Exercise 1.4

1. Which of the following product of matrices is conformable for multiplication?

Ans. (i) $\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \end{bmatrix}$

Number of Columns = Number of Rows

$\therefore$ product is possible.

(ii) $\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

Number of columns = Number of Rows.

$\therefore$ product is possible.

(iii) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$

Number of columns $\neq$ Number of Rows.

$\therefore$ product is not possible.

(iv) $\begin{bmatrix} 1 & 2 \\ 0 & -1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$

Number of columns = Number of Rows.

$\therefore$ product is possible.

(v) $\begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ -2 & 3 \end{bmatrix}$

Number of Columns = Number of Rows.

$\therefore$ Product is possible.

2. If $A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$, find (i) AB (ii) BA (if possible).

(i) $AB = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 6 \\ 5 \end{bmatrix}$

$= \begin{bmatrix} 3(6) + 0(5) \\ -1(6) + 2(5) \end{bmatrix}$

$= \begin{bmatrix} 18+0 \\ -6+10 \end{bmatrix} = \begin{bmatrix} 18 \\ 4 \end{bmatrix}$

(ii) $BA = \begin{bmatrix} 6 \\ 5 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$

$\therefore$ Product is not possible.

Because number of columns $\neq$ number of rows.

3. Find the following products.

Ans. (i) $\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$

$=[1(4)+2(0)]$

$=[4+0]$

$=[4]$

(ii) $\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}5\\-4\end{bmatrix}$

$=[1(5)+2(-4)]$

$=[5-8]$

$=[-3]$

(iii) $\begin{bmatrix}-3 & 0\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$

$=[-3(4)+0(0)]=[-12]$

(iv) $\begin{bmatrix}6 & 0\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$

$=[6(4)+(0)(0)]=[24]$

(v) $\begin{bmatrix}1 & 2\\-3 & 0\\6 & -1\end{bmatrix}\begin{bmatrix}4 & 5\\0 & -4\end{bmatrix}$

$=\begin{bmatrix}1(4)+2(0) & 1(5)+2(-4)\\-3(4)+0(0) & -3(5)+0(-4)\\6(4)+-1(0) & 6(5)+(-1)(-4)\end{bmatrix}$

$=\begin{bmatrix}4+0 & 5-8\\-12+0 & -15+0\\24+0 & 30+4\end{bmatrix}$

$=\begin{bmatrix}4 & -3\\-12 & -15\\24 & 34\end{bmatrix}$

4. Multiply the following matric

(a) $\begin{bmatrix}2 & 3\\1 & 1\\0 & -2\end{bmatrix}\begin{bmatrix}2 & -1\\3 & 0\end{bmatrix}$

(b) $\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}\begin{bmatrix}1 & 2\\3 & 4\\-1 & 1\end{bmatrix}$

(c) $\begin{bmatrix}1 & 2\\3 & 4\\-1 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}$

(d) $\begin{bmatrix}8 & 5\\6 & 4\end{bmatrix}\begin{bmatrix}2 & -\frac{5}{4}\\4 & 4\end{bmatrix}$

(e) $\begin{bmatrix}-1 & 2\\1 & 3\end{bmatrix}\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

Ans. (a) $\begin{bmatrix}2 & 3\\1 & 1\\0 & -2\end{bmatrix}\begin{bmatrix}2 & -1\\3 & 0\end{bmatrix}$

$=\begin{bmatrix}2(2)+3(3) & 2(-1)+3(0)\\1(2)+1(3) & 1(-1)+1(0)\\0(2)+(-2)(3) & 0(-1)+(-2)(0)\end{bmatrix}$

$=\begin{bmatrix}13 & -2\\5 & -1\\-6 & 0\end{bmatrix}$

(b) $\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}\begin{bmatrix}1 & 2\\3 & 4\\-1 & 1\end{bmatrix}$

$=\begin{bmatrix}1(1)+2(3)+3(-1) & 1(2)+2(4)+3(1)\\4(1)+15(3)+6(-1) & 4(2)+5(4)+6(1)\end{bmatrix}$

$=\begin{bmatrix}1+6-3 & 2+8+3\\4+15-6 & 8+20+6\end{bmatrix}$

$$=\begin{bmatrix}4 & 13\\13 & 34\end{bmatrix}$$

(c) $$\begin{bmatrix}1 & 2\\3 & 4\\-1 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}$$

$$=\begin{bmatrix}1(1)+2(4) & 1(2)+2(5) & 1(3)+2(6)\\3(1)+4(4) & 3(2)+4(5) & 3(3)+4(6)\\-1(1)+1(4) & -1(2)+1(5) & -1(3)+1(6)\end{bmatrix}$$

$$=\begin{bmatrix}9 & 12 & 15\\19 & 26 & 33\\3 & 3 & 3\end{bmatrix}$$

(d) $$\begin{bmatrix}8 & 5\\6 & 4\end{bmatrix}\begin{bmatrix}2 & -\frac{5}{2}\\-4 & 4\end{bmatrix}$$

$$=\begin{bmatrix}8(2)+5(-4) & 8\left(\frac{-5}{2}\right)+5(4)\\6(2)+4(-4) & 6\left(\frac{-5}{2}\right)+4(4)\end{bmatrix}$$

$$=\begin{bmatrix}16-20 & -20+20\\12-16 & -15+16\end{bmatrix}=\begin{bmatrix}-4 & 0\\-4 & 1\end{bmatrix}$$

(e) $$\begin{bmatrix}-1 & 2\\1 & 3\end{bmatrix}\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

$$=\begin{bmatrix}-1(0)+2(0) & -1(0)+2(0)\\1(0)+3(0) & 1(0)+3(0)\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

5. Let $A=\begin{bmatrix}-1 & 3\\2 & 0\end{bmatrix}$, $B=\begin{bmatrix}1 & 2\\-3 & -5\end{bmatrix}$ **and** $C=\begin{bmatrix}2 & 1\\1 & 3\end{bmatrix}$. **Verify whether**

(i) AB = BA.

(ii) A(BC) = (AB)C

(iii) A(B+C)=AB+AC

(iv) A(B−C)=AB−AC

Ans. (i) AB = BA.

To check whether AB = BA Or not

$$AB=\begin{bmatrix}-1 & 3\\2 & 0\end{bmatrix}\begin{bmatrix}1 & 2\\-3 & -5\end{bmatrix}$$

$$=\begin{bmatrix}-1(1)+3(-3) & -1(2)+3(-5)\\2(1)+0(-3) & 2(2)+0(-5)\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\2-0 & 4+0\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\2 & 4\end{bmatrix}$$

$$BA=\begin{bmatrix}1 & 2\\-3 & -5\end{bmatrix}\begin{bmatrix}-1 & 3\\2 & 0\end{bmatrix}$$

$$=\begin{bmatrix}1(-1)+2(2) & 1(3)+2(0)\\-3(-1)+-5(2) & -3(3)+(-5)(0)\end{bmatrix}$$

$$=\begin{bmatrix}-1+4 & 3+0\\3-10 & -9+0\end{bmatrix}$$

$$=\begin{bmatrix}3 & 3\\-7 & -9\end{bmatrix}$$

So **AB ≠ BA**

(ii) A(BC) = (AB)C

L.H.S = A(BC)

$$BC=\begin{bmatrix}1 & 2\\-3 & -5\end{bmatrix}\begin{bmatrix}2 & 1\\1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}1(2)+2(1) & 1(1)+2(3)\\-3(2)+-5(1) & -3(1)+-5(3)\end{bmatrix}$$

$$=\begin{bmatrix}2+2 & 1+6\\-6-5 & -3-15\end{bmatrix}$$

$$=\begin{bmatrix}4 & 7\\-11 & -18\end{bmatrix}$$

$$A(BC)=\begin{bmatrix}-1 & 3\\2 & 0\end{bmatrix}\begin{bmatrix}4 & 7\\-11 & -18\end{bmatrix}$$

$$=\begin{bmatrix}-1(4)+3(-11) & -1(7)+3(-18)\\2(4)+0(-11) & 2(7)+0(-18)\end{bmatrix}$$

$$=\begin{bmatrix}-4-33 & -7-54\\8+0 & 14+0\end{bmatrix}$$

$$=\begin{bmatrix}-37 & -61\\ 8 & 14\end{bmatrix}$$

R.H.S = (AB)C

$$(AB)=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$$

$$=\begin{bmatrix}-1(1)+3(-3) & -1(2)+3(-5)\\ 2(1)+0(-3) & 2(2)+0(-5)\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\ 2+0 & 4+0\end{bmatrix}=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}$$

$$(AB)C=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-10(2)+(-17)(1) & -10(1)+(-17)(3)\\ 2(2)+4(1) & 2(1)+4(3)\end{bmatrix}$$

$$=\begin{bmatrix}-20-17 & -10-51\\ 4+4 & 2+12\end{bmatrix}$$

$$=\begin{bmatrix}-37 & -61\\ 8 & 14\end{bmatrix}$$

Hence A(BC) = (AB)C

(iii) A (B + C) = AB + AC

L.H.S = A (B + C)

$$(B+C)=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}+\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}1+2 & 2+1\\ -3+1 & -5+3\end{bmatrix}=\begin{bmatrix}3 & 3\\ -2 & -2\end{bmatrix}$$

$$A(B+C)=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\begin{bmatrix}3 & 3\\ -2 & -2\end{bmatrix}=\begin{bmatrix}-3-6 & -3-6\\ 6+0 & 6+0\end{bmatrix}$$

$$=\begin{bmatrix}-9 & -9\\ 6 & 6\end{bmatrix}$$

L.H.S.

AB + AC

$$AB=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$$

$$=\begin{bmatrix}-1(1)+3(-3) & -1(2)+3(-5)\\ 2(1)+0(-3) & 2(2)+0(-5)\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\ 2+0 & 4+0\end{bmatrix}=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}$$

$$AC=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-1(2)+3(1) & -1(1)+3(3)\\ 2(2)+0(1) & 2(1)+0(3)\end{bmatrix}=\begin{bmatrix}1 & 8\\ 4 & 2\end{bmatrix}$$

$$AB+AC=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}+\begin{bmatrix}1 & 8\\ 4 & 2\end{bmatrix}$$

$$=\begin{bmatrix}-10+1 & -17+8\\ 2+4 & 4+2\end{bmatrix}=\begin{bmatrix}-9 & -9\\ 6 & 6\end{bmatrix}$$

6. For the matrices.

$$A=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix},B=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix},\ C=\begin{bmatrix}-2 & 6\\ 3 & -9\end{bmatrix}$$

Verify that(i)$(AB)^t=B^tA^t$(ii)$(BC)^t=C^tB^t$.

Ans. (i) $(AB)^t=B^tA^t$

L.H.S = $(AB)^t$

$$AB=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$$

$$=\begin{bmatrix}-1(1)+3(-3) & -1(2)+3(-5)\\ 2(1)+0(-3) & 2(2)+0(-5)\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\ 2+0 & 4+0\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}$$

$$(AB)^t=\begin{bmatrix}-10 & 2\\ -17 & 4\end{bmatrix}$$

R.H.S = B^tA^t

$$A^t=\begin{bmatrix}-1 & 2\\ 3 & 0\end{bmatrix}$$

$$B^t=\begin{bmatrix}1 & -3\\ 2 & -5\end{bmatrix}$$

$$B^tA^t=\begin{bmatrix}1 & -3\\ 2 & -5\end{bmatrix}\begin{bmatrix}-1 & 2\\ 3 & 0\end{bmatrix}$$

$$=\begin{bmatrix}1(-1)+(-3)(3) & 1(2)+(-3)(0)\\ 2(-1)+(-5)(3) & 2(2)+5(0)\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & 2-0\\ -2-15 & 4\end{bmatrix}=\begin{bmatrix}-10 & 2\\ -17 & 4\end{bmatrix}$$

L.H.S = R.H.S

Hence $(AB)^t=B^tA^t$

(ii) $(BC)^t = C^t B^t$

L.H.S $= (BC)^t$

$$BC = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}$$

$$= \begin{bmatrix} 1(-2)+2(3) & 1(6)+2(-9) \\ -3(-2)+-5(3) & -3(6)+-5(-9) \end{bmatrix}$$

$$= \begin{bmatrix} -2+6 & 6-18 \\ 6-15 & -18+45 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -12 \\ -9 & 27 \end{bmatrix}$$

$$(BC)^t = \begin{bmatrix} 4 & -9 \\ -12 & 27 \end{bmatrix}$$

R.H.S $= C^t B^t$

$$C^t = \begin{bmatrix} -2 & 3 \\ 6 & -9 \end{bmatrix}$$

$$B^t = \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix}$$

$$C^tB^t = \begin{bmatrix} -2 & 3 \\ 6 & -9 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} -2(1)+3(2) & -2(-3)+3(-5) \\ 6(1)+2(-9) & 6(-3)+-9(-5) \end{bmatrix}$$

$$= \begin{bmatrix} -2+6 & 6-15 \\ 6-18 & -18+45 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -9 \\ -12 & 27 \end{bmatrix}$$

L.H.S = R.H.S

Hence $(BC)^t = C^t B^t$