EXERCISE 6.8

1. **Find an equation of each of the following with respect to new parallel axes obtained by shifting the origin to the indicated points.**

(i) $x^2 + 16y - 16 = 0, O' (0, 1)$

Solution. $x^2 + 16y - 16 = 0 \quad \dots (1), \quad O' (0, 1) = (h, k)$

Equations of transformation are

$$x = X + h = X + 0 = X \quad , \quad y = Y + k = Y + 1$$

Substituting these values of x, y into (1), we have

$$X^2 + 16(Y + 1) - 16 = 0$$

or $\quad X^2 + 16Y + 16 - 16 = 0 \quad \Rightarrow \quad X^2 + 16Y = 0$

is the required transformed equation.

(ii) $4x^2 + y^2 + 16x - 10y + 37 = 0, O' (-2, 5)$

Solution. $4x^2 + y^2 + 16x - 10y + 37 = 0 \quad \dots (1), \quad O' (-2, 5) = (h, k)$

Equations of transformation are

$$x = X + h = X - 2 \quad , \quad y = Y + k = Y + 5$$

Substituting these values of x, y into (1), we have

$$4(X-2)^2+(Y+5)^2+16(X-2)-10(Y+5)+37 = 0$$

or $\quad 4(X^2-4X+4)+(Y^2+10Y+25)+16(X-2)-10(Y+5)+37 = 0$

or $\quad 4X^2-16X+16+Y^2+10Y+25+16X-32-10Y-50+37 = 0$

or $\quad 4X^2+Y^2-4 = 0$ is the required transformed equation.

(iii) $\quad \mathbf{9x^2+4y^2+18x-16y-11 = 0,\ O'(-1,2)}$

Solution. $9x^2+4y^2+18x-16y-11 = 0 \quad \ldots (1), \qquad O'(-1,2) = (h,k)$

Equations of transformation are

$$x = X+h = X-1, \quad y = Y+k = Y+2$$

Putting these values of x and y into (1), we have

$$9(X-1)^2+4(Y+2)^2+18(X-1)-16(Y+2)-11 = 0$$

or $\quad 9(X^2-2X+1)+4(Y^2+4Y+4)+18(X-1)-16(Y+2)-11=0$

or $\quad 9X^2-18X+9+4Y^2+16Y+16+18X-18-16Y-32-11 = 0$

or $\quad 9X^2+4Y^2-36 = 0$ is the required transformed equation.

(iv) $\quad \mathbf{x^2-y^2+4x+8y-11 = 0,\ O'(-2,4)}$

Solution. $x^2-y^2+4x+8y-11 = 0 \quad \ldots (1), \qquad O'(-2,4) = (h,k)$

Equations of transformation are

$$x = X+h = X-2, \quad y = Y+k = Y+4$$

Substituting these values of x and y into (1), we have

$$(X-2)^2-(Y+4)^2+4(X-2)+8(Y+4)-11 = 0$$

or $\quad X^2-4X+4-(Y^2+8Y+16)+4X-8+8Y+32-11 = 0$

or $\quad X^2-4X+4-Y^2-8Y-16+4X-8+8Y+32-11 = 0$

or $\quad X^2-Y^2+1 = 0$ is the required transformed equation.

(v) $\quad \mathbf{9x^2-4y^2+36x+8y-4 = 0,\ O'(-2,1)}$

Solution. $9x^2-4y^2+36x+8y-4 = 0 \quad \ldots (1) \quad O'(-2,1) = (h,k)$

Equations of transformation are

$$x = X+h = X-2, \quad y = Y+k = Y+1$$

Substituting these values of x and y into (1), we have

$$9(X-2)^2-4(Y+1)^2+36(X-2)+8(Y+1)-4 = 0$$

or $\quad 9(X^2-4X+4)-4(Y^2+2Y+1)+36(X-2)+8(Y+1)-4 = 0$

or $\quad 9X^2-36X+36-4Y^2-8Y-4+36X-72+8Y+8-4 = 0$

or $\quad 9X^2 - 4Y^2 - 36 = 0$ is the required transformed equation.

2. Find coordinates of the new origin (axes remaining parallel) so that first degree terms are removed from the transformed equation of each of the following. Also find the transformed equation.

(i) $\quad 3x^2 - 2y^2 + 24x + 12y + 24 = 0$

Solution. $3x^2 - 2y^2 + 24x + 12y + 24 = 0 \quad \ldots (1)$

Let the coordinates of the new origin be $O'(h, k)$.

Equations of transformation are

$$x = X + h, \quad y = Y + k$$

Substituting these values of x, y into (1), we get

$$3(X+h)^2 - 2(Y+k)^2 + 24(X+h) + 12(Y+k) + 24 = 0$$

or $\quad 3(X^2 + 2Xh + h^2) - 2(Y^2 + 2kY + k^2) + 24(X+h) + 12(Y+k) + 24 = 0$

or $\quad 3X^2 + 6hX + 3h^2 - 2Y^2 - 4kY - 2k^2 + 24X + 24h + 12Y + 12k + 24 = 0$

or $\quad 3X^2 - 2Y^2 + X(6h + 24) + Y(-4k + 12) + 3h^2 + 24h - 2k^2 + 12k + 24 = 0 \quad \ldots (2)$

To remove first degree terms, we have

$$6h + 24 = 0 \quad \Rightarrow \quad h = -4$$

and $\quad -4k + 12 = 0 \quad \Rightarrow \quad k = 3$

New origin is $O'(-4, 3)$. Putting $h = -4$ and $k = 3$ into (2), the transformed equation is

$$3X^2 - 2Y^2 + X[6(-4) + 24] + Y[-4(3) + 12] + 3(-4)^2 + 24(-4) - 2(3)^2 + 12(3) + 24 = 0$$

or $\quad 3X^2 - 2Y^2 + X(-24 + 24) + Y(-12 + 12) + 3(16) - 96 - 2(9) + 36 + 24 = 0$

or $\quad 3X^2 - 2Y^2 + X(0) + Y(0) + 48 - 96 - 18 + 36 + 24 = 0$

or $\quad 3X^2 - 2Y^2 - 6 = 0$

which is the required transformed equation with new origin at $(-4, 3)$

(ii) $\quad 25x^2 + 9y^2 + 50x - 36y - 164 = 0$

Solution. $25x^2 + 9y^2 + 50x - 36y - 164 = 0 \quad \ldots (1)$

Let the coordinates of the new origin be $O'(h, k)$. Equations of transformed are

$$x = X + h, \quad y = Y + k$$

Substituting these values of x, y into (1), we get

$$25(X+h)^2+9(Y+k)^2+50(X+h)-36(Y+k)-164 = 0$$

or $\quad 25(X^2+2Xh+h^2)+9(Y^2+2Yk+k^2)+50X+50h-36Y-36k-164=0$

or $\quad 25X^2+50Xh+25h^2+9Y^2+18Yk+9k^2+50X+50h-36Y-36k-164=0$

or $25X^2+9Y^2+(50h+50)X+(18k-36)Y+25h^2+9k^2+50h-36k-164=0$ ...(2)

To reove the first degree terms, we put

$$50h+50 = 0 \Rightarrow 50h = -50 \quad \Rightarrow \quad h = -1$$

and $\quad 18k-36 = 0 \Rightarrow 18k = 36 \quad \Rightarrow \quad k = 2$

$\therefore$ New origin is $O'(-1, 2)$.

Putting $h = -1$, $k = 2$ into (2), the transformed equation is

$$25X^2+9Y^2+(-50+50)X+(36-36)Y+25(-1)^2+9(2)^2+50(-1)-36(2)-164 = 0$$

or $\quad 25X^2+9Y^2+(0)X+(0)Y+25(1)+9(4)-50-72-164 = 0$

or $\quad 25X^2+9Y^2+25+36-50-72-164 = 0$

or $\quad 25X^2+9Y^2-225 = 0$

which is the required equation with new origin at $(-1, 2)$.

(iii) $\quad x^2-y^2-6x+2y+7 = 0$

Solution. $x^2-y^2-6x+2y+7 = 0$ ... (1)

Let the coordinates of the new origin be $O'(h,k)$. Equations of transformation are

$$x = X+h \quad , \quad y = Y+k$$

Substituting these values of x, y into (1), we get

$$(X+h)^2-(Y+k)^2-6(X+h)+2(Y+k)+7 = 0$$

or $\quad X^2+2Xh+h^2-(Y^2+2Yk+k^2)-6X-6h+2Y+2k+7 = 0$

or $\quad X^2-Y^2+X(2h-6)+Y(-2k+2)+h^2-k^2-6h+2k+7 = 0$ ... (2)

To remoe the first gdegree terms, we have

$$2h-6 = 0 \quad \Rightarrow \quad h = 3$$

and $\quad -2k+2 = 0 \quad \Rightarrow \quad k = 1$

New origin is $O'(3, 1)$. Putting $h = 3, k = 1$ into (2) the transformed equation is

$$X^2-Y^2+X(0)+Y(0)+(3)^2-(1)^2-6(3)+2(1)+7 = 0$$

or $\quad X^2 - Y^2 + 9 - 1 - 18 + 2 + 7 = 0$

or $\quad X^2 - Y^2 - 1 = 0$

which is the required equation with new origin at (3, 1).

3. In each of the following, find an equation referred to the new axes obtained by rotation of axes about the origin through the given angle:

(i) $\quad xy = 1, \ \theta = 45^\circ$

Solution. Here $\theta = 45^\circ$. Equations of transformation are

$$x = X\cos 45^\circ - Y\sin 45^\circ = X \cdot \frac{1}{\sqrt{2}} - Y \cdot \frac{1}{\sqrt{2}} = \frac{X - Y}{\sqrt{2}}$$

$$y = X\sin 45^\circ + Y\cos 45^\circ = X \cdot \frac{1}{\sqrt{2}} + Y \cdot \frac{1}{\sqrt{2}} = \frac{X + Y}{\sqrt{2}}$$

Substituting these values for x, y into the given equation, we have

$$\left(\frac{X - Y}{\sqrt{2}}\right)\left(\frac{X + Y}{\sqrt{2}}\right) = 1$$

or $\quad \dfrac{X^2 - Y^2}{2} = 1 \quad$ or $\quad X^2 - Y^2 = 2$

is the required transformed equation

(ii) $\quad 7x^2 - 8xy + y^2 - 9 = 0, \ \theta = \tan^{-1} 2$

Solution. Here $\theta = \tan^{-1} 2 \Rightarrow \tan\theta = 2 = \dfrac{2}{1}$

$$\Rightarrow \quad \text{base} = 2, \perp = 1 \text{ and hypotenuse} = \sqrt{4+1} = \sqrt{5}$$

$\therefore \quad \sin\theta = \dfrac{2}{\sqrt{5}} \quad$ and $\quad \cos\theta = \dfrac{1}{\sqrt{5}}$

Equations of transformation are

$$\left.\begin{aligned} x &= X\cos\theta - Y\sin\theta \\ y &= X\sin\theta + Y\cos\theta \end{aligned}\right\}$$

Using the above values

$$x = X \cdot \frac{1}{\sqrt{5}} - Y \cdot \frac{2}{\sqrt{5}} = \frac{X - 2Y}{\sqrt{5}}$$

$$y = X \cdot \frac{2}{\sqrt{5}} + Y \cdot \frac{1}{\sqrt{5}} = \frac{2X + Y}{\sqrt{5}}$$

Substituting these values for x, y into the given equation, we have

$$7\left(\frac{X-2Y}{\sqrt{5}}\right)^2-8\left(\frac{X-2Y}{\sqrt{5}}\right)\left(\frac{2X+Y}{\sqrt{5}}\right)+\left(\frac{2X+Y}{\sqrt{5}}\right)^2-9=0$$

By multiplying by 5, we get

$$7\left(\frac{X^2-4XY+4Y^2}{5}\right)-8\left(\frac{2X^2-3XY\quad 2Y^2}{5}\right)+\left(\frac{4X^2+4XY+Y^2}{5}\right)-9=0$$

or $\qquad 7X^2-28XY+28Y^2-16X^2+24XY+16Y^2+4X^2+4XY+Y^2-45=0$

or $\qquad -5X^2+45Y^2-45=0$

or $\qquad X^2-9Y^2+9=0$

which is the required transformed equation.

(iii) $\quad \mathbf{9x^2+12xy+4y^2-x-y=0\ ,\ \theta=\tan^{-1}\frac{2}{3}}$

Solution. Here $\theta=\tan^{-1}\frac{2}{3} \implies \tan\theta=\frac{2}{3}$

$\implies$ base $=3$, $\perp=2$ and hypotenuse $=\sqrt{9+4}=\sqrt{13}$

$\therefore \qquad \sin\theta=\frac{2}{\sqrt{13}}$ and $\cos\theta=\frac{3}{\sqrt{13}}$

Equations of transformation are

$$\left.\begin{aligned} x &= X\cos\theta - Y\sin\theta \\ y &= X\sin\theta + Y\cos\theta \end{aligned}\right\}$$

Putting values, we have

$$x = X\cdot\frac{3}{\sqrt{13}}-Y\cdot\frac{2}{\sqrt{13}}=\frac{3X-2Y}{\sqrt{13}}$$

$$y = X\cdot\frac{2}{\sqrt{13}}+Y\cdot\frac{3}{\sqrt{13}}=\frac{2X+3Y}{\sqrt{13}}$$

Substituting these values for x, y in to the given equations, we have

$$9\left(\frac{3X-2Y}{\sqrt{13}}\right)^2+12\left(\frac{3X-2Y}{\sqrt{13}}\right)\left(\frac{2X+3Y}{\sqrt{13}}\right)+4\left(\frac{2X+3Y}{\sqrt{13}}\right)^2-\left(\frac{3X-2Y}{\sqrt{13}}\right)$$

$$-\left(\frac{3X+2Y}{\sqrt{13}}\right)=0$$

or $$9\left(\frac{9X^2-12XY+4Y^2}{13}\right)+12\left(\frac{6X^2+5XY-6Y^2}{13}\right)$$

$$+ 4\left(\frac{4X^2 + 12XY + 9Y^2}{13}\right) - \left(\frac{3X - 2Y}{\sqrt{13}}\right) - \left(\frac{2X + 3Y}{\sqrt{13}}\right) = 0$$

or $$81X^2 - 108XY + 36Y^2 + 72X^2 + 60XY - 72Y^2 + 16X^2 + 48XY + 36Y^2$$
$$- 3\sqrt{13}\, X + 2\sqrt{13}\, Y - 2\sqrt{13}\, X - 3\sqrt{13}\, Y = 0 .$$

or $$169X^2 - 5\sqrt{13}\, X - \sqrt{13}\, Y = 0$$

or $$13\sqrt{13}\, X^2 - 5X - Y = 0$$

is the required transformed equation

(iv) $x^2 - xy + y^2 - 2\sqrt{2}\, x - 2\sqrt{2}\, y + 2 = 0, \; \theta = 45°$

Solution. Here $\theta = 45°$

Equations of transformation are

$$\left.\begin{aligned} x &= X\cos\theta - Y\sin\theta \\ y &= X\sin\theta + Y\cos\theta \end{aligned}\right\}$$

Putting values, ewe have

$$x = X\cos 45° - Y\sin 45° = X \cdot \frac{1}{\sqrt{2}} - Y \cdot \frac{1}{\sqrt{2}} = \frac{X - Y}{\sqrt{2}}$$

$$y = X\sin 45° + Y\cos 45° = X \cdot \frac{1}{\sqrt{2}} + Y \cdot \frac{1}{\sqrt{2}} = \frac{X + Y}{\sqrt{2}}$$

Substituting these expressions for x, y into the given equation, we have

$$\left(\frac{X - Y}{\sqrt{2}}\right)^2 - 2\left(\frac{X - Y}{\sqrt{2}}\right)\left(\frac{X + Y}{\sqrt{2}}\right) + \left(\frac{X + Y}{\sqrt{2}}\right)^2 - 2\sqrt{2}\left(\frac{X - Y}{\sqrt{2}}\right)$$
$$- 2\sqrt{2}\left(\frac{X + Y}{\sqrt{2}}\right) + 2 = 0$$

or $$\left(\frac{X^2 - 2XY + Y^2}{2}\right) - 2\left(\frac{X^2 - Y^2}{2}\right) + \left(\frac{X^2 + 2XY + Y^2}{2}\right)$$
$$- 2(X - Y) - 2(X + Y) + 2 = 0$$

(by $\times 2$): $X^2 - 2XY + Y^2 - 2X^2 + 2Y^2 + X^2 + 2XY + Y^2 - 4X + 4Y - 4X - 4Y + 4 = 0$

or $\quad 4Y^2 - 8X + 4 = 0 \quad$ or $\quad Y^2 - 2X + 1 = 0$

which is the required transformed equation.

4. Find measure of the angle through which the axes be rotated so that the product term XY is removed from the transformed equation. Also find the transformed equation.

(i) $2x^2 + 6xy + 10y^2 - 11 = 0$

Solution. Let the axes be rotated through an angle θ.

Equations of transformation are

$$x = X\cos\theta - Y\sin\theta \quad , \quad y = X\sin\theta + Y\cos\theta$$

Substituting into the given equation, we get

$$2(X\cos\theta - Y\sin\theta)^2 + 6(X\cos\theta - Y\sin\theta)(X\sin\theta + Y\cos\theta)$$
$$+ 10(X\sin\theta + Y\cos\theta)^2 - 11 = 0$$

or $$2(X^2\cos^2\theta - 2XY\sin\theta\cos\theta + Y^2\sin^2\theta)$$
$$+ 6(X^2\sin\theta\cos\theta - Y^2\sin\theta\cos\theta + XY\cos^2\theta - XY\sin^2\theta)$$
$$+ 10(X^2\sin^2\theta + 2XY\sin\theta\cos\theta + Y^2\cos^2\theta) - 11 = 0$$

or $$X^2(2\cos^2\theta + 6\sin\theta\cos\theta + 10\sin^2\theta)$$
$$+ XY(-4\sin\theta\cos\theta + 6\cos^2\theta - 6\sin^2\theta + 20\sin\theta\cos\theta)$$
$$+ Y^2(2\sin^2\theta - 6\sin\theta\cos\theta + 10\cos^2\theta) - 11 = 0 \qquad \text{(i)}$$

Since this equation is to be free from the product term, i.e.;

$$-4\sin\theta\cos\theta + 6\cos^2\theta - 6\sin^2\theta + 20\sin\theta\cos\theta = 0$$

or $$-6\sin^2\theta + 16\sin\theta\cos\theta + 6\cos^2\theta = 0$$

or $$3\tan^2\theta - 8\tan\theta - 3 = 0 \qquad [\text{by} \div \cos^2\theta]$$

$$\therefore \quad \tan\theta = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(-3)}}{2(3)} = \frac{8 \pm \sqrt{64+36}}{6}$$

$$= \frac{8 \pm \sqrt{100}}{6} = \frac{8 \pm 10}{6} = \frac{8+10}{6}, \frac{8-10}{6} = \frac{18}{6}, \frac{-2}{6}$$

$$= 3, \frac{-1}{3} \Rightarrow \tan\theta = 3 \text{ (as } \theta \text{ is in the first quadrant)}$$

$\Rightarrow$ base = 1, $\perp = 3$ and hypotenuse = $\sqrt{1+9} = \sqrt{10}$

$$\therefore \quad \sin\theta = \frac{3}{\sqrt{10}} \quad \text{and} \quad \cos\theta = \frac{1}{\sqrt{10}}$$

Substituting $\sin\theta = \dfrac{3}{\sqrt{10}}$ and $\cos\theta = \dfrac{1}{\sqrt{10}}$ in (i), then

$$X^2\left(\left(2\left(\frac{1}{\sqrt{10}}\right)\right)^2 + 6\left(\frac{3}{\sqrt{10}}\right)\left(\frac{1}{\sqrt{10}}\right) + 10\left(\frac{3}{\sqrt{10}}\right)^2\right)$$
$$+ XY\left(-4\left(\frac{3}{\sqrt{10}}\right)\left(\frac{1}{\sqrt{10}}\right) + 6\left(\frac{1}{\sqrt{10}}\right)^2 - 6\left(\frac{3}{\sqrt{10}}\right)^2 + 20\left(\frac{3}{\sqrt{10}}\right)\left(\frac{1}{\sqrt{10}}\right)\right)$$

$$+ Y^2\left(2\left(\frac{3}{\sqrt{10}}\right)^2 - 6\left(\frac{3}{\sqrt{10}}\right)\left(\frac{1}{\sqrt{10}}\right) + 10\left(\frac{1}{\sqrt{10}}\right)^2\right) - 11 = 0$$

or $$X^2\left(\frac{2}{10} + \frac{18}{10} + \frac{90}{10}\right) + XY\left(-\frac{12}{10} + \frac{6}{10} - \frac{54}{10} + \frac{60}{10}\right)$$

$$+ Y^2\left(\frac{18}{10} - \frac{18}{10} + \frac{10}{10}\right) - 1 = 0$$

or $$11X^2 + Y^2 - 11 = 0$$

(ii) $$xy + 4x - 3y - 10 = 0 \qquad \dots (1)$$

Solution. Let the axes be rotated through an angle θ.

Equation of transformation are

$$x = X\cos\theta - Y\sin\theta\ ;\ y = X\sin\theta + Y\cos\theta$$

Substituting into the given equation (1), we have

$$(X\cos\theta - Y\sin\theta)(X\sin\theta + Y\cos\theta) + 4(X\cos\theta - Y\sin\theta)$$

$$+ XY\cos^2\theta - XY\sin^2\theta - 3(X\sin\theta + Y\cos\theta) - 10 = 0$$

$$X^2\sin\theta\cos\theta + XY\cos^2\theta - XY\sin^2\theta - Y^2\sin\theta\cos\theta + 4X\cos\theta$$

$$-4Y\sin\theta - 3X\sin\theta - 3Y\cos\theta - 10 = 0$$

or $$X^2\sin\theta\cos\theta + XY(\cos^2\theta - \sin^2\theta) - Y^2\sin\theta\cos\theta + X(4\cos\theta - 3\sin\theta)$$

$$-Y(4\sin\theta + 3\cos\theta) - 10 = 0 \qquad \dots (2)$$

Since this equation is to be free from the product term XY

i.e., $$\cos^2\theta - \sin^2\theta = 0$$

or $$\cos^2\theta = \sin^2\theta \Rightarrow \tan^2\theta = 1 \Rightarrow \tan\theta = 1 \Rightarrow \theta = 45°$$

Thus axes be rotated through an angle of $45°$. So that XY term is removed from the transformed equation.

Setting $\theta = 45°$ in (i), the transformed equation is

$$X^2\sin 45°\cos 45° + XY(\cos^2 45° - \sin 2\,45°) - Y^2\sin 45°\cos 45°$$

$$+ X(4\cos 45° - 3\sin 45°) - Y(4\sin 45° + 3\cos 45°) - 10 = 0$$

$$X^2 \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + XY(0) - Y^2 \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + X\left(4 \cdot \frac{1}{\sqrt{2}} - 3 \cdot \frac{1}{\sqrt{2}}\right)$$

$$-Y\left(4 \cdot \frac{1}{\sqrt{2}} + 3 \cdot \frac{1}{\sqrt{2}}\right) - 10 = 0$$

or $$X^2 \cdot \frac{1}{2} - Y^2 \cdot \frac{1}{2} + X \cdot \frac{1}{\sqrt{2}} - Y \cdot \frac{7}{\sqrt{2}} - 10 = 0$$

or $\quad X^2 - Y^2 + \sqrt{2}\, X - 7\sqrt{2}\, Y - 20 = 0$

is the requied equation.

(iii) $\quad 5x^2 - 6xy + 5y^2 - 8 = 0 \qquad \ldots (1)$

Solution. Let the axes be rotated through an angle θ.

Equation of transformation are

$$x = X\cos\theta - Y\sin\theta \; ; \; y = X\sin\theta + Y\cos\theta$$

Substituting into the given equation, we get

$$5(X\cos\theta - Y\sin\theta)^2 - 6(X\cos\theta - Y\sin\theta)(X\sin\theta + Y\cos\theta) + 5(X\sin\theta + Y\cos\theta)^2 - 8 = 0$$

or
$$5(X^2\cos^2\theta - 2XY\sin\theta\cos\theta + Y^2\sin^2\theta) - 6(X^2\sin\theta\cos\theta + XY\cos^2\theta - XY\sin^2\theta - Y^2\sin\theta\cos\theta) + 5(X^2\sin^2\theta + 2XY\sin\theta\cos\theta + Y^2\cos^2\theta) - 8 = 0$$

or
$$X^2(5\cos^2\theta - 6\sin\theta\cos\theta + 5\sin^2\theta) + XY(-10\sin\theta\cos\theta - 6\cos^2\theta + 6\sin^2\theta + 10\sin\theta\cos\theta) + Y^2(5\sin^2\theta + 6\sin\theta\cos\theta + 5\cos^2\theta) - 8 = 0$$

Since this equation is to be free from the product term XY

i.e., $\quad -6\cos^2\theta - 6\sin^2\theta = 0$

or $\quad \sin^2\theta = \cos^2\theta \;\Rightarrow\; \tan^2\theta = 1 \;\Rightarrow\; \tan\theta = 1$

$\Rightarrow \quad \theta = 45^\circ$ (as θ is in the first quadrant)

Setting $\theta = 45^\circ$ in (1), the transformed equation is

$$X^2\left(5\cdot\left(\frac{1}{\sqrt{2}}\right)^2 - 6\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} + 5\cdot\left(\frac{1}{\sqrt{2}}\right)^2\right) + XY\left(-10\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} - 6\cdot\left(\frac{1}{\sqrt{2}}\right)^2 + 6\cdot\left(\frac{1}{\sqrt{2}}\right)^2 + 10\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}\right) + Y^2\left(5\cdot\left(\frac{1}{\sqrt{2}}\right)^2 + 6\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} + 5\cdot\left(\frac{1}{\sqrt{2}}\right)^2\right) - 8 = 0$$

or
$$X^2\left(\frac{5}{2} - \frac{6}{2} + \frac{5}{2}\right) + XY(0) + Y^2\left(\frac{5}{2} + \frac{6}{2} + \frac{5}{2}\right) - 8 = 0$$

or $\quad 2X^2 + 8Y^2 - 8 = 0 \;\Rightarrow\; X^2 + 4Y^2 - 4 = 0$

which is the required equation.