$$\tan\alpha = \frac{AC}{BC} = \frac{17.9}{7} = 255714$$

Solving for a gives us

$$\alpha = \tan^{-1}(2.55714)$$

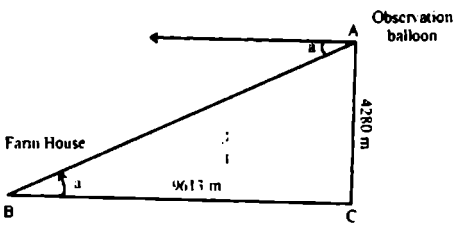
$$= (68.6666)^{\circ} = 68°40'$$

$$\Rightarrow \quad \alpha = 68°40'$$

Example 2

An observation balloon is 4280 meter above the ground and 9613 meter away from a farmhouse. Find the angle of depressor of the farmhouse as observed from the observation balloon.

Solution



For problems of this type the angle of elevation of A from B is considered equal to the angle of depression of B from A, as shown in the diagram.

$$\tan\alpha = \frac{AC}{BC} = \frac{4280}{9613} = 0.44523$$

$$\alpha = \tan^{-1}(0.44523) = 24°$$

So, angle of depression is 24°.

SOLVED EXERCISE 7.5

1. Find the angle of elevation of the sun if a 6 feet man casts a 3.5 feet shadow.

Solution

From the figure, we observe that α is the angle of elevation.

Using the fact that

$$\tan\alpha = \frac{\overline{BC}}{\overline{AC}}$$

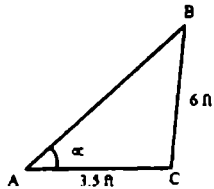
$$\tan\alpha = \frac{6}{3.5}$$

$\tan \alpha = 1.7143$

$\alpha = \tan^{-1}(1.7143)$

$\alpha = 57.94°$

or $\alpha = 59°44'37''$



2 A tree casts a 40 meter shadow when the angle of elevation of the sun is 25°. Find the height of the tree.

Solution

From the figure, we observe that α is the angle of elevation.
Using the fact that

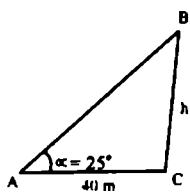
$$\tan \alpha = \frac{\overline{BC}}{\overline{AC}}$$

$$\tan 25° = \frac{h}{40}$$

$$h = 40 \tan 25°$$

$$= 40(0.44663)$$

$$= 18.652 \text{ m}$$



3. A 20 feet long ladder is leaning against a wall. The bottom of the ladder is 5 feet from the base of the wall. Find the acute angle (angle of elevation) the ladder makes with the ground.

Solution

From the figure, we observe that α is the angle of elevation.
Using the fact that

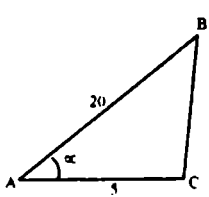
$$\cos \alpha = \frac{\overline{AC}}{\overline{AB}}$$

$$\cos \alpha = \frac{5}{20}$$

$$\alpha = \cos^{-1}(0.25)$$

$$\alpha = 75.5°$$

or $\alpha = 75°30'$



4. The base of a rectangle is 25 feet and the night of the rectangle is 13 feet. Find the angle that the diagonal of the rectangle makes with the base.

Solution

From the figure, we observe that $\propto$ is the angle of elevation.
Using the fact that

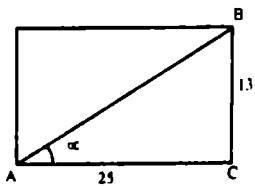
$$\tan \propto = \frac{\overline{BC}}{\overline{AC}}$$

$$\tan \propto = \frac{13}{25}$$

$$\propto = \tan^{-1}(0.52)$$

$$\propto = 27.47°$$

or $\propto = 275°28'28''$



5. A rocket is launched and climbs at a constant angle of 80°. Find the altitude of the rocket after it travels 5000 meter.

Solution

From the figure, we observe that $\propto$ is the angle of elevation.
Using the fact that

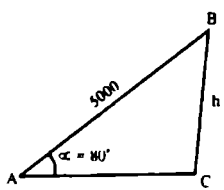
$$\sin \propto = \frac{\overline{BC}}{\overline{AB}}$$

$$\sin 80° = \frac{h}{5000}$$

$$h = 5000 \sin 80°$$

$$h = 5000(0.9848)$$

$$h = 4924.04 \text{ m}$$



6. An aeroplane pilot flying at an altitude of 4000m wishes to make an approach to an airport at an angle of 50° with the horizontal. How far from the airport will the plane be when the pilot begins to descend?

Solution

From the figure, we observe that $\propto$ is the angle of elevation.
Using the fact that

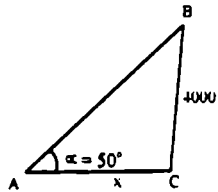
$$\tan \propto = \frac{\overline{BC}}{\overline{AC}}$$

$$\tan 50° = \frac{4000}{x}$$

$$x = \frac{4000}{\tan 50°}$$

$$x = \frac{4000}{1.1918}$$

$$x = 3356.4\text{m}$$



7. A guy wire (supporting wire) runs from the middle of a utility pole to the ground. The wire makes an angle of 78.2° with the ground and touch the ground 3 meters from the base of the pole. Find the height of the pole.

Solution

From the figure, we observe that α is the angle of elevation.
Using the fact that

$$\tan \alpha = \frac{\overline{BC}}{\overline{AC}}$$

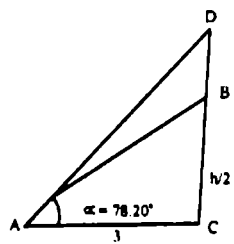
$$\tan 78.2° = \frac{h/2}{3}$$

$$\frac{h}{2} = 3 \tan 78.2°$$

$$\frac{h}{2} = 3(4.7867)$$

$$h = 6(407867)$$

$$h = 28.72 \text{ m}$$



8. A road is inclined at an angle 5.7°. Suppose that we drive 2 miles up this road starting from sea level. How high above sea level are we?

Solution

From the figure, we observe that α is the angle of elevation.
Using the fact that

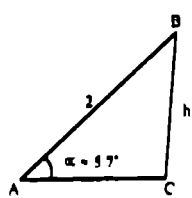
$$\cos \alpha = \frac{\overline{BC}}{\overline{AB}}$$

$$\cos 5.7° = \frac{hc}{2}$$

$$= 2\cos 5.7°$$

$$= 2(0.995)$$

$$= 1.99 \text{ miles}$$



9. **A television antenna of 8 feet height is located on the top of a house. From a point on the ground the angle of elevation to the top of the house is 17° and the angle of elevation to the top of the antenna is 21.8°. Find the height of the house.**

Solution

In Δ ABC

$$\tan 17^\circ = \frac{\overline{BC}}{\overline{AC}}$$

$$0.3057 = \frac{h}{AC}$$

$$AC = \frac{h}{0.3057} \qquad (1)$$

In Δ ADC

$$\tan 21.8^\circ = \frac{\overline{CD}}{\overline{AC}}$$

$$0.4 = \frac{\overline{BC} + \overline{BD}}{\overline{AC}}$$

$$0.4 = \frac{h + 8}{AC}$$

$$AC = \frac{h + 8}{0.4} \qquad (2)$$

Comparing eq. (1) and (2), we have

$$\frac{h}{0.3057} = \frac{h + 8}{0.4}$$

$$0.4h = 0.3057(h + 8)$$

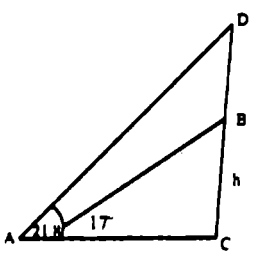
$$0.4h = 0.3057h + 2.4456$$

$$0.4h - 0.3057h = 2.4456$$

$$0.0943h = 2.4456$$

$$h = \frac{2.4456}{0.0943}$$

$$h = 25.94 \text{ feet}$$



10. **From an observation point, the angles of depression of two boats in line with this point are found to 30° and 45°. Find the distance between the two boats if the point of observation is 4000 feet high.**

Solution

In Δ BCD

$$\tan 45° = \frac{\overline{CD}}{\overline{BC}}$$

$$1 = \frac{4000}{\overline{BC}}$$

$$BC = 4000$$

In Δ ACD

$$\tan 30° = \frac{\overline{CD}}{\overline{AC}}$$

$$\frac{1}{\sqrt{3}} = \frac{4000}{\overline{AB} + \overline{BC}}$$

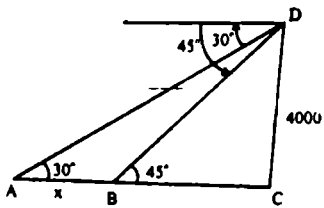
$$\frac{1}{\sqrt{3}} = \frac{4000}{\overline{AC} + 4000}$$

$$AC + 4000 = 4000\sqrt{3}$$

$$\overline{AC} = 4000\sqrt{3} - 4000$$

$$\overline{AC} = 6928.2 - 4000$$

$$\overline{AC} = 2928.2 \text{ feet}$$



11. Two ships, which are in line with the base of a vertical cliff, are 120 meters apart. The angles of depression from the top of the cliff to the ships are 30° and 45°G, as show in the diagram.

(a) Calculate the distance $\overline{BC}$

(b) Calculate the height $\overline{CD}$, of the cliff.

Solution

In Δ BCD

$$\tan 45° = \frac{\overline{CD}}{\overline{BC}}$$

$$1 = \frac{\overline{CD}}{\overline{BC}}$$

$$\overline{BC} = \overline{CD} \qquad \text{\_\_\_\_(1)}$$

In Δ ACD

$$\tan 30° = \frac{\overline{CD}}{\overline{AC}}$$

$$\frac{1}{\sqrt{3}} = \frac{\overline{CD}}{\overline{AB} + \overline{BC}}$$

$$\frac{1}{\sqrt{3}} = \frac{\overline{BC}}{\overline{AB} + \overline{BC}}$$

$$\frac{1}{\sqrt{3}} = \frac{\overline{BC}}{120 + \overline{BC}} \qquad \because \text{From (1) } \overline{BC} = \overline{CD}$$

$$\sqrt{3}\,\overline{BC} = 120 + \overline{BC}$$

$$\sqrt{3}\,\overline{BC} - \overline{BC} = 120$$

$$\left(\sqrt{3} - 1\right)\overline{BC} = 120$$

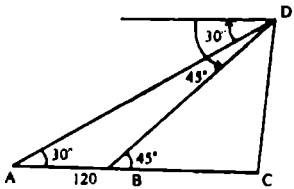
$$0.732\,\overline{BC} = 120$$

$$\overline{BC} = \frac{120}{0.732}$$

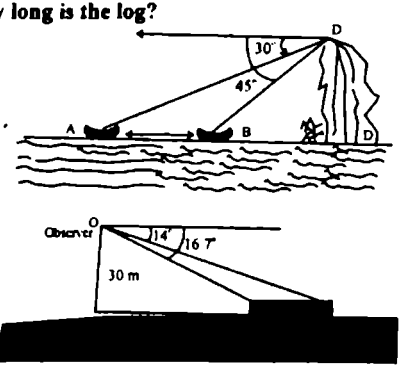
$$\overline{BC} = 164\,\text{m}$$

Put $\overline{BC} = 164$ in eq. (1), we get

$$\overline{CD} = 164\,\text{m}$$



12. Suppose that we are standing on a bridge 30 feet above a river watching a log (piece of wood) floating toward we. If the angle with the horizontal to the front of the log is 16.7° and angle with the horizontal to the back of the log is 14°, how long is the log?



Solution

In Δ BOC

$$\tan 16.7^\circ = \frac{\overline{OC}}{\overline{BC}}$$

$$0.3 = \frac{30}{BC}$$

$$\overline{BC} = \frac{30}{0.3}$$

$$\overline{BC} = 100\text{m}$$

In Δ AOC

$$\tan 14^\circ = \frac{\overline{OC}}{AC}$$

$$0.2493 = \frac{30}{AB + BC}$$

$$0.2493 = \frac{30}{x + 100}$$

$$0.2493(x + 100) = 30$$

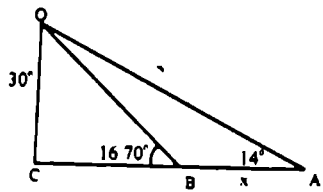
$$0.2493x + 24.93 = 30$$

$$0.2493x = 30 - 24.93$$

$$0.2493x = 5.07$$

$$x = \frac{5.07}{0.2493}$$

$$x = 20.33 \text{ m}$$



SOLVED MISCELLANEOUS EXERCISE • 7

Q1. Multiple Choice Questions

Four possible answers are given for the following questions. Tick (✓) the correct answer.

(i) The union of two non-collinear rays, which have common end point is called

(a) an angle (b) a degree (c) a minute (d) a radian

(ii) The system of measurement in which the angle is measured in radians is called

(a) CGS system (b) sexagesimal system

(c) MKS system (d) circular system

(iii) 20° =

(a) 360′ (b) 630′ (c) 1200′ (d) 3600′

(iv) $\frac{3\pi}{4}$ radians =

(a) 115° (b) 135° (c) 150° (d) 150°

(v) If $\tan\theta = \sqrt{3}$, then θ is equal to

(a) 90° (b) 45° (c) 60° (d) 30°

(vi) $\sec^2\theta$ =