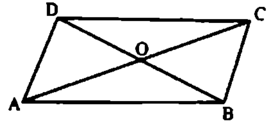
Given:

ABCD is a parallelogram with $\overline{AC}$ and $\overline{BD}$ are its diagonals.



To Prove

$$\overline{AB}^2+\overline{BC}^2+\overline{CD}^2+\overline{DA}^2=\overline{AC}^2+\overline{BD}^2$$

In ΔACD

$$\overline{DC}^2+\overline{AD}^2=2\overline{OD}^2+\overline{OA}^2 \quad \_\_\_\_\_\_ \text{(i)}$$

And In ΔABC

$$\overline{AB}^2+\overline{BC}^2=2\overline{OB}^2+\overline{OA}^2 \quad \_\_\_\_\_\_ \text{(ii)}$$

Adding (i) & (ii)

$$\overline{DC}^2+\overline{AD}^2+\overline{AB}^2+\overline{BC}^2=4\overline{OA}^2+2\overline{OD}^2+2\overline{OB}^2$$

$$\overline{AB}^2+\overline{BC}^2+\overline{CD}^2+\overline{AD}^2=4\overline{OA}^2+2\overline{OD}^2+2\overline{OD}^2 \qquad \left[\because \overline{OB}=\overline{OD}\right]$$

$$\overline{AB}^2+\overline{BC}^2+\overline{CD}^2+\overline{AD}^2=4\overline{OA}^2+4\overline{OD}^2$$

$$\overline{AB}^2+\overline{BC}^2+\overline{CD}^2+\overline{AD}^2=\left(2\overline{OA}\right)^2+\left(2\overline{OD}\right)^2$$

$$\overline{AB}^2+\overline{BC}^2+\overline{CD}^2+\overline{AD}^2=\overline{AC}^2+\overline{BD}^2$$

Hence proved

SOLVED MISCELLANEOUS EXERCISE 8

Q1. In a ΔABC, $m\angle A = 60^\circ$, prove that $(\overline{BC})^2=(\overline{AB})^2+\overline{AC}^2-m\overline{AB}\,.\,m\overline{AC}$.

Solution:

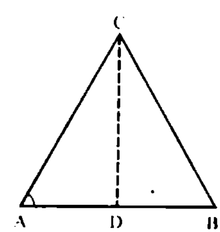
In a ΔABC, $m\angle A = 60^\circ$,

Given:

In a ΔABC, $m\angle A = 60^\circ$

Required:

$$\left(\overline{BC}\right)^2=\left(\overline{AB}\right)^2+\left(\overline{AC}\right)^2-\overline{AB}.\overline{AC}$$



Construction:

Draw $\overline{CD}\perp\overline{AB}$, so that the Projection of $\overline{AC}$ on AB.

Proof:

In right angle ΔACD

$\angle A = 60^\circ$ and $\angle ACD = 30^\circ$ (being complement of $\overline{CA}$)

And $\angle$ ACD, side opposite to $\angle = 30^\circ = \frac{1}{2}$ hyp $\overline{AC}$.

Now, according to the theorem, we have

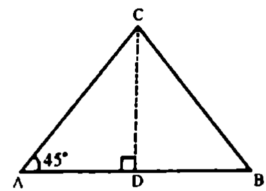
$\overline{BC}^2 = \overline{AB}^2 + \overline{AC}^2 - 2\,\overline{AB}\,.\,\overline{AD}$

$\Rightarrow \overline{BC}^2 = \overline{AB}^2 + \overline{AC}^2 - \overline{AB}\,.\,\overline{AC}$ $\quad [\because 2AD = AC]$.

Q2. In a ΔABC, $m\angle A = 45^\circ$, prove that $(\overline{BC})^2 = (\overline{AB})^2 + (\overline{AC})^2 - \sqrt{2}\; m\overline{AB}.m\overline{AC}$.

Solution:

In a ΔABC, m $\angle A = 45^\circ$.



Given:

In a ΔABC; $m\angle A = 45^\circ$.

Required:

$\overline{BC}^2 = AB^2 + AC^2 - \sqrt{2}\,\overline{AB}.\overline{AC}^2$

Construction:

Draw CD $\perp$ AB, so that the projection of AC on AB.

Proof:

In right angle ΔACD

$\angle A = 45^\circ$ and $\angle ACD = 45^\circ$ (being complement of $\angle A$)

And $\angle ACD$; side opposite to $\angle 45^\circ = \sqrt{2}$ .hyp. $\overline{AC}$

ΔABC is acute angled at A, so according to the theorem, we have

$\overline{BC}^2 = \overline{AB}^2 + \overline{AC}^2 - 2\overline{AB}.\overline{AD}$

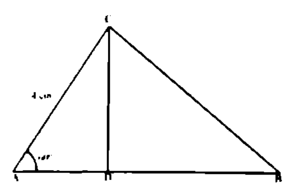
$\Rightarrow BC^2 = AB^2 + AC^2 - \sqrt{2}\;\overline{AB}.\overline{AD}\;\left[\because 2\overline{AD} = \sqrt{2}AC\right]$

Hence proved

Q3. In a ΔABC, calculate mBC when $m\overline{AB} = 5$ cm, $m\overline{AC} = 4$ cm, $m\angle A = 60^\circ$.

Solution:

We know that



$$\begin{aligned}\overline{BC}^2 &= \overline{AB}^2 + \overline{AC}^2 - \overline{AB}^2.AC\\ &= 5^2 + 4^2 - 5.4\\ &= 25 + 16 - 20\\ &= 21\end{aligned}$$

$m\overline{BC}^2 = \sqrt{21} = 4.58$cm

Q4. In a ΔABC, calculate $m\overline{AC}$ when $m\overline{AB} = 5$ cm, $mBC = 4\sqrt{2}$ cm, $m\angle B = 45°$.

Solution:

We know that

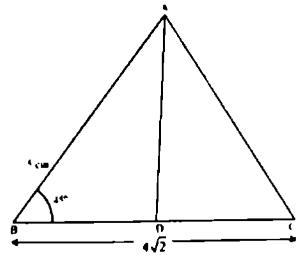
$$\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2 - \sqrt{2}\,\overline{AB}.\overline{BC}$$

$$= (5)^2 + \left(4\sqrt{2}\right)^2 - \sqrt{2}(5)\left(4\sqrt{2}\right)$$

$$= 25 + 32 - 40$$

$$= 57 - 40 = 17.$$

$$m\overline{AC}^2 = \sqrt{17}\text{cm} = 4.123\text{cm}$$



Q5. In a triangle ABC, $m\overline{BC} = 21$ cm, $m\overline{AC} = 17$ cm, $m\overline{AB} = 10$ cm. Measure the length of projection of $\overline{AC}$ upon $\overline{BC}$.

Solution:

C = 10 cm, a = 21 cm, b = 17 m, x = ?

We know that

$$C^2 = a^2 + b^2 - 2(a)(x)$$

$$(10)^2 = (21)^2 + (17)^2 - 2(21)(x)$$

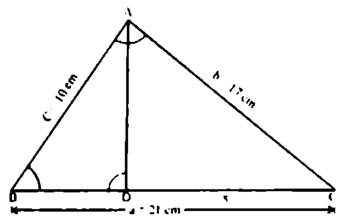
$$100 = 441 + 189 - 42x$$

$$42x = 441 + 189 - 42x$$

$$42x = 730 - 100$$

$$42x = 630$$

$$x = \frac{630}{40} = 15 \text{ cm}$$



Q6. In a triangle ABC, mBC = 21 cm. mAC = 17 cm, mAB = 10 cm. Calculate the projection of AB upon BC.

Solution:

C = 10 cm, a = 21 cm, b = 17 m, x = ?

We know that

$$b^2 = a^2 + c^2 - 2ax$$

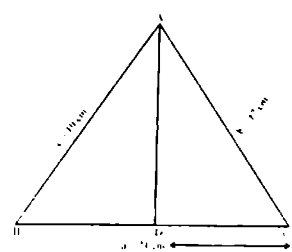
$$(17)^2 = (10)^2 + (21)^2 - 2(21)(x)$$

$$289 = 100 + 441 - 42x$$

$$289 = 541 - 42x$$

$$42x = 541 - 289$$

$$42x = 252$$



$$x = \frac{252}{42} = 6 \text{ cm}$$

Q7. In a ΔABC, a = 17 cm, b = 15 cm and c = 8 cm find $m\angle A$.

Solution:

Given:

In a ΔABC, $a = 17$ cm, $b = 15$ cm and, $c = 8$ cm

Required: $m\angle A = ?$

by Pythagoras theorem.

$$a^2 = b^2 + c^2$$

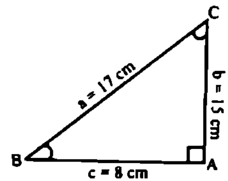
$$17^2 = 15^2 + 8^2$$

$$289 = 225 + 64$$

$$289 = 289$$

So, it satisfied, that given values are the sides of a right angled triangle.

$\therefore m\angle A = 90^\circ$



Q8. In a ΔABC, a = 17 cm, b = 15 cm and c = 8 cm find $m\angle B$.

Solution:

Given:

In a Δ ABC; $a = 17$ cm, $b = 15$ cm and $c = 8$ cm

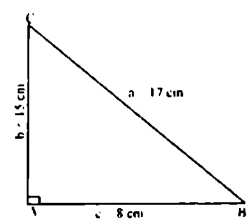
Required:

$m\angle B = ?$

We know that it is right angled triangle.

$$\text{Sin}\,(m\angle B) = \frac{b}{a} = \frac{15}{17} = 0.882$$

$$m\angle B = \sin^{-1}(0.882) = 61.90$$



Q9. Whether the triangle with sides 5 cm, 7 cm, 8 cm is acute, obtuse or right angled.

Solution:

Given:

$a = 5$cm; $b = 7$ cm; $c = 8$ cm

Case I:

$$c^2 = a^2 + b^2$$

$$8^2 = 5^2 + 7^2$$

$$64 = 25 + 49$$

$$64 = 74$$

It is not right angled triangle.

Case II:

Then.

$$
\begin{aligned}
b^2 &= a^2 + c^2 \\
(7)^2 &= (5)^2 + (8)^2 \\
49 &= (5)^2 + (8)^2 \\
49 &\neq 91
\end{aligned}
$$

Case III:

$$
\begin{aligned}
a^2 &= b^2 + c^2 \\
(5)^2 &= (7)^2 + (8)^2 \\
25 &= 49 + 64 \\
25 &\neq 113
\end{aligned}
$$

Which is not possible, so the given data shows that it is not obtuse triangle; It is acute angled triangle.

Q10. Whether the triangle with sides 8 cm, 15 cm, 17 cm is acute, obtuce or right angled.

Solution:

$a = 8;\quad b = 15;\quad c = 17$

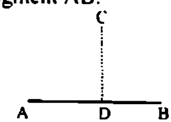
Case1: It is right angled.

$$
\begin{aligned}
c^2 &= a^2 + b^2 \\
17^2 &= 8^2 + 15^2 \\
289 &= 64 + 225 \\
289 &= 289
\end{aligned}
$$

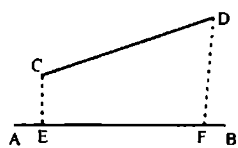
Hence; it is right angled triangle.

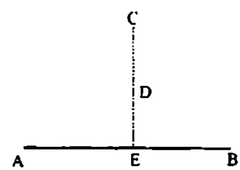
SUMMARY

- ✔ The projection of a given point on a line segment is the foot ⊥ of drawn from the point on that line segment. If $\overline{CD} \perp \overline{AB}$, then evidently D is the foot of perpendicular $\overline{CD}$ from tire point C on the line segment AB.



- ✔ The projection of a line segment $\overline{CD}$ on a line segment AB is the portion $\overline{EF}$ of the latter intercepted between foots of the perpendiculars drawn from C and D. However projection of a vertical line segment $\overline{CD}$ on a line segment AB is a point on $\overline{AB}$ which is of zero dimension.





- ✔ In an obtuse-angled triangle, the square on the side opposite to the obtuse angle is equal to the sum of the squares on the sides containing the obtuse angle together with twice the rectangle contained by one of the sides, and the projection on it of the other.
- ✔ In any triangle, the square on the side opposite to an acute angle is equal to the sum of the squares on the sides containing that acute angle diminished by twice the rectangle contained by one of those sides and the projeetion on it of the other.
- ✔ In any triangle, the sum of the squares on any two sides is equal to twice the square on half the third side together with twice the square on the median which bisects the third side (Apolloniu's Theorem).