

## Exercise 6.3

### Question # 1

Find A.M. between

(i)  $3\sqrt{5}$  and  $5\sqrt{5}$

(ii)  $x-3$  and  $x+5$

(iii)  $1-x+x^2$  and  $1+x+x^2$

#### Solution

(i)  $3\sqrt{5}$  and  $5\sqrt{5}$

Here  $a = 3\sqrt{5}$  and  $b = 5\sqrt{5}$ , so

$$\text{A.M.} = \frac{a+b}{2} = \frac{3\sqrt{5} + 5\sqrt{5}}{2} = \frac{8\sqrt{5}}{2} = 4\sqrt{5}$$

(ii)  $x-3$  and  $x+5$

Here  $a = x-3$  and  $b = x+5$

$$\text{A.M.} = \frac{a+b}{2} = \frac{x-3+x+5}{2} = \frac{2x+2}{2} = x+1$$

(iii)  $1-x+x^2$  and  $1+x+x^2$

Here  $a = 1-x+x^2$  and  $b = 1+x+x^2$

$$\text{A.M.} = \frac{a+b}{2} = \frac{1-x+x^2+1+x+x^2}{2} = \frac{2+2x^2}{2} = 1+x^2$$

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### Question # 2

If 5, 8 are two A.Ms between  $a$  and  $b$ , find  $a$  and  $b$ .

#### Solution

Since 5, 8 are two A.Ms between  $a$  and  $b$ .

Therefore  $a, 5, 8, b$  are in A.P.

Here  $a_1 = a$  and  $d = 8 - 5 = 3$

$$\text{Now } a_2 = a_1 + d \Rightarrow 5 = a + 3 \Rightarrow 5 - 3 = a \Rightarrow \boxed{a=2}$$

$$\text{Also } a_4 = a_1 + 3d \Rightarrow b = 2 + 3(3) \Rightarrow \boxed{b=11}$$

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### Question # 3

Find 6 A.Ms between 2 and 5.

#### solution

Let  $A_1, A_2, A_3, A_4, A_5$  and  $A_6$  are six A.Ms between 2 and 5.

Then  $2, A_1, A_2, A_3, A_4, A_5, A_6, 5$  are in A.P.

Here  $a_1 = 2$  and  $a_8 = 5$

$$\Rightarrow a_1 + 7d = 5 \Rightarrow 2 + 7d = 5$$

$$\Rightarrow 7d = 5 - 2 \Rightarrow 7d = 3 \Rightarrow d = \frac{3}{7}$$

$$\text{So } A_1 = a_2 = a_1 + d = 2 + \frac{3}{7} = \frac{17}{7}$$

$$A_2 = a_3 = a_1 + 2d = 2 + 2\left(\frac{3}{7}\right) = 2 + \frac{6}{7} = \frac{20}{7}$$

$$A_3 = a_4 = a_1 + 3d = 2 + 3\left(\frac{3}{7}\right) = 2 + \frac{9}{7} = \frac{23}{7}$$

$$A_4 = a_5 = a_1 + 4d = 2 + 4\left(\frac{3}{7}\right) = 2 + \frac{12}{7} = \frac{26}{7}$$

$$A_5 = a_6 = a_1 + 5d = 2 + 5\left(\frac{3}{7}\right) = 2 + \frac{15}{7} = \frac{29}{7}$$

$$A_6 = a_7 = a_1 + 6d = 2 + 6\left(\frac{3}{7}\right) = 2 + \frac{18}{7} = \frac{32}{7}$$

Hence  $\frac{17}{7}, \frac{20}{7}, \frac{23}{7}, \frac{26}{7}, \frac{29}{7}, \frac{32}{7}$  are six A.Ms between 2 and 5.

#### Question # 4

Find 4 A.Ms between  $\sqrt{2}$  and  $\frac{12}{\sqrt{2}}$ .

#### Solution

Suppose  $A_1, A_2, A_3$  and  $A_4$  are four A.Ms between  $\sqrt{2}$  and  $\frac{12}{\sqrt{2}}$ .

Then  $\sqrt{2}, A_1, A_2, A_3, A_4, \frac{12}{\sqrt{2}}$  are in A.P.

Here  $a_1 = \sqrt{2}$  and  $a_6 = \frac{12}{\sqrt{2}}$

$$\Rightarrow a_1 + 5d = \frac{12}{\sqrt{2}} \Rightarrow \sqrt{2} + 5d = \frac{12}{\sqrt{2}} \Rightarrow 5d = \frac{12}{\sqrt{2}} - \sqrt{2}$$

$$\Rightarrow 5d = \frac{12 - 2}{\sqrt{2}} \Rightarrow 5d = \frac{10}{\sqrt{2}}$$

$$\Rightarrow d = \frac{2}{\sqrt{2}} = \frac{(\sqrt{2})^2}{\sqrt{2}} \Rightarrow d = \sqrt{2}$$

$$\text{Now } A_1 = a_2 = a_1 + d = \sqrt{2} + \sqrt{2} = 2\sqrt{2}$$

$$A_2 = a_3 = a_1 + 2d = \sqrt{2} + 2\sqrt{2} = 3\sqrt{2}$$

$$A_3 = a_4 = a_1 + 3d = \sqrt{2} + 3\sqrt{2} = 4\sqrt{2}$$

$$A_4 = a_5 = a_1 + 4d = \sqrt{2} + 4\sqrt{2} = 5\sqrt{2}$$

Hence  $2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}, 5\sqrt{2}$  are four A.Ms between  $\sqrt{2}$  and  $\frac{12}{\sqrt{2}}$ .

#### Question # 5

Insert 7 A.Ms between 4 and 8.

**Solution** *Do yourself*

#### Question # 6

Find three A.Ms between 3 and 11

**Solution** *Do yourself*

### Question # 7

Find  $n$  so that  $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$  may be the A.M. between  $a$  and  $b$ .

#### Solution

Since we know that A.M. (i)

But we have given A.M.  $= \frac{a^n + b^n}{a^{n-1} + b^{n-1}} \dots\dots\dots$  (ii)

Comparing (i) and (ii)

$$\frac{a^n + b^n}{a^{n-1} + b^{n-1}} = \frac{a + b}{2}$$

$$\Rightarrow 2(a^n + b^n) = (a + b)(a^{n-1} + b^{n-1}) \quad \text{Cross } \times \text{ing}$$

$$\Rightarrow 2a^n + 2b^n = a^n + a^{n-1}b + ab^{n-1} + b^n$$

$$\Rightarrow 2a^n + 2b^n - a^n - b^n = a^{n-1}b + ab^{n-1}$$

$$\Rightarrow a^n + b^n = a^{n-1}b + ab^{n-1}$$

$$\Rightarrow a^n - a^{n-1}b = ab^{n-1} - b^n$$

$$\Rightarrow a^{n-1+1} - a^{n-1}b = ab^{n-1} - b^{n-1+1} \quad \because n = n-1+1$$

$$\Rightarrow a^{n-1}(a - b) = b^{n-1}(a - b)$$

$$\Rightarrow a^{n-1} = b^{n-1}$$

$$\Rightarrow \frac{a^{n-1}}{b^{n-1}} = 1$$

$$\Rightarrow \left(\frac{a}{b}\right)^{n-1} = \left(\frac{a}{b}\right)^0 \quad \because \left(\frac{a}{b}\right)^0 = 1$$

$$\Rightarrow n-1 = 0 \Rightarrow \boxed{n=1}$$

### Question # 8

*The solution is given at the end of Exercise 6.4.*

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