Review Exercise 7

Q3. Answer the following short questions.

i) Define a linear inequality in one variable.

Ans. **Linear Inequality in one variable**

Let a, b be real numbers, then a is greater than b if the difference $a - b$ is positive and we denote this order relation by the inequality $a > b$. An equivalent statement is that b is less than a, symbolized by $b < a$. Similarly, if $a - b$ is negative, then a is less than b and expressed in symbols as $a < b$.

ii) State the trichotomy and transitive properties of inequality.

Ans. **Trichotomy Property of inequality**

For any $a, b \in R$, one and only one of the following statements is true.

$$a < b \text{ or } a = b, \text{ or } a > b$$

Transitive Property of inequality

Let $a, b, c \in R$

i) If $a > b$ and $b > c$, then $a > c$

ii) If $a > b$ and $b < c$, then $a < c$

iii) **The formula relating degrees Fahrenheit to degrees Celcius is $F=\frac{9}{5}C+32$. For what value of C is $F<0$.**

Ans. According to formula "F" will be zero, if $\frac{9}{5}C+32=0$

$\frac{9}{5}C=-32$

$C=-\frac{32}{9}\times 5$

$C=-\frac{160}{9}$

To get $F<0$ i.e. negative $C<-\frac{160}{9}$

iv) **Seven times the sum of an integer and 12 is at least 50 and at most 60. Write and solve the inequality that expresses this relationship.**

Ans. Let the required integer be x then

$50\le x+12\le 60$

$50\le x+12$ and $x+12\le 60$

$50-12\le x$ and $x\le 60-12$

$38\le x$ and $x\le 48$

$38\le x\le 48$

Q4. **Solve each of the following and check for extraneous solution, if any.**

i) $\sqrt{2t+4}=\sqrt{t-1}$

Squaring both sides

$\left(\sqrt{2t+4}\right)^2=\left(\sqrt{t-1}\right)^2$

$2t+4=t-1$

$2t-t=-1-4$

$t=-5$

Check:

$\sqrt{2t+4}=\sqrt{t-1}$

$\sqrt{2(-5)+4}=\sqrt{-5-1}$

$\sqrt{-10+4}=\sqrt{-6}$

$\sqrt{-6}=\sqrt{-6}$ Which is true, so solution Set = $\{-5\}$

ii) $\sqrt{3x-1}-2\sqrt{8-2x}=0$

$\sqrt{3x-1}=2\sqrt{8-2x}$

Squaring both sides

$\left(\sqrt{3x-1}\right)^2=\left(2\sqrt{8-2x}\right)^2$

$3x-1=4(8-2x)$

$3x-1=32-8x$

$3x+8x=32+1$

$11x=33$

$x=\frac{\cancel{33}}{\cancel{11}}$

$x=3$

Check:

$\sqrt{3x-1}-2\sqrt{8-2x}=0$

$\sqrt{3(3)-1}-2\sqrt{8-2(3)}=0$

$\sqrt{9-1}-2\sqrt{8-6}=0$

$\sqrt{8}-2\sqrt{2}=0$

$2\sqrt{2}-2\sqrt{2}=0$

$0=0$ Which is true, so

solution set = $\{3\}$

Q5. Solve for x

i) $|3x+14|-2=5x$

$|3x+14|=5x+2$

$\pm(3x+14)=5x+2$

$3x+14=\pm(5x+2)$

$3x+14=5x+2$ or $3x+14=-5x-2$

$3x-5x=2-14$ or $3x+5x=-2-14$

$-2x=-12$ or $8x=-16$

$x=\frac{12}{2}$ or $x=-\frac{16}{8}$

$x=6$ or $x=-2$

Check:

Put $x=6$ in

$$|3x+14|-2=5x$$
$$|3(6)+14|-2=5(6)$$
$$|18+14|-2=30$$
$$|32|-2=30$$
$$30-2=30$$
$30=30$, which is true

Now put $x=-2$

$$|3(-2)+14|-2\neq 5(-2)$$
$$|-6+14|-2\neq -10$$
$$|8|-2\neq -10$$
$$8-2\neq -10$$
$6\neq -10$ which is not true

So, Solution Set = $\{6\}$

ii) $\frac{1}{3}|x-3|=\frac{1}{2}|x+2|$

$$\frac{|x-3|}{|x+2|}=\frac{3}{2}$$
$$\left|\frac{x-3}{x+2}\right|=\frac{3}{2}$$
$$\pm\left(\frac{x-3}{x+2}\right)=\frac{3}{2}$$

or $\frac{x-3}{x+2}=\pm\frac{3}{2}$

$\frac{x-3}{x+2}=\frac{3}{2}$ or $\frac{x-3}{x+2}=-\frac{3}{2}$

$2(x-3)=3(x+2)$ or $2(x-3)=-3(x+2)$

$2x-6=3x+6$ or $2x+3x=6-6$

$-x=12$ or $5x=0$

$x=-12$ or $x=0$

Check:

Put $x=-12$

$$\frac{1}{3}|x-3|=\frac{1}{2}|x+2|$$
$$\frac{1}{3}|-12-3|=\frac{1}{2}|-12+2|$$
$$\frac{1}{3}|-15|=\frac{1}{2}|-10|$$
$$\frac{15}{3}=\frac{10}{2}$$
$5=5$, which is true

Now put $x=0$

$$\frac{1}{3}|0-3|=\frac{1}{2}|0+2|$$
$$\frac{1}{3}|-3|=\frac{1}{2}|2|$$
$$\frac{3}{3}=\frac{2}{2}$$
$1=1$, which is true

So, Solution Set = $\{-12,0\}$

Q6. Solve the following inequality.

i) $-\frac{1}{3}x+5\leq 1$

$$-\frac{1}{3}x\leq 1-5$$
$$-\frac{1}{3}x\leq -4$$

Multiplying both sides by -3

$$x\geq 12$$

Solution Set = $\{x/x\geq 12\}$

ii) $-3<\frac{1-2x}{5}<1$

$-3<\frac{1-2x}{5}$ and $\frac{1-2x}{5}<1$

Multiplying both sides by 5

$-15<1-2x$ and $1-2x<5$

$-15-1<-2x$ and $-2x<5-1$

$-16<-2x$ and $-2x<4$

Multiplying both sides by – 1

$16>2x$ and $2x>-4$

$\frac{16}{2}>x$ and $x>\frac{-4}{2}$

$8>x$ and $x>-2$

$$8>x>-2$$

Solution Set = $\{x/8>x>-2\}$

Objective

1. *Which of the following is the solution of the inequality* $3-4x \le 11$?
   (a) $x \ge -8$
   (b) $x \ge -2$
   (c) $x \ge \frac{-14}{4}$
   (d) None of these
2. A statement involving any of the symbols <, > or ≤ or ≥ is called:
   (a) Equation (b) Identity
   (c) Inequality (d) Linear equation
3. $x =$ ________ is a solution of the inequality $-2 < x < \frac{3}{2}$
   (a) -5 (b) 3
   (c) 0 (d) $\frac{5}{2}$
4. If x is not larger than 10, then
   (a) $x \ge 8$ (b) $x \le 10$
   (c) $x < 10$ (d) $x > 10$
5. If the capacity c of an elevator is at most 1600 pounds, then __
   (a) $c < 1600$ (b) $c \ge 1600$
   (c) $c \le 1600$ (d) $c > 1600$
6. $x = 0$ is a solution of the inequality
   (a) $x > 0$ (b) $3x + 5 < 0$
   (c) $x + 2 < 0$ (d) $x - 2 < 0$
7. The linear equation in one variable x is:
   (a) $ax + b = 0$
   (b) $ax^2 + bx + c = 0$
   (c) $ax + by + c = 0$
   (d) $ax^2 + by^2 + c = 0$
8. *An inconsistent equation is that* whose solution set is:
   (a) Empty (b) Not empty
   (c) Zero (d) None of these
9. Absolute value of a real number a is defined as.
   (a) $|a| = \begin{cases} a & \text{if } a \ge 0 \\ -a, & \text{if } a < 0 \end{cases}$
   (b) $|a| = \begin{cases} a & \text{if } a \le 0 \\ -a & \text{if } a > 0 \end{cases}$
   (c) $|a| = \begin{cases} a & \text{if } a > 0 \\ -a & \text{if } a < 0 \end{cases}$
   (d) None of these
10. $|x| = a$ is equivalent to:
    (a) $x = a$ or $x = -a$
    (b) $x = \frac{1}{a}$ or $x = \frac{-1}{a}$
    (c) $x = a$ or $x = \frac{-1}{a}$
    (d) None of these
11. A linear inequality in one variable x is:
    (a) $ax + b > 0, \quad a \ne 0$
    (b) $ax^2 + bx + c < 0, a \ne 0$
    (c) $ax + by + c > 0, \quad a \ne 0$
    (d) $ax^2 + by^2 + c < 0, a \ne 0$
12. Law of Trichotomy is … $(a, b \in R)$
    (a) $a < b$ or $a = b$ or $a > b$
    (b) $a < b$ or $a = b$
    (c) $a < b$ or $a > b$
    (d) *None of these*

13. Transitive law is____

(a) $a < b$ and $b < c$, then $a < c$

(b) $a > b$ and $b < c$, then $a > c$

(c) $a > b$ and $b < c$, then $a > c$

(d) None of these

14. If $a > b$, $c > 0$ then:

(a) $ac < bc$ (b) $ac > bc$

(c) $ac = bc$ (d) None

15. If $a > b$, $c > 0$ then:

(a) $\frac{a}{c} > \frac{b}{c}$ (b) $\frac{a}{c} < \frac{b}{c}$

(c) $\frac{a}{c} = \frac{b}{c}$ (d) $\frac{b}{c} \neq \frac{b}{c}$

16. If $a > b$, $c < 0$, then:

(a) $\frac{a}{c} < \frac{b}{c}$ (b) $\frac{a}{c} > \frac{b}{c}$

(c) $\frac{a}{c} = \frac{b}{c}$ (d) $\frac{a}{c} \leq \frac{b}{c}$

17. If $a, b \in \mathbf{R}$ then:

(a) $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$ (b) $|ab| = \frac{|a|}{|b|}$

(c) $\left|\frac{b}{a}\right| = \frac{|b|}{|a|}$ (d) None of these

18. When the variable in an equation occurs under a radical, the equation is called a ___equation.

(a) Radical (b) Absolute value

(c) Linear (d) None of these

19. $|x|=0$ has only ___ solution.

(a) one (b) two

(c) three (d) none of these

20. The equation $|x|=2$ is equivalent to

(a) $x=2$ or $x=-2$

(b) $x = -2$ or $x = -2$

(c) $x = 2$ or $x = \frac{1}{2}$

(d) $x = 2$ or $x = \frac{-1}{2}$

21. An __ is equation that is satisfied by every number for which both sides are defined:

(a) Identity (b) Conditional

(c) Inconsistent (c) None

22. An__ equation is an equation whose solution set is the empty set:

(a) Identity (b) Conditional

(c) Inconsistent (d) None

23. A ___ equation is an equation that is satisfied by atleast one number but is not an identity:

(a) Identity (b) Conditional

(c) Inconsistent (d) None

24. $x + 4 = 4 + x$ is ___ equation:

(a) Identity (b) Conditional

(c) Inconsistent (d) None

25. $2x + 1 = 9$ is ___ equation:

(a) Identity (b) Conditional

(c) Inconsistent (d) None

26. $x = x + 5$ is ___ equation:

(a) Identity (b) Conditional

(c) Inconsistent (d) None

27. Equations having exactly the same solution are called ___ equations.

(a) equivalent (b) Linear

(c) Inconsistent (c) None

28. A solution that does not satisfy the original equation is called ____ solution:

(a) Extraneous (b) Root

(c) General (d) None

ANSWER KEY

1.	b	**2.**	c	**3.**	c	**4.**	b	**5.**	c
6.	d	**7.**	a	**8.**	a	**9.**	a	**10.**	a
11.	a	**12.**	a	**13.**	a	**14.**	b	**15.**	a
16.	a	**17.**	a	**18.**	a	**19.**	a	**20.**	a
21.	a	**22.**	c	**23.**	b	**24.**	a	**25.**	b
26.	c	**27.**	a	**28.**	a				