

5. Complete the circle with Centre O and radius ($\overline{OA} = \overline{OB} = \overline{OC} = \overline{OP} = \overline{OE} = \overline{OF}$). This will pass through all the points A, B, C, D, E and F on the given part of the circumference.

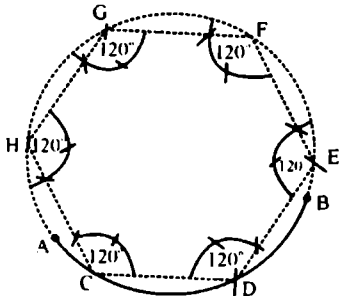
To complete the circle without finding the centre when a part of its circumference is given

Given:

\widehat{AB} is the part of circumference of a circle

Steps of Construction:

1. Take a chord \overline{CD} of reasonable length on the arc AB.
2. Construct an internal angle of 120° at point D and draw a line segment \overline{DE} equal to the length of \overline{CD} .
3. At point E again construct an internal angle of 120° and from point E draw line segment \overline{EF} of length equal to \overline{CD} etc.
4. Continue this practice until we reach at the starting point.
5. Now join the points D, E, F, G, H and C by arcs DE, EF and FG and HC all having length equal to the length of arc CD.



As a result we get a circle including the given part of circumference.

SOLVED EXERCISE 13.1

Q1. Divide an arc of any length.

(i) Into three equal parts.

Solution:

Steps of construction:

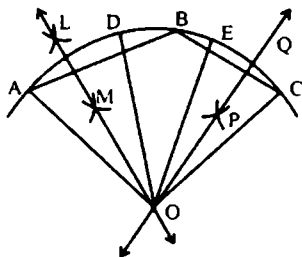
1. Draw an arc \widehat{ABC} .
2. Join A with B and B with C.
3. Draw \overline{LM} and \overline{PQ} right

bisectors of $\angle A$ and $\angle C$ respectively. \overline{LM} and \overline{PQ} intersect at point O.

4. Divide the arc ABC in three equal parts, such that $\widehat{AD} = \widehat{DE} = \widehat{EC}$

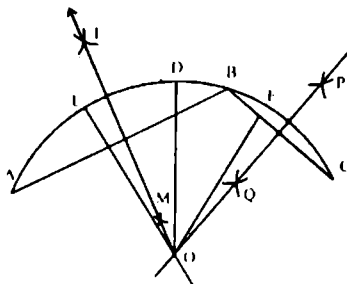
(ii) Into four equal parts

Solution:



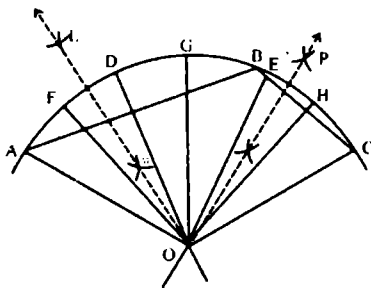
Steps of construction:

1. Draw an arc \widehat{ABC} .
2. Join A with B and B with C.
3. Divide the arc ABC in four equal parts, such that $\widehat{AE} = \widehat{ED} = \widehat{DF} = \widehat{FC}$.
4. Draw \overline{LM} and \overline{PQ} right bisectors of \overline{AB} and \overline{BC} respectively. \overline{LM} and \overline{PQ} intersect at point O.



(iii) Into six equal parts

Solution:



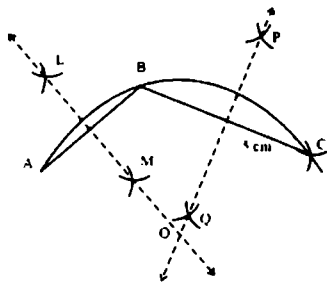
1. Draw an arc \widehat{ABC} .
2. Join A with B and B with C.
3. Divide the arc ABC in six equal parts, such that $\widehat{AF} = \widehat{FD} = \widehat{DG} = \widehat{GE} = \widehat{EH} = \widehat{HC}$.
4. Draw \overline{LM} and \overline{PQ} right bisectors of \overline{AB} and \overline{BC} respectively. \overline{LM} and \overline{PQ} intersect at point O.

2. Practically find the centre of an arc ABC.

Solution:

Steps of Construction:

1. Draw an arc \widehat{ABC} .
2. Join A with B and B with C.
3. Draw LM and PQ right bisectors of AB and BC respectively. LM and PQ intersect at point O.
4. O is the required centre of an arc ABC.

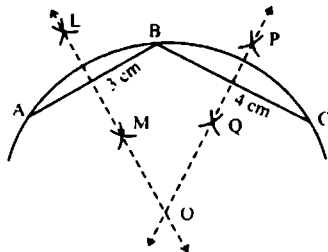


3. (i) If $|\overline{AB}| = 3 \text{ cm}$ and $|\overline{BC}| = 4 \text{ cm}$ are the lengths of two chords of an arc, then locate the centre of the arc.

Solution:

Steps of Construction:

1. Draw an arc \widehat{ABC} .
2. Draw $|\overline{AB}| = 3 \text{ cm}$ and $|\overline{BC}| = 4 \text{ cm}$
3. Draw \overline{LM} and \overline{PQ} right bisectors of \overline{AB} and \overline{BC} respectively. \overline{LM} and \overline{PQ} intersect at point O.
4. O is the required centre of an arc ABC.

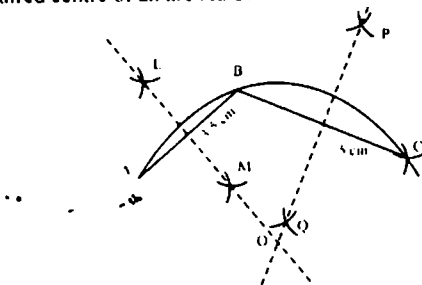


- (ii) If $|\overline{AB}| = 3.5 \text{ cm}$ and $|\overline{BC}| = 5 \text{ cm}$ are the lengths of two chords of an arc, then locate the centre of the arc.

Solution:

Steps of Construction:

1. Draw an arc \widehat{ABC} .
2. Draw $|\overline{AB}| = 3.5 \text{ cm}$ and $|\overline{BC}| = 5 \text{ cm}$.
3. Draw \overline{LM} and \overline{PQ} right bisectors of \overline{AB} and \overline{BC} respectively. \overline{LM} and \overline{PQ} intersect at point O.
4. O is the required centre of an arc ABC.

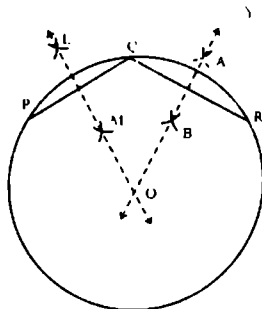


4. For an arc draw two perpendicular bisectors of the chords \overline{PQ} and \overline{QR} of this arc, construct a circle through P, Q and R.

Solution:

Steps of Construction:

1. Draw an arc \overline{ABC} .
2. Join P with Q and Q with R.
3. Draw \overline{LM} and \overline{PQ} right bisectors of \overline{PQ} and \overline{QR} respectively. \overline{LM} and \overline{AB} intersect at pint O.
4. O is the required centre of an arc ABC.
5. Draw a circle with radius $\overline{OP} = \overline{OQ} = \overline{OR}$ having centre at O, which is the required circle.

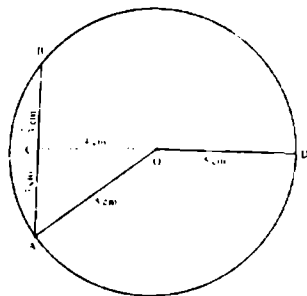


5. Describe circle of radius 5 cm passing through points A and B, 6 cm apart. Also find distance from the centre to the line segment AB.

Solution:

Steps of Construction:

1. Draw a circle of radius 5 cm passing through points A and B, 6 cm apart.
2. The distance from the centre O to the line segment AB is 4 cm.



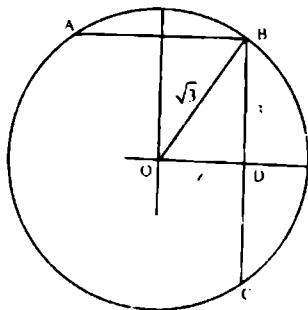
6. If $|\overline{AB}| = 4$ cm and $|\overline{BC}| = 6$ m, such that \overline{AB} is perpendicular to \overline{BC} , construct a circle through points A, B and C. Also measure its radius.

Solution:

Steps of Construction:

1. Draw $|\overline{AB}| = 4$ cm and $|\overline{BC}| = 6$ cm, such that $|\overline{AB}|$ is perpendicular to \overline{BC} .
2. Now in $\triangle OBD$, by Pythagoras theorem, we have

$$\begin{aligned} |\overline{OB}| &= \sqrt{|\overline{OD}|^2 + |\overline{BD}|^2} \\ &= \sqrt{(2)^2 + (3)^2} \\ &= \sqrt{4 + 9} \\ &= \sqrt{13} \quad (\text{Radius}) \end{aligned}$$



3. Draw a circle of radius $|\overline{OA}| = |\overline{OB}| = |\overline{OC}| = \sqrt{13}$ cm, which passes through points A, B and C.

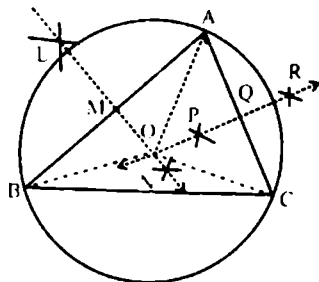
Circumscribe a circle about a given triangle:

Given:

Triangle ABC.

Steps of Construction:

1. Draw \overline{LMN} as perpendicular bisector of side \overline{AB} .
2. Draw \overline{PQR} as perpendicular bisector of side \overline{AC} .
3. \overline{LN} and \overline{PP} intersect at point O.
4. With centre O and radius
 $m \overline{OA} = m \overline{OB} = m \overline{OC}$, draw a circle.



This circle will pass through A, B and C whereas O is the circumcentre of the circumscribed circle.

Remember: The circle passing through the vertices of triangle ABC is known as **circumcircle**, its radius as **circumradius** and centre as **circumcentre**.

Inscribe a circle in a given triangle:

Given:

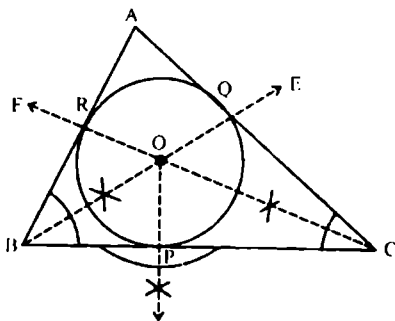
A triangle ABC.

Steps of Construction:

1. Draw \overline{BE} and \overline{CF} to bisect the angles ABC and ACB respectively. Rays \overline{BE} and \overline{CF} intersect each other at point O.
2. O is the centre of the inscribed circle.

From O draw \overline{OP} perpendicular to \overline{BC} .

With centre O and radius \overline{OP} draw a circle. This circle is the inscribed circle of triangle ABC.



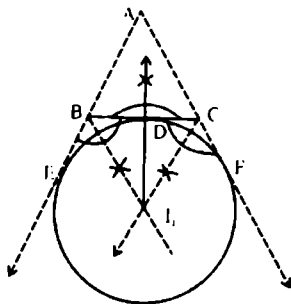
Remember:

A circle which touches the three sides of a triangle internally is known as **incircle**, its radius as **in-radius** and centre as **in centre**.

Describe a circle to a given triangle:

Given:

A triangle ABC



Steps of Construction:

1. Produce the sides \overline{AB} and \overline{AC} of $\triangle ABC$.
2. Draw bisectors of exterior angles $\angle ABC$ and $\angle ACB$.
These bisectors of exterior angles meet at I_1 .
3. From I_1 draw perpendicular on Side \overline{BC} of $\triangle ABC$.
Which I_1D intersect BC at D . I_1D is the radius of the escribed circle with centre at I_1 .
4. Draw the circle with radius I_1D and centre at I_1 that will touch the side BC of the $\triangle ABC$ externally and the produced sides AB and AC .

Escribed circle:

The circle touching one side of the triangle externally and two produced sides internally is called escribed circle (e-circle). The centre of e-circle is called **e-centre** and radius is called **e-radius**.

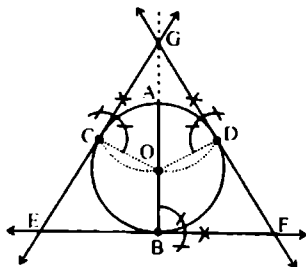
Circumscribe an equilateral triangle about a given circle

Given:

A circle with centre O of reasonable radius.

Steps of Construction:

1. Draw \overline{AB} , the diameter of the circle, for locating.
2. Draw an arc of radius $m\ OA$ with centre at A for locating points C and D on the circle.
3. Join O to the points C and D .
4. Draw tangents to the circle at points B, C and D .
5. These tangents intersect at points E, F and G .



(v) Inscribe an equilateral triangle in a given circle.

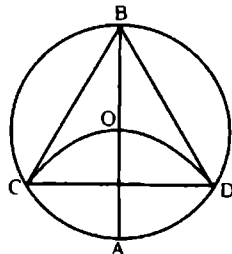
Given:

A circle with centre at O .

Steps of Construction:

1. Draw any diameter \overline{AB} of the circle.
2. Draw an arc of radius OA from point A . The arc cuts the circle at points C and D .
3. Join the points B, C and D to form straight line segments \overline{BC} , \overline{CD} and \overline{BD} .

Triangle BCD is the required inscribed equilateral triangle.



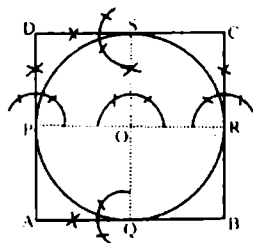
Circumscribe a square: about a given circle:

Given:

A circle with centre at O.

Steps of Construction:

1. Draw two diameters \overline{PQ} and \overline{OS} which bisect each other at right angle.
2. At points P, Q, R and S draw tangents to the circle.
3. Produce the tangents to meet each other at A, B, C and D. ABCD is the required circumscribed square.



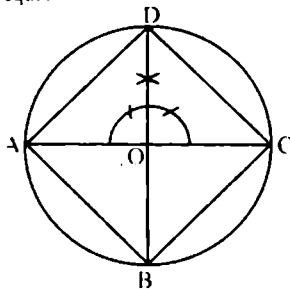
Inscribe a square in a given circle:

Given:

A circle, with centre at O.

Steps of Construction:

1. Through O draw two diameters \overline{AC} and \overline{BD} which bisect each other at right angle.
 2. Join A with B, B with C, C with D, and D with A.
- ABCD is the required square inscribed in the circle.



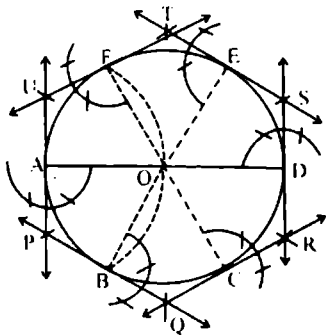
Circumscribe a regular hexagon about a given circle:

Given:

A circle with centre at O.

Steps of Construction:

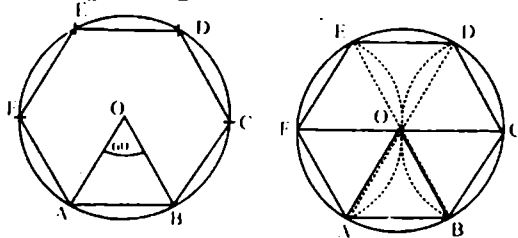
1. Draw any diameter \overline{AD} .
2. From point A draw an arc of radius \overline{AO} (the radius of the circle), which cuts the circle at points B and F.
3. Join B with O and extend it to meet the circle at E.
4. Join F with O and extend it to meet the circle at C.
5. Draw tangents to the circle at points A, B, C, D, E and F intersecting one another at points P, Q, R, S, T and U.



S, T and U respectively.

6. Thus PQRSTU is the circumscribed regular hexagon.

Inscribe a regular hexagon in a given circle.



Given:

A circle, with centre at O.

Steps of Construction:

1. Take any point A on the circle and point with O.
2. From point A, draw an arc of radius \overline{AO} which intersects the circle at points B and F.
3. Join (O and A with points B and F).
4. $\triangle OAB$ and $\triangle OAF$ are equilateral therefore $\angle AOB$ and $\angle AOF$ are of measure 60° i.e., $m\widehat{AO} = m\widehat{AB} = m\widehat{AF}$.
5. Produce \overline{FO} to meet the circle at C. Join B to C. Since in $\angle BOC = 60^\circ$ therefore $m\widehat{BC} = m\widehat{OA}$.
6. From C and E, draw arcs of radius \overline{OA} , which intersect the circle at points D and E.
7. Join C to D, D to E and E' to F ultimately. We have $m\widehat{OA} = m\widehat{OB} = m\widehat{OC} = m\widehat{OD} = m\widehat{OE} = m\widehat{OF}$

Thus the figure ABCDEF is a regular hexagon inscribed in the circle.

SOLVED EXERCISE 13.2

1. Circumscribe a circle about a triangle ABC with sides

$$|\overline{AB}| = 6 \text{ cm}, \quad |\overline{BC}| = 3 \text{ cm}, \quad |\overline{CA}| = 4 \text{ cm}$$

Also measure its circum radius.

Solution:

Given:

Three sides

$$|\overline{AB}| = 6 \text{ cm}, \quad |\overline{BC}| = 3 \text{ cm}, \quad |\overline{CA}| = 4 \text{ cm}.$$

Required: