In the name of God

Producer:

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Subject:

The accpetance-rejection methods in R

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Isuue:

A) In the rejection and acceptance algorithm, we want to make n=1 to a random number of Beta(4,3) distributions. If we consider the function g equal to 1 (for all values of y=> g(y)=1), for a = 4,3,2.5,2.073, repeat this algorithm 10,000 times and each time Repeat N (number of numbers generated up to Y = x) and finally for each a, we have 10,000 to N. Take their average and display them as points and draw vertical lines equal to the standard deviation of Ns next to the points.

B) In the rejection and acceptance algorithm, we want to make n=10000 to a random number of Beta(4,3) distributions. If we consider the function g equal to 1 (for all values of y), run this algorithm for a = 2.073 and get the value N.

C) In the rejection and acceptance algorithm, if we set N: the number of generated numbers to Y = X, then calculate the statistical distribution of the random variable N and also write the mathematical expectation and its variance.

Sole:

A) First, by forming a matrix(4*10000), each row of which represents a from the top to the bottom with different values specified in the vector c, and each column represents a repetition of the algorithm and we write the number N obtained in it.

```
> rm(list = ls())
> par(mfrow=c(1,1))
> Niha<-c(rep(0 ,40000))
> N.m<-matrix(Niha ,  nrow = 4 , byrow = 10000)
> c<-c(4,3,2.5,2.073)
> for (k in 1:4){for(i in 1:10000){
+
+ N<-0
+ a<-c[k]
+ n<-1
+ u<-runif(n)
+ y<-runif(n)
+ g<-function(y){g=1}
+ f<-function(y){f=pbeta(y,4,3)}
+ while(f(y)/a*g(y)<u)
+ {
+   N<-N+1
+   u<-runif(n)
+   y<-runif(n)
+ N.m[k,i]<-N
+ }}
+   }
```

Now by obtaining the mean and standard deviation in each row (for different a's) and plotting them so that our horizontal axis represents the value of a and the vertical axis represents the average number of iterations of the algorithm until the condition is accepted.

```
> z<-apply(N.m , 1 ,mean)
> x<-apply(N.m , 1 ,sd)
>
> plot(c , z,type="p" , col="red" ,xlab= "a value" , ylab = "N.bar" ,xlim = c(1,5) , ylim = c(-10,17) )
> for(i in 1:4){
+   segments(c[i],z[i]-x[i],x1=c[i] , y1 = z[i]+x[i] , col = "black")}
```

B) In this section, to create 10,000 random numbers from the Beta(4,3) distribution, we must form two 10,000-digit sequences of uniform distribution using the rejection and acceptance algorithm, and then perform the algorithm for each of these numbers, and each time these numbers are in pairs. Now they are compared, they calculate N and add, then we display N.

```
> N<-0
> a<-2.073
> n<-10000
> u<-runif(n)
> y<-runif(n)
> g<-function(y){g=1}
> f<-function(y){f=pbeta(y,4,3)}
> for(i in 1:10000){
+ if(f(y[i])/a*g(y[i])>=u[i]){
+   N<-N+1
+
+ }
+ }
> print(N)
[1] 2092
```

C) We know that:

$$y \sim unif(0,1)\,, \qquad u \sim unif(0,1) \;\; , g(y) = 1 \;\; , a = 4,3,2.5,2.073$$

For rejection in this algorithm we should have this:

$$1 > \frac{f(y)}{a * g(y)} \geq u > 0$$

$$P(acceptance) = P\left(\frac{f(y)}{a * g(y)} \geq u\right) \overset{g(y)=1}{\Longleftrightarrow} P\left(\frac{f(y)}{a} \geq u\right) = P(f(y) \geq a * u)$$

According to the distribution function is uniform, we are inverse and positive. We have:

$P(acceptance) = P\left(y \geq f^{-1}(a * u)\right)$

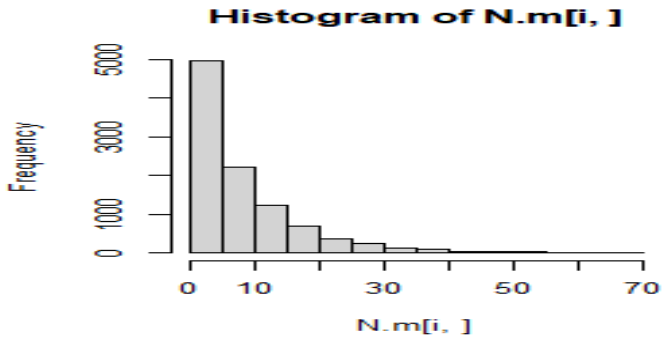
N: The number of times our algorithm was rejected until the first acceptance or the number of steps of the algorithm until the first victory

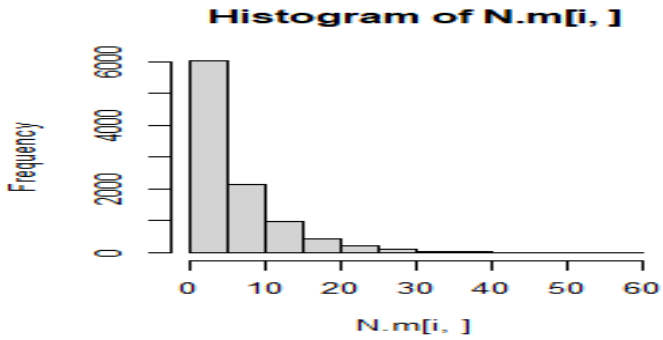
It is clear that:

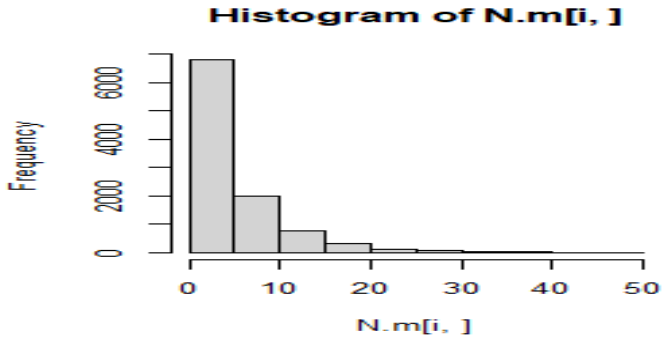
$$N \sim Ge(p) \qquad ; \; p = P(acceptance) = \; P\left(y \geq f^{-1}(a * u)\right) \;\; , q = p - 1$$

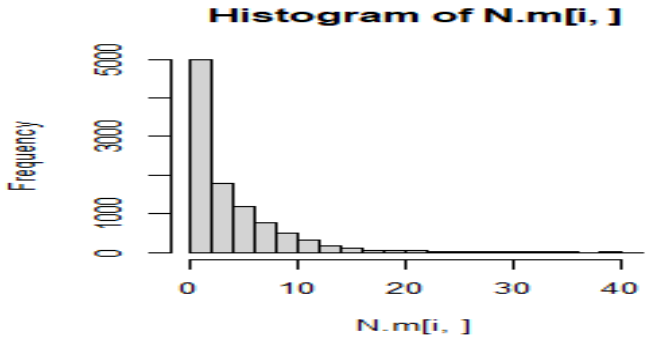
$$f_N(n) = pq^n \qquad , E[N] = \frac{q}{p} \;\; , Var(N) = \frac{q}{p^2}$$

We can see the N histogram with doffrents a value that they are such az Geometric distrubtion histogram:


Histogram of N.m[i,]
Frequency
0
1000
3000
5000
0
10
30
50
70
N.m[i,]


Histogram of N.m[i,]
Frequency
0
2000
4000
6000
0
10
20
30
40
50
60
N.m[i,]


Histogram of N.m[i,]
Frequency
0
2000
4000
6000
0
10
20
30
40
50
N.m[i,]


Histogram of N.m[i,]
Frequency
0
1000
3000
5000
0
10
20
30
40
N.m[i,]