Exercise: 5.1

Q.1 (i) Given inequality is

$2x+y \le 6 \rightarrow (1)$

Associated equation of (1) is

$2x+y=6 \rightarrow (2)$

for $y=0$, (2) $\Rightarrow 2x+0=6 \quad \therefore x=3$

for $x=0$, (2) $\Rightarrow 0+y=6 \quad \therefore y=6$

$\therefore$ line (2) cuts x-axis at $(3,0)$ and y-axis at $(0,6)$

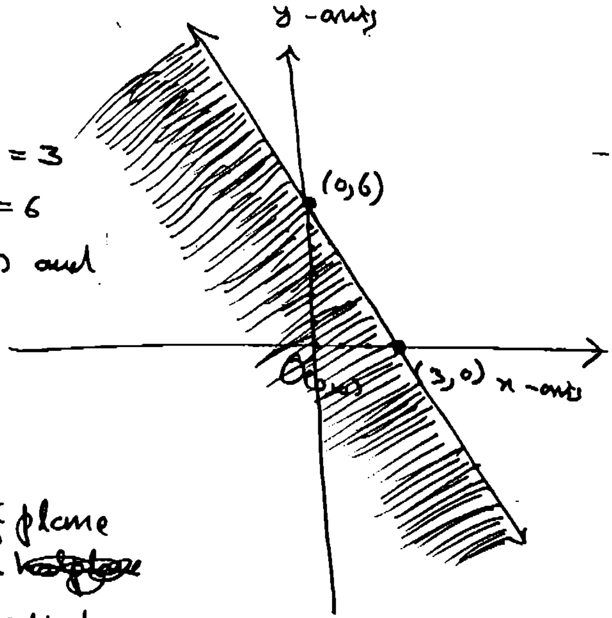
Putting $(0,0)$ in L.H.S of (1)

$2(0)+0=0+0=0<6$

$\therefore$ (1) is satisfied by $(0,0)$

$\therefore$ Graph of (1) is the closed half plane on the side of $(0,0)$, or closed ~~half plane~~ half plane containing $(0,0)$

The ~~graph~~ part of graph is shaded portion as shown in the figure.



(ii) Given inequality is

$3x+7y \ge 21 \rightarrow (1)$

The corresponding or associated equation of (1) is

$3x+7y=21 \rightarrow (2)$

For $y=0$, (2) $\Rightarrow 3x+0=21 \quad \therefore x=7$

For $x=0$, (2) $\Rightarrow 0+7y=21 \quad \therefore y=3$

$\therefore$ line (2) cuts x-axis at $(0,7)$ and y-axis at $(0,3)$

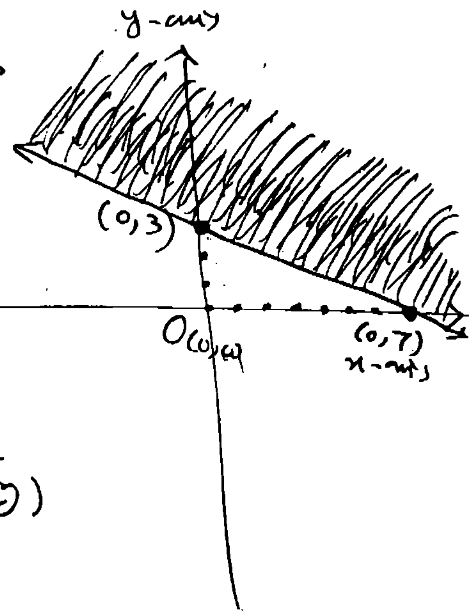
Now putting $(0,0)$ in L.H.S of (1)

$3(0)+7(0)=0+0=0<21$

i.e. $(0,0)$ does not satisfy (1)

$\therefore$ The graph of inequality (1) is the closed half plane (made by line (2)) which does not contain $(0,0)$.

$\therefore$ The part of graph is the shaded portion as shown in the figure.



(iii) Given inequality is

$$3x - 2y \geq 6 \longrightarrow (1)$$

Associated equation of (1) is

$$3x - 2y = 6 \longrightarrow (2)$$

For $y = 0$, (1) $\Rightarrow 3x - 0 = 6$ $\therefore x = 2$

For $x = 0$, (1) $\Rightarrow 0 - 2y = 6$ $\therefore y = -3$

$\therefore$ line (2) cuts x-axis at $(2, 0)$

and y-axis at $(0, -3)$

The line (2) divides xy-plane into two half planes.

Now we take $(0,0)$ as test point.

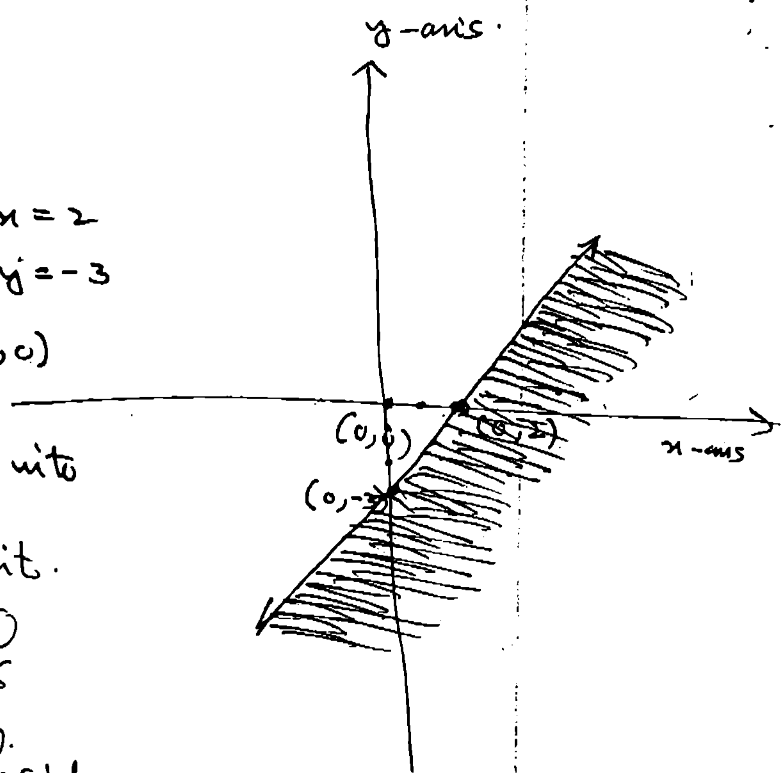
Putting $(0,0)$ in the L.H.S of (1)

$$3(0) - 2(0) = 0 - 0 = 0 < 6$$

i.e. $(0,0)$ does not satisfy (1).

$\therefore$ Graph of (1) is the closed half plane which does not contain $(0,0)$

$\therefore$ The part of graph is shown as shaded region as shown in the figure.



(iv) Given inequality is

$$5x - 4y \leq 20 \longrightarrow (1)$$

Associated equation of (1) is

$$5x - 4y = 20 \longrightarrow (2)$$

For $y = 0$, (2) $\Rightarrow 5x - 0 = 20$ $\therefore x = 4$

For $x = 0$, (2) $\Rightarrow 0 - 4y = 20$ $\therefore y = -5$

$\therefore$ The line (2) cuts the x-axis at $(4, 0)$

and y-axis at $(0, -5)$

The line (2) divides the xy-plane into two half planes.

Now we take $(0,0)$ as the test point.

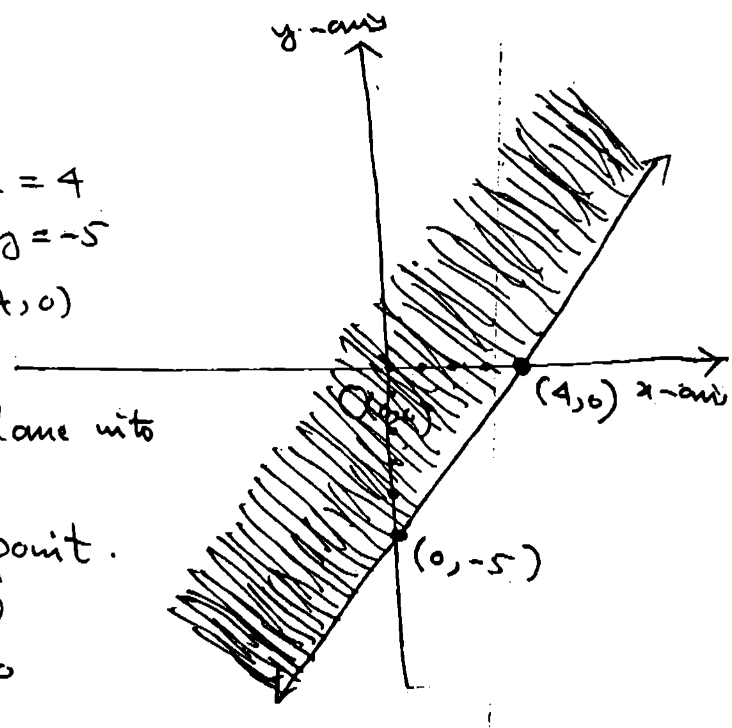
Putting $(0,0)$ in the L.H.S of (1)

$$5(0) - 4(0) = 0 - 0 = 0 < 20$$

i.e. $(0,0)$ satisfies (1)

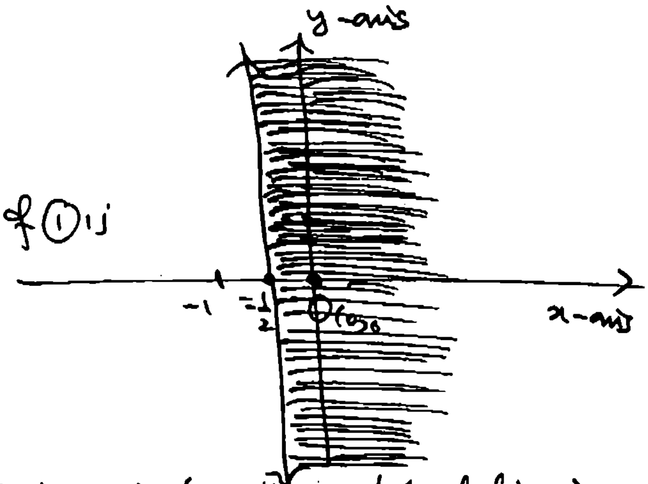
$\therefore$ The graph of (1) is the closed half plane containing $(0,0)$ or on the side of origin $(0,0)$.

$\therefore$ The part of graph of (1) is the shaded region as shown in the figure.



(v) Given inequality is

$2x+1 \geq 0 \rightarrow (1)$



The associated equation of (1) is

$2x+1=0 \rightarrow (2)$

(2) can be written as

$2x=-1$

or $x=-\frac{1}{2}$

which is a line parallel to y-axis (i.e the vertical line) at a distance $\frac{1}{2}$ on the left side of y-axis.

The line (2) divides the xy-plane into two half planes, i.e into right half plane and left half plane.

$(1) \Rightarrow 2x \geq -1$

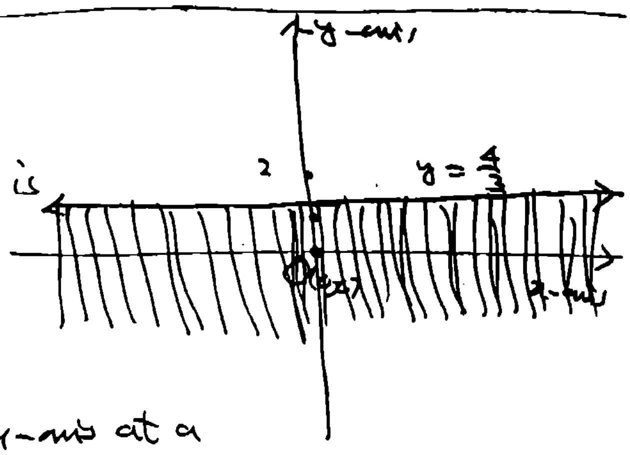
i.e $x \geq -\frac{1}{2}$

∴ The graph of the inequality (1) is the closed right half plane.

The part of graph of (1) is the shaded region as shown in the figure.

(vi) Given inequality is

$3y-4 \leq 0 \rightarrow (1)$



Associated equation of (1) is

$3y-4=0 \rightarrow (2)$

$(2) \Rightarrow 3y=4$

∴ $y=\frac{4}{3}$

i.e (2) is a line parallel to x-axis at a distance of $\frac{4}{3}$ from the origin above x-axis

The line (2) divides the xy-plane into two half planes, upper half plane and lower half plane.

As $(1) \Rightarrow 3y \leq 4$

∴ $y \leq \frac{4}{3}$

∴ The graph of the (1) is the closed low plane.

The part of the graph of (1) is the shaded region as shown in the figure.

Q. 2 i) Given inequalities are

$$\left.\begin{array}{l} 2x-3y \le 6 \rightarrow (1) \\ 2x+3y \le 12 \rightarrow (2) \end{array}\right] \rightarrow (I)$$

Associated equation of (1) is

$$2x-3y=6 \rightarrow (3)$$

Associated equation of (2) is

$$2x+3y=12 \rightarrow (4)$$

For $y=0$, (3) $\Rightarrow 2x-0=6 \quad \therefore x=3$

For $x=0$, (3) $\Rightarrow 0-3y=6 \quad \therefore y=-2$

$\therefore$ Line (3) cuts x-axis at $(3,0)$ and y-axis at $(0,-2)$

Now for $y=0$, (4) $\Rightarrow 2x+0=12 \quad \therefore x=6$

for $x=0$, (4) $\Rightarrow 0+3y=12 \quad \therefore y=4$

$\therefore$ Line (4) cuts x-axis at $(6,0)$ and y-axis $(0,4)$

Now we take $(0,0)$ as test point.

Putting $(0,0)$ in the L.H.S of (1)

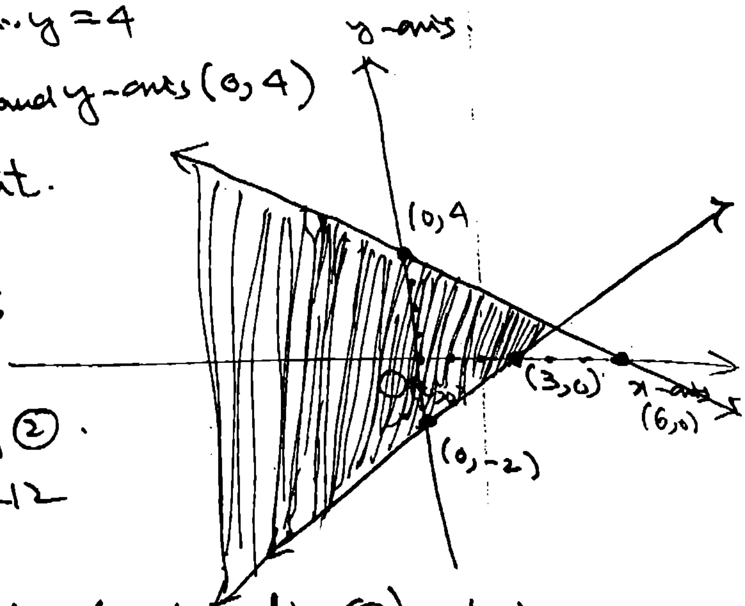
$$2(0)-3(0)=0-0=0<6$$

$\therefore (0,0)$ satisfies (1)

and putting $(0,0)$ in the L.H.S of (2).

$$2(0)+3(0)=0+0=0<12$$

$\therefore (0,0)$ also satisfies (2)



$\therefore$ Graph of (1) is the closed half plane (made by line (3)) containing $(0,0)$

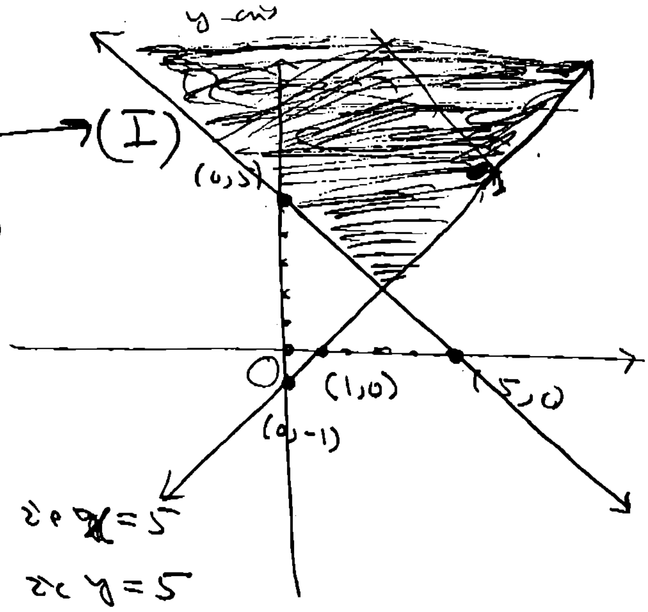
Also graph of (2) is the closed half plane (mad by line (4)) containing $(0,0)$.

The intersection of these two graphs is the solution set of system (I) as shown by shading.

(5)

(ii) Given inequalities are

$x+y \geq 5 \rightarrow (1)$
$-y+x \leq 1 \rightarrow (2)$ $\Big] \longrightarrow (I)$



Associated equation of (1) is $x+y=5 \longrightarrow (3)$

Associated equation of (2) is $-y+x=1 \longrightarrow (4)$

For $y=0$, (3) $\Rightarrow x+0=5$ i.e $x=5$

For $x=0$, (3) $\Rightarrow 0+y=5$ i.e $y=5$

$\therefore$ Line (3) cuts x-axis at (5,0) and y-axis at (0,5)

Now for $y=0$, (4) $\Rightarrow -0+x=1 \therefore x=1$

for $x=0$, (4) $\Rightarrow -y+0=1 \therefore y=-1$

$\therefore$ (4) cuts x-axis at (1,0) and y-axis at (0,-1).

Now we take (0,0) at the test point.

Putting (0,0) in the L.H.S of (1)

$0+0=0<5$

i.e (0,0) does not satisfy (1).

$\therefore$ The graph of (1) is the closed half plane (made by line (3)) not on the side of origin.

Now putting (0,0) in the L.H.S of (2)

$-0+0=0<1$

$\therefore$ (0,0) satisfies (2).

$\therefore$ Graph of (2) is the closed half plane (made by line (4)) on the side of origin.

The intersection of graphs of (1) and (2) is the solution set of system (I) as shown by shading in the figure.

(iii) Given inequalities are

$$\left.\begin{array}{l} 3x+7y \geq 21 \rightarrow (1) \\ x-y \leq 2 \rightarrow (2) \end{array}\right] \rightarrow (I)$$

The associated equation of (1) is

$$3x+7y=21 \rightarrow (3)$$

The associated equation of (2) is

$$x-y=2 \rightarrow (4)$$

For $y=0$, (3) $\Rightarrow 3x+0=21$ $\therefore x=7$

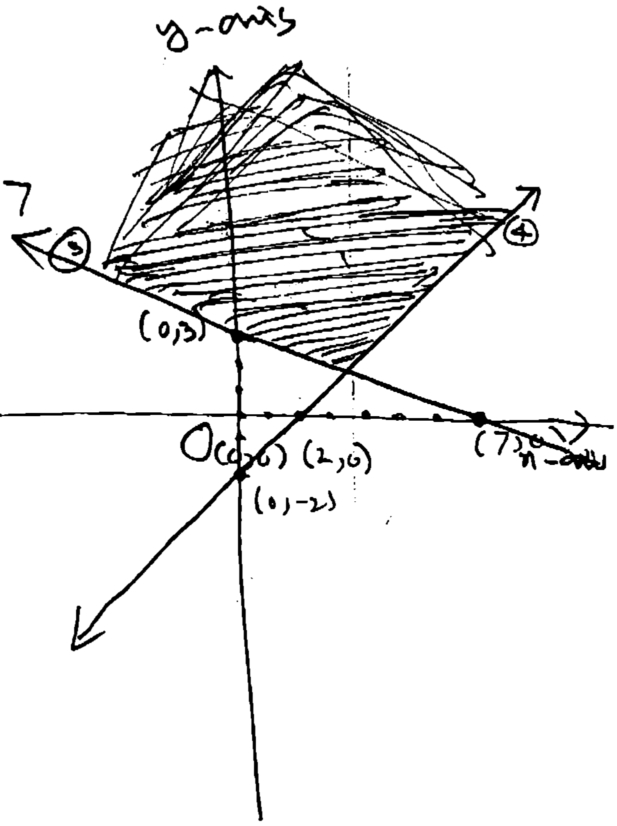
For $x=0$, (3) $\Rightarrow 0+7y=21$ $\therefore y=3$

$\therefore$ The line (3) cuts x-axis at $(7,0)$ and y-axis at $(0,3)$

For $y=0$, (4) $\Rightarrow x-0=2$ $\therefore x=2$

For $x=0$, (4) $\Rightarrow 0-y=2$ $\therefore y=-2$

$\therefore$ The line (4) cuts x-axis at $(2,0)$ and y-axis at $(0,-2)$.



Now we take $(0,0)$ as test point.

Putting $(0,0)$ in the L.H.S of (1)

$$3(0)+7(0)=0+0=0<21$$

$\therefore (0,0)$ does not satisfy (1).

$\therefore$ The graph of (1) is the closed half plane (made by line (3)) not on the side of origin.

Now putting $(0,0)$ in the L.H.S of (2).

$$0-0=0<2$$

i.e $(0,0)$ satisfies (2).

$\therefore$ The graph of (2) is the closed half plane (made by line (4)) on the side of origin.

The intersection of graphs of (1) and (2) is the solution set of system (I) as shown in the diagram by shading.

Given inequalities are

$$\left.\begin{aligned} 4x-3y &\le 12 \longrightarrow (1) \\ x &\ge -\tfrac{3}{2} \longrightarrow (2) \end{aligned}\right] \longrightarrow (I)$$

Associated equation of (1) is

$4x-3y=12 \longrightarrow (3)$

Associated equation of (2) is

$x=-\frac{3}{2} \longrightarrow (4)$

For $y=0$, (3) $\Rightarrow$ $4x-0=12$ $\therefore x=3$

For $x=0$, (3) $\Rightarrow$ $0-3y=12$ $\therefore y=-4$

$\therefore$ Line (3) cuts x-axis at $(3,0)$ and y-axis at $(0,-4)$

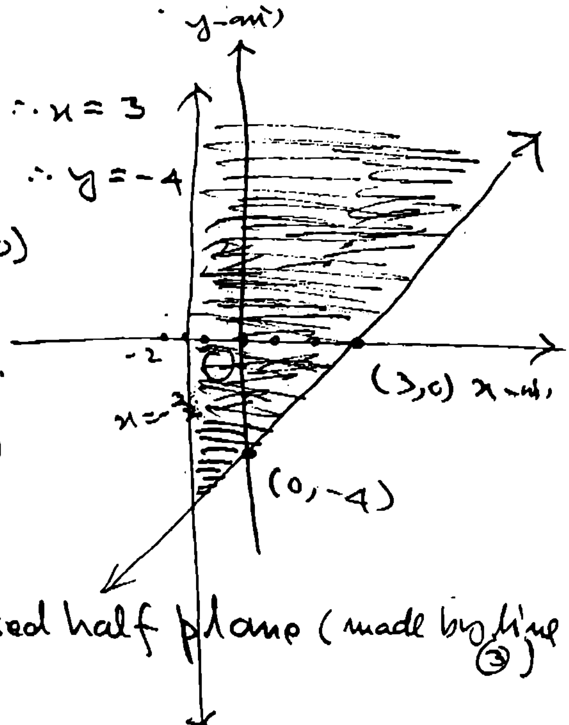
We take $(0,0)$ as test point.

Putting $(0,0)$ in L.H.S of (1)

$4(0)-3(0)=0-0=0<12$

$\therefore (0,0)$ satisfies (1)

$\therefore$ The graph of (1) is the closed half plane (made by line (3)) on the side of origin.



(4) is the line ∥ to y-axis at a distance of $\frac{3}{2}$ from the origin on the left side of y-axis.

Now the line (4) divides xy-plane into right half plane and left half plane

as $x \ge -\frac{3}{2}$

$\therefore$ Graph of (2) is closed right half plane.

The intersection of graphs of (1) and (2) is the solution set of system (I) as shown in the figure by shading.

(v) Given inequalities are

$$\left.\begin{array}{l} 3x+7y \geq 21 \longrightarrow (1) \\ y \leq 4 \longrightarrow (2) \end{array}\right\} \rightarrow (I)$$

The associated (or corresponding) equation of (1) is

$$3x+7y=21 \longrightarrow (3)$$

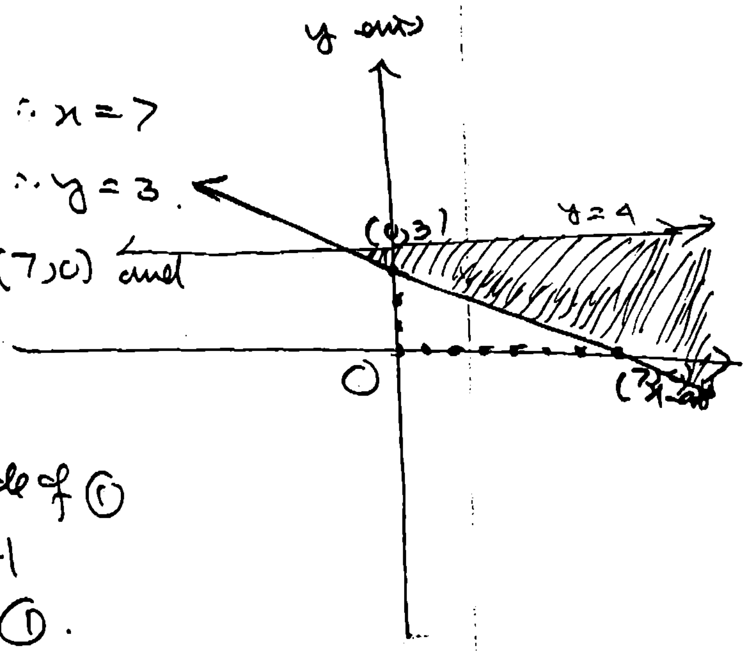
Associated equation of (2) is

$$y=4 \longrightarrow (4)$$

For $y=0$, (3) $\Rightarrow$ $3x+0=21$ $\therefore x=7$

For $x=0$, (3) $\Rightarrow$ $0+7y=21$ $\therefore y=3$.

$\therefore$ The line (3) cuts x-axis at $(7,0)$ and y-axis at $(0,3)$



We take $(0,0)$ as test point.

putting $(0,0)$ in the left side of (1)

$3(0)+7(0)=0+0=0<21$

$\therefore$ $(0,0)$ does not satisfy (1).

$\therefore$ Graph of (1) is the closed half plane (made by line (3)) not on the side of $(0,0)$

line (4) is a line parallel to x-axis. and is at distance of 4 units from the origin and above x-axis.

~~The graph of (2) is the~~

as $y \leq 4$

$\therefore$ graph of (2) is the closed lower half plane (made by line (4))

The intersection of graphs (1) and (2) is the solution set of system (I) as shown in the figure by shading.

Q.3: 1: Given inequalities are

$$\left.\begin{aligned} 2x-3y &\le 6 \rightarrow (1)\\ 2x+3y &\le 12 \rightarrow (2)\\ y &\geq 0 \rightarrow (3) \end{aligned}\right] \rightarrow (I)$$

Associated equations of (1), (2) and (3) are

$2x-3y=6 \rightarrow (4)$

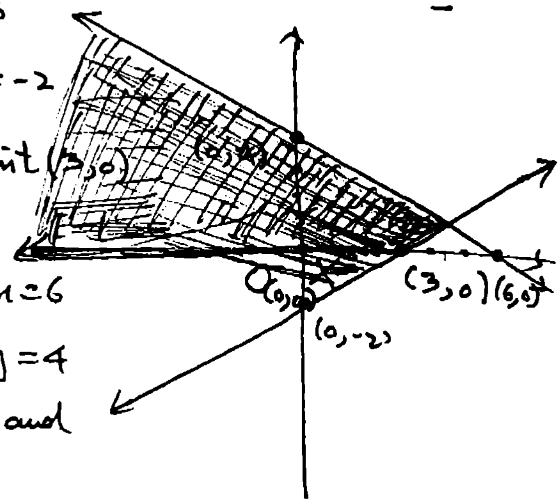
$2x+3y=12 \rightarrow (5)$

$y=0 \rightarrow (6)$

For $y=0$, (4) $\Rightarrow 2x-0=6 \quad \therefore x=3$

For $x=0$, (4) $\Rightarrow 0-3y=6 \quad \therefore y=-2$

$\therefore$ Line (4) cuts x-axis at the point $(3,0)$ and y-axis at the point $(0,-2)$



Now for $y=0$, (5) $\Rightarrow 2x+0=12 \quad \therefore x=6$

for $x=0$, (5) $\Rightarrow 0+3y=12 \quad \therefore y=4$

$\therefore$ Line (5) cuts x-axis at $(6,0)$ and y-axis at $(0,4)$.

Now we take test point $(0,0)$.

Putting $(0,0)$ in the L.H.S of (1)

$2(0)-3(0)=0-0=0<6$

$\therefore (0,0)$ satisfies (1).

$\therefore$ Graph of (1) is the closed half plane (made by line (4)) on the side of origin.

Now putting $(0,0)$ in the L.H.S of (2)

$2(0)+3(0)=0+0=0<12$

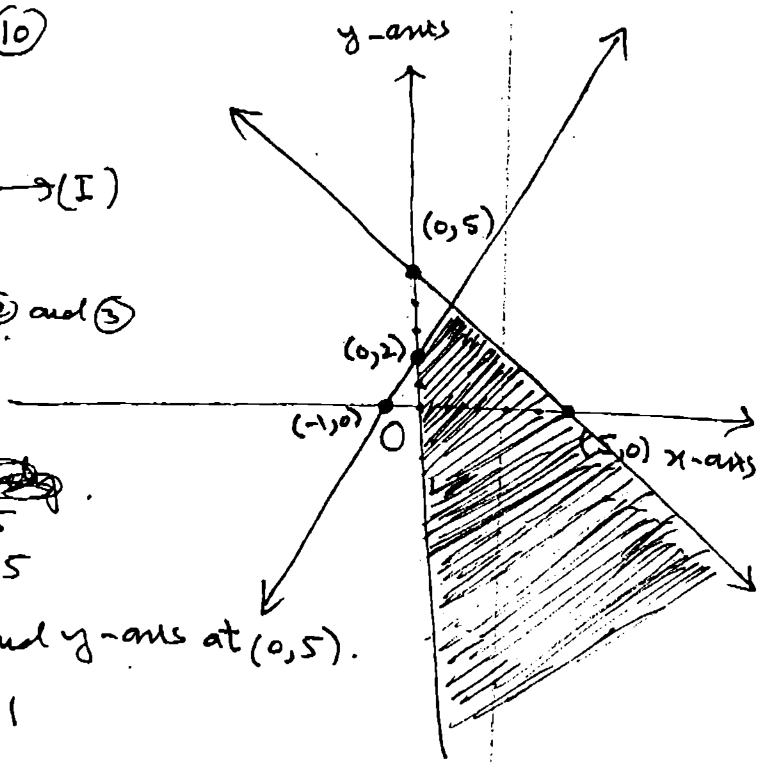
$\therefore$ (2) is also satisfied by $(0,0)$.

$\therefore$ The graph of (2) is the closed half plane (made by line (5)) on the side of $(0,0)$.

(6) is the equation of x-axis.

Graph of (3) is the closed upper half plane.

The intersection of the graphs of (1), (2) and (3) is the solution region of system (I) as shown in the figure by shading.



(ii) Given inequalities are

$$\left.\begin{array}{l} x+y \le 5 \rightarrow (1) \\ y-2x \le 2 \rightarrow (2) \\ x \geq 0 \rightarrow (3) \end{array}\right] \rightarrow (I)$$

Associated equations of (1), (2) and (3) are

$x+y=5 \rightarrow (4)$

$y-2x=2 \rightarrow (5)$

$x = 0 \rightarrow (6)$

For $y=0$, (4) $\Rightarrow x+0=5 \quad \therefore x=5$

For $x=0$, (4) $\Rightarrow 0+y=5 \quad \therefore y=5$

$\therefore$ Line (1) cuts x-axis at (5,0) and y-axis at (0,5).

For $y=0$, (5) $\Rightarrow 0-2x=2 \quad \therefore x=-1$

For $x=0$, (5) $\Rightarrow y-0=2 \quad \therefore y=2$

$\therefore$ Line (2) cuts x-axis at (-1,0) and y-axis at (0,2)

Now we take (0,0) as test point.

Putting (0,0) in the L.H.S of (1)

$0+0=0<5$

i.e (1) is satisfied by (0,0).

$\therefore$ Graph of (1) is the closed half plane (intercepted by line (4)) on the side of (0,0)

Putting (0,0) in the L.H.S of (2)

$0-2(0)=0-0=0<2$

$\therefore$ (2) is also satisfied by (0,0).

$\therefore$ Graph of (2) is the closed half plane (intercepted by line (5)) on the side of (0,0).

(6) is the equation of y-axis.

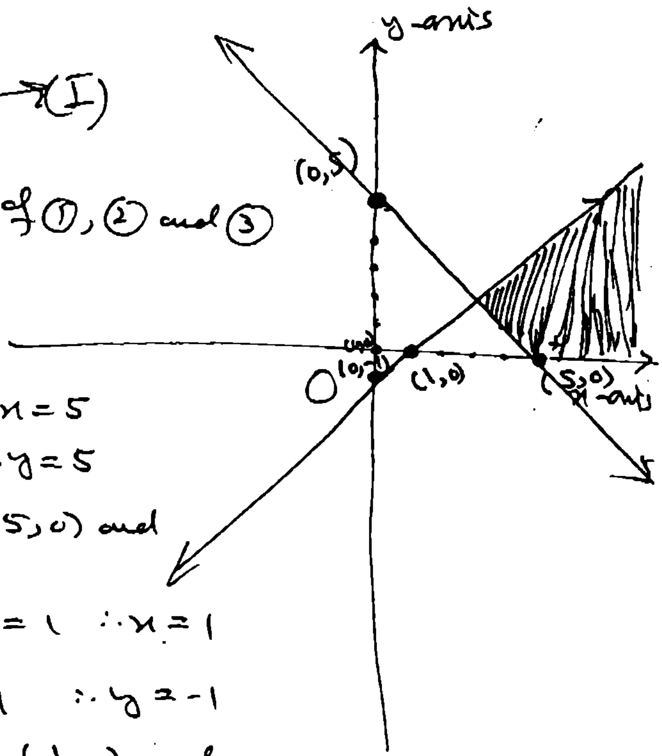
Now the line $x=0$, divides the xy-plane into right half plane and left half plane.

Now $x \geq 0 \Rightarrow$ the graph of (3) is the closed right half plane.

The intersection of graphs of (1), (2), (3) is the solution region of system (I) as shown in the figure by shading.

(iii) Given inequalities are

$$\left.\begin{array}{ll} x+y \geq 5 & \rightarrow (1) \\ x-y \geq 1 & \rightarrow (2) \\ y \geq 0 & \rightarrow (3) \end{array}\right] \rightarrow (I)$$



The associated equations of (1), (2) and (3)

are $x+y=5 \rightarrow (4)$

$x-y=1 \rightarrow (5)$

$\therefore y=0 \rightarrow (6)$

For $y=0$, (4) $\Rightarrow x+0=5 \quad \therefore x=5$

For $x=0$, (4) $\Rightarrow 0+y=5 \quad \therefore y=5$

$\therefore$ Line (4) cuts x-axis at $(5,0)$ and y-axis at $(0,5)$.

Now for $y=0$, (5) $\Rightarrow x-0=1 \quad \therefore x=1$

for $x=0$, (5) $\Rightarrow 0-y=1 \quad \therefore y=-1$

$\therefore$ Line (5) cuts x-axis at $(1,0)$ and y-axis at $(0,-1)$.

Now we take $(0,0)$ as the test point.

Putting $(0,0)$ in L.H.S of (1)

$0+0=0<5$

$\therefore (0,0)$ does not satisfy (1).

$\therefore$ Graph of (1) is the closed half plane (intersected by line (4)) not on the side of $(0,0)$.

Putting $(0,0)$ in the L.H.S of (2)

$0-0=0<1$

$\therefore (0,0)$ also does not satisfy (2).

$\therefore$ Graph of (2) is the closed half plane (by the division of xy-plane into two planes by line (5)) not on the side of $(0,0)$

$y=0$ is the line of x-axis.

$\therefore$ The graph of $y \geq 0$ is the closed upper half plane of the xy-plane.

The intersection of the graphs of (1), (2), (3) is the solution region of the system (I) as shown in the figure by shading.

(iv) Given inequalities are

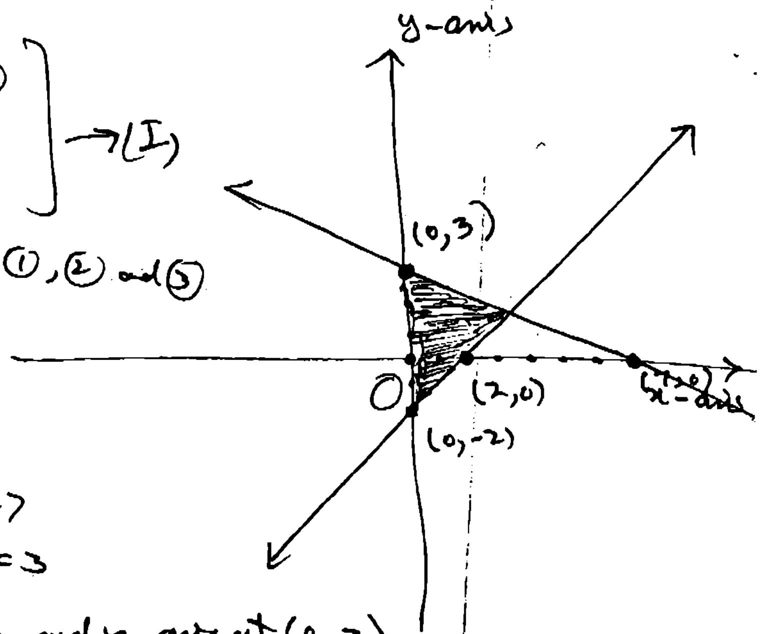
$$\left.\begin{array}{ll} 3x+7y \le 21 & \rightarrow (1) \\ x-y \le 2 & \rightarrow (2) \\ x \ge 0 & \rightarrow (3) \end{array}\right\} \rightarrow (I)$$

The associated equations of (1), (2) and (3) are

$3x+7y=21 \rightarrow (4)$

$x-y=2 \rightarrow (5)$

$x=0 \rightarrow (6)$



For $y=0$, (4) $\Rightarrow 3x+0=21 \therefore x=7$

For $x=0$, (4) $\Rightarrow 0+7y=21 \therefore y=3$

$\therefore$ Line (4) cuts x-axis at (7,0) and y-axis at (0,3)

For $y=0$, (5) $\Rightarrow x-0=2 \therefore x=2$

For $x=0$, (5) $\Rightarrow 0-y=2 \therefore y=-2$

$\therefore$ Line (5) cuts x-axis at (2,0) and y-axis (0,-2)

We take (0,0) as the test point.

Putting (0,0) in the L.H.S of (1).

$3(0)+7(0)=0+0=0<21$

$\therefore$ (0,0) satisfies (1).

$\therefore$ Graph of (1) is the closed half plane (made by line (4)) on the side of (0,0).

Putting (0,0) in the L.H.S of (2)

$0-0=0<2$

$\therefore$ Also (0,0) satisfies (2).

$\therefore$ Graph of (2) is the closed half plane (made by line (5)) on the side of (0,0).

and (6) is the equation of y-axis.

$\therefore$ The graph of $x \ge 0$, is the right closed half plane of xy-plane formed by y-axis.

The intersection of graphs of (1), (2), (3) is the solution region of system (I) as shown in the figure by shading.

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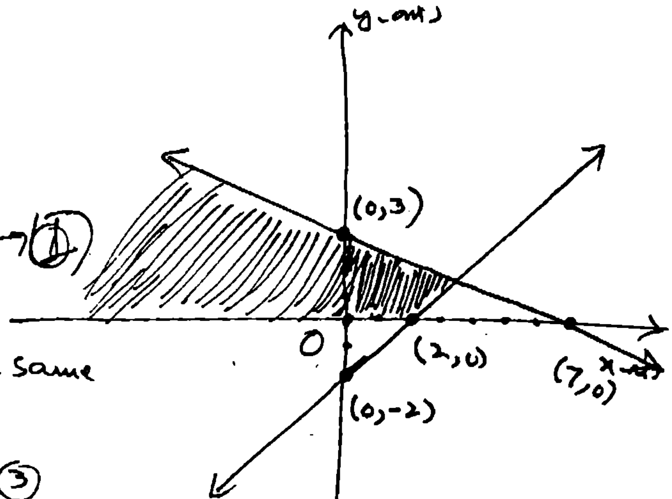
(v) Given inequalities are

$$\left.\begin{array}{l} 3x+7y \le 21 \rightarrow (1) \\ x-y \le 2 \rightarrow (2) \\ y \ge 0 \rightarrow (3) \end{array}\right] \rightarrow (I)$$



The graphs of (1) and (2) are same as in part (iv).

Here associated equation of (3) is $y=0$

and $y=0$ is the equation of x-axis.

$\therefore$ Graph of $y \ge 0$ is the closed upper half plane of xy-plane.

The intersection of graphs of (1), (2) and (3) is the solution region of system (I) as shown in the figure by shading.

(vi)

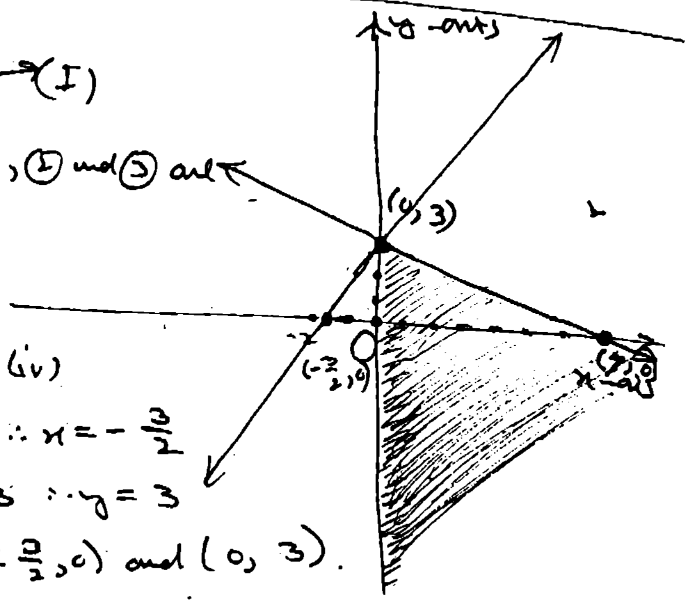
$$\left.\begin{array}{l} 3x+7y \le 21 \rightarrow (1) \\ 2x-y \ge -3 \rightarrow (2) \\ x \ge 0 \rightarrow (3) \end{array}\right] \rightarrow (I)$$

Associated equations of (1), (2) and (3) are

$3x+7y = 21 \rightarrow (4)$

$2x-y = -3 \rightarrow (5)$

$x = 0 \rightarrow (6)$



Graph of (1) is as in part (iv)

For $y=0$, (5) $\Rightarrow 2x-0=-3 \quad \therefore x = -\frac{3}{2}$

For $x=0$, (5) $\Rightarrow 0-y=-3 \quad \therefore y=3$

$\therefore$ Line (5) cuts x-axis at $(-\frac{3}{2},0)$ and $(0,3)$.

We take $(0,0)$ as the test point.

Putting $(0,0)$ in the L.H.S of (2).

$2(0)-0 = 0-0 = 0 > -3$

$\therefore (0,0)$ satisfies (2)

$\therefore$ The graph of (2) is the ~~half~~ closed half plane (formed by line (5)) on the side of $(0,0)$.

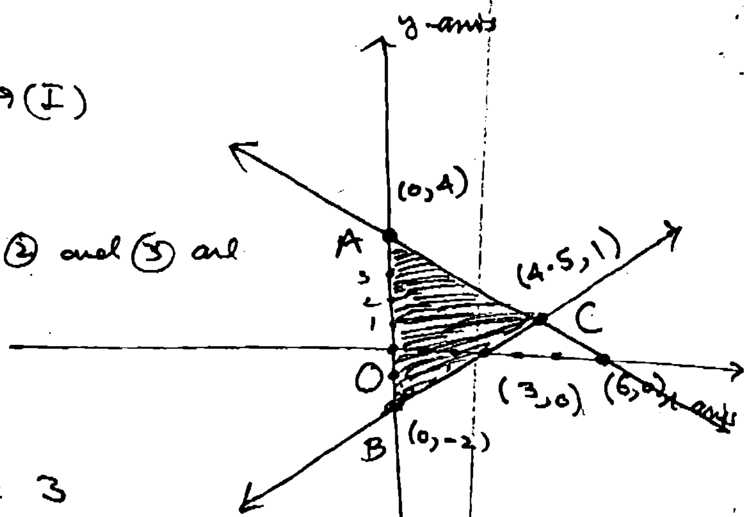
$x=0$ is the equation of y-axis.

$\therefore$ The graph of $x \ge 0$ is the closed right half plane of xy-plane.

The intersection of graphs of (1), (2) and (3) is the solution region of system (I) as shown in the figure by shading.

4 (i) Given inequalities are

$$\left.\begin{array}{l} 2x-3y \le 6 \rightarrow (1) \\ 2x+3y \le 12 \rightarrow (2) \\ x \ge 0 \longrightarrow (3) \end{array}\right] \rightarrow (I)$$



The associated equations of (1), (2) and (3) are

$2x-3y=6 \rightarrow (4)$

$2x+3y=12 \rightarrow (5)$

$x=0 \longrightarrow (6)$

For $y=0$, (4) $\Rightarrow 2x-0=6$ $\therefore x=3$

For $x=0$, (4) $\Rightarrow 0-3y=6$ $\therefore y=-2$

$\therefore$ Line (1) cuts x-axis at (3, 0) and y-axis at (0, -2)

For $y=0$, (5) $\Rightarrow 2x+0=12$ $\therefore x=6$

For $x=0$, (5) $\Rightarrow 0+3y=12$ $\therefore y=4$

$\therefore$ Line (2) cuts x-axis at (6, 0) and y-axis at (0, 4)

We take origin (0,0) as the test point.

Putting (0,0) in L.H.S of (1).

$2(0)-3(0)=0-0=0<6$

$\therefore$ (1) is satisfied by (0,0)

Putting (0,0) in L.H.S of (2)

$2(0)+3(0)=0+0=0<12$

$\therefore$ (0,0) satisfies (2).

$\therefore$ Graph of (1) is the closed half plane (made by line (4)) on the side of (0,0) and graph of (2) is the closed half plane (formed by line (5)) on the side of (0,0).

$x=0$ is the equation of y-axis.

$\therefore$ Graph of $x \ge 0$ is the close right half plane of xy-plane.

The intersection of graphs of (1), (2), (3) is the solution region as shown in the figure by shading.

Here corner points of solution region are A(0, 4), B(0, -2), C(4.5, 1)

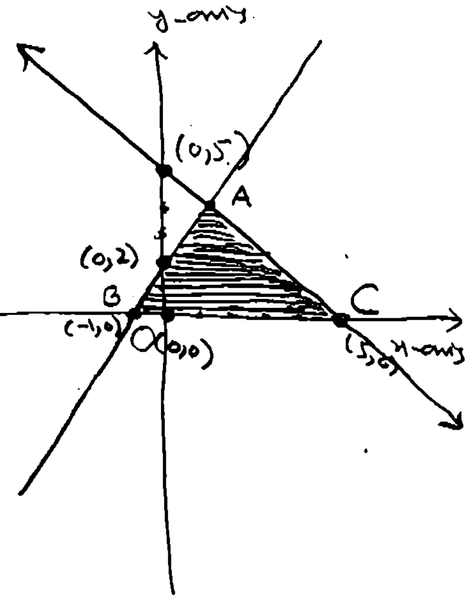
A is point of intersection of (5) and (6)

B is point of intersection of (4) and (6)

C is point of intersection of (4) and (5).

(ii) Given inequalities are

$x+y \le 5 \rightarrow (1)$
$-2x+y \le 2 \rightarrow (2)$
$y \ge 0 \rightarrow (3)$ $\Big\} \rightarrow (I)$



The associated equation of (1), (2) and (3) are

$x+y=5 \rightarrow (4)$
$-2x+y = 2 \rightarrow (5)$
$y=0 \rightarrow (6)$

For $y=0$, (4) $\Rightarrow x+0=5 \therefore x=5$
for $x=0$, (4) $\Rightarrow 0+y=5 \therefore y=5$
$\therefore$ Line (1) cuts x-axis at $(5,0)$ and y-axis at $(0,5)$
For $y=0$, (5) $\Rightarrow -2x+0=2 \therefore x=-1$
For $x=0$, (5) $\Rightarrow -0+y=2 \therefore y=2$
$\therefore$ Line (5) cuts x-axis at $(-1,0)$ and y-axis at $(0,2)$
We take $(0,0)$ as test point.
Putting $(0,0)$ in L.H.S of (1)
$0+0=0<5$
$\therefore (0,0)$ satisfies (1)
Now putting $(0,0)$ in L.H.S of (2)
$-2(0)+0 = 0+0=0<2$
$\therefore (0,0)$ also satisfies (2).
$\therefore$ Graph of (1) is the closed half plane (made by line (4)) on the side of origin $(0,0)$.
Graph of (2) is the closed half plane (made by line (5)) on the side of origin $(0,0)$.
$y=0$ is x-axis.
$\therefore$ Graph of $y \ge 0$ is the closed upper half plane of the xy-plane.
The intersection of graphs of (1), (2) and (3) is the solution region of system (I) as shown in the figure by shading.
Here A, B, C are the corner points of solution region.
$A(1,4)$ is the point of intersection of (4) and (5)
$B(-1,0)$ is the point of intersection of (5) and (6)
$C(5,0)$ is the point of intersection of (4) and (6).

(iii) Given inequalities are

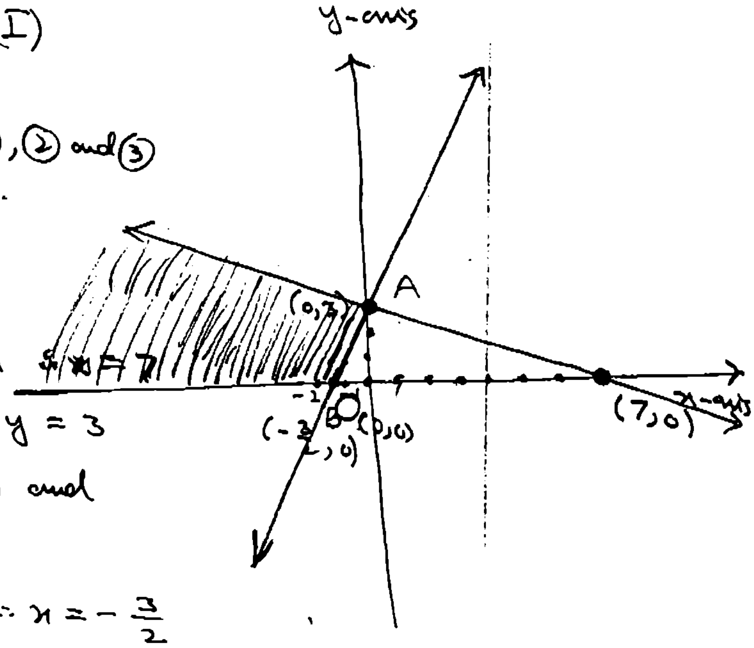
$$\left.\begin{array}{l} 3x+7y \le 21 \rightarrow (1) \\ 2x-y \le -3 \rightarrow (2) \\ y \ge 0 \longrightarrow (3) \end{array}\right] \rightarrow (I)$$

The associated equations of (1), (2) and (3) are

$3x+7y=21 \rightarrow (4)$

$2x-y=-3 \rightarrow (5)$

$y=0 \longrightarrow (6)$



For $y=0$, (4) $\Rightarrow$ $3x+0=21$ $\therefore x=7$

For $x=0$, (4) $\Rightarrow$ $0+7y=21$ $\therefore y=3$

$\therefore$ Line (4) cuts x-axis at $(7,0)$ and y-axis at $(0,3)$.

For $y=0$, (5) $\Rightarrow$ $2x-0=-3$ $\therefore x=-\frac{3}{2}$

For $x=0$, (5) $\Rightarrow$ $0-y=-3$ $\therefore y=3$

$\therefore$ Line (5) cuts x-axis at $(-\frac{3}{2},0)$ and y-axis at $(0,3)$

We take the test point $(0,0)$

Putting $(0,0)$ in the L.H.S of (1)

$3(0)+7(0)=0+0=0<21$

$\therefore$ $(0,0)$ satisfies (1). $\therefore$ The graph of (1) is the closed half plane (made by line (4)) on the side of $(0,0)$.

Now putting $(0,0)$ in the L.H.S of (2)

$2(0)-0=0-0=0 > -3$

$\therefore$ $(0,0)$ does not satisfy (2)

$\therefore$ The graph of (2) is the closed half plane (made by line (5)) not on the side of $(0,0)$.

$y=0$ is the x-axis.

$\therefore$ Graph of $y \ge 0$, is the upper half plane of xy-plane.

The intersection of graphs of (1), (2), (3) is the solution region ~~as shown~~ of system (I) as shown in the figure by shading the region.

Here A and B are only two corner points of solution region.

$A(0,3)$ is pt. of intersection of lines (4) and (5)

$B(-\frac{3}{2},0)$ is the pt of intersection of lines (5) and (6).

(iv) Given inequalities are

$$\left.\begin{array}{l} 3x+2y \geq 6 \to (1) \\ x+3y \leq 6 \to (2) \\ y \geq 0 \to (3) \end{array}\right] \to (I)$$

$9x+6y=18$
$2x+6y=12$
$7x=6$
$x=\frac{6}{7}$
$3y=6-\frac{6}{7}$
$=\frac{36}{7}$
$y=\frac{12}{7}$

The associated equations of (1), (2) and (3) are $3x+2y=6 \to (4)$

$x+3y=6 \to (5)$

$y=0 \to (6)$

For $y=0$, (4) $\Rightarrow 3x+0=6 \therefore x=2$

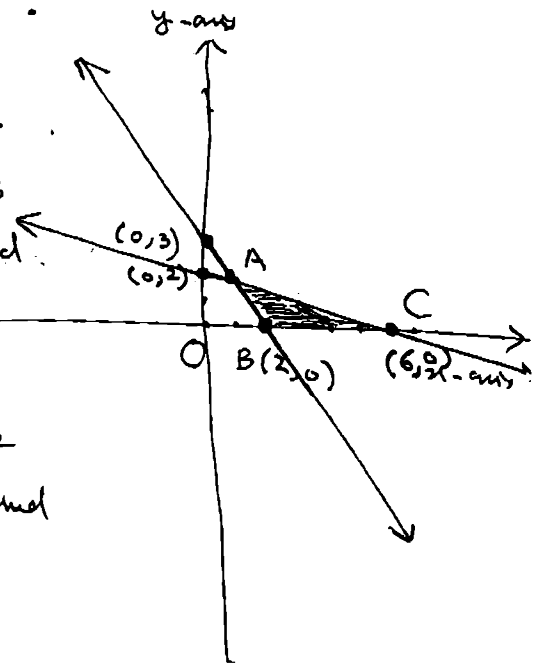
for $x=0$, (4) $\Rightarrow 0+2y=6 \therefore y=3$

$\therefore$ Line (4) cuts x-axis at $(2,0)$ and y-axis at $(0,3)$.

For $y=0$, (5) $\Rightarrow x+0=6 \therefore x=6$

for $x=0$, (5) $\Rightarrow 0+3y=6 \therefore y=2$

$\therefore$ Line (5) cuts x-axis at $(6,0)$ and y-axis at $(0,2)$



Now we take test point as $(0,0)$

Putting $(0,0)$ in L.H.S of (1)

$3(0)+2(0)=0+0=0<6$

$\therefore (0,0)$ does not satisfy (1).

$\therefore$ Graph of (1) is the closed half plane (made by line (4)) not on the side of $(0,0)$.

Now putting $(0,0)$ in L.H.S of (2)

$0+3(0)=0+0=0<6$

$\therefore (0,0)$ satisfies (2).

$\therefore$ Graph of (2) is the closed half plane (made by line (5)) on the side of $(0,0)$.

$y=0$ is the x-axis.

$\therefore$ Graph of $y \geq 0$ is the closed upper half plane of xy-plane.

The intersection of graph of (1), (2), (3) is the solution region of system (I) as shown in the figure by shading the region.

Here corner points of solution region are A, B, C.

$A\left(\frac{6}{7}, \frac{12}{7}\right)$ is the pt. of intersection of (4) and (5)

$B(2,0)$ is the point of intersection of (4) and (6)

$C(6,0)$ is the point of intersection of (5) and (6).

(v) Given inequalities are

$5x+7y \le 35 \to (1)$

$-x+3y \le 3 \to (2)$

$x \ge 0 \to (3)$

The associated equations of (1), (2) and (3)

are $5x+7y = 35 \to (4)$

$-x+3y = 3 \to (5)$

$x = 0 \to (6)$

$5x+7y=35$

$-5x+15y=15$

$\therefore 22y = 50 \quad \therefore y = \frac{25}{11}$

$x = 3y-3 = \frac{75}{11}-3 = \frac{42}{11}$

For $y=0$, (4) $\Rightarrow 5x+0=35 \therefore x=7$

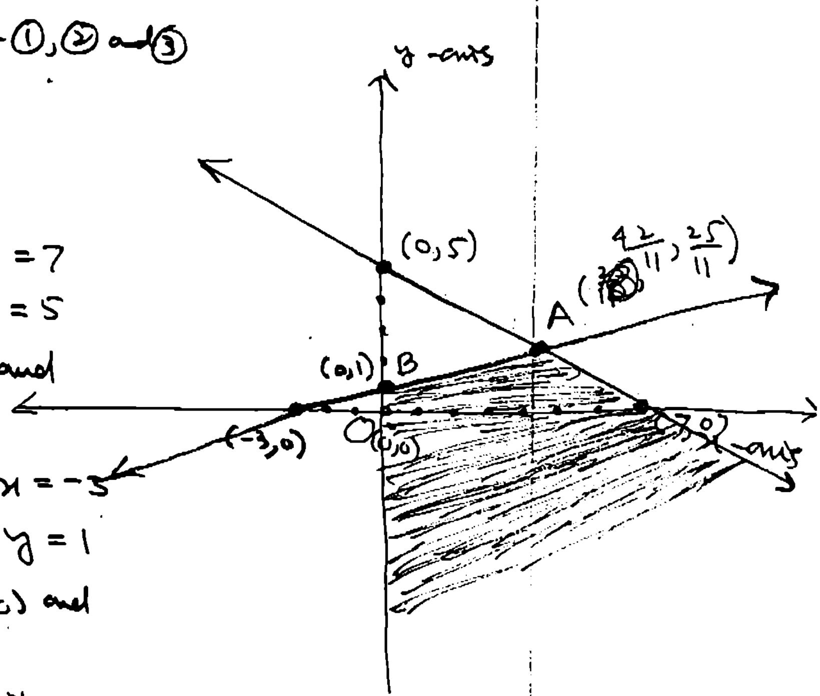
For $x=0$, (4) $\Rightarrow 0+7y=35 \therefore y=5$

$\therefore$ Line (1) cuts x-axis at $(7,0)$ and y-axis at $(0,5)$

For $y=0$, (5) $\Rightarrow -x+0=3 \therefore x=-3$

for $x=0$, (5) $\Rightarrow 0+3y=3 \therefore y=1$

$\therefore$ Line (5) cuts x-axis at $(-3,0)$ and y-axis at $(0,1)$



Now we take $(0,0)$ as test point.

Putting $(0,0)$ in the L.H.S of (1)

$5(0)+7(0) = 0+0 = 0 < 35$

$\therefore (0,0)$ satisfies (1)

$\therefore$ Graph of (1) is the closed half plane (made by line (4)) on the side of $(0,0)$.

Now putting $(0,0)$ in the L.H.S of (2)

$-0+3(0) = -0+0 = 0 < 3$

$\therefore (0,0)$ also satisfies (2)

$\therefore$ Graph of (2) is the closed half plane (made by line (5)) on the the side of $(0,0)$

$x=0$ is the y-axis.

$\therefore$ The graph of $x \ge 0$ is the closed right half plane of xy-plane.

The intersection of graphs of (1), (2) and (3) is the solution region of system (I) as shown in the figure by shading the region.

Here are only two corner points A and B.

$A(\frac{42}{11}, \frac{25}{11})$ is point of intersection of (4) and (5).

$B(0,1)$ is point of intersection of (5) and (6)

(vi) $5x+7y \le 35 \rightarrow ①$

$x-2y \le 2 \rightarrow ②$ $\Big] \rightarrow (I)$

$x \ge 0 \rightarrow ③$

$5x+7y=35$
$5x-10y=10$
$\underline{-\quad+\quad-}$
$17y=25$
$y=\frac{25}{17}$
$x=2+2y=2+\frac{50}{17}$
$=\frac{84}{17}$

The associated equations of ①, ② and ③

are $5x+7y=35 \rightarrow ④$

$x-2y = 2 \rightarrow ⑤$

$x=0 \rightarrow ⑥$

~~For y=0, ④ ⇒ 0+ ... =35~~

For $y=0$, ④ $\Rightarrow 5x+0=35 \quad \therefore x=7$

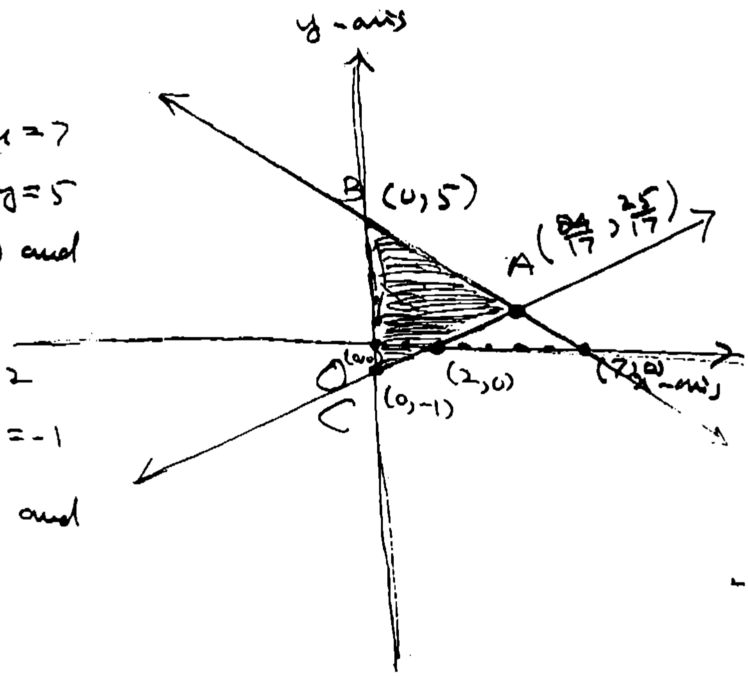
for $x=0$, ④ $\Rightarrow 0+7y=35 \quad \therefore y=5$

$\therefore$ Line ④ cuts x-axis at $(7,0)$ and y-axis at $(0,5)$

For $y=0$, ⑤ $\Rightarrow x-0=2 \quad \therefore x=2$

for $x=0$, ⑤ $\Rightarrow 0-2y=2 \quad \therefore y=-1$

$\therefore$ Line ⑤ cuts x-axis at $(2,0)$ and y-axis at $(0,-1)$



We take $(0,0)$ as test point.

Putting $(0,0)$ in the L.H.S of ①.

$5(0)+7(0)=0+0=0<35$

$\therefore (0,0)$ satisfies ①.

$\therefore$ Graph of ① is the closed half plane (made by line ④) on the side of $(0,0)$.

Putting $(0,0)$ in the L.H.S of ②

$0-2(0)=0-0=0<2$

$\therefore (0,0)$ also satisfies ②

$\therefore$ Graph of ② is the closed half plane (made by line ⑤) on the side of $(0,0)$

$x=0$ is the y-axis.

$\therefore$ Graph of $x \ge 0$ is the closed is the right half plane of xy-plane.

The intersection of graphs of ① ② and ③ is the solution region of system (I) is as shown in the figure by shading the region.

Here A, B, C are the corner points of solution

$A\left(\frac{84}{17}, \frac{25}{17}\right)$ is the point of intersection of ④ and ⑤

$B(0,5)$ is the point of intersection of ④ and ⑥

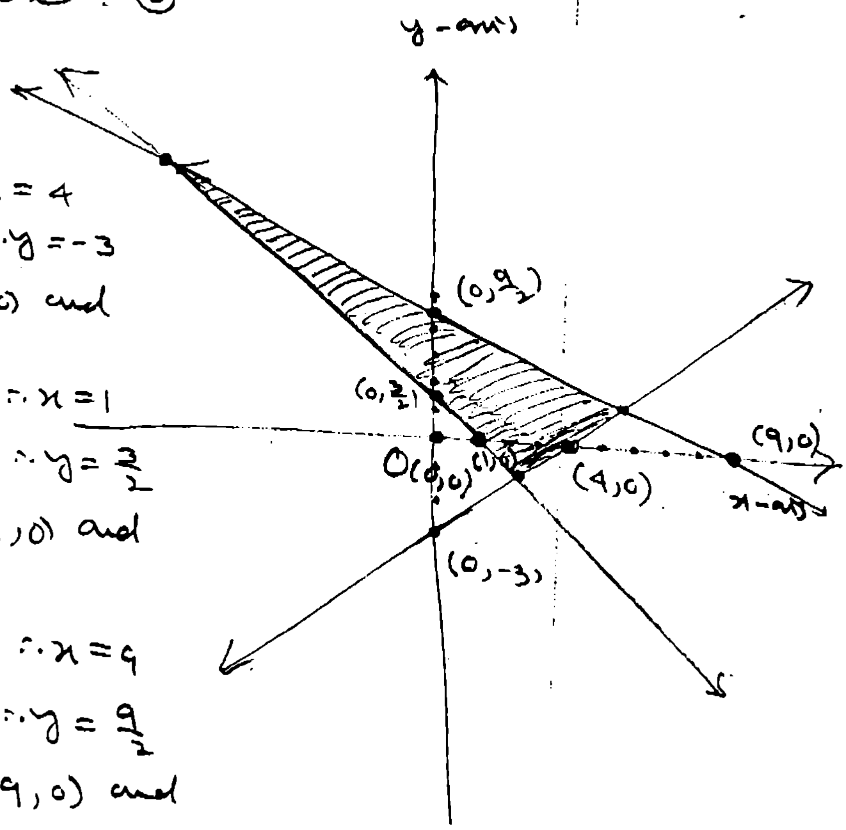
$C(0,-1)$ is the point of intersection of ⑤ and ⑥.

Q.5: (i) Given inequalities are

$$\left.\begin{aligned} 3x - 4y &\le 12 \rightarrow (1) \\ 3x+2y &\ge 3 \rightarrow (2) \\ x+2y &\le 9 \rightarrow (3) \end{aligned}\right] \rightarrow (I)$$

The associated equations of (1) (2) and (3) are

$$3x - 4y = 12 \rightarrow (4)$$
$$3x+2y = 3 \rightarrow (5)$$
$$x+2y = 9 \rightarrow (6)$$



For $y=0$, (4) $\Rightarrow 3x - 0 = 12 \therefore x = 4$

For $x=0$, (4) $\Rightarrow 0-4y=12 \therefore y=-3$

$\therefore$ Line (4) cuts x-axis at $(4,0)$ and y-axis at $(0,-3)$

For $y=0$, (5) $\Rightarrow 3x+0=3 \therefore x=1$

for $x=0$, (5) $\Rightarrow 0+2y=3 \therefore y=\frac{3}{2}$

$\therefore$ Line (5) cuts x-axis at $(1,0)$ and y-axis at $(0,\frac{3}{2})$.

For $y=0$, (6) $\Rightarrow x+0=9 \therefore x=9$

for $x=0$, (6) $\Rightarrow 0+2y=9 \therefore y=\frac{9}{2}$

$\therefore$ Line (6) cuts x-axis at $(9,0)$ and y-axis at $(0,\frac{9}{2})$

We take $(0,0)$ as test point.

Putting $(0,0)$ in L.H.S of (1)

$$3(0)-4(0)=0-0=0<12$$

$\therefore (0,0)$ satisfies (1).

$\therefore$ Graph of (1) is the closed half plane (made by line (4)) on the side of $(0,0)$.

Putting $(0,0)$ in the L.H.S of (2)

$$3(0)+2(0)=0+0=0<3$$

$\therefore (0,0)$ does not satisfy (2).

$\therefore$ Graph of (2) is the closed half plane (made by line (5)) not on the side of $(0,0)$.

Putting $(0,0)$ in L.H.S of (3)

$$0+2(0)=0+0=0<9$$

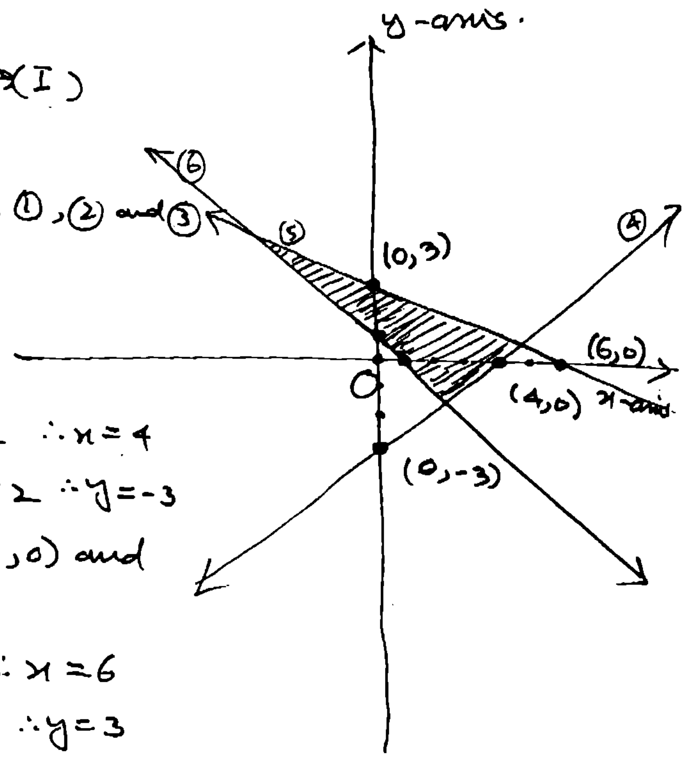
$\therefore (0,0)$ satisfies (3).

$\therefore$ Graph of (3) is the closed half plane (made by line (6)) on the side of $(0,0)$.

The intersection of graphs of (1), (2) and (3) is the solution region of system (I) as shown in the figure by shading the region.

(ii) Given inequalities are

$$\left.\begin{array}{l} 3x-4y\le 12 \to (1) \\ x+2y\le 6 \to (2) \\ x+y\ge 1 \to (3) \end{array}\right] \to (I)$$



Associated equations of (1), (2) and (3) are

$3x-4y=12 \to (4)$

$x+2y=6 \to (5)$

$x+y=1 \to (6)$

For $y=0$, (4) $\Rightarrow 3x-0=12 \quad \therefore x=4$

for $x=0$, (4) $\Rightarrow 0-4y=12 \quad \therefore y=-3$

$\therefore$ Line (4) cuts x-axis at $(4,0)$ and y-axis at $(0,-3)$

For $y=0$, (5) $\Rightarrow x+0=6 \quad \therefore x=6$

for $x=0$, (5) $\Rightarrow 0+2y=6 \quad \therefore y=3$

$\therefore$ Line (5) cuts x-axis at $(6,0)$ and y-axis at $(0,3)$

For $y=0$, (6) $\Rightarrow x+0=1$ i.e $x=1$

for $x=0$, (6) $\Rightarrow 0+y=1$ i.e $y=1$

$\therefore$ Line (6) cuts x-axis at $(1,0)$ and y-axis at $(0,1)$

We take $(0,0)$ as test point.

Putting $(0,0)$ in L.H.S of (1)

$3(0)-4(0)=0-0=0<12$

$\therefore (0,0)$ satisfies (1)

$\therefore$ Graph of (1) is the closed half plane (made by line (4)) on the side of origin $(0,0)$.

Putting $(0,0)$ in L.H.S of (2)

$0+2(0)=0+0=0<6$

$\therefore (0,0)$ satisfies (2).

$\therefore$ Graph of (2) is the closed half plane (made by line (5)) on the side of origin $(0,0)$

Putting $(0,0)$ in the L.H.S of (3).

$0+0=0<1$

$\therefore (0,0)$ does not satisfy (3)

$\therefore$ Graph of (3) is the closed half plane (made by line (6)) not on the side of origin $(0,0)$.

The intersection of graphs of (1), (2) and (3) is the solution region of system (I) as shown in the figure by shading the region.

(iii) Given inequalities are

$2x + y \le 4 \to (1)$

$2x - 3y \ge 12 \to (2)$

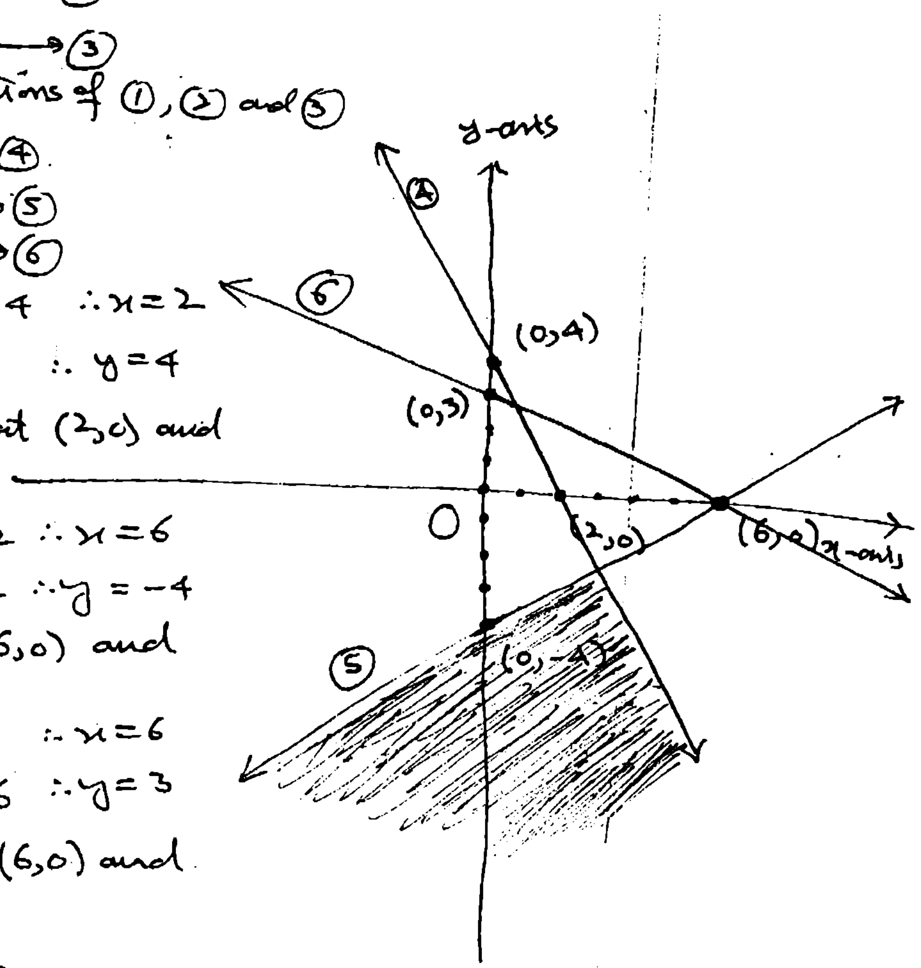
$x + 2y \le 6 \to (3)$

The associated equations of (1), (2) and (3) are

$2x + y = 4 \to (4)$

$2x - 3y = 12 \to (5)$

$x + 2y = 6 \to (6)$



For $y = 0$, (4) $\Rightarrow 2x + 0 = 4 \quad \therefore x = 2$

for $x = 0$, (4) $\Rightarrow 0 + y = 4 \quad \therefore y = 4$

$\therefore$ The line (4) cuts x-axis at (2,0) and y-axis at (0, 4)

For $y = 0$, (5) $\Rightarrow 2x - 0 = 12 \quad \therefore x = 6$

for $x = 0$, (5) $\Rightarrow 0 - 3y = 12 \quad \therefore y = -4$

$\therefore$ Line (5) cuts x-axis at (6,0) and y-axis at (0, -4).

for $y = 0$, (6) $\Rightarrow x + 0 = 6 \quad \therefore x = 6$

for $x = 0$, (6) $\Rightarrow 0 + 2y = 6 \quad \therefore y = 3$

$\therefore$ Line (6) cuts x-axis at (6,0) and y-axis at (0, 3)

We take (0,0) as test point.

Putting (0,0) in L.H.S of (1)

$2(0) + 0 = 0 + 0 = 0 < 4$

$\therefore$ (0,0) satisfies (1).

$\therefore$ Graph of (1) is the closed half plane (made by line (4)) on the side of origin O.

Putting (0,0) in L.H.S of (2)

$2(0) - 3(0) = 0 - 0 = 0 < 12$

$\therefore$ (0,0) does not satisfy (2).

$\therefore$ Graph of (2) is the closed half plane (made by line (5)) not on the side of origin O.

Putting (0,0) in L.H.S of (3)

$0 + 2(0) = 0 + 0 = 0 < 6$

$\therefore$ (0,0) satisfies (3).

$\therefore$ Graph of (3) is the closed half plane (made by line (6)) on the side of (0,0).

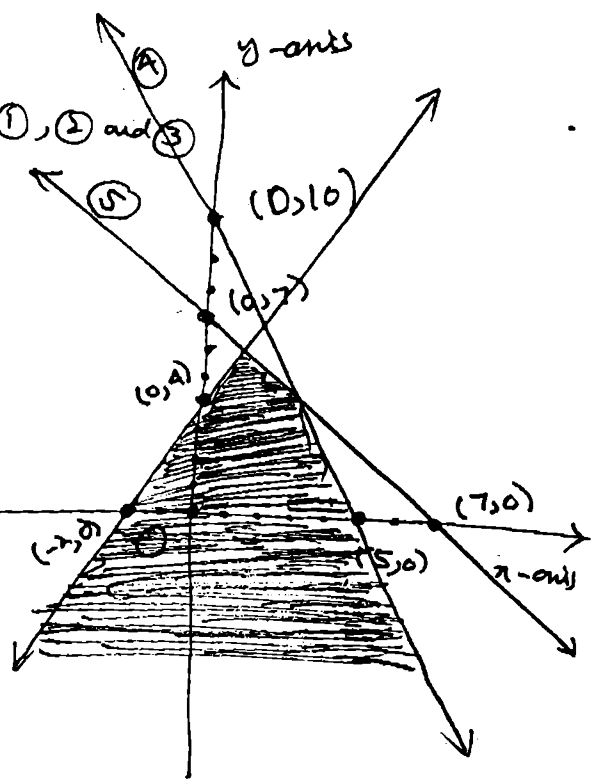
The intersection of graphs of (1), (2) and (3) is the solution region of system (I) as shown partly in the figure by shading the region.

(iv) Given inequalities are

$2x + y \le 10 \rightarrow (1)$

$x + y \le 7 \rightarrow (2)$

$-2x + y \le 4 \rightarrow (3)$



The associated equations of (1), (2) and (3) are

$2x + y = 10 \rightarrow (4)$

$x + y = 7 \rightarrow (5)$

$-2x + y = 4 \rightarrow (6)$

For $y = 0$, (4) $\Rightarrow 2x + 0 = 10 \therefore x = 5$

for $x = 0$, (4) $\Rightarrow 0 + y = 10 \therefore y = 10$

$\therefore$ Line (4) cuts x-axis at $(5, 0)$ and y-axis at $(0, 10)$.

For $y = 0$, (5) $\Rightarrow x + 0 = 7 \therefore x = 7$

for $x = 0$, (5) $\Rightarrow 0 + y = 7 \therefore y = 7$

$\therefore$ Line (5) cuts x-axis at $(7, 0)$ and y-axis at $(0, 7)$.

For $y = 0$, (6) $\Rightarrow -2x + 0 = 4 \therefore x = -2$

for $x = 0$, (6) $\Rightarrow -0 + y = 4 \therefore y = 4$

$\therefore$ Lines (6) cuts x-axis at $(-2, 0)$ and y-axis at $(0, 4)$

We take $(0, 0)$ as the test point.

Putting $(0,0)$ in L.H.S of (1)

$2(0) + 0 = 0 + 0 = 0 < 10$

$\therefore (0, 0)$ satisfies (1).

$\therefore$ Graph of (1) is the closed half plane (made by line (4)) on the side of $(0, 0)$.

Putting $(0, 0)$ in L.H.S of (2)

$0 + 0 = 0 < 7$

$\therefore (0, 0)$ satisfies (2).

$\therefore$ The graph of (2) is the closed half plane (made by line (5)) on the side of $(0, 0)$

Putting $(0, 0)$ in L.H.S of (3).

$-2(0) + 0 = -0 + 0 = 0 < 4$

$\therefore (0, 0)$ satisfies (3)

$\therefore$ The graph of (3) is the closed half plane (formed by line (6)) on the side of $(0, 0)$.

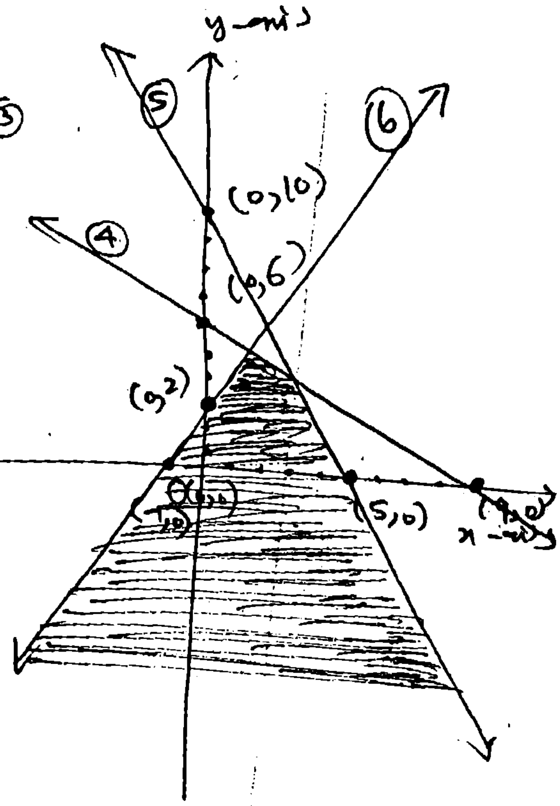
The intersection of graphs of (1), (2) and (3) is the solution region of system (I), the part of which is as shown in the figure by shading the region.

(v) Given inequalities are

$2x+3y \le 18 \rightarrow (1)$
$2x+y \le 10 \rightarrow (2)$
$-2x+y \le 2 \rightarrow (3)$

The associated equations of (1), (2) and (3) are

$2x+3y=18 \rightarrow (4)$
$2x+y=10 \rightarrow (5)$
$-2x+y=2 \rightarrow (6)$



For $y=0$, (4) $\Rightarrow 2x+0=18 \quad \therefore x=9$
for $x=0$, (4) $\Rightarrow 0+3y=18 \quad \therefore y=6$
$\therefore$ The line (4) cuts x-axis at (9,0) and y-axis at (0,6)

For $y=0$, (5) $\Rightarrow 2x+0=10 \quad \therefore x=5$
for $x=0$, (5) $\Rightarrow 0+y=10 \quad \therefore y=10$
$\therefore$ Line (5) cuts x-axis at (5,0) and y-axis at (0,10)

For $y=0$, (6) $\Rightarrow -2x+0=2 \quad \therefore x=-1$
for $x=0$, (6) $\Rightarrow -0+y=2 \quad \therefore y=2$
$\therefore$ Line (6) cuts x-axis at (-1,0) and y-axis at (0,2).

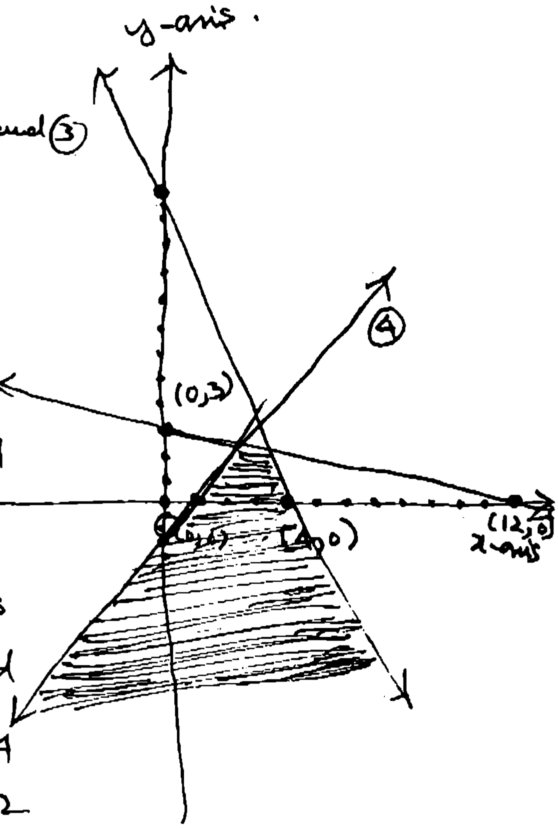
We take (0,0) as test point.
Putting (0,0) in L.H.S of (1)
$2(0)+3(0)=0+0=0<18$.
$\therefore$ (0,0) satisfies (1).
$\therefore$ The graph of (1) is the closed half plane (formed by line (4)) on the side of (0,0).
Putting (0,0) in L.H.S of (2)
$2(0)+0=0+0=0<10$
$\therefore$ (0,0) also satisfies (2)
$\therefore$ The graph of (2) is the closed half plane (made by line (5)) on the side of (0,0).
Putting (0,0) in L.H.S of (3)
$-2(0)+0=-0+0=0<2$
$\therefore$ (0,0) also satisfies (3).
$\therefore$ The graph of (3) is the closed half planed (made by line (6)) on the side of (0,0).
The intersection of graphs of (1), (2), (3) is the solution region of system (I), partly shown in the figure by shading the region.

Given inequalities are

$$\left.\begin{array}{l} 3x-2y \geq 3 \rightarrow (1) \\ x+4y \leq 12 \rightarrow (2) \\ 3x+y \leq 12 \rightarrow (3) \end{array}\right] \rightarrow (I)$$

The associated equations of (1), (2) and (3) are

$$3x-2y=3 \rightarrow (4)$$
$$x+4y=12 \rightarrow (5)$$
$$3x+y=12 \rightarrow (6)$$



For $y=0$, (4) $\Rightarrow 3x-0=3 \quad \therefore x=1$

for $x=0$, (4) $\Rightarrow 0-2y=3 \quad \therefore y=-\frac{3}{2}$

$\therefore$ Line (4) cuts x-axis at $(1,0)$ and y-axis at $(0,-\frac{3}{2})$

For $y=0$, (5) $\Rightarrow x+0=12 \quad \therefore x=12$

for $x=0$, (5) $\Rightarrow 0+4y=12 \quad \therefore y=3$

$\therefore$ Line (5) cuts x-axis at $(12,0)$ and y-axis at $(0,3)$.

For $y=0$, (6) $\Rightarrow 3x+0=12 \quad \therefore x=4$

for $x=0$, (6) $\Rightarrow 0+y=12 \quad$ i.e. $y=12$

$\therefore$ Line (6) cuts x-axis at $(4,0)$ and y-axis at $(0,12)$.

We take $(0,0)$ as the test point.

Putting $(0,0)$ in L.H.S of (1)

$3(0)-2(0)=0-0=0<3$

$\therefore (0,0)$ does not satisfy (1).

$\therefore$ Graph of (1) is the closed half plane (made by line (4)) not on the side of $(0,0)$.

Putting $(0,0)$ in L.H.S of (2)

$0+4(0)=0+0=0<12$

$\therefore (0,0)$ satisfies (2).

$\therefore$ The graph of (2) is the closed half plane (made by line (5)) on the side of $(0,0)$.

Putting $(0,0)$ in L.H.S of (3)

$3(0)+0=0+0=0<12$

$\therefore (0,0)$ also satisfies (3).

$\therefore$ Graph of (3) is the closed half plane (made by line (6)) on the side of $(0,0)$.

The intersection of graphs of (1), (2), (3) is the solution region of system (I) is partly shown in the figure by shading the region.