We know that $2k\pi + \theta - 0$ where $k \in Z$.

Now $\theta = 360° = 0° + (360°)\,1 = 0°$ where $k = 1$

So $\sin 360° = \sin 0° = 0$; $\text{cosec}\,360° = \dfrac{1}{\sin 360°} = \dfrac{1}{\sin 0°} = \dfrac{1}{0} = \infty$ (undefined)

$\cos 360^0 = \cos b° = 1$; $\sec 360° = \dfrac{1}{\cos 0°} = \dfrac{1}{1} = 1,$

$\tan 360^0 = \tan 0^0 = 0$; $\cot 360° = \dfrac{1}{\tan 0°} = \dfrac{1}{0} = \infty$ (undefined)

SOLVED EXERCISE 7.3

1. Locate each of the following angles in standard position using a protractor or fair free hand guess. Also find a positive and a negative angle conterminal with each given angle.

(i) 170°

Solution

Positive conterminal angle

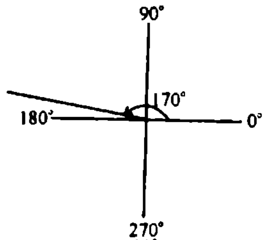
$= 360° + 170°$

$= 530°$

Negative Coterminal angle

$= 360 - 170°$

$= 190°$



(ii) 780°

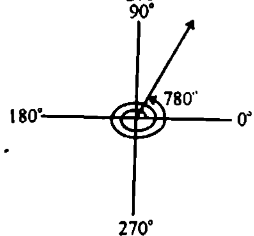
Solution

Positive conterminal angle

$= 780° - 360° - 360°$

$= 780° - 720°$

$= 60°$



(iii) −100°

Solution

Positive conterminal angle

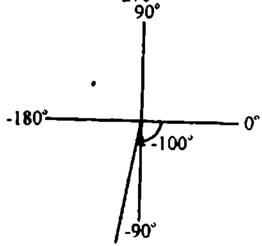
$= 360° - 100°$

$= 260°$

Negative Conterminal angle

$= -360° - 100°$

$= -460°$



(iv) -500°

Solution

Positive conterminal angle

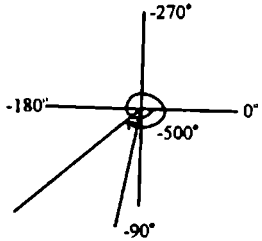
$= 720^\circ - 500^\circ$

$= 220^\circ$

Negative Conterminal angle

$= 360^\circ - 500^\circ$

$= -140^\circ$



2. Identify the closest quadrantal angles between which the following angles lies.

(i) 156°

Solution:

The closest quaurantangles between which 156° lies are 90° and 180°.

(ii) 318°

Solution:

The closest quadrantal angles between which 318° lies are 270° and 360°.

(iii) 572°

Solution

The closest quadrantal angles between which 572° lies are 540° and 630°.

(iv) -330°

Solution

The closest quadrantal angles between which -33° lies are 0° and 90°.

3. Write the closest quadrantal angles between which the angle lies. Write your answer in radian measure.

(i) $\frac{\pi}{3}$

Solution

The closest quadrantal angles between which $\frac{\pi}{3}$ lies are 0 and $\frac{\pi}{2}$.

(ii) $\frac{3\pi}{4}$

Solution

The closest quadrantal angles between which $\frac{3\pi}{4}$ lies are $\frac{\pi}{2}$ and π.

(iii) $\dfrac{-\pi}{4}$

Solution

The closest quadrantal angles between which $\dfrac{-\pi}{4}$ lies are 0 and $-\dfrac{\pi}{4}°$.

(iv) $\dfrac{-3\pi}{4}$

Solution

The closest quadrantal angles between which $\dfrac{-3\pi}{4}$ lies are $-\dfrac{\pi}{2}$ and $-\pi$.

4. In which quadrant (9 lie when

(i) $\sin\theta > 0$, $\tan\theta < 0$

Solution

II

(ii) $\cos\theta < 0$, $\sin\theta < 0$

Solution

III

(iii) $\sec\theta > 0$, $\sin\theta < 0$

Solution

IV

(iv) $\cos\theta < 0$, $\tan\theta < 0$

Solution

II

(v) $\operatorname{cosec}\theta > 0$, $\cos\theta > 0$

Solution

I

(vi) $\sin\theta < 0$, $\sec\theta < 0$

Solution

III

5. Fill in the blanks.

(i) $\cos(-150°) =$ .... $\cos 150°$

Solution

(ii) $\sin(-310^\circ) = \ldots\ldots\ldots \sin 310^\circ$

Solution

- ve

(iii) $\tan(-210^\circ) = \ldots\ldots\ldots \tan 210^\circ$

Solution

- ve

(iv) $\cot(-45^\circ) = \ldots\ldots\ldots \cot 45^\circ$

Solution

- ve

(v) $\sec(-60^\circ) = \ldots\ldots\ldots \sec 60^\circ$

Solution:

+ ve

(vi) $\operatorname{cosec}(-137^\circ) = \ldots\ldots\ldots \operatorname{cosec} 137^\circ$

Solution:

- ve

6. The given point P lies on the-terminal side of 0. Find quadrant of 6 and all six trigonometric ratios.

(I) (-2, 3)

Solution

$(-2,3)$

Here $x = -2$ and $y = 3$

So, the quadrant of θ is II.

Now, by Pythagorus theome,

We have

$$r = \sqrt{x^2 + y^2}$$

$$= \sqrt{(3-2)^2 + (3)^2}$$

$$= \sqrt{4+9}$$

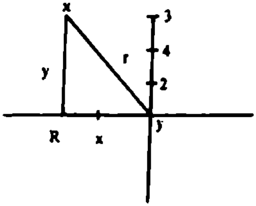
$$= \sqrt{13}$$

The six trigonometric ratios are

$$\operatorname{Sin}\theta = \frac{y}{r} = \frac{3}{\sqrt{13}} \qquad \operatorname{Cosec}\theta = \frac{r}{y} = \frac{\sqrt{13}}{3}$$

$$\operatorname{Cos}\theta = \frac{x}{r} = \frac{-2}{\sqrt{13}} \qquad \operatorname{Sec}\theta = \frac{r}{x} = \frac{\sqrt{13}}{-2}$$

$$\operatorname{Tan}\theta = \frac{y}{x} = \frac{3}{-2} \qquad \operatorname{Cot}\theta = \frac{x}{y} = \frac{-2}{3}$$



(ii) (-3, -4)

Solution

$\left(-3,-4\right)$

Here x = 3 and y = - 4

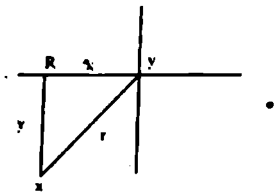
So, the quadrant of θ is III.

Now by Pythagoras theorem, we have

$r = \sqrt{x^2 + y^2}$

$= \sqrt{\left(-3\right)^2 + \left(-4\right)^2}$

$= \sqrt{9+16} = \sqrt{25} = 5$



The six trigonometric ratios are

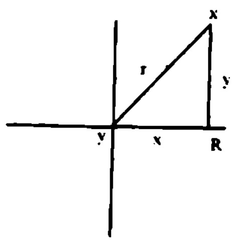
$\text{Sin}\ \theta = \frac{y}{r} = \frac{-4}{5}$ $\qquad$ $\text{Cosec}\theta = \frac{r}{y} = \frac{5}{-4}$

$\text{Cos}\ \theta = \frac{x}{r} = \frac{-3}{5}$ $\qquad$ $\text{Sec}\ \theta = \frac{r}{x} = \frac{5}{-3}$

$\text{Tan}\ \theta = \frac{y}{x} = \frac{-4}{-3} = \frac{4}{3}$ $\qquad$ $\text{Cot}\ \theta = \frac{x}{y} = \frac{-3}{-4} = \frac{3}{4}$

(iii) ($\sqrt{2}$, 1)

Solution



$\left(\sqrt{2},1\right)$

Here x = $\sqrt{2}$ and y = 1

So, the quadrant of θ is I

Now by Pythagoras theorem, we have

$$r = \sqrt{x^2 + y^2}$$
$$= \sqrt{\left(\sqrt{2}\right)^2 + (1)^2}$$
$$= \sqrt{2+1}$$
$$= \sqrt{3}$$

The six trigonometric ratios are

$\text{Sin } \theta = \dfrac{y}{r} = \dfrac{1}{\sqrt{3}}$ $\qquad$ $\text{Cosec}\theta = \dfrac{r}{y} = \dfrac{\sqrt{3}}{1} = \sqrt{3}$

$\text{Cos } \theta = \dfrac{x}{r} = \dfrac{\sqrt{2}}{\sqrt{3}} = \sqrt{\dfrac{2}{3}}$ $\qquad$ $\text{Sec } \theta = \dfrac{r}{x} = \dfrac{\sqrt{3}}{\sqrt{2}} = \sqrt{\dfrac{2}{3}}$

$\text{Tan } \theta = \dfrac{y}{x} = \dfrac{1}{\sqrt{2}}$ $\qquad$ $\text{Cot } \theta = \dfrac{x}{y} = \dfrac{\sqrt{2}}{1} = \sqrt{2}$

7. If $\cos\theta = \dfrac{-2}{3}$ and terminal arm of the angle θ is in quadrant II, find the values of remaining trigonometric functions.

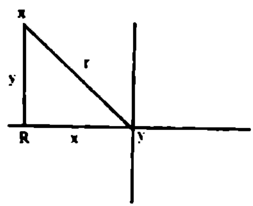
Solution

As $\text{Cos } \theta = -\dfrac{2}{3}$ and terminal arm of the angle θ is in quadrant II, so

$x = -2$ and $r = 3$

By Phthagoras theorem, we have

$$r^2 = x^2 + y^2$$
$$y^2 = r^2 - x^2$$
$$y = \sqrt{r^2 - x^2}$$
$$y = \sqrt{(3)^2 - (-2)^2}$$
$$y = \sqrt{9-4}$$
$$y = \sqrt{5}$$



The six trigonometric ratios are

$\text{Sin } \theta = \dfrac{y}{r} = \dfrac{1}{\sqrt{3}}$ $\qquad$ $\text{Cosec}\theta = \dfrac{r}{y} = \dfrac{\sqrt{3}}{1} = \sqrt{3}$

$\text{Cos } \theta = \dfrac{x}{r} = \dfrac{\sqrt{2}}{\sqrt{3}} = \sqrt{\dfrac{2}{3}}$ $\qquad$ $\text{Sec } \theta = \dfrac{r}{x} = \dfrac{\sqrt{3}}{\sqrt{2}} = \sqrt{\dfrac{2}{3}}$

$\text{Tan } \theta = \dfrac{y}{x} = \dfrac{1}{\sqrt{2}}$ $\qquad$ $\text{Cot } \theta = \dfrac{x}{y} = \dfrac{\sqrt{2}}{1} = \sqrt{2}$

8. If $\tan\theta = \frac{4}{3}$ and $\sin\theta < 0$, find the values of other trigonometric functions at θ.

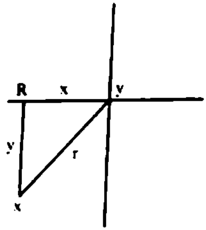
Solution

As $\tan\theta = \frac{4}{3}$ and $\sin\theta < 0$ (terminal arm of the angle θ is in quadrant III), so.

$x = 3$ and $y = 4$

By Phthagoras theorem, we have

$$r = \sqrt{x^2 + y^2}$$
$$= \sqrt{(3)^2 + (4)^2}$$
$$= \sqrt{9+16}$$
$$= \sqrt{25} = 5$$



The six trigonometric ratios are

$\sin\theta = -\frac{y}{r} = -\frac{4}{5}$

$\cos\theta = -\frac{x}{r} = -\frac{3}{5}$

$\operatorname{Cosec}\theta = \frac{r}{y} = -\frac{5}{4}$

$\sec\theta = -\frac{r}{x} = -\frac{5}{3}$

$\cot\theta = \frac{x}{y} = \frac{3}{4}$

9. If $\tan\theta = -\frac{1}{\sqrt{2}}$ and terminal side of the angle is not in quadrant II, find the values of $\tan\theta$, $\sec\theta$, and $\operatorname{cosec}\theta$.

As $\tan\theta = -\frac{1}{\sqrt{2}}$ and terminal said of the angle is in quadrant – IV, so.

$y = -1$ and $r = \sqrt{2}$

By Phthagoras theorem, we have

$$r^2 = x^2 + y^2$$
$$x^2 = r^2 - y^2$$
$$x = \sqrt{r^2 - y^2}$$
$$x = \sqrt{\left(\sqrt{2}\right)^2 - (-1)^2}$$
$$x = \sqrt{2-1}$$
$$x = \sqrt{1}$$
$$x = 1$$

Now

$$\tan\theta = \frac{y}{x} = \frac{-1}{1} = -1 \qquad \text{Sec}\theta = \frac{r}{x} = -\frac{\sqrt{2}}{1} = -\sqrt{2}$$

$$\text{Cosec}\theta = \frac{r}{y} = \frac{\sqrt{2}}{-1} = -\sqrt{2}$$

10. If $\text{cosec}\theta = \frac{13}{12}$ and $\sec\theta > 0$, find the remaining trigonometric functions.

As $\text{Cosec}\theta = \frac{13}{12}$ and $\text{Sec}\theta > 0$ (terminal arm of angle θ is in quadrant I), so

$y = 12$ and $r = 13$

By Pythagoras theorem, we have

$r^2 = x^2 + y^2$

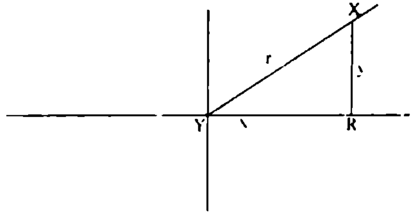
$x^2 = r^2 - y^2$

$= \sqrt{r^2 - y^2}$

$= \sqrt{\left(\sqrt{13}\right)^2 - (12)^2}$

$= \sqrt{169 - 144}$

$= \sqrt{25} = 5$



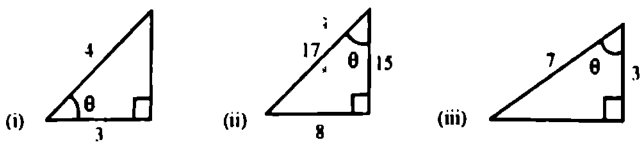
The remaining trigonometric ratios are

$$\text{Sin}\theta = \frac{y}{r} = \frac{12}{13}$$

$$\text{Cos}\theta = \frac{x}{r} = \frac{5}{13} \qquad \text{Sec}\theta = \frac{r}{x} = \frac{13}{5}$$

$$\tan\theta = \frac{y}{x} = \frac{12}{5} \qquad \text{Cot}\theta = \frac{x}{y} = \frac{5}{12}$$

11. Find the values of trigonometric functions at the indicated angle θ in the right triangle.



(i) As $x = 3$ and $r = 4$

By Pythagoras theorem, we have

$r^2 = x^2 + y^2$

$y^2 = r^2 - y^2$

$y = \sqrt{r^2 - x^2}$

$y = \sqrt{(4)^2 - (3)^2}$

$y = \sqrt{16 - 9}$

$y = \sqrt{7}$

The six trigonometric ratios are

$\text{Sin}\,\theta = \frac{y}{r} = \frac{\sqrt{7}}{4}$ $\qquad$ $\text{Cosec}\theta = \frac{r}{y} = \frac{4}{\sqrt{7}}$

$\text{Cos}\,\theta = \frac{x}{r} = \frac{3}{4}$ $\qquad$ $\text{Sec}\,\theta = \frac{r}{x} = \frac{4}{3}$

$\text{Tan}\,\theta = \frac{y}{x} = \frac{\sqrt{7}}{3}$ $\qquad$ $\text{Cot}\,\theta = \frac{x}{y} = \frac{3}{\sqrt{7}}$

(ii) As $r = 17, x = 15, y = 8$

The six trigonometric ratios are

$\text{Sin}\,\theta = \frac{y}{r} = \frac{8}{17}$ $\qquad$ $\text{Cosec}\theta = \frac{r}{y} = \frac{17}{8}$

$\text{Cos}\,\theta = \frac{x}{r} = \frac{15}{17}$ $\qquad$ $\text{Sec}\,\theta = \frac{r}{x} = \frac{17}{15}$

$\text{Tan}\,\theta = \frac{y}{x} = \frac{8}{15}$ $\qquad$ $\text{Cot}\,\theta = \frac{x}{y} = \frac{15}{8}$

(iii) As $x = 3$ and $r = 7$

By Pythagoras theorem, we have

$r^2 = x^2 + y^2$

$y^2 = r^2 - y^2$

$y = \sqrt{r^2 - x^2}$

$y = \sqrt{(7)^2 - (3)^2}$

$y = \sqrt{49 - 9}$

$y = \sqrt{40}$

$y = 2\sqrt{10}$

The six trigonometric ratios are

$\text{Sin}\,\theta = \frac{x}{r} = \frac{3}{7}$ $\qquad$ $\text{Cosec}\theta = \frac{r}{y} = \frac{7}{2\sqrt{10}}$

$\text{Cos}\,\theta = \frac{x}{r} = \frac{3}{7}$ $\qquad$ $\text{Sec}\,\theta = \frac{r}{x} = \frac{7}{3}$

$$\text{Tan}\,\theta = \frac{y}{x} = \frac{2\sqrt{10}}{3} \qquad \text{Cot}\,\theta = \frac{x}{y} = \frac{3}{2\sqrt{10}}$$

12. Find the values of the trigonometric functions. Do not use trigonometric tables or calculator.

(i) tan30°

Solution

$$\tan 30^\circ = \frac{1}{\sqrt{3}}$$

(ii) tan330°

Solution

We know that

$2k\pi + \theta$, θ where $k \in z$

$$\begin{aligned}\tan 330^\circ &= \tan(360^\circ - 30^\circ)\\ &= \tan(2(1)\pi - 30^\circ)\\ &= \tan(-30^\circ)\\ &= -\tan 30^\circ\\ &= -\frac{1}{\sqrt{3}}\end{aligned}$$

(iii) sec330

Solution

We know that .

$2k\pi + \theta = \theta$, where $k \in z$

$$\begin{aligned}\text{Sec}330^\circ &= \text{Sec}(360^\circ - 30^\circ)\\ &= \text{Sec}(2(1)\pi - 30^\circ)\\ &= \text{Sec}(-30^\circ)\\ &= \text{Sec}30^\circ\\ &= \frac{1}{\text{Sec}30^\circ}\\ &= \frac{2}{\sqrt{3}}\end{aligned}$$

(iv) $\cot\frac{\pi}{4}$

Solution

We know that

$2k\pi + \theta, \theta$ where $k \in z$

$$\cot\frac{\pi}{4} = \frac{1}{\tan\frac{\pi}{4}}$$

$$= \frac{1}{1} = 1$$

(v) $\cos\frac{2\pi}{3}$

Solution

We know that

$2k\pi + \theta, \theta$ where $k \in z$

$$\cos\left(\frac{2\pi}{3}\right) = \cos\left(2\left(\frac{1}{2}\right)\pi - \frac{\pi}{3}\right)$$

$$= \cos\left(-\frac{\pi}{3}\right)$$

$$= \cos\frac{\pi}{3}$$

$$= -\frac{1}{2} \because (\text{In quad. II} \cos\theta < 0)$$

(vi) $\operatorname{cosec}\frac{7\pi}{6}$

Solution

We know that

$2k\pi + \theta = \theta$

$$\operatorname{cosec}\left(\frac{2\pi}{3}\right) = \operatorname{cosec}\left(2\left(\frac{1}{2}\right) - \frac{\pi}{3}\right)$$

$$= \operatorname{cosec}\left(-\frac{\pi}{3}\right)$$

$$= \frac{1}{\sin(-\pi/3)}$$

$$= -\frac{1}{\sin(\pi/3)}$$

$$= \frac{1}{\sin(\pi/3)} \because (\text{In quad. II} \sin > 0)$$

$$=\frac{1}{\sqrt{3}/2}$$

$$=\frac{2}{\sqrt{3}}$$

(vii) cos (– 450°)

Solution

We know that

$2k\pi + \theta = \theta$

$$\text{Cos}(-450^\circ) = \text{Cos}(2(-1)\pi - 90^\circ)$$

$$= \text{Cos}(-90^\circ)$$

$$= \text{Cos}\,90^\circ = 0$$

(viii) tan (–9π)

Solution

We know that

$2k\pi + \theta = \theta$

$$\tan(-9\pi) = \tan(2(-5)\pi + \pi)$$

$$= \tan(\pi)$$

$$= 0$$

(ix) cos $\left(\frac{-5\pi}{6}\right)$

Solution

We know that

$2k\pi + \theta = \theta$

$$\cos\left(-\frac{5\pi}{6}\right) = \cos\left(2\left(-\frac{1}{2}\right)\pi + \frac{\pi}{6}\right)$$

$$= \cos\left(\frac{\pi}{6}\right)$$

$$= -\frac{\sqrt{3}}{2}$$

$\therefore$ (In quad. II cos < 0)

(x) sin $\frac{7\pi}{6}$

Solution

We know that

$2k\pi + \theta = \theta$

$$\sin\left(\frac{7\pi}{6}\right)=\sin\left(2\left(\frac{1}{2}\right)\pi+\frac{\pi}{6}\right)$$

$$=\sin\left(\frac{\pi}{6}\right)$$

$$=\frac{1}{2}\ \because\ (\text{In quad. III} \sin<0).$$

(xi) $\cot\left(\frac{7\pi}{6}\right)$

Solution

We know that

$2k\pi+\theta=0$

$$\sin\left(\frac{7\pi}{6}\right)=\sin\left(2\left(\frac{1}{2}\right)\pi+\frac{\pi}{6}\right)$$

$$=\sin\left(\frac{\pi}{6}\right)$$

$$=\frac{1}{\tan(\pi/6)}$$

$$=\frac{1}{\sqrt{3}}$$

(In quad. II $\cos<0$)

(xii) $\cos 225°$

Solution

We know that

$2k\pi+\theta=\theta$

$$\cos(225°)=\cos\left(2\left(\frac{1}{2}\right)\pi+45°\right)$$

$$=\cos 45°$$

$$=-\frac{1}{\sqrt{2}}$$

$\therefore$ (In quad. II $\cos<0$)

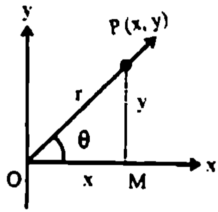
Trigonometric Identities:

Consider an angle $\angle MOP=\theta$ radian in standard position. Let point P (x, y) be on the terminal side of the angle. By Pythagorean theorem, we have from right triangle OMP.

$$OM^2+MP^2=OP^2$$

$$x^2+y^2=r^2 \quad \ldots\ldots \text{(i)}$$

Dividing both sides by r^2 we get



$$\frac{x^2}{r^2}+\frac{y^2}{r^2}=1$$

$$\Rightarrow \quad \left(\frac{x}{r}\right)^2+\left(\frac{y}{r}\right)^2=1$$

$$\Rightarrow \quad \left(\cos^2\theta\right)^2+\left(\sin\theta\right)^2=1$$

$$\therefore \quad \boxed{\cos^2\theta+\sin^2\theta=1} \qquad (1)$$

Dividing (i) by x^2, we have

$$\frac{x^2}{r^2}+\frac{y^2}{r^2}=\frac{r^2}{x^2}$$

$$\Rightarrow \quad 1+\left(\frac{y}{x}\right)^2+\left(\frac{r}{x}\right)^2 \qquad \because \tan\theta=\frac{y}{x} \text{ and } \sec\theta=\frac{r}{x}$$

$$\Rightarrow \quad 1+\left(\tan^2\theta\right)^2+\left(\sec\theta\right)^2=1$$

$$\therefore \quad \boxed{1+\tan^2\theta=\sec^2\theta} \text{ or } \sec^2\theta-\tan^2\theta=1 \quad (2)$$

Again dividing both sides of(i) byy^2, we get

$$\frac{x^2}{y^2}+\frac{y^2}{y^2}=\frac{r^2}{y^2}$$

$$\left(\frac{x}{y}\right)^2+1=\left(\frac{r}{y}\right)^2 \qquad \because \cot\theta=\frac{x}{y} \text{ and } \operatorname{cosec}\theta=\frac{r}{y}$$

$$\Rightarrow \quad \left(\cot\theta\right)^2+1=\left(\operatorname{cosec}\theta\right)^2$$

$$\therefore \quad \boxed{1+\cot^2\theta=\operatorname{cosec}^2\theta} \qquad \text{or } \operatorname{cosec}^2\theta-\cot^2\theta=1 \qquad (3)$$

The identities (1), (2) and (3) are also known as Pythagorean Identities.

The fundamental identities are used to simplify expressions involving rigonometric functions

Example 1:

Verify that $\cot\theta \sec\theta = \operatorname{cosec}\theta$

Solution

Expressing left hand side in terms of sine and cosine, we have

$$\text{L.H.S} = \cot\theta\sec\theta = \frac{\cos\theta}{\sin\theta}.\frac{1}{\cos\theta}$$
$$= \frac{1}{\sin\theta} = \operatorname{cosec}\theta$$
$$= \text{R.H.S}$$

Example 2

Verify that $\tan\theta + \tan^2\theta \tan^2\theta \sec^2\theta$

Solution

$$\text{L.H.S} = \tan^4\theta + \tan^2\theta = \tan^2\theta(\tan^2\theta + 1) \qquad \because \tan^2\theta + 1 = \sec^2\theta$$
$$= \tan^2\theta\sec^2\theta$$
$$= \text{R.H.S}$$

Example 3

Show that $\dfrac{\cot^2\alpha}{\operatorname{cosec}\alpha - 1} = \operatorname{cosec}\alpha + 1$

Solution

$$\frac{\cot^2\alpha}{\operatorname{cosec}\alpha - 1} \qquad \left(\begin{array}{l}\because \operatorname{cosec}^2\theta - \cot = 1\\ \cot^2\theta = \operatorname{cosec}^2\theta - 1\end{array}\right)$$

Solution

$$= \frac{(\operatorname{cosec}^2\alpha - 1)}{\operatorname{cosec}\alpha - 1} = \frac{(\operatorname{cosec}\alpha - 1)(\operatorname{cosec}\alpha + 1)}{(\operatorname{cosec}\alpha - 1)} = \operatorname{cosec}\alpha + 1 = \text{R.H.S}$$

Example 4

Express the trigonometric functions in terms of $\tan\theta$.

Solution

By using reciprocal identity, we can express $\cot\theta$ in terms of $\tan\theta$.

i.e., $\quad \cot\theta = \dfrac{1}{\tan\theta}$

By solving the identity $\quad 1 + \tan^2\theta \sec^2\theta$

We have expressed $\sec\theta$ in terms of $\tan\theta$.

$$\sec\theta = \pm\sqrt{\tan^2\theta + 1} \because$$

$$\cos\theta = \frac{1}{\sec\theta} \Rightarrow \cos\theta = \frac{1}{\pm\sqrt{\tan^2\theta + 1}}$$

Because $\sin\theta = \tan\theta\cos\theta$, we have

$$\sin\theta = \tan\theta\left(\frac{1}{\pm\sqrt{\tan^2\theta + 1}}\right) = \frac{\tan\theta}{\pm\sqrt{\tan^2\theta + 1}}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{\pm\sqrt{\tan^2\theta + 1}}{\tan\theta}$$