EXERCISE 4.2

Question # 1(i)

Since $P(x,y) = P(3,2)$

i.e. $x=3$ and $y=2$

$O'(h,k) = O'(1,3)$

i.e. $h=1$ and $k=3$

$\because\ X = x-h$

$= 3-1 = 2$

Also $Y = y-k$

$= 2-3 = -1$

Hence $(2,-1)$ is point P in $XY-$coordinates.

Question # 1(ii) & (iii)

Do yourself

Question # 1(iv)

Since $P(x,y) = P\left(\frac{3}{2},\frac{5}{2}\right)$

i.e. $x = \frac{3}{2}$ and $y = \frac{5}{2}$

$O'(h,k) = O'\left(-\frac{1}{2},\frac{7}{2}\right)$

i.e. $h=-\frac{1}{2}$ and $k=\frac{7}{2}$

$\because\ X = x-h$

$= \frac{3}{2}-\left(-\frac{1}{2}\right) = 2$

And $Y = y-k$

$= \frac{5}{2}-\frac{7}{2} = -1$

Hence $(2,-1)$ are coordinates of P in XY-axes.

Question # 2(i)

$\because\ P(X,Y) = P(8,10)$

$\Rightarrow\ X=8$ and $Y=10$

$O'(h,k) = O'(3,4)$

$\Rightarrow\ h=3$ and $k=4$

$\because\ X = x-h$

$\Rightarrow\ 8 = x-3$

$\Rightarrow\ x = 8+3 \quad \Rightarrow\ x = 11$

Also $Y = y-k$

$\Rightarrow\ 10 = y-4$

$\Rightarrow\ y = 10+4 \quad \Rightarrow\ y = 14$

Hence $(11,14)$ are coordinates of P in xy-axes.

Question # 2(ii)

Do yourself

Question # 2(iii)

$\because\ P(X,Y) = P\left(-\frac{3}{4},-\frac{7}{6}\right)$

$\Rightarrow\ X=-\frac{3}{4}$ and $Y=-\frac{7}{6}$

$O'(h,k) = O'\left(\frac{1}{4},-\frac{1}{6}\right)$

$\Rightarrow\ h=\frac{1}{4}$ and $k=-\frac{1}{6}$

$\because\ X = x-h$

$\Rightarrow\ -\frac{3}{4} = x-\frac{1}{4}$

$\Rightarrow\ x = -\frac{3}{4}+\frac{1}{4} \quad \Rightarrow\ x = -\frac{1}{2}$

Also $Y = y-k$

$\Rightarrow\ -\frac{7}{6} = y+\frac{1}{6}$

$\Rightarrow\ y = -\frac{7}{6}-\frac{1}{6} \quad \Rightarrow\ y = -\frac{4}{3}$

Hence $\left(-\frac{1}{2},-\frac{4}{3}\right)$ is the required point.

Question # 2(iv)

Do yourself

Rotation of Axes

Let (x,y) be the coordinates of point P in xy-coordinate system. If the axes are rotated through at angle of θ and (X,Y) are coordinate of P in new XY-coordinate system then

$X = x\cos\theta + y\sin\theta$

$Y = y\cos\theta - x\sin\theta$

Question # 3(i)

$\because\ P(x,y) = P(5,3)$

$\Rightarrow\ x=5\ \&\ y=3\ ,\ \theta=45^\circ$

Since $X = x\cos\theta + y\sin\theta$

$= 5\cos 45^\circ + 3\sin 45^\circ$

$= 5\left(\frac{1}{\sqrt{2}}\right)+3\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}(5+3)$

$= \frac{8}{\sqrt{2}} = \frac{4\times 2}{\sqrt{2}} = 4\sqrt{2}$

Now $Y = y\cos\theta - x\sin\theta$

$= 3\cos 45^\circ - 5\sin 45^\circ$

$= 3\left(\frac{1}{\sqrt{2}}\right)-5\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}(3-5)$

$= -2\left(\frac{1}{\sqrt{2}}\right) = -\sqrt{2}$

Hence the required point is $\left(4\sqrt{2},-\sqrt{2}\right)$.

Question # 3(ii)

$\because\ P(x,y) = P(3,-7)$

$\Rightarrow\ x=3\ \&\ y=-7\ ,\ \theta=30^\circ$

Since $X = x\cos\theta + y\sin\theta$

$= 3\cos 30^\circ - 7\sin 30^\circ$

$$\Rightarrow X = 3\left(\frac{\sqrt{3}}{2}\right) - 7\left(\frac{1}{2}\right)$$

$$= \frac{3\sqrt{3}-7}{2}$$

Now $Y = y\cos\theta - x\sin\theta$

$$= -7\cos 30^\circ - 3\sin 30^\circ$$

$$= -7\left(\frac{\sqrt{3}}{2}\right) - 3\left(\frac{1}{2}\right) = \frac{-7\sqrt{3}-3}{2}$$

Hence the required point is

$$\left(\frac{3\sqrt{3}-7}{2}, \frac{-7\sqrt{3}-3}{2}\right).$$

Question # 3(iii)

Do yourself

Question # 3(iv)

$\because P(x,y) = P(15,10)$

$\Rightarrow x = 15 \quad \& \quad y = 10$

Also $\theta = \tan^{-1}\left(\frac{1}{3}\right)$

$\Rightarrow \tan\theta = \frac{1}{3}$

$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{1/\sqrt{10}}{3/\sqrt{10}}$

$\Rightarrow \sin\theta = \frac{1}{\sqrt{10}}, \quad \cos\theta = \frac{3}{\sqrt{10}}$

$\tan\theta = \frac{y}{x} = \frac{1}{3}$

$\Rightarrow x = 3, \; y = 1$

$r = \sqrt{x^2+y^2}$

$= \sqrt{3^2+1^2}$

$= \sqrt{10}$

Now $X = x\cos\theta + y\sin\theta$

$$= 15\left(\frac{3}{\sqrt{10}}\right) + 10\left(\frac{1}{\sqrt{10}}\right)$$

$$= \frac{1}{\sqrt{10}}(45+10) = \frac{55}{\sqrt{10}}$$

$Y = y\cos\theta - x\sin\theta$

$$= 10\left(\frac{3}{\sqrt{10}}\right) - 15\left(\frac{1}{\sqrt{10}}\right)$$

$$= \frac{1}{\sqrt{10}}(30-15) = \frac{15}{\sqrt{10}}$$

Hence the required point is $\left(\frac{55}{\sqrt{10}}, \frac{15}{\sqrt{10}}\right)$.

Question # 3(iv) (Edition 2007)

$\because P(x,y) = P(15,10)$

$\Rightarrow x = 15 \quad \& \quad y = 10$

Also $\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{1/2}{\sqrt{3}/2}$

$\Rightarrow \sin\theta = \frac{1}{2}, \quad \cos\theta = \frac{\sqrt{3}}{2}$

$\tan\theta = \frac{y}{x} = \frac{1}{\sqrt{3}}$

$\Rightarrow x = \sqrt{3}, \; y = 1$

$r = \sqrt{x^2+y^2}$

$= \sqrt{\left(\sqrt{3}\right)^2+1^2}$

$= \sqrt{4} = 2$

Now $X = x\cos\theta + y\sin\theta$

$$= 15\left(\frac{\sqrt{3}}{2}\right) + 10\left(\frac{1}{2}\right)$$

$$= \frac{15\sqrt{3}+10}{2}$$

$Y = y\cos\theta - x\sin\theta$

$$= 10\left(\frac{\sqrt{3}}{2}\right) - 15\left(\frac{1}{2}\right)$$

$$= \frac{10\sqrt{3}-15}{2}$$

Hence the required point is

$$\left(= \frac{15\sqrt{3}+10}{2}, \frac{10\sqrt{3}-15}{2}\right).$$

Question # 4(i)

$\because P(X,Y) = P(-5,3)$

$\Rightarrow X = -5 \; \& \; Y = 3$

Also $\theta = 30^\circ$

Therefore $\sin\theta = \frac{1}{2} \quad \& \quad \cos\theta = \frac{\sqrt{3}}{2}$

Now $X = x\cos\theta + y\sin\theta$

$$\Rightarrow -5 = x\left(\frac{\sqrt{3}}{2}\right) + y\left(\frac{1}{2}\right)$$

$\Rightarrow \sqrt{3}x + y = -10$ …….. (i)

Also $Y = y\cos\theta - x\sin\theta$

$$\Rightarrow 3 = y\left(\frac{\sqrt{3}}{2}\right) - x\left(\frac{1}{2}\right)$$

$\Rightarrow 6 = \sqrt{3}y - x$

$\Rightarrow x = \sqrt{3}y - 6$ ……. (ii)

Putting value of x in (i)

$$\sqrt{3}\left(\sqrt{3}y-6\right) + y = -10$$

$\Rightarrow 3y - 6\sqrt{3} + y = -10$

$\Rightarrow 4y = -10 + 6\sqrt{3}$

$$\Rightarrow y = \frac{-10+6\sqrt{3}}{4}$$

$$= \frac{-5+3\sqrt{3}}{2}$$

Putting value of y in (ii)

$$x = \sqrt{3}\left(\frac{-5+3\sqrt{3}}{2}\right) - 6$$

$$= \frac{-5\sqrt{3}+9}{2} - 6 = \frac{-5\sqrt{3}+9-12}{2}$$

$$= \frac{-5\sqrt{3}-3}{2}$$

Hence $\left(\frac{-5\sqrt{3}-3}{2}, \frac{-5+3\sqrt{3}}{2}\right)$ is required point.

Question # 4(ii)

Do yourself

Question # 4(iii)

Since $P(X,Y) = P\left(-\frac{2}{13}, \frac{7}{13}\right)$

$\Rightarrow \quad X = -\frac{2}{13} \quad \& \quad Y = \frac{7}{13}$

Now $\theta = \arctan\frac{5}{12}$

$\Rightarrow \quad \tan\theta = \frac{5}{12}$

$\Rightarrow \quad \frac{\sin\theta}{\cos\theta} = \frac{5/13}{12/13}$

$\Rightarrow \quad \sin\theta = \frac{5}{13}$ and $\cos\theta = \frac{12}{13}$

Now do yourself as above.

$\tan\theta = \frac{y}{x} = \frac{5}{12}$

$\Rightarrow \quad x = 12 \ , \ y = 5$

$r = \sqrt{x^2 + y^2}$

$= \sqrt{12^2 + 5^2}$

$= \sqrt{169} = 13$