

Exercise

Q1. If $x + \frac{1}{x} = 3$ find

(i) $x^2 + \frac{1}{x^2}$

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$$\left(x + \frac{1}{x}\right)^2 = (3)^2$$

$$x^2 + \frac{1}{x^2} + 2 = 9$$

$$x^2 + \frac{1}{x^2} = 9 - 2$$

(ii) $x^4 + \frac{1}{x^4}$

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$$\left(x^2 + \frac{1}{x^2}\right)^2 = (7)^2$$

$$x^4 + \frac{1}{x^4} + 2 = 49$$

$$x^4 + \frac{1}{x^4} = 49 - 2$$

$$x^4 + \frac{1}{x^4} = 47$$

Q2. If $x - \frac{1}{x} = 2$ find

(i) $x^2 + \frac{1}{x^2}$

(ii) $x^4 + \frac{1}{x^4}$

(i) $x - \frac{1}{x} = 2$

Squaring

$$\left(x - \frac{1}{x}\right)^2 = (2)^2$$

$$x^2 + \frac{1}{x^2} - 2 = 4$$

$$x^2 + \frac{1}{x^2} = 4 + 2$$

$$x^2 + \frac{1}{x^2} = 6$$

(ii) $\left(x^2 + \frac{1}{x^2}\right) = (6)^2$

$$x^4 + \frac{1}{x^4} + 2 = 36$$

$$x^4 + \frac{1}{x^4} = 36 - 2$$

$$x^4 + \frac{1}{x^4} = 34$$

Q3. Find value of $x^3 + y^3$ and xy if

$x + y = 5$ and $x - y = 3$

$$4xy = (x + y)^2 - (x - y)^2$$

$$4xy = (5)^2 - (3)^2$$

Now

$$4xy = 25 - 9 = 16$$

$$xy = \frac{16}{4} = 4$$

$$x + y = 5$$

taking cube both sides

$$(x + y)^3 = (5)^3$$

$$x^3 + y^3 + 3xy(x + y) = 125$$

$$x^3 + y^3 + 3(4)(5) = 125$$

$$x^3 + y^3 + 60 = 125$$

$$x^3 + y^3 = 125 - 60$$

$$x^3 + y^3 = 65$$

Q4. If $P = 2 + \sqrt{3}$ find (i) $P + \frac{1}{P}$

(ii) $P - \frac{1}{P}$ (iii) $P^2 + \frac{1}{P^2}$ (iv) $P^2 - \frac{1}{P^2}$

$$P = 2 + \sqrt{3}$$

$$\frac{1}{P} = \frac{1}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}}$$

$$\frac{1}{P} = \frac{2 - \sqrt{3}}{(2)^2 - (\sqrt{3})^2} = \frac{2 - \sqrt{3}}{4 - 3} = 2 - \sqrt{3}$$

i) $P + \frac{1}{P} = 2 + \cancel{\sqrt{3}} + 2 - \cancel{\sqrt{3}} = 4$

ii) $P - \frac{1}{P} = \cancel{2} + \sqrt{3} - \cancel{2} + \sqrt{3} = 2\sqrt{3}$

iii) $P^2 + \frac{1}{P^2} = ?$

$$\left(P + \frac{1}{P}\right)^2 = (4)^2$$

$$P^2 + \frac{1}{P^2} + 2 = 16$$

$$P^2 + \frac{1}{P^2} = 16 - 2$$

$$P^2 + \frac{1}{P^2} = 14$$

iv) $P^2 - \frac{1}{P^2} = ?$

$$\begin{aligned}
 P^2 - \frac{1}{P^2} &= \left(P + \frac{1}{P}\right) \left(P - \frac{1}{P}\right) \\
 &= (4)(\sqrt{3}) \\
 &= 8\sqrt{3}
 \end{aligned}$$

Q5. If $q = \sqrt{5} + 2$ Find (i) $q + \frac{1}{q}$

(ii) $q - \frac{1}{q}$ (iii) $q^2 + \frac{1}{q^2}$ (iv) $q^2 - \frac{1}{q^2}$

Solution: $q = \sqrt{5} + 2$

$$\frac{1}{q} = \frac{1}{\sqrt{5}+2} \times \frac{\sqrt{5}-2}{\sqrt{5}-2}$$

$$\frac{1}{q} = \frac{\sqrt{5}-2}{(\sqrt{5})^2 - (2)^2}$$

$$\frac{1}{q} = \frac{\sqrt{5}-2}{1} = \sqrt{5} - 2$$

(i) $q + \frac{1}{q} = \sqrt{5} + 2 + \sqrt{5} - 2$
 $= 2\sqrt{5}$

(ii) $q - \frac{1}{q} = \sqrt{5} + 2 - \sqrt{5} + 2$
 $= 4$

(iii) $q^2 + \frac{1}{q^2}$
 $\left(q + \frac{1}{q}\right)^2 = (2\sqrt{5})^2$

$$q^2 + \frac{1}{q^2} + 2 = 20$$

$$q^2 + \frac{1}{q^2} = 20 - 2$$

$$q^2 + \frac{1}{q^2} = 18$$

(iv) $q^2 - \frac{1}{q^2} = \left(q + \frac{1}{q}\right) \left(q - \frac{1}{q}\right)$

$$\begin{aligned}
 &= (2\sqrt{5})(4) \\
 &= 8\sqrt{5}
 \end{aligned}$$

Q6. Simplify

i)
$$\begin{aligned}
 &\frac{\sqrt{a^2+2} + \sqrt{a^2-2}}{\sqrt{a^2+2} - \sqrt{a^2-2}} \\
 &= \frac{\sqrt{a^2+2} + \sqrt{a^2-2}}{\sqrt{a^2+2} - \sqrt{a^2-2}} \times \frac{\sqrt{a^2+2} + \sqrt{a^2-2}}{\sqrt{a^2+2} + \sqrt{a^2-2}} \\
 &= \frac{(\sqrt{a^2+2} + \sqrt{a^2-2})^2}{(\sqrt{a^2+2})^2 - (\sqrt{a^2-2})^2} \\
 &= \frac{(\sqrt{a^2+2})^2 + (\sqrt{a^2-2})^2 + 2(\sqrt{a^2+2})(\sqrt{a^2-2})}{a^2+2 - a^2+2} \\
 &= \frac{a^2+2 + a^2-2 + 2\sqrt{a^4-4}}{4} \\
 &= \frac{2a^2 + 2\sqrt{a^4-4}}{4} \\
 &= \frac{2(a^2 + \sqrt{a^4-4})}{4} \\
 &= \frac{a^2 + \sqrt{a^4-4}}{2}
 \end{aligned}$$

(ii)
$$\begin{aligned}
 &\frac{1}{a - \sqrt{a^2 - x^2}} - \frac{1}{a + \sqrt{a^2 - x^2}} \\
 &= \frac{1}{a - \sqrt{a^2 - x^2}} \times \frac{a + \sqrt{a^2 - x^2}}{a + \sqrt{a^2 - x^2}} \\
 &\quad - \frac{1}{a + \sqrt{a^2 - x^2}} \times \frac{a - \sqrt{a^2 - x^2}}{a - \sqrt{a^2 - x^2}} \\
 &= \frac{a + \sqrt{a^2 - x^2}}{(a)^2 - (\sqrt{a^2 - x^2})^2} - \frac{a - \sqrt{a^2 - x^2}}{(a)^2 - (\sqrt{a^2 - x^2})^2}
 \end{aligned}$$

$$= \frac{a + \sqrt{a^2 - x^2}}{\cancel{a^2} - \cancel{a^2} + x^2} - \frac{a - \sqrt{a^2 - x^2}}{\cancel{a^2} - \cancel{a^2} + x^2}$$

$$= \frac{a + \sqrt{a^2 - x^2}}{x^2} - \frac{a - \sqrt{a^2 - x^2}}{x^2}$$

$$= \frac{\cancel{a} + \sqrt{a^2 - x^2} - \cancel{a} + \sqrt{a^2 - x^2}}{x^2}$$

$$= \frac{2\sqrt{a^2 - x^2}}{x^2}$$

Objective

1. $4x + 3y - 2$ is an algebraic _____

- (a) Expression
(b) Sentence
(c) Equation
(d) In equation

2. The degree of polynomial

$$4x^4 + 2x^2y \text{ is } \underline{\hspace{2cm}}$$

- (a) 1 (b) 2
(c) 3 (d) 4

3. $a^3 + b^3$ is equal to _____

- (a) $(a-b)(a^2 + ab + b^2)$
(b) $(a+b)(a^2 - ab + b^2)$
(c) $(a-b)(a^2 - ab + b^2)$
(d) $(a-b)(a^2 + ab - b^2)$

4. $(3 + \sqrt{2})(3 - \sqrt{2})$ is equal to: _____

- (a) 7 (b) -7
(c) -1 (d) 1

5. Conjugate of Surd $a + \sqrt{b}$ is _____

- (a) $-a + \sqrt{b}$ (b) $a - \sqrt{b}$
(c) $\sqrt{a} + \sqrt{b}$ (d) $\sqrt{a} - \sqrt{b}$

6. $\frac{1}{a-b} - \frac{1}{a+b}$ is equal to

- (a) $\frac{2a}{a^2 - b^2}$ (b) $\frac{2b}{a^2 - b^2}$
(c) $\frac{-2a}{a^2 - b^2}$ (d) $\frac{-2b}{a^2 - b^2}$

7. $\frac{a^2 - b^2}{a + b}$ is equal to:

- (a) $(a-b)^2$ (b) $(a+b)^2$
(c) $a+b$ (d) $a-b$

8. $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})$ is equal to: _____

- (a) $a^2 + b^2$ (b) $a^2 - b^2$
(c) $a - b$ (d) $a + b$

9. The degree of the polynomial $x^2y^2 + 3xy + y^3$ is _____

- (a) 4 (b) 5
(c) 6 (d) 2

10. $x^2 - 4 = \underline{\hspace{2cm}}$

- (a) $(x-2)(x+2)$ (b) $(x-2)(x-2)$
(c) $(x+2)(x+2)$ (d) None

11. $x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)(\dots\dots\dots)$

- (a) $x^2 - 1 + \frac{1}{x^2}$ (b) $x^2 + 1 + \frac{1}{x^2}$
(c) $x^2 + 1 - \frac{1}{x^2}$ (d) $x^2 - 1 - \frac{1}{x^2}$

12. $2(a^2 + b^2) = \underline{\hspace{2cm}}$

- (a) $(a+b)^2 + (a-b)^2$
(b) $(a+b)^2 - (a-b)^2$
(c) $(a+b)^2$ (d) $4ab$

13. Order of surd $\sqrt[3]{x}$ is _____

- (a) 3 (b) $\frac{1}{3}$
(c) 0 (d) 1

14. $\frac{1}{2-\sqrt{3}} = \underline{\hspace{2cm}}$

- (a) $2+\sqrt{3}$ (b) $2-\sqrt{3}$
 (d) $-2+\sqrt{3}$ (d) $-2-\sqrt{3}$

15. $(a+b)^2 - (a-b)^2 = \underline{\hspace{2cm}}$

- (a) $2(a^2 + b^2)$ (b) $4ab$
 (c) $2ab$ (d) $3ab$

16. $\sqrt{14} \cdot \sqrt{35} = \underline{\hspace{2cm}}$

- (a) $\sqrt[4]{10}$ (b) $\sqrt[5]{10}$
 (c) $7\sqrt{10}$ (d) $8\sqrt{10}$

17. A surd which contains a single term is called surd.

- (a) Monomial
 (b) Binomial
 (c) Trinomial
 (d) None

ANSWER KEY

1.	a	2.	d	3.	b	4.	a	5.	b
6.	b	7.	d	8.	c	9.	a	10.	a
11.	a	12.	a	13.	a	14.	a	15.	b
16.	c	17.	a						