Exercise 2.4

Q1. Use laws of exponents to simplify

(i) $\dfrac{(243)^{-2/3}(32)^{-1/5}}{\sqrt{(196)^{-1}}}$

$=\dfrac{\sqrt{196}}{(243)^{2/3}(32)^{1/5}}$

$=\dfrac{\sqrt{14\times14}}{(3\times3\times3\times3\times3)^{2/3}(2\times2\times2\times2\times2)^{1/5}}$

$=\dfrac{\sqrt{(14)^2}}{\left(3^3\times3^2\right)^{2/3}\left(2^5\right)^{1/5}}$

$=\dfrac{14}{3^{\cancel{3}\times\frac{2}{\cancel{3}}}\times3^{2\times\frac{2}{3}}\times2^{\cancel{5}\times\frac{1}{\cancel{5}}}}$

$=\dfrac{\cancel{14}7}{3^2\times3^{2\times\frac{2}{3}}\times\cancel{2}}$

$=\dfrac{7}{3^2\times3^{\frac{4}{3}}}$

$=\dfrac{7}{3^2\times3^{\frac{3+1}{3}}}$

$=\dfrac{7}{3^2\times3^{\frac{3}{3}+\frac{1}{3}}}$

$=\dfrac{7}{3^2\times3\times3^{\frac{1}{3}}}$

$=\dfrac{7}{3^3\times\sqrt[3]{3}}$

$=\dfrac{7}{27\left(\sqrt[3]{3}\right)}$

(ii) $\left(2x^5y^{-4}\right)\left(-8x^{-3}.y^2\right)$

$=2(-8)x^{5-3}.y^{-4+2}$

$=-16x^2.y^{-2}$

$=-16\dfrac{x^2}{y^2}$

(iii) $\left(\dfrac{x^{-2}y^{-1}z^{-4}}{x^4y^{-3}z^0}\right)^{-3}$

$=\left(x^{-2-4}.y^{-1+3}.z^{-4+0}\right)^{-3}$

$=\left(x^{-6}.y^2.z^{-4}\right)^{-3}$

$=x^{-6(-3)}.y^{2(-3)}.z^{-4(-3)}$

$=x^{18}.y^{-6}.z^{12}$

$=\dfrac{x^{18}.z^{12}}{y^6}$

(iv) $\dfrac{(81)^n.3^5-(3)^{4n-1}(243)}{\left(9^{2n}\right)\left(3^3\right)}$

$=\dfrac{\left(3^4\right)^n.3^5-(3)^{4n-1}\left(3^5\right)}{\left(3^2\right)^{2n}\left(3^3\right)}$

$=\dfrac{3^{4n+5}-3^{4n-1+5}}{3^{4n+3}}$

$=\dfrac{3^{4n+3+2}-3^{4n+4}}{3^{4n+3}}$

$=\dfrac{3^{4n+3+2}-3^{4n+3+1}}{3^{4n+3}}$

$=\dfrac{3^{4n+3}.3^2-3^{4n+3}.3^1}{3^{4n+3}}$

$$=\frac{\cancel{3^{4n+3}}\left(3^2-3^1\right)}{\cancel{3^{4n+3}}}$$

$=9-3$

$=6$

Q2. Show that

$$\left(\frac{x^a}{x^b}\right)^{a+b}\times\left(\frac{x^b}{x^c}\right)^{b+c}\times\left(\frac{x^c}{x^a}\right)^{c+a}=1$$

Sol: L.H.S

$$=\left(\frac{x^a}{x^b}\right)^{a+b}\times\left(\frac{x^b}{x^c}\right)^{b+c}\times\left(\frac{x^c}{x^a}\right)^{c+a}$$

$$=\left(x^{a-b}\right)^{a+b}\times\left(x^{b-c}\right)^{b+c}\times\left(x^{c-a}\right)^{c+a}$$

$=x^{(a-b)(a+b)}\times x^{(b-c)(b+c)}\times x^{(c-a)(c+a)}$

$=x^{a^2-b^2}\times x^{b^2-c^2}\times x^{c^2-a^2}$

$=x^{\cancel{a^2}-\cancel{b^2}+\cancel{b^2}-\cancel{c^2}+\cancel{c^2}-\cancel{a^2}}$

$=x^0$

$=1$

$=$ R.H.S

Q3. Simplify

(i) $$\frac{2^{1/3}\times(27)^{1/3}\times(60)^{1/2}}{(180)^{1/2}\times(4)^{-1/3}\times(9)^{1/4}}$$

$$=\frac{2^{1/3}\times\left(3^3\right)^{1/3}\times\left(2^2\times3\times5\right)^{1/2}}{\left(2^2\times3^2\times5\right)^{1/2}\times\left(2^2\right)^{-1/3}\times\left(3^2\right)^{1/4}}$$

$$=\frac{2^{\frac{1}{3}}\times3^{3\times\frac{1}{3}}\times2^{2\times\frac{1}{2}}\times3^{\frac{1}{2}}\times5^{\frac{1}{2}}}{2^{2\times\frac{1}{2}}\times3^{2\times\frac{1}{2}}\times5^{\frac{1}{2}}\times2^{2\left(-\frac{1}{3}\right)}\times3^{2\times\frac{1}{4}}}$$

$$=\frac{2^{\frac{1}{3}}\times3\times2\times3^{\frac{1}{2}}\times5^{\frac{1}{2}}}{2\times3\times5^{\frac{1}{2}}\times2^{\frac{-2}{3}}\times3^{\frac{1}{2}}}$$

$=2^{\frac{1}{3}+1-1+\frac{2}{3}}\times3^{1+\frac{1}{2}-1-\frac{1}{2}}\times5^{\cancel{\frac{1}{2}}-\cancel{\frac{1}{2}}}$

$=2^{\frac{3}{3}}\times3^0\times5^0$

$=2\times1\times1$

$=2$

(ii) $$\sqrt{\frac{(216)^{\frac{2}{3}}\times(25)^{\frac{1}{2}}}{(.04)^{-\frac{1}{2}}}}$$

$$=\sqrt{\frac{\left(6^3\right)^{\frac{2}{3}}\times\left(5^2\right)^{\frac{1}{2}}}{\left(\frac{4}{100}\right)^{-\frac{1}{2}}}}$$

$$=\sqrt{\frac{6^{\cancel{3}\times\frac{2}{\cancel{3}}}\times5^{\cancel{2}\times\frac{1}{\cancel{2}}}}{\left(\frac{100}{4}\right)^{\frac{1}{2}}}}\qquad=\sqrt{\frac{6^2\times5}{(25)^{\frac{1}{2}}}}$$

$$=\sqrt{\frac{6^2\times5}{5^{\cancel{2}\times\frac{1}{\cancel{2}}}}}$$

$$=\sqrt{\frac{6^2\times\cancel{5}}{\cancel{5}}}$$

$=\sqrt{6^2}$

$=6$

(iii) $5^{2^3}\div\left(5^2\right)^3$

$=5^8\div5^6$

$=\frac{5^8}{5^6}$

$=5^{8-6}$

$=5^2$

$=25$

iv) $\left(x^3\right)^2 \div x^{3^2}$

$= x^6 \div x^9$

$= \dfrac{x^6}{x^9}$

$= \dfrac{1}{x^{9-6}} = \dfrac{1}{x^3}$