$$y = \frac{56}{8}$$

$$y = 7$$

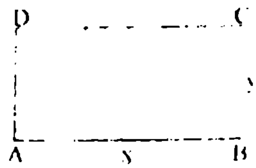
put $\quad y = 7$ in (A)

$$x - 7 = 4$$

$$x = 4 + 7 = 11$$

Integers are: 11 and 7

10. Find the dimensions of a rectangle, whose perimeter is 80cm and its area is $375cm^2$.



Solution:

Let x and y be the length and width respectively of the rectangle.
According to the given conditions.

Perimeter: $\quad 2x + 2y = 80$

or $\quad x + y = 40$ ....(A)

Area: $\quad xy = 375$ ....(B)

$$x = 40 - y \text{ (from A)}$$

Putting $\quad x = 40 - y$ in (B)

$$(40 - y)y = 375$$

$$40y - y^2 = 375$$

$$\Rightarrow \quad y^2 - 40y + 375 = 0$$

$$y^2 - 25y - 15y + 375 = 0$$

$$y(y - 25) - 15(y - 25) = 0$$

$$(y - 15)(y - 25) = 0$$

$y - 15 = 0 \quad$ gives $\quad y = 15$

Putting $\quad y = 15$ in (A)

$$x + 15 = 40$$

$$x = 40 - 15 = 25$$

Length = 25cm. Breadth = 15 cm.

SOLVED MISCELLANEOUS EXERCISE - 2

1. Multiple Choice Questions:
Four possible answers are given for the following questions. Tick (✓) the correct answer.

(i) If α, β are the roots of $3x^2 + 5x - 2 = 0$, then $\alpha + \beta$ is

(a) $\frac{5}{3}$ $\quad$ (b) $\frac{3}{5}$ $\quad$ (c) $\frac{-5}{3}$ $\quad$ (d) $\frac{-2}{3}$

(ii) If α, β are the roots of $7x^2 - x + 4 = 0$, then $\alpha\beta$ is

(a) $\frac{-1}{7}$ $\quad$ (b) $\frac{4}{7}$ $\quad$ (c) $\frac{7}{4}$ $\quad$ (d) $\frac{-4}{7}$

(iii) Roots of the equation $4x^2 - 5x + 2 = 0$ are

(a) irrational (b) imaginary (c) rational (d) none of these

(iv) Cube roots of −1 are
(a) $-1, -\omega, -\omega^2$ (b) $-1, \omega, -\omega^2$ (c) $-1, -\omega, \omega^2$ (d) $1, -\omega, -\omega^2$

(v) Sum of the cube roots of unity is
(a) 0 (b) 1 (c) −1 (d) 3

(vi) Product of cube roots of unity is
(a) 0 (b) 1 (c) −1 (d) 3

(vii) If $b^2 - 4ac < 0$, then the roots of $ax^2 + bx + c = 0$ are
(a) irrational (b) rational (c) imaginary (d) none of these

(viii) If $b^2 - 4ac > 0$, but not a perfect square then roots of $ax^2 + bx + c = 0$ are
(a) irrational (b) rational (c) imaginary (d) none of these

(ix) $\frac{1}{\alpha} + \frac{1}{\beta}$ is equal to
(a) $\frac{1}{\alpha}$ (b) $\frac{1}{\alpha} - \frac{1}{\beta}$ (c) $\frac{\alpha-\beta}{\alpha\beta}$ (d) $\frac{\alpha+\beta}{\alpha\beta}$

(x) $\alpha^2 + \beta^2$ is equal to
(a) $\alpha^2 - \beta^2$ (b) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$
(c) $(\alpha + \beta)^2 - 2\alpha\beta$ (d) $\alpha + \beta$

(xi) Two square roots of unity are
(a) $1, -1$ (b) $1, \omega$ (c) $1, -\omega$ (d) ω, ω^2

(xii) Roots of the equation $4x^2 - 4x + 1 = 0$ are
(a) real, equal (b) real, unequal (c) imaginary (d) irrational

(xiii) If α, β are the roots of $px^2 + qx + r = 0$, then sum of the roots 2α and 2β is
(a) $\frac{-q}{p}$ (b) $\frac{r}{p}$ (c) $\frac{-2q}{p}$ (d) $\frac{q}{-2p}$

(xiv) If α, β are the roots of $x^2 - x - 1 = 0$, then product of the roots 2α and 2β is
(a) −2 (b) 2 (c) 4 (d) −4

(xv) The nature of the roots of equation $ax^2 + bx + c = 0$ is determined by
(a) sum of the roots (b) product of the roots
(c) synthetic division (d) discriminant

(xvi) The discriminant of $ax^2 + bx + c = 0$ is
(a) $b^2 - 4ac$ (b) $b^2 + 4ac$ (c) $-b^2 + 4ac$ (d) $-b^2 - 4ac$

Answers:

(i)	c	(ii)	b	(iii)	b	(iv)	a	(v)	a
(vi)	b	(vii)	c	(viii)	d	(ix)	d	(x)	c
(xi)	a	(xii)	a	(xiii)	c	(xiv)	d	(xv)	d

(xvi)	a									

2. Write short answers of the following questions.

(i) Discuss the nature of the roots of the following equations.

(a) $x^2 + 3x + 5 = 0$ **(b)** $2x^2 - 7x + 3 = 0$

(c) $x^2 + 6x - 1 = 0$ **(d)** $16x^2 - 8x + 1 = 0$

Solution:

(a) $x^2 + 3x + 5 = 0$

Here, $a = 1, b = 3, c = 5$

$$\begin{aligned} \text{Disc.} &= b^2 - 4ac \\ &= (3)^2 - 4(1)(5) \\ &= 9 - 20 \\ &= -11 \end{aligned}$$

Disc. = is negative, therefore, roots are imaginary.

(b) $2x^2 + 7x + 3 = 0$

Here, $a = 2, b = -7, c = 3$

$$\begin{aligned} \text{Disc.} &= b^2 - 4ac \\ &= (-7)^2 - 4(2)(3) \\ &= 49 - 24 \\ &= 25 \end{aligned}$$

Disc. = is a perfect square, therefore, roots are real, rational and unequal.

(c) $x^2 + 6x - 1 = 0$

Here, $a = 1, b = 6, c = -1$

$$\begin{aligned} \text{Disc.} &= b^2 - 4ac \\ &= (6)^2 - 4(1)(-1) \\ &= 36 - 4 \\ &= 40 \qquad (+\text{ ve}) \end{aligned}$$

Roots are real, irrational, unequal.

(d) $16x^2 + 8x + 1 = 0$

Here, $a = 16, b = -8, c = 1$

$$\begin{aligned} \text{Disc.} &= b^2 - 4ac \\ &= (-8)^2 - 4(16)(1) \\ &= 64 - 64 = 0 \end{aligned}$$

Disc. = is zero, therefore, roots are real, rational and equal.

(ii) Find ω^2, if $\omega = \dfrac{-1+\sqrt{-3}}{2}$

Solution:

$$\omega = \frac{-1+\sqrt{-3}}{2}$$

$$\omega^2 = \left(\frac{-1+\sqrt{-3}}{2}\right)^2$$

$$\omega^2 = \frac{1+\left(\sqrt{-3}\right)^2 - 2\left(\sqrt{-3}\right)}{4}$$

$$\omega^2 = \frac{1-3-2\sqrt{-3}}{4}$$

$$\omega^2 = \frac{-2-2\sqrt{-3}}{4}$$

$$\omega^2 = \frac{2\left(-1-\sqrt{-3}\right)}{4}$$

$$\omega^2 = \frac{-1-\sqrt{-3}}{2}$$

(iii) Prove that the sum of the all cube roots of unity is zero.

Solution:

We know that cube roots of unity are:

$$1, \frac{-1+\sqrt{-3}}{2}, \frac{-1-\sqrt{-3}}{2}$$

Now, $1+\frac{-1+\sqrt{-3}}{2}+\frac{-1-\sqrt{-3}}{2}$

$$= \frac{2-1+\sqrt{-3}-1-\sqrt{-3}}{2}$$

$$= \frac{2-2}{2}$$

$$= \frac{0}{2} = 0 \qquad \text{proved.}$$

(iv) Find the product of complex cube roots of unity.

Solution:

Roots are: $1, \frac{-1+\sqrt{-3}}{2}, \frac{-1-\sqrt{-3}}{2}$

Product = $1, \left(\frac{-1+\sqrt{-3}}{2}\right), \left(\frac{-1-\sqrt{-3}}{2}\right)$

$$=\frac{(-1)^2-\left(\sqrt{-3}\right)^2}{4}$$
$$=\frac{1-(-3)}{4}$$
$$=\frac{1+3}{4}=\frac{4}{4}=1$$

(v) Show that $x^3+y^3=(x+y)(x+\omega y)(x+\omega^2 y)$

Solution:

R.H.S
$$=(x+y)(x+\omega y)(x+\omega^2 y)$$
$$=(x+y)[x^2+\omega^2 xy+\omega xy+\omega^3 y^2]$$
$$=(x+y)[x^2+(\omega^2+\omega)xy+(1)y^2]$$
$$=(x+y)[x^2+(-1)xy+y^2]$$
$$=(x+y)(x^2-xy+y^2)$$
$$=x^3-x^2y+xy^2+x^2y-xy^2+y^3$$
$$=x^3+y^3=\text{L.H.S}$$

(vi) Evaluate: $\omega^{37}+\omega^{38}+1$

Solution:

$$\omega^{37}+\omega^{38}+1$$
$$=(\omega^3)^{12}\omega+(\omega^3)^{12}\omega^2+1$$
$$=(1)^{12}\omega+(1)^{12}\omega^2+1$$
$$=(1)\omega+(1)\omega^2+1$$
$$=\omega+\omega^2+1$$
$$=1+\omega+\omega^2=0$$

(vii) Evaluate: $(1-\omega-\omega^2)^6$

Solution:

$$(1-\omega-\omega^2)^6$$
$$=[1-(\omega+\omega^2)]^6$$
$$=[1-(-1)]^6$$
$$=(1+1)^6$$
$$=(2)^6$$
$$=64$$

(viii) If ω is cube root of unity, form an equation whose roots are 3ω and $3\omega^2$.

Solution:

Roots are 3ω, $3\omega^2$

S = Sum of the roots $=3\omega+3\omega^2=3(\omega+\omega^2)$

P = Product of the roots $=3\omega\times 3\omega^2=9\omega^3$

Required Eq

$$x^2-Sx+P=0$$

$$x^2 - 3(\omega + \omega^2)x + 9\omega^3 = 0$$
$$x^2 - 3(-1)x + 9(1) = 0$$
$$x^2 + 3x + 9 = 0$$

(ix) Using synthetic division, find the remainder and quotient when $(x^3 + 3x^2 + 2) \div (x - 2)$

Solution:

Here, $P(x) = x^3 + 3x^2 + 2$
$= x^3 + 3x^2 + 0x + 2$
$x - a = x - 2$
$a = 2$

Using synthetic division:

	1	3	0	2
2	↓	2	10	20
	1	5	10	22

Quotient $= x^2 + 5x + 10$
Remainder = 22

(x) Using synthetic division, show that $x - 2$ is the factor of $x^3 + x^2 - 7x + 2$

Solution:

$P(x) = x^3 + x^2 - 7x + 2$
$x - a = x - 2$
$a = 2$

Using synthetic division:

	1	1	−7	2
2	↓	2	6	−2
	1	3	−1	0

R = 0, therefore, $x - 2$ is a factor of P(x).

(xi) Find the sum and product of the roots of the equation $2px^2 + 3qx - 4r = 0$

Solution:

Equation: $2px^2 + 3qx - 4r = 0$
Here, $a = 2p$, $b = 3q$, $c = -4r$

$$S = -\frac{b}{a} = -\frac{3q}{2p}$$

$$P = \frac{c}{a} = \frac{-4r}{2p} = -\frac{2r}{P}$$

(xii) Find $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ of the roots of the equation $x^2 - 4x + 3 = 0$

Solution:

$$x^2 - 4x + 3 = 0$$

Here, $S = \alpha + \beta = -\frac{-4}{1} = 4$

$$P = \alpha\beta \quad = \frac{3}{1} = +3$$

Now, $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

$$= \frac{\beta^2 + \alpha^2}{\alpha^2\beta^2}$$

$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

Putting values of $\alpha + \beta$ and $\alpha\beta$, we get

$$= \frac{(4)^2 - 2(3)}{(3)^2}$$

$$= \frac{16 - 6}{9} = \frac{10}{9}$$

(xiii) If α, β are the roots of $4x^2 - 3x + 6 = 0$ find

(a) $\alpha^2 + \beta^2$ **(b)** $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

(c) $\alpha - \beta$

Solution:

Equation: $4x^2 - 3x + 6 = 0$

Here, $S = \alpha + \beta = -\left(\frac{-3}{4}\right) = \frac{3}{4}$

$$P = \alpha\beta \quad = \frac{6}{4} \quad = \frac{3}{2}$$

(a) $\alpha^2 + \beta^2$

$$= (\alpha + \beta)^2 - 2$$

Putting values of $(\alpha + \beta)$ and $\alpha\beta$, we get

$$= \left(\frac{3}{4}\right)^2 - 2\left(\frac{3}{2}\right)$$

$$= \frac{9}{16} - 3$$

$$= \frac{9 - 48}{16} = \frac{-39}{16}$$

(b) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

$$= \frac{\alpha^2 + \beta^2}{\alpha\beta}$$

$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$

Putting values of $(\alpha + \beta)$ and $\alpha\beta$, we get

$$= \frac{\left(\frac{3}{4}\right)^2 - 2\left(\frac{3}{2}\right)}{\frac{3}{2}}$$

$$= \left(\frac{9}{16} - 3\right) \div \frac{3}{2}$$

$$= \left(\frac{9-48}{16}\right) \times \frac{2}{3}$$

$$= \frac{39}{16} \times \frac{2}{3}$$

$$= \frac{-13}{8}$$

(c) $\alpha - \beta$

Now, $(\alpha - \beta)^2$

$$= (\alpha + \beta)^2 - 4\alpha\beta$$

Putting values of $(\alpha + \beta)$ and $\alpha\beta$, we get

$$= \left(\frac{3}{4}\right)^2 - 4\left(\frac{3}{2}\right)$$

$$= \frac{9}{16} - 6$$

$$= \frac{9-96}{16}$$

$$(\alpha - \beta)^2 = \frac{-87}{16} \quad \text{(Taking sq. root)}$$

$$\alpha - \beta = \sqrt{\frac{-87}{16}} = \frac{\sqrt{-87}}{4}$$

(xiv) If α, β are the roots of $x^2 - 5x + 7 = 0$, find an equation whose roots are
(a) $-\alpha, -\beta$ (b) $2\alpha, 2\beta$

Solution:

Equation: $x^2 - 5x + 7 = 0$

Here, $a = 1, b = -5, c = 7$

S = Sum of the roots $= \alpha + \beta = -\frac{b}{a} = -\left(\frac{-5}{1}\right) = 5$ ....(A)

P = Product of the roots $= \alpha\beta = \frac{c}{a} = \frac{7}{1} = 7$ ....(B)

Part (a) $S = -\alpha - \beta$

$= -(\alpha + \beta) = -(5) = -5$ from A

$P = (-\alpha)(-\beta) = \alpha\beta = 7$ from B

Equation: $x^2 - Sx + P = 0$

$x^2 - (-5)x + 7 = 0$

$x^2 + 5x + 7 = 0$

Part (b) $S = 2\alpha + 2\beta$

$= 2(\alpha + \beta)$

$= 2(5)$ from (A)

$= 10$

$P = (2\alpha)(2\beta)$

$= 4\alpha\beta$

$= 4(7)$ from (B)

$= 28$

Equation: $x^2 - Sx + P = 0$

$x^2 - 10x + 28 =$

Q3. Fill in the blanks:

(i) The discriminant of $ax^2 + bx + c = 0$ is ________.

(ii) If $b^2 - 4ac = 0$, then roots of $ax^2 + bx + c = 0$ are ________.

(iii) If $b^2 - 4ac > 0$, then the roots of $ax^2 + bx + c = 0$ are ________.

(iv) If $b^2 - 4ac < 0$, then the roots of $ax^2 + bx + c = 0$ are ________.

(v) If $b^2 - 4ac > 0$and perfect square, then the roots of $ax^2 + bx + c = 0$ are ________.

(vi) If $b^2 - 4ac > 0$, and not a perfect square, then roots of $ax^2 + bx + c = 0$ are ________.

(vii) If α, β are the roots of $ax^2 + bx + c = 0$, then sum of the roots is ________.

(viii) If α, β are the roots of $ax^2 + bx + c = 0$, then product of the roots is ________.

(ix) If α, β are the roots of $7x^2 - 5x + 3 = 0$, then sum of the roots is ________.

(x) If α, β are the roots of $5x^2 + 3x - 9 = 0$, then product of the roots is ________.

(xi) Far a quadratic equation $ax^2 + bx + c = 0$, $\frac{1}{\alpha\beta}$ is equal to ________.

(xii) Cube roots of unity are ________.

(xiii) Under usual notation sum of the cube roots of unity is ________.

(xiv) If $1, \omega, \omega^2$ are the cube roots of unity, then ω^{-7} is equal to ________.

(xv) If α, β are the roots of the quadratic equation, then the quadratic equation is written as ________.

Answer:

(i)	$b^2 - 4ac$	(ii)	equal	(iii)	real
(iv)	imaginary	(v)	rational	(vi)	irrational
(vii)	$-\frac{b}{a}$	(viii)	$\frac{c}{a}$	(ix)	$\frac{5}{7}$
(x)	$-\frac{9}{5}$	(xi)	$\frac{1}{\alpha\beta}$	(xii)	$1, \omega, \omega^2$

(xiii)	zero	(xiv)	ω^2	(xv)	$x^2 - (\alpha + \beta)x + \alpha\beta = 0$
(xvi)	$x^2 + 2x + 4 = 0$				

SUMMARY

- **Discriminant** of the quadratic expression $ax^2 + bx + c$ is "$b^2 - 4ac$".
- The **cube roots** of unity are 1, $\frac{-1+\sqrt{-3}}{2}$ and $\frac{-1-\sqrt{-3}}{2}$.
- **Complex cube roots** of unity are ω and ω^2.
- **Properties of cube roots of unity.**
  - (a) The product of three cube roots of unity is one. i.e., $(1)(\omega)(\omega^2) = \omega^3 = 1$
  - (b) Each of the complex cube roots of unity is **reciprocal** of the other.
  - (c) Each of the complex cube roots of unity is the **square** of the other.
  - (d) The sum of all the cube roots of unity is zero, i.e., $1 + \omega + \omega^2 = 0$
- The **roots** of the quadratic equation $ax^2 + bx + c = 0, a \neq 0$ are

  $\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a}$ and $\beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$
- The **sum** and the **product** of the roots of $ax^2 + bx + c = 0, a \neq 0$ are

  $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$ respectively.
- **Symmetric functions** of the roots of quadratic equation are those functions in which the roots involved are such that the values of the expressions remain unaltered, when roots are interchanged.
- Formation of a quadratic equation if its roots are given;

  x^2 – (sum of the roots) x + product of the roots = 0

  $\Rightarrow \quad x^2 - (\alpha + \beta)x + \alpha\beta = 0.$
- **Synthetic division** is the process of finding the quotient and remainder, when a polynomial is divided by a linear polynomial.
- A system of equations having a common solution is called a system of **simultaneous equations.**

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