Exercise 9.3

Q1. Find the mid point of the line segment joining each of the following pairs of points.

a) $A(9,2)$, $B(7,2)$

If $R(x,y)$ is the desired midpoint then,

$$x=\frac{x_1+x_2}{2}=\frac{9+7}{2}=\frac{16}{2}=8$$

$$y=\frac{y_1+y_2}{2}=\frac{2+2}{2}=\frac{4}{2}=2$$

$\therefore\ R(x,y)=R(8,2)$

b) $A(2,6)$, $B(3,-6)$

If $R(x,y)$ is the desired midpoint then,

$$x=\frac{x_1+x_2}{2}=\frac{2+3}{2}=\frac{5}{2}$$

$$y=\frac{y_1+y_2}{2}=\frac{6-6}{2}=\frac{0}{2}=0$$

$$R(x,y)=R\left(\frac{5}{2},0\right)$$

c) $A(-8,1)$, $B(6,1)$

If $R(x,y)$ is the desired midpoint then

$$x=\frac{x_1+x_2}{2}=\frac{-8+6}{2}=\frac{-2}{2}=-1$$

$$y=\frac{y_1+y_2}{2}=\frac{1+1}{2}=\frac{2}{2}=1$$

$$\therefore\ R(x,y)=R(-1,1)$$

d) $A(-4,9),\ B(-4,-3)$

If $R(x,y)$ is the desired mid point then,

$$x=\frac{x_1+x_2}{2}=\frac{-4-4}{2}=\frac{-8}{2}=-4$$

$$y=\frac{y_1+y_2}{2}=\frac{9-3}{2}=\frac{6}{2}=3$$

$$R(x,y)=R(-4,3)$$

e) $A(3,-11),\ B(3,-4)$

If $R(x,y)$ is the desired midpoint then,

$$x=\frac{x_1+x_2}{2}=\frac{3+3}{2}=\frac{6}{2}=3$$

$$y=\frac{y_1+y_2}{2}=\frac{-11-4}{2}=\frac{-15}{2}=-7.5$$

$$\therefore\ R(x,y)=R(3,-7.5)$$

f) $A(0,0),\ B(0,-5)$

If $R(x,y)$ is the desired midpoint then,

$$x=\frac{x_1+x_2}{2}=\frac{0+0}{2}=0$$

$$y=\frac{y_1+y_2}{2}=\frac{0-5}{2}=\frac{-5}{2}=-2.5$$

$$\therefore\ R(x,y)=R(0,-2.5)$$

Q2. The end point P of a line segment PQ (–3,6) and its mid point is (5,8). Find the co-ordinates of the end point Q.

Sol: $(-3,6)$

If $R(x,y)$ is mid point then,

$$x=\frac{x_1+x_2}{2}\Rightarrow 5=\frac{-3+x_2}{2}$$

$$\Rightarrow 10=-3+x_2$$

$$x_2=10+3=13$$

and $y=\frac{y_1+y_2}{2}\Rightarrow 8=\frac{6+y_2}{2}$

$$\Rightarrow 16=6+y_2$$

$$y_2=10$$

$\therefore$ Coordinates of the end point $Q(13,10)$

Q3. Prove that midpoint of the hypotenuse of a right triangle is equidistant from its three vertices $P(-2,5),Q(1,3)$ and $R(-1,0)$

SOL. $|PQ|^2=(1+2)^2+(3-5)^2=9+4=13$

$$|QR|^2=(-1-1)^2+(0-3)^2=4+9=13$$

$$|PR|^2=(-1+2)^2+(0-5)^2=1+25=26$$

As $|PQ|^2+|QR|^2=|PR|^2$

Hence PR is the hypotenuse

If $M(x,y)$ is desired midpoint then,

$$x=\frac{-1+(-2)}{2}=\frac{-1-2}{2}=\frac{-3}{2}$$

$$y=\frac{5+0}{2}=\frac{5}{2}$$

$$\therefore\quad M(x,y)=M\left(\frac{-3}{2},\frac{5}{2}\right)$$

Now $|PM|=\sqrt{\left(\frac{-3}{2}+2\right)^2+\left(\frac{5}{2}-5\right)^2}$

$$=\sqrt{\left(\frac{-3+4}{2}\right)^2+\left(\frac{5}{2}-10\right)^2}$$

$$=\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{-5}{2}\right)^2}$$

$$=\sqrt{\frac{1}{4}+\frac{25}{4}}$$

$$=\sqrt{\frac{26}{4}}$$

$$|RM|=\sqrt{\left(-\frac{3}{2}+1\right)^2+\left(\frac{5}{2}-0\right)^2}$$

$$=\sqrt{\left(\frac{-3+2}{2}\right)^2+\left(\frac{5-0}{2}\right)^2}$$

$$=\sqrt{\left(\frac{-1}{2}\right)^2+\left(\frac{5}{2}\right)^2}=\sqrt{\frac{1}{4}+\frac{25}{4}}$$

$$=\sqrt{\frac{1+25}{4}}=\sqrt{\frac{26}{4}}$$

$$|QM|=\sqrt{\left(-\frac{3}{2}-1\right)^2+\left(\frac{5}{2}-3\right)^2}$$

$$=\sqrt{\left(\frac{-3-2}{2}\right)^2+\left(\frac{5-6}{2}\right)^2}$$

$$=\sqrt{\left(\frac{-5}{2}\right)^2+\left(-\frac{1}{2}\right)^2}=\sqrt{\frac{25}{4}+\frac{1}{4}}==\sqrt{\frac{26}{4}}$$

As $|PM|=|RM|=|QM|$

$\therefore$ M is equidistant from P, Q and R.

Q4. O (0, 0), A(3, 0) and B(3, 5) are three points in the plane, find M_1 and M_2 as midpoints of the line segments $\overline{AB}$ and $\overline{OB}$ respectively. Find $|M_1, M_2|$.

Sol: Let O (0,0), A(3,0), B(3,5) are three points in the plane. M_1 is the mid point of $\overline{OB}$ and M_2 is the mid-point of $\overline{AB}$

$$M(x,y)=M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$=M_1\left(\frac{0+3}{2},\frac{0+5}{2}\right)$$

$$=M_1\left(\frac{3}{2},\frac{5}{2}\right)$$

M_2 is midpoint of $\overline{AB}$ therefore

$$M_2\left(\frac{3+3}{2},\frac{0+5}{2}\right)=M_2\left(\frac{6}{2},\frac{5}{2}\right)$$

$$=M_2\left(3,\frac{5}{2}\right)$$

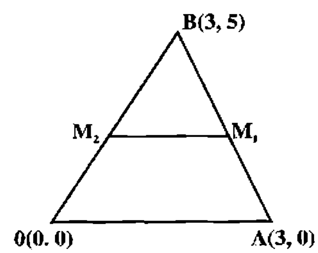
Now $\left(\frac{3}{2},\frac{5}{2}\right)$ and $\left(3,\frac{5}{2}\right)$ are midpoints we find $|M_1 M_2|$

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Then $|M_1 M_2|=\sqrt{\left(3-\frac{3}{2}\right)^2+\left(\frac{5}{2}-\frac{5}{2}\right)}$

$$=\sqrt{\left(\frac{6-3}{2}\right)+0}$$

$$=\sqrt{\left(\frac{3}{2}\right)^2+0}=\frac{3}{2}$$



Q5. Show that the diagonals of the parallelogram having vertices $A(1,2)$, $B(4,2)$, $C(-1,-3)$, $D(-4,-3)$ bisect each other.

Sol: If M_1 is desired midpoint of diagonal DB.

$$x=\frac{x_1+x_2}{2}=\frac{4-4}{2}=0$$

$$y=\frac{y_1+y_2}{2}=\frac{2-3}{2}=\frac{-1}{2}$$

$$M_1(x,y)=\left(0,-\frac{1}{2}\right)$$

If M_2 is desired midpoint of diagonal AC

$$x=\frac{x_1+x_2}{2}=\frac{1-1}{2}=0$$

$$y=\frac{y_1+y_2}{2}=\frac{2-3}{2}=\frac{-1}{2}$$

$$M_2(x,y)=\left(0,-\frac{1}{2}\right)$$

$\therefore$ As midpoints of the diagonals coincide hence diagonal bisect each other.

Q6. The vertices of a triangle are P(4,6), Q(–2,–4) and R(–8, 2) show that the length of line segment joining the mid points of line segment PR, QR is $\frac{1}{2}$ PQ.

Sol. If M_1 is desired midpoint of line segment PR.

$$x=\frac{x_1+x_2}{2}=\frac{4-8}{2}=\frac{-4}{2}=-2$$

$$y=\frac{y_1+y_2}{2}=\frac{6+2}{2}=\frac{8}{2}=4$$

$$M_1(x,y)=M_1(-2,4)$$

If M_2 is desired midpoint of line segment QR.

$$x=\frac{x_1+x_2}{2}=\frac{-2-8}{2}=\frac{-10}{2}=-5$$

$$y=\frac{y_1+y_2}{2}=\frac{-4+2}{2}=\frac{-2}{2}=-1$$

$$M_2(x,y)=M_2(-5,-1)$$

$$|M_1M_2|=\sqrt{(-5+2)^2+(-1-4)^2}$$

$$=\sqrt{(-3)^2+(-5)^2}$$

$$=\sqrt{9+25}=\sqrt{34}$$

$$|PQ|=\sqrt{(-2-4)^2+(-4-6)^2}$$

$$=\sqrt{(-6)^2+(-10)^2}$$

$$=\sqrt{36+100}=\sqrt{136}=\sqrt{34\times 4}$$

$$=2\sqrt{34}$$

As $2|M_1M_2|=|PQ|$

Hence $|M_1M_2|=\frac{1}{2}|PQ|$