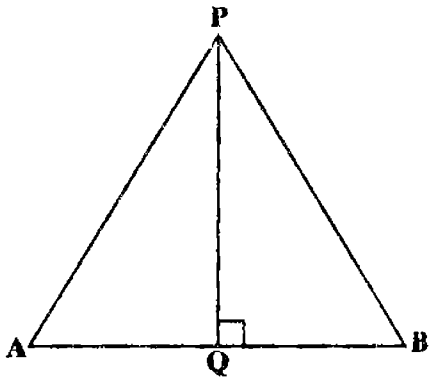
Exercise 10.4

1. In ΔPAB of figure, $\overline{PQ} \perp \overline{AB}$ and $\overline{PA} \cong \overline{PB}$, prove that $\overline{AQ} \cong \overline{BQ}$ and $\angle APQ \cong \angle BPQ$.



Given

In ΔPAB, $\overline{PQ} \perp \overline{AB}$ and $\overline{PA} \cong \overline{PB}$

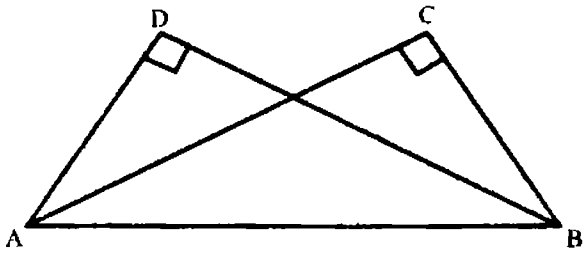
To Prove

$\overline{AQ} \cong \overline{BQ}$ and $\angle APQ \cong \angle BPQ$

Proof

Statements	Reasons
In $\Delta APQ \leftrightarrow \Delta BPQ$	
$\overline{PA} \cong \overline{PB}$	Given
$\overline{PQ} \cong \overline{PQ}$	Common
$\therefore$ $\Delta PAQ \cong \Delta PBQ$	H.S ≅ H.S
$\therefore$ $\overline{AQ} \cong \overline{BQ}$	Corresponding sides of congruent triangles
$\angle APQ \cong \angle BPQ$	Corresponding angles of the congruent triangles.

2. In the figure, $m\angle C = m\angle D = 90^\circ$ and $\overline{BC} \cong \overline{AD}$. Prove that $\overline{AC} \cong \overline{BD}$ and $\angle BAC \cong \angle ABD$.



Given

$m\angle C = m\angle D = 90^\circ$

$\overline{BC} \cong \overline{AD}$

To Prove

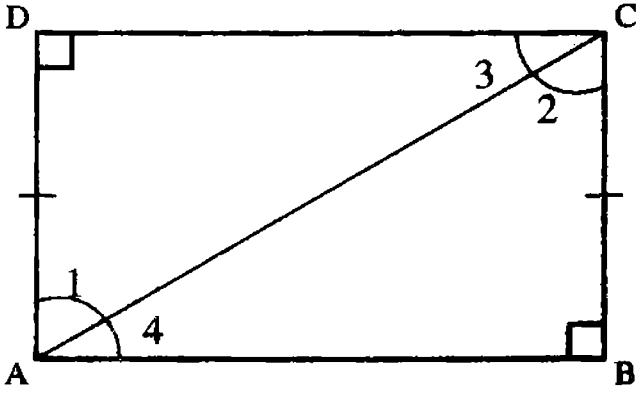
$\overline{AC} \cong \overline{BD}$

$\angle BAC \cong \angle ABD$

Proof

Statements		Reasons
In	$\Delta ABC \leftrightarrow \Delta ABD$	
	$m\angle C \cong m\angle D$	Each of 90°
	$\overline{BC} \cong \overline{AD}$	Given
	$\overline{AB} \cong \overline{AB}$	Common
$\therefore$	$\Delta ABC \cong \Delta ABD$	H.S ≅ H.S
	$\overline{AC} \cong \overline{BD}$	Corresponding sides of congruent triangles
$\therefore$	$\angle BAC \cong \angle ABD$	Corresponding angles of the congruent triangles

3. In the figure, $m\angle B = m\angle D = 90^\circ$ and $\overline{AD} \cong \overline{BC}$. Prove that ABCD is a rectangle.



Given

$m \angle B = m \angle D = 90^\circ$, $\overline{AD} \cong \overline{BC}$

Proof

ABCD is a rectangle

Statements		Reasons
In	$\Delta ABC \leftrightarrow \Delta ADC$	
	$m\angle B \cong m\angle D$	Each of 90°
	$\overline{AD} \cong \overline{BC}$	Given
	$\overline{AC} \cong \overline{AC}$	Common
$\therefore$	$\Delta ABC \cong \Delta ADC$	H.S $\cong$ H.S
	$\overline{AB} \cong \overline{DC}$	
	$\angle 1 \cong \angle 2$ ...(i)	
	$\angle 4 \cong \angle 3$ ...(ii)	
	$\angle 1 + \angle 4 = \angle 2 + m\angle 3$	
	$\angle A = \angle C = 90^\circ$	
	ABCD is a rectangle	By (i) and (ii)

4. Which of the following are true and which are false?

(i) A ray has two end points.

(ii) In a triangle, there can be only one right angle.

(iii) Three points are said to be collinear if they lie on same line.

(iv) Two parallel lines intersect at a point.

(v) Two lines can intersect only in one point.

(vi) A triangle of congruent sides has non-congruent angles.

Answers

(i) False (ii) True (iii) True

(iv) False (v) True (vi) False

5. If $\Delta ABC \cong \Delta LMN$, then

(i) $m\angle M \cong$ .....

(ii) $m\angle N \cong$ .....

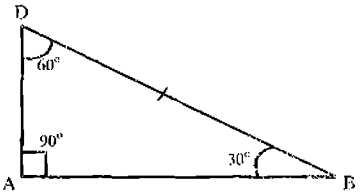
(iii) $m\angle A \cong$ .....

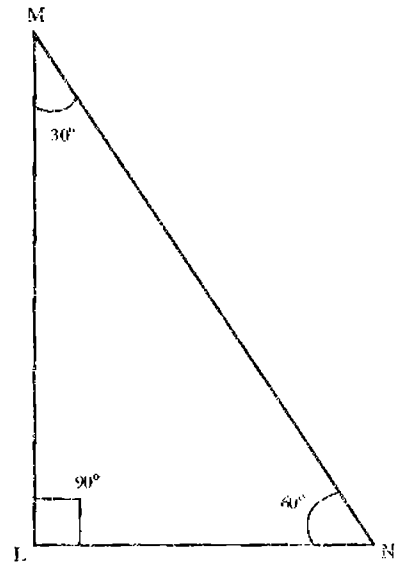
Answers

(i) $m\angle M \cong m\angle B$

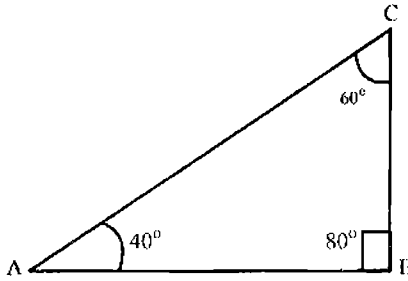
(ii) $m\angle N \cong m\angle C$

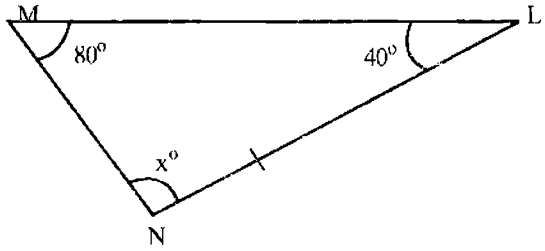
(iii) $m\angle A \cong m\angle L$





6. If $\Delta ABC \cong \Delta LMN$, then find the unknown x.

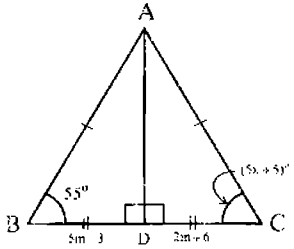




Answers

$x = 60^\circ$

7. Find the value of unknowns for the given congruent triangles.



$\Delta ABD \cong \Delta ACD$

$\overline{BD} \cong \overline{DC}$

$\Rightarrow \quad 5m - 3 = 2m + 6$

$5m - 2m = 3 + 6$

$3m = 9$

$m = \frac{9}{3} = 3$

Also

$\angle ACD \cong \angle ABD \Rightarrow$

Angles opposite to congruent sides are congruent

$5x + 5 = 55$

$5x = 55 - 5$

$5x = 50$

$x = \frac{50}{5}$

$x = 10$

8. If $\Delta PQR \cong \Delta ABC$, then find the unknowns.

$\Delta PQR \cong \Delta ABC$

$\overline{PQ} \cong \overline{AB}$

$x = 3$

$\overline{BC} \cong \overline{QR}$

$\Rightarrow z = 4$ cm

$\overline{AC} \cong \overline{PR}$

$y - 1 = 5$

$y = 5 + 1$

$y = 6$cm

$\therefore x = 3$cm, $y = 6$cm, $z = 4$cm