3$

$\boxed{B = -\frac{3}{2}}$

Putting values of A, B in (i)

$$\frac{3}{(x+1)(x-1)} = \frac{-3}{2(x+1)} + \frac{3}{2(x-1)}$$

$$\frac{2}{3}\left[\frac{1}{x-1} - \frac{1}{x+1}\right]$$

(viii) Resolve $\frac{x}{(x-3)^2}$ into partial fractions.

Ans: Let $\frac{x}{(x-3)^2} = \frac{A}{x-3} + \frac{B}{(x-3)^2}$ .....(i)

Multiplying by $(x-3)^2$, we get

$x = A(x-3) + B$ ....(ii)

Put $x - 3 = 0$ i.e; $x = 3$ in (ii)

$3 = B$

$B = 3$

Comparing coefficients of x

$\boxed{1 = A}$

Putting values of A, B in (i)

$$\frac{x}{(x-3)^2} = \frac{1}{x-3} + \frac{3}{(x-3)^2}$$

(ix) How we can make the partial fractions of $\frac{x}{(x+a)(x-a)}$?

Ans: Let $\frac{x}{(x+a)(x-a)} = \frac{A}{x+a} + \frac{B}{x-a}$ .....(i)

Multiplying by (x + a) (x − a), we get

$x = A(x-a) + B(x+a)$ ....(ii)

Put $x - a = 0$ i.e; $x = \alpha$ in (ii)

$a = B(a+a)$

$a = B(2a)$

$$\boxed{B = \frac{1}{2}}$$

Put $x + a = 0$ i.e;

$x = -a$ in (ii)

$-a = A(-a - a)$

$-a = A(-2a)$

$$A = \frac{-a}{-2a}$$

$$\boxed{A = \frac{1}{2}}$$

Putting values of A , B in (i)

$$\frac{x}{(x+a)(x-a)} = \frac{1}{2(x+a)} + \frac{B}{2(x-a)}$$

$$= \frac{1}{2}\left[\frac{1}{x+a} - \frac{1}{x-a}\right] \text{(Partial Fractions)}$$

(x) Whether $(x + 3)^2 = x^2 + 6x + 9$ is an identity?

Ans:

$(x + 3)^2 = x^2 + 6x + 9$ .... (i)

Put $x = 7$ in it

$(7 + 3)^2 = (7)^2 + 6(7) + 9$

$100 = 100$

Put $x = -7$ in (i)

$(-7 + 3)^2 = (-7)^2 + 6(-7) + 9$

$(-4)^2 = 49 - 42 + 9$

$16 = 58 - 42$

$16 = 16$

Yes, this is an identity.

It is true for every value of x.

SUMMARY

- A fraction is an indicated quotient of two numbers or algebraic expressions.
- An expression of the form $\frac{N(x)}{D(x)}$, with $D(x) \neq 0$ and $N(x)$ and $D(x)$ are polynomials in x with real coefficients, is called a rational fraction. Every fractional expression can be expressed as a quotient of two polynomials.
- A rational fraction $\frac{N(x)}{D(x)}$ with $D(x) \neq 0$ is called a proper fraction if degree of the polynomial $N(x)$ in the numerator is less than the degree of the polynomial $D(x)$, in the denominator.
- A rational fraction $\frac{N(x)}{D(x)}$ with $D(x) \neq 0$ is called an improper fraction if degree of the polynomial $N(x)$ is greater or equal to the degree of the polynomial $D(x)$.
- Partial fractions: Decomposition of resultant fraction $\frac{N(x)}{D(x)}$, with $D(x) \neq 0$, when

  (a) $D(x)$ consists of non-repeated linear factors.
  (b) $D(x)$ consists of repeated linear factors.
  (c) $D(x)$ consists of non-repeated irreducible quadratic factors.
  (d) $D(x)$ consists of repeated irreducible quadratic factors.