Factorize each of the following cubic polynomials by factor theorem.

Q.1 $x^3 - 2x^2 - x + 2$

Let $P(x) = x^3 - 2x^2 - x + 2$

Put $x = 1$

$P(1) = (1)^3 - 2(1)^2 - (1) + 2$

$\qquad = 1 - 2 - 1 + 2$

$\qquad = -3 + 3 = 0$

As, $R = 0$,

So $\quad (x - 1)$ is a factor

Put $x = -1$

$P(-1) = (-1)^3 - 2(-1)^2 - (-1) + 2$

$\qquad = -1 - 2 + 1 + 2$

As $R = 0$,

So $(x + 1)$ is the second factor of $p(x)$.

Put $x = 2$

$P(2) = (2)^3 - 2(2)^2 - (2) + 2$

$\qquad = 8 - 8 - 2 + 2$

$\qquad = 10 - 10$

$\qquad = 0$

As $R = 0$,

So $(x - 2)$ is the third factor

Hence $P(x) = x^3 - 2x^2 - x + 2$

$\qquad = (x - 1)(x + 1)(x - 2)$

Q.2 $x^3 - x^2 - 22x + 40$

Sol:

Let $P(x) = x^3 - x^2 - 22x + 40$

Put $x = 1$

$P(1) = (1)^3 - (1)^2 - 22(1) + 40$

$\qquad = 1 - 1 - 22 + 40$

$=18 \neq 0$

Hence x −1 is not a zero of P(x)

Put x =−1

$P(-1) = (-1)^3 - (-1)^2 - 22(-1) + 40$

$= -1 - 1 + 22 + 40$

$= 60 \neq 0$

Hence x =−1 is not a zero of P(x)

Put x = 2

$P(2) = (2)^3 - (2)^2 - 22(2) + 40$

$= 8 - 4 - 44 + 40 = 0$

Hence x − 2 is a zero of P(x)

So (x − 2) is a factor

Put x =−2

$P(-2) = (-2)^3 - (-2)^2 - 22(-2) + 40$

$= -8 - 4 + 44 + 40 = 72$

Hence x =− 2 is not a zero of P(x)

Put x = 3

$P(3) = (3)^3 - (3)^2 - 22(3) + 40$

$= 27 - 9 - 66 + 40$

$= 67 - 75$

$= -8 \neq 0$

Hence x = 3 is not a zero of P(x)

Put x =−3

$P(-3) = (-3)^3 - (-3)^2 - 22(-3) + 40$

$= -27 - 9 + 66 + 40$

$= 106 - 36$

$= 70 \neq 0$

Hence x =−3 is not a zero of P(x)

Put x = 4

$P(4) = (4)^3 - (4)^2 - 22(4) + 40$

$= 64 - 16 - 88 + 40$

$= 104 - 104$

$= 0$

Hence x = 4 is a zero of P(x)

So (x − 4) is second factor

Put x =− 4

$P(-4) = (-4)^3 - (-4)^2 - 22(-4) + 40$

$= -64 - 16 + 88 + 40$

$= -80 + 128$

$= 48 \neq 0$

So, x =− 4 is not a zero of P(x)

Put x = 5

$P(5) = (5)^3 - (5)^2 - 22(5) + 40$

$= 125 - 25 - 110 + 40$

$= 165 - 135$

$= 30 \neq 0$

So, x = 5 is not a zero of P(x)

Put x =−5

$P(-5) = (-5)^3 - (5)^2 - 22(-5) + 40$

$= -125 - 25 + 110 + 40$

$= -150 + 150$

$= 0$

So, x =− 5 is a zero of P(x)

Hence x + 5 is third factor of P(x)

Hence $P(x) = x^3 - x^2 - 22x + 40$

$= (x - 2)(x - 4)(x + 5)$

Q.3 $x^3 - 6x^2 + 3x + 10$

Sol:

Let , $P(x) = x^3 - 6x^2 - 6x^2 + 3x + 10$

Put x = 1

$P(1) = (1)^3 - 6(1)^2 + 3(1) + 10$

$= 1 - 6 + 3 + 10$

$= 14 - 6$

$= 8 \neq 0$

So, $x = 1$ is not a zero of P(x)

Put $x = -1$

$P(-1) = (-1)^3 - 6(-1)^2 + 3(-1) + 10$

$\qquad = -1 - 6 - 3 + 10$

$\qquad = -10 + 10$

$\qquad = 0$

So, $x = -1$ is a zero of P(x).

Hence $(x+1)$ is a factor of P(x)

Put $x = 2$

$P(2) = (2)^3 - 6(2)^2 + 3(2) + 10$

$\qquad = 8 - 24 + 6 + 10$

$\qquad = 24 - 24$

$\qquad = 0$

So, $x = 2$ is a zero of P(x).

Hence $(x-2)$ is second factor of P(x)

Put $x = -2$

$P(-2) = (-2)^3 - 6(-2)^2 + 3(-2) + 10$

$\qquad = -8 - 24 - 6 + 10$

$\qquad = -28 \neq 0$

So, $x = -2$ is not a zero of P(x)

Put $x = 3$

$P(3) = (3)^3 - 6(3)^2 + 3(3) + 10$

$\qquad = 27 - 6(9) + 9 + 10$

$\qquad = 46 - 54$

$\qquad = -8 \neq 0$

So, $x = 3$ is not a zero of P(x)

Put $x = -3$

$P(-3) = (-3)^3 - 6(-3)^2 + 3(-3) + 10$

$\qquad = -27 - 6(9) - 9 + 10$

$\qquad = -90 + 10$

$\qquad = -80 \neq 0$

So, $x = -3$ is not a zero of P(x)

Put $x = 4$

$P(4) = (4)^3 - 6(4)^2 + 3(4) + 10$

$\qquad = 64 - 6(16) + 12 + 10$

$\qquad = 86 - 96$

$\qquad = -10 \neq 0$

So, $x = 4$ is not a zero of P(x)

Put $x = -4$

$P(-4) = (-4)^3 - 6(-4)^2 + 3(-4) + 10$

$\qquad = -64 - 6(16) - 12 + 10$

$\qquad = -64 - 96 - 12 + 10$

$\qquad = -172 + 10$

$\qquad = -162$

$\qquad = -162 \neq 0$

Put $x = 5$

$P(5) = (5)^3 - 6(5)^2 + 3(5) + 10$

$\qquad = 125 - 150 + 15 + 10$

$\qquad = 150 - 150$

$\qquad = 0$

So, $x = 5$ is a zero of P(x)

Hence $(x-5)$ is third factor of P(x)

Hence $P(x) = x^3 - 6x^2 + 3x + 10$

$\qquad = (x+1)(x-2)(x-5)$

Q.4 $x^3 + x^2 - 10x + 8$

Sol:

Let $P(x) = x^3 + x^2 - 10x + 8$

Put $x = 1$

$P(1) = (1)^3 + (1)^2 - 10(1) + 8$

$\qquad = 1 + 1 - 10 + 8$

$\qquad = 0$

So, $x = 1$ is a zero of P(x)

Hence $(x-1)$ is a factor of $P(x)$

Put $x = -1$

$$P(-1) = (-1)^3 + (-1)^2 - 10(-1) + 8$$
$$= -1 + 1 + 10 + 8$$
$$= 18 \neq 0$$

So, $x = -1$ is not a zero of $P(x)$

Put $x = 2$

$$P(2) = (2)^3 + (2)^2 - 10(2) + 8$$
$$= 8 + 4 - 20 + 8$$
$$= 20 - 20$$
$$= 0$$

So, $x = 2$ is a zero of $P(x)$

Hence $x - 2$ is second factor of $P(x)$

Put $x = -2$

$$P(-2) = (-2)^3 + (-2)^2 - 10(-2) + 8$$
$$= -8 + 4 + 20 + 8$$
$$= 24 \neq 0$$

So, $x = -2$ is not a zero of $P(x)$

Put $x = 3$

$$P(3) = (3)^3 + (3)^2 - 10(3) + 8$$
$$= 27 + 9 - 30 + 8$$
$$= 44 - 30$$
$$= 14 \neq 0$$

Put $x = -3$

$$P(-3) = (-3)^3 + (-3)^2 - 10(-3) + 8$$
$$= -27 + 9 + 30 + 8$$
$$= -27 + 47$$
$$= 20 \neq 0$$

So, $x = -3$ is not a zero of $P(x)$

Put $x = 4$

$$P(4) = (4)^3 + (4)^2 - 10(4) + 8$$

$$= 64 + 16 - 40 + 8$$
$$= 88 - 40$$
$$= 48 \neq 0$$

So, $x = 4$ is not a zero of $P(x)$

Put $x = -4$

$$P(-4) = (-4)^3 + (-4)^2 - 10(-4) + 8$$
$$= -64 + 16 + 40 + 8$$
$$= -64 + 64$$
$$= 0$$

So, $x = -4$ is a zero of $P(x)$

Hence $x + 4$ is third factor of $P(x)$

Hence $P(x) = x^3 + x^2 - 10x + 8$
$$= (x-1)(x-2)(x+4)$$

Q.5 $x^3 - 2x^2 - 5x + 6$

Sol:

$$P(x) = x^3 - 2x^2 - 5x + 6$$

Put $x = 1$

$$P(1) = (1)^3 - 2(1)^2 - 5(1) + 6$$
$$= 1 - 2 - 5 + 6$$
$$= 7 - 7$$
$$= 0$$

So, $x = 1$ is a zero of $P(1)$

Hence $x - 1$ is a factor of $P(x)$

Put $x = -1$

$$P(-1) = (-1)^3 - 2(-1)^2 - 5(-1) + 6$$
$$= -1 - 2 + 5 + 6$$
$$= -3 + 11$$
$$= 8 \neq 0$$

So, $x = -1$ is not a zero of $P(x)$

Put $x = 2$

$$P(2) = (2)^3 - 2(2)^2 - 5(2) + 6$$

$=8-8-10+6$

$=-4\neq0$

So, $x=2$ is not a zero of $P(x)$

Put $x=-2$

$P(-2)=(-2)^3-2(-2)^2-5(-2)$

$=-8-8+10+6$

$=0$

So, $x=-2$ is a zero of $P(x)$

Hence $(x+2)$ is second factor of $P(x)$

Put $x=3$

$P(3)=(3)^3-2(3)^2-5(3)+6$

$=27-18-15+6$

$=33-33$

$=0$

So, $x=3$ is a zero of $P(x)$

Hence $(x-3)$ is third factor of $P(x)$

Hence $P(x)=x^3-2x^2-5x+6$

$=(x-1)(x+2)(x-3)$

Q.6 $x^3+5x^2-2x-24$

Sol:

Let $P(x)=x^3+5x^2-2x-24$

Put $x=1$

$P(1)=(1)^3+5(1)^2-2(1)-24$

$=1+5-2-24$

$=6-26$

$=-20\neq0$

So, $x=1$ is not a zero of $P(x)$

Put $x=-1$

$P(-1)=(-1)^3+5(-1)^2-2(-1)-24$

$=-1+5+2-24$

$=7-25$

$=-18\neq0$

So, $x=-1$ is not a zero of $P(x)$

Put $x=2$

$P(2)=(2)^3+5(2)^2-2(2)-24$

$=8+20-4-24$

$=28-28$

$=0$

So, $x=2$ is a zero of $P(x)$

Hence $(x-2)$ is a factor of $P(x)$

Put $x=-2$

$P(-2)=(-2)^3+5(-2)^2-2(-2)-24$

$=-8+5(4)+4-24$

$=-32+24$

$=-8\neq0$

So, $x=-2$ is not a zero of $P(x)$

Put $x=3$

$P(3)=(3)^3+5(3)^2-2(3)-24$

$=27+5(9)-6-24$

$=72-30$

$=42\neq0$

So, $x=3$ is not a zero of $P(x)$

Put $x=-3$

$P(-3)=(-3)^3+5(-3)^2-2(-3)-24$

$=-27+45+6-24$

$=51-51$

$=0$

So, $x=-3$ is a zero of $P(x)$

Hence $(x+3)$ is second factor of $P(x)$

Put $x=4$

$P(4)=(4)^3+5(4)^2-2(4)-24$

$=64+5(16)-8-24$

$=144-32$

$=112 \neq 0$

So, $x = 4$ is not a zero of P(x)

Put $x = -4$

$P(-4) = (-4)^3 + 5(-4)^2 - 2(-4) - 24$

$\qquad = -64 + 80 + 8 - 24$

$\qquad = 0$

So, $x = -4$ is a zero of P(x)

Hence $(x + 4)$ is third factor of P(x)

Hence $P(x) = x^3 + 5x^2 - 2x - 24$

$\qquad = (x - 2)(x + 3)(x + 4)$

Q. 7 $\quad 3x^3 - x^2 - 12x + 4$

Sol: $\quad P(x) = 3x^3 - x^2 - 12x + 4$

Put $x = 1$

$P(1) = 3(1)^3 - (1)^2 - 12(1) + 4$

$\qquad = 3 - 1 - 12 + 4$

$\qquad = 7 - 13$

$\qquad = -6 \neq 0$

So, $x = 1$ is not a zero of P(x)

Put $x = -1$

$P(-1) = 3(-1)^3 - (-1)^2 - 12(-1) + 4$

$\qquad = -3 - 1 + 12 + 4$

$\qquad = -4 + 16$

$\qquad = 12 \neq 0$

So, $x = -1$ is not a zero of P(x)

Put $x = 2$

$P(2) = 3(2)^3 - (2)^2 - 12(2) + 4$

$\qquad = 24 - 4 - 24 + 4$

$\qquad = 28 - 28$

$\qquad = 0$

So, $x = 2$ is a zero of P(x)

Hence $(x - 2)$ is a factor of P(x)

Put $x = -2$

$P(-2) = 3(-2)^3 - (-2)^2 - 12(-2) + 4$

$\qquad = -24 - 4 + 24 + 4$

$\qquad = -28 + 28$

$\qquad = 0$

So, $x = -2$ is a zero of P(x)

Hence $(x + 2)$ is second factor of P(x)

Put $3x = 1$

$x = \dfrac{1}{3}$

$P\left(\dfrac{1}{3}\right) = 3\left(\dfrac{1}{3}\right)^3 - \left(\dfrac{1}{3}\right)^2 - 12\left(\dfrac{1}{3}\right) + 4$

$\qquad = \cancel{3}\left(\dfrac{1}{27}\right) - \dfrac{1}{9} - 12\left(\dfrac{1}{3}\right) + 4$

$\qquad = \dfrac{1}{9} - \dfrac{1}{9} - 4 + 4$

$\qquad = 0$

So, $x = \dfrac{1}{3}$ is a zero of P(x)

Hence $(3x - 1)$ is third factor of P(x)

Hence $P(x) = 3x^3 - x^2 - 12x + 4$

$\qquad = (x - 2)(x + 2)(3x - 1)$

Q.8 $2x^3 + x^2 - 2x - 1$

Let $P(x) = 2x^3 + x^2 - 2x - 1$

Put $x = 1$

$P(1) = 2(1)^3 + (1)^2 - 2(1) - 1$

$\qquad = 2 + 1 - 2 - 1$

$\qquad = 3 - 3$

$\qquad = 0$

So, $x = 1$ is a zero of P(x)

Hence $(x - 1)$ is a factor of P(x)

Put $x = -1$

$P(-1)=2(-1)^3+(-1)^2-2(-1)-1$

$\qquad =-2+1+2-1$

$\qquad =-1+1$

$\qquad =0$

So, $x=-1$ is a zero of $P(x)$

Hence $(x+1)$ is second factor of $P(x)$

Put $2x=1$

$x=\dfrac{1}{2}$

$P\left(\dfrac{1}{2}\right)=2\left(\dfrac{1}{2}\right)^3+\left(\dfrac{1}{2}\right)^2-2\left(\dfrac{1}{2}\right)-1$

$\qquad =2\left(\dfrac{1}{8}\right)+\left(\dfrac{1}{2}\right)^2-2\left(\dfrac{1}{2}\right)-1$

$\qquad\qquad =\dfrac{1}{4}+\dfrac{1}{4}-1-1$

$\qquad\qquad =\dfrac{-3}{2}\neq 0$

So, $x-2$ is not a zero of $P(x)$

Put $\quad x=\dfrac{-1}{2}$

$P\left(\dfrac{-1}{2}\right)=2\left(\dfrac{-1}{2}\right)^3+\left(\dfrac{-1}{2}\right)^2-2\left(\dfrac{-1}{2}\right)-1$

$\qquad =2\left(-\dfrac{1}{8}\right)+\dfrac{1}{4}+1-1$

$\qquad =-\dfrac{1}{4}+\dfrac{1}{4}+1-1$

$\qquad =0$

So, $x=\dfrac{-1}{2}$ is a zero of $P(x)$

Hence $2x+1$ is third factor of $P(x)$

Hence $P(x)=2x^3+x^2-2x-1$

$\qquad\qquad =(x-1)(x+1)(2x+1)$