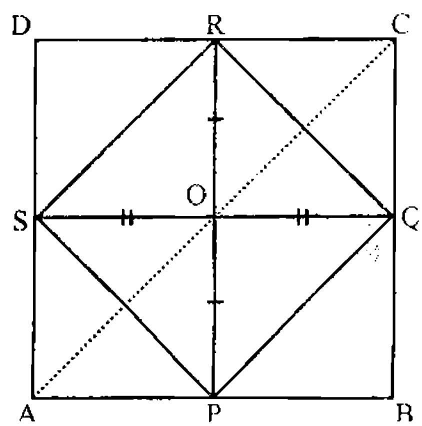
# EXERCISE 11.3

**(1) Prove that the line-segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.**




**Given**

ABCD is a quadrilateral.

P, Q, R, S are the mid-points of $\overline{AB}, \overline{BC}, \overline{CD}, \overline{DA}$ respectively.

P is joined to R, Q is joined to S. $\overline{SQ}, \overline{PR}$ intersect at point "O"

**To Prove**

$\overline{OP} \cong \overline{OR}, \overline{OS} \cong \overline{OQ}$

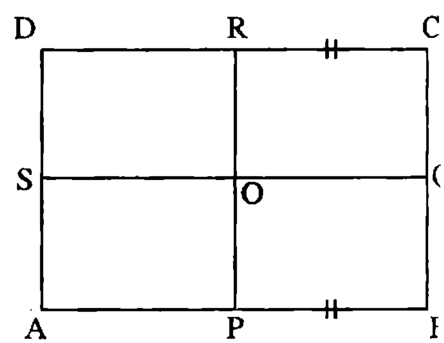
**Construction** Join P, Q, R, S in order, join A to C.

**Proof**

| Statements | Reasons |
|---|---|
| $\overline{SR} \parallel \overline{AC}$ (i) | In $\Delta ADC$. S, R are mid-points Of $\overline{AD}, \overline{DC}$ |
| $m\overline{SR} = \frac{1}{2} m\overline{AC}$ (ii) | |

| | |
|---|---|
| And $\overline{PQ} \parallel \overline{AC}$ (iii) | In $\Delta ABC$; P, Q are mid-points of $\overline{AB}, \overline{BC}$ |
| $m\overline{PQ} = \frac{1}{2} m\overline{AC}$ (iv) | |
| $\therefore$ $\overline{PQ} \parallel \overline{SR}$ (v) | from (i), and (iii) |
| $m\overline{PQ} = m\overline{SR}$ (vi) | From (ii) and (iv) |
| Similarly $\overline{PS} \parallel \overline{QR}$ | |
| $m\overline{PS} = m\overline{QR}$ | |
| Hence PQRS is a parallelogram | |
| Now $\overline{PR}$, $\overline{SQ}$ are the diagonals Of PQRS that intersect at point O. | |
| $\therefore$ $\overline{OP} \cong \overline{OR}$ | |
| $\therefore$ $\overline{OS} \cong \overline{OQ}$ | Diagonals of a parallelogram Bisect each other. |

**(2) Prove that the line-segments joining the mid-points of the opposite sides of a rectangle are the right-bisectors of each other.**




**Given**

ABCD is a rectangle.

and P, Q, R, S are the mid-points of sides $\overline{AB}, \overline{BC}$, $\overline{CD}$ and $\overline{DA}$, respectively.

P is joined to R, S to Q These intersect at "O"

**To Prove**

$\overline{OQ} \cong \overline{OS}, \overline{OR} \cong \overline{OP}$ and $\overline{RP} \perp \overline{SQ}$

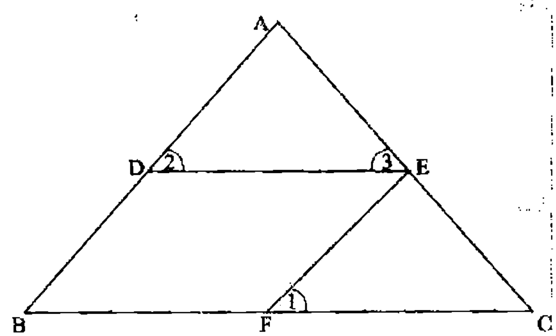
**Proof**

| Statements | Reasons |
|---|---|
| AB ∥ CD | opposite sides of rectangle |
| $\overline{AP} = \overline{DR}$ (i) | |
| $m\overline{AB} = m\overline{CD}$ | |
| $\frac{1}{2} m\overline{AB} = \frac{1}{2} m\overline{CD}$ | |
| $m\overline{AP} = m\overline{DR}$ (ii) | |
| $\therefore$ APRD is rectangle | |

| | |
|---|---|
| $\therefore$ $\overline{OR} \cong \overline{OP}$<br>Similarly $\overline{OQ} \cong \overline{OS}$<br>Now In rectangle APRD<br>$m\overline{DA} = m\overline{RP}$<br>$\frac{1}{2} m\overline{DA} = m\overline{RP}$<br>$m\overline{DS} = m\overline{RO}$<br>$\therefore$ $\overline{DS} \parallel \overline{RO}$,<br>Hence SORD is rectangle.<br>$\therefore$ $m\angle SOR = 90^\circ$, $\overrightarrow{RP} \perp \overline{SQ}$. | As $m\angle A = m\angle D = 90^\circ$ |

[Note: Diagonals of a rectangle are congruent.]

**(9) Prove that the line-segment passing through the mid-point of one side and parallel to another side of a triangle also bisects the third side.**




**Given** In $\Delta ABC$, D is mid-point of $\overline{AB}$, $\overline{DE} \parallel \overline{BC}$ which meets $\overline{AC}$ at E.

**Required** E is mid-point of $\overline{AC}$ and $\overline{EA} \cong \overline{EC}$

**Construction**

Take $\overline{EF} \parallel \overline{AB}$ which meets $\overline{BC}$ at F.

**Proof**

| Statements | Reasons |
|---|---|
| Now BDEF is parallelogram | $\overline{DE} \parallel \overline{BF}$ given, $\overline{EF} \parallel \overline{DB}$ const. |
| $\therefore$ $\overline{EF} \cong \overline{DB}$ (i) | Opposite sides of parallelogram |
| $\overline{EF} \cong \overline{AD}$ (ii) | Given |
| $\angle 1 \cong \angle B$ | Corresponding angles. |
| $\angle 2 \cong \angle B$ (iii) | Corresponding angles. |
| $\therefore$ $\angle 1 \cong \angle 2$ (iv) | Form (iii) |
| Now In $\Delta ADE \leftrightarrow \Delta EFC$ | |
| $\angle 1 \cong \angle 2$ | Form (iv) |
| $\angle 3 \cong \angle C$ | Corresponding angles. |
| $\overline{AD} \cong \overline{EF}$ | Form (ii) |
| Hence $\Delta ADE \cong \Delta EFC$ | A.A.S $\cong$ A.A.S |

| | |
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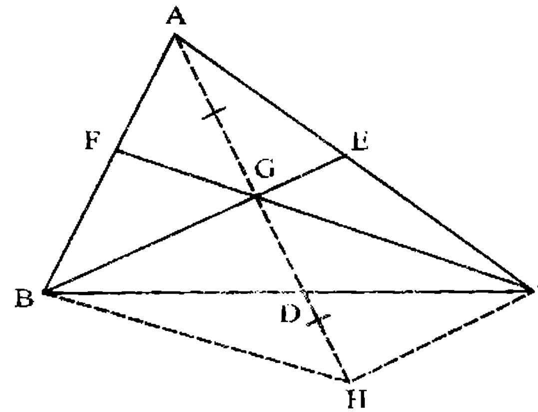
| $\therefore \quad \overline{AE} \cong \overline{CE}$ | Corresponding sides of congruent triangles. |

**Theorem**

The medians of a triangle are concurrent and their point of concurrency is the point of trisection of each median.




**Given**

$\Delta ABC$

**To Prove**

The medians of the $\Delta ABC$ are concurrent and the point of concurrency is the point of trisection of each median.

**Construction**

Draw two medians $\overline{BE}$ and $\overline{CF}$ of the $\Delta ABC$ which intersect each other at point G. Join A to G and produce it to point H such that $\overline{AG} \cong \overline{GH}$. Join H to the points B and C.

$\overline{AH}$ Intersects $\overline{BC}$ at the point D.

**Proof**

| Statements | Reasons |
|---|---|
| In $\Delta ACH$, | |
| $\overline{GE} \parallel \overline{HC}$, | G and E are mid-points of sides $\overline{AH}$ and $\overline{AC}$ respectively |
| or $\overline{BE} \parallel \overline{HC}$ .....(i) | G is a point of $\overline{BE}$ |
| Similarly $\overline{CF} \parallel \overline{HB}$ .....(ii) | |
| $\therefore$ BHCG is a parallelogram | from (i) and (ii) |
| and $m\overline{GD} = \frac{1}{2} m\overline{GH}$ .....(iii) | (Diagonals $\overline{BC}$ and $\overline{GH}$ of a parallelogram BHCG intersect each other at point D). |
| $\overline{BD} \cong \overline{CD}$ | |
| $\overline{AD}$ is a median of $\Delta ABC$ | |
| Medians $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ pass through the point G | (G is the intersecting point of $\overline{BE}$ and $\overline{CF}$ and $\overline{AD}$ pass through it.) |
| Now $\overline{GH} \cong \overline{AG}$ .....(iv) | Construction |

| | |
|---|---|
| $\therefore \quad m\overline{GD} = \frac{1}{2} m\overline{AG}$<br>and G is the point of trisection of $\overline{AD}$ –(v)<br>similarly it can be proved that G is also the point of trisection of $\overline{CF}$ and $\overline{BE}$. | from (iii) and (iv) |