Exercise 6.4

Q No 1. The series of integral multiple of 3 between 4 and 96 is

$6+9+12+15+\cdots\cdots+96$

Here $a_1 = 6$

$d = 9-6 = 12-9 = 3$

$a_n = 96$

Since $a_n = a_1+(n-1)d$

$\Rightarrow 96 = 6+(n-1)(3)$

$\Rightarrow 96 = 6+3n-3$

$\Rightarrow 96-6+3 = 3n$

$\Rightarrow 93 = 3n \Rightarrow \boxed{n=31}$

Now

$S_n = \frac{n}{2}(a_1+a_n)$ $\frac{n}{2}(2a_1+(n-1)d)$

$\Rightarrow S_{31} = \frac{31}{2}(6+96)$

$= \frac{31}{2}(102)$

$= (31)(51) = 1581$ Answer

Q No 2 i)

$-3+(-1)+1+3+5+\cdots\cdots+a_{16}$

Here $a_1 = -3$

$d = -1-(-3) = -1+3 = 2$

$n = 16$

Since $S_n = \frac{n}{2}(2a_1+(n-1)d)$

$\Rightarrow S_{16} = \frac{16}{2}(2(-3)+(16-1)(2))$

$= 8(-6+30)$

$= 8(24) = 192$ Answer

ii) $\frac{3}{\sqrt{2}}+2\sqrt{2}+\frac{5}{\sqrt{2}}+\cdots\cdots+a_{13}$

Here $a_1 = \frac{3}{\sqrt{2}}$

$d = 2\sqrt{2}-\frac{3}{\sqrt{2}} = \frac{4-3}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$n = 13$

Since $S_n = \frac{n}{2}(2a_1+(n-1)d)$

$\Rightarrow S_{13} = \frac{13}{2}\left(2\left(\frac{3}{\sqrt{2}}\right)+(13-1)\frac{1}{\sqrt{2}}\right)$

$= \frac{13}{2}\left(\frac{6}{\sqrt{2}}+\frac{12}{\sqrt{2}}\right)$

$= \frac{13}{\cancel{2}}\left(\frac{\cancel{18}^{9}}{\sqrt{2}}\right) = \frac{117}{\sqrt{2}}$ Answer

iii) $1.11+1.41+1.71+\cdots\cdots+a_{10}$

Here $a_1 = 1.11$

$d = 1.41-1.11 = 0.31$

$n = 10$

Do yourself

iv) $-8-3\frac{1}{2}+1+\cdots\cdots+a_{11}$

$-8-\frac{7}{2}+1+\cdots\cdots+a_{11}$

Here $a_1 = -8$

$d = -\frac{7}{2}-(-8) = -\frac{7}{2}+8$

$= \frac{9}{2}$; $n = 11$

Now Do yourself as (i)

v) $(x-a)+(x+a)+(x+3a)+\cdots\cdots$ to n terms

Here $a_1 = x-a$

$d = (x+a)-(x-a)$

$= x+a-x+a = 2a$

$n = n$.

Since $S_n = \frac{n}{2}[2a_1+(n-1)d]$

$\Rightarrow S_n = \frac{n}{2}[2(x-a)+(n-1)(2a)]$

$= \frac{n}{2}[2x-2a+2an-2a]$

$= \frac{n}{2}[2x+2an-4a]$

$= \frac{n}{\cancel{2}}\cancel{2}[x+an-2a]$

$= n[x+(n-2)a]$ Answer

vi) $\frac{1}{1-\sqrt{x}} + \frac{1}{1-x} + \frac{1}{1+\sqrt{x}} + \cdots\cdots$ to n term

Here $a_1 = \frac{1}{1-\sqrt{x}}$

$$d = \frac{1}{1-x} - \frac{1}{1-\sqrt{x}}$$

$$= \frac{1}{(1-\sqrt{x})(1+\sqrt{x})} - \frac{1}{1-\sqrt{x}}$$

$$= \frac{1-(1+\sqrt{x})}{(1-\sqrt{x})(1+\sqrt{x})}$$

$$= \frac{1-1-\sqrt{x}}{1-x} = \frac{-\sqrt{x}}{1-x}$$

& $n = n$

Since $S_n = \frac{n}{2}[2a_1+(n-1)d]$

$$\Rightarrow S_n = \frac{n}{2}\left[2\frac{1}{1-\sqrt{x}} + (n-1)\left(\frac{-\sqrt{x}}{1-x}\right)\right]$$

$$= \frac{n}{2}\left[\frac{2}{1-\sqrt{x}} - \frac{\sqrt{x}(n-1)}{1-x}\right]$$

$$= \frac{n}{2}\left[\frac{2}{1-\sqrt{x}} - \frac{\sqrt{x}(n-1)}{(1-\sqrt{x})(1+\sqrt{x})}\right]$$

$$= \frac{n}{2}\left[\frac{2(1+\sqrt{x}) - \sqrt{x}(n-1)}{(1-\sqrt{x})(1+\sqrt{x})}\right]$$

$$= \frac{n}{2}\left(\frac{2+2\sqrt{x} - \sqrt{x}(n-1)}{1-x}\right)$$

$$= \frac{n}{2}\left(\frac{2+(2-n+1)\sqrt{x}}{1-x}\right)$$

$$= \frac{n}{2}\left(\frac{2+(3-n)\sqrt{x}}{1-x}\right)$$ Answer

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vii) Do yourself as above

Q No. 3

$-7+(-5)+(-3)+\cdots\cdots$ amount to 65

Here $a_1 = -7$

$d = (-5)-(-7) = -5+7 = 2$

$S_n = 65$, $n = ?$

Since $S_n = \frac{n}{2}\{2a_1+(n-1)d\}$

$$\Rightarrow 65 = \frac{n}{2}\{2(-7)+(n-1)(2)\}$$

$$\Rightarrow 130 = n(-14+2n-2)$$

$$\Rightarrow 130 = n(2n-16)$$

$$\Rightarrow 130 = 2n^2-16n$$

$$\Rightarrow 2n^2-16n-130 = 0$$

$$\Rightarrow n^2-8n-65 = 0 \quad \div\text{ing by 2.}$$

$$\Rightarrow n^2-13n+5n-65 = 0$$

$$\Rightarrow n(n-13)+5(n-13) = 0$$

$$\Rightarrow (n-13)(n+5) = 0$$

$$\Rightarrow n-13 = 0 \quad \text{or} \quad n+5 = 0$$

$$n = 13 \quad \text{or} \quad n = -5$$

As n can not be -ive

so $\boxed{n = 13}$ Answer

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ii) Do yourself

Hint: you will get equation

$$3n^2-17n-288 = 0$$

you may use quadretic formula to find value of n.

Q No. 4 i)

$3+5-7+9+11-13+15+17-19+\cdots\cdots$

$\cdots\cdots+$ to 3n terms

$(3+5-7)+(9+11-13)+(15+17-19)+\cdots$

$\cdots$ to n terms

$1+7+13+\cdots\cdots$ to n terms

Here $a_1 = 1$, $d = 7-1 = 6$, $n = n$

Since $S_n = \frac{n}{2}\{2a_1+(n-1)d\}$

$$\Rightarrow S_n = \frac{n}{2}[2(1)+(n-1)(6)]$$

$$= \frac{n}{2}[2+6n-6] = \frac{n}{2}(6n-4)$$

$$= \frac{n}{\cancel{2}}\,\cancel{2}(3n-2)$$

$$= n(3n-2)$$ Answer

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ii) Do yourself as above

Q No. 5 Since $a_r = 3r+1$

put $r=1$, $a_1 = 3(1)+1 = 4$

put $r=2$, $a_2 = 3(2)+1 = 7$

So $d = a_2 - a_1 = 7-4 = 3$

also $n = 20$, $S_n = ?$

Since $S_n = \frac{n}{2}(2a_1+(n-1)d)$

$\Rightarrow S_{20} = \frac{20}{2}(2(4)+(20-1)(3))$

$= 10(8+57) = 10(65)$

$= 650$ Answer

Q No 6. $S_n = n(2n-1)$

put $n = 1, 2, 3, 4$

$S_1 = 1(2(1)-1) = 1(2-1) = 1$

$S_2 = 2(2(2)-1) = 2(4-1) = 6$

$S_3 = 3(2(3)-1) = 3(6-1) = 15$

$S_4 = 4(2(4)-1) = 4(8-1) = 28$

Now

$a_1 = S_1 = 1$

$a_2 = S_2 - S_1 = 6-1 = 5$

$a_3 = S_3 - S_2 = 15-6 = 9$

$a_4 = S_4 - S_3 = 28-15 = 13$

Hence required series is

$1+5+9+13+\cdots\cdots$

Q No. 7

Consider a_1, a_1' are the first terms and d, d' are the common differences of two series in A.P.

Now we gave given

$S_n : S_n' = 3n+2 : n+1$

$$\Rightarrow \frac{S_n}{S_n'} = \frac{3n+2}{n+1}$$

$$\Rightarrow \frac{\frac{n}{2}(2a_1+(n-1)d)}{\frac{n}{2}(2a_1'+(n-1)d')} = \frac{3n+2}{n+1}$$

$$\Rightarrow \frac{2a_1+(n-1)d}{2a_1'+(n-1)d'} = \frac{3n+2}{n+1}$$

$$\Rightarrow \frac{\cancel{2}\{a_1+(\frac{n-1}{2})d\}}{\cancel{2}\{a_1'+(\frac{n-1}{2})d'\}} = \frac{3n+2}{n+1}$$

$$\Rightarrow \frac{a_1+(\frac{n-1}{2})d}{a_1'+(\frac{n-1}{2})d'} = \frac{3n+2}{n+1} \quad \text{——(i)}$$

For 8th term

Consider $\frac{n-1}{2} = 7$

$\Rightarrow n-1 = 14$

$\Rightarrow n = 14+1 = 15$

putting in (i)

$$\frac{a_1+7d}{a_1'+7d'} = \frac{3(15)+2}{15+1}$$

$$\Rightarrow \frac{a_8}{a_8'} = \frac{47}{16} \qquad \because a_8 = a_1+7d$$

$\Rightarrow a_8 : a_8' = 47 : 16$ or $\frac{47}{16}$

Answer

Q No. 8

Since

$S_2 = S_{2n} = \frac{2n}{2}(2a_1+(2n-1)d)$ ——(i)

$S_3 = S_{3n} = \frac{3n}{2}(2a_1+(3n-1)d)$ ——(ii)

$S_5 = S_{5n} = \frac{5n}{2}(2a_1+(5n-1)d)$ ——(iii)

Now

$R.H.S = 5(S_3 - S_2)$

$\cancel{=5\left(\frac{3n}{2}(2a_1+(3n-1)d)-\frac{2n}{2}(2a_1+\right.}$

$= 5\left(\frac{3n}{2}(2a_1+(3n-1)d) - \frac{2n}{2}(2a_1+(2n-1)d)\right)$

$= \frac{5n}{2}\left(3(2a_1+(3n-1)d) - 2(2a_1+(2n-1)d)\right)$

$= \frac{5n}{2}(6a_1+3(3n-1)d - 4a_1 - 2(2n-1)d)$

$= \frac{5n}{2}(2a_1+[3(3n-1)-2(2n-1)]d)$

$= \frac{5n}{2}(2a_1+(9n-3-4n+2)d)$

$\Rightarrow R.H.S = \frac{5n}{2}(2a_1+(5n-1)d)$

$= S_5 = L.H.S$ from (iii)

Hence $S_5 = 5(S_3 - S_2)$ proved

QNo 9 The series of integers which are neither divisible by 5 nor by 2 is

$1+3+7+9+11+13+17+19+21+23+27+29+\cdots+991+993+997+999$ (400 terms)

$(1+3+7+9)+(11+13+17+19)+(21+23+27+29)+\cdots+(991+993+997+999)$ (100 terms)

$20+60+100+\cdots+3980$ (100 terms)

here $a_1 = 20$, $d = 60-20 = 40$

$n = 100$

Since

$S_n = \frac{n}{2}(2a_1+(n-1)d)$

$\Rightarrow S_{100} = \frac{100}{2}(2(20)+(100-1)(40))$

$= 50(40+3960)$

$= 50(4000) = 200000$ Answer

QNo 10 $50S_9 = 63S_8$, $a_1 = 2$

$\Rightarrow 50\left(\frac{9}{2}(2a_1+(9-1)d)\right) = 63\left(\frac{8}{2}(2a_1+(8-1)d)\right)$

$\Rightarrow 50\left(\frac{9}{2}(2a_1+8d)\right) = 63(4(2a_1+7d))$

$\Rightarrow 225(2a_1+8d) = 252(2a_1+7d)$

$\because a_1 = 2$

$\Rightarrow 225(2(2)+8d) = 252(2(2)+7d)$

$\Rightarrow 225(4+8d) = 252(4+7d)$

$\Rightarrow 900+1800d = 1008+1764d$

$\Rightarrow 1800d - 1764d = 1008 - 900$

$\Rightarrow 36d = 108 \Rightarrow d = \frac{108}{36} = 3$

Now $S_9 = \frac{9}{2}(2a_1+(9-1)d)$

$\Rightarrow S_9 = \frac{9}{2}(2(2)+8(3))$

$= \frac{9}{2}(4+24) = \frac{9}{2}(28)$

$= 126$ Answer

QNo 11

Since $S_9 = 171$

$\Rightarrow \frac{9}{2}(2a_1+(9-1)d) = 171$

$\Rightarrow \frac{9}{2}(2a_1+8d) = 171$

$\Rightarrow \frac{9}{\cancel{2}}\cdot\cancel{2}(a_1+4d) = 171$

$\Rightarrow 9a_1+36d = 171$ ——— (i)

Now $a_8 = 31$

$\Rightarrow a_1+7d = 31$ ——— (ii)

Multiplying eq (ii) by 9 and subtracting from (i)

$$\begin{array}{r} \cancel{9a_1} + 36d = 171 \\ -\cancel{9a_1} \pm 63d = \pm 279 \\ \hline -27d = -108 \end{array}$$

$\Rightarrow d = \frac{-108}{-27} \Rightarrow \boxed{d = 4}$

put $d = 4$ in eq (ii)

$a_1 + 7(4) = 31$

$\Rightarrow a_1 + 28 = 31$

$\Rightarrow a_1 = 31-28 \Rightarrow \boxed{a_1 = 3}$

Now

$a_2 = a_1 + d = 3+4 = 7$

$a_3 = a_1 + 2d = 3+2(4) = 11$

$a_4 = a_1 + 3d = 3+3(4) = 15$

Hence the required series is

$3+7+11+15+\cdots\cdots$

Q No 12 Since

$$S_9 + S_7 = 203 \text{ ——— (i)}$$

also $S_9 - S_7 = 49$ ——— (ii)

adding (i) and (ii)

$$S_9 + \cancel{S_7} = 203$$
$$S_9 - \cancel{S_7} = 49$$
$$2S_9 = 252$$

$$\Rightarrow S_9 = 126$$

If a_1 be the first term and 'd' be the common difference then

$$\frac{9}{2}[2a_1 + (9-1)d] = 126$$

$$\Rightarrow 9[2a_1 + 8d] = 252$$

$$\Rightarrow 18a_1 + 72d = 252$$

$$\Rightarrow 18[a_1 + 4d] = 252$$

$$\Rightarrow a_1 + 4d = 14 \text{ ——— (iii)}$$

Now -ing (i) and (ii)

$$\cancel{S_9} + S_7 = 203$$
$$\underline{-\cancel{S_9} \mp S_7 = -49}$$
$$2S_7 = 154$$

$$\Rightarrow S_7 = 77$$

$$\Rightarrow \frac{7}{2}[2a_1 + (7-1)d] = 77$$

$$\Rightarrow 7[2a_1 + 6d] = 154$$

$$\Rightarrow 14a_1 + 42d = 154$$

$$\Rightarrow 14(a_1 + 3d) = 154$$

$$\Rightarrow a_1 + 3d = 11 \text{ ——— (iv)}$$

Subtracting (iii) & (iv)

$$\cancel{a_1} + 4d = 14$$
$$\underline{-\cancel{a_1} \pm 3d = -11}$$
$$d = 3$$

putting in (iii)

$$a_1 + 4(3) = 14$$

$$\Rightarrow a_1 + 12 = 14 \Rightarrow a_1 = 14 - 12$$

$$\Rightarrow a_1 = 2$$

Now

$$a_2 = a_1 + d = 2 + 3 = 5$$

$$a_3 = a_1 + 2d = 2 + 2(3) = 8$$

$$a_4 = a_1 + 3d = 2 + 3(3) = 11$$

Thus the required series is

$$2 + 5 + 8 + 11 + \cdots\cdots$$

Q No. 13

Since $\dfrac{S_9}{S_7} = \dfrac{18}{11}$

$$\Rightarrow 11 S_9 = 18 S_7$$

$$\Rightarrow 11 \cdot \frac{9}{2}[2a_1 + (9-1)d] = \overset{9}{\cancel{18}} \cdot \frac{7}{\cancel{2}}[2a_1 + (7-1)d]$$

$$\Rightarrow \frac{99}{2}[2a_1 + 8d] = 63[2a_1 + 6d]$$

$$\Rightarrow 99a_1 + 396d = 126a_1 + 378d$$

$$\Rightarrow 99a_1 - 126a_1 = 378d - 396d$$

$$\Rightarrow -27a_1 = -18d$$

$$\Rightarrow a_1 = \frac{-18}{-27}d$$

$$\Rightarrow a_1 = \frac{2}{3}d \text{ ——— (i)}$$

also $a_7 = 20$

$$\Rightarrow a_1 + 6d = 20$$

putting value of a_1 in above

$$\frac{2}{3}d + 6d = 20$$

$$\Rightarrow \frac{20}{3}d = 20$$

$$\Rightarrow d = \frac{3}{20} \cdot 20 \Rightarrow \boxed{d = 3}$$

putting in (i)

$$a_1 = \frac{2}{3}(3) \Rightarrow \boxed{a_1 = 2}$$

Now

$$a_2 = a_1 + d = 2 + 3 = 5$$

$$a_3 = a_1 + 2d = 2 + 2(3) = 8$$

$$a_4 = a_1 + 3d = 2 + 3(3) = 11$$

Thus the required series is

$$2 + 5 + 8 + 11 + \cdots\cdots$$

Q No 14 Let the number in A.P are $a-d,\ a,\ a+d$.

By given condition

$a-\cancel{d}+a+a+\cancel{d}=24$

$\Rightarrow 3a=24 \Rightarrow a=8$

also by given condition

$(a-d).a.(a+d)=440$

$\Rightarrow a(a^2-d^2)=440$

putting $a=8$ in above

$8((8)^2-d^2)=440$

$\Rightarrow 8(64-d^2)=440$

$\Rightarrow 512-8d^2=440$

$\Rightarrow 512-440=8d^2$

$\Rightarrow 8d^2=72 \Rightarrow d^2=\frac{72}{8}$

$\Rightarrow d^2=9 \Rightarrow d=\pm 3$

When $a=8,\ d=3$

$a-d=8-3=5$

$a=8$

$a+d=8+3=11$

When $a=8\ \ d=-3$

$a-d=8-(-3)=8+3=11$

$a=8$

$a+d=8+(-3)=8-3=5$

hence 5, 8, 11 OR 11, 8, 5 are the required number

Q No 15 Consider four numbers $a-3d,\ a-d,\ a+d,\ a+3d$ are in A.P then

$a-\cancel{3d}+a-\cancel{d}+a+\cancel{d}+a+\cancel{3d}=32$

$\Rightarrow 4a=32 \Rightarrow \boxed{a=8}$

also

$(a-3d)^2+(a-d)^2+(a+d)^2+(a+3d)^2=276$

$\Rightarrow a^2-\cancel{6ad}+9d^2+a^2-\cancel{2ad}+d^2$

$+a^2+\cancel{2ad}+d^2+a^2+\cancel{6ad}+9d^2=276$

$\Rightarrow 4a^2+20d^2=276$

$4(8)^2+20d^2=276$

$\Rightarrow 256+20d^2=276$

$\Rightarrow 20d^2=276-256$

$\Rightarrow 20d^2=20$

$\Rightarrow d^2=1 \Rightarrow d=\pm 1$

When $a=8,\ d=1$

$a-3d=8-3(1)=8-3=5$

$a-d=8-(1)=8-1=7$

$a+d=8+(1)=8+1=9$

$a+3d=8+3(1)=8+3=11$

When $a=8,\ d=-1$

$a-3d=8-3(-1)=8+3=11$

$a-d=8-(-1)=8+1=9$

$a+d=8+(-1)=8-1=7$

$a+3d=8+3(-1)=8-3=5$

hence 5, 7, 9, 11 OR 11, 9, 7, 5 are required number

Q No. 16

Do yourself.

Consider $a-2d,\ a-d,\ a,\ a+d$ and $a+2d$ as five number in A.P.

Q No 17

Since $a_6+a_8=40$

$\Rightarrow a_1+5d+a_1+7d=40$

$\Rightarrow 2a_1+12d=40$

$\Rightarrow 2(a_1+6d)=40$

$\Rightarrow a_1+6d=20$ ——— (i)

Also $a_4 . a_7=220$

$\Rightarrow (a_1+3d)(a_1+6d)=220$

$\Rightarrow (a_1+3d)(20)=220$ From (i)

$\Rightarrow a_1+3d=\frac{\cancel{220}}{\cancel{20}}$

Subtracting (i) & (ii)

$$\cancel{a_1} + 6d = 20$$
$$\underline{-\cancel{a_1} \pm 3d = -11}$$
$$3d = 9 \Rightarrow d = 3$$

putting in (ii)

$$a_1 + 3(3) = 11$$
$$a_1 + 9 = 11 \Rightarrow a_1 = 11 - 9$$
$$\Rightarrow a_1 = 2$$

Now

$$a_2 = a_1 + d = 2 + 3 = 5$$
$$a_3 = a_1 + 2d = 2 + 2(3) = 8$$
$$a_4 = a_1 + 3d = 2 + 3(3) = 11$$

Thus the required A.P is

$2, 5, 8, 11, \ldots\ldots$

Q No. 18

Since a^2, b^2, c^2 are in A.P

therefore

$$b^2 - a^2 = c^2 - b^2$$
$$\Rightarrow (b-a)(b+a) = (c-b)(c+b) \text{ --- (i)}$$

Now to show $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P Consider

$$d = \frac{1}{c+a} - \frac{1}{b+c}$$
$$= \frac{b+c-c-a}{(c+a)(b+c)}$$
$$\Rightarrow d = \frac{b-a}{(c+a)(b+c)} \text{ --- (ii)}$$

Also

$$d = \frac{1}{a+b} - \frac{1}{c+a}$$
$$= \frac{c+a-a-b}{(a+b)(c+a)}$$
$$= \frac{c-b}{(a+b)(c+a)}$$

From eq (i) $\frac{(b-a)(b+a)}{(c+b)} = c - b$

$$d = \frac{\frac{(b-a)(b+a)}{(c+b)}}{(a+b)(c+a)}$$
$$= \frac{(b-a)(b\cancel{+a})}{(c+b)(\cancel{a+b})(c+a)}$$
$$= \frac{b-a}{(c+b)(c+a)}$$
$$\Rightarrow d = \frac{b-a}{(c+a)(b+c)} \text{ --- (iii)}$$

From (ii) and (iii)

$$d = d$$

Hence $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$

are in A.P

Q No. 8 (Ex 6.3)

Let $A_1, A_2, A_3, \ldots, A_n$ be the n A.Ms between a & b then $a, A_1, A_2, A_3, \ldots, A_n, b$ are in A.P.

Here $a_1 = a$

$$a_{n+2} = b$$
$$\Rightarrow a_1 + (n+2-1)d = b$$
$$\Rightarrow a_1 + (n+1)d = b$$
$$\Rightarrow (n+1)d = b - a$$
$$\Rightarrow d = \frac{b-a}{n+1}$$

Now

$$A_1 = a_2 = a_1 + d = a + \frac{b-a}{n+1}$$
$$A_2 = a_3 = a_1 + 2d = a + 2\left(\frac{b-a}{n+1}\right)$$
$$A_3 = a_4 = a_1 + 3d = a + 3\left(\frac{b-a}{n+1}\right)$$
$$\vdots$$
$$A_n = a_{n+1} = a_1 + nd = a + n\left(\frac{b-a}{n+1}\right)$$

P.T.O

Now Sum of n A.Ms

$$= A_1 + A_2 + A_3 + \cdots + A_n$$

$$= a + \frac{b-a}{n+1} + a + 2\left(\frac{b-a}{n+1}\right) + a + 3\left(\frac{b-a}{n+1}\right) + \cdots + a + n\left(\frac{b-a}{n+1}\right)$$

$$= (a+a+a+\cdots+a) + \frac{b-a}{n+1} + 2\left(\frac{b-a}{n+1}\right) + 3\left(\frac{b-a}{n+1}\right) + \cdots + n\left(\frac{b-a}{n+1}\right)$$

$$= na + \frac{b-a}{n+1}(1+2+3+\cdots+n)$$

$a_1 = 1,\ d = 2-1 = 1,\ n = n$

$$= na + \frac{b-a}{n+1}\left[\frac{n}{2}\left(2(1) + (n-1)(1)\right)\right]$$

$$= na + \frac{b-a}{n+1}\left(\frac{n}{2}(2+n-1)\right)$$

$$= na + \frac{b-a}{\cancel{(n+1)}}\left[\frac{n}{2}\cancel{(n+1)}\right]$$

$$= na + (b-a)\left(\frac{n}{2}\right)$$

$$= n\left(a + \frac{b-a}{2}\right)$$

$$= n\left[\frac{2a+b-a}{2}\right]$$

$$= n\left(\frac{a+b}{2}\right)$$

$$= n(\text{A.M. between } a \,\&\, b)$$

Hence sum of n A.Ms between a & b is n times their A.Ms.

proved

End.