Exercise 9.2

Question # 1 & 2

Do yourself

Question # 3

In which quadrant are the terminal arms of the angle lie when

(i) $\sin\theta < 0$ and $\cos\theta > 0$

(ii) $\cot\theta > 0$ and $\operatorname{cosec}\theta > 0$

Solutions

(i) Since $\sin\theta < 0$ so θ lies in *IIIrd* or *IVth* quadrant.

Also $\cos\theta > 0$ so θ lies in *Ist* or *IVth* quadrant.

$\Rightarrow$ θ lies in *IVth* quadrant

(ii) Since $\cot\theta > 0$ so θ lies in *Ist* or *IIIrd* quadrant.

Also $\csc\theta > 0$ so θ lies in *Ist* or *IInd* quadrant

$\Rightarrow$ θ lies in *Ist* quadrant.

Question # 3 (iii), (iv) and ……

Do yourself as above

Question # 4

Find the values of the remaining trigonometric functions:

(i) $\sin\theta = \dfrac{12}{13}$ and the terminal arm of the angle is in quad. I.

(ii) $\cos\theta = \dfrac{9}{41}$ and the terminal arm of the angle is in quad. IV.

(iv) $\tan\theta = -\dfrac{1}{3}$ and the terminal arm of the angle is in quad. II.

Solutions

(i) Since $\sin^2\theta + \cos^2\theta = 1$

$$\Rightarrow \cos^2\theta = 1 - \sin^2\theta$$

$$\Rightarrow \cos\theta = \pm\sqrt{1-\sin^2\theta}$$

As terminal ray lies in *Ist* quadrant so $\cos\theta$ is +ive.

$$\cos\theta = \sqrt{1-\sin^2\theta}$$

$$\Rightarrow \cos\theta = \sqrt{1-\left(\frac{12}{13}\right)^2} \qquad \because\ \sin\theta = \frac{12}{13}$$

$$= \sqrt{1-\frac{144}{169}} = \sqrt{\frac{25}{169}} \qquad \Rightarrow \boxed{\cos\theta = \frac{5}{13}}$$

Now

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{12/13}{5/13} = \frac{12}{13}\cdot\frac{13}{5} \qquad \Rightarrow \boxed{\tan\theta = \frac{12}{5}}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{12/13} = \frac{13}{12} \quad \Rightarrow \quad \boxed{\csc\theta = \frac{13}{12}}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{5/13} = \frac{13}{5} \quad \Rightarrow \quad \boxed{\sec\theta = \frac{13}{5}}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{12/5} = \frac{5}{12} \quad \Rightarrow \quad \boxed{\cot\theta = \frac{5}{12}}$$

(ii) Since $\sin^2\theta + \cos^2\theta = 1$

$$\Rightarrow \sin^2\theta = 1 - \cos^2\theta$$

$$\Rightarrow \sin\theta = \pm\sqrt{1-\cos^2\theta}$$

As terminal ray lies in *IVth* quadrant so $\sin\theta$ is –ive .

$$\sin\theta = -\sqrt{1-\cos^2\theta}$$

$$\Rightarrow \sin\theta = -\sqrt{1-\left(\frac{9}{41}\right)^2}$$

$$= -\sqrt{1-\frac{81}{1681}} = -\sqrt{\frac{1600}{1681}} = -\frac{40}{41} \quad \Rightarrow \quad \boxed{\sin\theta = -\frac{40}{41}}$$

Now

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-40/41}{9/41} = -\frac{40}{41}\cdot\frac{41}{9} = -\frac{40}{9} \quad \Rightarrow \quad \boxed{\tan\theta = -\frac{40}{9}}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{-40/41} = -\frac{41}{40} \quad \Rightarrow \quad \boxed{\csc\theta = -\frac{40}{41}}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{9/11} = \frac{41}{9} \quad \Rightarrow \quad \boxed{\sec\theta = \frac{41}{9}}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-40/9} = -\frac{9}{40} \quad \Rightarrow \quad \boxed{\cot\theta = -\frac{9}{40}}$$

(iv) Since $\sec^2\theta = 1 + \tan^2\theta$

$$\Rightarrow \sec\theta = \pm\sqrt{1+\tan^2\theta}$$

As terminal ray is in *IInd* quadrant so $\sec\theta$ is –ive.

$$\Rightarrow \sec\theta = -\sqrt{1-\tan^2\theta}$$

$$\Rightarrow \sec\theta = -\sqrt{1+\left(-\frac{1}{3}\right)^2} = -\sqrt{1+\frac{1}{9}} = -\sqrt{\frac{10}{9}}$$

$$\Rightarrow \boxed{\sec\theta = -\frac{\sqrt{10}}{3}}$$

Now $\cos\theta = \dfrac{1}{\sec\theta} = \dfrac{1}{-\sqrt{10}/3} = -\dfrac{3}{\sqrt{10}}$ $\quad\Rightarrow \boxed{\cos\theta = -\dfrac{3}{\sqrt{10}}}$

Also $\dfrac{\sin\theta}{\cos\theta} = \tan\theta$

$$\Rightarrow \sin\theta = (\tan\theta)(\cos\theta) = \left(-\frac{1}{3}\right)\left(-\frac{3}{\sqrt{10}}\right) \quad \Rightarrow \boxed{\sin\theta = \frac{1}{\sqrt{10}}}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{1/\sqrt{10}} \quad \Rightarrow \boxed{\csc\theta = \sqrt{10}}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-1/3} \quad \Rightarrow \boxed{\cot\theta = -3}$$

Question # 4 (iii) and (v)

Do yourself as above.

Question # 5

If $\cot\theta = \dfrac{15}{8}$ and terminal arm of the angle is not in quad. I, find the values of $\cos\theta$ and $\operatorname{cosec}\theta$.

Solution

As $\cot\theta$ is +ive and it is not in *Ist* quadrant, so it is in *IIIrd* quadrant

($\cot\theta$ +ive in *Ist* and *IIIrd* quadrant)

Now $\qquad \csc^2\theta = 1 + \cot^2\theta$

$$\Rightarrow \csc\theta = \pm\sqrt{1+\cot^2\theta}$$

As terminal ray is in *IIIrd* quadrant so $\csc\theta$ is –ive.

$$\csc\theta = -\sqrt{1+\cot^2\theta}$$

$$\Rightarrow \csc\theta = -\sqrt{1+\left(\frac{15}{8}\right)^2} = -\sqrt{1+\frac{225}{64}} \qquad \because \cot\theta = \frac{15}{8}$$

$$= -\sqrt{\frac{289}{64}} \quad \Rightarrow \boxed{\csc\theta = -\frac{17}{8}}$$

$$\sin\theta = \frac{1}{\csc\theta} = \frac{1}{-17/8} \quad \Rightarrow \boxed{\sin\theta = -\frac{8}{17}}$$

Now $\dfrac{\cos\theta}{\sin\theta} = \cot\theta$

$$\Rightarrow \cos\theta = \cot\theta\,\sin\theta = \left(\frac{15}{8}\right)\left(-\frac{8}{17}\right) \quad \Rightarrow \boxed{\cos\theta = -\frac{15}{17}}$$

Question # 6

If $\operatorname{cosec}\theta = \dfrac{m^2+1}{2m}$ and $\left(0<\theta<\dfrac{\pi}{2}\right)$, find the values of the remaining trigonometric function.

Solution

Since $0<\theta<\dfrac{\pi}{2}$ therefore terminal ray lies in *Ist* quadrant.

Now $\qquad 1+\cot^2\theta = \csc^2\theta$

$\Rightarrow\ \cot^2\theta = \csc^2\theta - 1$

$\Rightarrow\ \cot\theta = \pm\sqrt{\csc^2\theta - 1}$

As terminal ray of θ is in *Ist* quadrant so $\cot\theta$ is +ive.

$$\cot\theta = \sqrt{\csc^2\theta - 1}$$

$$\Rightarrow\ \cot\theta = \sqrt{\left(\frac{m^2+1}{2m}\right)^2 - 1} = \sqrt{\frac{(m^2+1)^2}{(2m)^2} - 1} \qquad \because\ \csc\theta = \frac{m^2+1}{2m}$$

$$= \sqrt{\frac{m^4+2m^2+1}{4m^2} - 1} = \sqrt{\frac{m^4+2m^2+1-4m^2}{4m^2}} = \sqrt{\frac{m^4-2m^2+1}{4m^2}}$$

$$= \sqrt{\frac{(m^2-1)^2}{(2m)^2}} = \frac{m^2-1}{2m} \qquad \Rightarrow \boxed{\cot\theta = \frac{m^2-1}{2m}}$$

$$\sin\theta = \frac{1}{\csc\theta} = \frac{1}{\left(\frac{m^2+1}{2m}\right)} = \frac{2m}{\left(m^2+1\right)} \qquad \Rightarrow \boxed{\sin\theta = \frac{2m}{m^2+1}}$$

Now $\quad \dfrac{\cos\theta}{\sin\theta} = \cot\theta \qquad \Rightarrow\ \cos\theta = (\cot\theta)(\sin\theta)$

$$\Rightarrow\ \cos\theta = \left(\frac{m^2-1}{2m}\right)\left(\frac{2m}{m^2+1}\right) \qquad \Rightarrow \boxed{\cos\theta = \left(\frac{m^2-1}{m^2+1}\right)}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{m^2-1}{m^2+1}} \qquad \Rightarrow \boxed{\sec\theta = \left(\frac{m^2+1}{m^2-1}\right)}$$

$$\tan\theta = \frac{1}{\cot\theta} = \frac{1}{\frac{m^2-1}{2m}} \qquad \Rightarrow \boxed{\tan\theta = \left(\frac{2m}{m^2-1}\right)}$$

Question # 7

If $\tan\theta = \dfrac{1}{\sqrt{7}}$ and the terminal arm of the angle is not is the II quad. Find the value of $\dfrac{\csc^2\theta - \sec^2\theta}{\csc^2\theta + \sec^2\theta}$.

Solution

Since $\tan\theta$ is +ive and terminal arm is not in the *IIIrd* quadrant, therefore terminal arm lies in *Ist* quadrant.

Now $\quad \sec^2\theta = 1+\tan^2\theta$

$$\Rightarrow \sec\theta = \pm\sqrt{1+\tan^2\theta}$$

as terminal arm is in the first quadrant so $\sec\theta$ is +ive.

$$\sec\theta = \sqrt{1+\tan^2\theta}$$

$$\sec\theta = \sqrt{1+\left(\frac{1}{\sqrt{7}}\right)^2} = \sqrt{1+\frac{1}{7}} = \sqrt{\frac{8}{7}} \quad \Rightarrow \boxed{\sec\theta = \frac{2\sqrt{2}}{\sqrt{7}}}$$

Now $\quad \cos\theta = \dfrac{1}{\sec\theta} = \dfrac{1}{2\sqrt{2}/\sqrt{7}} \quad \Rightarrow \boxed{\cos\theta = \dfrac{\sqrt{7}}{2\sqrt{2}}}$

Now $\quad \dfrac{\sin\theta}{\cos\theta} = \tan\theta \qquad \Rightarrow \sin\theta = (\tan\theta)(\cos\theta)$

$$\Rightarrow \sin\theta = \left(\frac{1}{\sqrt{7}}\right)\left(\frac{\sqrt{7}}{2\sqrt{2}}\right) \quad \Rightarrow \boxed{\sin\theta = \frac{1}{2\sqrt{2}}}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{1/2\sqrt{2}} \quad \Rightarrow \boxed{\csc\theta = 2\sqrt{2}}$$

Now $\quad \dfrac{\csc^2\theta - \sec^2\theta}{\csc^2\theta + \sec^2\theta} = \dfrac{\left(2\sqrt{2}\right)^2 - \left(2\sqrt{2}/\sqrt{7}\right)^2}{\left(2\sqrt{2}\right)^2 + \left(2\sqrt{2}/\sqrt{7}\right)^2} = \dfrac{8-\frac{8}{7}}{8+\frac{8}{7}}$

$$= \frac{48/7}{64/7} = \frac{48}{7}\cdot\frac{7}{64} = \frac{3}{4} \quad \textit{Answer}$$

Question # 8

If $\cot\theta = \dfrac{5}{2}$ and the terminal arm of the angle is in the I quad., find the value of $\dfrac{3\sin\theta + 4\cos\theta}{\cos\theta - \sin\theta}$

Solution

Since $\quad \csc^2\theta = 1+\cot^2\theta$

$$\Rightarrow \csc\theta = \pm\sqrt{1+\cot^2\theta}$$

As terminal ray is in *Ird* quadrant so $\csc\theta$ is +ive.

$$\csc\theta = \sqrt{1+\cot^2\theta} = \sqrt{1+\left(\frac{5}{2}\right)^2} = \sqrt{1+\frac{25}{4}} = \sqrt{\frac{29}{4}} = \frac{\sqrt{29}}{2}$$

Now $\sin\theta = \dfrac{1}{\csc\theta} = \dfrac{1}{{}^{\sqrt{29}}\!/\!_{2}} \qquad \Rightarrow \boxed{\sin\theta = \dfrac{2}{\sqrt{29}}}$

Now $\dfrac{\cos\theta}{\sin\theta} = \cot\theta \quad \Rightarrow \cos\theta = (\cot\theta)(\sin\theta)$

$$\Rightarrow \cos\theta = \left(\frac{5}{2}\right)\left(\frac{2}{\sqrt{29}}\right) \qquad \Rightarrow \boxed{\cos\theta = \frac{5}{\sqrt{29}}}$$

Now $\dfrac{3\sin\theta + 4\cos\theta}{\cos\theta - \sin\theta} = \dfrac{3\left({}^{2}\!/\!_{\sqrt{29}}\right) + 4\left({}^{5}\!/\!_{\sqrt{29}}\right)}{{}^{5}\!/\!_{\sqrt{29}} - {}^{2}\!/\!_{\sqrt{29}}} = \dfrac{{}^{6}\!/\!_{\sqrt{29}} + {}^{20}\!/\!_{\sqrt{29}}}{{}^{5}\!/\!_{\sqrt{29}} - {}^{2}\!/\!_{\sqrt{29}}}$

$$= \frac{\frac{6+20}{\sqrt{29}}}{\frac{5-2}{\sqrt{29}}} = \frac{{}^{26}\!/\!_{\sqrt{29}}}{{}^{3}\!/\!_{\sqrt{29}}} = \frac{26}{\sqrt{29}} \cdot \frac{\sqrt{2}}{3} = \frac{26}{3} \quad \textit{Answer}$$