Exercise 3.4

Q1. Use log tables to find the values of

i) 0.8176×13.64

Sol: Let $x = 0.8176 \times 13.64$

Taking log of both sides

$\log x = \log\ 0.8176 \times 13.64$

$\log x = \log 0.8176 + \log 13.64$

$= \bar{1}.9125 + 1.1348$

$= -1 + 0.9125 + 1.1348$

$\log x = 1.0473$

Characteristics = 1

Mantissa = .0473

x = antilog 1.0473 = 11.15

ii) $(789.5)^{\frac{1}{8}}$

Sol: Let $x = (789.5)^{\frac{1}{8}}$

Taking log of both sides

$\log x = \log(789.5)^{\frac{1}{8}}$

$= \frac{1}{8}\log(789.5)$

$= \frac{1}{8}(2.8974)$

$\log x = 0.3622$

Characteristics = 0

Mantissa = .3622

x = antilog 0.3622 = 2.302

iii) $\dfrac{0.678 \times 9.01}{0.0234}$

Let $x = \dfrac{0.678 \times 9.01}{0.0234}$

Taking log of both sides

$\log x = \log \dfrac{0.678 \times 9.01}{0.0234}$

$= \log 0.678 + \log 9.01 - \log 0.0234$

$= \bar{1}.8312 + 0.9547 - (\bar{2}.3692)$

$= -1 + 0.8312 + 0.9547 - (-2 + 0.3692)$

$= -1 + 0.8312 + 0.9547 + 2 - 0.3692$

$\log x = 2.4167$

Characteristics = 2

Mantissa = .4167

x = antilog 2.4167 = 261.0

iv) $\sqrt[5]{2.709} \times \sqrt[7]{1.239}$

Sol: Let $x = \sqrt[5]{2.709} \times \sqrt[7]{1.239}$

Taking log of both sides

$\log x = \log(2.709)^{\frac{1}{5}} \times (1.239)^{\frac{1}{7}}$

$= \log(2.709)^{\frac{1}{5}} + \log(1.239)^{\frac{1}{7}}$

$= \frac{1}{5}\log(2.709) + \frac{1}{7}\log(1.239)$

$= \frac{1}{5}(0.4328) + \frac{1}{7}(0.0931)$

$= 0.0866 + 0.0133$

$\log x = 0.0999$

Characteristics = 0

Mantissa = .0999

x = antilog 0.0999

$x = 1.259$

v) $\dfrac{(1.23)(0.6975)}{(0.0075)(1278)}$

Sol: Let $x = \dfrac{(1.23)(0.6975)}{(0.0075)(1278)}$

Taking log of both sides

$\log x = \log \dfrac{(1.23)(0.6975)}{(0.0075)(1278)}$

$= \log 1.23 + \log 0.6975 - \log 0.0075 - \log 1278$

$= 0.0899 + \bar{1}.8435 - \bar{3}.8751 - 3.1065$

$= 0.0899 - 1 + 0.8435 + 3 - 0.8751 - 3.1065$

$\log x = -1.0482$

$= -2 + 2 - 1.0482$

$= -2 + 0.9518$

$\log x = \bar{2}.9518$

Characteristics = $\bar{2}$

Mantissa = .9518

x = antilog $\bar{2}.9518$ = 0.0895

vi) $\sqrt[3]{\dfrac{0.7214 \times 20.37}{60.8}}$

Let $x = \sqrt[3]{\dfrac{0.7214 \times 20.37}{60.8}}$

$x = \left(\dfrac{0.7214 \times 20.37}{60.8}\right)^{\frac{1}{3}}$

Taking log of both sides

$\log x = \log\left(\dfrac{0.7214 \times 20.37}{60.8}\right)^{\frac{1}{3}}$

$= \dfrac{1}{3}\log\left(\dfrac{0.7214 \times 20.37}{60.8}\right)$

$= \dfrac{1}{3}(\log 0.7214 + \log 20.37 - \log 60.8)$

$= \dfrac{1}{3}(\bar{1}.8582 + 1.3090 - 1.7839)$

$= \dfrac{1}{3}(-1 + 0.8582 + 1.3090 - 1.7839)$

$= \dfrac{1}{3}(-0.6167)$

$\log x = -0.2056$

$= -1 + 1 - 0.2056$

$= -1 + 0.7944$

$\log x = \bar{1}.7944$

Characteristics = $\bar{1}$

Mantissa = .7944

x = antilog $\bar{1}.7944$

= 0.6229

vii) $\dfrac{83 \times \sqrt[3]{92}}{127 \times \sqrt[5]{246}}$

Sol: Let $x = \dfrac{83 \times \sqrt[3]{92}}{127 \times \sqrt[5]{246}}$

$x = \dfrac{83 \times (92)^{\frac{1}{3}}}{127 \times (246)^{\frac{1}{5}}}$

Taking log of both sides

$\log x = \log \dfrac{83 \times (92)^{\frac{1}{3}}}{127 \times (246)^{\frac{1}{5}}}$

$= \log 83 + \log (92)^{\frac{1}{3}} - \log 127 - \log (246)^{\frac{1}{5}}$

$= \log 83 + \dfrac{1}{3}\log (92) - \log 127 - \dfrac{1}{5}\log (246)$

$= 1.9191 + \dfrac{1}{3}(1.9638) - 2.1038 - \dfrac{1}{5}(2.391)$

$= 1.9191 + 0.6546 - 2.1038 - 0.4782$

$\log x = -0.0083$

$= -1 + 1 - 0.0083$

$= -1 + 0.9917$

$\log x = \bar{1}.9917$

Characteristics = $\bar{1}$

Mantissa = .9917

x = antilog $\bar{1}.9917$ = 0.9811

viii) $\dfrac{(438)^3 \sqrt{0.056}}{(388)^4}$

Sol: Let $x = \dfrac{(438)^3 \sqrt{0.056}}{(388)^4}$

$x = \dfrac{(438)^3 \times (0.056)^{\frac{1}{2}}}{(388)^4}$

Taking log of both sides

$$\log x = \log \frac{(438)^3 \times (0.056)^{\frac{1}{2}}}{(388)^4}$$

$$= \log(438)^3 + \log(0.056)^{\frac{1}{2}} - \log(388)^4$$

$$= 3\log(438) + \frac{1}{2}\log(0.056) - 4\log(388)$$

$$= 3(2.6415) + \frac{1}{2}(\bar{2}.7482) - 4(2.5888)$$

$$= 3(2.6415) + \frac{1}{2}(-2 + 0.7482) - 4(2.5888)$$

$$= 7.9245 + \frac{1}{2}(-1.2518) - 10.3552$$

$$= 7.9245 - 0.6259 - 10.3552$$

$$\log x = -3.0566$$

$$= -4 + 4 - 3.0566$$

$$= -4 + 0.9434$$

$$\log x = \bar{4}.9434$$

Characteristic = $\bar{4}$

Mantissa = .9434

x = antilog $\bar{4}.9434$ = 0.0008778

Q2. A gas is expanding according to the law $PV^n = C$. Find C when P=80, V=3.1 and $n = \frac{5}{4}$.

Sol: $PV^n = C$

Taking log of both sides:

$$\log PV^n = \log C$$

$$\log P + \log V^n = \log C$$

$$\log C = \log P + n \log V$$

Putting $P = 80$, $V = 3.1$ and $n = \frac{5}{4}$

$$\log C = \log 80 + \frac{5}{4}\log 3.1$$

$$= 1.9031 + \frac{5}{4}(0.4914)$$

$$= 1.9031 + 0.6143$$

$$\log C = 2.5174$$

Characteristic = 2

Mantissa = .5174

C = antilog 2.5174

C = 329.2 unit

Q3. The formula $p = 90(5)^{-\frac{q}{10}}$ applies to the demand of a product, where 'q' is the number of units and p is the price of one unit. How many units will be demanded if the price is Rs. 18.00?

Sol: $p = 90(5)^{-\frac{q}{10}}$

$q = ?$ and $p = Rs.\ 18.00$

As $p = 90(5)^{-\frac{q}{10}}$

$$18 = 90(5)^{\frac{-q}{10}}$$

Taking log of both sides

$$\log 18 = \log 90(5)^{-\frac{q}{10}}$$

$$\log 18 = \log 90 + \log(5)^{-\frac{q}{10}}$$

$$\log 18 - \log 90 = \frac{-q}{10}\log 5$$

$$10(\log 18 - \log 90) = -q \log 5$$

$$10(1.2553 - 1.9542) = -q(0.6990)$$

$$-6.989 = -q(0.6990)$$

$\Rightarrow$ $q(0.6990) = 6.989$

$$q = \frac{6.989}{0.6990}$$

$$q = 9.998$$

$q = 10$ approximately

So 10 units will be demanded

OR

$$p = 90\ (5)^{-q/10}$$

Taking log of both sides

$$\log p = \log 90\ (5)^{-q/10}$$

$$\log p = \log 90 + \log\ (5)^{-q/10}$$

$$\log p = \log 90 - \frac{q}{10} \log 5$$

$$\frac{q}{10} \log 5 = \log 90 - \log p$$

$$\frac{q}{10} \log 5 = \log 90 - \log 18$$

$$\frac{q}{10} \log 5 = \log \frac{90}{18}$$

$$\frac{q}{10} \log 5 = \log 5$$

$$\frac{q}{10} = \frac{\log 5}{\log 5}$$

$$\frac{q}{10} = 1$$

$q = 10$ Units

Q4. **If** $A = \pi r^2$

$$\pi = \frac{22}{7},\ r = 15,\ A = ?$$

As $A = \pi r^2$

Taking log of both sides

$$\log A = \log \pi r^2$$

$$= \log \pi + \log r^2$$

$$= \log \pi + 2 \log r$$

$$= \log \frac{22}{7} + 2 \log 15$$

$$= \log 22 - \log 7 + 2 \log 15$$

$$= 1.3424 - 0.8451 + 2(1.1761)$$

$$= 1.3424 - 0.8451 + 2.3522$$

$$\log A = 2.8495$$

Characteristics = 2

Mantissa = .8495

A = antilog 2.8495

A = 707.1

Q5. **If** $v = \frac{1}{3}\pi r^2 h$, **find** v **when** $\pi = \frac{22}{7}, r = 2.5$ **and** $h = 4.2$

Sol: $v = \frac{1}{3}\pi r^2 h$

$\pi = \frac{22}{7}$, $r = 2.5$ and $h = 4.2$, $v = ?$

As $v = \frac{1}{3}\pi r^2 h$

Taking log of both sides

$$\log v = \log \frac{1}{3}\pi r^2 h$$

$$= \log \frac{1}{3} + \log \pi + \log r^2 + \log h$$

$$= \log \frac{1}{3} + \log \frac{22}{7} + 2 \log r + \log h$$

$$= \log 1 - \log 3 + \log 22 - \log 7 + 2 \log 2.5 + \log 4.2$$

$$= 0 - 0.4771 + 1.3424 - 0.8451 + 2(0.3979) + 0.6232$$

$$\log v = 1.4392$$

Characteristics = 1

Mantissa = .4392

v = antilog 1.4392

v = 27.49