Exercise 6.2

Q No.1 i) $a_1 = 5$ and other three consecutive terms are 23, 26, 29

Since $a_1 = 5$ & $d = 26 - 23 = 3$

Now $a_2 = a_1 + d = 5 + 3 = 8$

$a_3 = a_2 + d = 8 + 3 = 11$

$a_4 = a_3 + d = 11 + 3 = 14$

hence 5, 8, 11, 14 are first four terms of A.P

Q No.1 (ii) $a_5 = 17$ and $a_9 = 37$

Consider a_1 be the first term and 'd' be the common difference

Since $a_5 = 17$

$\Rightarrow a_1 + (5-1)d = 17$

$\Rightarrow a_1 + 4d = 17$ —— (i)

also $a_9 = 37$

$\Rightarrow a_1 + (9-1)d = 37$

$\Rightarrow a_1 + 8d = 37$ —— (ii)

Subtracting (i) and (ii)

$$\begin{array}{r} \cancel{a_1} + 4d = 17 \\ \underline{\cancel{a_1} + 8d = 37} \\ -4d = -20 \end{array}$$

$\Rightarrow d = 5$

putting value of d in (i)

$a_1 + 4(5) = 17$

$\Rightarrow a_1 + 20 = 17$

$\Rightarrow a_1 = 17 - 20$

$\Rightarrow a_1 = -3$

So $a_2 = a_1 + d = -3 + 5 = 2$

$a_3 = a_2 + d = 2 + 5 = 7$

$a_4 = a_3 + d = 7 + 5 = 12$

hence −3, 2, 7, 12 are first four terms of A.P.

iii) $3a_7 = 7a_4$ & $a_{10} = 33$

Suppose a_1 be the first term and d be the common difference

Since $3a_7 = 7a_4$

$\Rightarrow 3(a_1 + 6d) = 7(a_1 + 3d)$

$\Rightarrow 3a_1 + 18d = 7a_1 + 21d$

$\Rightarrow 3a_1 + 18d - 7a_1 - 21d = 0$

$\Rightarrow -4a_1 - 3d = 0$

$\Rightarrow 4a_1 + 3d = 0$ —— (i)

also $a_{10} = 33$

$\Rightarrow a_1 + 9d = 33$ —— (ii)

xing eq (ii) by 4 & subtracting from (i)

$$\begin{array}{r} \cancel{4a_1} + 3d = 0 \\ \underline{\cancel{4a_1} + 36d = 132} \\ -33d = -132 \end{array}$$

$\Rightarrow d = \frac{-132}{-33} = 4$

putting value of d in (ii)

$a_1 + 9(4) = 33$

$\Rightarrow a_1 + 36 = 33$

$\Rightarrow a_1 = 33 - 36 \Rightarrow a_1 = -3$

So $a_2 = a_1 + d = -3 + 4 = 1$

$a_3 = a_2 + d = 1 + 4 = 5$

$a_4 = a_3 + d = 5 + 4 = 9$

hence −3, 1, 5, 9 are the first four terms of A.P.

Q No.2 $a_{n-3} = 2n - 5$

$\Rightarrow a_{n-3} = 2n - 6 + 1$

$= 2(n-3) + 1$

Replacing n−3 by n

$a_n = 2n + 1$

Answer

Q No 3 Suppose a_1 be the first term and d be common difference of A.P

Since $a_5 = 16$

$\Rightarrow a_1 + 4d = 16$ —— (i)

also $a_{20} = 46$

$\Rightarrow a_1 + 19d = 46$ —— (ii)

Subtracting (i) & (ii)

$\cancel{a_1} + 4d = 16$

$\cancel{a_1} + 19d = 46$

$- \quad - \quad -$

$-15d = -30$

$\Rightarrow d = 2$

putting value of d in (i)

$a_1 + 4(2) = 16$

$\Rightarrow a_1 + 8 = 16$

$\Rightarrow a_1 = 16 - 8 \Rightarrow a_1 = 8$

Now

$a_{12} = a_1 + 11d$

$= 8 + 11(2)$

$= 8 + 22 = 30$ Answer

Q No 4

$x, 1, 2-x, 3-2x, \ldots\ldots$

here $a_1 = 1$

and $d = a_2 - a_1$

$= 1 - x$

Since $a_{13} = a_1 + 12d$

$= x + 12(1-x)$

$= x + 12 - 12x$

$\Rightarrow a_{13} = 12 - 11x$ Answer

Q No 5 Same as Q No 3

Q No 6

$5, 2, -1, \ldots\ldots$ is, -85

here $a_1 = 5$

$d = a_2 - a_1 = 2 - 5 = -3$

$a_n = -85$, $n = ?$

Since

$a_n = a_1 + (n-1)d$

$\Rightarrow -85 = 5 + (n-1)(-3)$

$\Rightarrow -85 = 5 - 3n + 3$

$\Rightarrow 3n = 5 + 3 + 85$

$\Rightarrow 3n = 93$

$\Rightarrow \boxed{n = 31}$ Answer

Q No 7 Same as above

Q No 8 $a_1 = 11$, $a_n = 68$

$d = 3$, $n = ?$

Since $a_n = a_1 + (n-1)d$

$\Rightarrow 68 = 11 + (n-1)3$

Now solve yourself as above

Q No 9

Since $a_n = 3n - 1$

put $n = 1$

$a_1 = 3(1) - 1 = 3 - 1 = 2$

put $n = 2$

$a_2 = 3(2) - 1 = 6 - 1 = 5$

put $n = 3$

$a_3 = 3(3) - 1 = 9 - 1 = 8$

put $n = 4$

$a_4 = 3(4) - 1 = 12 - 1 = 11$

Thus

$2, 5, 8, 11, \ldots\ldots$

is the required A.P

Q No. 10 $17, 13, 9, \ldots\ldots$

$a_1 = 17$, $d = 13 - 17 = -4$

i) Suppose -19 be the nth term of A.P. i.e $a_n = -19$

Since $a_n = a_1 + (n-1)d$

$\Rightarrow -19 = 17 + (n-1)(-4)$

$\Rightarrow -19 = 17 - 4n + 4$

$\Rightarrow 4n = 17 + 4 + 19$

$= 40$

$\Rightarrow n = 10$

Thus -19 is the 10th term of A.P.

ii) Suppose 2 be the nth term of A.P i.e $a_n = 2$

Since $a_n = a_1 + (n-1)d$

$\Rightarrow 2 = 17 + (n-1)(-4)$

$\Rightarrow 2 = 17 - 4n + 4$

$\Rightarrow 4n = 17 + 4 - 2$

$= 19$

$\Rightarrow n = \frac{19}{4}$ which is a rational. therefore 2 is not the term of A.P

Q No. 11.

Let a_1 be the first term and d be the common difference

now $a_p = l$:

$\Rightarrow a_1 + (p-1)d = l$

$a_q = m$

$\Rightarrow a_1 + (q-1)d = m$

$a_r = n$

$\Rightarrow a_1 + (r-1)d = n$

i) $L.H.S = l(q-r) + m(r-p) + n(p-q)$

$= [a_1 + (p-1)d](q-r) + [a_1 + (q-1)d](r-p)$
$+ [a_1 + (r-1)d](p-q)$

$= (a_1 + pd - d)(q-r) + (a_1 + qd - d)(r-p)$
$+ (a_1 + rd - d)(p-q)$

$= \cancel{a_1q} + \cancel{pqd} - \cancel{qd} - \cancel{a_1r} - \cancel{prd} + \cancel{rd}$
$+ \cancel{a_1r} + \cancel{qrd} - \cancel{qr} - \cancel{a_1p} - \cancel{pqd} + pq$
$+ \cancel{a_1p} + \cancel{prd} - pd - \cancel{a_1q} - \cancel{qrd} + \cancel{qd}$

$= 0 = R.H.S$ proved

ii) $L.H.S = p(m-n) + q(n-l) + r(l-m)$

$= p[\cancel{a_1} + (q-1)d - \cancel{a_1} - (r-1)d]$
$+ q[\cancel{a_1} + (r-1)d - \cancel{a_1} - (p-1)d]$
$+ r[\cancel{a_1} + (p-1)d - \cancel{a_1} - (q-1)d]$

$= p[qd - \cancel{d} - rd + \cancel{d}]$
$+ q[rd - \cancel{d} - pd + \cancel{d}]$
$+ r[pd - \cancel{d} - qd + \cancel{d}]$

$= pqd - prd + qrd - pqd$
$+ prd - qrd = 0 = R.H.S$

proved

Q No. 12

$\left(\frac{4}{3}\right)^2, \left(\frac{7}{3}\right)^2, \left(\frac{10}{3}\right)^2, \ldots\ldots$

We first find the nth term of $4, 7, 10, \ldots\ldots$

$a_1 = 4$, $d = 7 - 4 = 3$

So $a_n = a_1 + (n-1)d$

$= 4 + (n-1)3$

$= 4 + 3n - 3 = 3n + 1$

hence nth term of given sequence is $\left(\frac{3n+1}{3}\right)^2$

P.T.O

Q No. 13 Since $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P. therefore

$$d = \frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$$

$$\Rightarrow \frac{1}{b} + \frac{1}{b} = \frac{1}{a} + \frac{1}{c}$$

$$\Rightarrow \frac{1+1}{b} = \frac{c+a}{ac}$$

$$\Rightarrow \frac{2}{b} = \frac{a+c}{ac}$$

$$\Rightarrow \frac{b}{2} = \frac{ac}{a+c}$$

$$\Rightarrow b = \frac{2ac}{a+c}$$ proved

Q No 14 Since $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P therefore

$$d = \frac{1}{b} - \frac{1}{a} \quad \text{---(i)}$$

also

$$d = \frac{1}{c} - \frac{1}{b} \quad \text{---(ii)}$$

Comparing (i) and (ii)

$$\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$$

$$\Rightarrow \frac{1}{b} + \frac{1}{b} = \frac{1}{c} + \frac{1}{a}$$

$$\Rightarrow \frac{1+1}{b} = \frac{a+c}{ac}$$

$$\Rightarrow \frac{2}{b} = \frac{a+c}{ac}$$

$$\Rightarrow \frac{b}{2} = \frac{ac}{a+c}$$

$$\Rightarrow b = \frac{2ac}{a+c}$$

putting value of b in eq (i)

$$d = \frac{1}{\frac{2ac}{a+c}} - \frac{1}{a} = \frac{a+c}{2ac} - \frac{1}{a}$$

$$= \frac{a+c-2c}{2ac} = \frac{a-c}{2ac}$$

Hence the common difference is $\frac{a-c}{2ac}$

END.