

مشق 2.4

تجزی کیجیے۔

$$8x^3 - y^3 \quad -1$$

حل:

$$8x^3 - y^3 = (2x)^3 - y^3$$

$$\begin{aligned} a^3 - b^3 &= (a - b)(a^2 + ab + b^2) \\ &= (2x - y)(4x^2 + 2xy + y^2) \end{aligned}$$

چونکہ
اس لیے

$$27x^3 + 1 \quad -2$$

حل:

$$\begin{aligned} 27x^3 + 1 &= (3x)^3 + (1)^3 \\ a^3 + b^3 &= (a + b)(a^2 - ab + b^2) \\ &= (3x + 1)(9x^2 - 3x + 1) \end{aligned}$$

چونکہ

$$1 - 343x^3 \quad -3$$

حل:

$$\begin{aligned} 1 - 343x^3 &= (1)^3 - (7x)^3 \\ &= (1 - 7x)(1 + 7x + 49x^2) \end{aligned}$$

$$\therefore a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$a^3b^3 + 512 \quad -4$$

حل:

$$\begin{aligned} a^3b^3 + 512 &= (ab)^3 + (8)^3 \\ \therefore a^3 + b^3 &= (a + b) (a^2 - ab + b^2) \\ &= (ab + 8) [(ab)^2 - (ab)(8) + (8)^2] \\ &= (ab + 8) (a^2b^2 - 8ab + 64) \end{aligned}$$

$$27 - 1000y^3 \quad -5$$

حل:

$$\begin{aligned} 27 - 1000y^3 &= (3)^3 - (10y)^3 \\ \therefore a^3 - b^3 &= (a - b) (a^2 + ab + b^2) \\ &= (3 - 10y) [(3)^2 + (3)(10y) + (10y)^2] \\ &= (3 - 10y) (9 + 30y + 100y^2) \end{aligned}$$

$$27x^3 - 64y^3 \quad -6$$

حل:

$$\begin{aligned} 27x^3 - 64y^3 &= (3x)^3 - (4y)^3 \\ \therefore a^3 - b^3 &= (a - b) (a^2 + ab + b^2) \\ &= (3x - 4y) [(3x)^2 + (3x)(4y) + (4y)^2] \\ &= (3x - 4y) (9x^2 + 12xy + 16y^2) \end{aligned}$$

پس

$$x^3y^3 + z^3 \quad -7$$

حل:

$$\begin{aligned} x^3y^3 + z^3 &= (xy)^3 + z^3 \\ \therefore a^3 + b^3 &= (a + b) (a^2 - ab + b^2) \\ &= (xy + z) (x^2y^2 - xyz + z^2) \end{aligned}$$

پس

$$216p^3 - 343 \quad -8$$

حل:

$$\begin{aligned} 216p^3 - 343 &= (6p)^3 - (7)^3 \\ \therefore a^3 - b^3 &= (a - b) (a^2 + ab + b^2) \\ &= (6p - 7) [(6p)^2 + (6p)(7) + (7)^2] \\ &= (6p - 7) (36p^2 + 42p + 49) \end{aligned}$$

پس

$$8x^3 - \frac{1}{27} \quad -9$$

حل:

$$\begin{aligned} 8x^3 - \frac{1}{27} &= (2x)^3 - \left(\frac{1}{3}\right)^3 \\ \therefore a^3 - b^3 &= (a - b) (a^2 + ab + b^2) \\ &= \left(2x - \frac{1}{3}\right) \left(4x^2 + \frac{2}{3}x + \frac{1}{9}\right) \end{aligned}$$

$$a^3 + b^3 + a + b \quad -10$$

حل:

$$\begin{aligned} a^3 + b^3 + a + b &= (a^3 + b^3) + (a + b) \\ &= (a + b) (a^2 - ab + b^2) + (a + b) \quad \text{کو مشترک لینے سے} \\ &= (a + b) (a^2 - ab + b^2 + 1) \end{aligned}$$

$$a - b - a^3 + b^3 \quad -11$$

حل:

$$\begin{aligned} a - b - a^3 + b^3 &= (a - b) - (a^3 - b^3) \\ &= (a - b) - (a - b) (a^2 + ab + b^2) \quad \text{کو مشترک لینے سے} \\ &= (a - b) [1 - (a^2 + ab + b^2)] \end{aligned}$$

$$x - 8xy^3 \quad -12$$

حل:

$$\begin{aligned} x - 8xy^3 &= x (1 - 8y^3) \\ &= x [1^3 - (2y)^3] \\ &= x [(1 - 2y) (1 + 2y + 4y^2)] \\ &= x (1 - 2y) (1 + 2y + 4y^2) \end{aligned}$$

$$x^{12} - y^{12} \quad -13$$

حل:

$$\begin{aligned} x^{12} - y^{12} &= (x^6)^2 - (y^6)^2 \\ \therefore a^2 - b^2 &= (a + b) (a - b) \\ &= (x^6 + y^6) (x^6 - y^6) \\ &= [(x^2)^3 + (y^2)^3] [(x^3)^2 - (y^3)^2] \quad \therefore a^3 \pm b^3 = (a \pm b) (a^2 \mp ab + b^2) \\ &= (x^2 + y^2) (x^4 - x^2y^2 + y^4) [(x^3 + y^3) (x^3 - y^3)] \\ &= (x^2 + y^2) (x^4 - x^2y^2 + y^4) (x + y) (x^2 - xy + y^2) (x - y) (x^2 + xy + y^2) \\ &= (x + y) (x - y) (x^2 + y^2) (x^2 - xy + y^2) (x^2 + xy + y^2) (x^4 - x^2y^2 + y^4) \end{aligned}$$

$$1 - \frac{64p^3}{q^3} \quad -14$$

حل:

$$\begin{aligned} 1 - \frac{64p^3}{q^3} &= (1)^3 - \left(\frac{4p}{q}\right)^3 \\ &= \left(1 - \frac{4p}{q}\right) \left(1 + \frac{4p}{q} + \frac{16p^2}{q^2}\right) \end{aligned}$$

$$1 + 64u^3 \quad -15$$

حل:

$$\begin{aligned} 1 + 64u^3 &= (1)^3 + (4u)^3 \\ &= (1 + 4u) (1 - 4u + 16u^2) \end{aligned}$$

$$8x^3 - 6x - 9y + 27y^3 \quad -16$$

حل:

$$\begin{aligned} 8x^3 - 6x - 9y + 27y^3 &= 8x^3 + 27y^3 - 6x - 9y \\ &= (2x)^3 + (3y)^3 - 3(2x + 3y) \\ &= (2x + 3y) (4x^2 - 6xy + 9y^2) - 3(2x + 3y) \end{aligned}$$

(2x + 3y) مشترک لینے سے

$$= (2x + 3y) (4x^2 - 6xy + 9y^2 - 3)$$

پس

$$z^3 + 125 \quad -17$$

حل:

$$\begin{aligned} z^3 + 125 &= z^3 + 5^3 \\ &= (z + 5) (z^2 - 5z + 25) \end{aligned}$$

$$x^9 + y^9 \quad -18$$

حل:

$$\begin{aligned} x^9 + y^9 &= (x^3)^3 + (y^3)^3 \\ &= (x^3 + y^3) (x^6 - x^3y^3 + y^6) \\ &= (x + y) (x^2 - xy + y^2) (x^6 - x^3y^3 + y^6) \end{aligned}$$

$$m^6 - n^6 \quad -19$$

حل:

$$\begin{aligned} m^6 - n^6 &= (m^3)^2 - (n^3)^2 \\ &= (m^3 + n^3) (m^3 - n^3) \\ &= (m + n) (m^2 - mn + n^2) (m - n) (m^2 + mn + n^2) \\ &= (m + n) (m - n) (m^2 + mn + n^2) (m^2 - mn + n^2) \end{aligned}$$

$$64x^7 - xa^6 \quad -20$$

حل:

$$\begin{aligned} 64x^7 - xa^6 &= x (64x^6 - a^6) \\ &= x [(2x)^6 - a^6] \\ &= x [(2x)^3]^2 - [(a^3)^2] \quad \therefore a^2 - b^2 = (a + b) (a - b) \\ &= x [(2x)^3 - a^3] [(2x)^3 + a^3] \quad \therefore a^3 \pm b^3 = (a \pm b) (a^2 \mp ab + b^2) \\ &= x (2x - a) (4x^2 + 2ax + a^2) (2x + a) (4x^2 - 2ax + a^2) \\ &= x (2x - a) (2x + a) (4x^2 + 2ax + a^2) (4x^2 - 2ax + a^2) \end{aligned}$$

$$x^3 - 27a^3 \quad -21$$

حل:

$$\begin{aligned} x^3 - 27a^3 &= x^3 - (3a)^3 \\ &= (x - 3a) (x^2 + 3ax + 9a^2) \end{aligned}$$

$$x^3 + 27a^3 \quad -22$$

حل:

$$\begin{aligned} x^3 + 27a^3 &= x^3 + (3a)^3 \\ &= (x + 3a) (x^2 - 3ax + 9a^2) \end{aligned}$$