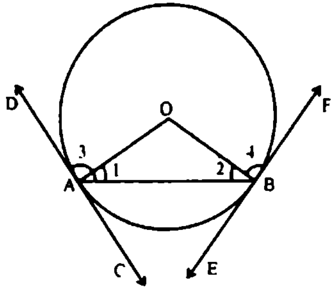


Given:

$\overline{AB}$ is the chord of a circle with centre O.

CAD is the tangent at pointy and EBF is another tangent at point B.

To prove:

$m\angle BAD = m\angle ABF$

Construction:

Join O with A and O with B so that we form a ΔOAB
then write $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$ as shown in the Figure.

Proof:

Statements	Reasons
In Δ OAB	
$\therefore m\overline{OA} = m\overline{OB}$	Radii of the same circle.
$\therefore m\angle 1 = m\angle 2$ (i)	Angles opp. to equal sides of ΔOAB
Also $\overline{OA} \perp \overline{CD}$	Radius is $\perp$ to the tangent line
$\therefore m\angle 3 = \angle OAD = 90°$ (ii)	
Similarly $\overline{OB} \perp \overline{EF}$	Radius is $\perp$ to the tangent
$\therefore m\angle A = m\angle OBF = 90°$ (iii)	Using (ii) and (iii)
Hence $m\angle 3 = m\angle 4$ (iv)	Adding (i) and (iv)
$\Rightarrow$ $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 4$	
i.e., $m\angle BAD = m\angle ABF$	

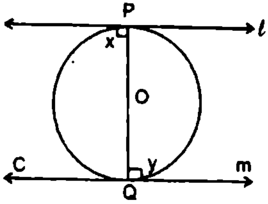
SOLVED EXERCISE 10.1

1. Prove that the tangents drawn at the ends of a diameter in a given circle must be parallel and conversely.

Solution:

Given:

Let ℓ and m be two tangents to the circle at the end points of a diameter $\overline{PQ}$.



To prove: $\ell \parallel m$

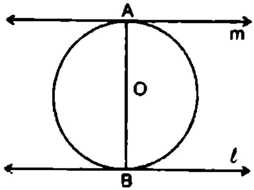
Proof:

Statements	Reasons
$OP \perp \ell$, $OQ \perp m$	$\therefore$ A tangent at any point of a circle is $\perp$ to the radius through the point of contact.
$\angle x = 90^\circ$, $\angle y = 90^\circ$	
$\Rightarrow m\angle x = m\angle y = 90^\circ$	
But:	
$m\angle x$ and $\angle y$ are alternate angle.	
Hence, $\ell \parallel m$.	

Conversely: parallel tangents of a circle must pass through its centre.

Given:

Let ℓ and m are tangent to the circle at the ends of diameter $\overline{AB}$.
To the centre O. and $AB \perp \ell$ and $AB \perp m$.



To prove:

$\overline{AB}$ passes through the centre (diameter)

Proof:

If $\overline{AB}$ does not pass through the centre join $\overline{OB}$.
$\overline{OB}$ is radius and ℓ is a tangent at B.

So that

$\overline{OB} \perp \ell$ or
But $\overline{AB} \perp \ell$. (given).
$\therefore$ $\overline{OB}$ coincides with $\overline{AB}$.

Hence, $\overline{AB}$ passes through the centre.

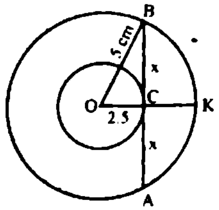
2. The diameters of two concentric circles are 10 cm and 5cm respectively. Look for the length of any chord of the outer circle which touches the inner one.

Solution:

In a triangle OCB.

$$\left(\overline{OB}\right)^2 = \left(\overline{OC}\right)^2 + \left(\overline{CB}\right)^2$$

$$\Rightarrow \quad \left(\overline{CB}\right)^2 = \left(\overline{OB}\right)^2 - \left(\overline{OC}\right)^2$$
$$= (5)^2 - (2.5)^2$$
$$= 25 - 6.25$$
$$= 18.75$$
$$\Rightarrow \quad \overline{CB} = \sqrt{18.75}$$
$$\overline{AB} = 2\overline{CB} \; 2x = 2\sqrt{18.75} = 8.7\text{cm}$$

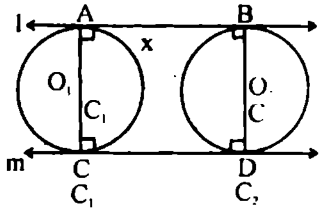


3. AB and CD are the common tangents drawn to the pair of circles. If A and C are the points of tangency of 1st circle where B and D are the points of tangency of 2nd circle, then prove that AC || BD.

Solution:

Given:

Two circle C_1 and C_2. Points of tangency of C_1 is A and C and points of tangency of C_2 is B and D



To Prove:

$\overline{AC} \parallel \overline{BC}$

Proof:

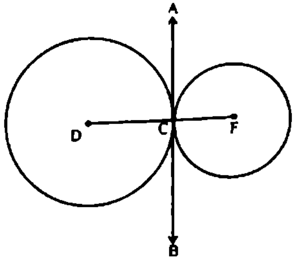
Statements	Reasons
In circle "C_1"	Tangent is perpendicular to the circle
and $\ell \parallel m$	
$m\angle CAB = 90^\circ$ ________ (i)	
and in circle "C_2"	
$\ell \parallel m$	Proved
and $m\angle ABC = 90^\circ$ ________ (ii)	Tangent is perpendicular to the circle
$\Rightarrow \angle CAB \cong \angle ABD$	by (i) 4 (ii)
Similarly $\angle ACD \cong BDC$	
Therefore:	
ABCD is rectangle	
$\therefore \overline{AC} \parallel \overline{BC}$	Parallel sides of a rectangle.

THEOREM 4 (A)

10.1 (iv) If two circles touch externally then the distance between their centres is equal to the sum of their radii.

Given:

Two circles with centres D and F respectively touch each other externally at point. So that $\overline{CD}$ and $\overline{CF}$ are respectively the radii of the two circles.



To prove:

(i) Point C lies on the join of centres D and F.

(ii) $m\overline{DF} = m\overline{DC} + m\overline{CF}$

Construction:

Draw $\overline{ACB}$ as a common tangent to the pair of circles at C.

Join C with D and C with F.

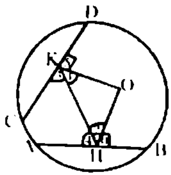
Proof:

Statements	Reasons
Both circles touch externally at C whereas $\overline{CD}$ is radial segment and ACB is the common tangent.	
$\therefore$ $m\angle ACD = 90°$ (i)	Radial segment CD $\perp$ the tangent line AB
Similarly CF is radial segment and ACS is the common tangent	
$m\angle ACF = 90°$ (ii)	Radial segment CF $\perp$ the tangent line AB
$m\angle ACD + m\angle ACF$ $90° + 90°$	Adding (i) and (ii)
$m\angle DCF = 180°$ (iii)	Sum of supplementary adjacent angles
Hence DCF is a straight line with point C between D and F	
so that $m\overline{DF} = m\overline{DC} + m\overline{CF}$	

SOLVED EXERCISE 10.2

1. **AB and CD are two equal chords in a circle with centre 0. H and K are respectively the mid points of the chords. Prove that HK makes equal angles with AB and CD.**

Solution:



Given:

$\overline{AB}$ and $\overline{CD}$ are equal chords of a circle with centre O.

To prove:

(i) $m\angle AHK = m\angle CKH$

(ii) $m\angle BHK = m\angle DKH$