Exercise 9.2

Q1. Show whether the points with vertices $(5,-2),(5,4)$ and $(-4,1)$ are vertices of an equilateral triangle or an isosceles triangle?

SOL. Let P(5,-2) , Q(5,4), R(-4,1)

$$|PQ|=\sqrt{(5-5)^2+(4+2)^2}=\sqrt{(0)^2+(6)^2}=\sqrt{36}=6$$

$$|QR|=\sqrt{(-4-5)^2+(1-4)^2}=\sqrt{81+9}=\sqrt{90}$$

$$|PR|=\sqrt{(-4-5)^2+(1+2)^2}=\sqrt{81+9}=\sqrt{90}$$

Since $|QR|=|PR|=\sqrt{90}$ and

$|PQ|=6\neq\sqrt{90}$

So the non collinear points P,Q,R form an isosceles triangle PQR

Q2. Show whether or not the points with vertices $(-1,1),(5,4),(2,-2)$ and $(-4,1)$ form a square.

Sol. Let $A(-1,1),B(5,4),C(2,-2),D(-4,1)$

Since $|AB|=\sqrt{(5+1)^2+(4-1)^2}$

$$=\sqrt{6^2+3^2}=\sqrt{36+9}=\sqrt{45}$$

$$|BC|=\sqrt{(2-5)^2+(-2-4)^2}$$

$$|BC|=\sqrt{(-3)^2+(-6)^2}=\sqrt{9+36}=\sqrt{45}$$

$$|CD|=\sqrt{(-4-2)^2+(1+2)^2}$$

$$=\sqrt{(-6)^2+(3)^2}=\sqrt{36+9}=\sqrt{45}$$

$$|DA|=\sqrt{(-4+1)^2+(1-1)^2}$$

$$=\sqrt{(-3)^2+(0)^2}=\sqrt{9}=3$$

Hence $|AB|=|BC|=|CD|=\sqrt{45}$

but $|DA|\neq\sqrt{45}$

Hence given points do not form a square.

Q3. Show whether or not the points with coordinates $(1,3)(4,2)$, and $(-2,6)$ are vertices of a right triangle.

Sol. Let $P(1,3)$, $Q(4,2)$ and $R(-2,6)$

$$|PQ|=\sqrt{(4-1)^2+(2-3)^2}$$

$$=\sqrt{(3)^2+(1)^2}=\sqrt{9+1}=\sqrt{10}$$

$$|QR| = \sqrt{(-2-4)^2 + (6-2)^2}$$
$$= \sqrt{36+16} = \sqrt{52}$$
$$|PR| = \sqrt{(-2-1)^2 + (6-3)^2}$$
$$|BC| = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

Now $|PQ|^2 + |QR|^2 = \left(\sqrt{10}\right)^2 + \left(\sqrt{52}\right)^2$
$$= 10 + 52 = 62$$

and $|PR|^2 = \left(\sqrt{18}\right)^2 = 18$
$$|PQ|^2 + |QR|^2 \neq |PR|^2$$

So triangle is not right angled

Q4. Use the distance formula to prove whether or not the points $(1,1), (-2,-8)$ and $(4,10)$ lie on a straight line.

Let $A(1,1)$, $B(-2,-8)$, $C(4,10)$

Since $|AB| = \sqrt{(-2-1)^2 + (-8-1)^2}$
$$= \sqrt{(-3)^2 + (-9)^2} = \sqrt{9+81} = \sqrt{90} = 3\sqrt{10}$$
$$|BC| = \sqrt{(4+2)^2 + (10+8)^2}$$
$$|BC| = \sqrt{(6)^2 + (18)^2}$$
$$= \sqrt{36+324} = \sqrt{360}$$
$$= \sqrt{2\times2\times2\times3\times3\times5} = 6\sqrt{10}$$
$$|AC| = \sqrt{(4-1)^2 + (10-1)^2}$$
$$= \sqrt{(3)^2 + (9)^2} = \sqrt{9+81} = \sqrt{90} = 3\sqrt{10}$$
$$|AB| + |AC| = 3\sqrt{10} + 3\sqrt{10}$$
$$= 6\sqrt{10} = |BC|$$

So $|AB| + |AC| = |BC|$ the points A, B and C are collinear.

Q5. Find K given that the point $(2,K)$ is equidistance from $(3,7)$ and $(9,1)$.

Sol. Let $P(2,K), Q(3,7)$ and $R(9,1)$
$$|PQ| = \sqrt{(3-2)^2 + (7-K)^2}$$
$$= \sqrt{1^2 + (7-K)^2} = \sqrt{1 + (7-K)^2}$$
$$= \sqrt{1 + 49 - 2(7)k + k^2}$$
$$= \sqrt{50 - 14k + k^2}$$
$$|PR| = \sqrt{(9-2)^2 + (1-K)^2}$$
$$= \sqrt{49 + 1 - 2(1)k + k^2}$$
$$= \sqrt{50 - 2k + k^2}$$

As point P is equidistant from Q and

So $|PQ| = |PR|$
$$\sqrt{50 - 14k + k^2} = \sqrt{50 - 2k + k^2}$$
$$50\text{-}14\text{ k} + \text{k}^2 = 50 - 2\text{k} + \text{k}^2$$
$$-12k = 0 \Rightarrow k = 0$$

Q6. Use distance formula to verify that the points $A(0,7)$, $B(3,-5)$, $C(-2,15)$ are collinear.

So $|AB| = \sqrt{(3-0)^2 + (-5-7)^2}$
$$= \sqrt{9 + (-12)^2} = \sqrt{9+144}$$
$$= \sqrt{153} = 12.37$$
$$|BC| = \sqrt{(-2-3)^2 + (15+5)^2}$$
$$= \sqrt{25+400} = \sqrt{425} = 20.62$$
$$|CA| = \sqrt{(-2-0)^2 + (15-7)^2}$$
$$= \sqrt{4+64} = \sqrt{68} = 8.25$$

As $|AB| + |CA| = |BC|$

So given points are collinear with A between B and C.

Q7. Verify whether or not the points $O(0,0)$, $A(\sqrt{3},1)$, $B(\sqrt{3}-1)$ are vertices of a equilateral triangle.

Sol. $|OA| = \sqrt{(\sqrt{3}-0)+(1-0)^2}$

$= \sqrt{(\sqrt{3})^2+(1)^2}$

$= \sqrt{3+1} = \sqrt{4} = 2$

$|AB| = \sqrt{(\sqrt{3}-\sqrt{3})^2+(-1-1)^2}$

$= \sqrt{(0)^2+(-2)^2} = \sqrt{0+4} = 2$

$|OB| = \sqrt{(\sqrt{3}-0)^2+(-1-0)^2}$

$= \sqrt{(\sqrt{3})^2+(-1)^2}$

$= \sqrt{3+1} = \sqrt{4} = 2$

As $|OA| = |AB| = |OB| = 2$

Hence points are not collinear.

$\therefore$ the triangle OAB is equilateral

Q8. Show that the points A(-6,-5), B(5,-5), C(5,-8), D(-6,-8) are vertices of a rectangle. Find the lengths of its diagonals. Are they equal ?.

Sol. $|AB| = \sqrt{(5+6)^2+(-5+5)^2}$

$= \sqrt{(11)^2+(0)^2} = \sqrt{121} = 11$

$|BC| = \sqrt{(5-5)^2+(-8+5)^2}$

$= \sqrt{(0)^2+(-3)^2} = \sqrt{9} = 3$

$|DC| = \sqrt{(5+6)^2+(-8+8)^2}$

$= \sqrt{(11)^2+(0)^2} = \sqrt{121} = 11$

$|AD| = \sqrt{(-6+6)^2+(-8+5)^2}$

$= \sqrt{(-3)^2} = \sqrt{9} = 3$

Since $|AB| = |DC| = 11$ and

$|AD| = |BC| = 3$ opposite sides are equal

Diagonal $|AC| = \sqrt{(5+6)^2+(-8+5)^2}$

$= \sqrt{11^2+3^2} = \sqrt{121+9} = \sqrt{130}$

Diagonal $|BD| = \sqrt{(-6-5)^2+(-8+5)^2}$

$= \sqrt{11^2+3^2} = \sqrt{121+9} = \sqrt{130}$

$|AD|^2+|DC|^2 = |AC|^2$

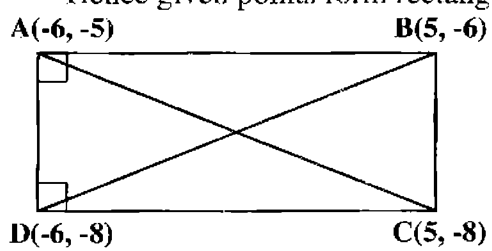
$\therefore \quad \angle ADC = 90^\circ$

Also $|AB|^2+|AD|^2 = |BD|^2$

$\therefore \angle BAD = 90^\circ$

$|AC| = |BD| = \sqrt{130}$

Hence given points form rectangle



As $|AC| = |BD| = \sqrt{130}$

Hence diagonals are equal.

Q9. Show that the points $M(-1,4)$, $N(-5,3)$, $P(1,-3)$ and $Q(5,-2)$ are the vertices of a parallelogram.

SOL. $|PQ| = \sqrt{(5-1)^2+(-2+3)^2}$

$= \sqrt{(4)^2+(1)^2} = \sqrt{16+1} = \sqrt{17}$

$$|MN| = \sqrt{(-5+1)^2 + (3-4)^2}$$

$$= \sqrt{(-4)^2 + (-1)^2} = \sqrt{16+1} = \sqrt{17}$$

$$|NP| = \sqrt{(1+5)^2 + (-3-3)^2}$$

$$= \sqrt{(6)^2 + (-6)^2} = \sqrt{36+36} = \sqrt{72}$$

$$|MQ| = \sqrt{(5+1)^2 + (-2-4)^2}$$

$$= \sqrt{6^2 + (-6)^2} = \sqrt{36+36} = \sqrt{72}$$

Since $|PQ| = |MN| = \sqrt{17}$

and $|NP| = |MQ| = \sqrt{72}$

So opposite sides, of quadrilateral MNPQ are equal.

$$|NQ| = \sqrt{(-5-5)^2 + (3+2)^2}$$

$$= \sqrt{(-10)^2 + (5)^2}$$

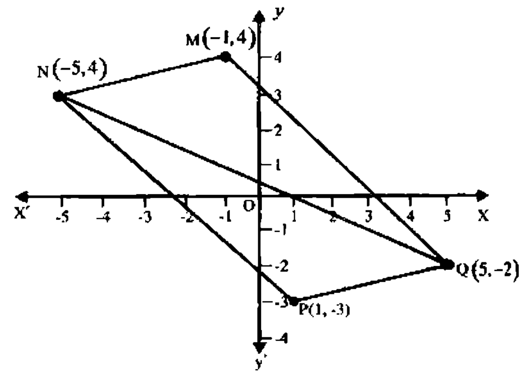
$$= \sqrt{100+25} = \sqrt{125} = 5\sqrt{5}$$

$$|PN|^2 + |PQ|^2 = \left(\sqrt{72}\right)^2 + \left(\sqrt{17}\right)^2$$

$$= 72 + 17 = 89$$

$$|PN|^2 + |PQ|^2 \neq |NQ|^2$$

The measure of angle at $P \neq 90^o$



Hence given points form a parallelogram.

Q10. Find the length of the diameter of the circle having centre at $C(-3,6)$ and passing through $P(1,3)$.

SOL. Length of radius=

$$|PC| = \sqrt{(-3-1)^2 + (6-3)^2}$$

$$= \sqrt{(-4)^2 + (3)^2}$$

$$= \sqrt{16+9}$$

$$= \sqrt{25}$$

$$= 5$$

Length of diameter $= 2r = 2(r) = 10$