EXERCISE 1.4

Question # 1:

$(i)\qquad f(x)=2x^2+x-5 \qquad\qquad c=1$

$$\lim_{x\to1^-} f(x)=\lim_{x\to1^-}\left(2x^2+x-5\right)=2(1)^2+1-5=2+1-5=-2$$

$$\lim_{x\to1^+} f(x)=\lim_{x\to1^+}\left(2x^2+x-5\right)=2(1)^2+1-5=2+1-5=-2$$

$$\Rightarrow \qquad \lim_{x\to1^{-1}} f(x)=\lim_{x\to1^+} f(x)=-2 \qquad\qquad \therefore \qquad \lim_{x\to1} f(x)=-2$$

$(ii)\qquad f(x)=\dfrac{x^2-9}{x-3} \qquad\qquad C=-3$

$$\lim_{x\to-3^-} f(x)=\lim_{x\to-3^-}\frac{x^2-9}{x-3}=\frac{\lim\limits_{x\to-3^-}\left(x^2-9\right)}{\lim\limits_{x\to-3^-}(x-3)}=\frac{(-3)^2-9}{-3-3}=\frac{9-9}{-6}=\frac{0}{-6}=0$$

$$Now \qquad \lim_{x\to-3^+} f(x)=\lim_{x\to-3^+}\frac{x^2-9}{x-3}=\frac{\lim\limits_{x\to-3^+}\left(x^2-9\right)}{\lim\limits_{x\to-3^+}(x-3)}=\frac{(-3)^2-9}{-3-3}=\frac{9-9}{-6}=\frac{0}{-6}=0$$

$$\Rightarrow \qquad \lim_{x\to-3^-} f(x)=\lim_{x\to-3^+} f(x)=0 \qquad\qquad \therefore \qquad \lim_{x\to-3} f(x)=0$$

$(iii)\qquad f(x)=|x-5| \qquad\qquad C=5$

$$\lim_{x\to5^-} f(x)=\lim_{x\to5^-}|x-5| \qquad\qquad |x-5|=\pm(x-5)$$

$-(x-5)$		$+(x-5)$	
$-\infty$	5		$+\infty$

$$=\lim_{x\to5^-}\left[-(x-5)\right]=-\lim_{x\to5}(x-5)=-(5-5)=0$$

$$\lim_{x\to5^+} f(x)=\lim_{x\to5^+}|x-5|=\lim_{x\to5^+}(x-5)=5-5=0$$

$$\Rightarrow \qquad \lim_{x\to5^-} f(x)=\lim_{x\to5^+} f(x)=0$$

$$\lim_{x\to5} f(x)=0$$

Question # 2:

Discuss the continuity of $\quad f(x) \quad at \quad x=c$

$$(i)\qquad f(x)=\begin{cases}2x+5 & if \quad x\le 2\\ 4x+1 & if \quad x>2\\ & c=2\end{cases}$$

We have to discuss the continuity of $f(x) \quad at \quad x=2$

$(a)\qquad f(2)=2(2)+5=4+5=9 \qquad ....(1)$

$(b)\qquad \lim\limits_{x\to2} f(x)=?$

$f(x)=2x+5$		$f(x)=4x+1$
$-\infty$	2	$+\infty$

$$\lim_{x\to2^-} f(x)=\lim_{x\to2}(2x+5)=2(2)+5=4+5=9$$

$$and \qquad \lim_{x\to2^+} f(x)=\lim_{x\to2}(4x+1)=4(2)+1=8+1=9$$

$$\therefore \qquad \lim_{x\to2} f(x)=9 \qquad ....(2)$$

(c) *from* (1) *and* (2) *we get*

$\lim_{x\to 2} f(x) = f(2)$

$\therefore$ $f(x)$ *is continous at* $x = 2$

$$(ii) \quad f(x)=\begin{cases} 3x-1 & if \quad x<1 \\ 4 & x=1 \quad c=2 \\ 2x & x>1 \end{cases}$$

if $c = 2$ $f(c) = f(2)$

is not defined so given function is discontinous

(ii) *Correction*

$$f(x)=\begin{cases} 3x-1 & if \quad x<1 \\ 4 & if \quad x=1 \\ 2x & if \quad x>1 \end{cases}$$

$c = 1$ $(correction)$

$$\begin{array}{ccc} & f(1)=4 & \\ \hline -\infty & 1 & +\infty \\ f(x)=3x-1 & & f(x)=2x \end{array}$$

(a) $f(1) = 4$ $(given)$

(b) $\lim_{x\to 1} f(x) = ?$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1}(3x-1) = 3(1)-1 = 2$

and $\lim_{x\to 1^+} f(x) = \lim_{x\to 1}(2x) = 2(1) = 2$

$\Rightarrow$ $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = 2$

$\therefore$ $\lim_{x\to 1} f(x) = 2$ ....(2)

(c) *From* (1) *and* (2) *we get*

$\lim_{x\to 1} f(x) \neq f(1)$

$\therefore$ $f(x)$ *is discontinous at* $x = 1$

$$(iii) \quad f(x)=\begin{cases} 3x-1 & if \quad x<1 \\ & \\ 2x & if \quad x>1 \end{cases} \qquad c=1$$

(a) $f(1)$ *is not defined*

$\therefore$ $f(x)$ *is discontinous at* $x = 1$

Question # 3:

Given that

$$f(x)=\begin{cases} 3x & if & x\le -2 \\ x^2-1 & if & -2<x<2 \\ 3 & if & x\ge 2 \end{cases}$$

$$\begin{array}{ccccc} -\infty & & & & +\infty \\ \hline f(x)=3x & -2 & f(x)=x^2-1 & 2 & f(x)=3 \end{array}$$

(i) *We check continuity at* $x = 2$

(a) $f(2) = 3$ ....(1) $(given)$

$(b)\quad \lim_{x\to 2} f(x) = ?$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2}\left(x^2-1\right) = (2)^2 - 1 = 4-1 = 3$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2}(3) = 3$

$\Rightarrow \quad \lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = 3$

$\therefore \quad \lim_{x\to 2} f(x) = 3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$

$(c)\quad$ *From* (1) *and* (2), *we get*

$\lim_{x\to 2} f(x) = f(2)$

$\therefore \quad f(x)$ *is continuous at* $x = 2$

$(ii)\quad$ *At* $x = -2$

$(a)\quad f(-2) = 3(-2) = -6 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$

$(b)\quad \lim_{x\to -2} f(x) = ?$

$\lim_{x\to -2^-} f(x) = \lim_{x\to -2}(3x) = 3(-2) = -6$

and $\quad \lim_{x\to -2^+} f(x) = \lim_{x\to -2}\left(x^2-1\right) = (-2)^2 - 1 = 4-1 = 3$

$\Rightarrow \quad \lim_{x\to -2^-} f(x) \neq \lim_{x\to -2^+} f(x) \qquad \Rightarrow \lim_{x\to -2} f(x)$ *does not exist*

$\therefore \quad f(x)$ *is discontinuous at* $x = -2$

Question # 4:

Given that

$$f(x) = \begin{cases} x+2 & x \le -1 \\ c+2 & x > -1 \end{cases}$$

$c = ?$

$-\infty \qquad\qquad\qquad\qquad\qquad\qquad +\infty$

$f(x) = x+2 \qquad\qquad -1 \qquad\qquad f(x) = c+2$

$\because \quad \lim_{x\to -1} f(x) \quad$ *exists*

$\therefore \quad \lim_{x\to -1^-} f(x) = \lim_{x\to -1^+} f(x)$

$\Rightarrow \quad \lim_{x\to -1}(x+2) = \lim_{x\to -1}(c+2)$

$\Rightarrow \quad -1+2 = c+2$

$\Rightarrow \quad 1 = c+2$

$\Rightarrow \quad c = 1-2 \qquad \Rightarrow \qquad c = -1$

Question # 5:

(i)

$$f(x) = \begin{cases} mx & if & x < 3 \\ n & if & x = 3 \\ -2x+9 & if & x > 3 \end{cases}$$

here $\quad f(3) = n \quad$ (*given*)

$\because \quad f(x)$ *is continuous at* $x = 3$

$\therefore \quad \lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = f(3)$

$\Rightarrow \quad \lim_{x\to 3}(mx) = \lim_{x\to 3}(-2x+9) = n$

$$\Rightarrow \quad (m)(3) = -2(3)+9 = n$$
$$\Rightarrow \quad 3m = -6+9 = n$$
$$\Rightarrow \quad 3m = 3 = n$$
$$\Rightarrow \quad 3m = 3 \quad , \quad n = 3$$
$$\Rightarrow \quad m = 1 \quad , \quad n = 3$$

$$(ii) \quad f(x) = \begin{cases} mx & if \quad x < 4 \\ x^2 & if \quad x \geq 4 \end{cases}$$

$$here \quad f(4) = (4)^2 = 16$$
$$\because \quad f(x) \quad \text{is continuous at} \quad x = 4$$
$$\because \quad \lim_{x\to 4^-} f(x) = \lim_{x\to 4^+} f(x) = f(4)$$
$$\Rightarrow \quad \lim_{x\to 4}(mx) = \lim_{x\to 4}(x^2) = 16$$
$$\Rightarrow \quad 4m = (4)^2 = 16$$
$$\Rightarrow \quad 4m = 16 = 16 \quad \Rightarrow \quad 4m = 16$$
$$\Rightarrow \quad m = 4$$

Question # 6:

Given that

$$f(x) = \begin{cases} \dfrac{\sqrt{2x+5}-\sqrt{x+7}}{x-2} & x \neq 2 \\ K & x = 2 \end{cases}$$

$$K = ?$$

$$here \quad f(2) = K \quad given$$
$$\because \quad f(x) \quad \text{is continuous at} \quad x = 2$$
$$\because \quad \lim_{x\to 2} f(x) = f(2)$$

$$\lim_{x\to 2}\frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2} = K \quad \Rightarrow \quad \lim_{x\to 2}\frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}\cdot\frac{\sqrt{2x+5}+\sqrt{x+7}}{\sqrt{2x+5}+\sqrt{x+7}} = K$$

$$\Rightarrow \quad \lim_{x\to 2}\frac{\left(\sqrt{2x+5}\right)^2-\left(\sqrt{x+7}\right)^2}{(x-2)\left(\sqrt{2x+5}+\sqrt{x+7}\right)} = K \quad \Rightarrow \quad \frac{(2x+5)-(x+7)}{(x-2)\left(\sqrt{2x+5}+\sqrt{x+7}\right)} = K$$

$$\Rightarrow \quad \frac{2x+5-x-7}{(x-2)\left(\sqrt{2x+5}+\sqrt{x+7}\right)} = K \quad \Rightarrow \quad \frac{(x-2)}{(x-2)\left(\sqrt{2x+5}+\sqrt{x+7}\right)} = K$$

$$\Rightarrow \quad \lim_{x\to 2}\frac{1}{\sqrt{2x+5}+\sqrt{x+7}} = K \quad \Rightarrow \quad \frac{1}{\lim_{x\to 2}\left[\sqrt{2x+5}+\sqrt{x+7}\right]} = K$$

$$\Rightarrow \quad \frac{1}{\sqrt{2(2)+5}+\sqrt{2+7}} = K \quad \Rightarrow \quad \frac{1}{\sqrt{9}+\sqrt{9}} = K$$

$$\Rightarrow \quad \frac{1}{3+3} = K$$

$$\Rightarrow \quad K = \frac{1}{6}$$