

Exercise 4.3

1. Express each of the following surd in the simplest form.

$$\begin{aligned} \text{(i)} \quad & \sqrt{180} \\ &= \sqrt{2 \times 2 \times 3 \times 3 \times 5} \\ &= 2 \times 3 \sqrt{5} \\ &= 6\sqrt{5} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad & 3\sqrt{162} \\ &= 3\sqrt{2 \times 3 \times 3 \times 3 \times 3} \\ &= 3(3 \times 3)\sqrt{2} \\ &= 27\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad & \frac{3}{4} \sqrt[3]{128} \\ &= \frac{3}{4} (128)^{\frac{1}{3}} \\ &= \frac{3}{4} (2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2)^{\frac{1}{3}} \end{aligned}$$

$$\begin{aligned} &= \frac{3}{4} (2^3 \times 2^3 \times 2)^{\frac{1}{3}} \\ &= \frac{3}{4} (2^3)^{\frac{1}{3}} \times (2^3)^{\frac{1}{3}} \times 2^{\frac{1}{3}} \\ &= \frac{3}{4} (2)(2) \times \sqrt[3]{2} \\ &= 3\sqrt[3]{2} \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad & \sqrt[5]{96x^6y^7z^8} \\ &= \sqrt[5]{2 \times 2 \times 2 \times 2 \times 2 \times 3 \times x^6y^7z^8} \\ &= (2^5 \times 3 \times x^5 \cdot x \cdot y^5 \cdot y^2 \cdot z^5 \cdot z^3)^{\frac{1}{5}} \\ &= (2^5)^{\frac{1}{5}} (3)^{\frac{1}{5}} (x^5)^{\frac{1}{5}} \cdot x^{\frac{1}{5}} \cdot (y^5)^{\frac{1}{5}} \cdot (y^2)^{\frac{1}{5}} \cdot (z^5)^{\frac{1}{5}} (z^3)^{\frac{1}{5}} \\ &= 2^{\frac{1}{5}} \cdot 3^{\frac{1}{5}} \cdot x^{\frac{1}{5}} \cdot x^{\frac{1}{5}} \cdot y^{\frac{2}{5}} \cdot z^{\frac{3}{5}} \\ &= 2xyz^{\frac{3}{5}} \cdot x^{\frac{1}{5}} \cdot y^{\frac{2}{5}} \cdot z^{\frac{3}{5}} \end{aligned}$$

$$= 2xyz\sqrt[5]{3xy^2z^3}$$

2. Simplify

$$\begin{aligned} \text{(i)} \quad \frac{\sqrt{18}}{\sqrt{3} \cdot \sqrt{2}} &= \frac{\sqrt{3 \cdot 3 \cdot 2}}{\sqrt{3} \cdot \sqrt{2}} = \frac{3\cancel{\sqrt{2}}}{\sqrt{3} \cdot \cancel{\sqrt{2}}} \\ &= \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\cancel{3}\sqrt{3}}{\cancel{3}} = \sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad \frac{\sqrt{21} \times \sqrt{9}}{\sqrt{63}} &= \frac{\sqrt{3 \times 7} \times \sqrt{3 \times 3}}{\sqrt{3 \times 3 \times 7}} \\ &= \frac{\sqrt{3 \times 7 \times 3 \times 3}}{\sqrt{3 \times 3 \times 7}} \\ &= \frac{\cancel{3}\sqrt{21}}{\cancel{3}\sqrt{7}} = \sqrt{\frac{21}{7}} \\ &= \sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad \sqrt[5]{243x^5y^{10}z^{15}} \\ &= \left(3^5 \cdot x^5 y^{10} z^{15}\right)^{\frac{1}{5}} \\ &= (3^5)^{\frac{1}{5}} (x^5)^{\frac{1}{5}} (y^{10})^{\frac{1}{5}} (z^{15})^{\frac{1}{5}} \\ &= 3xy^2z^3 \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad \frac{4}{5}\sqrt[3]{125} \\ &= \frac{4}{5} \left(\cancel{5}^{\cancel{3}} \right)^{\frac{1}{3}} \\ &= 4 \end{aligned}$$

$$\begin{aligned} \text{(v)} \quad \sqrt{21} \times \sqrt{7} \times \sqrt{3} \\ &= \sqrt{3 \times 7} \times \sqrt{7} \times \sqrt{3} \\ &= \sqrt{3 \times 7 \times 7 \times 3} = \left(3^2 \times 7^2\right)^{\frac{1}{2}} \\ &= (3^2)^{\frac{1}{2}} \times (7^2)^{\frac{1}{2}} \\ &= 3 \times 7 \end{aligned}$$

$$= 21$$

3. Simplify by combining similar terms:

$$\begin{aligned} \text{(i)} \quad \sqrt{45} - 3\sqrt{20} + 4\sqrt{5} \\ &= \sqrt{9 \times 5} - 3\sqrt{4 \times 5} + 4\sqrt{5} \\ &= 3\sqrt{5} - 6\sqrt{5} + 4\sqrt{5} \\ &= (3 - 6 + 4)\sqrt{5} \\ &= (-3 + 4)\sqrt{5} \\ &= \sqrt{5} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad 4\sqrt{12} + 5\sqrt{27} - 3\sqrt{75} + \sqrt{300} \\ &= 4\sqrt{3 \times 4} + 5\sqrt{3 \times 3 \times 3} - 3\sqrt{3 \times 5 \times 5} \\ &\quad + \sqrt{3 \times 2 \times 5 \times 2 \times 5} \\ &= 8\sqrt{3} + 15\sqrt{3} - 15\sqrt{3} + 10\sqrt{3} \\ &= (8 + \cancel{15} - \cancel{15} + 10)\sqrt{3} \\ &= 18\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad \sqrt{3}(2\sqrt{3} + 3\sqrt{3}) \\ &= \sqrt{3}((2+3)\sqrt{3}) \\ &= \sqrt{3}(5\sqrt{3}) \\ &= 5\sqrt{3} \times \sqrt{3} \\ &= 5(\sqrt{3 \times 3}) \\ &= 5(3) \\ &= 15 \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad 2(6\sqrt{5} - 3\sqrt{5}) \\ &= 2((6-3)\sqrt{5}) \\ &= 2(3\sqrt{5}) \\ &= 6\sqrt{5} \end{aligned}$$

4. Simplify:

$$\begin{aligned} \text{(i)} \quad (3 + \sqrt{3})(3 - \sqrt{3}) \\ &= (3)^2 - (\sqrt{3})^2 \end{aligned}$$

$$= 9 - 3$$

$$= 6$$

$$\begin{aligned} \text{(ii)} \quad & (\sqrt{5} + \sqrt{3})^2 \\ &= (\sqrt{5})^2 + (\sqrt{3})^2 + 2\sqrt{5}\sqrt{3} \\ &= 5 + 3 + 2\sqrt{15} \\ &= 8 + 2\sqrt{15} \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad & (\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3}) \\ &= (\sqrt{5})^2 - (\sqrt{3})^2 \\ &= 5 - 3 \\ &= 2 \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad & \left(\sqrt{2} + \frac{1}{\sqrt{3}} \right) \left(\sqrt{2} - \frac{1}{\sqrt{3}} \right) \\ &= (\sqrt{2})^2 - \left(\frac{1}{\sqrt{3}} \right)^2 \end{aligned}$$

$$= 2 - \frac{1}{3}$$

$$= \frac{6-1}{3} = \frac{5}{3}$$

$$\begin{aligned} \text{(v)} \quad & (\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})(x + y) \\ & (x^2 + y^2) \\ &= \left((\sqrt{x})^2 - (\sqrt{y})^2 \right) ((x + y)(x^2 + y^2)) \\ &= (x - y)(x + y)(x^2 + y^2) \\ &= (x^2 - y^2)(x^2 + y^2) \\ &= (x^2)^2 - (y^2)^2 \\ &= x^4 - y^4 \end{aligned}$$