

## Exercise 12.5

### Question # 1

Solve the following triangle  $ABC$  in which:

$$b = 95, \quad c = 34, \quad \alpha = 52^\circ.$$

#### Solution

$$b = 95, \quad c = 34, \quad \alpha = 52^\circ.$$

By law of cosine

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc \cos \alpha \\ &= (95)^2 + (34)^2 - 2(95)(34) \cos 52^\circ \\ &= 9025 + 1156 - 6460(0.6157) = 6203.578 \end{aligned}$$

$$\Rightarrow a = \sqrt{6203.578} \Rightarrow \boxed{a = 78.76}$$

Again by law of cosine

$$\begin{aligned} \cos \beta &= \frac{c^2 + a^2 - b^2}{2ca} = \frac{(34)^2 + (78.76)^2 - (95)^2}{2(34)(78.76)} \\ &= \frac{1156 + 6203.138 - 9025}{5355.68} = -\frac{1665.862}{5355.68} = -0.311 \end{aligned}$$

$$\beta = \cos^{-1}(-0.311) \Rightarrow \boxed{\beta = 108^\circ 7' 20''}$$

Since in any triangle

$$\alpha + \beta + \gamma = 180$$

$$\Rightarrow \gamma = 180 - \alpha - \beta = 180^\circ - 52^\circ - 108^\circ 7' 20'' \Rightarrow \boxed{\gamma = 19^\circ 52' 40''}$$

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### Question # 2

Solve the following triangle  $ABC$  in which:

$$b = 12.5, \quad c = 23, \quad \alpha = 38^\circ 20'$$

#### Solution

$$b = 12.5, \quad c = 23, \quad \alpha = 38^\circ 20'$$

By law of cosine

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc \cos \alpha \\ &= (12.5)^2 + (23)^2 - 2(12.5)(23) \cos 38^\circ 20' \\ &= 156.25 + 529 - 575(0.7844) = 234.21 \end{aligned}$$

$$\Rightarrow a = \sqrt{234.21} \Rightarrow \boxed{a = 15.304}$$

Again by law of cosine

$$\begin{aligned} \cos \beta &= \frac{c^2 + a^2 - b^2}{2ca} = \frac{(23)^2 + (15.304)^2 - (12.5)^2}{2(23)(15.304)} \\ &= \frac{529 + 234.21 - 156.25}{703.984} = \frac{606.96}{703.984} = 0.8622 \end{aligned}$$

$$\beta = \cos^{-1}(0.8622) \Rightarrow \boxed{\beta = 30^\circ 26'}$$

Since in any triangle

$$\alpha + \beta + \gamma = 180$$

$$\Rightarrow \gamma = 180 - \alpha - \beta = 180^\circ - 38^\circ 20' - 30^\circ 26' \Rightarrow \boxed{\gamma = 111^\circ 14'}$$

### Question # 3

Solve the following triangle  $ABC$  in which:

$$a = \sqrt{3} - 1 = 0.732, \quad b = \sqrt{3} + 1 = 2.732, \quad \gamma = 60^\circ$$

**Solution**

$$a = \sqrt{3} - 1 = 0.732, \quad b = \sqrt{3} + 1 = 2.732, \quad \gamma = 60^\circ$$

By law of cosine

$$\begin{aligned} c^2 &= a^2 + b^2 - ab \cos \gamma \\ &= (0.732)^2 + (2.732)^2 - 2(0.732)(2.732) \cos 60^\circ \\ &= 0.5358 + 7.4638 - 1.9998 = 5.9998 \approx 6 \end{aligned}$$

$$\Rightarrow \boxed{c = \sqrt{6} = 2.449}$$

Again by law of cosines

$$\begin{aligned} \cos \alpha &= \frac{b^2 + c^2 - a^2}{2bc} = \frac{(2.732)^2 + (2.449)^2 - (0.732)^2}{2(2.732)(2.449)} \\ &= \frac{7.4638 + 5.9976 - 0.5358}{13.3813} = \frac{12.9256}{13.3813} = 0.9659 \end{aligned}$$

$$\Rightarrow \alpha = \cos^{-1}(0.9659) \Rightarrow \boxed{\alpha = 15^\circ}$$

Since in any triangle

$$\alpha + \beta + \gamma = 180$$

$$\Rightarrow \beta = 180 - \alpha - \gamma = 180 - 15 - 60 \Rightarrow \boxed{\beta = 105^\circ}$$

### Question # 4

Solve the following triangle  $ABC$  in which:

$$a = 3, \quad b = 6, \quad \gamma = 36^\circ 20'$$

**Solution**

*Do yourself as above*

### Question # 5

Solve the following triangle  $ABC$  in which:

$$a = 7, \quad b = 3, \quad \gamma = 38^\circ 13'$$

**Solution**

*Do yourself as above*

### Question # 6

Solve the following triangle, using first law of tangent and then law of sines:

$$a = 36.21, \quad b = 42.09, \quad \gamma = 44^\circ 29'$$

**Solution**

$$\begin{aligned} \text{Since} \quad & \alpha + \beta + \gamma = 180 \\ \Rightarrow & \alpha + \beta = 180 - \gamma \end{aligned}$$

$$= 180 - 44^\circ 29'$$

$$\Rightarrow \alpha + \beta = 135^\circ 31' \dots\dots\dots (i)$$

By law of tangent

$$\frac{a-b}{a+b} = \frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(\frac{\alpha+\beta}{2}\right)} \Rightarrow \frac{36.21-42.09}{36.21+42.09} = \frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(\frac{135^\circ 31'}{2}\right)}$$

$$\Rightarrow \frac{-5.88}{78.3} = \frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan(67^\circ 45')} \Rightarrow -0.0751 = \frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{2.4443}$$

$$\Rightarrow \tan\left(\frac{\alpha-\beta}{2}\right) = -0.0751(2.4443)$$

$$= -0.1836$$

$$\Rightarrow \frac{\alpha-\beta}{2} = \tan^{-1}(-0.1836)$$

$$\Rightarrow \frac{\alpha-\beta}{2} = -10^\circ 24' \Rightarrow \alpha - \beta = -20^\circ 48' \dots\dots\dots (ii)$$

Adding (i) & (ii)

$$\begin{array}{rcl} \alpha + \beta & = & 135^\circ 31' \\ \alpha - \beta & = & -20^\circ 48' \\ \hline 2\alpha & = & 114^\circ 43' \Rightarrow \boxed{\alpha = 57^\circ 22'} \end{array}$$

Putting value of  $\alpha$  in eq. (i)

$$57^\circ 22' + \beta = 135^\circ 22'$$

$$\Rightarrow \beta = 135^\circ 22' - 57^\circ 22' \Rightarrow \boxed{\beta = 78^\circ 9'}$$

Now by law of sine

$$\frac{c}{\sin \gamma} = \frac{a}{\sin \alpha} \Rightarrow \frac{c}{\sin 44^\circ 29'} = \frac{36.21}{\sin 57^\circ 22'}$$

$$\Rightarrow c = \frac{36.21}{\sin 57^\circ 22'} \cdot \sin 44^\circ 29'$$

$$= \frac{36.21}{0.8421} \cdot 0.7007 \Rightarrow \boxed{c = 30.13}$$

### Question # 7, 8 & 9

Solve the following triangle, using first law of tangent and then law of sines:

- (7)  $a = 93$ ,  $c = 101$ ,  $\alpha = 80^\circ$       (8)  $b = 14.8$ ,  $c = 16.1$ ,  $\alpha = 42^\circ 45'$   
 (9)  $a = 319$ ,  $b = 168$ ,  $\alpha = 110^\circ 22'$

**Solution**

*Do yourself as above*

**Question # 10**

Solve the following triangle, using first law of tangent and then law of sines:

$$b = 61 \quad , \quad c = 32 \quad , \quad \alpha = 59^\circ 30'$$

**Solution**

$$b = 61 \quad , \quad c = 32 \quad , \quad \alpha = 59^\circ 30'$$

Since  $\alpha + \beta + \gamma = 180$

$$\Rightarrow \beta + \gamma = 180 - \alpha$$

$$= 180 - 59^\circ 30'$$

$$\Rightarrow \beta + \gamma = 120^\circ 30' \dots\dots\dots (i)$$

By law of tangent

$$\frac{b-c}{b+c} = \frac{\tan\left(\frac{\beta-\gamma}{2}\right)}{\tan\left(\frac{\beta+\gamma}{2}\right)} \Rightarrow \frac{61-32}{61+32} = \frac{\tan\left(\frac{\beta-\gamma}{2}\right)}{\tan\left(\frac{120^\circ 30'}{2}\right)}$$

$$\Rightarrow \frac{29}{93} = \frac{\tan\left(\frac{\beta-\gamma}{2}\right)}{\tan(60^\circ 15')} \Rightarrow 0.3118 = \frac{\tan\left(\frac{\beta-\gamma}{2}\right)}{1.7496}$$

$$\Rightarrow \tan\left(\frac{\beta-\gamma}{2}\right) = 0.3118(1.7496)$$

$$= 0.5455$$

$$\Rightarrow \frac{\beta-\gamma}{2} = \tan^{-1}(0.5455)$$

$$\Rightarrow \frac{\beta-\gamma}{2} = 28^\circ 37' \quad \Rightarrow \beta - \gamma = 57^\circ 14' \dots\dots\dots (ii)$$

Adding (i) & (ii)

$$\beta + \gamma = 120^\circ 30'$$

$$\beta - \gamma = 57^\circ 14'$$

$$\underline{\hspace{1cm}} \quad 2\beta = 177^\circ 44' \quad \Rightarrow \quad \boxed{\beta = 88^\circ 52'}$$

Putting value of  $\alpha$  in eq. (i)

$$88^\circ 52' + \gamma = 120^\circ 30'$$

$$\Rightarrow \gamma = 120^\circ 30' - 88^\circ 52' \quad \Rightarrow \quad \boxed{\gamma = 31^\circ 38'}$$

Now by law of sine

$$\frac{c}{\sin \gamma} = \frac{a}{\sin \alpha} \Rightarrow \frac{32}{\sin 31^\circ 38'} = \frac{a}{\sin 59^\circ 30'}$$

$$\Rightarrow c = \frac{32}{\sin 31^\circ 38'} \cdot \sin 59^\circ 30'$$

$$= \frac{32}{0.5244} \cdot 0.8616 \quad \Rightarrow \quad \boxed{c = 52.57}$$

**Question # 11**

Measure of two sides of the triangle are in the ratio 3 : 2 and they include an angle of measure  $57^\circ$ . Find the remaining two angles.

**Solution**

Let  $a : b = 3 : 2$

i.e.  $\frac{a}{b} = \frac{3}{2} \Rightarrow a = \frac{3}{2}b$

and  $\gamma = 57^\circ$

Since  $\alpha + \beta + \gamma = 180$

$\Rightarrow \alpha + \beta = 180 - \gamma$

$= 180 - 57 \Rightarrow \alpha + \beta = 123^\circ \dots\dots\dots (i)$

By law of tangent

$$\begin{aligned} \frac{a-b}{a+b} &= \frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(\frac{\alpha+\beta}{2}\right)} \Rightarrow \frac{\frac{3}{2}b-b}{\frac{3}{2}b+b} = \frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(\frac{123^\circ}{2}\right)} \\ \Rightarrow \frac{\frac{1}{2}b}{\frac{5}{2}b} &= \frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan(61^\circ 30')} \Rightarrow \frac{1}{5} = \frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{1.8418} \\ \Rightarrow \tan\left(\frac{\alpha-\beta}{2}\right) &= \frac{1}{5}(1.8418) \\ &= 0.3684 \\ \Rightarrow \frac{\alpha-\beta}{2} &= \tan^{-1}(0.3684) = 20^\circ 13' \\ \Rightarrow \alpha - \beta &= 40^\circ 26' \dots\dots\dots (ii) \end{aligned}$$

Adding (i) & (ii)

$$\begin{aligned} \alpha + \beta &= 123^\circ \\ \alpha - \beta &= 40^\circ 27' \\ \hline 2\alpha &= 163^\circ 27' \Rightarrow \boxed{\alpha = 81^\circ 44'} \end{aligned}$$

Putting value of  $\alpha$  in eq. (i)

$$\begin{aligned} 81^\circ 44' + \beta &= 123^\circ \\ \Rightarrow \beta &= 123^\circ - 81^\circ 44' \Rightarrow \boxed{\beta = 41^\circ 16'} \end{aligned}$$

**Question # 12**

Two forces of 40N and 30N are represented by  $\overrightarrow{AB}$  and  $\overrightarrow{BC}$  which are inclined at an angle of  $147^\circ 25'$ . Find  $\overrightarrow{AC}$ , the resultant of  $\overrightarrow{AB}$  and  $\overrightarrow{BC}$ .

**Solution**

Since  $\overrightarrow{AB} = c = 40N$

$$\begin{aligned}\overrightarrow{BC} &= a = 30N \\ m\angle B &= \beta = 147^\circ 25' \\ \overrightarrow{AC} &= b = ?\end{aligned}$$

By law of cosine

$$\begin{aligned}b^2 &= c^2 + a^2 - 2ca \cos \beta \\ &= (40)^2 + (30)^2 - 2(40)(30) \cos 147^\circ 25' \\ &= 1600 + 900 - 2400(-0.8426) \\ &= 4522.26\end{aligned}$$

$$\Rightarrow b = \sqrt{4522.26} = 67.248$$

$$\text{i.e. } \overrightarrow{AC} = 67.248N$$

