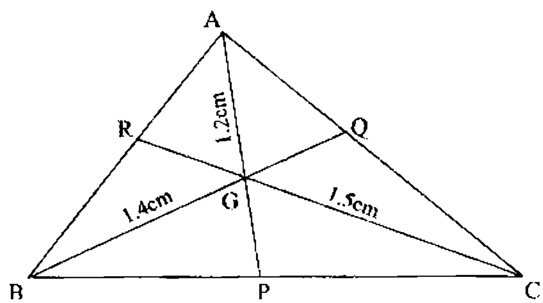
EXERCISE 11.4

(1) The distances of the point of concurrency of the medians of a triangle from its vertices are respectively 1.2cm; 1.4 cm and 1.5 cm. Find the lengths of its medians.



Solution: Let ABC be a triangle with center of gravity at G where $m\overline{AG}=1.2cm$, $\overline{BG}=1.4cm$, $m\overline{CG}=1.5cm$

Required To find the length of AP, BQ, CR

Proof:

$$m\overline{AP} = \frac{3}{2}\times(mAG)$$

$$= \frac{3}{2}\times1.2=1.8cm$$

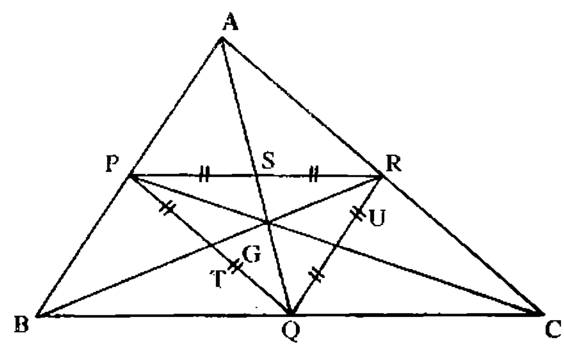
$$m\overline{BQ} = \frac{3}{2}\times(m\overline{BG})$$

$$= \frac{3}{2}\times1.4=2.1cm$$

$$m\overline{CR} = \frac{3}{2}\times(mCG)$$

$$= \frac{3}{2}\times1.5=2.25cm$$

(2) Prove that the point of concurrency of the medians of a triangle and the triangle which is made by joining the mid-points of its sides is the same.



Given

In ΔABC, $\overline{AQ}, \overline{BR}, \overline{CP}$ are its medians that are concurrent at point G.

ΔPQR is formed by joining mid-points of $\overline{AB}$, $\overline{BC}$, $\overline{CA}$

To Prove

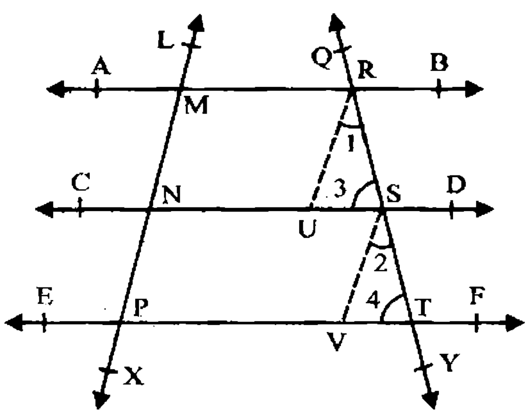
Point G is point of concurrency of triangle PQR.

Proof

Statements	Reasons
$\overline{PR} \parallel \overline{BC}$	P, R are mid-points of $\overline{AB}$ and $\overline{AC}$
$\Rightarrow$ $\overline{PR} \parallel \overline{BQ}$ (i)	
$\overline{RQ} \parallel AB$	P, Q are mid-points of $\overline{AB}$ and $\overline{BC}$
$\Rightarrow$ $\overline{RQ} \parallel \overline{PB}$ (ii)	
$\therefore$ PBQR is a parallelogram.	
$\overline{BR}$, $\overline{PQ}$ are its diagonals, that bisect each other at T.	
T is mid-point $\overline{PQ}$, similarly	
S is mid-point of $\overline{PR}$ and U is mid-point of $\overline{PQ}$.	

Theorem

If three or more parallel lines make congruent segments on a transversal, they also intercept congruent segments on any other line that cuts them.



Given

$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$

The transversal $\overleftrightarrow{LX}$ intersects $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ at the points M, N and P respectively, such that $\overline{MN} \cong \overline{NP}$. The transversal $\overleftrightarrow{QY}$ intersects them at points R, S and T respectively.

To Prove

$\overline{RS} \cong \overline{ST}$

Construction

From R, draw $\overline{RU} \parallel \overline{LX}$, which meets $\overline{CD}$ at U. From S, draw $\overline{SV} \parallel \overline{LX}$ which meets $\overline{EF}$ at V. as shown in the figure let the angles be labeled as

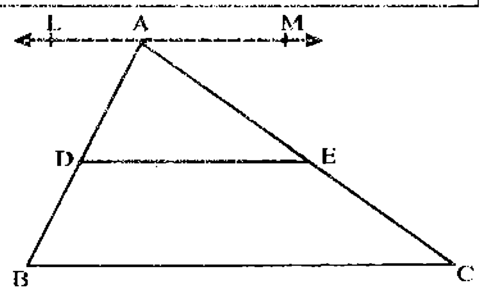
$\angle 1, \angle 2, \angle 3$ and $\angle 4$

Proof

Statements	Reasons
MNUR is a parallelogram	$\overline{RU} \parallel \overline{LX}$ (construction)
	$\overline{AB} \parallel \overline{CD}$ (given)
$\therefore$ $\overline{MN} \cong \overline{RU}$ .....(i)	(opposite sides of a parallelogram)

Similarly,	
$\overline{NP} \cong \overline{SV}$ .....(ii)	
But $\overline{MN} \cong \overline{NP}$ .....(iii)	Given
$\therefore$ $\overline{RU} \cong \overline{SV}$	{from (i), (ii) and (iii)}
Also $\overline{RU} \parallel \overline{SV}$	Each is $\parallel \overleftrightarrow{LX}$ (construction)
$\therefore$ $\angle 1 \cong \angle 2$	Corresponding angles
and $\angle 3 \cong \angle 4$	Corresponding angles
In $\Delta RUS \leftrightarrow \Delta SVT$,	
$\overline{RU} \cong \overline{SV}$	Proved
$\angle 1 \cong \angle 2$	Proved
$\angle 3 \cong \angle 4$	Proved
$\therefore$ $\Delta RUS \cong \Delta SVT$	S.A.A. $\cong$ S.A.A.
Hence $\overline{RS} \cong \overline{ST}$	(corresponding sides of a congruent triangles)

Corollaries : (i) A line, through the mid-point of one side, parallel to another side of a triangle, bisects the third side.



Given : In ΔABC, D is the mid-point of $\overline{AB}$. $\overline{DE} \parallel \overline{BC}$ which cuts $\overline{AC}$ at E.

To prove

$\overline{AE} \cong \overline{EC}$

Construction

Through A, draw $\overleftrightarrow{LM} \parallel \overline{BC}$.

Proof

Statements	Reasons
Intercepts cut by $\overleftrightarrow{LM}$, $\overleftrightarrow{DE}$, $\overleftrightarrow{BC}$ on $\overleftrightarrow{AC}$ are congruent. i.e., $\overline{AC} \cong \overline{EC}$	{Intercepts cut by parallels $\overleftrightarrow{LM}$, $\overline{DE}$, $\overline{BC}$ on AB are congruent (given)}

(ii) The parallel line from the mid-point of one non-parallel side of a trapezium to the parallel sides bisects the other non-parallel side.

(iii) If one side of a triangle is divided into congruent segments, the line drawn from the point of division parallel to the other side will make congruent segments on third side.