EXERCISE 4.4

◈ Point of intersection of lines

Let $l_1:\ a_1x+b_1y+c_1=0$

$l_2:\ a_2x+b_2y+c_2=0$ be non-parallel lines.

Let $P(x_1,y_1)$ be the point of intersection of l_1 and l_2. Then

$$a_1x_1+b_1y_1+c_1=0\ldots\ldots\ldots\ldots(i)$$
$$a_2x_1+b_2y_1+c_2=0\ldots\ldots\ldots\ldots(ii)$$

Solving (i) and (ii) simultaneously, we have

$$\frac{x_1}{b_1c_2-b_2c_1}=\frac{-y_1}{a_1c_2-a_2c_1}=\frac{1}{a_1b_2-a_2b_1}$$

$$\Rightarrow \frac{x_1}{b_1c_2-b_2c_1}=\frac{1}{a_1b_2-a_2b_1} \quad\text{and}\quad \frac{-y_1}{a_1c_2-a_2c_1}=\frac{1}{a_1b_2-a_2b_1}$$

$$\Rightarrow x_1=\frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1} \quad\text{and}\quad y_1=-\frac{a_1c_2-a_2c_1}{a_1b_2-a_2b_1}$$

Hence $\left(\dfrac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1},-\dfrac{a_1c_2-a_2c_1}{a_1b_2-a_2b_1}\right)$ is the point of intersection of l_1 and l_2.

◈ Equation of line passing through the point of intersection.

Let $l_1:\ a_1x+b_1y+c_1=0$

$l_2:\ a_2x+b_2y+c_2=0$

Then equation of line passing through the point of intersection of l_1 and l_2 is

$$l_1+k\,l_2=0 \quad,\ \text{where } k \text{ is constant.}$$

i.e. $\quad a_1x+b_1y+c_{11}+k\left(a_2x+b_2y+c_2\right)=0$

◈ Question # 1

(a) $\quad l_1:\ x-2y+1=0$

$l_2:\ 2x-y+2=0$

Slope of $l_1=m_1=-\dfrac{1}{-2}=\dfrac{1}{2}$

Slope of $l_2=m_2=-\dfrac{2}{-1}=2$

$\because\ m_1\neq m_2$, therefore lines are intersecting.

Now if (x,y) is the point of intersection of l_1 and l_2 then

$$\frac{x}{(-2)(2)-(-1)(1)}=\frac{-y}{(1)(2)-(2)(1)}=\frac{1}{(1)(-1)-(2)(-2)}$$

$$\Rightarrow \frac{x}{-4+1}=\frac{-y}{2-2}=\frac{1}{-1+4}$$

$$\Rightarrow \frac{x}{-3}=\frac{-y}{0}=\frac{1}{3}$$

$$\Rightarrow \frac{x}{-3}=\frac{1}{3} \quad\text{and}\quad \frac{-y}{0}=\frac{1}{3}$$

$$\Rightarrow x=\frac{-3}{3} \quad\text{and}\quad y=-\frac{0}{3}$$

$$\Rightarrow x=-1 \quad\text{and}\quad y=0$$

Hence $(-1,0)$ is the point of intersection.

(b)
$$l_1:\ 3x+y+12=0$$
$$l_2:\ x+2y-1=0$$

Slope of $l_1 = m_1 = -\dfrac{3}{1} = -3$

Slope of $l_2 = m_2 = -\dfrac{1}{2}$

$\because\ m_1 \neq m_2$, therefore lines are intersecting.

Now if (x,y) is the point of intersection of l_1 and l_2 then
$$\frac{x}{-1-24} = \frac{-y}{-3-12} = \frac{1}{6-1}$$
$$\Rightarrow \frac{x}{-25} = \frac{-y}{-15} = \frac{1}{5}$$
$$\Rightarrow \frac{x}{-25} = \frac{1}{5} \quad \text{and} \quad \frac{-y}{-15} = \frac{1}{5}$$
$$\Rightarrow x = \frac{-25}{5} = -5 \quad \text{and} \quad y = \frac{15}{5} = 3$$

Hence $(-5,3)$ is the point of intersection.

(c) *Do yourself as above.*

◈ Question # 2 (a)

Let $l_1:\ 2x+5y-8=0$

$l_2:\ 3x-4y-6=0$

Equation of line passing through point of intersection of l_1 and l_2 is
$$2x+5y-8+k(3x-4y-6)=0 \ldots\ldots\ldots\ldots (i)$$

Since $(2,-9)$ lies on (i) therefore put $x=2$ and $y=-9$ in (i)
$$2(2)+5(-9)-8+k(3(2)-4(-9)-6)=0$$
$$\Rightarrow 4-45-8+k(6+36-6)=0$$
$$\Rightarrow -49+36k=0$$
$$\Rightarrow 36k=49 \qquad \Rightarrow k=\frac{49}{36}$$

Putting value of k in (i)
$$2x+5y-8+\frac{49}{36}(3x-4y-6)=0$$
$$\Rightarrow 72x+180y-288+49(3x-4y-6)=0 \qquad \times\text{ing by } 36$$
$$\Rightarrow 72x+180y-288+147x-196y-294=0$$
$$\Rightarrow 219x-16y-582=0$$ is the required equation.

◈ Question # 2 (b)

Let $l_1:\ x-y-4=0$

$l_2:\ 7x+y+20=0$

$l_3:\ 6x+y-14=0$

Let l_4 be a line passing through point of intersection of l_1 and l_2, then
$$l_4:\ l_1+k\,l_2=0$$
$$\Rightarrow x-y-4+k(7x+y+20)=0 \ldots\ldots\ldots\ldots (i)$$
$$\Rightarrow (1+7k)x+(-1+k)y+(-4+20k)=0$$

Slope of $l_4 = m_1 = -\dfrac{1+7k}{-1+k}$

Slope of $l_3 = m_2 = -\frac{6}{1} = -6$

(i) If l_3 and l_4 are parallel then

$$m_1 = m_2$$
$$\Rightarrow -\frac{1+7k}{-1+k} = -6$$
$$\Rightarrow 1+7k = 6(-1+k) \qquad \Rightarrow 1+7k = -6+6k$$
$$\Rightarrow 7k-6k = -6-1 \qquad \Rightarrow k = -7$$

Putting value of k in (i)

$$x-y-4-7(7x+y+20) = 0$$
$$\Rightarrow x-y-4-49x-7y-140 = 0$$
$$\Rightarrow -48x-8y-144 = 0$$
$$\Rightarrow 6x+y+18 = 0$$

is the required equation

(ii) If l_3 and l_4 are $\perp$ then

$$m_1 m_2 = -1$$
$$\Rightarrow \left(-\frac{1+7k}{-1+k}\right)(-6) = -1$$
$$\Rightarrow 6(1+7k) = -(-1+k) \qquad \Rightarrow 6+42k = 1-k$$
$$\Rightarrow 42k+k = 1-6 \qquad \Rightarrow 43k = -5 \qquad \Rightarrow k = -\frac{5}{43}$$

Putting in (i) we have

$$x-y-4-\frac{5}{43}(7x+y+20) = 0$$
$$\Rightarrow 43x-43y-172-5(7x+y+20) = 0$$
$$\Rightarrow 43x-43y-172-35x-5y-100 = 0$$
$$\Rightarrow 8x-48y-272 = 0$$
$$\Rightarrow x-6y-34 = 0$$ is the required equation.

◈ *Question # 2(c)*

Suppose $l_1:\ x+2y+3 = 0$

$l_2:\ 3x+4y+7 = 0$

Equation of line passing through the intersection of l_1 and l_2 is given by:

$$x+2y+3+k(3x+4y+7) = 0 \quad \ldots\ldots\ldots\ldots (i)$$
$$\Rightarrow (1+3k)x+(2+4k)y+(3+7k) = 0$$
$$\Rightarrow (1+3k)x+(2+4k)y = -(3+7k)$$
$$\Rightarrow \frac{(1+3k)x}{-(3+7k)} + \frac{(2+4k)y}{-(3+7k)} = 1$$
$$\Rightarrow \frac{x}{-(3+7k)/(1+3k)} + \frac{y}{-(3+7k)/(2+4k)} = 1$$

Which is two-intercept form of equation of line with

$$x\text{-intercept} = \frac{-(3+7k)}{(1+3k)} \quad \text{and} \quad y\text{-intercept} = \frac{-(3+7k)}{(2+4k)}$$

We have given

$$x\text{-intercept} = y\text{-intercept}$$
$$\Rightarrow \frac{-(3+7k)}{(1+3k)} = \frac{-(3+7k)}{(2+4k)}$$

$$\Rightarrow \frac{1}{(1+3k)} = \frac{1}{(2+4k)} \quad \Rightarrow (2+4k) = (1+3k)$$

$$\Rightarrow 4k-3k = 1-2 \quad \Rightarrow k=-1$$

Putting value of k in (i)

$$x+2y+3-1(3x+4y+7) = 0$$

$$\Rightarrow x+2y+3-3x-4y-7 = 0 \quad \Rightarrow -2x-2y-4 = 0$$

$$\Rightarrow x+y+2 = 0$$

is the required equation.

◈ *Question # 3*

Let $l_1: \ 16x-10y-33=0$

$l_2: \ 12x+14y+29=0$

$l_3: \ x-y+4=0$

$l_4: \ x-7y+2=0$

For point of intersection of l_1 and l_2

$$\frac{x}{-290+462} = \frac{-y}{464+396} = \frac{1}{224+120}$$

$$\Rightarrow \frac{x}{172} = \frac{-y}{860} = \frac{1}{334}$$

$$\Rightarrow \frac{x}{172} = \frac{1}{334} \quad \text{and} \quad \frac{-y}{860} = \frac{1}{334}$$

$$\Rightarrow x = \frac{172}{334} = \frac{1}{2} \quad \text{and} \quad y = -\frac{860}{334} = -\frac{5}{2}$$

$\Rightarrow \left(\frac{1}{2}, -\frac{5}{2}\right)$ is a point of intersection of l_1 and l_2.

For point of intersection of l_3 and l_4.

$$\frac{x}{-2+28} = \frac{-y}{2-4} = \frac{1}{-7+1}$$

$$\Rightarrow \frac{x}{26} = \frac{-y}{-2} = \frac{1}{-6}$$

$$\Rightarrow \frac{x}{26} = \frac{1}{-6} \quad \text{and} \quad \frac{-y}{-2} = \frac{1}{-6}$$

$$\Rightarrow x = \frac{26}{-6} = -\frac{13}{3} \quad \text{and} \quad y = \frac{2}{-6} = -\frac{1}{3}$$

$\Rightarrow \left(-\frac{13}{3}, -\frac{1}{3}\right)$ is a point of intersection of l_3 and l_4.

Now equation of line passing through $\left(\frac{1}{2}, -\frac{5}{2}\right)$ and $\left(-\frac{13}{3}, -\frac{1}{3}\right)$

$$y + \frac{5}{2} = \frac{-\frac{1}{3} + \frac{5}{2}}{-\frac{13}{3} - \frac{1}{2}}\left(x - \frac{1}{2}\right)$$

$$\Rightarrow y + \frac{5}{2} = \frac{\frac{13}{6}}{-\frac{29}{6}}\left(x - \frac{1}{2}\right) \quad \Rightarrow y + \frac{5}{2} = -\frac{13}{29}\left(x - \frac{1}{2}\right)$$

$$\Rightarrow 29y + \frac{145}{2} = -13x + \frac{13}{2} \qquad \Rightarrow 13x - \frac{13}{2} + 29y + \frac{145}{2} = 0$$

$$\Rightarrow 13x + 29y + 66 = 0$$

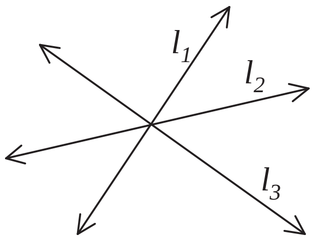
is the required equation.

◈ *Three Concurrent Lines*

Suppose $l_1:\ a_1x+b_1y+c_1=0$

$l_2:\ a_2x+b_2y+c_2=0$

$l_3:\ a_3x+b_3y+c_3=0$



If l_1, l_2 and l_3 are concurrent (intersect at one point) then

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}=0$$

See proof on book at page 208

◈ *Question # 4*

Assume that $l_1:\ y=m_1x+c_1$

$\Rightarrow\ m_1x-y+c_1=0$

$l_2:\ y=m_2x+c_2$

$\Rightarrow\ m_2x-y+c_2=0$

$l_3:\ y=m_3x+c_3$

$\Rightarrow\ m_3x-y+c_3=0$

If l_1, l_2 and l_3 are concurrent then

$$\begin{vmatrix} m_1 & -1 & c_1 \\ m_2 & -1 & c_2 \\ m_3 & -1 & c_3 \end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix} m_1 & -1 & c_1 \\ m_2-m_1 & 0 & c_2-c_1 \\ m_3-m_1 & 0 & c_3-c_1 \end{vmatrix}=0 \qquad \text{by} \quad \begin{matrix} R_2-R_1 \\ R_3-R_1 \end{matrix}$$

Expanding by C_2

$$-(-1)\big[(m_2-m_1)(c_3-c_1)-(m_3-m_1)(c_2-c_1)\big]+0-0=0$$

$$\Rightarrow \big[(m_2-m_1)(c_3-c_1)-(m_3-m_1)(c_2-c_1)\big]=0$$

$$\Rightarrow (m_2-m_1)(c_3-c_1)=(m_3-m_1)(c_2-c_1)$$

is the required condition.

◈ *Question # 5*

Let $l_1:\ 2x-3y-1=0$

$l_2:\ 3x-y-5=0$

$l_3:\ 3x+py+8=0$

Since l_1, l_2 and l_3 meets at a point i.e. concurrent therefore

$$\begin{vmatrix} 2 & -3 & -1 \\ 3 & -1 & -5 \\ 3 & p & 8 \end{vmatrix}=0$$

$$\Rightarrow\ 2(-8+5p)+3(24+15)-1(3p+3)=0$$

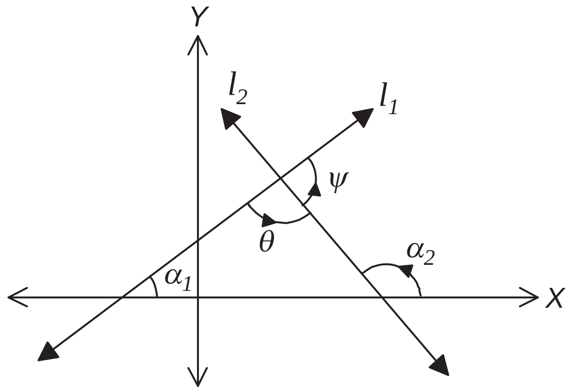
$$\Rightarrow\ -16+10p+72+45-3p-3=0$$

$$\Rightarrow\ 7p+98=0 \quad \Rightarrow\ 7p=-98$$

$$\Rightarrow\ p=-\frac{98}{7} \quad \Rightarrow\ \boxed{p=-14}$$

◈ *Angle between lines*

Let l_1 and l_2 be two lines. If α_1 and α_2 be inclinations and m_1 and m_2 be slopes of lines l_1 and l_2 respectively, Let θ be a angle from line l_1 to l_2 then θ is given by



$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

See proof on book at page 219

◈ *Question # 6*

Let $l_1: \; 4x - 3y - 8 = 0$

$l_2: \; 3x - 4y - 6 = 0$

$l_3: \; x - y - 2 = 0$

To check l_1, l_2 and l_3 are concurrent, let

$$\begin{vmatrix} 4 & -3 & -8 \\ 3 & -4 & -6 \\ 1 & -1 & -2 \end{vmatrix}$$

$$= 4(8-6) + 3(-6+6) - 8(-3+4)$$

$$= 4(2) + 3(0) - 8(1)$$

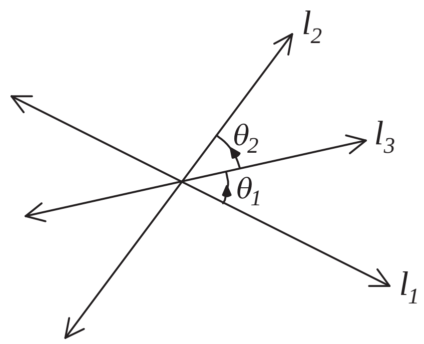
$$= 8 + 0 - 8 = 0$$

Hence l_1, l_2 and l_3 are concurrent.

Slope of $l_1 = m_1 = -\dfrac{4}{-3} = \dfrac{4}{3}$

Slope of $l_2 = m_2 = -\dfrac{3}{-4} = \dfrac{3}{4}$

Slope of $l_3 = m_3 = -\dfrac{1}{-1} = 1$



Now let θ_1 be angle from l_1 to l_3 and θ_2 be a angle from l_3 to l_2. Then

$$\tan\theta_1 = \frac{m_3 - m_1}{1 + m_3 m_1} = \frac{1 - \frac{4}{3}}{1 + (1)\left(\frac{4}{3}\right)} = \frac{-\frac{1}{3}}{\frac{7}{3}} = -\frac{1}{7} \quad \ldots\ldots\ldots\ldots (i)$$

And $$\tan\theta_2 = \frac{m_2 - m_3}{1 + m_2 m_3} = \frac{\frac{3}{4} - 1}{1 + \left(\frac{3}{4}\right)(1)} = \frac{-\frac{1}{4}}{\frac{7}{4}} = -\frac{1}{7} \quad \ldots\ldots\ldots\ldots (ii)$$

From (*i*) and (*ii*)

$$\tan\theta_1 = \tan\theta_2 \quad \Rightarrow \quad \theta_1 = \theta_2$$

$\Rightarrow$ l_3 bisect the angle formed by the first two lines.

◈ *Question # 7*

Given vertices of triangles are $A(-2,3)$, $B(-4,1)$ and $C(3,5)$.

(i) Centroid of triangle is the intersection of medians and is given by

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

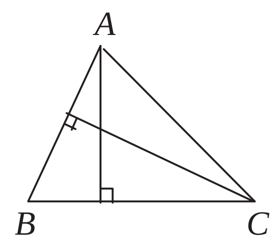
$$= \left(\frac{-2 - 4 + 3}{3}, \frac{3 + 1 + 5}{3}\right) = \left(\frac{-3}{3}, \frac{9}{3}\right) = (-1,3)$$

Hence $(-1,3)$ is the centroid of the triangle.

(ii) Orthocentre is the point of intersection of altitudes.

Slope of $\overline{AB} = m_1 = \frac{1-3}{-4+2} = \frac{-2}{-2} = 1$

Slope of $\overline{BC} = m_2 = \frac{5-1}{3+4} = \frac{4}{7}$



Since altitudes are $\perp$ to sides therefore

Slope of altitude on $\overline{AB} = -\frac{1}{m_1} = -\frac{1}{1} = -1$

Slope of altitude on $\overline{BC} = -\frac{1}{m_2} = -\frac{1}{4/7} = -\frac{7}{4}$

Equation of altitude on $\overline{AB}$ with slope -1 from $C(3,5)$

$$y-5 = -1(x-3)$$

$$\Rightarrow\ y-5 = -x+3 \quad \Rightarrow\ x-3+y-5=0$$

$$\Rightarrow\ x+y-8=0 \ \text{..........}\ (i)$$

Now equation of altitude on $\overline{BC}$ with slope $-\frac{7}{4}$ from $A(-2,3)$

$$y-3 = -\frac{7}{4}(x+2)$$

$$\Rightarrow\ 4y-12 = -7x-14 \quad \Rightarrow\ 7x+14+4y-12=0$$

$$\Rightarrow\ 7x+4y+2=0 \ \text{.............}\ (ii)$$

For point of intersection of (i) and (ii)

$$\frac{x}{2+32} = \frac{-y}{2+56} = \frac{1}{4-7}$$

$$\Rightarrow\ \frac{x}{34} = \frac{-y}{58} = \frac{1}{-3}$$

$$\Rightarrow\ \frac{x}{34} = \frac{1}{-3} \quad \text{and} \quad \frac{-y}{58} = \frac{1}{-3}$$

$$\Rightarrow\ x = -\frac{34}{3} \quad \text{and} \quad y = -\frac{58}{-3} = \frac{58}{3}$$

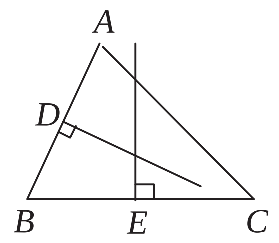
Hence $\left(-\frac{34}{3}, \frac{58}{3}\right)$ is orthocentre of triangle ABC.

(iii) Circumcentre of the triangle is the point of intersection of perpendicular bisector.

Let D and E are midpoints of side $\overline{AB}$ and $\overline{BC}$ respectively.

Then coordinate of $D = \left(\frac{-4-2}{2}, \frac{1+3}{2}\right) = \left(\frac{-6}{2}, \frac{4}{2}\right) = (-3,2)$

Coordinate of $E = \left(\frac{-4+3}{2}, \frac{1+5}{2}\right) = \left(\frac{-1}{2}, \frac{6}{2}\right) = \left(-\frac{1}{2}, 3\right)$



Slope of $\overline{AB} = m_1 = \frac{1-3}{-4+2} = \frac{-2}{-2} = 1$

Slope of $\overline{BC} = m_2 = \frac{5-1}{3+4} = \frac{4}{7}$

Slope of $\perp$ bisector on $\overline{AB} = -\frac{1}{m_1} = -\frac{1}{1} = -1$

Slope of $\perp$ bisector on $\overline{BC} = -\frac{1}{m_2} = -\frac{1}{4/7} = -\frac{7}{4}$

Now equation of $\perp$ bisector having slope -1 through $D(-3,2)$

$$y-2 = -1(x+3)$$

$$\Rightarrow\ y-2=-x-3 \qquad \Rightarrow\ x+3+y-2=0$$
$$\Rightarrow\ x+y+1=0\ \ldots\ldots\ldots\ldots\ (iii)$$

Now equation of $\perp$ bisector having slope $-\frac{7}{4}$ through $E\left(-\frac{1}{2},3\right)$

$$y-3=-\frac{7}{4}\left(x+\frac{1}{2}\right) \quad \Rightarrow\ 4y-12=-7x-\frac{7}{2}$$
$$\Rightarrow\ 7x+\frac{7}{2}+4y-12=0 \quad \Rightarrow\ 7x+4y-\frac{17}{2}=0$$
$$\Rightarrow\ 14x+8y-17=0\ \ldots\ldots\ldots\ldots\ldots\ (iv)$$

For point of intersection of (iii) and (iv)

$$\frac{x}{-17-8}=\frac{-y}{-17-14}=\frac{1}{8-14}$$
$$\Rightarrow\ \frac{x}{-25}=\frac{-y}{-31}=\frac{1}{-6}$$
$$\Rightarrow\ \frac{x}{-25}=\frac{1}{-6} \quad \text{and} \quad \frac{-y}{-31}=\frac{1}{-6}$$
$$\Rightarrow\ x=\frac{-25}{-6}=\frac{25}{6} \quad \text{and} \quad y=-\frac{31}{6}$$

Hence $\left(\frac{25}{6},-\frac{31}{6}\right)$ is the circumcentre of the triangle.

Now to check $(-1,3)$, $\left(-\frac{34}{3},\frac{58}{3}\right)$ and $\left(\frac{25}{6},-\frac{31}{6}\right)$ are collinear, let

$$\begin{vmatrix} -1 & 3 & 1 \\ -34/3 & 58/3 & 1 \\ 25/6 & -31/6 & 1 \end{vmatrix}$$
$$=-1\left(\frac{58}{3}+\frac{31}{6}\right)-3\left(-\frac{34}{3}-\frac{25}{6}\right)+1\left(\frac{1054}{18}-\frac{1450}{18}\right)$$
$$=-1\left(\frac{49}{2}\right)-3\left(-\frac{31}{2}\right)+1(-22)$$
$$=-\frac{49}{2}+\frac{93}{2}-22=0$$

Hence centroid, orthocentre and circumcentre of triangle are collinear.

◈ *Question # 8*

Let $l_1:\ 4x-3y-8=0$

$l_2:\ 3x-4y-6=0$

$l_3:\ x-y-2=0$

To check lines are concurrent, let

$$\begin{vmatrix} 4 & -3 & -8 \\ 3 & -4 & -6 \\ 1 & -1 & -2 \end{vmatrix}$$
$$=4(8-6)+3(-6+6)-8(-3+4)$$
$$=4(2)+3(0)-8(1) \quad =8+0-8\ =0$$

Hence l_1, l_2 and l_3 are concurrent.

For point of concurrency, we find intersection of l_1 and l_2 (You may choose any two lines)

$$\frac{x}{18-32}=\frac{-y}{-24+24}=\frac{1}{-16+9}$$

$$\Rightarrow \frac{x}{-14}=\frac{-y}{0}=\frac{1}{-7}$$

$$\Rightarrow \frac{x}{-14}=\frac{1}{-7} \quad \text{and} \quad \frac{-y}{0}=\frac{1}{-7}$$

$$\Rightarrow x=\frac{-14}{-7}=2 \quad \text{and} \quad y=-\frac{0}{-7}=0$$

Hence $(2,0)$ is the point of concurrency.

◈ Question # 9

Let $l_1:\ x-2y-6=0$

$l_2:\ 3x-y+3=0$

$l_3:\ 2x+y-4=0$

For point of intersection of l_1 and l_2

$$\frac{x}{-6-6}=\frac{-y}{3+18}=\frac{1}{-1+6}$$

$$\Rightarrow \frac{x}{-12}=\frac{-y}{21}=\frac{1}{5}$$

$$\Rightarrow \frac{x}{-12}=\frac{1}{5} \quad \text{and} \quad \frac{-y}{21}=\frac{1}{5}$$

$$\Rightarrow x=-\frac{12}{5} \quad \text{and} \quad y=-\frac{21}{5}$$

$\Rightarrow \left(-\frac{12}{5},-\frac{21}{5}\right)$ is the point of intersection of l_1 and l_2.

For point of intersection of l_2 and l_3.

$$\frac{x}{4-3}=\frac{-y}{-12-6}=\frac{1}{3+2}$$

$$\Rightarrow \frac{x}{1}=\frac{-y}{-18}=\frac{1}{5}$$

$$\Rightarrow \frac{x}{1}=\frac{1}{5} \quad \text{and} \quad \frac{-y}{-18}=\frac{1}{5}$$

$$\Rightarrow x=\frac{1}{5} \quad \text{and} \quad y=\frac{18}{5}$$

$\Rightarrow \left(\frac{1}{5},\frac{18}{5}\right)$ is the point of intersection of l_2 and l_3.

Now for point of intersection of l_1 and l_3

$$\frac{x}{8+6}=\frac{-y}{-4+12}=\frac{1}{1+4}$$

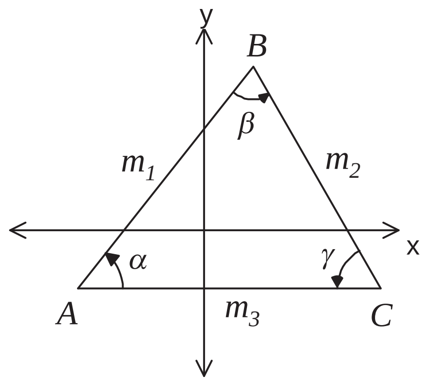
$$\Rightarrow \frac{x}{14}=\frac{-y}{8}=\frac{1}{5}$$

$$\Rightarrow \frac{x}{14}=\frac{1}{5} \quad \text{and} \quad \frac{-y}{8}=\frac{1}{5}$$

$$\Rightarrow x=\frac{14}{5} \quad \text{and} \quad y=-\frac{8}{5}$$

$\Rightarrow \left(\frac{14}{5},-\frac{8}{5}\right)$ is the point of intersection of l_1 and l_3.

Hence $\left(-\frac{12}{5},-\frac{21}{5}\right)$, $\left(\frac{1}{5},\frac{18}{5}\right)$ and $\left(\frac{14}{5},-\frac{8}{5}\right)$ are vertices of triangle made by l_1 , l_2 and l_3. We say these vertices as A,B and C respectively.



Slope of side $AB = m_1 = \dfrac{\frac{18}{5}+\frac{21}{5}}{\frac{1}{5}+\frac{12}{5}} = \dfrac{\frac{39}{5}}{\frac{13}{5}} = \dfrac{39}{13} = 3$

Slope of side $BC = m_2 = \dfrac{-\frac{8}{5}-\frac{18}{5}}{\frac{14}{5}-\frac{1}{5}} = \dfrac{-\frac{26}{5}}{\frac{13}{5}} = -\dfrac{26}{13} = -2$

Slope of side $CA = m_3 = \dfrac{-\frac{21}{5}+\frac{8}{5}}{-\frac{12}{5}-\frac{14}{5}} = \dfrac{-\frac{13}{5}}{-\frac{26}{5}} = \dfrac{13}{26} = \dfrac{1}{2}$

Let α,β and γ denotes angles of triangle at vertices A,B and C respectively. Then

$$\tan\alpha = \frac{m_1-m_3}{1+m_1m_3} = \frac{3-\frac{1}{2}}{1+(3)\left(\frac{1}{2}\right)} = \frac{\frac{5}{2}}{\frac{5}{2}} = 1$$

$$\Rightarrow\ \alpha = \tan^{-1}(1) \quad \Rightarrow \boxed{\alpha = 45^\circ}$$

Now $\quad \tan\beta = \dfrac{m_2-m_1}{1+m_2m_1} = \dfrac{-2-3}{1+(-3)(3)} = \dfrac{-5}{-5} = 1$

$$\Rightarrow\ \beta = \tan^{-1}(1) \quad \Rightarrow \boxed{\beta = 45^\circ}$$

Now $\quad \tan\gamma = \dfrac{m_3-m_2}{1+m_3m_2} = \dfrac{\frac{1}{2}+2}{1+\left(\frac{1}{2}\right)(-2)} = \dfrac{\frac{5}{2}}{0} = \infty$

$$\Rightarrow\ \gamma = \tan^{-1}(\infty) \quad \Rightarrow \boxed{\gamma = 90^\circ}.$$

◈ Question # 10 (a)

Since l_1: joining $(2,7)$ and $(7,10)$

Therefore slope of $l_1 = m_1 = \dfrac{10-7}{7-2} = \dfrac{3}{5}$

Also l_2: joining $(1,1)$ and $(-5,3)$

Therefore slope of $l_2 = m_2 = \dfrac{3-1}{-5-1} = \dfrac{2}{-6} = -\dfrac{1}{3}$

Let θ be a angle from l_1 to l_2 then

$$\tan\theta = \frac{m_2-m_1}{1+m_1m_2} = \frac{-\frac{1}{3}-\frac{3}{5}}{1+\left(\frac{3}{5}\right)\left(-\frac{1}{3}\right)}$$

$$= \frac{-\frac{14}{15}}{1-\frac{1}{5}} = \frac{-\frac{14}{15}}{\frac{4}{5}} = -\frac{14}{15}\times\frac{5}{4} = -\frac{7}{6}$$

$$\Rightarrow\ -\tan\theta = \frac{7}{6} \quad \Rightarrow\ \tan(180-\theta) = \frac{7}{6} \qquad \because\ \tan(180-\theta) = -\tan\theta$$

$$\Rightarrow\ 180-\theta = \tan^{-1}\left(\frac{7}{6}\right) = 49.4$$

$$\Rightarrow\ \theta = 180-49.4 \quad \Rightarrow \boxed{\theta = 130.6^\circ}$$

Now acute angle between lines = $180-130.6 = 49.4^\circ$

◈ Question # 10(b)

Do yourself as above.

◈ Question # 10(c)

Since l_1: joining $(1,-7)$ and $(6,-4)$

Therefore slope of $l_1 = m_1 = \dfrac{-4+7}{6-1} = \dfrac{3}{5}$

Also l_2: joining $(-1,2)$ and $(-6,-1)$

Therefore slope of $l_2 = m_2 = \dfrac{-1-2}{-6+1} = \dfrac{-3}{-5} = \dfrac{3}{5}$

Let θ be a angle from l_1 to l_2 then

$$\tan\theta = \frac{m_2 - m_1}{1+m_1m_2} = \frac{3/5 - 3/5}{1+\left(3/5\right)\left(3/5\right)} = \frac{0}{1+9/25} = 0$$

$$\Rightarrow \theta = \tan^{-1}(0) \quad \Rightarrow \boxed{\theta = 0^\circ}$$

Also acute angle between lines = 0°

◈ Question # 10(d)

Since l_1: joining $(-9,-1)$ and $(3,-5)$

Therefore slope of $l_1 = m_1 = \dfrac{-5+1}{3+9} = \dfrac{-4}{12} = -\dfrac{1}{3}$

Also l_2: joining $(2,7)$ and $(-6,-7)$

Therefore slope of $l_2 = m_2 = \dfrac{-7-7}{-6-2} = \dfrac{-14}{-8} = \dfrac{7}{4}$

Let θ be a angle from l_1 to l_2 then

$$\tan\theta = \frac{m_2 - m_1}{1+m_1m_2} = \frac{7/4 - \left(-1/3\right)}{1+\left(7/4\right)\left(-1/3\right)}$$

$$= \frac{7/4 + 1/3}{1 - 7/12} = \frac{25/12}{5/12} = \frac{25}{12} \times \frac{12}{5} = 5$$

$$\Rightarrow \theta = \tan^{-1}(5) \quad \Rightarrow \boxed{\theta = 78.69^\circ}$$

Also acute angle between lines $= 78.69^\circ$

◈ Question # 11(a)

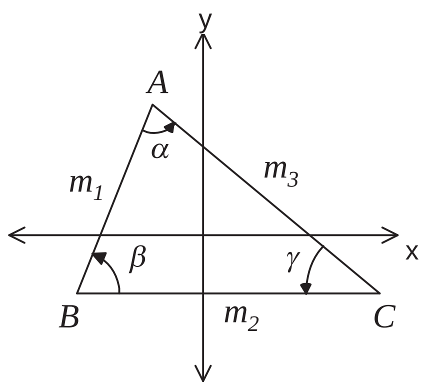
Given vertices $A(-2,11)$, $B(-6,-3)$ and $C(4,-9)$

Let m_1, m_2 and m_3 denotes the slopes of side AB, BC and CA respectively. Then

$$m_1 = \frac{-3-11}{-6+2} = \frac{-14}{-4} = \frac{7}{2}$$

$$m_2 = \frac{-9+3}{4+6} = \frac{-6}{10} = -\frac{3}{5}$$

$$m_3 = \frac{11+9}{-2-4} = \frac{20}{-6} = -\frac{10}{3}$$



Let α, β and γ denotes angles of triangle at vertex A, B and C respectively. Then

$$\tan\alpha = \frac{m_3 - m_1}{1+m_3m_1} = \frac{-10/3 - 7/2}{1+\left(-10/3\right)\left(7/2\right)}$$

$$= \frac{-41/6}{1-35/3} = \frac{-41/6}{-32/3} = \frac{41}{6} \times \frac{3}{32} = \frac{41}{64}$$

$$\Rightarrow \alpha = \tan^{-1}\left(\frac{41}{64}\right) \qquad \Rightarrow \boxed{\alpha = 32.64^\circ}$$

$$\tan\beta = \frac{m_1 - m_2}{1+m_1m_2} = \frac{7/2 - \left(-3/5\right)}{1+\left(7/2\right)\left(-3/5\right)}$$

$$= \frac{7/2 + 3/5}{1 - 21/10} = \frac{41/10}{-11/10} = -\frac{41}{10}\times\frac{10}{11} = -\frac{41}{11}$$

$$\Rightarrow -\tan\beta = \frac{41}{11} \qquad \Rightarrow \tan\left(180-\beta\right) = \frac{41}{11} \qquad \because \tan(180-\theta) = -\tan\theta$$

$$\Rightarrow 180-\beta = \tan^{-1}\left(\frac{41}{11}\right) = 74.98$$

$$\Rightarrow \beta = 180 - 74.98 \qquad \Rightarrow \boxed{\beta = 105.02}$$

$$\tan\gamma = \frac{m_2 - m_3}{1+m_2m_3} = \frac{-3/5 - \left(-10/3\right)}{1+\left(-3/5\right)\left(-10/3\right)}$$

$$= \frac{-3/5 + 10/3}{1+2} = \frac{41/15}{3} = \frac{41}{15\times 3} = \frac{41}{45}$$

$$\Rightarrow \gamma = \tan^{-1}\left(\frac{41}{45}\right) \qquad \Rightarrow \boxed{\gamma = 42.34^\circ}$$

◈ *Question # 11(b)*

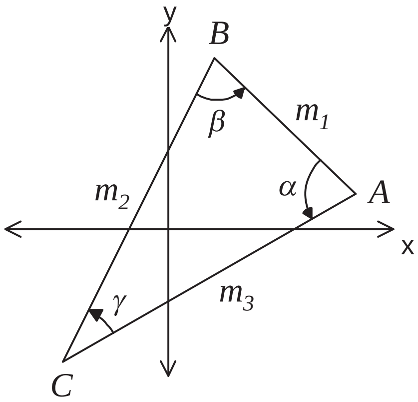
Given vertices $A(6,1)$, $B(2,7)$ and $C(-6,-7)$

Let m_1, m_2 and m_3 denotes the slopes of side AB, BC and CA respectively. Then

$$m_1 = \frac{7-1}{2-6} = \frac{6}{-4} = -\frac{3}{2}$$

$$m_2 = \frac{-7-7}{-6-2} = \frac{-14}{-8} = \frac{7}{4}$$

$$m_3 = \frac{1+7}{6+6} = \frac{8}{12} = \frac{2}{3}$$



Let α, β and γ denotes angles of triangle at vertex A, B and C respectively. Then

$$\tan\alpha = \frac{m_3 - m_1}{1+m_3m_1} = \frac{2/3 - \left(-3/2\right)}{1+\left(2/3\right)\left(-3/2\right)}$$

$$= \frac{2/3 + 3/2}{1-1} = \frac{13/6}{0} = \infty$$

$$\Rightarrow \alpha = \tan^{-1}(\infty) \qquad \Rightarrow \boxed{\alpha = 90^\circ}$$

$$\tan\beta = \frac{m_1 - m_2}{1+m_1m_2} = \frac{-3/2 - 7/4}{1+\left(-3/2\right)\left(7/4\right)}$$

$$= \frac{-13/4}{1-21/8} = \frac{-13/4}{-13/8} = \frac{13}{4}\times\frac{8}{13} = 2$$

$$\Rightarrow \beta = \tan^{-1}(2) \qquad \Rightarrow \boxed{\beta = 63.43^\circ}$$

$$\tan\gamma = \frac{m_2 - m_3}{1 + m_2 m_3} = \frac{\frac{7}{4} - \frac{2}{3}}{1 + \left(\frac{7}{4}\right)\left(\frac{2}{3}\right)}$$

$$= \frac{\frac{13}{12}}{1 + \frac{7}{6}} = \frac{\frac{13}{12}}{\frac{13}{6}} = \frac{13}{12} \times \frac{6}{13} = \frac{1}{2}$$

$$\Rightarrow \gamma = \tan^{-1}\left(\frac{1}{2}\right) \qquad \Rightarrow \boxed{\gamma = 26.57^\circ}$$

◈ *Question # 11(c) & (d)*

Do yourself as above.

◈ *Question # 12*

Given vertices are $A(5,2)$, $B(-2,3)$, $C(-3,-4)$ and $D(4,-5)$

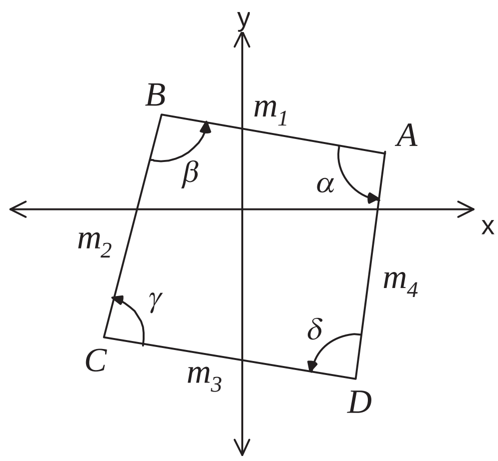
Let m_1, m_2, m_3 and m_4 be slopes of side AB, BC, CD and DA. Then

$$m_1 = \frac{3-2}{-2-5} = \frac{1}{-7}$$

$$m_2 = \frac{-4-3}{-3+2} = \frac{-7}{-1} = 7$$

$$m_3 = \frac{-5+4}{4+3} = \frac{-1}{7}$$

$$m_4 = \frac{2+5}{5-4} = \frac{7}{1} = 7$$



Now suppose α, β, γ and δ are angels of quadrilateral at vertices A, B, C and D respectively. Then

$$\tan\alpha = \frac{m_4 - m_1}{1 + m_4 m_1} = \frac{7 - \left(-\frac{1}{7}\right)}{1 + (7)\left(-\frac{1}{7}\right)} = \frac{7 + \frac{1}{7}}{1 - 1} = \frac{\frac{50}{7}}{0} = \infty$$

$$\Rightarrow \alpha = \tan^{-1}(\infty) \qquad \Rightarrow \boxed{\alpha = 90^\circ}$$

$$\tan\beta = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{-\frac{1}{7} - 7}{1 + \left(-\frac{1}{7}\right)(7)} = \frac{-\frac{50}{7}}{0} = \infty$$

$$\Rightarrow \beta = \tan^{-1}(\infty) \qquad \Rightarrow \boxed{\beta = 90^\circ}$$

$$\tan\gamma = \frac{m_2 - m_3}{1 + m_2 m_3} = \frac{7 - \left(-\frac{1}{7}\right)}{1 + (7)\left(-\frac{1}{7}\right)} = \frac{7 + \frac{1}{7}}{1 - 1} = \frac{\frac{50}{7}}{0} = \infty$$

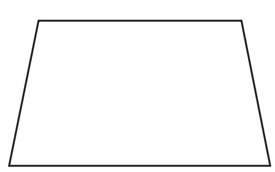
$$\Rightarrow \gamma = \tan^{-1}(\infty) \qquad \Rightarrow \boxed{\gamma = 90^\circ}$$

$$\tan\delta = \frac{m_3 - m_4}{1 + m_3 m_4} = \frac{-\frac{1}{7} - 7}{1 + \left(-\frac{1}{7}\right)(7)} = \frac{-\frac{50}{7}}{0} = \infty$$

$$\Rightarrow \delta = \tan^{-1}(\infty) \qquad \Rightarrow \boxed{\delta = 90^\circ}$$

◈ *Trapezium*

If any two opposite sides of the quadrilateral are parallel then it is called *trapezium*.



◈ Question # 13

Given vertices are $A(-1,-1)$, $B(-3,0)$, $C(3,7)$ and $D(1,8)$

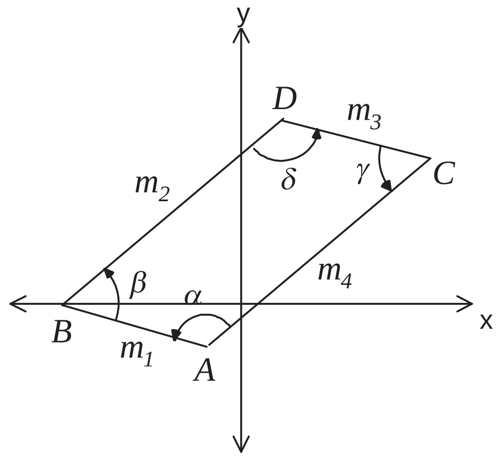
Let m_1, m_2, m_3 and m_4 be slopes of side $\overline{AB}, \overline{BD}, \overline{DC}$ and $\overline{CA}$. Then

$$m_1 = \frac{0+1}{-3+1} = \frac{1}{-2}$$

$$m_2 = \frac{8-0}{1+3} = \frac{8}{4} = 2$$

$$m_3 = \frac{7-8}{3-1} = \frac{-1}{2}$$

$$m_4 = \frac{-1-7}{-1-3} = \frac{-8}{-4} = 2$$



Since $m_2 = m_4$ or $m_1 = m_3$

Hence A, B, C and D are vertices of trapezium.

Now suppose α, β, γ and δ are angels of quadrilateral at vertices A, B, C and D respectively. Then

Now do yourself as above in Question # 12

◈ Question # 14

Let $l_1 :\ 7x - y - 10 = 0$

$l_2 :\ 10x + y - 41 = 0$

$l_3 :\ 3x + 2y + 3 = 0$

For intersection of l_1 and l_2

$$\frac{x}{41+10} = \frac{-y}{-287+100} = \frac{1}{7+10}$$

$$\Rightarrow \frac{x}{51} = \frac{-y}{-187} = \frac{1}{17}$$

$$\Rightarrow \frac{x}{51} = \frac{1}{17} \quad \text{and} \quad \frac{y}{187} = \frac{1}{17}$$

$$\Rightarrow x = \frac{51}{17} = 3 \quad \text{and} \quad y = \frac{187}{17} = 11$$

$\Rightarrow (3,11)$ is the point of intersection of l_1 and l_2.

Now for point of intersection of l_2 and l_3

$$\frac{x}{3+82} = \frac{-y}{30+123} = \frac{1}{20-3}$$

$$\Rightarrow \frac{x}{85} = \frac{-y}{153} = \frac{1}{17}$$

$$\Rightarrow \frac{x}{85} = \frac{-y}{153} = \frac{1}{17}$$

$$\Rightarrow \frac{x}{85} = \frac{1}{17} \quad \text{and} \quad \frac{-y}{153} = \frac{1}{17}$$

$$\Rightarrow x = \frac{85}{17} = 5 \quad \text{and} \quad y = -\frac{153}{17} = -9$$

$\Rightarrow (5,-9)$ is the point of intersection of l_2 and l_3.

For point of intersection of l_1 and l_3

$$\frac{x}{-3+20} = \frac{-y}{21+30} = \frac{1}{14+3}$$

$$\Rightarrow \frac{x}{17} = \frac{-y}{51} = \frac{1}{17}$$

$$\Rightarrow \frac{x}{17}=\frac{1}{17} \quad \text{and} \quad \frac{-y}{51}=\frac{1}{17}$$

$$\Rightarrow x=\frac{17}{17}=1 \quad \text{and} \quad y=-\frac{51}{17}=3$$

$\Rightarrow (1,-3)$ is the point of intersection of l_1 and l_3.

Now area of triangle having vertices $(3,11)$, $(5,-9)$ and $(1,-3)$ is given by:

$$\frac{1}{2}\begin{vmatrix} 3 & 11 & 1 \\ 5 & -9 & 1 \\ 1 & -3 & 1 \end{vmatrix}$$

$$=\frac{1}{2}\left|3(-9+3)-11(5-1)+1(-15+9)\right|$$

$$=\frac{1}{2}\left|3(-6)-11(4)+1(-6)\right| \quad =\frac{1}{2}\left|-18-44-6\right|$$

$$=\frac{1}{2}\left|-68\right| \quad =\frac{1}{2}(68) \quad =34 \text{ sq. unit}$$

◈ Question # 15 *Same Question # 7(c)*

◈ Question # 16(a)

$$x+3y-2=0$$
$$2x-y+4=0$$
$$x-11y+14=0$$

In matrix form

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Coefficient matrix of the system is

$$A=\begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix}$$

$$\Rightarrow |A|=1(-14+44)-3(28-4)-2(-22+1)$$
$$=1(30)-3(24)-2(-21)$$
$$=30-72+42=0$$

Hence given lines are concurrent.

◈ Question # 16(b)

$$2x+3y+4=0$$
$$x-2y-3=0$$
$$3x+y-8=0$$

In matrix form

$$\begin{bmatrix} 2 & 3 & 4 \\ 1 & -2 & -3 \\ 3 & 1 & -8 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Coefficient matrix of the system is

$$A=\begin{bmatrix} 2 & 3 & 4 \\ 1 & -2 & -3 \\ 3 & 1 & -8 \end{bmatrix}$$

$$\Rightarrow |A|=2(16+3)-3(-8+9)+4(1+6)$$
$$=2(19)-3(1)+4(7) \quad =38-3+28 \quad =63\neq 0$$

Hence given lines are not concurrent.

◈ Question # 16(c)

Do yourself as above

◈ Question # 17(a)

$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & 1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x-0+1 \\ 2x+0+1 \\ 0-y+1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \Rightarrow \begin{bmatrix} x+1 \\ 2x+1 \\ -y+1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Equating the elements

$$x+1=0$$
$$2x+1=0$$
$$-y+1=0$$

are the required equation of lines.

Coefficients matrix of the system

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & 1 \\ 0 & -1 & 2 \end{bmatrix}$$

$$\Rightarrow \det A = 1(0+1) - 0 - 1(-2-0)$$
$$= 1+2 = 3 \neq 0$$

Hence system is not concurrent.

(b)

Do yourself as above.
