EXERCISE 7.3

Question # 1

(i) $\underline{u} = 3\hat{\underline{i}} + \hat{\underline{j}} - \hat{\underline{k}}$, $\underline{v} = 2\hat{\underline{i}} - \hat{\underline{j}} + \hat{\underline{k}}$

$$|\underline{u}| = \sqrt{(3)^2 + (1)^2 + (-1)^2} = \sqrt{9+1+1} = \sqrt{11}$$

$$|\underline{v}| = \sqrt{(2)^2 + (-1)^2 + (1)^2} = \sqrt{4+1+1} = \sqrt{6}$$

$$\underline{u}\cdot\underline{v} = \left(3\hat{\underline{i}} + \hat{\underline{j}} - \hat{\underline{k}}\right)\cdot\left(2\hat{\underline{i}} - \hat{\underline{j}} + \hat{\underline{k}}\right)$$

$$= (3)(2) + (1)(-1) + (-1)(1) = 6 - 1 - 1 = 4$$

Now $\underline{u}\cdot\underline{v} = |\underline{u}||\underline{v}|\cos\theta$

$$\Rightarrow \cos\theta = \frac{\underline{u}\cdot\underline{v}}{|\underline{u}||\underline{v}|} = \frac{4}{\sqrt{11}\times\sqrt{6}} \qquad \Rightarrow \boxed{\cos\theta = \frac{4}{\sqrt{66}}}$$

(ii) *Do yourself as above*

(iii) $\underline{u} = [-3,5] = -3\hat{\underline{i}} + 5\hat{\underline{j}}$, $\underline{v} = [6,-2] = 6\hat{\underline{i}} - 2\hat{\underline{j}}$

Now do yourself as above

(iv) $\underline{u} = [1,-3,1] = \hat{\underline{i}} - 3\hat{\underline{j}} + \hat{\underline{k}}$, $\underline{v} = 2\hat{\underline{i}} + 4\hat{\underline{j}} + \hat{\underline{k}}$ *Now do yourself as (i)*

Question # 2

(i) $\underline{a} = \hat{\underline{i}} - \hat{\underline{k}}$, $\underline{b} = \hat{\underline{j}} + \hat{\underline{k}}$

$$|\underline{a}| = \sqrt{(1)^2 + (0)^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2}$$

$$|\underline{b}| = \sqrt{(0)^2 + (1)^2 + (1)^2} = \sqrt{1+1} = \sqrt{2}$$

$$\underline{a}\cdot\underline{b} = \left(\hat{\underline{i}} - \hat{\underline{k}}\right)\cdot\left(\hat{\underline{j}} + \hat{\underline{k}}\right) = (1)(0) + (0)(1) + (-1)(1) = 0 + 0 - 1 = -1$$

Since $\underline{a}\cdot\underline{b} = |\underline{a}||\underline{b}|\cos\theta$

So projection of $\underline{a}$ along $\underline{b}$ $= |\underline{a}|\cos\theta = \dfrac{\underline{a}\cdot\underline{b}}{|\underline{b}|} = \dfrac{-1}{\sqrt{2}}$

Also projection of $\underline{b}$ along $\underline{a}$ $= |\underline{b}|\cos\theta = \dfrac{\underline{a}\cdot\underline{b}}{|\underline{a}|} = \dfrac{-1}{\sqrt{2}}$

(ii) *Do yourself as above*

Question # 3

(i) *Do yourself as (ii) below*

(ii) $\underline{u} = \alpha\hat{\underline{i}} + 2\alpha\hat{\underline{j}} - \hat{\underline{k}}$, $\underline{v} = \hat{\underline{i}} + \alpha\hat{\underline{j}} + 3\hat{\underline{k}}$

Since $\underline{u}$ and $\underline{v}$ are perpendicular therefore $\underline{u}.\underline{v} = 0$

$$\Rightarrow \left(\alpha\hat{\underline{i}} + 2\alpha\hat{\underline{j}} - \hat{\underline{k}}\right)\cdot\left(\hat{\underline{i}} + \alpha\hat{\underline{j}} + 3\hat{\underline{k}}\right) = 0$$

$$\Rightarrow (\alpha)(1) + (2\alpha)(\alpha) + (-1)(3) = 0$$

$$\Rightarrow \alpha + 2\alpha^2 - 3 = 0 \qquad \Rightarrow 2\alpha^2 + \alpha - 3 = 0$$

$$\Rightarrow 2\alpha^2 + 3\alpha - 2\alpha - 3 = 0 \qquad \Rightarrow \alpha(2\alpha+3) - 1(2\alpha+3) = 0$$

$$\Rightarrow (2\alpha+3)(\alpha-1) = 0$$

$$\Rightarrow 2\alpha + 3 = 0 \quad \text{or} \quad \alpha - 1 = 0$$

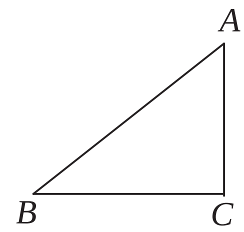
$$\Rightarrow \alpha = -\frac{3}{2} \quad \text{or} \quad \alpha = 1$$

Question # 4

Given vertices: $A(1,-1,0)$, $B(-2,2,1)$ and $C(0,2,z)$

$\overrightarrow{CA}=(1-0)\hat{\underline{i}}+(-1-2)\hat{\underline{j}}+(0-z)\hat{\underline{k}} \quad =\hat{\underline{i}}-3\hat{\underline{j}}-z\hat{\underline{k}}$

$\overrightarrow{CB}=(-2-0)\hat{\underline{i}}+(2-2)\hat{\underline{j}}+(1-z)\hat{\underline{k}} \quad =-2\hat{\underline{i}}+(1-z)\hat{\underline{k}}$



Now $\overrightarrow{CA}$ is $\perp$ to $\overrightarrow{CB}$ therefore $\overrightarrow{CA}\cdot\overrightarrow{CB}=0$

$\Rightarrow \left(\hat{\underline{i}}-3\hat{\underline{j}}-z\hat{\underline{k}}\right)\cdot\left(-2\hat{\underline{i}}+(1-z)\hat{\underline{k}}\right)=0$

$\Rightarrow (1)(-2)+(-3)(0)+(-z)(1-z)=0$

$\Rightarrow -2+0-z+z^2=0 \qquad \Rightarrow z^2-z-2=0$

$\Rightarrow z^2-2z+z-2=0 \qquad \Rightarrow z(z-2)+1(z-2)=0$

$\Rightarrow (z-2)(z+1)=0$

$\Rightarrow z-2=0 \quad \text{or} \quad z+1=0$

$\Rightarrow z=2 \quad \text{or} \quad z=-1$

Question # 5

Suppose $\underline{v}=a_1\hat{\underline{i}}+a_2\hat{\underline{j}}+a_3\hat{\underline{k}}$

Since $\underline{v}\cdot\hat{\underline{i}}=0 \quad \Rightarrow \left(a_1\hat{\underline{i}}+a_2\hat{\underline{j}}+a_3\hat{\underline{k}}\right)\cdot\hat{\underline{i}}=0$

$\Rightarrow a_1\hat{\underline{i}}\cdot\hat{\underline{i}}+a_2\hat{\underline{j}}\cdot\hat{\underline{i}}+a_3\hat{\underline{k}}\cdot\hat{\underline{i}}=0$

$\Rightarrow a_1(1)+a_2(0)+a_3(0)=0 \qquad \Rightarrow a_1=0$

Also $\underline{v}\cdot\hat{\underline{j}}=0 \quad \Rightarrow \left(a_1\hat{\underline{i}}+a_2\hat{\underline{j}}+a_3\hat{\underline{k}}\right)\cdot\hat{\underline{j}}=0$

$\Rightarrow a_1\hat{\underline{i}}\cdot\hat{\underline{j}}+a_2\hat{\underline{j}}\cdot\hat{\underline{j}}+a_3\hat{\underline{k}}\cdot\hat{\underline{j}}=0$

$\Rightarrow a_1(0)+a_2(1)+a_3(0)=0 \qquad \Rightarrow a_2=0$

Also $\underline{v}\cdot\hat{\underline{k}}=0 \quad \Rightarrow \left(a_1\hat{\underline{i}}+a_2\hat{\underline{j}}+a_3\hat{\underline{k}}\right)\cdot\hat{\underline{k}}=0$

$\Rightarrow a_1\hat{\underline{i}}\cdot\hat{\underline{k}}+a_2\hat{\underline{j}}\cdot\hat{\underline{k}}+a_3\hat{\underline{k}}\cdot\hat{\underline{k}}=0$

$\Rightarrow a_1(0)+a_2(0)+a_3(1)=0 \qquad \Rightarrow a_3=0$

Hence

$$\underline{v}=(0)\hat{\underline{i}}+(0)\hat{\underline{j}}+(0)\hat{\underline{k}}=0$$

Question # 6 (i)

Let $\underline{a}=3\hat{\underline{i}}-2\hat{\underline{j}}+\hat{\underline{k}}$, $\underline{b}=\hat{\underline{i}}-3\hat{\underline{j}}+5\hat{\underline{k}}$ and $\underline{c}=2\hat{\underline{i}}+\hat{\underline{j}}-4\hat{\underline{k}}$

Now $\underline{b}+\underline{c}=\hat{\underline{i}}-3\hat{\underline{j}}+5\hat{\underline{k}}+2\hat{\underline{i}}+\hat{\underline{j}}-4\hat{\underline{k}}$

$=3\hat{\underline{i}}-2\hat{\underline{j}}+\hat{\underline{k}} \; =\underline{a}$

Hence $\underline{a}$, $\underline{b}$ and $\underline{c}$ form a triangle.

Now $\underline{a}\cdot\underline{b}=\left(3\hat{\underline{i}}-2\hat{\underline{j}}+\hat{\underline{k}}\right)\cdot\left(\hat{\underline{i}}-3\hat{\underline{j}}+5\hat{\underline{k}}\right)$

$=(3)(1)+(-2)(-3)+(1)(5) \; =4+6+5 \; =15$

$\underline{b}\cdot\underline{c}=\left(\hat{\underline{i}}-3\hat{\underline{j}}+5\hat{\underline{k}}\right)\cdot\left(2\hat{\underline{i}}+\hat{\underline{j}}-4\hat{\underline{k}}\right)$

$=(1)(2)+(-3)(1)+(5)(-4)=2-3-20=-21$

$\underline{c}\cdot\underline{a}=\left(2\hat{\underline{i}}+\hat{\underline{j}}-4\hat{\underline{k}}\right)\cdot\left(3\hat{\underline{i}}-2\hat{\underline{j}}+\hat{\underline{k}}\right)$

$=(2)(3)+(1)(-2)+(-4)(1) \; =6-2-4 \; =0$

Since $\underline{c}\cdot\underline{a}=0$ therefore $\underline{c}\perp\underline{a}$

Hence $\underline{a}$, $\underline{b}$ and $\underline{c}$ represents sides of right triangle.

Question # 6(ii)

Given: $P(1,3,2)$, $Q(4,1,4)$ and $R(6,5,5)$

$$\overrightarrow{PQ}=(4-1)\underline{\hat{i}}+(1-3)\underline{\hat{j}}+(4-2)\underline{\hat{k}}\ =3\underline{\hat{i}}-2\underline{\hat{j}}+2\underline{\hat{k}}$$

$$\overrightarrow{QR}=(6-4)\underline{\hat{i}}+(5-1)\underline{\hat{j}}+(5-4)\underline{\hat{k}}\ =2\underline{\hat{i}}+4\underline{\hat{j}}+\underline{\hat{k}}$$

$$\overrightarrow{RP}=(1-6)\underline{\hat{i}}+(3-5)\underline{\hat{j}}+(2-5)\underline{\hat{k}}\ =-5\underline{\hat{i}}-2\underline{\hat{j}}-3\underline{\hat{k}}$$

Now

$$\overrightarrow{PQ}+\overrightarrow{QR}+\overrightarrow{RP}$$

$$=3\underline{\hat{i}}-2\underline{\hat{j}}+2\underline{\hat{k}}+2\underline{\hat{i}}+4\underline{\hat{j}}+\underline{\hat{k}}-5\underline{\hat{i}}-2\underline{\hat{j}}-3\underline{\hat{k}}\ =0$$

Hence P,Q and R are vertices of triangle.

Now

$$\overrightarrow{PQ}\cdot\overrightarrow{QR}=\left(3\underline{\hat{i}}-2\underline{\hat{j}}+2\underline{\hat{k}}\right)\cdot\left(2\underline{\hat{i}}+4\underline{\hat{j}}+\underline{\hat{k}}\right)$$

$$=(3)(2)+(-2)(4)+(2)(1)\ =6-8+2\ =0$$

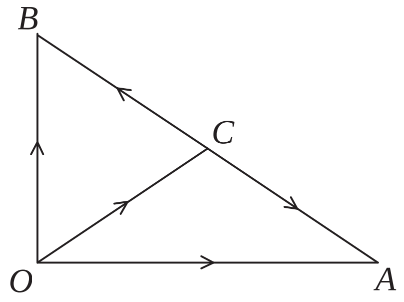
$$\Rightarrow\ \overrightarrow{PQ}\ \perp\ \overrightarrow{QR}$$

Hence P,Q and R are vertices of right triangle.

Question # 7

Suppose a right triangle OAB. Let C be a midpoint of hypotenuse AB, then

$$\overrightarrow{CA}=-\overrightarrow{CB}\quad\Rightarrow\ \left|\overrightarrow{CA}\right|=\left|\overrightarrow{CB}\right|\ \ldots\ldots\ldots\ldots..\ (i)$$



Now $\overrightarrow{OA}=\overrightarrow{OC}+\overrightarrow{CA}$

$\overrightarrow{OB}=\overrightarrow{OC}+\overrightarrow{CB}$

Since $\overrightarrow{OA}\perp\overrightarrow{OB}$ therefore $\overrightarrow{OA}\cdot\overrightarrow{OB}=0$

$$\Rightarrow\left(\overrightarrow{OC}+\overrightarrow{CA}\right)\cdot\left(\overrightarrow{OC}+\overrightarrow{CB}\right)=0$$

$$\Rightarrow\left(\overrightarrow{OC}-\overrightarrow{CB}\right)\cdot\left(\overrightarrow{OC}+\overrightarrow{CB}\right)=0\qquad\because\ \overrightarrow{CA}=-\overrightarrow{CB}$$

$$\Rightarrow\overrightarrow{OC}\cdot\left(\overrightarrow{OC}+\overrightarrow{CB}\right)-\overrightarrow{CB}\cdot\left(\overrightarrow{OC}+\overrightarrow{CB}\right)=0$$

$$\Rightarrow\overrightarrow{OC}\cdot\overrightarrow{OC}+\overrightarrow{OC}\cdot\overrightarrow{CB}-\overrightarrow{CB}\cdot\overrightarrow{OC}-\overrightarrow{CB}\cdot\overrightarrow{CB}=0$$

$$\Rightarrow\left|\overrightarrow{OC}\right|^2+\overrightarrow{OC}\cdot\overrightarrow{CB}-\overrightarrow{OC}\cdot\overrightarrow{CB}-\left|\overrightarrow{CB}\right|^2=0\qquad\because\ \overrightarrow{OC}\cdot\overrightarrow{CB}=\overrightarrow{CB}\cdot\overrightarrow{OC}$$

$$\Rightarrow\left|\overrightarrow{OC}\right|^2-\left|\overrightarrow{CB}\right|^2=0$$

$$\Rightarrow\left|\overrightarrow{OC}\right|^2=\left|\overrightarrow{CB}\right|^2\qquad\Rightarrow\left|\overrightarrow{OC}\right|=\left|\overrightarrow{CB}\right|\ \ldots\ldots\ldots\ldots\ldots\ (ii)$$

Combining (i) and (ii), we have

$$\left|\overrightarrow{OC}\right|=\left|\overrightarrow{CA}\right|=\left|\overrightarrow{CB}\right|$$

Hence midpoint of hypotenuse of right triangle is equidistant from its vertices.

Question # 8

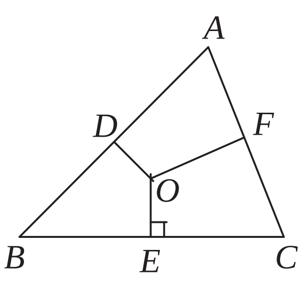
Let A,B and C be a vertices of a triangle having position vectors $\underline{a}$, $\underline{b}$ and $\underline{c}$ respectively.

Also consider D,E and F are midpoints of sides $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$, then

p.v of $D\ =\overrightarrow{OD}=\dfrac{\underline{a}+\underline{b}}{2}$

p.v of $E\ =\overrightarrow{OE}=\dfrac{\underline{b}+\underline{c}}{2}$

p.v of $F\ =\overrightarrow{OF}=\dfrac{\underline{c}+\underline{a}}{2}$



Let right bisector on $\overline{AB}$ and $\overline{BC}$ intersect at point O, which is an origin.
Since $\overrightarrow{OD}$ is $\perp$ to $\overrightarrow{AB}$

Therefore $\overrightarrow{OD} \cdot \overrightarrow{AB} = 0$

$$\Rightarrow \left(\frac{\underline{a}+\underline{b}}{2}\right) \cdot \left(\underline{b}-\underline{a}\right) = 0 \qquad \Rightarrow \frac{1}{2}\left(\underline{b}+\underline{a}\right) \cdot \left(\underline{b}-\underline{a}\right) = 0$$

$$\Rightarrow \left(\underline{b}+\underline{a}\right) \cdot \left(\underline{b}-\underline{a}\right) = 0 \qquad \Rightarrow \underline{a} \cdot \left(\underline{b}-\underline{a}\right) + \underline{b} \cdot \left(\underline{b}-\underline{a}\right) = 0$$

$$\Rightarrow \underline{a} \cdot \underline{b} - \underline{a} \cdot \underline{a} + \underline{b} \cdot \underline{b} - \underline{b} \cdot \underline{a} = 0$$

$$\Rightarrow \underline{a} \cdot \underline{b} - |\underline{a}|^2 + |\underline{b}|^2 - \underline{a} \cdot \underline{b} = 0 \qquad \because \ \underline{a} \cdot \underline{b} = \underline{b} \cdot \underline{a}$$

$$\Rightarrow |\underline{b}|^2 - |\underline{a}|^2 = 0 \ \ldots\ldots\ldots\ldots\ldots \ (i)$$

Also $\overrightarrow{OE}$ is $\perp$ to $\overrightarrow{BC}$

Therefore $\overrightarrow{OE} \cdot \overrightarrow{BC} = 0 \quad \Rightarrow \left(\dfrac{\underline{b}+\underline{c}}{2}\right) \cdot \left(\underline{c}-\underline{b}\right) = 0$

Similarly solving as above, we get

$$|\underline{c}|^2 - |\underline{b}|^2 = 0 \ldots\ldots\ldots\ldots\ldots \ (ii)$$

Adding (i) and (ii), we have

$$|\underline{b}|^2 - |\underline{a}|^2 + |\underline{c}|^2 - |\underline{b}|^2 = 0 + 0$$

$$\Rightarrow |\underline{c}|^2 - |\underline{a}|^2 = 0$$

$$\Rightarrow (\underline{c}+\underline{a}) \cdot (\underline{c}-\underline{a}) = 0$$

$$\Rightarrow \left(\frac{\underline{c}+\underline{a}}{2}\right) \cdot (\underline{c}-\underline{a}) = 0$$

$$\Rightarrow \overrightarrow{OF} \cdot \overrightarrow{AC} = 0 \quad \Rightarrow \overrightarrow{OF} \text{ is } \perp \text{ to } \overrightarrow{AC}$$

i.e. $\overrightarrow{OF}$ is also right bisector of $\overrightarrow{AC}$.

Hence perpendicular bisector of the sides of the triangle are concurrent.

Question # 9

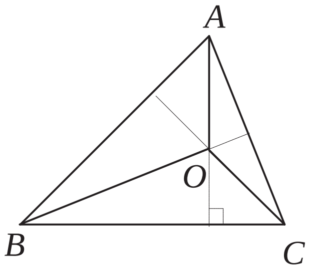
Consider A, B and C are vertices of triangle having position vectors $\underline{a}, \underline{b}$ and $\underline{c}$ respectively. Let altitude on $\overrightarrow{AB}$ and $\overrightarrow{BC}$ intersect at origin $O(0,0)$.

Since $\overrightarrow{OC}$ is perpendicular to $\overrightarrow{AB}$

$$\Rightarrow \overrightarrow{OC} \cdot \overrightarrow{AB} = 0$$

$$\Rightarrow \underline{c} \cdot (\underline{b}-\underline{a}) = 0$$

$$\Rightarrow \underline{c} \cdot \underline{b} - \underline{c} \cdot \underline{a} = 0 \ \ldots\ldots\ldots\ldots\ldots (i)$$



Also $\overrightarrow{OA}$ is perpendicular to $\overrightarrow{BC}$

$$\Rightarrow \overrightarrow{OA} \cdot \overrightarrow{BC} = 0$$

$$\Rightarrow \underline{a} \cdot (\underline{c}-\underline{b}) = 0$$

$$\Rightarrow \underline{a} \cdot \underline{c} - \underline{a} \cdot \underline{b} = 0 \ \ldots\ldots\ldots\ldots\ldots (ii)$$

Adding (i) and (ii)

$$\underline{c} \cdot \underline{b} - \underline{c} \cdot \underline{a} + \underline{a} \cdot \underline{c} - \underline{a} \cdot \underline{b} = 0 + 0$$

$$\Rightarrow \underline{c} \cdot \underline{b} - \underline{c} \cdot \underline{a} + \underline{c} \cdot \underline{a} - \underline{a} \cdot \underline{b} = 0 \qquad \because \ \underline{a} \cdot \underline{c} = \underline{c} \cdot \underline{a}$$

$$\Rightarrow \underline{c} \cdot \underline{b} - \underline{a} \cdot \underline{b} = 0$$

$$\Rightarrow (\underline{c}-\underline{a}) \cdot \underline{b} = 0$$

$$\Rightarrow \overrightarrow{AC} \cdot \overrightarrow{OB} = 0 \qquad \because \ \overrightarrow{AC} = \underline{c} - \underline{a}$$

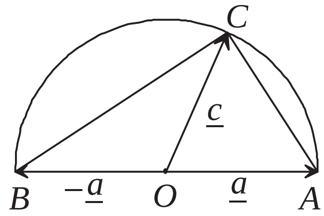
$$\Rightarrow \overrightarrow{AC} \text{ is perpendicular to } \overrightarrow{OB}.$$

Hence altitude of the triangle are concurrent.

Question # 10

Consider a semicircle having centre at origin $O(0,0)$ and A, B are end points of diameter having position vectors $\underline{a}$, $-\underline{a}$ respectively. Let C be any point on a circle having position vector $\underline{c}$.

Clearly radius of semicircle $=|\underline{a}|=|-\underline{a}|=|\underline{c}|$



Now $\overrightarrow{AC}=\underline{c}-\underline{a}$

$\overrightarrow{BC}=\underline{c}-(-\underline{a})=\underline{c}+\underline{a}$

Consider

$$\begin{aligned}\overrightarrow{AC}\cdot\overrightarrow{BC} &= (\underline{c}-\underline{a})\cdot(\underline{c}+\underline{a})\\ &= \underline{c}\cdot(\underline{c}+\underline{a})-\underline{a}\cdot(\underline{c}+\underline{a})\\ &= \underline{c}\cdot\underline{c}+\underline{c}\cdot\underline{a}-\underline{a}\cdot\underline{c}-\underline{a}\cdot\underline{a}\\ &= |\underline{c}|^2+\underline{a}\cdot\underline{c}-\underline{a}\cdot\underline{c}+|\underline{a}|^2 \qquad \because\ \underline{a}\cdot\underline{c}=\underline{c}\cdot\underline{a}\\ &= |\underline{c}|^2-|\underline{a}|^2\\ &= |\underline{c}|^2-|\underline{c}|^2=0 \qquad \because\ |\underline{a}|=|\underline{c}|\end{aligned}$$

This show $\overrightarrow{AC}$ is $\perp$ to $\overrightarrow{BC}$ i.e. $\angle ACB=90^\circ$

Hence angle in a semi circle is a right angle.

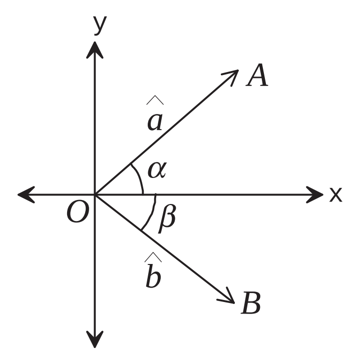
Question # 11

Consider two unit vectors $\underline{\hat{a}}$ and $\underline{\hat{b}}$ making angle α and $-\beta$ with $+$ive $x-$axis.

Then $\underline{\hat{a}}=OA=\cos\alpha\,\underline{\hat{i}}+\sin\alpha\,\underline{\hat{j}}$

and $\underline{\hat{b}}=OB=\cos(-\beta)\,\underline{\hat{i}}+\sin(-\beta)\,\underline{\hat{j}}$

$=\cos\beta\,\underline{\hat{i}}-\sin\beta\,\underline{\hat{j}}$



Now

$$\underline{\hat{a}}\cdot\underline{\hat{b}}=\left(\cos\alpha\,\underline{\hat{i}}+\sin\alpha\,\underline{\hat{j}}\right)\cdot\left(\cos\beta\,\underline{\hat{i}}-\sin\beta\,\underline{\hat{j}}\right)$$

$\Rightarrow\ \underline{\hat{a}}\cdot\underline{\hat{b}}=\cos\alpha\cos\beta-\sin\alpha\sin\beta$ ....(i)

But we have $\angle AOB=\alpha+\beta$

$\Rightarrow\ \underline{\hat{a}}\cdot\underline{\hat{b}}=|\underline{\hat{a}}||\underline{\hat{b}}|\cos(\alpha+\beta)$

$=(1)(1)\cos(\alpha+\beta) \qquad \because\ |\underline{\hat{a}}|=|\underline{\hat{b}}|=1$

$\Rightarrow\ \underline{\hat{a}}\cdot\underline{\hat{b}}=\cos(\alpha+\beta)$ ....(ii)

Comparing (i) and (ii), we have

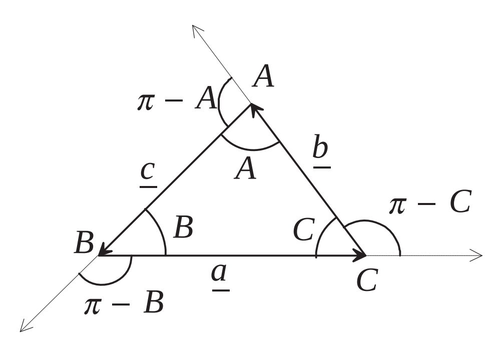
$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

Question # 12

Consider $\underline{a}$, $\underline{b}$ and $\underline{c}$ are vectors along the sides of triangle BC, CA and AB, also let $|\underline{a}|=a$, $|\underline{b}|=b$ and $|\underline{c}|=c$

then form triangle,

$\underline{a}+\underline{b}+\underline{c}=0$ ....(i)



(i) $\Rightarrow\ \underline{b}=-\underline{a}-\underline{c}$

Taking dot product of above with $\underline{b}$, we have

$$\begin{aligned}\underline{b}\cdot\underline{b}&=(-\underline{a}-\underline{c})\cdot\underline{b}\\ |\underline{b}|^2&=-\underline{a}\cdot\underline{b}-\underline{c}\cdot\underline{b}\\ &=-|\underline{a}||\underline{b}|\cos(\pi-C)-|\underline{c}||\underline{b}|\cos(\pi-A)\\ &=|\underline{a}||\underline{b}|\cos C+|\underline{c}||\underline{b}|\cos A \qquad \because\ \cos(\pi-B)=-\cos B\end{aligned}$$

$$\Rightarrow\ b^2 = ab\cos C + cb\cos A$$
$$\Rightarrow\ b = a\cos C + c\cos A \qquad \div\text{ing by } b$$

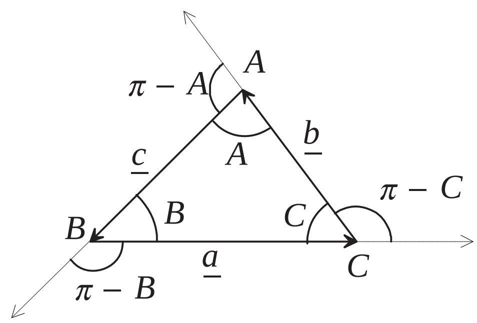
(ii) From equation (i)

$$\underline{c} = -\underline{a} - \underline{b}$$

Taking dot product of above equation with $\underline{c}$.

$$\underline{c}\cdot\underline{c} = \left(-\underline{a} - \underline{b}\right)\cdot\underline{c}$$

Now do yourself as above.



(iii) From equation (i)

$$\underline{b} = -\underline{a} - \underline{c}$$

Taking dot product of above equation with $\underline{b}$

$$\underline{b}\cdot\underline{b} = (-\underline{a} - \underline{c})\cdot\underline{b}$$
$$= (-\underline{a} - \underline{c})\cdot(-\underline{a} - \underline{c}) \qquad \because\ \underline{b} = -\underline{a} - \underline{c}$$
$$\left|\underline{b}\right|^2 = -\underline{a}\cdot\left(-\underline{a} - \underline{c}\right) - \underline{c}\cdot\left(-\underline{a} - \underline{c}\right)$$
$$= \underline{a}\cdot\underline{a} + \underline{a}\cdot\underline{c} + \underline{c}\cdot\underline{a} + \underline{c}\cdot\underline{c}$$
$$= \underline{a}\cdot\underline{a} + \underline{a}\cdot\underline{c} + \underline{a}\cdot\underline{c} + \underline{c}\cdot\underline{c} \qquad \because\ \underline{a}\cdot\underline{b} = \underline{b}\cdot\underline{a}$$
$$= \underline{a}\cdot\underline{a} + 2\,\underline{a}\cdot\underline{c} + \underline{c}\cdot\underline{c}$$
$$= \left|\underline{a}\right|^2 + 2\left|\underline{a}\right|\left|\underline{c}\right|\cos\left(\pi - B\right) + \left|\underline{c}\right|^2$$
$$\Rightarrow\ b^2 = a^2 + ac\left(-\cos B\right) + c^2 \qquad \because\ \cos\left(\pi - B\right) = -\cos B$$

Hence $\quad b^2 = c^2 + a^2 - 2ca\cos B$

(iv) From equation (i)

$$\underline{c} = -\underline{a} - \underline{b}$$

Taking dot product of above equation with $\underline{c}$

$$\underline{c}\cdot\underline{c} = \left(-\underline{a} - \underline{b}\right)\cdot\underline{c}$$
$$= (-\underline{a} - \underline{b})\cdot(-\underline{a} - \underline{b}) \qquad \because\ \underline{c} = -\underline{a} - \underline{b}$$

Now do yourself as above (iii)