EXERCISE 2.7

Question # 1 (i)

$$y = 2x^5 - 3x^4 + 4x^3 + x - 2$$

Diff. w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}\left(2x^5 - 3x^4 + 4x^3 + x - 2\right)$$

$$\Rightarrow y_1 = 2\left(5x^4\right) - 3\left(4x^3\right) + 4\left(3x^2\right) + 1 - 0$$

$$= 10x^4 - 12x^3 + 12x^2 + 1$$

Again diff. w.r.t x

$$\frac{dy_1}{dx} = \frac{d}{dx}\left(10x^4 - 12x^3 + 12x^2 + 1\right)$$

$$\Rightarrow y_2 = 10\left(4x^3\right) - 12\left(3x^2\right) + 12\left(2x\right) + 0$$

$$= 40x^3 - 36x^2 + 24x \quad \textit{Ans.}$$

Question # 1(ii)

$$y = \left(2x+5\right)^{\frac{3}{2}}$$

Diff. w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}\left(2x+5\right)^{\frac{3}{2}}$$

$$\Rightarrow y_1 = \frac{3}{2}\left(2x+5\right)^{\frac{3}{2}-1}\frac{d}{dx}\left(2x+5\right)$$

$$= \frac{3}{2}\left(2x+5\right)^{\frac{1}{2}}\left(2\right) = 3\left(2x+5\right)^{\frac{1}{2}}$$

Again diff. w.r.t x

$$\frac{dy_1}{dx} = 3\frac{d}{dx}\left(2x+5\right)^{\frac{1}{2}} \quad \Rightarrow y_2 = 3\cdot\frac{1}{2}\left(2x+5\right)^{-\frac{1}{2}}(2) \quad \Rightarrow y_2 = \frac{3}{\sqrt{2x+5}}$$

Question # 1(iii)

$$y = \sqrt{x} + \frac{1}{\sqrt{x}} \quad \Rightarrow y = \left(x\right)^{\frac{1}{2}} + \left(x\right)^{-\frac{1}{2}}$$

Diff. w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}\left[\left(x\right)^{\frac{1}{2}} + \left(x\right)^{-\frac{1}{2}}\right] \quad \Rightarrow y_1 = \frac{1}{2}\left(x\right)^{-\frac{1}{2}} - \frac{1}{2}\left(x\right)^{-\frac{3}{2}}$$

Again diff. w.r.t x

$$\frac{dy_1}{dx} = \frac{1}{2}\frac{d}{dx}\left[\left(x\right)^{-\frac{1}{2}} - \left(x\right)^{-\frac{3}{2}}\right]$$

$$\Rightarrow y_2 = \frac{1}{2}\left[-\frac{1}{2}\left(x\right)^{-\frac{3}{2}} + \frac{3}{2}\left(x\right)^{-\frac{5}{2}}\right]$$

$$= \frac{1}{4}\left[-\frac{1}{x^{\frac{3}{2}}} + \frac{3}{x^{\frac{5}{2}}}\right] = \frac{1}{4}\left[\frac{-x+3}{x^{\frac{5}{2}}}\right] \quad \text{or} \quad y_2 = \frac{3-x}{4x^{\frac{5}{2}}}$$

Question # 2(i)

$$y = x^2 e^{-x}$$

Diff. w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}x^2 e^{-x}$$

$$\Rightarrow\ y_1 = x^2\frac{d}{dx}e^{-x} + e^{-x}\frac{d}{dx}x^2$$

$$= x^2e^{-x}(-1) + e^{-x}(2x) \quad = e^{-x}\left(-x^2+2x\right)$$

Again diff. w.r.t x

$$\frac{dy_1}{dx} = \frac{d}{dx}e^{-x}\left(-x^2+2x\right)$$

$$y_2 = e^{-x}\frac{d}{dx}\left(-x^2+2x\right) + \left(-x^2+2x\right)\frac{d}{dx}e^{-x}$$

$$= e^{-x}(-2x+2) + \left(-x^2+2x\right)e^{-x}(-1) \quad = e^{-x}\left(-2x+2+x^2-2x\right)$$

$$= e^{-x}\left(x^2-4x+2\right)$$

Question # 2(ii)

$$y = \ln\left(\frac{2x+3}{3x+2}\right)$$

$$\Rightarrow\ y = \ln(2x+3) - \ln(3x+2)$$

Diff. w.r.t x

$$\Rightarrow\ \frac{dy}{dx} = \frac{d}{dx}\ln(2x+3) - \frac{d}{dx}\ln(3x+2)$$

$$\Rightarrow\ y_1 = \frac{1}{2x+3}(2) - \frac{1}{3x+2}(3)$$

$$= 2(2x+3)^{-1} - 3(3x+2)^{-1}$$

Again diff. w.r.t x

$$\frac{dy_1}{dx} = 2\frac{d}{dx}(2x+3)^{-1} - 3\frac{d}{dx}(3x+2)^{-1}$$

$$\Rightarrow\ y_2 = 2\left[-(2x+3)^{-2}(2)\right] - 3\left[-(3x+2)^{-2}(3)\right]$$

$$\Rightarrow\ y_2 = -\frac{4}{(2x+3)^2} + \frac{9}{(3x+2)^2} \quad \textit{Ans.}$$

$$\text{OR}\quad y_2 = \frac{-4(3x+2)^2 + 9(3x+2)^2}{(2x+3)^2(3x+2)^2} = \frac{-4\left(9x^2+12x+4\right)+9\left(4x^2+12x+9\right)^2}{(2x+3)^2(3x+2)^2}$$

$$= \frac{-36x^2-48x-16+36x^2+108x+81}{(2x+3)^2(3x+2)^2} = \frac{60x+65}{(2x+3)^2(3x+2)^2} \quad \textit{Ans.}$$

Question # 2(iii)

$$y = \sqrt{\frac{1-x}{1+x}}$$

Diff. w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1-x}{1+x}\right)^{\frac{1}{2}}$$

By solving, you will get (differentiate here)

$$\Rightarrow\ y_1 = \frac{-1}{(1-x)^{\frac{1}{2}}(1+x)^{\frac{3}{2}}} \quad = -(1-x)^{-\frac{1}{2}}(1+x)^{-\frac{3}{2}}$$

Again diff. w.r.t x

$$\frac{dy_1}{dx} = -\frac{d}{dx}\left[(1-x)^{-\frac{1}{2}}(1+x)^{-\frac{3}{2}}\right]$$

$$\Rightarrow\ y_2 = -(1-x)^{-\frac{1}{2}}\frac{d}{dx}(1+x)^{-\frac{3}{2}} - (1+x)^{-\frac{3}{2}}\frac{d}{dx}(1-x)^{-\frac{1}{2}}$$

$$= -(1-x)^{-\frac{1}{2}}\left(-\frac{3}{2}(1+x)^{-\frac{5}{2}}(1)\right)-(1+x)^{-\frac{3}{2}}\left(-\frac{1}{2}(1-x)^{-\frac{3}{2}}(-1)\right)$$

$$= \frac{3}{2(1-x)^{\frac{1}{2}}(1+x)^{\frac{5}{2}}}-\frac{1}{2(1+x)^{\frac{3}{2}}(1-x)^{\frac{3}{2}}} = \frac{3(1-x)-(1+x)}{2(1-x)^{\frac{3}{2}}(1+x)^{\frac{5}{2}}}$$

$$= \frac{3-3x-1-x}{2(1-x)^{\frac{3}{2}}(1+x)^{\frac{5}{2}}} = \frac{2-4x}{2(1-x)^{\frac{3}{2}}(1+x)^{\frac{5}{2}}} = \frac{1-2x}{(1-x)^{\frac{3}{2}}(1+x)^{\frac{5}{2}}} \quad \textit{Ans.}$$

Question # 3(i)

$$x^2+y^2 = a^2$$

Diff. w.r.t x

$$\frac{d}{dx}\left(x^2+y^2\right) = \frac{d}{dx}a^2 \quad \Rightarrow 2x+2y\frac{dy}{dx} = 0$$

$$\Rightarrow 2y\,y_1 = -2x \quad \Rightarrow y_1 = -\frac{x}{y}$$

Again diff. w.r.t x

$$\Rightarrow \frac{dy_1}{dx} = -\frac{d}{dx}\left(\frac{x}{y}\right) \quad \Rightarrow y_2 = -\left(\frac{y\frac{dx}{dx}-x\frac{dy}{dx}}{y^2}\right)$$

$$\Rightarrow y_2 = -\left(\frac{y(1)-x\left(-\frac{x}{y}\right)}{y^2}\right) \qquad \because \frac{dy}{dx}=-\frac{x}{y}$$

$$= -\left(\frac{y+\frac{x^2}{y}}{y^2}\right) = -\left(\frac{\frac{y^2+x^2}{y}}{y^2}\right) = -\left(\frac{x^2+y^2}{y^3}\right) \quad \textit{Ans.}$$

OR $\quad y_2 = -\dfrac{a^2}{y^3} \qquad \because x^2+y^2 = a^2$

Question # 3(ii)

$$x^3-y^3 = a^3$$

Diff. w.r.t x

$$\frac{d}{dx}\left(x^3-y^3\right) = \frac{d}{dx}a^3$$

$$3x^2-3y^2\frac{dy}{dx} = 0 \quad \Rightarrow -3y^2y_1 = -3x^2 \quad \Rightarrow y_1 = \frac{x^2}{y^2}$$

Again diff. w.r.t x

$$\Rightarrow \frac{dy_1}{dx} = \frac{d}{dx}\left(\frac{x^2}{y^2}\right)$$

$$\Rightarrow y_2 = \frac{y^2\frac{d}{dx}(x^2)-x^2\frac{d}{dx}(y^2)}{\left(y^2\right)^2}$$

$$= \frac{y^2(2x)-x^2\left(2y\frac{dy}{dx}\right)}{y^4} = \frac{2xy^2-2x^2y\left(\frac{x^2}{y^2}\right)}{y^4} \qquad \because \frac{dy}{dx} = \frac{x^2}{y^2}$$

$$= \frac{2xy^2 - \frac{2x^4}{y}}{y^4} = \frac{\frac{2xy^3 - 2x^4}{y}}{y^4} = \frac{-2x\left(x^3 - y^3\right)}{y^5} \quad \textit{Ans.}$$

OR $\quad y_2 = \dfrac{-2x\left(a^3\right)}{y^5} \qquad \because\ x^3 - y^3 = a^3$

$$\Rightarrow y_2 = -\frac{2a^3x}{y^5}$$

Question # 3(iii)

$$x = a\cos\theta \quad , \quad y = a\sin\theta$$

Diff. x w.r.t θ

$$\frac{dx}{d\theta} = a\frac{d}{d\theta}\cos\theta$$
$$= -a\sin\theta$$
$$\Rightarrow \frac{d\theta}{dx} = -\frac{1}{a\sin\theta}$$

Diff y w.r.t θ

$$\frac{dy}{d\theta} = a\frac{d}{d\theta}\sin\theta$$
$$= a\cos\theta$$

Now by chain rule

$$\frac{dy}{dx} = \frac{dy}{d\theta}\cdot\frac{d\theta}{dx}$$
$$= a\cos\theta\cdot\frac{-1}{a\sin\theta} \qquad \Rightarrow y_1 = -\cot\theta$$

Now diff. y_1 w.r.t θ

$$\frac{dy_1}{dx} = -\frac{d}{dx}\cot\theta$$
$$\Rightarrow y_2 = +\operatorname{cosec}^2\theta\frac{d\theta}{dx}$$
$$= \operatorname{cosec}^2\theta.\left(-\frac{1}{a\sin\theta}\right)$$
$$\Rightarrow y_2 = \frac{-1}{a\sin^3\theta}$$

Question # 3(iv)

$$x = at^2 \quad , \quad y = bt^4$$

Diff. x w.r.t t

$$\frac{dx}{dt} = a\frac{d}{dt}t^2$$
$$= 2at$$
$$\Rightarrow \frac{dt}{dx} = \frac{1}{2at}$$

Diff. y w.r.t t

$$\frac{dy}{dt} = b\frac{d}{dt}(t^4)$$
$$= 4bt^3$$

Now by chain rule

$$\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx}$$
$$= 4bt^3\cdot\frac{1}{2at} \qquad \Rightarrow y_1 = \frac{2b}{a}t^2$$

Now diff. y_1 w.r.t x

$$\frac{dy_1}{dx} = \frac{2b}{a}\frac{d}{dt}\left(t^2\right)$$
$$\Rightarrow y_2 = \frac{2b}{a}(2t) \qquad \Rightarrow y_2 = \frac{4bt}{a}$$

Question # 3(v)

$$x^2+y^2+2gx+2fy+c=0$$

$$\Rightarrow \frac{d}{dx}\left(x^2+y^2+2gx+2fy+c\right)=\frac{d}{dx}(0)$$

$$\Rightarrow 2x+2y\frac{dy}{dx}+2g(1)+2f\frac{dy}{dx}+0=0 \quad \Rightarrow (2y+2f)\frac{dy}{dx}+(2x+2g) = 0$$

$$\Rightarrow (2y+2f)\frac{dy}{dx} = -(2x+2g) \quad \Rightarrow \frac{dy}{dx} = -\frac{(2x+2g)}{(2y+2f)}$$

$$\Rightarrow y_1 = -\frac{x+g}{y+f}$$

Again diff. w.r.t x

$$\frac{dy_1}{dx} = -\frac{d}{dx}\left(\frac{x+g}{y+f}\right)$$

$$\Rightarrow y_2 = -\left(\frac{(y+f)\frac{d}{dx}(x+g)-(x+g)\frac{d}{dx}(y+f)}{(y+f)^2}\right)$$

$$= -\frac{(y+f)(1)-(x+g)\frac{dy}{dx}}{(y+f)^2} = -\frac{(y+f)-(x+g)\left(-\frac{x+g}{y+f}\right)}{(y+f)^2}$$

$$= -\frac{\frac{(y+f)^2+(x+g)^2}{y+f}}{(y+f)^2} = -\frac{(y+f)^2+(x+g)^2}{(y+f)^3} \qquad \textit{Ans.}$$

OR $$y_2 = -\frac{y^2+2yf+f^2+x^2+2xg+g^2}{(y+f)^3}$$

$$= -\frac{\left(x^2+y^2+2gx+2fy+c\right)-c+f^2+g^2}{(y+f)^3}$$

$$= -\frac{0-c+f^2+g^2}{(y+f)^3} \qquad \because x^2+y^2+2gx+2fy+c=0$$

$$\Rightarrow y_2 = \frac{c-f^2-g^2}{(y+f)^3} \qquad \textit{Ans.}$$

Question # 4(i)

$$y = \sin 3x$$

Diff. w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}(\sin 3x) \quad \Rightarrow y_1 = \cos 3x\,(3) \quad \Rightarrow y_1 = 3\cos 3x$$

Again diff. w.r.t x

$$\frac{dy_1}{dx} = 3\frac{d}{dx}\cos 3x \quad \Rightarrow y_2 = 3\left(-\sin 3x\,(3)\right) \quad \Rightarrow y_2 = -9\sin 3x$$

Again diff. w.r.t x

$$\frac{dy_2}{dx} = -9\frac{d}{dx}\sin 3x \quad \Rightarrow y_3 = -9\cos 3x\,(3) \quad \Rightarrow y_3 = -27\cos 3x$$

Again diff. w.r.t x

$$\frac{dy_3}{dx} = -27\frac{d}{dx}\cos 3x \quad \Rightarrow y_4 = -27\left(-\sin 3x\,(3)\right) \quad \Rightarrow \boxed{y_4 = 81\sin 3x}$$

Question # 4(ii)

$$y = \cos^3 x$$

Diff w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}\left(\cos^3 x\right)$$

$$\Rightarrow y_1 = 3\left(\cos^2 x\right)\frac{d}{dx}\cos x \qquad \Rightarrow y_1 = 3\left(\cos^2 x\right)\left(-\sin x\right)$$

$$\Rightarrow y_1 = 3\left(1-\sin^2 x\right)\left(-\sin x\right) \qquad \Rightarrow y_1 = -3\sin x + 3\sin^3 x$$

Again diff. w.r.t x

$$\frac{dy_1}{dx} = -3\frac{d}{dx}\sin x + 3\frac{d}{dx}\sin^3 x$$

$$\Rightarrow y_2 = -3\cos x + 9\sin^2 x\frac{d}{dx}\sin x$$

$$\Rightarrow y_2 = -3\cos x + 9\left(1-\cos^2 x\right)\cos x$$

$$= -3\cos x + 9\cos x - 9\cos^3 x = 6\cos x - 9\cos^3 x$$

Again diff. w.r.t x

$$\frac{dy_2}{dx} = 6\frac{d}{dx}\cos x - 9\frac{d}{dx}\cos^3 x$$

$$\Rightarrow y_3 = 6\left(-\sin x\right) - 9\left(-3\sin x + 3\sin^3 x\right) \qquad \because \frac{d}{dx}(\cos^3 x) = -3\sin x + 3\sin^3 x$$

$$= -6\sin x + 27\sin x - 27\sin^3 x = 21\sin x - 27\sin^3 x$$

Again diff. w.r.t x

$$\frac{dy_3}{dx} = 21\frac{d}{dx}\sin x - 27\frac{d}{dx}\sin^3 x$$

$$\Rightarrow y_4 = 21\left(\cos x\right) - 27\left(3\sin^2 x\right)\frac{d}{dx}\sin x$$

$$= 21\cos x - 81\sin^2 x\left(\cos x\right) = 21\cos x - 81\left(1-\cos^2 x\right)\left(\cos x\right)$$

$$= 21\cos x - 81\cos x + 81\cos^3 x = -60\cos x + 81\cos^3 x$$

Alternative:

$$y = \cos^3 x$$

Since $\cos 3x = 4\cos^3 x - 3\cos x$

$$\Rightarrow \cos 3x - 3\cos x = 4\cos^3 x \quad \Rightarrow \cos^3 x = \frac{1}{4}\left(\cos 3x - 3\cos x\right)$$

Therefore

$$y = \frac{1}{4}\left(\cos 3x - 3\cos x\right)$$

Now diff. w.r.t x

$$\Rightarrow \frac{dy}{dx} = \frac{1}{4}\left(\frac{d}{dx}\cos 3x - 3\frac{d}{dx}\cos x\right)$$

Do yourself

Question # 4(iv)

$$y = \ln\left(x^2 - 9\right)$$

$$= \ln\left[\left(x+3\right)\left(x-3\right)\right] = \ln\left(x+3\right) + \ln\left(x-3\right)$$

Diff. w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}\ln(x+3) + \frac{d}{dx}\ln(x-3)$$

$$\Rightarrow y_1 = \frac{1}{x+3} + \frac{1}{x-3}$$
$$= (x+3)^{-1} + (x-3)^{-1}$$

Again diff w.r.t x

$$\frac{dy_1}{dx} = \frac{d}{dx}(x+3)^{-1} + \frac{d}{dx}(x-3)^{-1}$$
$$\Rightarrow y_2 = -(x+3)^{-2} - (x-3)^{-2}$$

Again diff. w.r.t x

$$\frac{dy_2}{dx} = -\frac{d}{dx}(x+3)^{-2} - \frac{d}{dx}(x-3)^{-2} \quad \Rightarrow y_3 = 2(x+3)^{-3} + 2(x-3)^{-3}$$

Again diff. w.r.t x

$$\frac{dy_3}{dx} = 2\frac{d}{dx}(x+3)^{-3} + 2\frac{d}{dx}(x-3)^{-3}$$
$$\Rightarrow y_4 = 2\left(-3(x+3)^{-4}\right) + 2\left(-3(x-3)^{-4}\right)$$
$$= \frac{-6}{(x+3)^4} + \frac{-6}{(x+3)^4} = -6\left[\frac{1}{(x+3)^4} + \frac{1}{(x+3)^4}\right] \quad \textit{Ans.}$$

Question # 5

$x = \sin\theta$ …… (i) , $y = \sin m\theta$ ……..(ii)

From (i) $\theta = \sin^{-1}x$, putting in (ii)

$$y = \sin\left(m\sin^{-1}x\right)$$

Diff. w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}\sin m\left(\sin^{-1}x\right)$$
$$\Rightarrow y_1 = \cos\left(m\sin^{-1}x\right)\frac{d}{dx}m\sin^{-1}x$$
$$= \cos\left(m\sin^{-1}x\right)\cdot m\frac{1}{\sqrt{1-x^2}}$$
$$\Rightarrow y_1\sqrt{1-x^2} = m\cos\left(m\sin^{-1}x\right)$$

Taking square on both sides.

$$y_1^2\left(1-x^2\right) = m^2\cos^2\left(m\sin^{-1}x\right)$$
$$\Rightarrow y_1^2\left(1-x^2\right) = m^2\left(1-\sin^2(m\sin^{-1}x)\right) \qquad \because \cos^2 x = 1-\sin^2 x$$
$$\Rightarrow y_1^2\left(1-x^2\right) = m^2\left(1-y^2\right) \qquad \text{From } (ii)$$

Now again diff. w.r.t x

$$\frac{d}{dx}y_1^2\left(1-x^2\right) = m^2\frac{d}{dx}\left(1-y^2\right)$$
$$\Rightarrow y_1^2\frac{d}{dx}\left(1-x^2\right) + \left(1-x^2\right)\frac{d}{dx}y_1^2 = m^2\left(0-2y\frac{dy}{dx}\right)$$
$$\Rightarrow y_1^2(-2x) + \left(1-x^2\right)2y_1\frac{dy_1}{dx} = -2m^2y\frac{dy}{dx}$$
$$\Rightarrow -2xy_1^2 + \left(1-x^2\right)2y_1y_2 = -2m^2y\,y_1$$
$$\Rightarrow 2y_1\left(-xy_1 + \left(1-x^2\right)y_2\right) = 2y_1\left(-m^2y\right)$$
$$\Rightarrow -xy_1 + \left(1-x^2\right)y_2 = -m^2y$$
$$\Rightarrow \left(1-x^2\right)y_2 - xy_1 + m^2y = 0 \qquad \textit{Proved}$$

Question # 6

$$y = e^x \sin x$$

Diff. w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}e^x \sin x$$

$$= e^x \frac{d}{dx}\sin x + \sin x \frac{d}{dx}e^x$$

$$= e^x \cos x + \sin x\, e^x \quad = e^x(\cos x + \sin x)$$

Again diff. w.r.t x

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}e^x(\cos x + \sin x)$$

$$\Rightarrow \frac{d^2y}{dx^2} = e^x \frac{d}{dx}(\cos x + \sin x) + (\cos x + \sin x)\frac{d}{dx}e^x$$

$$= e^x(-\sin x + \cos x) + (\cos x + \sin x)e^x \quad = e^x(-\sin x + \cos x + \cos x + \sin x)$$

$$= e^x(2\cos x) = 2e^x \cos x$$

Now

$$\text{L.H.S} = \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$$

$$= 2e^x \cos x - 2e^x(\cos x + \sin x) + 2e^x \sin x$$

$$= 2e^x(\cos x - \cos x - \sin x + \sin x)$$

$$= 0$$

i.e. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$ *Proved*

Question # 7

$$y = e^{ax}\sin bx$$

Diff. w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}e^{ax}\sin bx$$

$$= e^{ax}\frac{d}{dx}\sin bx + \sin bx \frac{d}{dx}e^{ax} \quad = e^{ax}\cos bx(b) + \sin bx\, e^{ax}(a)$$

$$= e^{ax}(b\cos bx + a\sin bx)$$

Again diff. w.r.t x

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}e^{ax}(b\cos bx + a\sin bx)$$

$$\Rightarrow \frac{d^2y}{dx^2} = e^{ax}\frac{d}{dx}(b\cos bx + a\sin bx) + (b\cos bx + a\sin bx)\frac{d}{dx}e^{ax}$$

$$= e^{ax}(-b\sin bx(b) + a\cos bx(b)) + (b\cos bx + a\sin bx)e^{ax}(a)$$

$$= e^{ax}(-b^2\sin bx + ab\cos bx + ab\cos bx + a^2\sin bx)$$

$$= e^{ax}(2ab\cos bx + a^2\sin bx - b^2\sin bx)$$

$$= e^{ax}(2ab\cos bx + 2a^2\sin bx - a^2\sin bx - b^2\sin bx)$$

$$= e^{ax}\left[2a(b\cos bx + a\sin bx) - (a^2 + b^2)\sin bx\right]$$

$$= 2ae^{ax}(b\cos bx + 2a\sin bx) - (a^2 + b^2)e^{ax}\sin bx$$

$$\Rightarrow \frac{d^2y}{dx^2} = 2a\frac{dy}{dx} - (a^2 + b^2)y \quad \Rightarrow \frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + (a^2 + b^2)y = 0$$ *Proved*

Question # 8

$$y = \left(Cos^{-1}x\right)^2$$

Diff. w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}\left(Cos^{-1}x\right)^2 \quad \Rightarrow y_1 = 2\left(Cos^{-1}x\right)\frac{d}{dx}Cos^{-1}x$$

$$\Rightarrow y_1 = 2\left(Cos^{-1}x\right)\cdot\frac{-1}{\sqrt{1-x^2}} \quad \Rightarrow y_1\sqrt{1-x^2} = -2\left(Cos^{-1}x\right)$$

On squaring both sides

$$y_1^2\left(1-x^2\right) = 4\left(Cos^{-1}x\right)^2$$

$$\Rightarrow y_1^2\left(1-x^2\right) = 4y \qquad \because\ y = \left(Cos^{-1}x\right)^2$$

Again diff. w.r.t x

$$\frac{d}{dx}y_1^2\left(1-x^2\right) = 4\frac{dy}{dx}$$

$$\Rightarrow \left(1-x^2\right)\frac{d}{dx}y_1^2 + y_1^2\frac{d}{dx}\left(1-x^2\right) = 4y_1$$

$$\Rightarrow \left(1-x^2\right)\cdot 2y_1\frac{dy_1}{dx} + y_1^2\left(-2x\right) = 4y_1 \quad \Rightarrow 2y_1\left[\left(1-x^2\right)y_2 - xy_1\right] = 4y_1$$

$$\Rightarrow \left(1-x^2\right)y_2 - xy_1 - 2y_1 = 0$$

Question # 9

$$y = a\cos\left(\ln x\right) + b\sin\left(\ln x\right)$$

Diff. w.r.t x

$$\frac{dy}{dx} = a\frac{d}{dx}\cos\left(\ln x\right) + b\frac{d}{dx}\sin\left(\ln x\right)$$

$$= a\left[-\sin\left(\ln x\right)\right]\frac{d}{dx}(\ln x) + b\cos\left(\ln x\right)\frac{d}{dx}(\ln x)$$

$$= -a\sin\left(\ln x\right)\frac{1}{x} + b\cos\left(\ln x\right)\frac{1}{x}$$

$$\Rightarrow x\frac{dy}{dx} = -a\sin\left(\ln x\right) + b\cos\left(\ln x\right)$$

Again diff. w.r.t x

$$\frac{d}{dx}\left[x\frac{dy}{dx}\right] = -a\frac{d}{dx}\sin\left(\ln x\right) + b\frac{d}{dx}\cos\left(\ln x\right)$$

$$\Rightarrow x\frac{d}{dx}\left(\frac{dy}{dx}\right) + \frac{dy}{dx}\cdot\left(\frac{dx}{dx}\right) = -a\cos\left(\ln x\right)\frac{d}{dx}\left(\ln x\right) + b\left(-\sin\left(\ln x\right)\right)\frac{d}{dx}\left(\ln x\right)$$

$$\Rightarrow x\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot(1) = -a\cos\left(\ln x\right)\cdot\frac{1}{x} - b\sin\left(\ln x\right)\cdot\frac{1}{x}$$

$$\Rightarrow x\frac{d^2y}{dx^2} + \frac{dy}{dx} = -\frac{1}{x}\left(a\cos\left(\ln x\right) + b\sin\left(\ln x\right)\right) \quad \Rightarrow x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -y$$

$$\Rightarrow x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0 \quad \textit{Proved}$$