EXERCISE 2.7

1 Complete the Table indicating by a tick mark those properties which are satisfied by the specified set of numbers.

Property ↓ \ Set of Nos →		Natural	whole	Integers	Rational	Reals
Closure	+	✓	✓	✓	✓	✓
	x	✓	✓	✓	✓	✓
Associative	+	✓	✓	✓	✓	✓
	x	✓	✓	✓	✓	✓
Identity	+	No	✓	✓	✓	✓
	x	✓	✓	✓	✓	✓
Inverse	+	No	No	✓	✓	✓
	x	No	No	No	No	No
Commutative	+	✓	✓	✓	✓	✓
	x	✓	✓	✓	✓	✓

2. What are the field axioms? In what respect does the field of real nos differ from Complex No

Sol. FIELD: A field $F(+,\cdot)$ is a set having atleast two elements and two binary operation $+, \cdot$ defined on F s.t following axioms satisfied

(i) $(F, +)$ is an abelian group.

(ii) $(F - \{0\}, \cdot)$ is an abelian group under multiplication

(iii) $\forall a, b, c \in F$ the right distributive Law holds that is $(a+b)c = ac + bc$

As the set 'R' and 'C' are two different sets so they form two different fields

3. Show that the adjoining table is multiplication of elements of the set of residue classes of modulo 5.

×	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

Sol. In the given table the result of simple multiplication of numbers in the table unless the result is less than '5' and where the result of multiplication of the numbers is equal to '5' or exceeds '5' it is divided by '5' and the remainder obtained after division division is inserted in the table. Hence the given table is the multiplication of the elements of the set of residue classes modulo 5.

4. Prepare a table of addition of the elements of the set of residue classes modulo 4.

Sol. consider the set of residues $\{0, 1, 2, 3\}$
Add the elements as in ordinary addition unless the result is less than '4' where the sum is 4 or greater we divide it out by '4' and insert the remainder only in the table.

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

5. Which of the following binary operations shown in tables (a) or (b) is commutative?

*	a	b	c	d
a	a	c	b	d
b	b	c	b	a
c	c	d	b	c
d	d	a	b	b

*	a	b	c	d
a	a	c	b	d
b	c	d	b	a
c	b	b	a	c
d	d	a	c	d

Sol (i) As the multiplication table is not symmetric that is when rows and columns interchange the table changes. Hence binary operation is not commutative.

(ii) As the multiplication table is symmetric that is when rows and columns are interchange the table remains unchanged. Hence the binary operation is commutative.

6. Supply the missing elements of the third row of given table so that the operation $\dot{\times}$ may be associative

$\dot{\times}$	a	b	c	d
a	a	c	b	d
b	b	a	c	d
c	–	–	–	–
d	d	c	c	d

Sol. Let x, y, z, t be the required numbers

$\dot{\times}$	a	b	c	d
a	a	c	b	d
b	b	a	c	d
c	x	y	z	t
d	d	c	c	d

Then $(d \dot{\times} b) \dot{\times} a = d \dot{\times} (b \dot{\times} a)$

$c \dot{\times} a = d \dot{\times} b$

$x = c$

$(a \dot{\times} b) \dot{\times} b = a \dot{\times} (b \dot{\times} b)$

$c \dot{\times} b = a \dot{\times} a$

$y = a$

$(d \dot{\times} b) \dot{\times} c = d \dot{\times} (b \dot{\times} c)$

$c \dot{\times} c = d \dot{\times} c$

$z = c$

$(d \dot{\times} b) \dot{\times} d = d \dot{\times} (b \dot{\times} d)$

$c \dot{\times} d = d \dot{\times} d \Rightarrow t = a$

7. What operation represented by the table? Name the identity element of the set if it exists. Is the operation associative? Find the inverses of $0, 1, 2, 3$ if they exist.

$\dot{\times}$	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

Sol. In this multiplication table binary operation is addition of the elements of the set of residue classes modulo '4'. Identity element of this Set is '0' Yes this binary operation is associative

Inverse of $0 = 0$ Inverse of $1 = 3$

Inverse of $3 = 1$ Inverse of $2 = 2$