Solution:

$$-(\ell+m)-\ell x^2+(2\ell+m)x=0, \ell=0$$

$$-\ell x^2+(2\ell+m)x-(\ell+m)=0$$

$$-\left[\ell x^2-(2\ell+m)x+(\ell+m)\right]=0$$

$$\Rightarrow \ell x^2-(2\ell+m)x+(\ell+m)=0$$

Compare it with, we have

$$ax^2+bx+c=0$$

Here $a=\ell$, $b=-(2\ell+m)$, $c=(\ell+m)$

Now $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-\left[-(2\ell+m)\pm\sqrt{\left[-(2\ell+m)\right]^2-4(\ell)(\ell+m)}\right]}{2\ell}$$

$$x=\frac{(2\ell+m)\pm\sqrt{(2\ell+m)^2-4\ell(\ell+m)}}{2\ell}$$

$$x=\frac{(2\ell+m)\pm\sqrt{4\ell^2+4\ell m+m^2-4\ell^2-4\ell m}}{2\ell}$$

$$x=\frac{(2\ell+m)\pm\sqrt{m^2}}{2\ell}$$

$$x=\frac{(2\ell+m)\pm m}{2\ell}$$

$$x=\frac{2\ell+2m}{.2\ell}, \qquad x=\frac{2\ell+2m-m}{2\ell}$$

$$x=\frac{2\ell+2m}{2l} \qquad =\frac{2l}{2l}$$

$$=\frac{2(l+m)}{2\ell} \qquad =\ell$$

$$=\frac{\ell+m}{\ell}$$

Thus, solution set = $\left\{\ell, \dfrac{\ell+m}{\ell}\right\}$

SOLVED EXERCISE 1.3

Q1. Solve the following equations.

(1) $2x^4-11x^2-5=0$

Solution:

$2x^4 - 11x^2 - 5 = 0$ ______ (i)

Let $x^2 = y$. then $x^4 = y^2$

So eq. (i) becomes

$2y^2 - 11y + 5 = 0$

$2y^2 - 10y - y + 5 = 0$

$2y(y - 5) - 1(y - 5) = 0$

$(2y - 1)(y - 5) = 0$

Either $2y - 1 = 0$ or $y - 5 = 0$

$2y = 1$ $y = 5$

Put $y = \frac{1}{2}$ in $x^2 = y$, we get

$x^2 = \frac{1}{2}$

$\sqrt{x^2} = \pm\sqrt{\frac{1}{2}}$

$x = \pm\frac{1}{\sqrt{2}}$

Put $y = 5$ in $x^2 = y$, we get

$x^2 = 5$

$\sqrt{x^2} = \pm\sqrt{5}$

$x = \pm\sqrt{5}$

Thus, solution set $= \left\{\pm\frac{1}{\sqrt{2}}, \pm\sqrt{5}\right\}$

(2) $2x^4 = 9x^2 - 4$

Solution:

$2x^4 = 9x^2 - 4$

$2x^4 - 9x^2 + 4 = 0$ ______ (i)

Let $x^2 = y$. then $x^4 = y^2$

So eq. (i) becomes

$2y^2 - 9y + 4 = 0$

$2y^2 - 8y - y + 4 = 0$

$2y(y - 4) - 1(y - 4) = 0$

$(2y - 1)(y - 4) = 0$

Either $2y - 1 = 0$ or $y - 4 = 0$

$2y = 1$ $y = 4$

$y = \frac{1}{2}$

Put $y = \frac{1}{2}$ in $x^2 = y$, we get

Put $y = 4$ in $x^2 = y$, we get

$$x^2 = y \qquad\qquad x^2 = 4$$

$$\sqrt{x^2} = \pm\sqrt{\frac{1}{2}} \qquad\qquad \sqrt{x^2} = \pm\sqrt{4}$$

$$x = \pm\frac{1}{\sqrt{2}} \qquad\qquad x = \pm 2$$

Thus, solution set $= \left\{\pm\frac{1}{\sqrt{2}}, \pm 2\right\}$

(3) $5x^{1/2} = 7x^{1/4} - 2$

Solution:

$$5x^{1/2} = 7x^{1/4} - 2$$

$$5x^{\frac{1}{2}} - 7x^{\frac{1}{4}} + 2 = 0 \text{\_\_\_\_\_\_}(i)$$

Let $x^{\frac{1}{4}} = y$. then $x^{\frac{1}{2}} = y^2$

So eq. (1) becomes

$$5y^2 - 7y + 2 = 0$$
$$5y^2 - 5y - 2y + 2 = 0$$
$$5y(y-1) - 2(y-1) = 0$$
$$(5y-2)(y-1) = 0$$

Either $5y - 2 = 0$ or $y - 1 = 0$

$$5y = 2 \qquad\qquad y = 1$$

$$y = \frac{2}{5}$$

Put $y = \frac{2}{5}$ in $x^{\frac{1}{4}} = y$, we get

$$x^{\frac{1}{4}} = y$$

$$x^{\frac{1}{4}} = \frac{2}{5}$$

Taking power '4' on both sides, we get

$$\left(x^{\frac{1}{4}}\right)^4 = \left(\frac{2}{3}\right)^4$$

$$x = \frac{2^4}{5^4}$$

$$x = \frac{16}{625}$$

Put $y = 1$ in $x^{\frac{1}{2}} = y$, we get

$$x^{\frac{1}{4}} = y$$

$$x^{\frac{1}{4}} = 1$$

Taking power '4' on both sides, we get

$$\left(x^{\frac{1}{4}}\right)^4 = (1)^4$$

$$\left(x^{\frac{1}{4}}\right)^4 = 1$$

$$x = 1$$

Thus, solution set = $\left\{\frac{16}{625}, 1\right\}$

(1) $x^{\frac{2}{3}} + 54 = 15x^{\frac{1}{3}}$

Solution:

$$x^{\frac{2}{3}} + 54 = 15x^{\frac{1}{3}}$$

$$x^{\frac{2}{3}} - 15x^{\frac{1}{3}} + 54 = 0 \_\_\_\_\_\_ (i)$$

Let $x^{\frac{1}{2}} = y$. Then $x^{\frac{2}{3}} = y^2$

So eq (i) becomes

$$y^2 - 15y + 54 = 0$$

$$y^2 - 9y - 6y + 54 = 0$$

$$y(y-9) - 6(y-9) = 0$$

$$(y-6)(y-9) = 0$$

Either $y - 9 = 0$ or $y - 6 = 0$

$y = 9$ $\quad$ $y = 6$

Put $y = 9$ in $x^{\frac{1}{3}} = y$, we get

$$x^{\frac{1}{3}} = y$$

$$x^{\frac{1}{3}} = 9$$

Taking cube on both

We get

$$\left(x^{\frac{1}{3}}\right)^3 = (9)^3$$

$$x = 729$$

Put $y = 6$ in $x^{\frac{1}{3}} = y$, we get

$$x^{\frac{1}{3}} = y$$

$$x^{\frac{1}{3}} = 6$$

Taking cube on both

We get

$$\left(x^{\frac{1}{3}}\right)^3 = (6)^3$$

$$x = 216$$

Thus, solution set = $\{729, 216\}$

(5) $3x^{-2} + 5 = 8x^{-1}$

Solution:

$$3x^{-2} + 5 = 8x^{-1}$$

$$3x^{-2} - 8x^{-1} + 5 = 0 \_\_\_\_\_\_ (i)$$

Let $x^{-1} = y$ = Then $x^{-2} = y^2$

So eq. (i) becomes

$$3y^2 - 8y + 5 = 0$$

$$3y^2 - 5y - 3y + 5 = 0$$

$$y(3y-5) - 1(3y-5) = 0$$

$$(y-1)(3y-5) = 0$$

Either $y - 1 = 0$ or $3y - 5 = 0$

$y = 1$ $\qquad$ $3y = 5$

$$y = \frac{5}{3}$$

Put $y = 1$ in $x^{-1} = y$, we get

$$x^{-1} = y$$
$$x^{-1} = 1$$
$$\frac{1}{x} = 1$$
$$\text{or } x = 1$$

Put $y = \frac{5}{3}$ in $x^{-1} = y$, we get

$$x^{-1} = y$$
$$x^{-1} = \frac{5}{3}$$
$$\frac{1}{x} = \frac{5}{3}$$
$$\text{or } x = \frac{5}{3}$$

Thus, solution set = $\left\{1, \frac{3}{5}\right\}$

6. $\qquad (2x^2 + 1) + \dfrac{3}{2x^2 + 1} = 4$

Solution:

$$(2x^2 + 1) + \frac{3}{2x^2 + 1} = 4$$

$$\left(2x^2 + 1\right) + \frac{3}{2x^2 + 1} = 4 \quad \_\_\_\_(i)$$

Let $2x^2 + 1 = y$

So eq. (i) becomes

$$y + \frac{3}{y} = 4$$

Multiplying both sides by 'y', we get

$$y^2 + 3 = 4y$$
$$y^2 - 4y + 3 = 0$$
$$y^2 - 3y - y + 3 = 0$$
$$y(y - 3) - 1(y - 3) = 0$$
$$(y - 1)(y - 3) = 0$$

Either $\qquad y - 1 = 0 \qquad$ or $\qquad y - 3 = 0$

$\qquad\qquad y = 1 \qquad\qquad\qquad y = 3$

Put $y = 1$ in $2x^2 + 1 = y$, we get

Put $y = 3$ in $2x^2 + 1 = y$, we get

$$2x^2+1=1$$
$$2x^2+1=1-1$$
$$2x^2=0$$
$$x^2=0$$
$$\Rightarrow \quad x=0$$

$$2x^2+1=3$$
$$2x^2+1=3-1$$
$$2x^2=2$$
$$x^2=1$$
$$\Rightarrow \quad x=\pm 1$$

Thus, solution set = $\{-1, 0, 1\}$

(7) $\dfrac{x}{x-3}+4\left(\dfrac{x-3}{x}\right)=4$

Solution:

$$\frac{x}{x-3}+4\left(\frac{x-3}{x}\right)=4 \qquad \text{\_\_\_\_\_(i)}$$

Let $\quad \dfrac{x}{x-3}=y$

So eq. (i) becomes

$$y+4\left(\frac{1}{y}\right)=4$$

Multiplying both sides by 'y', we get

$$y^2+4=4y$$
$$y^2-4y+4=0$$
$$(y)^2-2(y)(2)+(2)^2=0$$
$$(y-2)^2=0$$
$$\Rightarrow y-2=0$$

Put $y=2$ in $\dfrac{x}{x-3}=y$, we get

$$\frac{x}{x-3}=y$$
$$\frac{x}{x-3}=2$$
$$2(x-3)=x$$
$$2x-6=x$$
$$2x-x=6$$
$$x=6$$

Thus, solution set = $\{6\}$

8. $\dfrac{4x+1}{4x-1}+\dfrac{4x-1}{4x-1}=2\dfrac{1}{6}$

Solution:

$$\frac{4x+1}{4x-1}+\frac{4x-1}{4x-1}=2\frac{1}{6}$$

$$\frac{4x+1}{4x-1}+\frac{4x-1}{4x-1}=\frac{13}{6} \quad \text{———(i)}$$

Let $\dfrac{4x+1}{4x-1}=y$

So eq. (i) becomes

$$y+\frac{1}{y}=\frac{13}{6}$$

Multiplying both sides by ' 6y', we get

$$6y^2+6=13y$$

$$6y^2-13y+6=0$$

$$6y^2-9y-4y+6=0$$

$$3y(2y-3)-2(2y-3)=0$$

$$(3y-2)(2y-3)=0$$

Either $3y-2=0$ or $2y-3=0$

$3y=2$ | $2y=3$

$y=\dfrac{2}{3}$ | $y=\dfrac{3}{2}$

Put $y=\dfrac{2}{3}$ in $\dfrac{4x+1}{4x-1}=y$, we get

$$\frac{4x+1}{4x-1}=y$$

$$\frac{4x+1}{4x-1}=\frac{2}{3}$$

$$3(4x+1)=2(4x-1)$$

$$12x+3=8x-2$$

$$12x-8x=-2-3$$

$$4x=-5$$

$$x=-\frac{5}{4}$$

Put $y=\dfrac{3}{2}$ in $\dfrac{4x+1}{4x-1}=y$, we get

$$\frac{4x+1}{4x-1}=y$$

$$\frac{4x+1}{4x-1}=\frac{3}{2}$$

$$3(4x-1)=2(4x+1)$$

$$12x-3=8x+2$$

$$12x-8x=-2+3$$

$$4x=5$$

$$x=\frac{5}{4}$$

Thus, solution set $=\left\{\pm\dfrac{5}{4}\right\}$

9. $\dfrac{x-a}{x+a}-\dfrac{x+a}{x-a}=\dfrac{7}{12}$

Solution:

$$\frac{x-a}{x+a}-\frac{x+a}{x-a}=\frac{7}{12} \qquad \text{\_\_\_\_\_\_\_\_ (i)}$$

Let $\quad \frac{x-a}{x+a}=y$

So eq (i) becomes

$$y-\frac{1}{y}=\frac{7}{12}$$

Multiplying both sides by 12y, we get

$$12y^2-12=7y$$

$$12y^2-7y-12=0$$

$$12y^2-16y+9y-12=0$$

$$4y(3y-4)+3(3y-4)=0$$

Either $4y+3=0$ or $3y-4=0$

$4y=-3 \qquad 3y=4$

$y=-\frac{3}{4} \qquad y=\frac{4}{3}$

Put $= y-\frac{3}{4}$ in $\frac{x-a}{x+a}=y$, we get

$$\frac{x-a}{x+a}=y$$

$$\frac{x-a}{x+a}=-\frac{3}{4}$$

$$4(x-a)=-3(x+a)$$

$$4x-4a=-3x-3a$$

$$4x+3x=4a-3a$$

$$7x=a$$

$$x=\frac{a}{7}$$

Put $= y-\frac{4}{3}$ in $\frac{x-a}{x+a}=y$, we get

$$\frac{x-a}{x+a}=y$$

$$\frac{x-a}{x+a}=\frac{4}{3}$$

$$4(x+a)=3(x-a)$$

$$4x+4a=3x-3a$$

$$4x-3x=-4a-3a$$

$$x=-7a$$

Thus, solution set = $\left\{-7a, \frac{a}{7}\right\}$

(10)* $x^4-2x^3-2x^2+2x+1=0$

Solution:

$$x^4-2x^3-2x^2+2x+1=0$$

Dividing each term by x^2, we get

$$\frac{x^4}{x^2}-2\frac{x^3}{x^2}-2\frac{x^2}{x^2}+\frac{2x}{x^2}+\frac{1}{x^2}=0$$

$$x^2-2x-2+\frac{2}{x}+\frac{1}{x^2}=0$$

$$x^2+\frac{1}{x^2}-2x+\frac{2}{x}-2=0$$

$$\left(x^2+\frac{1}{x^2}\right)-2\left(x-\frac{2}{x}\right)-2=0 \_\_\_\_\_ \text{(i)}$$

Let $x-\frac{1}{x}=y$

$$\left(x-\frac{1}{x}\right)^2=y^2$$

$$x^2+\frac{1}{x^2}-2=y^2$$

$$x^2+\frac{1}{x^2}=y^2+2$$

So eq. (i) becomes

$$y^2+2-2y-2=0$$

$$y^2-2y=0$$

$$y(y-2)=0$$

Either $y=0$ or $y-2=0, \Rightarrow y=2$

Put $y=0$ in $x-\frac{1}{x}=y$, we get

$$x-\frac{1}{x}=y$$

$$x-\frac{1}{x}=0$$

$$\Rightarrow x^2-1=0$$

$$x^2=1$$

$$x=\pm 1$$

Put $y=2$ in $x-\frac{1}{x}=y$, we get

$$x-\frac{1}{x}=y$$

$$x-\frac{1}{x}=2$$

$$\Rightarrow x^2-1=2x$$

$$x^2-2x-1=0$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4+4}}{2}$$

$$x = \frac{2 \pm \sqrt{8}}{2}$$

$$x = \frac{2 \pm 2\sqrt{2}}{2}$$

$$x = \frac{2 \pm 2\sqrt{2}}{2}$$

$$x = 1 \pm \sqrt{2}$$

Thus, solution set = $\{\pm 1, 1 \pm \sqrt{2}\}$

(11) $2x^4 + x^3 - 6x^2 + x + 2 = 0$

Solution:

$$2x^4 + x^3 - 6x^2 + x + 2 = 0$$

Dividing both sides by x^2, we get

$$\frac{2x^4}{x^2} + \frac{x^3}{x^2} - \frac{6x^2}{x^2} + \frac{x}{x^2} + \frac{2}{x^2} = 0$$

$$2x^2 + x - 6 + \frac{1}{x} + \frac{2}{x^2} = 0$$

$$2x^2 + \frac{2}{x^2} + x + \frac{1}{x} - 6 = 0$$

$$2\left(x^2 + \frac{2}{x^2}\right) + \left(x + \frac{1}{x}\right) - 6 = 0 \_\_\_\_\_\_ (i)$$

Let $x + \frac{1}{x} = y$

$$\left(x + \frac{1}{x}\right)^2 = y^2$$

$$x^2 + \frac{1}{x^2} + 2 = y^2$$

$$x^2 + \frac{1}{x^2} = y^2 - 2$$

So eq. (i) becomes

$$2\left(y^2-2\right)+y-6=0$$
$$2y^2-4+y-6=0$$
$$2y^2+y-10=0$$
$$2y^2+5y-4y-10=0$$
$$y(2y+5)-2(2y+5)=0$$
$$(y-2)(2y+5)=0$$

Either $y-2=0$ or $2y+5=0$

$y=2$ $\quad$ $2y=-5$

$$y=-\frac{5}{2}$$

Put $y=2$ in $x+\frac{1}{x}=y$, we get

$$x+\frac{1}{x}=y$$
$$x+\frac{1}{x}=2$$
$$\Rightarrow \quad x^1+1=2x$$
$$x^2-2x+1=0$$
$$(x-2)^2=0$$
$$\Rightarrow \quad x-1=0$$
$$x=1$$

Put $y=-\frac{5}{2}$ in $x+\frac{1}{x}=y$, we get

$$x+\frac{1}{x}=y$$
$$x+\frac{1}{x}=-\frac{5}{2}$$
$$\Rightarrow \quad x^2+1=2=-5x$$
$$2x^2+5x+2=0$$
$$2x^2+4x+x+2=0$$
$$2x(x+2)+1(x+2)=0$$

Either $2x+1=0$ $x+2=0$

$2x=-1$ $\quad$ $x=-2$

$$x=-\frac{1}{2}$$

Thus, solution set = $\left\{1,-2,-\frac{1}{2}\right\}$

(12) $4.2^{2x+1}-9.2^x+1=0$

Solution:

$$4.2^{2x+1}-9.2^x+1=0$$
$$4.2^{2x}.2^1-9.2^x+1=0 \quad \_\_\_\_\_(i)$$

Let $\quad 2^x=y$ Then $2^{2x}=y^2$

So eq. (i) becomes.

$$4.y^2.2 - 9.y + 1 = 0$$
$$8y^2 - 9y + 1 = 0$$
$$8y^2 - 8y - y + 1 = 0$$
$$8y(y-1) - 1(y-1) = 0$$
$$(8y-1)(y-1) = 0$$

Either $8y - 1 = 0$ or $y - 1 = 0$

$8y = 1$ $\quad$ $y = 1$

$$y = \frac{1}{8}$$

Put $y = \frac{1}{8}$ in $2^x = y$; we get

$$2^x = y$$
$$2^x = \frac{1}{8}$$
$$2^x = \frac{1}{2^3}$$
$$2^x = 2^3$$
$$\Rightarrow x = -3$$

Put $y = 1$ in $2^x = y$, we get

$$2^x = y$$
$$2^x = 1$$
$$2^x = 2 = 0$$
$$\Rightarrow \quad x = 0$$

Thus, solution set $= \{-3, 0\}$

13. $\quad 3^{2x+2} = 12.3^x - 3$

Solution:

$$3^{2x+2} = 12.3^x - 3$$
$$3^{2x}.3^2 - 12.3^x + 3 = 0 \_\_\_\_\_\_ (i)$$

Let $\quad 3^x = y$. Then $3^{2x} = y^2$

So eq. (i) becomes

$$y^2.9 - 12y + 3 = 0$$
$$9y^2 - 12y + 3 = 0$$
$$9y^2 - 9y - 3y + 3 = 0$$
$$9y(y-1) - 3(y-1) = 0$$
$$(9y-3)(y-1) = 0$$

Either $9y - 3 = 0$ or $y - 1 = 0$

$9y = 3$ $\quad$ $y = 1$

$$y = \frac{3}{9}$$
$$y = \frac{1}{3}$$

Put $y = \frac{1}{3}$ in $3^x = y$, we get

$$3^x = y$$
$$3^x = \frac{1}{3}$$
$$3^x = 3^{-1}$$
$$\Rightarrow x = -1$$

Put $y = 1$ in $3^x = y$, we get

$$3^x = y$$
$$3^x = 1$$
$$3^x = 3^0$$
$$\Rightarrow x = 0$$

Thus, solution set = $\{-1, 0\}$

(14) $2^x + 64.2^{-x} - 20 = 0$

Solution:

$$2^x + 64.2^{-x} - 20 = 0 \_\_\_\_\_\_ (i)$$

Let $2^x = y$. Then $2^{-x} = \frac{1}{y}$

So eq (i) becomes

$$y - 64.\frac{1}{y} = 20 = 0$$
$$\Rightarrow \quad y^2 - 64 - 20y = 0$$
$$y^2 - 20y - 64 = 0$$
$$y^2 - 16y - 4y - 64 = 0$$
$$y(y - 16) - 4(y - 16) = 0$$
$$(y - 4)(y - 16) = 0$$

Either $y - 4 = 0$ or $y - 16 = 0$

$y = 4$ $\quad$ $y = 16$

Put $y = 4$ in $2^x = y$, we get

$$2^x = y$$
$$2^x = 4$$
$$2^x = 2^2$$
$$\Rightarrow \quad x = 2$$

Put $y = 16$ in $2^x = y$, we get

$$2^x = y$$
$$2^x = 16$$
$$2^x = 2^4$$
$$\Rightarrow \quad x = 4$$

Thus, solution set = $\{2, 4\}$

(15) $(x + 1)(x + 3)(x - 5)(x - 7) = 192$

Solution:

$$(x + 1)(x + 3)(x - 5)(x - 7) = 192$$

As $1 - 5 = 3 - 7$

So $$[(x+1)(x-5)][(x+3)(x-7)] = 192$$
$$[x^2 - 5x + x - 5][x^2 - 7x + 3x - 21] = 192$$
$$(x^2 - 4x - 5)(x^2 - 4x - 21) = 192 \_\_\_\_\_ (i)$$

$$(y-5)(y-21)=192$$
$$y^2-21y-5y+105=192$$
$$y^2-26y+105-192=0$$
$$y^2-26y-87=0$$
$$y^2-29y+3y-87=0$$
$$(y+3)(y-29)=0$$

Either $y+3=0$ or $y-29=0$

$y=-3$ $\quad$ $y=29$

Put $y=-3$ in $x^2-4x=y$, we get

$$x^2-4x=y$$
$$x^2-4x=-3$$
$$x^2-4x+3=0$$
$$x^2-3x-x+3=0$$
$$x(x-3)-1(x-3)=0$$
$$(x-1)(x-3)=0$$

Either $x-1=0$ or $x-3=0$

$x=1$ $\quad$ $x=3$

Put $y=29$ in $x^2-4x=y$, we get

$$x^2-4x=y$$
$$x^2-4x=29$$
$$x^2-4x-29=0$$

Here $a=1, b=-4, c=-29$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-29)}}{2(1)}$$
$$x=\frac{4\pm\sqrt{16+116}}{2}$$
$$x=\frac{4\pm\sqrt{132}}{2}$$
$$x=\frac{4\pm2\sqrt{33}}{2}$$
$$x=\frac{2\left(2\pm\sqrt{33}\right)}{2}$$
$$x=2\pm\sqrt{33}$$

Thus, solution set $=\left\{1,3,2\pm\sqrt{33}\right\}$

(16) $(x-1)(x-2)(x-8)(x+5)360=0$

Solution:

$$(x-1)(x-2)(x-8)(x+5)360=0$$

As $\quad -1-2=-8+5$

$$-3=-3$$

So $\quad \left[(x-1)(x-2)\right]\left[(x-8)(x+5)\right]+360=0$

$$\left[x^2-2x-x+2\right]\left[x^2+5x-8x-40\right]+360=0$$

$$\left(x^2-3x+2\right)\left(x^2-3x-40\right)+360=0 \_\_\_\_\_ (i)$$

Let $\quad x^2-3x=y$

So eq (i) become

$$(y+2)(y-40)+360=0$$

$$y^2-40y+2y-80+360=0$$

$$y^2-38y+280=0$$

$$y^2-28y-10y+280=0$$

$$y(y-28)-10(y-28)=0$$

$$(y-10)(y-28)=0$$

$$(y-10)(y-28)=0$$

Either $\quad y-10=0 \quad$ or $\quad y-28=0$

$y=10 \qquad y=28$

Put $y=10$ in $x^2-3x=y$, we get

$$x^2-3x=y$$

$$x^2-3x=10$$

$$x^2-3x-10=0$$

$$x^2-5x+2x-10=0$$

$$x(x-5)+2(x-5)=0$$

$$(x+2)(x-5)=0$$

Either $\quad x+2=0 \quad$ or $\quad x-5=0$

$x=-2 \qquad x=5$

Put $y=10$ in $x^2-3x=y$, we get

$$x^2-3x=y$$

$$x^2-3x=28$$

$$x^2-3x-28=0$$

$$x^2-7x+4x-28=0$$

$$x(x-7)+4(x-7)=0$$

$$(x+4)(x-7)=0$$

Either $\quad x+4=0 \quad$ or $\quad x-7=0$

$x=-4 \qquad x=7$

Thus, solution set = $\{-4, -2, 5, 7\}$

Radical equations:

An equation involving expression under the radical sign is called a radical equation.

e.g., $\sqrt{x+3}=x+1$ and $\sqrt{x-1}=\sqrt{x-2}+1$