Type of Premium and policy value calculations
Example 7.18

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Problem 1

An insurer issues a whole life insurance policy to a life aged 50. The sum insured of 100 000$ is payable at the end of the year of death. Level premiums are payable annually in advance throughout the term of the contract. All premiums and policy values are calculated using the Standard Select Survival Model, and an interest rate of 5% per year effective. Initial expenses are 50% of the gross premium plus 250$. Renewal expenses are 3% of the gross premium plus 25$ at each premium date after the first.
Calculate:
(a) the expense loading, P^e, and
(b) ${}_{10}V^e$, ${}_{10}V^n$ and ${}_{10}V^g$.

Solve 1

Solution

Step 1: Define the Random Loss Variable

The random loss variable L_t at time t is defined as:

$$L_t = B_t - P_t - E_t,$$

where:

- B_t: the benefits variable at time t,
- P_t: the premiums variable at time t,
- E_t: the expenses variable at time t.

Step 2: Components of the Loss Variables

1. Benefits

The benefits are \$100,000, payable at the end of the year of death. The present value of benefits as time t is given by:

$$B_t = 100,000 \cdot \sum_{k=1}^{n} v^k \cdot q_x,$$

where:

- $n = \infty$ (until death),
- $v = (1+i)^{-1}$ is the annullay discount factor, $i = 0.05$ annually,
- q_x is the probability of dying in the 1 year.

2. Premiums

The annually premium is \$. The present value of premiums at time t is:

$$P_t = \sum_{k=1}^{n} \cdot v^k \cdot \, p_x.$$

where:

- $n = \infty$,
- p_x is the probability of survival a person with x age for 1 year.

3. Expenses

- Inital expenses:
  according to the example information we have:

  $$0.5P^g + 250\$$$

  the above value will be pay just at t = 0.

- renewal expenses:
  we know that renewal expenses are : $0.03p^g + 25\$$.

Expenses are 10% of each gross premium paid, so the monthly expense is:

$$\text{Expense per premium payment} = 0.1 \cdot P = 46.$$

So if we want tot calculate the expense loading or p^e we should calculate the gross premium at the step 1.

Step 3: Expected Present Value (EPV) of the Loss Random Variable

Using the survival model, the gross premium policy value at time t is the expected value of the loss random variable:

$$V_t = \text{EPV}(Z_t) - \text{EPV}(P_t) - \text{EPV}(E_t).$$

Step 4: Components of the Expected Present Value (EPV) of the Loss Random Variable

1. Expected Present Value of Benefits (Z_t)

The benefits are 100,000\$, payable at the end of the month of death. The expected present value (EPV) of these benefits at time t is denoted by:

$$EPV(B)_t = S \cdot \, A_{[50]} = 100,000 \cdot \, A_{[50]}.$$

where:

- $A_{[50]}$: The expected present value of a 1-unit insurance benefit, payable at the end of the year of death, for a life aged 50.

- The symbol $A_{[50]}$ incorporates the annually interest rate and survival probabilities, as follows:

$$A_{[50]} = \sum_{k=1}^{n} v^k \cdot q_{50}.$$

- $v = (1+i)^{-1}$: The annual discount factor
- $n = \infty$: The total number of years for the whole life insurance.
- q_{50}: The probability that the life dies in the (k)th year for a person with age 50.

2. Expected Present Value of Premiums (P_t)

The premiums are payable annually until death. The expected present value (EPV) of these premiums at time t is denoted by:

$$EPV(P)_t = P \cdot \ddot{a}_{[50]} = P \cdot \ddot{a}_{[50]}.$$

where:

- $\ddot{a}_{[50]}$: The expected present value of a 1-unit premium paid annually in advance for a life aged 50, until death, with payments made 1 times per year(immediately).
- The symbol $\ddot{a}_{[50]}$ incorporates the annually interest rate and survival probabilities, as follows:

$$\sum_{k=1}^{\infty} v^k \cdot {}_kp_{50}$$

  where:

  - $v = (1+i)^{-1}$: The annual discount factor
  - $n = \infty$: The total number of year premiums,
  - ${}_kp_{50}$: The probability that the life age 50 dont death until 50 + k age.

3. Expected Present Value of Expenses (E_t)

we know that at the t =0 there is no renewal expense because its the first time of contract and only we have initial expense but in the other premium date its equal to the $0.03p^g + 25\$$ as an annually annuity until death. so we have two part for renewal calculation here, the first part is from t=1 to ∞ and the second part is t = 0 premium that we should not pay that. attention please:

$$EPV(E)_t = (0.03p^g + 25)\ddot{a}_{[50]} - (0.03p^g + 25) + (0.5p^g + 250).$$

where:

- $\ddot{a}_{[50]}$: The expected present value of a 1-unit premium paid annually in advance for a life aged 50, until death, with payments made 1 times per year(immediately).
- The symbol p^g is gross premium.

Step 5: Premiums Value Calculation at Times $t = 0$

The policy value at any time t is given by the general formula:

$$tV_x + EPV_t(\text{future premiums}) = EPV_t(\text{future benefits}) + EPV_t(\text{future expenses}),$$

where:

- tV_x: The policy value at time t for a life aged $x + t$.
- EPV_t(future premiums): The expected present value of premiums still to be paid after time t.
- EPV_t(future benefits): The expected present value of benefits payable after time t.
- EPV_t(future expenses): The expected present value of expenses after time t we should attention that the expenses are two part here, the first part is initial expenses and the second part is renewal expenses.

To calculate gross premium at $t = 0$ we should use general formula:

$$\begin{aligned}
p^g a\ddot{_{[50]}} &= 100,000 \cdot\ A_{[50]} + (0.03p^g + 25)\ddot{a}_{[50]} - (0.03p^g + 25) + (0.5p^g + 250) \\
<=> p^g a\ddot{_{[50]}} &= 100,000 \cdot\ A_{[50]} + 0.03p^g\ddot{a}_{[50]} + 25\ddot{a}_{[50]} - 0.03p^g + 25 + 0.5p^g + 250 \\
<=> p^g a\ddot{_{[50]}} &= 100,000 \cdot\ A_{[50]} + 0.03p^g\ddot{a}_{[50]} + 25\ddot{a}_{[50]} + 0.47p^g + 225 \\
<=> p^g a\ddot{_{[50]}} - 0.03p^g\ddot{a}_{[50]} - 0.47p^g &= 100,000 \cdot\ A_{[50]} + 25\ddot{a}_{[50]} + 225 \\
<=> p^g(a\ddot{_{[50]}} - 0.03\ddot{a}_{[50]} - 0.47) &= 100,000 \cdot\ A_{[50]} + 25\ddot{a}_{[50]} + 225 \\
<=> p^g(0.97a\ddot{_{[50]}} - 0.47) &= 100,000 \cdot\ A_{[50]} + 25\ddot{a}_{[50]} + 225 \\
<=> p^g &= \frac{100,000 \cdot\ A_{[50]} + 25\ddot{a}_{[50]} + 225}{(0.97a\ddot{_{[50]}} - 0.47)} = 1219.09\$
\end{aligned}$$

Now P^e can be calculated by finding the EPV of future expenses, and calculating the level premium to fund those expenses – that is

$$P^e\ddot{a}_{[50]} = 25\ddot{a}_{[50]} + 225 + 0.03P^e\ddot{a}_{[50]} + 0.47P^e.$$

Alternatively, we can calculate the net premium,

$$P^n = 100000A_{[50]}/\ddot{a}_{[50]} = \$1110.65,$$

and use $P^e = P^e - P^n$. Either method gives $P^e = \$108.43$. Compare the expense premium with the incurred expenses. The annual renewal expenses, payable at each premium date after the first, are \$61.57. The rest of the expense loading, \$46.86 at each premium date, reimburses the acquisition expenses, which total \$859.54.

Step 6: Different Type of Policy Values Calculation at Times $t = 10$

we know that for calculating policy values we have 3 type of them according to the premiums.

- ${}_tV^n$ net premium policy value
- ${}_tV^g$ gross premium policy value

- ${}_tV^e$ expense policy value

and we have bottem formula according to the book:

- ${}_tV^n = EPV(futureBenefits) - EPV(futurenetpremium)$
- ${}_tV^g = EPV(futureBenefits) + EPV(futureexpenses) - EPV(futuregrosspremium)$
- ${}_tV^e = EPV(futureexpenses) - EPV(futureexpensesloading)$
- ${}_tV^g =_t V^n +_t V^e$

so we can replace the above formula with calculated values at step 5. Thus,

- ${}_{10}V^e = 0.03p^g\ddot{a}_{[60]} + 25\ddot{a}_{[60]} - p^e\ddot{a}_{[60]} = -46.86\ddot{a}_{[60]} = -698.42\$$
- ${}_{10}V^n = 100,000 \cdot\ A_{[60]} - p^n\ddot{a}_{[60]} = 12474.94\$$
- ${}_{10}V^g = 100,000 \cdot\ A_{[60]} + 25\ddot{a}_{[60]} - 0.97p^g\ddot{a}_{[60]} = 11776.52\$$