EX 6·7

(1) (i) $y^2 = 4ax$ at $(at^2, 2at)$

$y^2 = 4ax$ —— (1)

Differentiating both sides of (1) w.r.t x

$\frac{d}{dx}(y^2) = \frac{d}{dx}(4ax)$

$\Rightarrow 2y\frac{dy}{dx} = 4a \Rightarrow \frac{dy}{dx} = \frac{2a}{y}$

Slope of Tangent to (1) at $(at^2, 2at)$

$= m = \frac{dy}{dx}\Big]_{(at^2, 2at)} = \frac{2a}{2at} = \frac{1}{t}$

Slope of Normal to (1) at $(at^2, 2at)$

$= m' = -\frac{1}{m} = -t$.

Now Equation of Tangent at $(at^2, 2at)$

$y - 2at = \frac{1}{t}(x - at^2)$.

$\Rightarrow yt - 2at^2 = x - at^2$

$\Rightarrow \boxed{yt = x + at^2}$

and Equation of Normal at $(at^2, 2at)$

$y - 2at = -t(x - at^2)$

$y - 2at = -tx + at^3$

$\Rightarrow \boxed{y = -tx + 2at + at^3}$

(ii) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at $(a\cos\theta, b\sin\theta)$

$\Rightarrow b^2x^2 + a^2y^2 = a^2b^2$ —— (1)

Differentiating (1) w.r.t x we get

$\frac{d}{dx}(b^2x^2 + a^2y^2) = \frac{d}{dx}(a^2b^2)$

$\Rightarrow 2b^2x + 2a^2y\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{b^2x}{a^2y}$

Slope of Tangent at $(a\cos\theta, b\sin\theta) = m$

$= -\frac{b^2 \cdot a\cos\theta}{a^2 \cdot b\sin\theta} = -\frac{b\cos\theta}{a\sin\theta}$

and Slope of Normal to (1) at $(a\cos\theta, b\sin\theta)$

$= m' = -\frac{1}{m} = \frac{a\sin\theta}{b\cos\theta}$

Now Equation of Tangent to (1) at $(a\cos\theta, b\sin\theta)$ becomes

$y - b\sin\theta = -\frac{b\cos\theta}{a\sin\theta}(x - a\cos\theta)$

$\Rightarrow ya\sin\theta - ab\sin^2\theta = -xb\cos\theta + ab\cos^2\theta$

$\Rightarrow xb\cos\theta + ya\sin\theta = ab\cos^2\theta + ab\sin^2\theta$

$= ab(\cos^2\theta + \sin^2\theta)$

$\Rightarrow xb\cos\theta + ya\sin\theta = ab$

$\Rightarrow \frac{xb\cos\theta}{ab} + \frac{ya\sin\theta}{ab} = \frac{ab}{ab}$

$\Rightarrow \frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$.

Equation of Normal to (1) at $(a\cos\theta, b\sin\theta)$ becomes

$y - b\sin\theta = \frac{a\sin\theta}{b\cos\theta}(x - a\cos\theta)$

$\Rightarrow \frac{y - b\sin\theta}{a\sin\theta} = \frac{x - a\cos\theta}{b\cos\theta}$

$\Rightarrow \frac{y}{a\sin\theta} - \frac{b}{a} = \frac{x}{b\cos\theta} - \frac{a}{b}$

$\Rightarrow \frac{y}{a\sin\theta} - \frac{x}{b\cos\theta} = -\frac{a}{b} + \frac{b}{a}$

$\frac{y}{a}\operatorname{cosec}\theta - \frac{x}{b}\sec\theta = \frac{-a^2+b^2}{ab}$

$\Rightarrow \frac{x}{b}\sec\theta - \frac{y}{a}\operatorname{cosec}\theta = \frac{a^2-b^2}{ab}$

$\Rightarrow ax\sec\theta - by\operatorname{cosec}\theta = a^2 - b^2$.

(iii) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at $(a\sec\theta, b\tan\theta)$

$\Rightarrow b^2x^2 - a^2y^2 = a^2b^2$ —— (1)

Differentiating both sides of (1) w.r.t x.

$2b^2x - 2a^2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{b^2x}{a^2y}$

Slope of Tangent to (1) at $(a\sec\theta, b\tan\theta)$

$= m = \frac{dy}{dx}\Big]_{(a\sec\theta, b\tan\theta)} = \frac{b^2 \cdot a\sec\theta}{a^2 \cdot b\tan\theta}$

$\Rightarrow m = \frac{b\sec\theta}{a\tan\theta}$

Slope of the Normal to (1) at $(a\sec\theta, b\tan\theta)$

$= m' = -\frac{1}{m} = -\frac{a\tan\theta}{b\sec\theta}$

$\therefore$ Equation of Tangent to (1) at $(a\sec\theta, b\tan\theta)$

$$y - b\tan\theta = \frac{b\sec\theta}{a\tan\theta}(x - a\sec\theta)$$

$$\Rightarrow ay\tan\theta - ab\tan^2\theta = bx\sec\theta - ab\sec^2\theta$$

$$\Rightarrow bx\sec\theta - ay\tan\theta = ab\sec^2\theta - ab\tan^2\theta$$

$$= ab\{\sec^2\theta - \tan^2\theta\}$$

$$= ab\{1 + \tan^2\theta - \tan^2\theta\}$$

$$bx\sec\theta - ay\tan\theta = ab$$

$$\frac{bx\sec\theta}{ab} - \frac{ay\tan\theta}{ab} = \frac{ab}{ab}$$

$$\frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1$$

Equation of Normal to (1) at $(a\sec\theta, b\tan\theta)$ becomes

$$y - b\tan\theta = -\frac{a\tan\theta}{b\sec\theta}(x - a\sec\theta)$$

$$\Rightarrow \frac{y - b\tan\theta}{a\tan\theta} = -\left(\frac{x - a\sec\theta}{b\sec\theta}\right)$$

$$\Rightarrow \frac{y}{a\tan\theta} - \frac{b\tan\theta}{a\tan\theta} = -\frac{x}{b\sec\theta} + \frac{a\sec\theta}{b\sec\theta}$$

$$\Rightarrow \frac{x}{b\sec\theta} + \frac{y}{a\tan\theta} = \frac{a\sec\theta}{b\sec\theta} + \frac{b\tan\theta}{a\tan\theta}$$

$$\Rightarrow \frac{x}{b}\cos\theta + \frac{y}{a}\cot\theta = \frac{a}{b} + \frac{b}{a}$$

$$\Rightarrow \frac{x}{b}\cos\theta + \frac{y}{a}\cot\theta = \frac{a^2+b^2}{ab}$$

$$\Rightarrow ax\cos\theta + by\cot\theta = a^2 + b^2$$

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2 (i) $3x^2 = -16y$ — (1)

when $y = -3$

From (1) $3x^2 = -16(-3) \Rightarrow 3x^2 = 48$

$\Rightarrow x^2 = 16 \Rightarrow x = \pm 4$.

Thus we have to find Equations of Tangents at $(4,-3)$ & $(-4,-3)$

Differentiating both sides of (1) w.r.t x

$$\frac{d}{dx}(3x^2) = \frac{d}{dx}(-16y)$$

$$\Rightarrow 6x = -16\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = -\frac{3x}{8}$$

At the point $(4,-3)$

$$\left.\frac{dy}{dx}\right]_{(4,-3)} = -\frac{3(4)}{8} = -\frac{3}{2}$$

Now Equation of Tangent to (1) at $(4,-3)$ with Slope $-\frac{3}{2}$ becomes $y-(-3) = -\frac{3}{2}(x-4)$

$$\Rightarrow 2(y+3) = -3x+12$$

$$\Rightarrow -3x+12 = 2y+6$$

$$\Rightarrow -3x-2y+6 = 0$$

$$\Rightarrow 3x+2y-6 = 0$$

At the point $(-4,-3)$

$$\left.\frac{dy}{dx}\right]_{(-4,-3)} = -\frac{3(-4)}{8} = \frac{3}{2}$$

Now Equation of the Tangent to (1) at $(-4,-3)$ with Slope $\frac{3}{2}$ becomes

$$y+3 = \frac{3}{2}(x+4)$$

$$\Rightarrow 2y+6 = 3x+12$$

$$\Rightarrow 3x-2y+6 = 0$$

(ii) $3x^2 - 7y^2 = 20$ — (1)

put $y = -1$ in (1) we get

$3x^2 - 7 = 20 \Rightarrow 3x^2 = 27 \Rightarrow x^2 = 9$

$\Rightarrow x = \pm 3$

Thus we have to find Equations of Tangents at the points $(3,-1)$ and $(-3,-1)$

Now Differentiating both sides of (1) w.r.t 'x' we have

$$\frac{d}{dx}(3x^2 - 7y^2) = \frac{d}{dx}(20)$$

$$3\frac{d}{dx}(x^2) - 7\frac{d}{dx}(y^2) = 0$$

$$3(2x) - 7(2)y\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{6x}{14y} \Rightarrow \frac{dy}{dx} = \frac{3x}{7y}$$

At the point $(3,-1)$

Slope of the Tangent to (1) at $(3,-1)$

$$= \left.\frac{dy}{dx}\right]_{(3,-1)} = \frac{3(3)}{7(-1)} = -\frac{9}{7}$$

Thus Equation of the Tangent at $(3,-1)$ becomes

$$y+1 = -\frac{9}{7}(x-3)$$

$$\Rightarrow 7y+7 = -9x+27 \Rightarrow \boxed{9x+7y-20=0}$$

At the point $(-3,-1)$

Slope of the tangent to ① at $(-3,-1)$

$= \frac{dy}{dx}\Big]_{(-3,-1)} = \frac{3(-3)}{7(-1)} = \frac{9}{7}$

Thus Equation of Tangent at $(-3,-1)$ becomes $y+1 = \frac{9}{7}(x+3)$

$\Rightarrow 7y+7 = 9x+27$

$\Rightarrow 9x-7y+20=0$

(iii) $3x^2-7y^2+2x-y-48=0$ — ①

putting $x=4$ in ① we have

$48-7y^2+8-y-48=0$

$\Rightarrow 7y^2+y-8=0$

$\Rightarrow 7y^2-7y+8y-8=0$

$\Rightarrow 7y(y-1)+8(y-1)=0$

$\Rightarrow (y-1)(7y+8)=0$

$\Rightarrow y-1=0$ or $7y+8=0$

$y=1$ | $7y=-8$

$y=-\frac{8}{7}$

Thus we have to find Eqs. of Tangents at $(4,1)$ & $(4,-\frac{8}{7})$

Now differentiating both sides of ① w.r.t 'x' we have

$\frac{d}{dx}(3x^2-7y^2+2x-y-48) = \frac{d}{dx}(0)$

$\Rightarrow 6x-14y\frac{dy}{dx}+2-\frac{dy}{dx}=0$

$(14y+1)\frac{dy}{dx} = 6x+2$

$\Rightarrow \frac{dy}{dx} = \frac{6x+2}{14y+1}$

Slope of Tangent to ① at $(4,1)$

$= \frac{dy}{dx}\Big]_{(4,1)} = \frac{24+2}{14+1} = \frac{26}{15}$

Thus Equation of Tangent at $(4,1)$ with Slope $\frac{26}{15}$ becomes

$y-1 = \frac{26}{15}(x-4)$

$\Rightarrow 15(y-1) = 26(x-4)$

$\Rightarrow 15y-15 = 26x-104$

$\Rightarrow 26x-15y-89=0$

Slope of Tangent at $(4,-\frac{8}{7})$

$= \left[\frac{dy}{dx}\right]_{(4,-8/7)} = \frac{6(4)+2}{14(-\frac{8}{7})+1}$

$= \frac{24+2}{-16+1} = -\frac{26}{15}$

Thus equation of Tangent to ① at $(4,-\frac{8}{7})$ becomes

$y+\frac{8}{7} = -\frac{26}{15}(x-4)$

$\Rightarrow 15(y+\frac{8}{7}) = -26(x-4)$

$15y+\frac{120}{7} = -26x+104$

$26x+15y = 104-\frac{120}{7}$

$= \frac{728-120}{7} = \frac{608}{7}$

$26x+15y-\frac{608}{7}=0$

$\Rightarrow 13x+\frac{15}{y}y-\frac{304}{7}=0$

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③ (i) $x^2+y^2=25$. $(7,-1)$

$\Rightarrow x^2+y^2=5^2$ — ①

Here $a=5$

Equations of Tangents to ① from any point are of the form

$y = mx \pm a\sqrt{1+m^2}$ $\forall m \in \mathbb{R}$ — ②

put $a=5$ in ② we have

$y = mx \pm 5\sqrt{1+m^2}$ — ③

As ③ passes through $(7,-1)$

$\therefore -1 = 7m \pm 5\sqrt{1+m^2}$ — (A)

$-1-7m = \pm 5\sqrt{1+m^2}$ — ④

Squaring both sides of ④ we get

$1+49m^2+14m = 25(1+m^2)$

$49m^2+14m+1 = 25+25m^2$

$49m^2+14m+1-25-25m^2=0$

$24m^2+14m-24=0$

$\Rightarrow 12m^2+7m-12=0$

$\Rightarrow 12m^2+16m-9m-12=0$.

$\Rightarrow 4m(3m+4)-3(3m+4)=0$

$\Rightarrow (3m+4)(4m-3)=0$

$\Rightarrow 3m+4=0$ or $4m-3=0$

$m=-\frac{4}{3}$ | $m=\frac{3}{4}$

Now $m=-\frac{4}{3}$ Satisfies the equation

$-1 = 7m + 5\sqrt{1+m^2}$

Thus the equation of tangent is

$Y = mx + 5\sqrt{1+m^2}$.

i.e $Y = -\frac{4}{3}x + 5\sqrt{1+(-\frac{4}{3})^2}$

$= -\frac{4x}{3} + \frac{5(5)}{3}$

$\Rightarrow 3Y = -4x + 25$

$\Rightarrow \boxed{4x + 3Y - 25 = 0}$

and $m = \frac{3}{4}$ satisfies the Eq.

$-1 = 7m - 5\sqrt{1+m^2}$

Thus the equation of tangent is

$Y = mx - 5\sqrt{1+m^2}$

i.e $Y = \frac{3}{4}x - 5\sqrt{1+\frac{9}{16}}$

$Y = \frac{3}{4}x - \frac{25}{4} \Rightarrow 4y = 3x - 25$

$\Rightarrow \boxed{3x - 4y - 25 = 0}$

(ii) $y^2 = 12x$ —(1) through $(1, 4)$

Here $4a = 12 \Rightarrow \boxed{a = 3}$

Equations of tangents to (1) are of the form

$Y = mx + \frac{a}{m}$,$m \in R$

If (2) passes through $(1, 4)$

Then $4 = m + \frac{3}{m} \Rightarrow m^2 + 3 = 4m$

$\Rightarrow m^2 - 4m + 3 = 0$

$\Rightarrow m^2 - m - 3m + 3 = 0$

$m(m-1) - 3(m-1) = 0$

$(m-1)(m-3) = 0$

$\Rightarrow m - 1 = 0$ or $m - 3 = 0$

$m = 1$ $m = 3$

For $m = 1$.

Equation of tangent (2) becomes

$Y = x + 3 \Rightarrow \boxed{x - y + 3 = 0}$

For $m = 3$

Equation of tangent (2) becomes

$Y = 3x + \frac{3}{3} \Rightarrow \boxed{3x - y + 1 = 0}$

(iii) $x^2 - 2y^2 = 2$ $(1, -2)$

$\frac{x^2}{2} - \frac{2 \cdot y^2}{2} = \frac{2}{2}$

$\frac{x^2}{2} - \frac{y^2}{1} = 1$ —(1)

Here $a^2 = 2$, $b^2 = 1$

Equations of tangents to (1) are of the form $Y = mx \pm \sqrt{a^2m^2 - b^2}$

i.e $Y = mx \pm \sqrt{2m^2 - 1}$ —(2)

If (2) passes through $(1, -2)$

Then $-2 = m \pm \sqrt{2m^2 - 1}$

$\Rightarrow -2 - m = \pm\sqrt{2m^2 - 1}$ —(3)

Squaring both sides of (3) we have

$4 + m^2 + 4m = 2m^2 - 1$

$\Rightarrow 2m^2 - m^2 - 4m - 1 - 4 = 0$

$\Rightarrow m^2 - 4m - 5 = 0$

$\Rightarrow (m+1)(m-5) = 0$

$\Rightarrow m + 1 = 0$ or $m - 5 = 0$

$m = -1$ $m = 5$

Now $\boxed{m = -1}$ satisfies the equation

$-2 - m = -\sqrt{2m^2 - 1}$

$\Rightarrow$ Equation of tangent is

$Y = mx - \sqrt{2m^2 - 1}$

i.e $Y = -x - \sqrt{2-1} \Rightarrow Y = -x - 1$

$\Rightarrow x + y + 1 = 0$

and $m = 5$ satisfies the equation

$-2 - m = -\sqrt{2m^2 - 1}$

Thus for $m = 5$ Equation of tangent is

$Y = 5x - \sqrt{2(5)^2 - 1}$

$\Rightarrow Y = 5x - 7 \Rightarrow \boxed{5x - y - 7 = 0}$

4) $y^2 = 8x$ —(1)

$2x + 3y - 10 = 0$ —(2)

Differentiating both sides of (1) w.r.t 'x' we have

$\frac{d}{dx}(y^2) = \frac{d}{dx}(8x)$

$\Rightarrow 2y\frac{dy}{dx} = 8 \Rightarrow \frac{dy}{dx} = \frac{4}{y}$.

Thus Slope of Tangent to ① $= \frac{4}{y}$

and Slope of Normal to ① $= -\frac{y}{4}$

Slope of line ② $= -\frac{2}{3}$.

Since Normal to ① is parallel to line ②

$\therefore \; -\frac{y}{4} = -\frac{2}{3} \Rightarrow \boxed{y = \frac{8}{3}}$

putting $y = \frac{8}{3}$ in ① we get

$(\frac{8}{3})^2 = 8x \Rightarrow 8x = \frac{64}{9}$

$\Rightarrow \boxed{x = \frac{8}{9}}$

Now slope of Normal to ① at $(\frac{8}{9}, \frac{8}{3}) = -\frac{8/3}{4} = -\frac{8}{3} \times \frac{1}{4}$

$= -\frac{2}{3}$

Now Required equation of Normal is $y - \frac{8}{3} = -\frac{2}{3}(x - \frac{8}{9})$

$y - \frac{8}{3} = -\frac{2}{3}x + \frac{16}{27}$

$\Rightarrow 27y - 72 = -18x + 16$

$\Rightarrow 18x + 27y - 88 = 0$

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⑤ $\frac{x^2}{4} + \frac{y^2}{1} = 1$ —— ①

$2x - 4y + 5 = 0$ —— ②

From ① $a^2 = 4, \quad b^2 = 1$

and Slope of line ② $= \frac{2}{4} = \frac{1}{2}$.

As The tangents are parallel ②

$\therefore$ Slope of Tangent to ① ∥ to ② $= \frac{1}{2}$

Now Required equations of Tangents are $y = mx \pm \sqrt{a^2m^2 + b^2}$

$\Rightarrow y = \frac{1}{2}x \pm \sqrt{4(\frac{1}{4}) + 1}$

$y = \frac{1}{2}x \pm \sqrt{2} \Rightarrow 2y = x \pm 2\sqrt{2}$

$\Rightarrow x - 2y \pm 2\sqrt{2} = 0$

i.e $x - 2y + 2\sqrt{2} = 0$ and

$x - 2y - 2\sqrt{2} = 0$.

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⑥ $9x^2 - 4y^2 = 36$

$\Rightarrow \frac{9x^2}{36} - \frac{4y^2}{36} = \frac{36}{36}$

$\Rightarrow \frac{x^2}{4} - \frac{y^2}{9} = 1$ —— ①

$5x - 2y + 7 = 0$ —— ②

From ① $a^2 = 4, \quad b^2 = 9$

Slope of the line ② $= \frac{5}{2}$.

Now Slope of Tangent parallel to ② $= \frac{5}{2}$.

Thus required equations of Tangents to ① and parallel to ② are

$y = \frac{5}{2}x \pm \sqrt{4(\frac{25}{4}) - 9}$

$y = \frac{5}{2}x \pm 4 \Rightarrow 2y = 5x \pm 8$

$\Rightarrow 5x - 2y \pm 8 = 0$

$\Rightarrow 5x - 2y + 8 = 0, \quad 5x - 2y - 8 = 0$

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⑦ (i) $x^2 = 80y$ —— ①

$x^2 + y^2 = 81$ —— ②

Let $y = mx + c$ —— ③

be common Tangent to ① and ②

using ③ in ① we have

$x^2 = 80(mx + c)$

$\Rightarrow x^2 - 80mx - 80c = 0$ —— ④

If ③ is Tangent to ① Then ④ has Equal roots.

$\Rightarrow$ Discriminant of ④ $= 0$

$\Rightarrow (-80m)^2 - 4(1)(-80c) = 0$

$\Rightarrow 6400m^2 + 320c = 0$

$\Rightarrow 320c = -6400m^2$

$\Rightarrow c = -\frac{6400m^2}{320}$

$\Rightarrow c = -20m^2$ —— ⑤

If ③ is Tangent to ② Then

$c^2 = 81(1 + m^2)$ —— ⑥

using ⑤ in ⑥ we have

$(-20m^2)^2 = 81(1 + m^2)$

$\Rightarrow 400m^4 - 81m^2 - 81 = 0$ —— ⑦

$\Rightarrow 400m^4 - 225m^2 + 144m^2 - 81 = 0$

$\Rightarrow 25m^2(16m^2-9) + 9(16m^2-9) = 0$

$\Rightarrow (16m^2-9)(25m^2+9) = 0$

$\Rightarrow 16m^2 - 9 = 0$ or $25m^2 + 9 = 0$

$\Rightarrow 16m^2 = 9$ | $25m^2 = -9$

$m^2 = \frac{9}{16}$ | $m^2 = -\frac{9}{25}$

$\Rightarrow m = \pm\frac{3}{4}$ | Neglecting Negative value of m^2 ($\because m^2$ is not Negative)

using $m = \pm\frac{3}{4}$ in (5) we have

$c = -20\left(\frac{9}{16}\right) = -\frac{45}{4}$

Thus the Required Equations of Common Tangents become

$y = \pm\frac{3}{4}x - \frac{45}{4}$

$\Rightarrow 4y = \pm 3x - 45$

$\Rightarrow \pm 3x - 4y - 45 = 0$ Ans.

7(ii) $y^2 = 16x$ —— (1) $4a = 16 \Rightarrow a = 4$

$x^2 = 2y$ —— (2)

Let $y = mx + c$ —— (3)

be the equation of common Tangent to (1) and (2).

Now if (3) is Tangent to (1)

Then $c = \frac{a}{m} \Rightarrow c = \frac{4}{m}$ —— (4)

using (4) in (3) we have

$y = mx + \frac{4}{m}$ —— (5)

using (5) in (2) we have

$x^2 = 2\left(mx + \frac{4}{m}\right)$

$x^2 = 2mx + \frac{8}{m}$

$\Rightarrow x^2 - 2mx - \frac{8}{m} = 0$ —— (6)

Now (3) is Tangent if roots of (6) are equal. Then

Discriminant of (6) $= 0$

$\Rightarrow (-2m)^2 - 4(1)\left(-\frac{8}{m}\right) = 0$

$\Rightarrow 4m^2 + \frac{32}{m} = 0$

$\Rightarrow m^2 + \frac{8}{m} = 0$

$\Rightarrow m^3 + 8 = 0 \Rightarrow m^3 + 2^3 = 0$

$\Rightarrow (m+2)(m^2 - 2m + 4) = 0$

$\Rightarrow m + 2 = 0$ or $m^2 - 2m + 4$

$\Rightarrow \boxed{m = -2}$ | $m = \frac{2 \pm \sqrt{4-16}}{2}$ (Neglecting complex roots)

putting $m = -2$ in (4) we have

$c = \frac{4}{-2} \Rightarrow \boxed{c = -2}$

Thus required equation of common tangent:

$y = -2x - 2$

$\Rightarrow \boxed{2x + y + 2 = 0}$ Ans.

OK

7(ii) $y^2 = 16x$ —(1) $x^2 = 2y$ —(2)

Let $y = mx + c$ —(3) be the common Tangent to (1) and (2)

If (3) is Tangent to (1) Then

$c = \frac{a}{m} \Rightarrow c = \frac{4}{m}$ —(4)

using (3) in (2) we get

$x^2 = 2(mx + c)$

$\Rightarrow x^2 - 2mx - 2c = 0$ —(5)

If (3) is tangent to (2) Then roots of (5) are equal

$\Rightarrow$ Discriminant of (5) $= 0$

$\Rightarrow (-2m)^2 - 4(1)(-2c) = 0$

$4m^2 + 8c = 0$ —(6)

using (4) in (6) we get

$4m^2 + 8\left(\frac{4}{m}\right) = 0$

$\Rightarrow m^2 + \frac{8}{m} = 0 \Rightarrow m^3 + 8 = 0$

$\Rightarrow (m+2)(m^2 - 2m + 4) = 0$

$\Rightarrow m + 2 = 0$ or $m^2 - 2m + 4 = 0$

$\boxed{m = -2}$ $\because m^2 - 2m + 4 = 0$ gives Imaginary values

putting $m = -2$ in (4) we have

$c = \frac{4}{-2} \Rightarrow \boxed{c = -2}$

Thus required equation of common tangent becomes

$y = -2x - 2$

$\Rightarrow 2x + y + 2 = 0$ Ans.

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8 (i) $\frac{x^2}{18} + \frac{y^2}{8} = 1$ —— ①

$\frac{x^2}{3} - \frac{y^2}{3} = 1$ —— ②

By multiplying eq ② by $\frac{1}{6}$ and then subtracting it from ① we have

$\frac{x^2}{18} + \frac{y^2}{8} = 1$

$\underline{\pm\frac{x^2}{18} \mp \frac{y^2}{18} = \frac{1}{6}}$

$\frac{y^2}{8} + \frac{y^2}{18} = 1 - \frac{1}{6}$

$\Rightarrow \frac{9y^2 + 4y^2}{72} = \frac{6-1}{6}$

$\Rightarrow \frac{13y^2}{72} = \frac{5}{6} \Rightarrow \frac{13y^2}{12} = 5$

$\Rightarrow 13y^2 = 60 \Rightarrow y^2 = \frac{60}{13}$

$\Rightarrow \boxed{y = \pm\sqrt{\frac{60}{13}}}$

putting $y^2 = \frac{60}{13}$ in ② we get

$\frac{x^2}{3} - \frac{\frac{60}{13}}{3} = 1 \Rightarrow \frac{x^2}{3} = 1 + \frac{60}{13} \times \frac{1}{3}$

$\Rightarrow \frac{x^2}{3} = 1 + \frac{20}{13} = \frac{13+20}{13} = \frac{33}{13}$

$\Rightarrow x^2 = \frac{99}{13} \Rightarrow \boxed{x = \pm\sqrt{\frac{99}{13}}}$

Thus required points of intersection of given conics

$\left(\pm\sqrt{\frac{99}{13}}, \pm\sqrt{\frac{60}{13}}\right)$ Ans.

(ii) $x^2 + y^2 = 8$ —— ①

$x^2 - y^2 = 1$ —— ②

By Adding

$2x^2 = 9$

$\Rightarrow x^2 = \frac{9}{2} \Rightarrow x = \pm\sqrt{\frac{9}{2}}$

putting $x^2 = \frac{9}{2}$ in ① we get

$\frac{9}{2} + y^2 = 8$

$y^2 = 8 - \frac{9}{2} \Rightarrow y^2 = \frac{7}{2}$

$\Rightarrow y = \pm\sqrt{\frac{7}{2}}$

Thus required points are

$\left(\pm\sqrt{\frac{9}{2}}, \pm\sqrt{\frac{7}{2}}\right)$.

(iii) $3x^2 - 4y^2 = 12$ —— ①

$-2x^2 + 3y^2 = 7$ —— ②

Multiplying Equation ① by 2 and Equation ② by 3 we have

$6x^2 - 8y^2 = 24$ —— ③

$-6x^2 + 9y^2 = 21$ —— ④

By Adding

$y^2 = 45$

$\Rightarrow y = \pm\sqrt{45}$

putting $y^2 = 45$ in ① we have

$3x^2 - 4(45) = 12$

$\Rightarrow x^2 - 60 = 4 \Rightarrow x^2 = 64$

$\Rightarrow x = \pm 8$

Thus required points of intersection of given conics are

$(\pm 8, \pm\sqrt{45})$ Ans.

(iv) $3x^2 + 5y^2 = 60$ —— ①

$9x^2 + y^2 = 124$ —— ②

By multiplying eq ① by 3 and then subtracting ② from it we have

$9x^2 + 15y^2 = 180$

$\underline{-9x^2 \pm y^2 = -124}$

$14y^2 = 56 \Rightarrow y^2 = 4$

$\Rightarrow y = \pm 2$

putting $y^2 = 4$ in ① we get

$3x^2 + 20 = 60 \Rightarrow 3x^2 = 40$

$\Rightarrow x^2 = \frac{40}{3} \Rightarrow x = \pm\sqrt{\frac{40}{3}}$

Thus the points of intersection of the given conics are

$\left(\pm\sqrt{\frac{40}{3}}, \pm 2\right)$ Ans.

(v) $4x^2+y^2=16$ — ①

$x^2+y^2+y+8=0$ — ②

By multiplying equation ② by 4 and then subtracting ① from it we have

$$
\begin{array}{r}
4x^2+4y^2+4y+32=0 \\
\underline{\underset{-}{4x^2}\ \underset{-}{+}\ y^2 \qquad = \underset{-}{16}} \\
3y^2+4y+32=-16
\end{array}
$$

$$3y^2+4y+48=0.$$

$$y=\frac{-4\pm\sqrt{16-4(3)(48)}}{6}$$

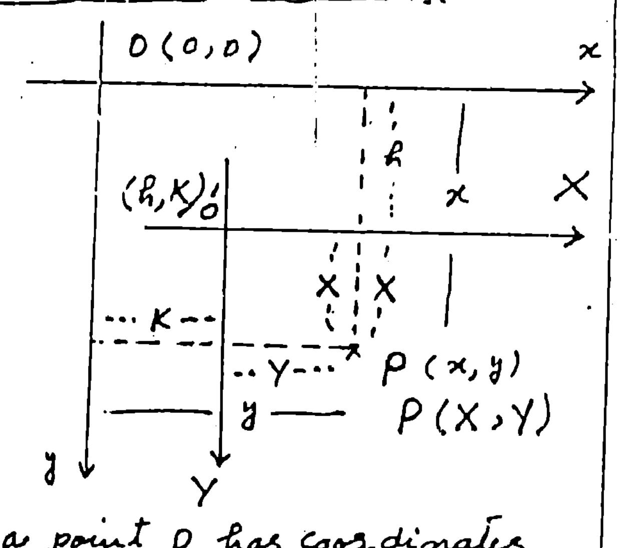
$$y=\frac{-4\pm\sqrt{16-576}}{6}=\frac{-4\pm\sqrt{-560}}{6}$$

$$y=\frac{-4\pm\sqrt{560}\,i}{6}$$

As the values of y are complex (Imaginary)

so no real points of intersection of given conics exist.

<u>Translation of Axes</u>.



If a point P has coordinates (x,y) referred to the xy-system and has coordinates (X,Y) referred to the translated axes $O'X$, $O'Y$ through $O'(h,K)$

then $x=X+h$, $y=Y+K$

Also $X=x-h$, $Y=y-K$