Exercise 7.1

Q1. Solve the following equations.

i) $\frac{2}{3}x-\frac{1}{2}x=x+\frac{1}{6}$

Sol: Multiplying both sides by 6

$$^{2}\cancel{6}\left(\frac{2}{\cancel{3}}x\right)-{}^{3}\cancel{6}\left(\frac{1}{\cancel{2}}x\right)=6(x)+\cancel{6}\left(\frac{1}{\cancel{6}}\right)$$

$$4x-3x=6x+1$$

$$x=6x+1$$

$$-1=6x-x$$

$$-1=5x$$

$$\Rightarrow \quad \boxed{x=-\frac{1}{5}}$$

Check:

Substituting $x=-\frac{1}{5}$ in the given equation

$$\frac{2}{3}\left(-\frac{1}{5}\right)-\frac{1}{2}\left(-\frac{1}{5}\right)=-\frac{1}{5}+\frac{1}{6}$$

$$-\frac{2}{15}+\frac{1}{10}=-\frac{1}{5}+\frac{1}{6}$$

$$\frac{-4+3}{30}=\frac{-6+5}{30}$$

$$-\frac{1}{30}=-\frac{1}{30}$$ which is true

Hence solution set $=\left\{-\frac{1}{5}\right\}$

ii) $\frac{x-3}{3}-\frac{x-2}{2}=-1$

Multiplying both sides by 6

$$^{2}\cancel{6}\left(\frac{x-3}{\cancel{3}}\right)-{}^{3}\cancel{6}\left(\frac{x-2}{\cancel{2}}\right)=6(-1)$$

$$2x-\cancel{6}-3x+\cancel{6}=-6$$

$$-x=-6$$

$$\boxed{x=6}$$

Check:

Substituting $x=6$ in the given equation

$$\frac{6-3}{3}-\frac{6-2}{2}=-1$$

$$\frac{3}{3}-\frac{4}{2}=-1$$

$$1-2=-1$$

$-1=-1$ which is true, so solution set $=\{6\}$

iii) $\frac{1}{2}\left(x-\frac{1}{6}\right)+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left(\frac{1}{2}-3x\right)$

$$\frac{1}{2}x-\frac{1}{12}+\frac{2}{3}=\frac{5}{6}+\frac{1}{6}-\frac{1}{\cancel{3}}(\cancel{3}x)$$

Multiplying both sides by 12

$$\cancel{12}\left(\frac{1}{\cancel{2}}x\right)-\cancel{12}\left(\frac{1}{\cancel{12}}\right)+{}^{4}\cancel{12}\left(\frac{2}{\cancel{3}}\right)={}^{2}\cancel{12}\left(\frac{5}{\cancel{6}}\right)+{}^{2}\cancel{12}\left(\frac{1}{\cancel{6}}\right)-12(x)$$

$$6x-1+8=10+2-12x$$

$$6x+7=12-12x$$

$$6x+12x=12-7$$

$$18x=5$$

$$\boxed{x=\frac{5}{18}}$$

Check:

Substituting $x=\frac{5}{18}$ in the given equation

$$\frac{1}{2}\left(\frac{5}{18}-\frac{1}{6}\right)+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left(\frac{1}{2}-\cancel{3}\times\frac{5}{{}^{6}\cancel{18}}\right)$$

$$\frac{1}{2}\left(\frac{5-3}{18}\right)+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left(\frac{3-5}{6}\right)$$

$$\frac{1}{2}\left(\frac{2}{18}\right)+\frac{2}{3}=\frac{5}{6}-\frac{2}{18}$$

$$\frac{1+12}{18}=\frac{15-2}{18}$$

$$\frac{13}{18}=\frac{13}{18}$$ which is true, so

Solution set $=\left\{\frac{5}{18}\right\}$

(iv) $x+\frac{1}{3}=2\left(x-\frac{2}{3}\right)-6x$

$$x+\frac{1}{3}=2x-\frac{4}{3}-6x$$

Multiplying both sides by 3

$$3x+\cancel{3}\times\frac{1}{\cancel{3}}=3(2x)-\cancel{3}\left(\frac{4}{\cancel{3}}\right)-3(6x)$$

$$3x+1=6x-4-18x$$

$$3x+1=-12x-4$$

$$15x=-5$$

$$x=-\frac{5}{15}$$

$$\boxed{x=-\frac{1}{3}}$$

Check:

Substituting $x=-\frac{1}{3}$ in the given equation

$$-\frac{1}{3}+\frac{1}{3}=2\left(-\frac{1}{3}-\frac{2}{3}\right)-\cancel{6}\left(-\frac{1}{\cancel{3}}\right)$$

$$0 = 2\left(-\frac{\cancel{3}}{\cancel{3}}\right)+2$$

$0 = -2+2$

$0 = 0$ which is true, so

Solution set $= \left\{-\frac{1}{3}\right\}$

v) $\frac{5(x-3)}{6} - x = 1 - \frac{x}{9}$

Multiplying both sides by 18

$$^{3}\cancel{18} \times \frac{5(x-3)}{\cancel{6}} - 18x = 18 - {}^{2}\cancel{18}\left(\frac{x}{\cancel{9}}\right)$$

$15(x-3)-18x = 18-2x$

$15x-45-18x = 18-2x$

$15x-18x+2x = 18+45$

$-x = 63$

$\Rightarrow \quad x = -63$

Check:

Substituting $x = -63$ in the given equation

$$\frac{5(-63-3)}{6} - (-63) = 1 - \frac{(-63)}{9}$$

$$5\frac{\left(\overset{11}{\cancel{-66}}\right)}{\cancel{6}} + 63 = 1 + \frac{\cancel{63}^{7}}{\cancel{9}}$$

$-55+63 = 1+7$

$8 = 8$ which is true, so

Solution set $= \{-63\}$

vi) $\frac{x}{3x-6} = 2 - \frac{2x}{x-2}$

$$\frac{x}{3(x-2)} = 2 - \frac{2x}{x-2}$$

Multiplying both sides by $3(x-2)$

$$\cancel{3}(x-2) \times \frac{x}{\cancel{3}(x-2)} = 2 \times 3(x-2) - \frac{2x}{x-2} \times 3(x-2)$$

$x = 6x-12-6x$

$\boxed{x = -12}$

Check:

Substituting $x = -12$ in the given equation

$$\frac{-12}{3(-12)-6} = 2 - \frac{2(-12)}{-12-2}$$

$$\frac{-12}{-36-6} = 2 - \frac{(\cancel{-24})}{\cancel{-14}}$$

$$\frac{-12}{-42} = 2 - \frac{12}{7}$$

$$\frac{2}{7} = \frac{14-12}{7}$$

$\frac{2}{7} = \frac{2}{7}$ which is true, so

Solution Set $= \{-12\}$

vii) $\frac{2x}{2x+5} = \frac{2}{3} - \frac{5}{4x+10}$, $x \neq -\frac{5}{2}$

$$\frac{2x}{2x+5} = \frac{2}{3} - \frac{5}{2(2x+5)}$$

Multiplying both sides by $6(2x+5)$

$$6\cancel{(2x+5)} \times \frac{2x}{\cancel{2x+5}} = \frac{2}{\cancel{3}} \times {}^{2}\cancel{6}\,(2x+5) - \frac{5}{\cancel{2(2x+5)}} \times {}^{3}\cancel{6}\,\cancel{(2x+5)}$$

$12x = 8x+20-15$

$12x-8x = 5$

$4x = 5$

$4x = 5$

$x = \frac{5}{4}$

Check:

Substituting $x = \frac{5}{4}$ in the given equation

$$\frac{\cancel{2}\left(\frac{5}{\cancel{4}}\right)}{\cancel{2}\left(\frac{5}{\cancel{4}}\right)+5} = \frac{2}{3} - \frac{5}{\cancel{4}\left(\frac{5}{\cancel{4}}\right)+10}$$

$$\frac{\frac{5}{2}}{\frac{5+10}{2}} = \frac{2}{3} - \frac{5}{15}$$

$$\frac{5}{15} = \frac{2}{3} - \frac{1}{3}$$

$\frac{1}{3} = \frac{1}{3}$ which is true, so

Solution set $= \left\{\frac{5}{4}\right\}$

viii) $\frac{2x}{x-1} + \frac{1}{3} = \frac{5}{6} + \frac{2}{x-1}$, $x \neq 1$

Multiplying both sides by $6(x-1)$

$$6(x-1) \times \frac{2x}{x-1} + {}^{2}\cancel{6}(x-1) \times \frac{1}{\cancel{3}}$$

$$= \cancel{6}(x-1) \times \frac{5}{\cancel{6}} + 6(x-1) \times \frac{2}{x-1}$$

$12x + 2x - 2 = 5x - 5 + 12$

$12x + 2x - 5x = 2 - 5 + 12$

$9x = 9$

$x = \frac{9}{9}$

$\boxed{x = 1}$

Check:

Substituting $x = 1$ in the given equator

$$\frac{2(1)}{1-1} + \frac{1}{3} = \frac{5}{6} + \frac{2}{1-1}$$

$$\frac{2}{0} + \frac{1}{3} = \frac{5}{6} + \frac{2}{0}$$

As $\frac{2}{0}$ is undefined, so $x = 1$ cannot be a solution thus the given equation has no solution.

ix) $\frac{2}{x^2-1} - \frac{1}{x+1} = \frac{1}{x+1}$, $x \neq \pm 1$

$$\frac{2}{(x+1)(x-1)} - \frac{1}{x+1} = \frac{1}{x+1}$$

Multiplying both sides by $(x+1)(x-1)$

$$\cancel{(x+1)(x-1)} \times \frac{2}{\cancel{(x+1)(x-1)}}$$

$$-\cancel{(x+1)}(x-1) \times \frac{1}{\cancel{x+1}} = \frac{1}{\cancel{x+1}} \times \cancel{(x+1)}(x-1)$$

$2 - x + 1 = x - 1$

$2 + 1 + 1 = x + x$

$2x = 4$

$x = \frac{4}{2}$

$x = 2$

Check:

Substituting $x = 2$ in the given equation

$$\frac{2}{(2)^2 - 1} - \frac{1}{2+1} = \frac{1}{2+1}$$

$$\frac{2}{4-1} - \frac{1}{3} = \frac{1}{3}$$

$$\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

$\frac{1}{3} = \frac{1}{3}$ which is true, so

Solution Set $= \{2\}$

x) $\frac{2}{3x+6} = \frac{1}{6} - \frac{1}{2x+4}$, $x \neq -2$

$$\frac{2}{3(x+2)} = \frac{1}{6} - \frac{1}{2(x+2)}$$

Multiplying both sides by $6(x+2)$

$${}^{2}\cancel{6}\cancel{(x+2)} \times \frac{2}{\cancel{3}\cancel{(x+2)}} =$$

$$\frac{1}{\cancel{6}} \times \cancel{6}(x+2) - \frac{1}{\cancel{2}\cancel{(x+2)}} \times {}^{3}\cancel{6}\cancel{(x+2)}$$

$4 = x + 2 - 3$

$4 = x - 1$

$4 + 1 = x$

$\boxed{x = 5}$

Check:

Substituting $x = 5$ in the given equation

$$\frac{2}{3(5)+6} = \frac{1}{6} - \frac{1}{2(5)+4}$$

$$\frac{2}{15+6} = \frac{1}{6} - \frac{1}{10+4}$$

$$\frac{2}{21} = \frac{1}{6} - \frac{1}{14}$$

$$\frac{2}{21} = \frac{7-3}{42}$$

$$\frac{2}{21} = \frac{4}{42}$$

$\frac{2}{21} = \frac{2}{21}$ which is true, so

Solution Set = $\{5\}$

Q2. Solve each question and check for extraneous solution, if any.

i) $\sqrt{3x+4} = 2$

Squaring both sides

$\left(\sqrt{3x+4}\right)^2 = (2)^2$

$3x + 4 = 4$

$3x = 4 - 4$

$3x = 0$

$x = \frac{0}{3}$

$\boxed{x = 0}$

Check:

Substituting $x = 0$ in the given equation

$\sqrt{3x+4} = 2$

$\sqrt{3(0)+4} = 2$

$\sqrt{0+4} = 2$

$\sqrt{4} = 2$

$2 = 2$ which is true, so

Solution Set = $\{0\}$

ii) $\sqrt[3]{2x-4} - 2 = 0$

$\sqrt[3]{2x-4} = 2$

Taking cube of both sides

$\left(\sqrt[3]{2x-4}\right)^3 = (2)^3$

$2x - 4 = 8$

$2x = 8 + 4$

$2x = 12$

$x = \frac{12}{2}$

$\boxed{x = 6}$

Check:

Putting $x = 6$ in the given equation.

$\sqrt[3]{2x-4} - 2 = 0$

$\sqrt[3]{2(6)-4} - 2 = 0$

$\sqrt[3]{12-4} - 2 = 0$

$\sqrt[3]{8} - 2 = 0$

$\sqrt[3]{2^3} - 2 = 0$

$2 - 2 = 0$

$0 = 0$ which is true, so

Solution Set = $\{6\}$

iii) $\sqrt{x-3} - 7 = 0$

or $\sqrt{x-3} = 7$

Squaring both sides

$\left(\sqrt{x-3}\right)^2 = (7)^2$

$x - 3 = 49$

$x = 49 + 3$

$x = 52$

Check:

Putting $x = 52$ in the given equation

$$\sqrt{x-3}-7=0$$
$$\sqrt{52-3}-7=0$$
$$\sqrt{49}-7=0$$
$$7-7=0$$
$$0=0 \quad \text{which true, so}$$

Solution Set = $\{52\}$

iv) $2\sqrt{t+4}=5$

$$\sqrt{t+4}=\frac{5}{2}$$

Squaring both sides

$$\left(\sqrt{t+4}\right)^2=\left(\frac{5}{2}\right)^2$$
$$t+4=\frac{25}{4}$$
$$t=\frac{25}{4}-4$$
$$=\frac{25-16}{4}$$
$$\boxed{t=\frac{9}{4}}$$

Check:

Putting $t=\frac{9}{4}$ in the given equation.

$$2\sqrt{t+4}=5$$
$$2\sqrt{\frac{9}{4}+4}=5$$
$$2\sqrt{\frac{9+16}{4}}=5$$
$$2\sqrt{\frac{25}{4}}=5$$
$$\cancel{2}\left(\frac{5}{\cancel{2}}\right)=5$$
$$5=5 \quad \text{which is true, so}$$

Solution Set = $\left\{\frac{9}{4}\right\}$

v) $\sqrt[3]{2x+3}=\sqrt[3]{x-2}$

Taking cube of both sides

$$\left(\sqrt[3]{2x+3}\right)^3=\left(\sqrt[3]{x-2}\right)^3$$
$$2x+3=x-2$$
$$2x-x=-2-3$$
$$\boxed{x=-5}$$

Check:

Putting $x=-5$ in the given equation.

$$\sqrt[3]{2x+3}=\sqrt[3]{x-2}$$
$$\sqrt[3]{2(-5)+3}=\sqrt[3]{-5-2}$$
$$\sqrt[3]{-10+3}=\sqrt[3]{-7}$$
$$\sqrt[3]{-7}=\sqrt[3]{-7} \quad \text{which is true, so}$$

Solution Set = $\{-5\}$

vi) $\sqrt[3]{2-t}=\sqrt[3]{2t-28}$

Taking cube of both sides

$$\left(\sqrt[3]{2-t}\right)^3=\left(\sqrt[3]{2t-28}\right)^3$$
$$2-t=2t-28$$
$$2+28=2t+t$$
$$3t=30$$
$$t=\frac{\cancel{30}}{\cancel{3}}$$
$$\boxed{t=10}$$

Check:

Putting $t=3$ in the given equation

$$\sqrt[3]{2-t}=\sqrt[3]{2t-28}$$
$$\sqrt[3]{2-10}=\sqrt[3]{2\times10-28}$$

$\sqrt[3]{-8} = \sqrt[3]{20-28}$

$\sqrt[3]{-8} = \sqrt[3]{-8}$ which is true, so

Solution Set = $\{10\}$

vii) $\sqrt{2t+6} - \sqrt{2t-5} = 0$ or

$\sqrt{2t+6} = \sqrt{2t-5}$

Squaring both sides

$\left(\sqrt{2t+6}\right)^2 = \left(\sqrt{2t-5}\right)^2$

$2t+6 = 2t-5$

$2t - 2t + 6 = -5$

$6 = -5$ which is not possible, so

Solution Set = $\{\ \}$

viii) $\sqrt{\dfrac{x+1}{2x+5}} = 2, \quad x \neq -\dfrac{5}{2}$

Squaring both sides

$\left(\sqrt{\dfrac{x+1}{2x+5}}\right)^2 = (2)^2$

$\dfrac{x+1}{2x+5} = 4$

$x+1 = 4(2x+5)$

$x+1 = 8x+20$

$1-20 = 8x-x$

$-19 = 7x$

$\Rightarrow \quad \boxed{x = -\dfrac{19}{7}}$

Check:

Putting $x = -\dfrac{19}{7}$ in the given equation

$$\sqrt{\frac{x+1}{2x+5}} = 2$$

$$\sqrt{\frac{\frac{-19}{7}+1}{2\left(-\frac{19}{7}\right)+5}} = 2$$

$$\sqrt{\frac{\frac{-19+7}{7}}{\frac{-38+35}{7}}} = 2$$

$$\sqrt{\frac{-12}{-3}} = 2$$

$\sqrt{4} = 2$

$2 = 2$ which is true, so

Solution Set = $\left\{\dfrac{-19}{7}\right\}$