Exercise 6.3

Q1. Use factorization to find the square root of the following expressions.

i) $4x^2-12xy+9y^2$

$=(2x)^2-2(2x)(3y)+(3y)^2$

$=(2x-3y)^2$

Hence $\sqrt{4x^2-12xy+9y^2}$

$=\sqrt{(2x-3y)^2}$

$=\pm(2x-3y)$

ii) $x^2-1+\dfrac{1}{4x^2}$

$=(x)^2-2(x)\left(\dfrac{1}{2x}\right)+\left(\dfrac{1}{2x}\right)^2$

Hence $\sqrt{x^2-1+\dfrac{1}{4x^2}}$

$=\sqrt{\left(x-\dfrac{1}{2x}\right)^2}$

$=\pm\left(x-\dfrac{1}{2x}\right)$

iii) $\dfrac{1}{16}x^2-\dfrac{1}{12}xy+\dfrac{1}{36}y^2$

$=\left(\dfrac{1}{4}x\right)^2-2\left(\dfrac{1}{4}x\right)\left(\dfrac{1}{6}y\right)+\left(\dfrac{1}{6}y\right)^2$

$=\left(\dfrac{1}{4}x-\dfrac{1}{6}y\right)^2$

Hence $\sqrt{\dfrac{1}{16}x^2-\dfrac{1}{12}xy+\dfrac{1}{36}y^2}$

$=\sqrt{\left(\dfrac{1}{4}x-\dfrac{1}{6}y\right)^2}$

$=\pm\left(\dfrac{1}{4}x-\dfrac{1}{6}y\right)$

iv) $4(a+b)^2-12(a^2-b^2)+9(a-b)^2$

$=[2(a+b)]^2-2\times2(a+b)\times3(a-b)+[3(a-b)]^2$

$=[2(a+b)-3(a-b)]^2$

$=(-a+5b)^2$

$=(5b-a)^2$

Hence $\sqrt{4(a+b)^2-12(a^2-b^2)+9(a-b)^2}$

$=\sqrt{(5b-a)^2}$

$=\pm(5b-a)$

v) $\dfrac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^4}$

$=\dfrac{(2x^3)^2-2(2x^3)(3y^3)+(3y^3)^2}{(3x^2)^2+2(3x^2)(4y^2)+(4y^2)^2}$

$$=\frac{\left(2x^3-3y^3\right)^2}{\left(3x^2+4y^2\right)^2}$$

Hence $\sqrt{\frac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^4}}$

$$=\sqrt{\left(\frac{2x^3-3y^3}{3x^2+4y^2}\right)^2}$$

$$=\pm\left(\frac{2x^3-3y^3}{3x^2+4y^2}\right)$$

vi) $\left(x+\frac{1}{x}\right)^2-4\left(x-\frac{1}{x}\right) \qquad (x\neq 0)$

$$=(x)^2+\left(\frac{1}{x}\right)^2+2(\cancel{x})\left(\frac{1}{\cancel{x}}\right)-4\left(x-\frac{1}{x}\right)$$

$$=x^2+\frac{1}{x^2}+2-4\left(x-\frac{1}{x}\right)\ldots\ldots\text{(i)}$$

Let $x-\frac{1}{x}=a$

Squaring $\left(x-\frac{1}{x}\right)^2=(a)^2$

$$x^2+\frac{1}{x^2}-2=a^2$$

$$x^2+\frac{1}{x^2}=a^2+2$$

So expression (i) becomes

$$=a^2+2+2-4a$$

$$=a^2-4a+4$$

$$=(a)^2-2(a)(2)+(2)^2$$

$$=(a-2)^2$$

Putting value of 'a'

$$=\left(x-\frac{1}{x}-2\right)^2$$

Hence $=\sqrt{\left(x-\frac{1}{x}-2\right)^2}$

$$=\pm\left(x-\frac{1}{x}-2\right)$$

vii) $\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12\ldots\text{(i)}$

Let $x+\frac{1}{x}=a$

Squaring $\left(x+\frac{1}{x}\right)^2=(a)^2$

$$x^2+\frac{1}{x^2}+2=a^2$$

$$x^2+\frac{1}{x^2}=a^2-2$$

So expression (i) becomes

$$=\left(a^2-2\right)^2-4(a)^2+12$$

$$=\left(a^2\right)^2-2\left(a^2\right)(2)+(2)^2-4a^2+12$$

$$=a^4-4a^2+4-4a^2+12$$

$$=a^4-8a^2+16$$

$$=\left(a^2\right)^2-2\left(a^2\right)(4)+(4)^2$$

$$=\left(a^2-4\right)^2$$

Putting values of a^2

$$=\left(x^2+\frac{1}{x^2}+2-4\right)^2$$

$$=\left(x^2+\frac{1}{x^2}-2\right)^2$$

Hence $=\sqrt{\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12}$

$$=\sqrt{\left(x^2+\frac{1}{x^2}-2\right)^2}$$

$$=\pm\left(x^2+\frac{1}{x^2}-2\right)$$

viii) $\left(x^2+3x+2\right)\left(x^2+4x+3\right)\left(x^2+5x+6\right)$

$$=\left(x^2+x+2x+2\right)\left(x^2+x+3x+3\right)\left(x^2+2x+3x+6\right)$$

$$=\left[x(x+1)+2(x+1)\right]\left[x(x+1)+3(x+1)\right]\left[x(x+2)+3(x+2)\right]$$

$$=(x+1)(x+2)(x+1)(x+3)(x+2)(x+3)$$

$$=(x+1)^2(x+2)^2(x+3)^2$$

Hence

$$\sqrt{\left(x^2+3x+2\right)\left(x^2+4x+3\right)\left(x^2+5x+6\right)}$$

$$=\sqrt{(x+1)^2(x+2)^2(x+3)^2}$$

$$=\pm(x+1)(x+2)(x+3)$$

ix) $\left(x^2+8x+7\right)\left(2x^2-x-3\right)\left(2x^2+11x-21\right)$

$$=\left(x^2+x+7x+7\right)\left(2x^2+2x-3x-3\right)\left(2x^2+14x-3x-21\right)$$

$$=\left[x(x+1)+7(x+1)\right]\left[2x(x+1)-3(x+1)\right]$$

$$\left[2x(x+7)-3(x+7)\right]$$

$$=(x+1)(x+7)(x+1)(2x-3)(x+7)(2x-3)$$

$$=(x+1)^2(x+7)^2(2x-3)^2$$

Hence

$$\sqrt{\left(x^2+8x+7\right)\left(2x^2-x-3\right)\left(2x^2+11x-21\right)}$$

$$=\sqrt{(x+1)^2(x+7)^2(2x-3)^2}$$

$$=\pm(x+1)(x+7)(2x-3)$$

Q2. Use division method to find the square root of the following expressions.

i) $4x^2+12xy+9y^2+16x+24y+16$

$$\begin{array}{r|l}
 & 2x+3y+4 \\
\hline
2x & 4x^2+12xy+9y^2+16x+24y+16 \\
 & \underline{4x^2} \\
\hline
4x+3y & 12xy+9y^2+16x+24y+16 \\
 & 12xy\pm 9y^2 \\
\hline
4x+6y+4 & 16x+24y+16 \\
 & 16x\pm 24y\pm 16 \\
\hline
 & 0
\end{array}$$

Hence the square root of given expression is

$\pm(2x+3y+4)$

ii) $x^4-10x^3+37x^2-60x+36$

$$\begin{array}{r|l}
 & x^2-5x+6 \\
\hline
x^2 & x^4-10x^3+37x^2-60x+36 \\
 & -x^4 \\
\hline
2x^2-5x & -10x^3+37x^2-60x+36 \\
 & \mp 10x^3\pm 25x^2 \\
\hline
2x^2-10x+6 & -12x^2-60x+36 \\
 & -12x^2\mp 60x\pm 36 \\
\hline
 & 0
\end{array}$$

Hence $\sqrt{x^4-10x^3+37x^2-60x+36}$

$$=\pm\left(x^2-5x+6\right)$$

iii) $9x^4-6x^3+7x^2-2x+1$

$$
\begin{array}{r|l}
 & 3x^2 - x + 1 \\
\hline
3x^2 & 9x^4 - 6x^3 + 7x^2 - 2x + 1 \\
 & -9x^4 \\
\hline
6x^2 - x & \cancel{-6x^3} + 7x^2 - 2x + 1 \\
 & \cancel{\mp 6x^3} \pm x^2 \\
\hline
6x^2 - 2x + 1 & \cancel{6x^2} - \cancel{2x} + \cancel{1} \\
 & \cancel{-6x^2} \mp \cancel{2x} \pm \cancel{1} \\
\hline
 & 0
\end{array}
$$

Hence $\sqrt{9x^4 - 6x^3 + 7x^2 - 2x + 1}$

$$= \pm\left(3x^2 - x + 1\right)$$

iv) $4 + 25x^2 - 12x - 24x^3 + 16x^4$

In descending order

$$= 16x^4 - 24x^3 + 25x^2 - 12x + 4$$

$$
\begin{array}{r|l}
 & 4x^2 - 3x + 2 \\
\hline
4x^2 & 16x^4 - 24x^3 + 25x^2 - 12x + 4 \\
 & -16x^4 \\
\hline
8x^2 - 3x & \cancel{-24x^3} + 25x^2 - 12x + 4 \\
 & \cancel{\mp 24x^3} \pm 9x^2 \\
\hline
8x^2 - 6x + 2 & \cancel{16x^2} - \cancel{12x} + \cancel{4} \\
 & \cancel{-16x^2} \mp \cancel{12x} \pm \cancel{4} \\
\hline
 & 0
\end{array}
$$

Hence $\sqrt{16x^4 - 24x^3 + 25x^2 - 12x + 4}$

$$= \pm\left(4x^2 - 3x + 2\right)$$

v) $\dfrac{x^2}{y^2} - 10\dfrac{x}{y} + 27 - 10\dfrac{y}{x} + \dfrac{y^2}{x^2}$

$(x \neq 0,\ y \neq 0)$

Hence

$$
\begin{array}{r|l}
 & \dfrac{x}{y} - 5 + \dfrac{y}{x} \\
\hline
\dfrac{x}{y} & \dfrac{x^2}{y^2} - 10\dfrac{x}{y} + 27 - 10\dfrac{y}{x} + \dfrac{y^2}{x^2} \\
 & -\dfrac{x^2}{y^2} \\
\hline
\dfrac{2x}{y} - 5 & \cancel{-10\dfrac{x}{y}} + 27 - 10\dfrac{y}{x} + \dfrac{y^2}{x^2} \\
 & \cancel{\mp 10\dfrac{x}{y}} \pm 25 \\
\hline
\dfrac{2x}{y} - 10 + \dfrac{y}{x} & \cancel{2} - \cancel{10\dfrac{y}{x}} + \cancel{\dfrac{y^2}{x^2}} \\
 & \cancel{2} \mp \cancel{10\dfrac{y}{x}} \pm \cancel{\dfrac{y^2}{x^2}} \\
\hline
 & 0
\end{array}
$$

$$\sqrt{\frac{x^2}{y^2} - 10\frac{x}{y} + 27 - 10\frac{y}{x} + \frac{y^2}{x^2}}$$

The required square root

$$= \pm\left(\frac{x}{y} - 5 + \frac{y}{x}\right)$$

Q3. Find the value of $'k'$ for which the following expression will become a perfect square?

i) $4x^4 - 12x^3 + 37x^2 - 42x + k$

$$
\begin{array}{r|l}
 & 2x^2 - 3x + 7 \\
\hline
2x^2 & 4x^4 - 12x^3 + 37x^2 - 42x + k \\
 & -4x^4 \\
\hline
4x^2 - 3x & \cancel{-12x^3} + 37x^2 - 42x + k \\
 & \cancel{\mp 12x^3} \pm 9x^2 \\
\hline
4x^2 - 6x + 7 & \cancel{28x^2} - \cancel{42x} + k \\
 & \cancel{-28x^2} \mp \cancel{42x} \pm 49 \\
\hline
 & k - 49
\end{array}
$$

As given that the given expression is a perfect square, so

$$\text{Remainder} = 0$$

$$k - 49 = 0$$

$\boxed{k=49}$

ii) $x^4-4x^3+10x^2-kx+9$

$$\begin{array}{r|l} & x^2-2x+3 \\ \hline x^2 & x^4-4x^3+10x^2-kx+9 \\ & -x^4 \\ \hline 2x^2-2x & -4x^3+10x^2-kx+9 \\ & \mp 4x^3 \pm 4x^2 \\ \hline 2x^2-4x+3 & 6x^2-kx+9 \\ & -6x^2 \mp 12x \pm 9 \\ \hline & (-k+12)x \end{array}$$

As given that the given expression is a perfect square, so

$$\text{Remainder}=0$$
$$(-k+12)x=0$$

As $x\neq 0$, so $-k+12=0$

$$\Rightarrow \boxed{k=12}$$

Q4. Find the values of 'l' and 'm' for which the following expression will become perfect square.

i) $x^4+4x^3+16x^2+lx+m$

$$\begin{array}{r|l} & x^2+2x+6 \\ \hline x^2 & x^4+4x^3+16x^2+lx+m \\ & -x^4 \\ \hline 2x^2+2x & 4x^3+16x^2+lx+m \\ & -4x^3 \pm 4x^2 \\ \hline 2x^2+4x+6 & 12x^2+lx+m \\ & -12x^2 \pm 24x \pm 36 \\ \hline & (l-24)x+(m-36) \end{array}$$

As the given expression is to be a perfect square, so

$$\text{Remainder}=0$$
$$(l-24)x+(m-36)=0$$

As $x\neq 0$, so $l-24=0$ and $m-36=0$

$\Rightarrow \boxed{l=24}$ and $\boxed{m=36}$

ii) $49x^4-70x^3+109x^2+lx-m$

$$\begin{array}{r|l} & 7x^2-5x+6 \\ \hline 7x^2 & 49x^4-70x^3+109x^2+lx-m \\ & -49x^4 \\ \hline 14x^2-5x & -70x^3+109x^2+lx-m \\ & \mp 70x^3 \pm 25x^2 \\ \hline 14x^2-10x+6 & 84x^2+lx-m \\ & -84x^2 \mp 60x \pm 36 \\ \hline & (l+60)x-m-36 \end{array}$$

As the given expression is to be a perfect square, so

$$(l+60)x-m-36=0$$

As $x\neq 0$, so $l+60=0$ and $-m-36=0$

$\Rightarrow \boxed{l=-60}$ and $\boxed{m=-36}$

Q5. To make the expression $9x^4-12x^3+22x^2-13x+12$ a perfect square.

i) What should be added to it?

ii) What should be subtracted from it?

iii) What should be the value of 'x'?

$$\begin{array}{r|l} & 3x^2-2x+3 \\ \hline 3x^2 & 9x^4-12x^3+22x^2-13x+12 \\ & -9x^4 \\ \hline 6x^2-2x & -12x^3+22x^2-13x+12 \\ & \mp 12x^3 \pm 4x^2 \\ \hline 6x^2-4x+3 & 18x^2-13x+12 \\ & -18x^2 \mp 12x \pm 9 \\ \hline & -x+3 \end{array}$$

To make the given expression a complete square

i) $x-3$ should be added

ii) $-x+3$ should be subtracted

iii) For value of 'x'

Remainder = 0

$-x+3=0$

$\boxed{x=3}$

Q6. Find H.C.F of following by factorization

$8x^4 - 128, 12x^3 - 96$.

Solution:

$8x^4 - 128 = 8(x^4 - 16)$
$= 8((x^2)^2 - (4)^2)$
$= 8(x^2 + 4)(x^2 - 4)$
$= 8(x^2+4)(x+2)(x-2)$

$12x^3 - 96 = 12(x^3 - 8)$
$= 12(x^3 - 2^3)$
$= 12(x-2)(x^2+2x+4)$

Common factor $= 4(x-2)$

H.C.F $= 4(x-2)$

Q7. Find H.C.F of following by division method.

$y^3 + 3y^2 - 3y - 9, y^3 + 3y^2 - 8y - 24$

Solution:

$$\begin{array}{r|l} & 1 \\ y^3+3y^2-3y-9 & y^3+3y^2-8y-24 \\ & -y^3 \pm 3y^2 \mp 3y \mp 9 \\ \hline & -5y-15 \\ & -5(y+3) \end{array}$$

$$\begin{array}{r|l} & y^2-3 \\ (y+3) & y^3+3y^2-3y-9 \\ & -y^3 \pm 3y^2 \\ \hline & -3y-9 \\ & \mp 3y \pm 9 \\ \hline & \times \end{array}$$

H.C.F = $y + 3$

Q8. Find L.C.M of following by factorization.

$12x^2\ 75, 6x^2 - 13x - 5, 4x^2 - 20x + 25$

Solution:

$12x^2 - 75 = 3(4x^2 - 25)$
$= 3((2x)^2 - (5)^2)$
$= 3(2x+5)(2x-5)$

$6x^2 - 13x - 5 = 6x^2 - 15x + 2x - 5$
$= 3x(2x - 5) + 1(2x - 5)$
$= (3x + 1)(2x - 5)$

$4x^2 - 20x + 25 = (2x)^2 + (5)^2 - 2(2x)(5)$
$= (2x - 5)^2$
$= (2x - 5)(2x - 5)$

L.C.M $= (2x - 5)^2 \times 3(2x + 5)(3x+1)$
$= 3(2x - 5)^2(2x+5)(3x+1)$

Q9. If H.C.F of $x^4+3x^3+5x^2+26x+56$ and $x^4 + 2x^3 - 4x^2 - x + 28$ is $x^2 + 5x + 7$, find the

Solution:

$$\text{L.C.M} = \frac{(x^4+3x^3+5x^2+26x+56)(x^4+2x^3-4x^2-x+28)}{x^2+5x+7}$$

$$\begin{array}{r|l} & x^2-2x+8 \\ x^2+5x+7 & \cancel{x^4}+3x^3+5x^2+26x+56 \\ & \cancel{-x^4} \pm 5x^3 \pm 7x^2 \\ \hline & \cancel{-2x^3}-2x^2+26x+56 \\ & \cancel{\mp 2x^3} \mp 10x^2 \mp 14x \\ \hline & 8x^2+40x+56 \\ & -8x^2 \pm 40x \pm 56 \\ \hline & \times \end{array}$$

L.C.M

$=(x^2-2x+8)(x^4+2x^3-4x^2-x+28)$

Q10. Simplify

(i) $\dfrac{3}{x^3+x^2+x+1} - \dfrac{3}{x^3-x^2+x-1}$

$$\frac{3}{(x^2+1)(x+1)} - \frac{3}{(x^2+1)(x-1)}$$

$$= \frac{3(x-1)-3(x+1)}{(x^2+1)(x+1)(x-1)}$$

$$= \frac{\cancel{3x}-3-\cancel{3x}-3}{(x^2+1)(x+1)(x-1)}$$

$$= \frac{-6}{(x^2+1)(x+1)(x-1)}$$

$$= \frac{-6}{(x^2+1)(x^2-1)}$$

$$= \frac{-6}{x^4-1} = \frac{6}{1-x^4} \text{ Ans.}$$

(ii) $\dfrac{a+b}{a^2-b^2} \div \dfrac{a^2-ab}{a^2-2ab+b^2}$

$$= \frac{a+b}{(a-b)(a+b)} \div \frac{a(a-b)}{(a-b)^2}$$

$$= \frac{1}{a-b} \div \frac{a}{a-b}$$

$$= \frac{1}{\cancel{a-b}} \times \frac{\cancel{a-b}}{a}$$

$$= \frac{1}{a}$$

Q11. Find square root by using factorization

$$\left(x^2+\frac{1}{x^2}\right)+10\left(x+\frac{1}{x}\right)+27 \qquad (x \neq 0)$$

Solution:

$$= \left(x^2+\frac{1}{x^2}\right)+10\left(x+\frac{1}{x}\right)+25+2$$

$$= x^2+\frac{1}{x^2}+2+10\left(x+\frac{1}{2}\right)+25$$

$$= \left(x+\frac{1}{x}\right)^2+10\left(x+\frac{1}{x}\right)+25$$

Let $x+\dfrac{1}{x}=a$

$$= a^2+10a+25$$

$$= (a+5)^2$$

Taking square root

$$= \sqrt{\left[\pm(a+5)\right]^2}$$

$$= \pm(a+5)$$

$$= \pm\left(x+\frac{1}{x}+5\right)$$

Q12. Find square root by using division method.

$$\frac{4x^2}{y^2}+\frac{20x}{y}+13-\frac{30y}{x}+\frac{9y^2}{x^2} \quad (x, y \neq 0)$$

Solution:

	$\frac{2x}{y}+5-\frac{3y}{x}$
$\frac{2x}{y}$	$\frac{4x^2}{y^2}+\frac{20x}{y}+13-\frac{30y}{x}+\frac{9y^2}{x^2}$
	$-\frac{4x^2}{y^2}$
$\frac{4x}{y}+5$	$\frac{20}{y}x+13$
	$-\frac{20}{y}x\pm25$
$\frac{4x}{y}+10-\frac{3y}{x}$	$-12-\frac{30y}{x}+\frac{9y^2}{x^2}$
	$\mp12\mp\frac{30y}{x}\pm\frac{9y^2}{x^2}$
	×

Required square root $= \pm\left(\dfrac{2x}{y}+5-\dfrac{3y}{x}\right)$