Exercise 1.5

1. Find the determinant of the following matrices.

Ans. (i) $A=\begin{bmatrix}-1 & 1\\ 2 & 0\end{bmatrix}$

$|A|=\begin{vmatrix}-1 & 1\\ 2 & 0\end{vmatrix}$

$=-1(0)-2(1)$

$=0-2=-2$

(ii) $B=\begin{bmatrix}1 & 3\\ 2 & -2\end{bmatrix}$

$|B|=1(-2)-2(3)$

$=-2-6$

$=-8$

(iii) $C=\begin{bmatrix}3 & 2\\ 3 & 2\end{bmatrix}$

$|C|=3(2)-3(2)$

$=6-6=0$

(iv) $D=\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}$

$|D|=3(4)-1(2)$

$=12-2=10$

2. Find which of the following matrices are singular or non-singular?

Ans. (i) $A=\begin{bmatrix}3 & 6\\ 2 & 4\end{bmatrix}$

$|A|=\begin{vmatrix}3 & 6\\ 2 & 4\end{vmatrix}$

$=3(4)-2(6)$

$=12-12$

$=0$ singular

(ii) $B=\begin{bmatrix}4 & 1\\ 3 & 2\end{bmatrix}$

$|B|=\begin{vmatrix}4 & 1\\ 3 & 2\end{vmatrix}$

$=4(2)-3(1)=8-3=5$ non-singular

(iii) $C=\begin{bmatrix}7 & -9\\ 3 & 5\end{bmatrix}$

$|C|=\begin{vmatrix}7 & -9\\ 3 & 5\end{vmatrix}$

$=7(5)-3(-9)$

$=35+27$

$=62\neq 0$ non-singular

(iv) $D=\begin{bmatrix} 5 & -10 \\ -2 & 4 \end{bmatrix}$

$|D| = \begin{vmatrix} 5 & -10 \\ -2 & 4 \end{vmatrix}$

$=5(4)-(-2)(-10)$

$=20-20$

$=0$ singular

3. Find the multiplicative inverse (if it exists) of each.

Ans. (i) $A=\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$

$|A| = \begin{vmatrix} -1 & 3 \\ 2 & 0 \end{vmatrix}$

$=-1(0)-2(3)$

$=-6$

$\text{Adj}A = \begin{bmatrix} 0 & -3 \\ -2 & -1 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}\text{adj } A$

$=\frac{1}{-6}\begin{bmatrix} 0 & -3 \\ -2 & -1 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 0 & 3 \\ 2 & 1 \end{bmatrix}$

$=\begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{6} \end{bmatrix}$

(ii) $B=\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$

$|B| = \begin{vmatrix} 1 & 2 \\ -3 & -5 \end{vmatrix}$

$=1(-5)-(-3)(2)$

$=-5+6$

$=1 \neq 0$

$\text{Adj } B=\begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix}$

$B^{-1}=\frac{1}{|B|}\text{adj}B$

$=\frac{1}{1}\begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix}$

(iii) $C=\begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}$

$|C| = \begin{vmatrix} -2 & 6 \\ 3 & -9 \end{vmatrix}$

$=-2(-9)-3(6)$

$=18-18=0$

C^{-1} does not exist.

(iv) $D=\begin{bmatrix} \frac{1}{2} & \frac{3}{4} \\ 1 & 2 \end{bmatrix}$

$|D| = \begin{vmatrix} \frac{1}{2} & \frac{3}{4} \\ 1 & 2 \end{vmatrix}$

$=\frac{1}{2}(2)-1\left(\frac{3}{4}\right)$

$=1-\frac{3}{4}$

$=\frac{4-3}{4}=\frac{1}{4}\neq 0$

$\text{Adj } D = \begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$

$D^{-1} = \frac{1}{|D|}\text{adj.D}$

$=\frac{1}{1/4}\begin{bmatrix} 2 & -3/4 \\ -1 & 1/2 \end{bmatrix}$

$=4\begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 8 & -3 \\ -4 & 2 \end{bmatrix}$

4. If $A=\begin{bmatrix}1 & 2\\ 4 & 6\end{bmatrix}$ and $B=\begin{bmatrix}3 & -1\\ 2 & -2\end{bmatrix}$, then

(i) **$A(\text{Adj } A) = (\text{Adj } A)\, A = (\det A)I$**

(ii) **$BB^{-1} = I = B^{-1}B$**

Ans. (i) $A(\text{Adj } A)=(\text{Adj } A)\, A = (\det A)I$

$$A=\begin{bmatrix}1 & 2\\ 4 & 6\end{bmatrix}$$

$$\text{Adj} A=\begin{bmatrix}6 & -2\\ -4 & 1\end{bmatrix}$$

$$A(\text{Adj}A)=\begin{bmatrix}1 & 2\\ 4 & 6\end{bmatrix}\begin{bmatrix}6 & -2\\ -4 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1(6)+2(-4) & 1(-2)+2(1)\\ 4(6)+6(-4) & 4(-2)+6(1)\end{bmatrix}$$

$$=\begin{bmatrix}6-8 & -2+2\\ 24-24 & -8+6\end{bmatrix}$$

$$=\begin{bmatrix}-2 & 0\\ 0 & -2\end{bmatrix}$$

Now $(\text{Adj}A)A=\begin{bmatrix}6 & -2\\ -4 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\ 4 & 6\end{bmatrix}$

$$=\begin{bmatrix}6(1)+-2(4) & 6(2)+-2(6)\\ -4(1)+1(4) & -4(2)+1(6)\end{bmatrix}$$

$$=\begin{bmatrix}6-8 & 12-12\\ -4+4 & -8+6\end{bmatrix}$$

$$=\begin{bmatrix}-2 & 0\\ 0 & -2\end{bmatrix}$$

Also $(\det A)I$

$$\det A =\begin{vmatrix}1 & 2\\ 4 & 6\end{vmatrix}$$

$$=1(6)-2(4)=6-8=-2$$

$$(\det A)I=-2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}-2 & 0\\ 0 & -2\end{bmatrix}$$

Hence: $A(\text{Adj}A) = (\text{Adj}A)\, A = (\det A)I$

(ii) $B=\begin{bmatrix}3 & -1\\ 2 & -2\end{bmatrix}$

$$|B|=\begin{vmatrix}3 & -1\\ 2 & -2\end{vmatrix}=3(2)-2(-1)$$

$$=-6+2=-4\neq 0$$

$$\text{Adj}B =\begin{bmatrix}-2 & 1\\ -2 & 3\end{bmatrix}$$

$$B^{-1}=\frac{1}{|B|}\text{Adj}B$$

$$=\frac{1}{-4}\begin{bmatrix}-2 & 1\\ -2 & 3\end{bmatrix}=\frac{1}{4}\begin{bmatrix}2 & -1\\ 2 & -3\end{bmatrix}$$

$$BB^{-1}=\frac{1}{4}\begin{bmatrix}3 & -1\\ 2 & -2\end{bmatrix}\begin{bmatrix}2 & -1\\ 2 & -3\end{bmatrix}$$

$$=\frac{1}{4}\begin{bmatrix}3(2)+(-1)(2) & 3(-1)+(-1)(-3)\\ 2(2)+(-2)(2) & 2(-1)+(-2)(-3)\end{bmatrix}$$

$$=\frac{1}{4}\begin{bmatrix}6-2 & -3+3\\ 4-4 & -2+6\end{bmatrix}$$

$$=\frac{1}{4}\begin{bmatrix}4 & 0\\ 0 & 4\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$$

Similarly:

$$B^{-1}B=\frac{1}{4}\begin{bmatrix}2 & -1\\ 2 & -3\end{bmatrix}\begin{bmatrix}3 & -1\\ 2 & -2\end{bmatrix}$$

$$=\frac{1}{4}\begin{bmatrix}2(3)+(-1)(2) & 2(-1)+(-1)(-2)\\ 2(3)+(-3)(2) & 2(-1)+(-3)(-2)\end{bmatrix}$$

$$=\frac{1}{4}\begin{bmatrix}4 & 0\\ 0 & 4\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$$

Hence: $BB^{-1} = I = B^{-1}B$

5. Determine whether the given matrices are multiplicative inverses of each other.

Ans. (i) $\begin{bmatrix}3 & 5\\ 4 & 7\end{bmatrix}$ and $\begin{bmatrix}7 & -5\\ -4 & 3\end{bmatrix}$

$$\begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix}\begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix}$$

$$=\begin{bmatrix} 3(7)+5(-4) & 3(-5)+5(3) \\ 4(7)+7(-4) & 4(-5(+7(3) \end{bmatrix}$$

$$=\begin{bmatrix} 21-20 & -15+15 \\ 28-28 & -20+21 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}=I$$

$\therefore$ Given matrices are multiplicative inverse of each other.

(ii) $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $\begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$$

$$=\begin{bmatrix} 1(-3)+2(2) & 1(2)+2(-1) \\ 2(-3)+3(2) & 2(2)+3(-1) \end{bmatrix}$$

$$=\begin{bmatrix} -3+4 & 2-2 \\ -6+6 & 4-3 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}=I$$

6. If $A=\begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}$, $B=\begin{bmatrix} -4 & -2 \\ 1 & -1 \end{bmatrix}$, $D=\begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}$**, then verify that**

(i) $(AB)^{-1}=B^{-1}A^{-1}$

(ii) $(DA)^{-1}=A^{-1}D^{-1}$

Ans. (i) $(AB)^{-1}=B^{-1}A^{-1}$

L.H.S $=(AB)^{-1}$

$$AB=\begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -4 & -2 \\ 1 & -1 \end{bmatrix}$$

$$=\begin{bmatrix} 4(-4)+0(1) & 4(-2)+0(-1) \\ -1(-4)+2(1) & -1(-2)+2(-1) \end{bmatrix}$$

$$=\begin{bmatrix} -16 & -8 \\ 6 & 0 \end{bmatrix}$$

$$|AB|=\begin{vmatrix} -16 & -8 \\ 6 & 0 \end{vmatrix}$$

$$=-16(0)-6(-8)$$

$$=0+48=48\neq 0$$

$$\text{Adj}(AB)=\begin{vmatrix} 0 & 8 \\ -6 & -16 \end{vmatrix}$$

$$(AB)^{-1}=\frac{1}{|AB|}\text{Adj}(AB)$$

$$=\frac{1}{48}\begin{bmatrix} 0 & 8 \\ -6 & -16 \end{bmatrix}=\begin{bmatrix} 0 & \frac{8}{48} \\ \frac{-6}{48} & \frac{-16}{48} \end{bmatrix}=\begin{bmatrix} 0 & \frac{1}{6} \\ \frac{-1}{8} & -\frac{1}{3} \end{bmatrix}$$

R.H.S $=B^{-1}A^{-1}$

$$B=\begin{bmatrix} -4 & -2 \\ 1 & -1 \end{bmatrix}$$

$$|B|=-4(-1)-1)(-2)=4+2=6$$

$$B^{-1}=\frac{1}{|B|}\text{Adj}B=\frac{1}{6}\begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix}$$

$$A=\begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}$$

$$|A|=\begin{vmatrix} 4 & 0 \\ -1 & 2 \end{vmatrix}=4(2)-(-1)(0)=8$$

$$A^{-1}=\frac{1}{|A|}\text{Adj}A=\frac{1}{8}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$B^{-1}A^{-1}=\frac{1}{6}\begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix}\frac{1}{8}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$=\frac{1}{48}\begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$=\frac{1}{48}\begin{bmatrix} -1(2)+2(1) & -1(0)+2(4) \\ -1(2)+-4(1) & -1(0)+-4(4) \end{bmatrix}$$

$$=\frac{1}{48}\begin{bmatrix} 0 & 8 \\ -6 & -16 \end{bmatrix}=\begin{bmatrix} 0 & \frac{8}{48} \\ -\frac{6}{48} & -\frac{16}{48} \end{bmatrix}$$

$$=\begin{bmatrix} 0 & \frac{1}{6} \\ -\frac{1}{8} & -\frac{1}{3} \end{bmatrix}$$

L.H.S = R.H.S

Hence: $(AB)^{-1}=B^{-1}A^{-1}$

(ii) $\mathbf{(DA)^{-1} = A^{-1} D^{-1}}$

L.H.S $= (DA)^{-1}$

$$DA = \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3(4)+1(-1) & -2(0)+1(2) \\ -2(4)+2(-1) & -2(0)+2(2) \end{bmatrix}_1$$

$$= \begin{bmatrix} 12-1 & 0+2 \\ -8-2 & 0+4 \end{bmatrix} = \begin{bmatrix} 11 & 2 \\ -10 & 4 \end{bmatrix}$$

$$|DA| = \begin{vmatrix} 11 & 2 \\ -10 & 4 \end{vmatrix}$$

$$= 11(4) - (-10)(2)$$

$$= 44 + 20$$

$$= 64$$

$$\text{Adj}(DA) = \begin{bmatrix} 4 & -2 \\ 10 & 11 \end{bmatrix}$$

$$(DA)^{-1} = \frac{1}{DA}\text{Adj}(DA)$$

$$= \frac{1}{64}\begin{bmatrix} 4 & -2 \\ 10 & 11 \end{bmatrix} = \begin{bmatrix} \frac{4}{64} & \frac{-2}{64} \\ \frac{10}{64} & \frac{11}{64} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{16} & \frac{-1}{32} \\ \frac{5}{32} & \frac{11}{64} \end{bmatrix}$$

R.H.S $= A^{-1}D^{-1}$

$$A = \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 4 & 0 \\ -1 & 2 \end{vmatrix}$$

$$= 4(2) - (-1)(0)$$

$$= 8 \neq 0$$

$$A^{-1} = \frac{1}{|A|}\text{AdjA}$$

$$= \frac{1}{8}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$D = \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}$$

$$|D| = 3(2) - (-2)(1)$$

$$= 6 + 2 = 8$$

$$D^{-1} = \frac{1}{|D|}\text{AdjD}$$

$$= \frac{1}{8}\begin{bmatrix} 2 & -1 \\ 2 & 3 \end{bmatrix}$$

$$A^{-1}D^{-1} = \frac{1}{8}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix} \cdot \frac{1}{8}\begin{bmatrix} 2 & -1 \\ 2 & 3 \end{bmatrix}$$

$$A^{-1}D^{-1} = \frac{1}{64}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 2 & 3 \end{bmatrix}$$

$$= \frac{1}{64}\begin{bmatrix} 2(2)+0(2) & 2(-1)+0(3) \\ 1(2)+4(2) & 1(-1)+4(3) \end{bmatrix}$$

$$= \frac{1}{64}\begin{bmatrix} 4+0 & -2+0 \\ 2+8 & -1+12 \end{bmatrix}$$

$$= \frac{1}{64}\begin{bmatrix} 4 & -2 \\ 10 & 11 \end{bmatrix} = \begin{bmatrix} \frac{4}{64} & \frac{-2}{64} \\ \frac{10}{64} & \frac{11}{64} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{16} & \frac{-1}{32} \\ \frac{5}{32} & \frac{11}{64} \end{bmatrix}$$

L.H.S = R.H.S

Hence: $(DA)^{-1} = A^{-1}D^{-1}$