

## Exercise 6.5

### Question # 1

A man deposits in a bank Rs.10 in the first month; Rs.15 in the second month; Rs.20 in the third month and so on. Find how much he will have deposited in the bank by the 9th month.

#### Solution

The sequence of the deposits is

10, 15, 20, ..... to 9 terms

Here  $a_1 = 10$ ,  $d = 15 - 10 = 5$ ,  $n = 9$

$$\text{Since } S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

$$\Rightarrow S_9 = \frac{9}{2}[2(10) + (9-1)(5)] = \frac{9}{2}[20 + 40] = \frac{9}{2}(60) = 270$$

Hence the total amount he deposits is Rs. 270.

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### Question # 2

378 trees are planted in rows in the shape of an isosceles triangle, the numbers in successive rows decreasing by one from the base to the top. How many trees are there in the row which forms the base of the triangle?

#### Solution

The sequence of the trees from top to base row is

1, 2, 3, .....

Let  $n$  be the total number of trees in base row then

$$a_1 = 1, d = 2 - 1 = 1, n = n, S_n = 378$$

$$\text{Now } S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

$$\Rightarrow 378 = \frac{n}{2}[2(1) + (n-1)(1)] \Rightarrow 756 = n[2 + n - 1]$$

$$\Rightarrow 756 = n(n+1) \Rightarrow 756 = n^2 + n$$

$$\Rightarrow n^2 + n - 756 = 0$$

$$\text{So } n = \frac{-1 \pm \sqrt{(-1)^2 - 4(1)(-756)}}{2(1)}$$

$$\Rightarrow n = \frac{-1 \pm \sqrt{1 + 3024}}{2} = \frac{-1 \pm \sqrt{3025}}{2} = \frac{-1 \pm 55}{2}$$

$$\text{So } n = \frac{-1 + 55}{2} = \frac{54}{2} = 27 \quad \text{or} \quad n = \frac{-1 - 55}{2} = \frac{-56}{2} = -28$$

Since  $n$  can never be negative therefore  $n = 27$

$$\text{Now } a_n = a_1 + (n-1)d$$

$$\Rightarrow a_{27} = (1) + (27-1)(1) = 1 + 26 = 27$$

Thus the numbers of trees in the base row are 27.

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### Question # 3

A man borrows Rs.1100 and agree to repay with a total interest of Rs.230 in 14 installments, each installment being less than the preceding by Rs.10. What should be his first installment?

#### Solution

Let the first installment be  $x$  then the sequence of installment will be

$x, x-10, x-20, \dots$

$$\text{Here } a_1 = x, d = -10, n = 14 \text{ and } S_n = 1100 + 230 = 1330$$

$$\begin{aligned}\text{Now } S_n &= \frac{n}{2}[2a_1 + (n-1)d] \\ \Rightarrow 1330 &= \frac{14}{2}[2x + (14-1)(-10)] \Rightarrow 1330 = 7[2x - 130] \Rightarrow 1330 = 14x - 910 \\ \Rightarrow 1330 + 910 &= 14x \Rightarrow 2240 = 14x \Rightarrow x = \frac{2240}{14} = 160\end{aligned}$$

Hence the first installment is 160.

#### Question # 4

A clock strikes once when its hour hand is at one, twice when it is at two and so on. How many times does the clock strike in twelve hour?

#### Solution

The sequence of the strikes is

1, 2, 3, ....., 12

Here  $a_1 = 1$ ,  $d = 2 - 1 = 1$ ,  $n = 12$ ,  $a_n = 12$

$$\text{Now } S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

$$\Rightarrow S_n = \frac{12}{2}[2(1) + (12-1)(1)] = \frac{12}{2}[2 + 11] = \frac{12}{2}[13] = 78$$

Hence clock strikes 78 hours in twelve strikes.

#### Question # 5

A student saves Rs. 12 at the end to the first week and goes on increasing his saving Rs. 4 weekly. After how many weeks will be able to save Rs. 2100?

#### Solution

The sequence of the savings is

12, 16, 20, .....

Total Savings = 2100

So here  $a_1 = 12$ ,  $d = 16 - 12 = 4$ ,  $S_n = 2100$ ,  $n = ?$

$$\text{Since } S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

$$\Rightarrow 2100 = \frac{n}{2}[2(12) + (n-1)(4)] \Rightarrow 4200 = n[24 + 4n - 4]$$

$$\Rightarrow 4200 = n[4n + 20] \Rightarrow 4200 = 4n^2 + 20n$$

$$\Rightarrow 4n^2 + 20n - 4200 = 0 \Rightarrow 4(n^2 + 5n - 1050) = 0$$

$$\Rightarrow n^2 + 5n - 1050 = 0$$

$$\begin{aligned}\Rightarrow n &= \frac{-5 \pm \sqrt{(5)^2 - 4(1)(-1050)}}{2(1)} = \frac{-5 \pm \sqrt{25 + 4200}}{2} \\ &= \frac{-5 \pm \sqrt{4225}}{2} = \frac{-5 \pm 65}{2}\end{aligned}$$

$$\text{So } n = \frac{-5 - 65}{2} = \frac{-70}{2} = -35 \quad \text{or} \quad n = \frac{-5 + 65}{2} = \frac{60}{2} = 30$$

As  $n$  can never be negative therefore  $n = 30$

Thus student will save Rs. 2100 in 30 weeks.

#### Question # 5

An object falling from rest, falls 9 meters during the first second, 27 meters during the next second, 45 meters during the third second and so on.

(i) How far will it fall during the fifth second?

(ii) How far will it fall up to the fifth second?

**Solution**

The sequence of the falls is

$$9, 27, 45, \dots\dots\dots$$

(i)  $a_1 = 9, d = 27 - 9 = 18, a_5 = ?$

Since  $a_5 = a_1 + 4d = 9 + 4(18) = 9 + 72 = 81$

Hence in fifth second the object will fall 81 meters.

(ii) Here  $a_1 = 9, d = 27 - 9 = 18, n = 5, S_5 = ?$

Since  $S_n = \frac{n}{2}[2a_1 + (n-1)d]$

$$\Rightarrow S_5 = \frac{5}{2}[2(9) + (5-1)(18)] = \frac{5}{2}[18 + 72] = \frac{5}{2}(90) = 225$$

Thus up to 5<sup>th</sup> second the object will fall 225 meters.

**Question # 7**

An investor earned Rs. 6000 for year 1980 and Rs. 12000 for year 1990 on the same investment. If his earning have increased by the same amount each year, how much income he has received from the investment over the past eleven years?

**Solution**

Here  $a_1 = 6000, a_{11} = 12000, n = 11$

Now  $S_n = \frac{n}{2}[a_1 + a_n]$

$$\Rightarrow S_{11} = \frac{11}{2}[6000 + 12000] = \frac{11}{2}(18000) = 99000$$

Hence he will receive Rs. 99000 in past eleven years.

**Question # 8**

The sum of interior angles of polygons having sides 3,4,5,.....etc. from an A.P. Find the sum of the interior angles for a 16 sided polygon.

**Solution**

Since the sum of angles of 3 sided polygon (triangle) =  $a_1 = \pi$

Sum of angles of 4 sided polygon (quadrilateral) =  $a_2 = 2\pi$

Sum of the angles of 5 sided polygon (pentagon) =  $a_3 = 3\pi$

So

The sum of interior angles of 16 side polygon =  $a_{14} = ?$

Here  $a_1 = \pi, d = a_2 - a_1 = 2\pi - \pi = \pi, n = 14$

Since  $a_n = a_1 + (n-1)d$

$$\Rightarrow a_{14} = \pi + (14-1)(\pi) = \pi + 13\pi = 14\pi$$

Hence sum of interior angles of 16 side polygon is  $14\pi$ .

**Question # 9**

The prize money Rs60,000 will be distributed among the eight teams according to their positions determined in the match-series. The award increases by the same amount for each higher position. If the last place tem is given Rs.4000, how much will be awarded to the first place team?

**Solution**

Let  $a_1$  denotes the prize money for the last position

Then  $a_1 = 4000, S_n = 60000, n = 8, a_n = ?$

Since  $S_n = \frac{n}{2}(a_1 + a_n)$

$$\Rightarrow 60000 = \frac{8}{2}(4000 + a_n) \Rightarrow 60000 = 4(4000 + a_n)$$

$$\Rightarrow 60000 = 16000 + 4a_n \quad \Rightarrow 60000 - 16000 = 4a_n \quad \Rightarrow 44000 = 4a_n$$

$$\Rightarrow a_n = \frac{44000}{4} = 11000$$

Hence the team at 1<sup>st</sup> place will get 11000 Rs.

### Question # 10

An equilateral triangular base is filled by placing eight balls in the first row, 7 balls in the second row and so on with one ball in the last row, After this base layer, second layer is formed by placing 7 balls in the first row, 6 balls in the second row and so on with one ball in its last row. Continuing this process, a pyramid of balls is formed with one ball on top. How many balls are there in the pyramid?

#### *Solution*

Balls in the first layer =  $8 + 7 + 6 + \dots + 2 + 1$

$$= \frac{8}{2} [2(8) + (8-1)(-1)] = 4(16-7) = 36$$

Balls in the second layer =  $7 + 6 + 5 + \dots + 2 + 1$

$$= \frac{7}{2} [2(7) + (7-1)(-1)] = \frac{7}{2} [14-6] = \frac{7}{2} [8] = 28$$

Balls in the third layer =  $6 + 5 + 4 + 3 + 2 + 1 = 21$

Balls in the fourth layer =  $5 + 4 + 3 + 2 + 1 = 15$

Balls in the fifth layer =  $4 + 3 + 2 + 1 = 10$

Balls in the sixth layer =  $3 + 2 + 1 = 6$

Balls in the seventh layer =  $2 + 1 = 3$

Balls in the eighth layer =  $1$

Hence the number of balls in pyramid

$$= 36 + 28 + 21 + 15 + 10 + 6 + 3 + 1 = 120$$