Exercise 6.6

Q.No 1 $3, 6, 12, \ldots$

$a_1 = 3$, $r = \frac{a_2}{a_1} = \frac{6}{3} = 2$, $n = 5$

Since $a_n = a_1 r^{n-1}$

$\Rightarrow a_5 = (3)(2)^{5-1}$

$= (3)(2)^4 = (3)(16)$

$= 48$ Answer

Q.No 2 $1+i, 2, \frac{4}{1+i}, \ldots$

$a_1 = 1+i$, $r = \frac{a_2}{a_1} = \frac{2}{1+i}$, $n = 11$

Since $a_n = a_1 r^{n-1}$

$\Rightarrow a_{11} = (1+i)\left(\frac{2}{1+i}\right)^{11-1}$

$= (1+i)\left(\frac{2}{1+i}\right)^{10}$

$= (1+i)\left(\frac{2}{1+i}\cdot\frac{1-i}{1-i}\right)^{10}$

$= (1+i)\left(\frac{2(1-i)}{(1)^2-(i)^2}\right)^{10}$

$= (1+i)\left(\frac{2(1-i)}{1+1}\right)^{10}$ $\because i^2 = -1$

$= (1+i)\left(\frac{\cancel{2}(1-i)}{\cancel{2}}\right)^{10}$

$= (1+i)(1-i)^{10}$

$= (1+i)\left[(1-i)^2\right]^5$

$= (1+i)\left[(1)^2 - 2(1)(i) + (i)^2\right]^5$

$= (1+i)\left[1-2i-1\right]^5$ $\because i^2 = -1$

$= (1+i)(-2i)^5$

$= (1+i)(-2)^5(i)^5$

$= (1+i)(-32)\, i^4 \cdot i$

$= (1+i)(-32)(i^2)^2 \cdot i$

$= (1+i)(-32)(-1)^2 \cdot i$

$= (1+i)(-32)(1)\cdot i$

$= -32i(1+i)$

$= -32i - 32i^2$

$= -32i - 32(-1) = -32i + 32$

$= 32(-i+1) = 32(1-i)$ Ans.

Q.No 3 $1+i, 2i, -2+2i, \ldots$

$a_1 = 1+i$, $r = \frac{a_2}{a_1} = \frac{2i}{1+i}$, $n = 12$

Since $a_n = a_1 r^{n-1}$

$\Rightarrow a_{12} = (1+i)\left(\frac{2i}{1+i}\right)^{12-1}$

$= (1+i)\left(\frac{2i}{1+i}\cdot\frac{1-i}{1-i}\right)^{11}$

$= (1+i)\left(\frac{2i-2i^2}{(1)^2-(i)^2}\right)^{11}$

$= (1+i)\left(\frac{2i+2}{1+1}\right)^{11}$ $\because i^2 = -1$

$= (1+i)\left(\frac{\cancel{2}(i+1)}{\cancel{2}}\right)^{11}$

$= (1+i)(1+i)^{11} = (1+i)^{12}$

$= \left[(1+i)^2\right]^6 = \left[1+2i+i^2\right]^6$

$= \left[1+2i-1\right]^6 = (2i)^6$

$= 2^6 \cdot (i)^6 = 64(i^2)^3$

$= 64(-1)^3 = -64$ Answer

Q.No. 4 Do yourself as Q.No. 2 & 3.

Q.No. 5 Here $a_1 = 12000$

depreciation $= 5\%$

therefore $r = 1 - \frac{5}{100} = 1 - 0.05$

$= 0.95$

$n = 5$

Since $a_n = a_1 r^{n-1}$

$\Rightarrow a_5 = (12000)(0.95)^{5-1}$

$= (12000)(0.95)^4$

$= (12000)(0.8145)$

$= 9774.08$

Thus value of automobile at the end of 4 year is 9774.08

Q No. 6 $x^2-y^2, x+y, \frac{x+y}{x-y}, \ldots$ is $\frac{x+y}{(x-y)^9}$

Here $a_1 = x^2-y^2$

$$r = \frac{x+y}{x^2-y^2} = \frac{\cancel{x+y}}{(x-y)\cancel{(x+y)}} = \frac{1}{x-y}$$

$n = ?$, $a_n = \frac{x+y}{(x-y)^9}$

Since
$$a_n = a_1 r^{n-1}$$

$$\Rightarrow \frac{x+y}{(x-y)^9} = (x^2-y^2)\left(\frac{1}{x-y}\right)^{n-1}$$

$$\Rightarrow \frac{\cancel{(x+y)}}{(x-y)^9} = \cancel{(x+y)}(x-y)\frac{1}{(x-y)^{n-1}}$$

$$\Rightarrow \frac{1}{(x-y)^9} = \frac{1}{(x-y)^{n-1-1}}$$

$$\Rightarrow \frac{1}{(x-y)^9} = \frac{1}{(x-y)^{n-2}}$$

$$\Rightarrow \left(\frac{1}{x-y}\right)^9 = \left(\frac{1}{x-y}\right)^{n-2}$$

$$\Rightarrow 9 = n-2 \Rightarrow 9+2 = n$$

$$\Rightarrow \boxed{n = 11} \quad \text{Answer}$$

Q No 7 Since a, b, c, d are in G.P

therefore $\frac{b}{a} = \frac{c}{b} = \frac{d}{c}$

So $\frac{b}{a} = \frac{c}{b} \Rightarrow b^2 = ac$ — (i)

$\frac{c}{b} = \frac{d}{c} \Rightarrow c^2 = bd$ — (ii)

$\frac{b}{a} = \frac{d}{c} \Rightarrow bc = ad$ — (iii)

i) To show $a-b, b-c, c-d$ are in G.P

Let $r = \frac{b-c}{a-b}$ — (1)

Also $r' = \frac{c-d}{b-c}$

$$= \frac{c-d}{b-c} \cdot \frac{a-b}{a-b}$$

$$= \frac{ac-ad-bc+bd}{(b-c)(a-b)}$$

$$= \frac{b^2-bc-bc+c^2}{(b-c)(a-b)} \qquad \because ac = b^2,\ \cancel{ad = bc},\ bd = c^2$$

$$= \frac{b^2-2bc+c^2}{(b-c)(a-b)}$$

$$= \frac{(b-c)^2}{(b-c)(a-b)} = \frac{b-c}{a-b} \text{ — (2)}$$

from (1) & (2)
$$r = r'$$

therefore $a-b, b-c, c-d$ ar in G.P.

ii) To show $a^2-b^2, b^2-c^2, c^2-d^2$ are in G.P.

Let $r = \frac{b^2-c^2}{a^2-b^2}$ — (1)

Also
$$r' = \frac{c^2-d^2}{b^2-c^2}$$

$$= \frac{c^2-d^2}{b^2-c^2} \cdot \frac{a^2-b^2}{a^2-b^2}$$

$$= \frac{a^2c^2-a^2d^2-b^2c^2+b^2d^2}{(b^2-c^2)(a^2-b^2)}$$

$$= \frac{(ac)^2-(ad)^2-(bc)^2+(bd)^2}{(b^2-c^2)(a^2-b^2)}$$

$$= \frac{(b^2)^2-(bc)^2-(bc)^2+(c^2)^2}{(b^2-c^2)(a^2-b^2)} \quad \text{From (i), (ii) \& (iii)}$$

$$= \frac{b^4-2b^2c^2+c^4}{(b^2-c^2)(a^2-b^2)}$$

$$= \frac{(b^2-c^2)^2}{(b^2-c^2)(a^2-b^2)}$$

$$= \frac{b^2-c^2}{a^2-b^2} \text{ — (2)}$$

from (1) and (2)
$$r = r'$$

Hence $a^2-b^2, b^2-c^2, c^2-d^2$ are in G.P.

iii) Do Yourself

Hint:- $r = \frac{b^2+c^2}{a^2+b^2}$ — (1)

Also $r = \frac{c^2+d^2}{b^2+c^2} = \frac{c^2+d^2}{b^2+c^2} \cdot \frac{a^2+b^2}{a^2+b^2}$

(Same as ii)

Q.No. 8 $a_1, a_1r^2, a_1r^4, \ldots\ldots\ldots$

The sequence of reciprocal of the term is

$\frac{1}{a_1}, \frac{1}{a_1r^2}, \frac{1}{a_1r^4}, \ldots\ldots\ldots$

To show this is in G.P. Let

$$r_1 = \frac{a_2}{a_1} = \frac{1/a_1r^2}{1/a_1} = \frac{1}{\cancel{a_1}r^2}\cdot\cancel{a_1} = \frac{1}{r^2} \quad \text{---(i)}$$

Also

$$r_1' = \frac{a_3}{a_2} = \frac{1/a_1r^4}{1/a_1r^2} = \frac{1}{\cancel{a_1}r^{\cancel{4}}}\cdot\cancel{a_1}\cancel{r^2}$$

$$= \frac{1}{r^2} \quad \text{---(ii)}$$

From (i) & (ii)

$$r_1' = r_1$$

therefore the sequence of reciprocal of the term of G.P is also in G.P

Q.No. 9 Let a_1 be the first term and r be the common ~~difference~~ ratio.

Since $\frac{a_5}{a_3} = \frac{4}{9}$

$$\Rightarrow \frac{\cancel{a_1}r^{5-1}}{\cancel{a_1}r^{3-1}} = \frac{4}{9} \Rightarrow \frac{r^4}{r^2} = \frac{4}{9}$$

$$\Rightarrow r^2 = \frac{4}{9} \Rightarrow r = \pm\frac{2}{3}$$

Also

$$a_2 = \frac{4}{9}$$

$$\Rightarrow a_1r^{2-1} = \frac{4}{9} \Rightarrow a_1r = \frac{4}{9}$$

When $r = \frac{2}{3}$

$$a_1\left(\frac{2}{3}\right) = \frac{4}{9} \Rightarrow a_1 = \frac{\overset{2}{\cancel{4}}}{\underset{3}{\cancel{9}}}\cdot\frac{\cancel{3}}{\cancel{2}}$$

$$\Rightarrow a_1 = \frac{2}{3}$$

When $r = -\frac{2}{3}$

$$a_1\left(-\frac{2}{3}\right) = \frac{4}{9} \Rightarrow a_1 = \frac{4}{9}\left(-\frac{3}{2}\right)$$

$$\Rightarrow a_1 = -\frac{2}{3}$$

Since $a_n = a_1r^{n-1}$

When $a_1 = \frac{2}{3}$ and $r = \frac{2}{3}$

$$a_n = \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)^{n-1}$$

$$= \left(\frac{2}{3}\right)^n$$

When $a_1 = -\frac{2}{3}$ and $r = -\frac{2}{3}$

$$a_n = \left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)^{n-1}$$

$$= \left(-\frac{2}{3}\right)^n \text{ or } (-1)^n\left(\frac{2}{3}\right)^n$$

Q.No. 10 Consider $\frac{a_1}{r}, a_1, a_1r$ are three consective terms in G.P. by given condition.

$$\frac{a_1}{r} + a_1 + a_1r = 26$$

$$\Rightarrow a_1\left(\frac{1}{r} + 1 + r\right) = 26$$

$$\Rightarrow a_1(1 + r + r^2) = 26r \quad \text{xing by } r \quad \text{---(i)}$$

Also we have given

$$\left(\frac{a_1}{\cancel{r}}\right)(a_1)(a_1\cancel{r}) = 216$$

$$\Rightarrow a_1^3 = 216$$

$$\Rightarrow a_1^3 = (6)^3 \Rightarrow \boxed{a_1 = 6}$$

putting in eq (i)

$$6(1 + r + r^2) = 26r$$

$$\Rightarrow 6 + 6r + 6r^2 - 26r = 0$$

$$\Rightarrow 6r^2 - 20r + 6 = 0$$

$$\Rightarrow 2(3r^2 - 10r + 3) = 0$$

$$\Rightarrow 3r^2 - 10r + 3 = 0$$

$$r = \frac{10 \pm \sqrt{100 - 4(3)(3)}}{2(3)}$$

$$= \frac{10 \pm \sqrt{64}}{6} = \frac{10 \pm 8}{6}$$

$$r = \frac{10+8}{6} \quad \text{or} \quad r = \frac{10-8}{6}$$

$$= \frac{18}{6} = 3, \quad = \frac{2}{6} = \frac{1}{3}$$

When $a_1 = 6$, $r = 3$

$$\frac{a_1}{r} = \frac{6}{3} = 2$$

$$a_1 = 6$$

$$a_1r = (6)(3) = 18$$

When $a_1 = 6$, $r = 1/3$

$\frac{a_1}{r} = \frac{6}{1/3} = 6 \times 3 = 18$

$a_1 = 6$

$a_1 r = 6 \times \frac{1}{3} = 2$

Hence 2, 6, 18 OR 18, 6, 2 are required number in G.P.

Q.No.11 Let the four terms in G.P are $a_1, a_1r, a_1r^2, a_1r^3$

By given condition.

$a_1 + a_1r + a_1r^2 + a_1r^3 = 80$

$\Rightarrow a_1(1+r+r^2+r^3) = 80$

$\Rightarrow a_1[1(1+r)+r^2(1+r)] = 80$

$\Rightarrow a_1(1+r)(1+r^2) = 80$ —— (i)

Also we have given

$\frac{a_1r + a_1r^3}{2} = 30$

$\Rightarrow \frac{a_1r(1+r^2)}{2} = 30$

$\Rightarrow a_1r(1+r^2) = 60$ —— (ii)

From eq (i)

$1+r^2 = \frac{80}{a_1(1+r)}$ putting in (ii)

$a_1r \cdot \frac{80}{a_1(1+r)} = 60$

$\Rightarrow \frac{80r}{1+r} = 60 \Rightarrow 80r = 60(1+r)$

$\Rightarrow 80r = 60 + 60r$

$\Rightarrow 80r - 60r = 60 \Rightarrow 20r = 60$

$\Rightarrow r = \frac{60}{20} \Rightarrow \boxed{r = 3}$

putting value of r in eq (i)

$a_1(1+3)(1+(3)^2) = 80$

$\Rightarrow a_1(4)(10) = 80 \Rightarrow 40a_1 = 80$

$\Rightarrow a_1 = \frac{80}{40} \Rightarrow \boxed{a_1 = 2}$

So

$a_1r = (2)(3) = 6$

$a_1r^2 = (2)(3)^2 = 2 \times 9 = 18$

$a_1r^3 = (2)(3)^3 = 2 \times 27 = 54$

hence 2, 6, 18, 54 are required number

Q.No.12 Since $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in G.P

therefore $r = \frac{1/b}{1/a} = \frac{a}{b}$ —— (i)

Also $r = \frac{1/c}{1/b} = \frac{b}{c}$ —— (ii)

xing (i) & (ii)

$r \cdot r = \frac{a}{b} \cdot \frac{b}{c} \Rightarrow r^2 = \frac{a}{c}$

$\Rightarrow r = \pm\sqrt{\frac{a}{c}}$ proved

Q.No.13 Let a_1-d, a_1, a_1+d are three numbers in A.P.

By given condition

$a_1 - d + a_1 + a_1 + d = 21$

$\Rightarrow 3a_1 = 21 \Rightarrow \boxed{a_1 = 7}$

Now a_1-d-1, a_1-4, a_1+d-3 are in G.P therefore

$r = \frac{a_1-4}{a_1-d-1} = \frac{a_1+d-3}{a_1-4}$

$\Rightarrow (a_1-4)^2 = (a_1+d-3)(a_1-d-1)$

put $a_1 = 7$

$(7-4)^2 = (7+d-3)(7-d-1)$

$\Rightarrow (3)^2 = (d+4)(6-d)$

$\Rightarrow 9 = 6d + 24 - d^2 - 4d$

$\Rightarrow 9 - 6d - 24 + d^2 + 4d = 0$

$\Rightarrow d^2 - 2d - 15 = 0$

$\Rightarrow d^2 - 5d + 3d - 15 = 0$

$\Rightarrow d(d-5) + 3(d-5) = 0$

$\Rightarrow (d-5)(d+3) = 0$

$d-5 = 0$, $d+3 = 0$

$d = 5$, $d = -3$

When $a_1 = 7$, $d = 5$	When $a_1 = 7$, $d = 3$
$a_1 - d = 7 - 5 = 2$	$a_1 - d = 7 + 3 = 10$
$a_1 = 7$	$a_1 = 7$
$a_1 + d = 7 + 5 = 12$	$a_1 + d = 7 - 3 = 4$

hence 2, 7, 12 or 10, 7, 4 are required number

Q.No.14 Hint, Consider number a_1-d, a_1, a_1+d then a_1-d+1, a_1+4, a_1+d+15 are in G.P

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******************** The End ********************