Exercise 12.4

1. Solve the triangle ABC if : $\beta = 60^\circ$, $\gamma = 15^\circ$, $b = \sqrt{6}$.

Solution. We know that $\alpha+\beta+\gamma = 180^\circ$

Now $\alpha = 180-\beta-\gamma = 180^\circ - 60^\circ - 15^\circ = 105^\circ$

By Law of Sines : $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} \Rightarrow a = b.\dfrac{\sin\alpha}{\sin\beta}$

$= \sqrt{6}.\dfrac{\sin 105^\circ}{\sin 60^\circ} = \sqrt{6}.\dfrac{0.9659}{0.8660} = \sqrt{6}\times 1.1153 = 2.731.$

Again, $\dfrac{c}{\sin\gamma} = \dfrac{b}{\sin\beta} \Rightarrow c = b.\dfrac{\sin\gamma}{\sin\beta}$

$= \sqrt{6}.\dfrac{\sin 15^\circ}{\sin 60^\circ} = \sqrt{6}.\dfrac{0.2588}{0.8660} = \sqrt{6}\times 0.2989 = 0.732$

Hence $\boxed{\alpha = 105^\circ\ ,\ a = 2.73\ , c = 0.73}$

2. $\beta = 52^\circ$, $\gamma = 89^\circ 35'$, $a = 89.35$.

Solution. We know that $\alpha+\beta+\gamma = 180^\circ$

Now $\alpha = 180-\beta-\gamma = 180^\circ - 52^\circ - 89^\circ 35' = 38^\circ 25'$

By Law of Sines : $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} \Rightarrow b = a.\dfrac{\sin\beta}{\sin\alpha}$

$= 89.35.\dfrac{\sin 52^\circ}{\sin 38^\circ 25'} = \sqrt{6}.\dfrac{0.7880}{0.6213} = 89.35\times 1.268 = 113.31.$

Again, $\dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma} \Rightarrow c = b.\dfrac{\sin\gamma}{\sin\beta}$

$= 113.31.\dfrac{\sin 89^\circ 35'}{\sin 52^\circ} = 113.31.\dfrac{0.999}{0.788} = 113.31\times 1.269 = 143.79$

Hence $\boxed{\alpha = 38^\circ 25'\ ,\ b = 113.31\ , c = 143.79}$

3. $b = 125$, $\gamma = 53^\circ$, $\alpha = 47^\circ$

Solution. We know that $\alpha+\beta+\gamma = 180^\circ$

Now $\beta = 180-\alpha-\gamma = 180^\circ - 47^\circ - 53^\circ = 80^\circ$

By Law of Sines : $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta}$

$\Rightarrow a = b.\dfrac{\sin\alpha}{\sin\beta} = 125.\dfrac{\sin 47^\circ}{\sin 80^\circ} = 125.\dfrac{0.731}{0.985} = 125\times 0.742 = 93$

Again, $\dfrac{c}{\sin\gamma} = \dfrac{b}{\sin\beta}$

$\Rightarrow c = b.\dfrac{\sin\gamma}{\sin\beta} = 152.\dfrac{\sin 53^\circ}{\sin 80^\circ} = 125.\dfrac{0.7986}{0.985} = 125\times 0.811 = 101$

Hence $\boxed{\beta = 80^\circ\ ,\ a = 93\ , c = 101}$

4. $c = 16.1$, $\alpha = 42^\circ 45'$, $\gamma = 74^\circ 32'$

Solution. We know that $\alpha+\beta+\gamma = 180^\circ$

Now $\beta = 180-\alpha-\gamma = 180^\circ - 42^\circ 45' - 74^\circ 32' = 62^\circ 43'$

Now $\dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma} \Rightarrow b = c.\dfrac{\sin\beta}{\sin\gamma}$

$= 16.1\times\dfrac{\sin 62^\circ 43'}{\sin 74^\circ 32'} = 16.1\times\dfrac{0.899}{0.964} = 16.1\times 0.922 = 14.85$

$$\text{Again: } \frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \Rightarrow a = b.\frac{\sin\alpha}{\sin\beta}$$

$$= 14.85 \times \frac{\sin 42^\circ.45'}{\sin 62^\circ\, 43'} = 14.85 \times \frac{0.678}{0.899} = 14.85 \times 0.7635 = 11.34$$

Hence $\boxed{\beta = 62^\circ\, 43',\ a = 11.34,\ b = 14.85}$

5. $a = 53$, $\beta = 88^\circ\, 36'$, $\gamma = 31^\circ\, 54'$

Solution. We know that $\alpha + \beta + \gamma = 180^\circ$

Now $\alpha = 180 - \beta - \gamma = 180^\circ - 88^\circ\, 36' - 31^\circ\, 54' = 59^\circ\, 30'$

$$\text{By Law of Sines : } \frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \Rightarrow b = a.\frac{\sin\beta}{\sin\alpha}$$

$$= 53 \times \frac{\sin 88^\circ\, 36'}{\sin 59^\circ\, 30'} = 53 \times \frac{0.9997}{0.8616} = 53 \times 1.16 = 61.49$$

$$\text{Again, } \frac{c}{\sin\gamma} = \frac{b}{\sin\beta} \Rightarrow c = b.\frac{\sin\gamma}{\sin\beta}$$

$$= 61.5 \times \frac{\sin 31^\circ\, 54'}{\sin 88^\circ\, 36'} = 61.5 \times \frac{0.528}{0.9997} = 61.5 \times 0.5286 = 32.51$$

Hence $\boxed{\alpha = 59^\circ\, 30',\ b = 61.49,\ c = 32.51}$