# EXERCISE 4.3

**Q.1** $3x^2+2x-\sqrt{3x^2+2x-1}=7$

Put $\sqrt{3x^2+2x-1}=y$

$\Rightarrow 3x^2+2x-1=y^2$

$3x^2+2x=y^2+1$

Then given equation takes form

$y^2+1-y=7$

$y^2-y+1-7=0$

$y^2-y-6=0$

$y^2+2y-3y-6=0$

$y(y+2)-3(y+2)=0$

$(y+2)(y-3)=0$

$y+2=0,\ y-3=0$

$y=-2,\ y=3$

If $y=-2$ Then,

$\sqrt{3x^2+2x-1}=-2$

$3x^2+2x-1=4$

$3x^2+2x-1-4=0$

$3x^2+2x-5=0$

$3x^2-3x+5x-5=0$

$3x(x-1)+5(x-1)=0$

$(x-1)(3x+5)=0$

$x-1=0,\ 3x+5=0$

$x=1,\ x=-5/3$

If $y=3$ Then

$\sqrt{3x^2+2x-1}=3$

$3x^2+2x-1=9$

$3x^2+2x-1-9=0$

$3x^2+2x-10=0$

Using $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-2\pm\sqrt{(2)^2-4(3)(-10)}}{2(3)}$

$x=\frac{-2\pm\sqrt{4+120}}{6}$

$x=\frac{-2\pm\sqrt{124}}{6}$

$x=\frac{-2\pm\sqrt{4\times31}}{6}\Rightarrow x=\frac{-2\pm2\sqrt{31}}{6}$

$x=2\left(\frac{-1\pm\sqrt{31}}{6}\right)\Rightarrow x=\frac{-1\pm\sqrt{31}}{3}$

On checking we found that $1$ and $-5/3$ are extraneous roots. Hence

S.S is $\left\{\frac{-1\pm\sqrt{31}}{3}\right\}$.

**Q.2** $x^2 - \frac{x}{2} - 7 = x - 3\sqrt{2x^2 - 3x + 2}$

Multiplying by 2

$2x^2 - x - 14 = 2x - 6\sqrt{2x^2 - 3x + 2}$

$2x^2 - x - 2x - 14 + 6\sqrt{2x^2 - 3x + 2} = 0$

$2x^2 - 3x - 14 + 6\sqrt{2x^2 - 3x + 2} = 0$

put $\sqrt{2x^2 - 3x + 2} = y$

$\Rightarrow 2x^2 - 3x + 2 = y^2$

$2x^2 - 3x = y^2 - 2$

Given equation takes form

$y^2 - 2 - 14 + 6y = 0$

$y^2 + 6y - 16 = 0$

$y^2 - 2y + 8y - 16 = 0$

$y(y-2) + 8(y-2) = 0$

$(y-2)(y+8) = 0$

$y - 2 = 0, \quad y + 8 = 0$

$y = 2, \quad y = -8$

If $y = 2$ then

$\sqrt{2x^2 - 3x + 2} = 2$

Squaring

$2x^2 - 3x + 2 = 4$

$2x^2 - 3x + 2 - 4 = 0$

$2x^2 - 3x - 2 = 0$

$2x^2 + x - 4x - 2 = 0$

$x(2x+1) - 2(2x+1) = 0$

$(x-2)(2x+1) = 0$

$x - 2 = 0, \quad 2x + 1 = 0$

$x = 2, \quad x = -\frac{1}{2}$

If $y = -8$ then

$\sqrt{2x^2 - 3x + 2} = -8$

Squaring

$2x^2 - 3x + 2 = 64$

$2x^2 - 3x + 2 - 64 = 0$

$2x^2 - 3x - 62 = 0$

Using $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-62)}}{2(2)}$

$x = \frac{3 \pm \sqrt{9 + 496}}{4}$

$x = \frac{3 \pm \sqrt{505}}{4}$

On checking we found $\frac{3 \pm \sqrt{505}}{4}$ is an extraneous root.

Hence S.S $\{2, -\frac{1}{2}\}$

**Q.3** $\sqrt{2x+8} + \sqrt{x+5} = 7$

Squaring both sides

$(\sqrt{2x+8} + \sqrt{x+5})^2 = (7)^2$

$(\sqrt{2x+8})^2 + (\sqrt{x+5})^2 + 2\sqrt{2x+8}\sqrt{x+5} = 49$

$2x + 8 + x + 5 + 2\sqrt{(2x+8)(x+5)} = 49$

$3x + 13 + 2\sqrt{2x^2 + 10x + 8x + 40} = 49$

$2\sqrt{2x^2 + 18x + 40} = 49 - 13 - 3x$

$2\sqrt{2x^2 + 18x + 40} = 36 - 3x$

Squaring both sides again

$4(2x^2 + 18x + 40) = (36 - 3x)^2$

$8x^2 + 72x + 160 = 1296 + 9x^2 - 216x$

$9x^2 - 8x^2 - 216x - 72x + 1296 - 160 = 0$

$x^2 - 288x + 1136 = 0$

$x^2 - 4x - 284x + 1136 = 0$

$x(x-4) - 284(x-4) = 0$

$(x-4)(x-284) = 0$

$x - 4 = 0, \quad x - 284 = 0$

$x = 4, \quad x = 284$

On checking we found 284 is an extraneous root. So

S.S is $\{4\}$

**Q.4** $\sqrt{3x+4} = 2 + \sqrt{2x-4}$

$\sqrt{3x+4} - \sqrt{2x-4} = 2$

Squaring both sides

$(\sqrt{3x+4} - \sqrt{2x-4})^2 = (2)^2$

$(\sqrt{3x+4})^2 + (\sqrt{2x-4})^2 - 2\sqrt{3x+4}\sqrt{2x-4} = 4$

$3x + 4 + 2x - 4 - 2\sqrt{(3x+4)(2x-4)} = 4$

$5x - 2\sqrt{6x^2 - 12x + 8x - 16} = 4$

$-2\sqrt{6x^2-4x-16} = 4-5x$

Squaring again

$4(6x^2-4x-16) = (4-5x)^2$

$24x^2-16x-64 = 16+25x^2-40x$

$25x^2-24x^2-40x+16x+16+64=0$

$x^2-24x+80=0$

$x^2-4x-20x+80=0$

$x(x-4)-20(x-4)=0$

$(x-4)(x-20)=0$

$x-4=0, \quad x-20=0$

$x=4, \quad x=20$

On checking we found that no root is an extraneous root.

Hence S.S $\{4, 20\}$

**Q.5** $\sqrt{x+7} + \sqrt{x+2} = \sqrt{6x+13}$

Squaring on both sides.

$(\sqrt{x+7} + \sqrt{x+2})^2 = (\sqrt{6x+13})^2$

$(\sqrt{x+7})^2 + (\sqrt{x+2})^2 + 2\sqrt{x+7}\sqrt{x+2} = 6x+13$

$x+7+x+2+2\sqrt{(x+7)(x+2)} = 6x+13$

$2x+9+2\sqrt{x^2+2x+7x+14} = 6x+13$

$2\sqrt{x^2+9x+14} = 6x-2x+13-9$

$2\sqrt{x^2+9x+14} = 4x+4$

$\sqrt{x^2+9x+14} = 2x+2$

Squaring again

$(\sqrt{x^2+9x+14})^2 = (2x+2)^2$

$x^2+9x+14 = 4x^2+8x+4$

$4x^2-x^2+8x-9x+4-14=0$

$3x^2-x-10=0$

$3x^2-6x+5x-10=0$

$3x(x-2)+5(x-2)=0$

$(x-2)(3x+5)=0$

$x-2=0, \quad 3x+5=0$

$x=2, \quad x=-5/3$

On checking we found that $-5/3$ is an extraneous root. Hence

S.S $\{2\}$

**Q6** $\sqrt{x^2+x+1} - \sqrt{x^2+x-1} = 1$

put $\sqrt{x^2+x+1} = a$, $\sqrt{x^2+x-1} = b$

Given equation takes form

$a-b=1 \rightarrow (1)$

To find $a$ and $b$ we find $a^2-b^2$ as

$a^2-b^2 = (\sqrt{x^2+x+1})^2 - (\sqrt{x^2+x-1})^2$

$a^2-b^2 = x^2+x+1-(x^2+x-1)$

$a^2-b^2 = x^2+x+1-x^2-x+1$

$a^2-b^2 = 2 \quad \because a^2-b^2=(a-b)(a+b)$

$(a-b)(a+b)=2 \rightarrow (2)$

putting value of $a-b$ from (1) in (2)

$1(a+b)=2 \Rightarrow a+b=2 \rightarrow (3)$

Adding (1) and (3)

$a-b=1$

$a+b=2$

$2a=3 \Rightarrow a=3/2$

putting values of $a$ in (1)

$\frac{3}{2}-1=b \Rightarrow b=\frac{3-2}{2}=1/2$

If $a=3/2$ then

$\sqrt{x^2+x+1} = 3/2$

$2\sqrt{x^2+x+1} = 3$

Squaring

$4(x^2+x+1)=9$

$4x^2+4x+4-9=0$

$4x^2+4x-5=0$

If $b=1/2$ then

$\sqrt{x^2+x-1} = 1/2$

$2\sqrt{x^2+x-1} = 1$

Squaring

$4(x^2+x-1)=1$

$4x^2+4x-4-1=0$

$4x^2+4x-5=0$

Solving any one

$4x^2+4x-5=0$

Using $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-4\pm\sqrt{(4)^2-4(4)(-5)}}{2(4)}$

$x=\frac{-4\pm\sqrt{16+80}}{8} \Rightarrow x=\frac{-4\pm\sqrt{96}}{8}$

$x=\frac{-4\pm\sqrt{16\times6}}{8} \rightarrow x=\frac{-4\pm4\sqrt{6}}{8}$

$x=\frac{4(-1\pm\sqrt{6})}{8} \rightarrow x=\frac{-1\pm\sqrt{6}}{2}$

On checking we found that no root is an extraneous root. So

S.S $\left\{\frac{-1\pm\sqrt{6}}{2}\right\}$

## Q.7

$\sqrt{x^2+2x-3}+\sqrt{x^2+7x-8}=\sqrt{5(x^2+3x-4)}$

Factorizing the expressions under the radical sign.

$\sqrt{x^2-x+3x-3}+\sqrt{x^2-x+8x-8}=\sqrt{5(x^2-x+4x-4)}$

$\sqrt{x(x-1)+3(x-1)}+\sqrt{x(x-1)+8(x-1)}=\sqrt{5[x(x-1)+4(x-1)]}$

$\sqrt{(x-1)(x+3)}+\sqrt{(x-1)(x+8)}=\sqrt{5(x-1)(x+4)}$

$\sqrt{(x-1)(x+3)}+\sqrt{(x-1)(x+8)}-\sqrt{(x-1)(x+4)5}=0$

$\sqrt{x-1}\left\{\sqrt{x+3}+\sqrt{x+8}-\sqrt{5(x+4)}\right\}=0$

$\sqrt{x-1}=0$, $\sqrt{x+3}+\sqrt{x+8}-\sqrt{5(x+4)}=0$

$x-1=0$, $\sqrt{x+3}+\sqrt{x+8}=\sqrt{5(x+4)}$

$x=1$, Squaring both sides

$\left(\sqrt{x+3}+\sqrt{x+8}\right)^2=\left(\sqrt{5(x+4)}\right)^2$

$\left(\sqrt{x+3}\right)^2+\left(\sqrt{x+8}\right)^2+2\sqrt{x+3}\sqrt{x+8}=5(x+4)$

$x+3+x+8+2\sqrt{(x+3)(x+8)}=5x+20$

$2x+11+2\sqrt{x^2+8x+3x+24}=5x+20$

$2\sqrt{x^2+11x+24}=5x-2x+20-11$

$2\sqrt{x^2+11x+24}=3x+9$

Squaring again

$4(x^2+11x+24)=(3x+9)^2$

$4x^2+44x+96=9x^2+54x+81$

$9x^2-4x^2+54x-44x+81-96=0$

$5x^2+10x-15=0$

Dividing by 5

$x^2+2x-3=0$

$x^2-x+3x-3=0$

$x(x-1)+3(x-1)=0$

$(x-1)(x+3)=0$

$x-1=0$, $x+3=0$

$x=1$, $x=-3$

On checking we found that no root is extraneous root. So

S.S is $\{1,-3\}$.

## Q.8

$\sqrt{2x^2-5x-3}+3\sqrt{2x+1}=\sqrt{2x^2+25x+12}$

Factorizing the expressions under radical sign.

$\sqrt{2x^2+x-6x-3}+3\sqrt{2x+1}=\sqrt{2x^2+x+24x+12}$

$\sqrt{x(2x+1)-3(2x+1)}+3\sqrt{2x+1}=\sqrt{x(2x+1)+12(2x+1)}$

$\sqrt{(2x+1)(x-3)}+3\sqrt{2x+1}-\sqrt{(2x+1)(x+12)}=0$

$\sqrt{2x+1}\left\{\sqrt{x-3}+3-\sqrt{x+12}\right\}=0$

$\sqrt{2x+1}=0$, $\sqrt{x-3}+3-\sqrt{x+12}=0$

$2x+1=0$, $\sqrt{x-3}+3=\sqrt{x+12}$

$x=-1/2$, Squaring

$\left(\sqrt{x-3}+3\right)^2=\left(\sqrt{x+12}\right)^2$

$\left(\sqrt{x-3}\right)^2+(3)^2+2(3)\sqrt{x-3}=x+12$

$x-3+9+6\sqrt{x-3}=x+12$

$x+6+6\sqrt{x-3}=x+12$

$6\sqrt{x-3}=x-x+12-6$

$6\sqrt{x-3}=6 \rightarrow \sqrt{x-3}=1$

$x-3=1 \Rightarrow x=4$

On checking we found no root is an extraneous root. Hence

S.S is $\{-\frac{1}{2}, 4\}$

## Q.9

$\sqrt{3x^2-5x+2}+\sqrt{6x^2-11x+5}=\sqrt{5x^2-9x+4}$

Factorizing the expressions under radical sign.

$\sqrt{3x^2-3x-2x+2}+\sqrt{6x^2-6x-5x+5}=\sqrt{5x^2-5x-4x+4}$

$\sqrt{3x(x-1)-2(x-1)}+\sqrt{6x(x-1)-5(x-1)}=\sqrt{5x(x-1)-4(x-1)}$

$\sqrt{(x-1)(3x-2)}+\sqrt{(x-1)(6x-5)}=\sqrt{(x-1)(5x-4)}$

$\sqrt{(x-1)(3x-2)}+\sqrt{(x-1)(6x-5)}-\sqrt{(x-1)(5x-4)}=0$

$\sqrt{x-1}\{\sqrt{3x-2}+\sqrt{6x-5}-\sqrt{5x-4}\}=0$

$\sqrt{x-1}=0, \quad \sqrt{3x-2}+\sqrt{6x-5}-\sqrt{5x-4}=0$

$x-1=0, \quad \sqrt{3x-2}+\sqrt{6x-5}=\sqrt{5x-4}$

$x=1,$ Squaring both sides

$(\sqrt{3x-2}+\sqrt{6x-5})^2=(\sqrt{5x-4})^2$

$(\sqrt{3x-2})^2+(\sqrt{6x-5})^2+2\sqrt{3x-2}\sqrt{6x-5}=5x-4$

$3x-2+6x-5+2\sqrt{(3x-2)(6x-5)}=5x-4$

$9x-7+2\sqrt{18x^2-15x-12x+10}=5x-4$

$2\sqrt{18x^2-27x+10}=5x-9x-4+7$

$2\sqrt{18x^2-27x+10}=3-4x$

Squaring again

$4(18x^2-27x+10)=(3-4x)^2$

$72x^2-108x+40=9+16x^2-24x$

$72x^2-16x^2-108x+24x+40-9=0$

$56x^2-84x+31=0$

Using $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(-84)\pm\sqrt{(-84)^2-4(56)(31)}}{2(56)}$

$x=\frac{84\pm\sqrt{7056-6944}}{112} \Rightarrow x=\frac{84\pm\sqrt{112}}{112}$

$x=\frac{84\pm\sqrt{16\times7}}{112}$

$x=\frac{84\pm4\sqrt{7}}{112} \Rightarrow x=\frac{4(21\pm\sqrt{7})}{112}$

$x=\frac{21\pm\sqrt{7}}{28}$

On checking we found that no root is an extraneous root. So

S.S is $\left\{\frac{21\pm\sqrt{7}}{28}\right\}$

## Q.10

$(x+4)(x+1)=\sqrt{x^2+2x-15}+3x+31$

$x^2+x+4x+4=\sqrt{x^2+2x-15}+3x+31$

$x^2+5x+4=\sqrt{x^2+2x-15}+3x+31$

$x^2+5x-3x+4-31-\sqrt{x^2+2x-15}=0$

$x^2+2x-27-\sqrt{x^2+2x-15}=0$

put $\sqrt{x^2+2x-15}=y$

$\Rightarrow x^2+2x-15=y^2$

$x^2+2x=y^2+15$

Given equation takes form

$y^2+15-27-y=0$

$y^2-y-12=0$

$y^2+3y-4y-12=0$

$y(y+3)-4(y+3)=0$

$(y-4)(y+3)=0$

$y-4=0, \quad y+3=0$

$y=4, \quad y=-3$

If $y=4$ then

$\sqrt{x^2+2x-15}=4$

$x^2+2x-15=16$

$x^2+2x-15-16=0$

$x^2+2x-31=0$

$a=1, b=2, c=-31$

Using $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

If $y=-3$ then

$\sqrt{x^2+2x-15}=-3$

$x^2+2x-15=9$

$x^2+2x-15-9=0$

$x^2+2x-24=0$

$x^2-4x+6x-24=0$

$x(x-4)+6(x-4)=0$

$x = \frac{-2 \pm \sqrt{(12)^2 - 4(1)(-31)}}{2(1)}$

$x = \frac{-2 \pm \sqrt{4 + 124}}{2}$

$x = \frac{-2 \pm \sqrt{128}}{2}$

$x = \frac{-2 \pm 8\sqrt{2}}{2}$

$(x-4)(x+6) = 0$

$x - 4 = 0, \quad x + 6 = 0$

$x = 4, \quad x = -6$

$\rightarrow x = \frac{-2 \pm \sqrt{64 \times 2}}{2}$

$\rightarrow x = \frac{2(-1 \pm 4\sqrt{2})}{2}$

$$x = -1 \pm 4\sqrt{2}$$

On checking we found that 4 and $-6$ are extraneous roots. Hence

S.S is $\{-1 \pm 4\sqrt{2}\}$

## Q.11

$$\sqrt{3x^2 - 2x + 9} + \sqrt{3x^2 - 2x - 4} = 13$$

put $\sqrt{3x^2 - 2x + 9} = a$, $\sqrt{3x^2 - 2x - 4} = b$

Given equation takes form

$a + b = 13 \rightarrow (1)$

To find a and b we find $a^2 - b^2$ as

$a^2 - b^2 = \left(\sqrt{3x^2 - 2x + 9}\right)^2 - \left(\sqrt{3x^2 - 2x - 4}\right)^2$

$a^2 - b^2 = 3x^2 - 2x + 9 - (3x^2 - 2x - 4)$

$a^2 - b^2 = 3x^2 - 2x + 9 - 3x^2 + 2x + 4$

$a^2 - b^2 = 13 \qquad \because a^2 - b^2 = (a+b)(a-b)$

$(a+b)(a-b) = 13 \rightarrow (2)$

putting value of $a+b$ from (1) in (2)

$13(a-b) = 13 \rightarrow a - b = 1 \rightarrow (3)$

Adding (1) and (3)

$$\begin{array}{r} a + b = 13 \\ a - b = 1 \\ \hline 2a = 14 \end{array} \rightarrow a = 7$$

putting value of a in (1)

$7 + b = 13 \rightarrow b = 6$

If $a = 7$ then $\sqrt{3x^2 - 2x + 9} = 7$

If $b = 6$ then $\sqrt{3x^2 - 2x - 4} = 6$

$3x^2 - 2x + 9 = 49$

$3x^2 - 2x + 9 - 49 = 0$

$3x^2 - 2x - 40 = 0$

$3x^2 - 2x - 4 = 36$

$3x^2 - 2x - 4 - 36 = 0$

$3x^2 - 2x - 40 = 0$

Solving any one

$3x^2 - 2x - 40 = 0$

$3x^2 - 12x + 10x - 40 = 0$

$3x(x-4) + 10(x-4) = 0$

$(x-4)(3x+10) = 0$

$x - 4 = 0, \quad 3x + 10 = 0$

$x = 4, \quad x = -10/3$

On checking we found no root is extraneous root. So.

S.S is $\{4, -10/3\}$

## Q.12

$$\sqrt{5x^2 + 7x + 2} - \sqrt{4x^2 + 7x + 18} = x - 4$$

put $\sqrt{5x^2 + 7x + 2} = a$, $\sqrt{4x^2 + 7x + 18} = b$

Given equation takes form

$a - b = x - 4 \rightarrow (1)$

To find a and b we find $a^2 - b^2$ as

$a^2 - b^2 = \left(\sqrt{5x^2 + 7x + 2}\right)^2 - \left(\sqrt{4x^2 + 7x + 18}\right)^2$

$a^2 - b^2 = 5x^2 + 7x + 2 - (4x^2 + 7x + 18)$

$a^2 - b^2 = 5x^2 + 7x + 2 - 4x^2 - 7x - 18$

$a^2 - b^2 = x^2 - 16$

$a^2 - b^2 = (x)^2 - (4)^2 \qquad \because a^2 - b^2 = (a-b)(a+b)$

$(a-b)(a+b) = (x-4)(x+4) \rightarrow (2)$

putting value of $a-b$ from (1) in (2)

$(x-4)(a+b) = (x-4)(x+4)$

$\rightarrow a + b = x + 4 \rightarrow (3)$

Adding eq (1) and (3)

$$\begin{array}{r} a - b = x - 4 \\ a + b = x + 4 \\ \hline 2a = 2x \end{array} \rightarrow a = x$$

putting value of a in (1)

$x - b = x - 4 \rightarrow b = 4$

If $a = x$ Then

$\sqrt{5x^2+7x+2} = x$

$5x^2+7x+2 = x^2$

$5x^2-x^2+7x+2 = 0$

$4x^2+7x+2 = 0$

If $b = 4$ Then

$\sqrt{4x^2+7x+18} = 4$

$4x^2+7x+18 = 16$

$4x^2+7x+18-16 = 0$

$4x^2+7x+2 = 0$

Solving any one

$4x^2+7x+2 = 0$

Using $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-7 \pm \sqrt{(7)^2-4(4)(2)}}{2(4)}$

$x = \frac{-7 \pm \sqrt{49-32}}{8} \Rightarrow x = \frac{-7 \pm \sqrt{17}}{8}$

On checking we found that no root is extraneous root. Hence

S.S is $\left\{ \frac{-7 \pm \sqrt{17}}{8} \right\}$