

Exercise 7.7

Question # 1

If simple space = $\{1, 2, 3, \dots, 9\}$, Event $A = \{2, 4, 6, 8\}$ and Event $B = \{1, 3, 5\}$, find $P(A \cup B)$.

Solution

Sample space = $\{1, 2, 3, \dots, 9\}$ then $n(S) = 9$

Since event $A = \{2, 4, 6, 8\}$ then $n(A) = 4$

Also event $B = \{1, 3, 5\}$ then $n(B) = 3$

$$\text{Now } P(A \cup B) = P(A) + P(B) = \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} = \frac{4}{9} + \frac{3}{9} = \frac{7}{9}$$

Question # 2

A box contains 10 red, 30 white and 20 black marbles. A marble is drawn at random. Find the probability that it is either red or white.

Solution

Red marble = 10, White marble = 30, Black marble = 20

Total marble = $10 + 30 + 20 = 60$

Therefore $n(S) = 60$

Let A be the event that the marble is red then $n(A) = 10$

And let B be the event that the marble is white then $n(B) = 30$

Since A and B are mutually exclusive event therefore

$$\text{Probability} = P(A \cup B) = P(A) + P(B) = \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} = \frac{10}{60} + \frac{30}{60} = \frac{40}{60} = \frac{2}{3}$$

Question # 3

A natural number is chosen out of the first fifty numbers. What is the probability that the chosen numbers is a multiple of 3 or of 5?

Solution

Since sample space is first fifty natural number so $S = \{1, 2, 3, \dots, 50\}$

Then $n(S) = 50$

Let A be the event that the chosen number is a multiple of 3 then

$A = \{3, 6, 9, \dots, 48\}$ so $n(A) = 16$

If B be the event that the chosen number is multiple of 5 then

$B = \{5, 10, 15, \dots, 50\}$ so $n(B) = 10$

Now $A \cap B = \{15, 30, 45\}$ so $n(A \cap B) = 3$

Since A and B are not mutually exclusive event therefore

Probability = $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)}$$

$$= \frac{16}{50} + \frac{10}{50} - \frac{3}{50} = \frac{16+10-3}{50} = \frac{23}{50}$$

Question # 4

A card is drawn from a deck of 52 playing cards. What is the probability that it is a diamond card or an ace?

Solution

Total number of cards = 52,

therefore possible outcomes = $n(S) = 52$

Let A be the event that the card is a diamond card.

Since there are 13 diamond card in the deck therefore $n(A) = 13$

Now let B the event that the card is an ace card.

Since there are 4 ace cards in the deck therefore $n(B) = 4$

Since one diamond card is also an ace card therefore A and B are not mutually exclusive event and $n(A \cap B) = 1$

Now probability = $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\begin{aligned} &= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)} \\ &= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{13+4-1}{52} = \frac{16}{52} = \frac{4}{13} \end{aligned}$$

Question # 5

A die is thrown twice. What is the probability that the sum of the number of dots shown is 3 or 11.

Solution

When die is thrown twice the possible outcomes are

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

This shows possible outcomes = $n(S) = 36$

Let A be the event that the sum is 3

Then the favourable outcomes are (1, 2) and (2, 1), i.e. $n(A) = 2$

Now let B the event that the sum is 11

Then the favourable outcomes are (5, 6) and (6, 5), i.e. $n(B) = 2$

Since A and B are mutually exclusive events therefore

$$\text{Probability} = P(A \cup B) = P(A) + P(B) = \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} = \frac{2}{36} + \frac{2}{36} = \frac{4}{36} = \frac{1}{9}$$

Question # 6

Two dice are thrown. What is the probability that the sum of the number of dots appearing on them is 4 or 6.

Solution *Do yourself as above*

Question # 7

Two dice are thrown simultaneously. If event A is that sum of the number of dots shown in an odd number and event B is that the number of dots shown on at least one die is 3. Find $P(A \cup B)$.

Solution

When two dice are thrown the possible outcomes are

[See the dice table of Question # 5]

This shows possible outcomes = $n(S) = 36$

Since A be the event that the sum of dots is an odd number

Then favourable outcomes are

(1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (3, 6),
(4, 1), (4, 3), (4, 5), (5, 2), (5, 4), (5, 6), (6, 1), (6, 3), (6, 5)

i.e. favourable outcomes = $n(A) = 18$

Since B is the event that the least one die has 3 dot on it therefore

favourable outcomes are (1, 3), (2, 3), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 3), (5, 3), (6, 3) i.e. favourable outcomes = $n(B) = 11$

Since A and B have common outcome (2, 3), (3, 2), (3, 4), (3, 6), (4, 3), (6, 3)

i.e. $n(A \cap B) = 6$

Now probability = $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\begin{aligned} &= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)} \\ &= \frac{18}{36} + \frac{11}{36} - \frac{6}{36} = \frac{18+11-6}{36} = \frac{23}{36} \end{aligned}$$

Question # 8

There are 10 girls and 20 boys in a class. Half of the boys and half of the girls have blue eyes. Find the probability that one student chosen as monitor is either a girl or has blue eyes.

Solution

Number of girls = 10, Number of boys = 20

Total number of students = $10 + 20 = 30$

Since half of the girls and half of the boys have blue eyes

Therefore students having blue eyes = $5 + 10 = 15$

Let A be event that monitor of the class is a student of blue eyes then

$n(A) = 15$

Now Let B be the event that the monitor of the class is girl then $n(B) = 10$

Since 5 girls have blue eyes therefore A and B are not mutually exclusive

Therefore $n(A \cap B) = 5$

Now probability = $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\begin{aligned} &= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)} \\ &= \frac{15}{30} + \frac{10}{30} - \frac{5}{30} = \frac{15+10-5}{30} = \frac{20}{30} = \frac{2}{3} \quad \text{Answer} \end{aligned}$$