Exercise 2.1

Q1. Identify which of the following are relational and irrational numbers.

(i) $\sqrt{3}$ Irrational Number

(ii) $\frac{1}{6}$ Rational Number

(iii) π Irrational Number

(iv) $\frac{15}{2}$ Rational Number

(v) 7.25 Rational Number

(vi) $\sqrt{29}$ Irrational Number

Q2. Convert the following fractions into decimal fraction.

(i) $\frac{17}{25}$

Sol: $\frac{17}{25} = 0.68$

(ii) $\frac{19}{4}$

Sol: $\frac{19}{4} = 4.75$

(iii) $\frac{57}{8}$

Sol: $\frac{57}{8} = 7.125$

(iv) $\frac{205}{18}$

Sol: $\frac{205}{18} = 11.3889$

(v) $\frac{5}{8}$

Sol: $\frac{5}{8} = 0.625$

(vi) $\frac{25}{38}$

Sol: $\frac{25}{38} = 0.65789$

Q2. Which of the following statements are true and which are false?

(i) $\frac{2}{3}$ is an irrational number. False

(ii) π is an irrational number. True

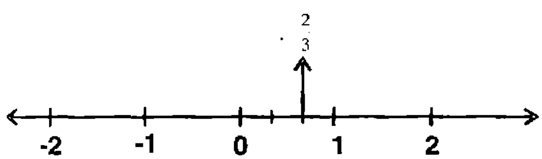
(iii) $\frac{1}{9}$ is a terminating fraction. False

(iv) $\frac{3}{4}$ is a terminating fraction. True

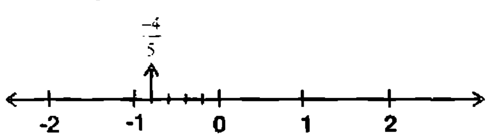
(v) $\frac{4}{5}$ is a recurring fraction. False

Q4. Represent the following numbers on the number line.

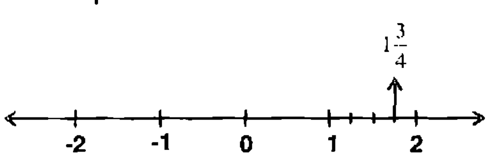
(i) $\frac{2}{3}$



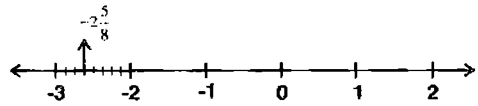
(ii) $-\frac{4}{5}$



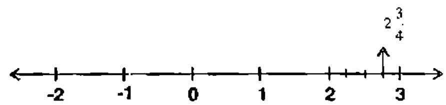
(iii) $1\frac{3}{4}$



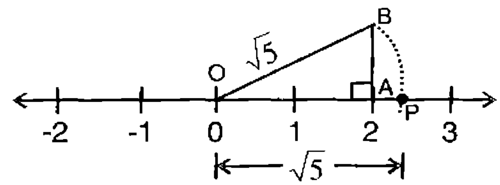
(iv) $-2\frac{5}{8}$



(v) $2\frac{3}{4}$



(vi) $\sqrt{5}$



By Pythagoras theorem

$$OB = \sqrt{(2)^2 + (1)^2} = \sqrt{4+1} = \sqrt{5}$$

By drawing an arc with centre at O and radius OB = $\sqrt{5}$ we get point P representing $\sqrt{5}$ on the number line.

Q5. Give a rational number between $\frac{3}{4}$ and $\frac{5}{9}$.

Ans. The required rational number is the mean of two given numbers, so the required number

$$= \frac{\frac{3}{4} + \frac{5}{9}}{2}$$

$$= \frac{1}{2}\left(\frac{3}{4} + \frac{5}{9}\right)$$

$$= \frac{1}{2}\left(\frac{27+20}{36}\right)$$

$$= \frac{47}{72}$$

Q6. Express the following recurring decimals as the rational number $\frac{p}{q}$, where p, q are integers and $q \neq 0$

(i) $0.\overline{5}$

Sol: Let $x = 0.\overline{5}$

$x = 0.55555\ldots\ldots\ldots$ (i)

Multiplying both sides by 10

$10x = 10(0.5555\ldots\ldots)$

$10x = 5.5555\ldots\ldots\ldots$ (ii)

Subtracting (i) from (ii)

$10x - x = (5.5555\ldots) - (0.5555\ldots)$

$9x = 5$

$x = \frac{5}{9}$

Hence $0.\overline{5} = \frac{5}{9}$

(ii) $0.\overline{13}$

Sol: Let $x = 0.\overline{13}$

$x = 0.13131313\ldots\ldots\ldots$ (i)

Multiplying both sides by 100

$100x = 100(0.131313\ldots\ldots)$

$100x = 13.131313\ldots\ldots\ldots$ (ii)

Subtracting (i) from (ii)

$100x - x = (13.1313\ldots\ldots) - (0.1313\ldots)$

$99x = 13$

$x = \frac{13}{99}$

Hence $0.\overline{13} = \frac{13}{99}$

(iii) $0.\overline{67}$

Let $x = 0.\overline{67}$

$x = 0.676767\ldots\ldots\ldots$ (i)

Multiplying both sides by 100

$100x = 100(0.676767\ldots\ldots)$

$100x = 67.676767\ldots\ldots$ (ii)

Subtracting (i) from (ii)

$100x - x = (67.676767\ldots) - (0.676767\ldots)$

$99x = 67$

$x = \frac{67}{99}$

Hence $0.\overline{67} = \frac{67}{99}$

Properties of Real numbers with respect to Addition and Multiplication

a. Properties of real numbers under addition are as follows:

(i) Closure Property

$a + b \in R, \forall a, b \in R$

e.g., if -3 and $5 \in R$

then $-3 + 5 = 2 \in R$

(ii) Commutative Property

$a + b = b + a, \forall a, b \in R$

e.g., if $2, 3 \in R$

then $2 + 3 = 3 + 2$

or $5 = 5$

(iii) Associative Property

$(a + b) + c = a + (b + c), \forall a, b, c \in R$

e.g., if $5, 7, 3 \in R$

then $(5 + 7) + 3 = 5 + (7 + 3)$

or $12 + 3 = 5 + 10$

or $15 = 15$

(iv) Additive Identity

There exists a unique real number 0 called additive identity such that

$a + 0 = a = 0 + a, \quad \forall a \in R$

(v) Additive Inverse

For every $a \in R$, there exists a unique real number $-a$ called the additive inverse of a such that

$a + (-a) = 0 = (-a) + a$

e.g., additive inverse of 3 is -3 since $3 + (-3) = 0 = (-3) + (3)$

b. Properties of real numbers under multiplication are as follows:

(i) **Closure Property**

$ab \in R, \quad \forall a, b \in R$

e.g., if $-3, 5 \in R$

then $(-3)(5) \in R$

or $-15 \in R$

(ii) **Commutative Property:**

$ab = ba, \quad \forall a, b \in R$

e.g., if $\frac{1}{3}, \frac{3}{2} \in R$

then $\left(\frac{1}{3}\right)\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)\left(\frac{1}{3}\right)$

or $\frac{1}{2} = \frac{1}{2}$

(iii) **Associative Property:**

$(ab)c = a(bc), \forall a, b, c \in R$

e.g., if $2, 3, 5 \in R$

then $(2 \times 3) \times 5 = 2 \times (3 \times 5)$

or $6 \times 5 = 2 \times 15$

or $30 = 30$

(iv) **Multiplicative Identity:**

There exists a unique real number 1, called the multiplicative identity such that

$a.1 = a = 1.a \; \forall a \in R$

(v) **Multiplicative Inverse**

For every non-zero real number, there exists a unique real number a^{-1} or $\frac{1}{a}$, called multiplicative inverse of a, such that

$aa^{-1} = 1 = a^{-1}a$

or $a \times \frac{1}{a} = 1 = \frac{1}{a} \times a$

e.g., if $5 \in R$, then $\frac{1}{5} \in R$ such that

$$5 \times \frac{1}{5} = 1 = \frac{1}{5} \times 5$$

So, 5 and $\frac{1}{5}$ are multiplicative inverse of each other.

(vi) **Multiplication is Distributive over Addition and Subtraction**

For all $a, b, c \in R$

$a(b + c) = ab + ac$ (Left distributive law)

$(a + b)c = ac + bc$ (Right distributive law)

e.g., if $2, 3, 5 \in R$, then

$2(3 + 5) = 2 \times 3 + 2 \times 5$

or $2 \times 8 = 6 + 10$

or $16 = 16$

And for all $a, b, c \in R$

$a(b - c) = ab - ac$ (Left distributive law)

$(a - b)c = ac - bc$ (Right distributive law)

e.g., if $2, 5, 3 \in R$, then

$2(5 - 3) = 2 \times 5 - 2 \times 3$

or $2 \times 2 = 10 - 6$

or $4 = 4$

(b) Properties of Equality of Real Numbers:

Properties of equality of real numbers are as follows:

(i) Reflexive Property

$a = a, \; \forall a \in R$

(ii) Symmetric Property

If $a = b$, then $b = a$, $\forall a, b \in R$

(iii) Transitive Property

If $a = b$ and $b = c$, then $a = c$, $\forall a, b, c \in R$

(iv) Additive Property

If $a = b$, then $a + c = b + c$, $\forall a, b, c \in R$

(v) Multiplicative Property

If $a = b$, then $ac = bc$, $\forall a, b, c \in R$

(vi) Cancellation Property for Addition

If $a+c=b+c$, then $a=b$, $\forall a,b,c \in R$

(vii) Cancellation property for Multiplication

If $ac = bc$, $c \neq 0$ then $a = b$, $\forall$ $a, b, c \in R$

(c) Properties of Inequalities of Real numbers

Properties of inequalities of real numbers are as follows:

(i) Trichotomy Property

$\forall$ $a, b \in R$

$a < b$ or $a = b$ or $a > b$

(ii) Transitive Property

$\forall$ $a, b, c \in R$

(a) $a < b$ *and* $b < c \Rightarrow a < c$

(b) $a>b$ *and* $b>c \Rightarrow a > c$

(iii) Multiplicative Property

(a) $\forall$ $a, b, c \in R$ and $c > 0$

(i) $a > b \Rightarrow ac > bc$ (ii) $a < b \Rightarrow ac < bc$

(i) $a > b \Rightarrow ca > cb$ (ii) $a < b \Rightarrow ca < cb$

(b) $\forall$ $a, b, c \in R$ and $c < 0$

(i) $a > b \Rightarrow ac < bc$ (ii) $a < b \Rightarrow ac > bc$

(i) $a > b \Rightarrow ca < cb$ (ii) $a < b \Rightarrow ca > cb$

(iv) Multiplicative Inverse Property:

$\forall$ $a, b \in R$ and $a \neq 0$, $b \neq 0$

(a) $a < b \Leftrightarrow \frac{1}{a} > \frac{1}{b}$

(b) $a > b \Leftrightarrow \frac{1}{a} < \frac{1}{b}$

(v) Additive property:

$\forall$ **a, b c $\in$ R**

(a) $a < b \Rightarrow a + c < b+c$

$a < b \Rightarrow c + a < c+b$

(b) $a > b \Rightarrow a + c > b + c$

$a > b \Rightarrow c + a > c + b$