and (ii) $a : a - b = c : c - d$

(5) Theorem of Componendo dividendo

If $a : b = e : d$, then

(i) $a + b : a - b = c + d : c - d$

and (ii) $a - b : a + b = c - d : c + d$

SOLVED EXERCISE 3.4

1. Prove that $a : b = c : d$, if

(i) $\dfrac{4a+5b}{4a-5b} = \dfrac{4c+5d}{4c-5d}$

Solution:

Given $\dfrac{4a+5b}{4a-5b} = \dfrac{4c+5d}{4c-5d}$

By componendo-dividendo theorem, we have

$$\frac{(4a+5b)+(4c+5b)}{(4a-5b)-(4c-5b)} = \frac{(4c+5d)+(4c-5d)}{(4c+5d)-(4c-5d)}$$

$$\frac{4a+5b+4a-5b}{4a+5b-4a+5b} = \frac{4c+5d+4c-5d}{4c+5d-4c-5d}$$

$$\frac{8a}{10b} = \frac{8c}{10d}$$

Multiplying both sides by $\dfrac{18}{4}$, we get

$$\frac{a}{b} = \frac{c}{d}$$

$\Rightarrow$ $a : b = c : d$

Hence proved

(ii) $\dfrac{2a+9b}{2a-9b} = \dfrac{2c+9d}{2c-9d}$

Solution:

Given

$$\frac{2a+9b}{2a-9b} = \frac{2c+9d}{2c-9d}$$

By componendo-dividendo theorem, we have

$$\frac{(2a+9b)+(2a+9b)}{(2a+9b)-(2a-9b)}=\frac{(2c+9d)+(2c-9d)}{(2c+9d)-(2c-9d)}$$

$$\frac{2a+9b+2a-9b}{2a+9b-2a+9b}=\frac{2c+9d+2c-9d}{2c+9d=2c+9d}$$

$$\frac{4a}{18b}=\frac{4c}{18d}$$

Multiplying both sides by $\frac{18}{4}$, we get

$$\frac{a}{b}=\frac{c}{d}$$

$\Rightarrow$ $a:b=c:d$

Hence proved

(iii) $\frac{ac^2+bd^2}{ac^2-bd^2}=\frac{c^3+d^3}{c^3-d^3}$

Solution:

Given

$$\frac{ac^2+bd^2}{ac^2-bd^2}=\frac{c^3+d^3}{c^3-d^3}$$

By componendo-dividendo theorem, we have

$$\frac{(ac^2+bd^2)+(ac^2-bd^2)}{(ac^2+bd^2)-(ac^2-bd^2)}=\frac{(c^3+d^3)+(c^3-d^3)}{(c^3+d^3)-(c^3-d^3)}$$

$$\frac{ac^2+bd^2+ac^2-bd^2}{ac^2+bd^2-ac^2+bd^2}=\frac{c^3+d^3+c^3-d^3}{c^3+d^3-c^3+d^3}$$

$$\frac{2ac^2}{2bd^2}=\frac{2c^3}{2d^3}$$

$$\Rightarrow \quad \frac{ac^2}{bd^2}=\frac{c^3}{d^3}$$

Multiplying both sides by $\frac{d^2}{c^2}$, we get

$$\frac{a}{b}=\frac{c}{d}$$

$\Rightarrow$ $a:b=c:d$

Hence proved

(iv) $\frac{a^2c+b^2d}{a^2c-b^2d}=\frac{ac^2+bd^2}{ac^2-bd^2}$

Solution:

Given

$$\frac{a^2c + b^2d}{a^2c - b^2d} = \frac{ac^2 + bd^2}{ac^2 - bd^2}$$

By componendo-dividendo theorem, we have

$$\frac{(a^2c + b^2d) + (a^2c - b^2d)}{(a^2c + b^2d) - (a^2c - b^2d)} = \frac{(ac^2 + bd^2) + (ac^2 - bd^2)}{(ac^2 + bd^2) - (ac^2 - bd^2)}$$

$$\frac{a^2c + b^2d + a^2c - b^2d}{a^2c + b^2d - a^2c + b^2d} = \frac{ac^2 + bd^2 + ac^2 - bd^2}{ac^2 + bd^2 - ac^2 + bd^2}$$

$$\frac{2a^2c}{2b^2d} = \frac{2ac^2}{2bd^2}$$

$$\Rightarrow \quad \frac{a^3c}{b^2d} = \frac{ac^2}{bd^2}$$

Multiplying both sides by $\frac{bd}{ac}$, we get

$$\frac{a}{b} = \frac{c}{d}$$

$\Rightarrow \quad a : b = c : d$

Hence proved

(v) $pa + qb : pa - qb = pc + qd : pc - qd$

Solution:

Given

$$pa + qb : pa - qb = pc + qd : pc - qd$$

$$\frac{pa + qb}{pa - qb} = \frac{pc + qd}{pc - qd}$$

By componendo-dividendo theorem, we have

$$\frac{(pa + qb) + (pa - qb)}{(pa + qb) - (pa - qb)} = \frac{(pc + qd) + (pc - qd)}{(pc + qd) - (pc - qd)}$$

$$\frac{pa + qb + pa - qb}{pa + qb - pa + qb} = \frac{pc + qd + pc - qd}{pc + qd - pc + qd}$$

$$\frac{2pa}{2qb} = \frac{2pc}{2qd}$$

Multiplying both sides by $\frac{2q}{2p}$, we get

$$\frac{a}{b} = \frac{c}{d}$$

$\Rightarrow \quad a : b = c : d$

Hence proved

(vi) $\dfrac{a+b+c+d}{a+b-c-d}=\dfrac{a-b+c-d}{a-b-c+d}$

Solution:

Given

$$\frac{a+b+c+d}{a+b-c-d}=\frac{a-b+c-d}{a-b-c+d}$$

$$\frac{(a+b+c+d)+(a+b-c-d)}{(a+b+c+d)-(a+b-c-d)}=\frac{(a-b+c-d)+(a-b+c+d)}{(a-b+c-d)-(a-b-c+d)}$$

$$\frac{a+b+c+d+a+b-c-d}{a+b+c+d-a-b+c+d}=\frac{a-b+c-d+a-b-c+d}{a-b+c-d-a+b+c-d}$$

$$\frac{2a+2b}{2c+2d}=\frac{2a-2b}{2c-2d}$$

$$\frac{2(a+b)}{2(c+d)}=\frac{2(a-b)}{2(c-d)}$$

$$\frac{a+b}{c+d}=\frac{a-b}{c-d}$$

or $\quad \dfrac{a+b}{c-d}=\dfrac{c+d}{c-d}$

By componendo-dividendo theorem, we have

$$\frac{(a+b)+(a-b)}{(a+b)-(a-b)}=\frac{(c+d)+(c-d)}{(c+d)-(c-d)}$$

$$\frac{a+b+a-b}{a+b-a+b}=\frac{c+d+c-d}{c+d-c+d}$$

$$\frac{2a}{2b}=\frac{2c}{2d}$$

$\Rightarrow \quad \dfrac{a}{b}=\dfrac{c}{d}$

$\Rightarrow \quad a:b=c:d$

Hence proved

(vii) $\dfrac{2a+3b+2c+3d}{2a+3b-2c-3d}=\dfrac{2a-3b+2c-3d}{2a-3b-2c+3d}$

Solution:

Given

$$\frac{(2a+3b+2c+3d)+(2a+3b-2c-3d)}{(2a+3b+2c+3d)-(2a+3b-2c-3d)}=\frac{(2a-3b+2c-3d)+(2a-3b-2c+3d)}{(2a-3b+2c-3d)-(2a-3b-2c+3d)}$$

$$\frac{2a+3b+2c+3d+2a+3b-2c-3d}{2a+3b+2c+3d-2a-3b+2c+3d}=\frac{2a-3b+2c-3d+2a-3b-2c+3d}{2a-3b-2c+3d-2a+3b+2c-3d}$$

$$\frac{4a+6b}{4c+6d}=\frac{4a-6b}{4c-6d}$$

or $$\frac{4a+6b}{4c-6d}=\frac{4c+6d}{4c-6d}$$

By componendo-dividendo theorem, we have

$$\frac{(4a+6b)+(4a-6b)}{(4a+6b)-(4a-6b)}=\frac{(4c+6d)+(4c-6d)}{(4c+6d)-(4c-6d)}$$

$$\frac{4a+6b+4a-6b}{4a+6b-4a+6b}=\frac{4c+6d+4c-6d}{4c+6d-4c+6d}$$

$$\frac{8a}{12b}=\frac{8c}{12d}$$

Multiplying both sides by $\frac{12}{8}$, we get

$$\frac{a}{b}=\frac{c}{d}$$

$\Rightarrow$ $a:b=c:d$

Hence proved

(viii) $$\frac{a^2+b^2}{a^2-b^2}=\frac{ac+bd}{ac-bd}$$

Solution:

Given

$$\frac{a^2+b^2}{a^2-b^2}=\frac{ac+bd}{ac-bd}$$

By componendo-dividendo theorem, we have

$$\frac{(a^2+b^2)+(a^2-b^2)}{(a^2-b^2)-(a^2-b^2)}=\frac{(ac+bd)+(ac+bd)}{(ac+bd)-(ac-bd)}$$

$$\frac{a^2+b^2+a^2-b^2}{a^2+b^2-a^2+b^2}=\frac{ac+bd+ac-bd}{ac+bd-ac+bd}$$

$$\frac{2a^2}{2b^2}=\frac{2ac}{2bd}$$

Multiplying both sides by $\frac{2b}{2a}$, we get

$$\frac{a}{b} = \frac{c}{d}$$

$\Rightarrow \quad a : b = c : d$

Hence proved

2. Using theorem of componendo-dividendo

(i) Find the value of $\frac{x+2y}{x-2y} + \frac{x+2z}{x-2z}$**, if** $x = \frac{4yz}{y+z}$

Solution:

Given $x = \frac{4yz}{y+z}$ or $x = \frac{(2y)(2z)}{y+z}$

or $\frac{x}{2y} = \frac{2z}{y+z}$

Applying compondendo-dividendo theorem, we get.

$$\frac{x+2y}{x-2y} = \frac{2z+(y+z)}{2z-(y+z)}$$

$$\frac{x+2y}{x-2y} = \frac{2z+y+z}{2z-y-z}$$

$$\frac{x+2y}{x-2y} = \frac{y+3z}{z-y} \qquad \text{———— (i)}$$

Now $x = \frac{4yz}{y+z}$ or $x = \frac{(2y)(2z)}{y+z}$

or $\frac{x}{2z} = \frac{2y}{y+z}$

Applying compondo-dividendo theorem, we get.

$$\frac{x+2z}{x-2z} = \frac{2y+(y+z)}{2y-(y+z)}$$

$$\frac{x+2z}{x-2z} = \frac{2y+y+z}{2y-y-z}$$

$$\frac{x+2z}{x-2z} = \frac{3y+z}{y-z}$$

Adding eq. (i) and eq. (ii), we get

$$\frac{x+2y}{x-2y}+\frac{x+2z}{x-2z}=\frac{y+3z}{z-y}+\frac{3y+z}{y-z}$$

$$=\frac{y+3z}{z-y}+\frac{3y+z}{-(z-y)}$$

$$=\frac{y+3z}{z-y}-\frac{3y+z}{z-y}$$

$$=\frac{(y+3z)-(3y+z)}{z-y}$$

$$=\frac{y+3z-3y-z}{z-y}$$

$$=\frac{2z-2y}{z-y}$$

$$=\frac{2(z-y)}{z-y}=2$$

(ii) Find the value of $\frac{m+5n}{m-5n}+\frac{m+5p}{m-5p}$, if $m=\frac{10np}{n+p}$

Solution:

Given $m=\frac{10np}{n+p}$ or $x=\frac{(2p)(5n)}{n+p}$

or $\frac{xm}{5n}=\frac{2p}{n+p}$

Applying componendo-dividendo theorem, we get.

$$\frac{m+5n}{m-5n}=\frac{2p+(n+p)}{2p-(n+p)}$$

$$\frac{m+5n}{m-5n}=\frac{2p+n+p}{2p-n-p}$$

$$\frac{m+5n}{m-5n}=\frac{3p+n}{p-n} \quad \text{\_\_\_\_\_\_(i)}$$

Now $m=\frac{10np}{n+p}$ or $m=\frac{(5p)(2n)}{n+p}$

or $\frac{m}{5p}=\frac{2n}{n+p}$

Applying componendo – dividend theorem, we get.

$$\frac{m+5n}{m-5n}=\frac{2n+(n+p)}{2n-(n+p)}$$

$$\frac{m+5n}{m-5n}=\frac{2n+n+p}{2n-n-p}$$

$$\frac{m+5n}{m-5n}+\frac{3n+p}{n-p} \quad \text{\_\_\_\_\_\_\_\_(ii)}$$

Adding eq. (i) and eq. (ii), we get.

$$\frac{m+5n}{m-5n}=\frac{m+5p}{m-5p}=\frac{3p+n}{p-n}+\frac{2n+p}{n-p}$$

$$=\frac{3p+n}{p-n}+\frac{2n+p}{-(p-n)}$$

$$=\frac{3p+n}{p-n}=\frac{2n+p}{p-n}$$

$$=\frac{(3p+n)-(3n+p)}{p-n}$$

$$=\frac{3p+n-3n-p}{p-n}$$

$$=\frac{2p-2n}{p-n}$$

$$=\frac{2(p-n)}{p-n}=2$$

(iii) Find the value of $\frac{x-6a}{x+6a}-\frac{x-6b}{x+6b}$, if $x=\frac{12ab}{a-b}$

Solution

Given $\quad x=\frac{12ab}{a-b} \qquad$ or $\quad x=\frac{(6a)(2b)}{a-b}$

or $\quad \frac{x}{6a}=\frac{2b}{a-b}$

Applying componendo-dividendo theorem, we get.

$$\frac{x+6a}{x-6a}=\frac{2b+(a-b)}{2b-(a-b)}$$

$$\frac{x+6a}{x-6a}=\frac{2b+a-b}{2b-a+b}$$

$$\frac{x+6a}{x-6a}=\frac{a+b}{3b-a}$$

or $\quad \frac{x-6a}{x+6a}=\frac{3b-a}{a+b}$ ______(i)

Now $\quad x=\frac{12ab}{a=b} \quad$ or $\quad x=\frac{(2a)(6b)}{a-b}$

or $\frac{x}{6b}=\frac{2a}{a-b}$

Applying componendo- dividend theorem, we get.

$$\frac{x+6b}{x-6b}=\frac{2a+(a-b)}{2a-(a-b)}$$

$$\frac{x+6b}{x-6b}=\frac{2a+a-b}{2a-a+b}$$

$$\frac{x+6b}{x-6b}=\frac{3a+b}{a+b} \text{ ______(ii)}$$

Subtrabtract eg. (*ii*) from eq. (i), we have.

$$\frac{x-6a}{x+6a}-\frac{x+6b}{x-6b}=\frac{3b-a}{a+b}-\frac{3a-b}{a+b}$$

$$=\frac{(3b-a)-(3a-b)}{a-b}$$

$$=\frac{3b-a-3a+b}{a-b}$$

$$=\frac{4b-4a}{a+b}$$

$$=\frac{4(b-a)}{a+b}$$

(iv) Find the value of $\frac{x-3y}{x+3y}-\frac{x-3z}{x+3z}$, if $x=\frac{3yz}{y-z}$

Solution:

Given $x=\frac{3yz}{y-z}$

or $\quad \frac{x}{3y}=\frac{z}{y-z}$

Applying componendo- dividend theorem, we get.

$$\frac{x+3y}{x-3y}=\frac{z+(y-z)}{z-(y-z)}$$

$$\frac{x+3y}{x-3y}=\frac{z+y-z}{z-y+z}$$

$$\frac{x+3y}{x-3y}=\frac{y}{2z-y}$$

$$\text{or}\ \frac{x+3y}{x-3y}=\frac{2z-y}{y}\quad \text{\_\_\_\_\_\_\_\_(i)}$$

Now $x=\dfrac{3yz}{y-z}$

$$\frac{x}{3z}=\frac{y}{y-z}$$

Applying componendo-dividendo theorem, we get.

$$\frac{x+3z}{x-3z}=\frac{y+(y-z)}{y-(y-z)}$$

$$\frac{x+3z}{x-3z}=\frac{y+y-z}{y-y+z}$$

$$\frac{x+3z}{x-3z}=\frac{2y-z}{z}\quad \text{\_\_\_\_\_\_\_\_(ii)}$$

Subtracting eq. (ii) from eq. (i), we get.

$$\frac{x-3y}{x+3y}-\frac{x+3z}{x-3z}=\frac{2z-y}{y}-\frac{2y-z}{z}$$

$$=\frac{z(2z-y)-y(2y-z)}{yz}$$

(v) Find the value of $\dfrac{s-3p}{s+3p}+\dfrac{s-3q}{s+3q}$, if $s=\dfrac{6pq}{s-3q}$

Solution:

Given $s=\dfrac{6pq}{s-3q}$ or $s=\dfrac{(3p)(2q)}{p-q}$ or $\dfrac{s}{3p}=\dfrac{2q}{p-q}$

Applying componendo-dividendo theorem, we get.

$$\frac{s+3p}{s-3p}=\frac{2q+(p-q)}{2q-(p-q)}$$

$$\frac{s+3p}{s-3p}=\frac{2q+p-q}{2q-p+q}$$

$$\frac{s+3p}{s-3p}=\frac{p+q}{3q-p}$$

$$\frac{s+3p}{s-3p}=\frac{3q-p}{p+q} \quad \text{\_\_\_\_\_\_\_\_(i)}$$

Now $S=\frac{6pq}{p-q}$ or $S=\frac{(3q)(2p)}{p-q}$

or $\frac{S}{3q}=\frac{2p}{p-q}$

Applying componendo-dividendo theorem, we get.

$$\frac{s+3p}{s-3p}=\frac{2p+(p-q)}{2p-(p-q)}$$

$$\frac{s+3p}{s-3p}=\frac{2p+p-q}{2p-p+q}$$

$$\frac{s+3p}{s-3p}=\frac{3p-p}{p+q} \quad \text{\_\_\_\_\_\_\_\_(ii)}$$

Adding eq, (i) and eq, (ii), we get.

$$\frac{s-3p}{s+3p}+\frac{s+3p}{s-3p}=\frac{3q-p}{p+q}+\frac{3p-p}{p+q}$$

$$=\frac{3q-p+3p-q}{p+q}$$

$$=\frac{2p=2q}{p+q}$$

$$=\frac{2(p+q)}{p+q}=2$$

(vi) Solve $\frac{(x-2)^2-(x-4)^2}{(x-2)^2+(x-4)^2}=\frac{12}{13}$

Solution:

$$\frac{(x-2)^2-(x-4)^2}{(x-2)^2+(x-4)^2}=\frac{12}{13}$$

Applying componendo-dividendo theorem, we get.

$$\frac{\left[(x-2)^2-(x-4)^2\right]+\left[(x-2)^2+(x-4)^2\right]}{\left[(x-2)^2-(x-4)^2\right]-\left[(x-2)^2+(x-4)^2\right]}=\frac{12+13}{12-13}$$

$$\frac{(x-2)^2-(x-4)^2+(x-2)^2+(x-4)^2}{(x-2)^2+(x-4)^2-(x-2)^2-(x-4)^2}=\frac{25}{-1}$$

$$\frac{2(x-2)^2}{-2(x-4)^2}=-25$$

$$\Rightarrow \quad \frac{(x-2)^2}{(x-4)^2}=25$$

Taking square root on both sides, we get.

$$\sqrt{\frac{(x-2)^2}{(x-4)^2}}=\pm\sqrt{25}$$

$$\frac{x-2}{x-4}=\pm 5$$

$$\Rightarrow \frac{x-2}{x-4}=-5 \quad \text{or} \quad \frac{x-2}{x-4}=5$$

$$x-2=-5(x-4) \qquad 5(x-4)=x-2$$

$$x-2=-5x+20 \qquad 5x-20=x-2$$

$$x+5x=2+20 \quad 5x-x=20-2$$

$$6x=22 \qquad 4x=18$$

$$x=\frac{22}{6} \qquad x=\frac{18}{4}$$

$$x=\frac{11}{3} \qquad x=\frac{9}{2}$$

Thus, solution set $=\left\{\frac{9}{2},\frac{11}{3}\right\}$

(vii) Solve $\dfrac{\sqrt{x^2+2}+\sqrt{x^2-2}}{\sqrt{x^2+2}-\sqrt{x^2-2}}=2$

Solution:

$$\frac{\sqrt{x^2+2}+\sqrt{x^2-2}}{\sqrt{x^2+2}-\sqrt{x^2-2}}=\frac{2}{1}$$

Applying componendo-dividendo theorem, we get.

$$\frac{\left[\sqrt{x^2+2}+\sqrt{x^2-2}\right]+\left[\sqrt{x^2+2}-\sqrt{x^2-2}\right]}{\left[\sqrt{x^2+2}-\sqrt{x^2-2}\right]-\left[\sqrt{x^2+2}-\sqrt{x^2-2}\right]}=\frac{2+1}{2-1}$$

$$\frac{\sqrt{x^2+2}+\sqrt{x^2-2}+\sqrt{x^2+2}-\sqrt{x^2-2}}{\sqrt{x^2+2}-\sqrt{x^2-2}-\sqrt{x^2+2}+\sqrt{x^2-2}}=\frac{3}{1}$$

$$\frac{2\sqrt{x^2+2}}{2\sqrt{x^2-2}}=3$$

Squaring both sides, we get.

$$\frac{x^2+2}{x^2-2}=9$$

$$9\left(x^2-2\right)=x^2+2$$

$$9x^2-18=x^2+2$$

$$9x^2-x^2=18+2$$

$$8x^2=20$$

$$x^2=\frac{20}{8}$$

$$x^2=\frac{5}{2}$$

$$x=\pm\sqrt{\frac{5}{2}}$$

Thus. solution set $=\left\{\pm\sqrt{\frac{5}{2}}\right\}$

(viii) Solve $\dfrac{\sqrt{x^2+8p^2}+\sqrt{x^2-p^2}}{\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}}=\dfrac{1}{3}$

Solution:

$$\frac{\sqrt{x^2+8p^2}+\sqrt{x^2-p^2}}{\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}}=\frac{1}{3}$$

Applying componendo-dividendo theorem. we get.

$$\frac{\left[\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}\right]+\left[\sqrt{x^2+8p^2}+\sqrt{x^2-p^2}\right]}{\left[\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}\right]-\left[\sqrt{x^2+8p^2}+\sqrt{x^2-p^2}\right]}=\frac{1+3}{1-3}$$

$$\frac{\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}+\sqrt{x^2+8p^2}+\sqrt{x^2-p^2}}{\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}-\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}}=\frac{4}{-2}$$

$$\frac{2\sqrt{x^2+8p^2}}{-2\sqrt{x^2-p^2}}=-2$$

$$\Rightarrow \quad \frac{\sqrt{x^2+8p^2}}{\sqrt{x^2-p^2}}=2$$

Squaring both sides, we get.

$$\frac{x^2+8p^2}{x^2-p^2}=4$$

$$4\left(x^2-p^2\right)=x^2+8p^2$$

$$4x^2-4p^2=x^2+8p^2$$

$$4x^2-x^2=8p^2+4p^2$$

$$3x^2=12p^2$$

$$x^2=4p^2$$

$$\Rightarrow \quad x=\pm 2p$$

Thus, solution set = $\{+2p, -2p\}$

(ix) Solve $\dfrac{(x+5)^3-(x-3)^3}{(x+5)^3+(x-3)^3}=\dfrac{13}{14}$

Solution:

$$\frac{(x+5)^3-(x-3)^3}{(x+5)^3+(x-3)^3}=\frac{13}{14}$$

Applying componendo-dividendo theorem, we get.

$$\frac{\left[(x+5)^3-(x-3)^3\right]+\left[(x+5)^3+(x-3)^3\right]}{\left[(x+5)^3-(x-3)^3\right]-\left[(x+5)^3+(x-3)^3\right]}=\frac{13+14}{13-14}$$

$$\frac{(x+5)^3-(x-3)^3+(x+5)^3+(x-3)^3}{(x+5)^3-(x-3)^3-(x+5)^3-(x-3)^3}=\frac{27}{-1}$$

$$\frac{2(x+5)^3}{-2(x-3)^3}=-27$$

$$\Rightarrow \quad \frac{(x+5)^3}{(x-3)^3}=27$$

$$\frac{(x+5)^3}{(x-3)^3}=(3)^3$$

Taking power '$\frac{1}{3}$' on both sides, we get.

$$\frac{\left[(x+5)^3\right]^{\frac{1}{3}}}{\left[(x+5)^3\right]^{\frac{1}{3}}}=\left[(3)^3\right]^{\frac{1}{3}}$$

$$\frac{x+5}{x-3}=3$$

$$3(x-3)=x+5$$

$$3x-9=x+5$$

$$3x-x=9+5$$

$$2x=14$$

$$\Rightarrow \quad x=7$$

Thus, solution set = $\{7\}$

(i) Joint variation

A combination of direct and inverse variations of one or more than one variables forms joint variation,

If a variable y varies directly as x and varies inversely as z.

Then $y \propto x$ and $y \propto \frac{1}{z}$

In joint variation, we write it as

$$y \propto \frac{x}{z}$$

i.e., $\quad y = k\frac{x}{z}$

Where $k \neq 0$ is the constant of variation.

SOLVED EXERCISE 3.5

1. If s varies directly as u^2 and inversely as v and s = 7 when M = 3, v = 2. Find the value of s when u = 6 and v = 10.

Solution:

Given that s varies directly as u^2, so

$$S \propto u^2$$

Also given that S varies inversely as V, so

$$S \propto \frac{1}{V}$$