Gross Premium Policy Value Calculations
Example 7.10

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Problem 1

A life aged 50 purchases a 10-year term insurance with sum insured \$500,000 payable at the end of the month of death. Level monthly premiums, each of amount $P = 460$, are payable for at most five years.
Calculate the (gross premium) policy values at durations 2.75, 3, and 6.5 years using the following basis:

- **Survival model:** Standard Select Survival Model
- **Interest:** 5% per year
- **Expenses:** 10% of each gross premium

Solve 1

Solution

Step 1: Define the Random Loss Variable

The random loss variable L_t at time t is defined as:

$$L_t = B_t - P_t - E_t,$$

where:

- B_t: the benefits variable at time t,
- P_t: the premiums variable at time t,
- E_t: the expenses variable at time t.

Step 2: Components of the Loss Variables

1. Benefits

The benefits are \$500,000, payable at the end of the month of death. The present value of benefits as time t is given by:

$$B_t = 500,000 \cdot \sum_{k=1}^{n} v^{k/12} \cdot {}_{t+k/12}q_x,$$

where:

- $n = 120$ months (10 years),
- $v = (1+i)^{-1}$ is the monthly discount factor, $i = 0.05$ annually,
- ${}_{t+k/12}q_x$ is the probability of dying in the $(k/12)$th month.

2. Premiums

The monthly premium is \$460. The present value of premiums at time t is:

$$P_t = \sum_{k=1}^{m} 4 * P \cdot v^{k/4} \cdot \ {}_{t+k/4}p_x = \sum_{k=1}^{m} 4 * 460 \cdot v^{k/4} \cdot \ {}_{t+k/4}p_x.$$

where:

- $m = 20$ Quarters (5 years maximum of Quarterly premiums),
- ${}_{t+k/4}p_x$ is the probability of survival to the $(k/4)$th Quarter.

3. Expenses

Expenses are 10% of each gross premium paid, so the monthly expense is:

$$\text{Expense per premium payment} = 0.1 \cdot P = 46.$$

The present value of expenses at time t is:

$$E_t = \sum_{k=1}^{m} 4 * 0.1P \cdot v^{k/4} \cdot \ {}_{t+k/4}p_x = \sum_{k=1}^{m} 4 * 46 \cdot v^{k/4} \cdot \ {}_{t+k/4}p_x.$$

Step 3: Expected Present Value (EPV) of the Loss Random Variable

Using the survival model, the gross premium policy value at time t is the expected value of the loss random variable:

$$V_t = \text{EPV}(Z_t) - \text{EPV}(P_t) - \text{EPV}(E_t).$$

Step 4: Components of the Expected Present Value (EPV) of the Loss Random Variable

1. Expected Present Value of Benefits (Z_t)

The benefits are \$500,000, payable at the end of the month of death. The expected present value (EPV) of these benefits at time t is denoted by:

$$EPV(B)_t = S \cdot \ \ddot{A}^{(12)}_{x:\overline{10}|} = 500,000 \cdot \ \ddot{A}^{(12)}_{x:\overline{10}|}.$$

where:

- $\ddot{A}^{(12)}_{x:\overline{10}|}$: The expected present value of a 1-unit insurance benefit, payable at the end of the month of death, for a life aged x, with payments made 12 times per year.
- The symbol $\ddot{A}^{(12)}_{x:\overline{10}|}$ incorporates the monthly interest rate and survival probabilities, as follows:

$$\ddot{A}^{(12)}_{x:\overline{n}|} = \sum_{k=1}^{n} v^{k/12} \cdot \ {}_{k/12}q_x.$$

- $v = (1+i)^{-1}$: The annual discount factor, with $i = 0.05$.
- $n = 120$: The total number of months for the 10-year term insurance.
- ${}_{k/12}q_x$: The probability that the life dies in the $(k/12)$th month.

2. Expected Present Value of Premiums (P_t)

The premiums are \$460, payable Quarterly for a maximum of 5 years (20 Quarter). The expected present value (EPV) of these premiums at time t is denoted by:

$$EPV(P)_t = 4 * P \cdot \ddot{a}^{(4)}_{x:\overline{5}|} = 4 * 460 \cdot \ddot{a}^{(4)}_{x:\overline{5}|}.$$

where:

- $\ddot{a}^{(12)}_{x:\overline{n}|}$: The expected present value of a 1-unit premium paid Quarterly in advance for a life aged x, for a maximum of n years, with payments made 4 times per year.
- The symbol $\ddot{a}^{(4)}_{x:\overline{n}|}$ incorporates the monthly interest rate and survival probabilities, as follows:

  $$\ddot{a}^{(4)}_{x:\overline{n}|} = \sum_{k=0}^{nm-1} v^{k/4} \cdot {}_{k/4}p_x,$$

  where:

  - $v = (1+i)^{-1}$: The annual discount factor, with $i = 0.05$,
  - $nm = 5 * 4 = 20$: The total number of Quarter premiums (5 years),
  - ${}_{k/4}p_x$: The probability that the life survives to the $(k/4)$th month.

3. Expected Present Value of Expenses (E_t)

The expenses are 10% of each premium, i.e., \$46, payable Quarterly for a maximum of 5 years (20 Quarte). The expected present value (EPV) of these expenses at time t is denoted by:

$$EPV(E)_t = 4 * 0.1P \cdot \ddot{a}^{(4)}_{x:\overline{5}|} = 4 * 46 \cdot \ddot{a}^{(4)}_{x:\overline{5}|}.$$

where:

- $\ddot{a}^{(4)}_{x:\overline{n}|}$: The expected present value of a 1-unit expense paid Quarterly in advance for a life aged x, for a maximum of n=6 years, with payments made 4 times per year.
- The symbol $\ddot{a}^{(4)}_{x}$ is the same as used in the premium calculation:

  $$\ddot{a}^{(4)}_{x:\overline{n}|} = \sum_{k=0}^{nm-1} v^{k/4} \cdot {}_{k/4}p_x,$$

  where:

  - $v = (1+i)^{-1}$: The annual discount factor, with $i = 0.05$,
  - $nm = 5 * 4 = 20$: The total number of monthly expense payments (5 years),
  - ${}_{k/4}p_x$: The probability that the life survives to the $(k/4)$th month.

Step 5: Policy Value Calculation at Times $t = 2.75$, $t = 3$, and $t = 6.5$

The policy value at any time t is given by the general formula:

$$tV_x + EPV_t(\text{future premiums}) = EPV_t(\text{future benefits}) + EPV_t(\text{future expenses}),$$

where:

- tV_x: The policy value at time t for a life aged $x + t$.
- EPV_t(future premiums): The expected present value of premiums still to be paid after time t.
- EPV_t(future benefits): The expected present value of benefits payable after time t.
- EPV_t(future expenses): The expected present value of expenses after time t.

To calculate the policy value at times $t = 2.75$, $t = 3$, and $t = 6.5$: The policy value at $t = 2.75$ is given by the general formula:

$$2.75V_{50} = EPV_{2.75}(\text{B}) + EPV_{2.75}(\text{E}) - EPV_{2.75}(\text{P})$$

Now we will place the values that we calculated above for t = 2.75.

$$2.75V_{50} = 500,000 \cdot \ddot{A}^{(12)}_{52.75:\overline{7.25}|} + 4 * 0.1P \cdot \ddot{a}^{(4)}_{52.75:\overline{2.25}|} - 4 * P \cdot \ddot{a}^{(4)}_{52.75:\overline{2.25}|}$$

$$= 500,000 \cdot \ddot{A}^{(12)}_{52.75:\overline{7.25}|} - 4 * 0.9P \cdot \ddot{a}^{(4)}_{52.75:\overline{2.25}|} = 3091.02\$$$

Similarly for t = 3.

$$3V_{50} = 500,000 \cdot \ddot{A}^{(12)}_{53:\overline{7}|} + 4 * 0.1P \cdot \ddot{a}^{(4)}_{53:\overline{2}|} - 4 * P \cdot \ddot{a}^{(4)}_{53:\overline{2}|}$$

$$= 500,000 \cdot \ddot{A}^{(12)}_{53:\overline{7}|} - 4 * 0.9P \cdot \ddot{a}^{(4)}_{53:\overline{2}|} = 3357.94\$$$

And finally similarly for t = 3. we should note that there is no premium in t = 6 so we dont have premiums and expenses!.

$$6.5V_{50} = 500,000 \cdot \ddot{A}^{(12)}_{56.5:\overline{3.5}|}$$

$$= 500,000 \cdot \ddot{A}^{(12)}_{56.5:\overline{3.5}|} = 4265.63\$$$