timeseries test

Mehrab Atighi

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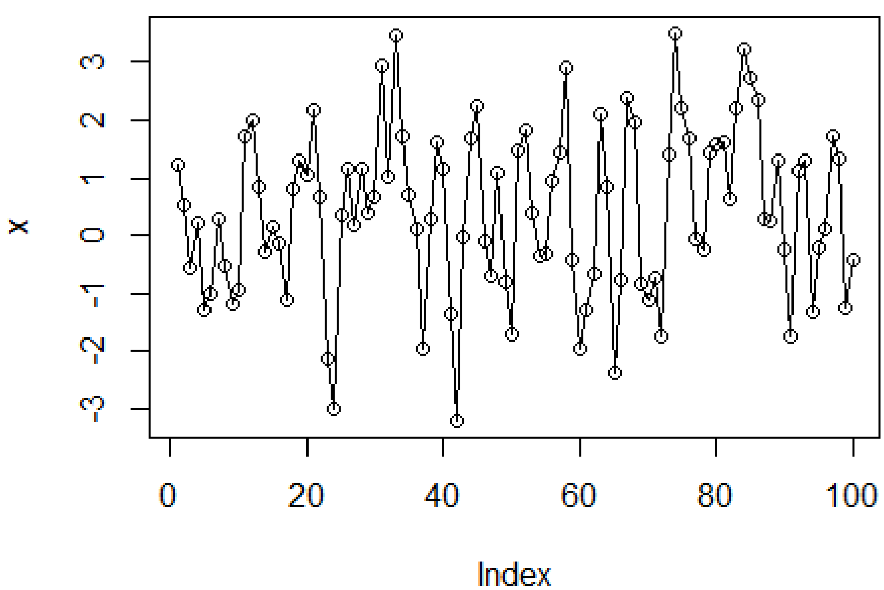
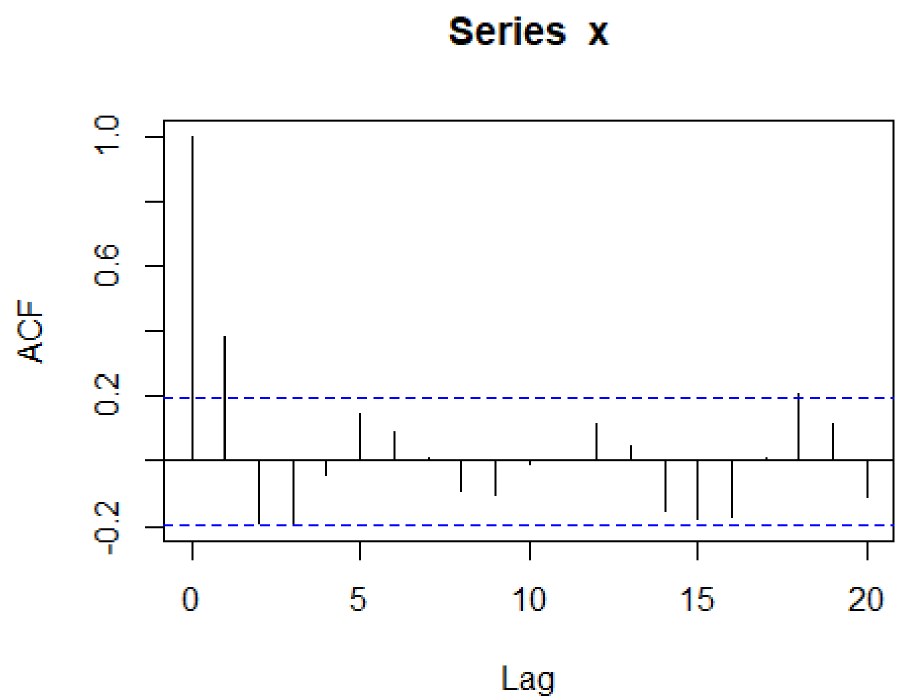
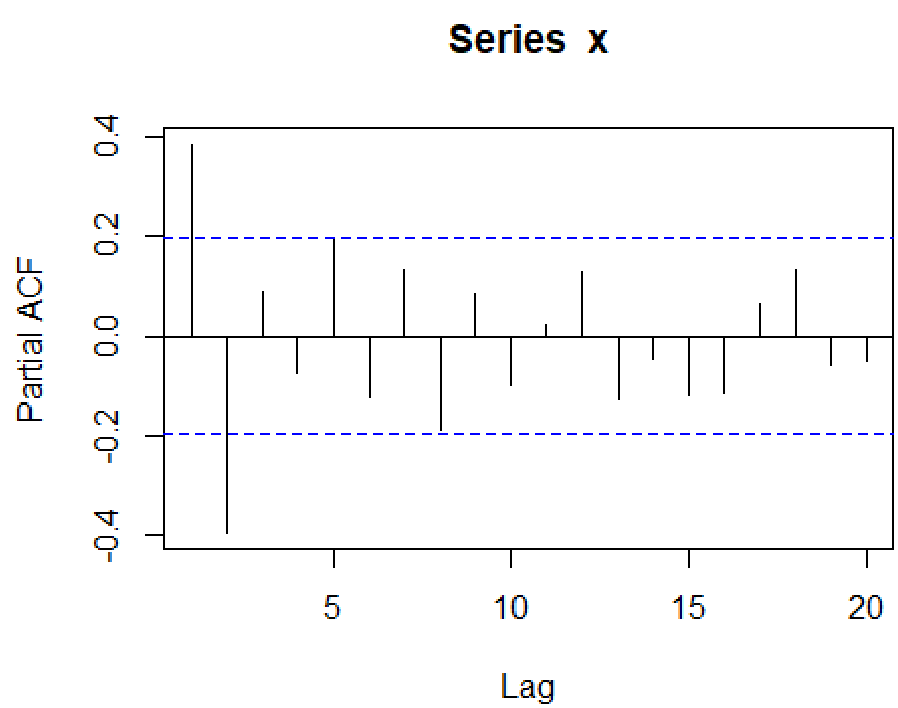
```
x = c(1.236,0.529,-0.552,0.198,-1.273,-1.006,0.291,-0.533,-1.175,-0.944,
      1.713,2.006,0.854,-0.282,0.141,-0.151,-1.129,0.822,1.307,1.048,
      2.157,0.66,-2.133,-2.996,0.367,1.143,0.187,1.170,0.389,0.666,
      2.928,1.026,3.477,1.723,0.720,0.118,-1.954,0.272,1.617,1.152,
      -1.365,-3.203,-0.017,1.698,2.230,-0.098,-0.704,1.094,-0.782,-1.717,
      1.461,1.807,0.4,-0.354,-0.326,0.941,1.446,2.886,-0.416,-1.963,
      -1.28,-0.655,2.088,0.844,-2.361,-0.761,2.393,1.951,-0.831,-1.111,
      -0.735,-1.756,1.386,3.5,2.205,1.675,-0.068,-0.254,1.450,1.565,
      1.624,0.634,2.203,3.201,2.741,2.352,0.297,0.259,1.308,-0.222,
      -1.723,1.117,1.306,-1.333,-0.195,0.114,1.703,1.342,-1.234,-0.401)
#a)

plot(x , type = "o")
```




```
#b)
a = acf(x)
```




```
p = pacf(x)
```




```
a$acf[1:5]
```

```
## [1]  1.00000000  0.38378644 -0.18889489 -0.18796471 -0.04284637
p$acf[1:5]
## [1]  0.38378644 -0.39425798  0.08945649 -0.07486718  0.19362712
```

[illegible] 3991218182 [illegible]

یک مدل ARMA(1,1) در نظر می‌گیریم 1

$p, q > ?$ $\to p, q$ $\Rightarrow p=1 \to q=1$

$q - p = 2$

بررسی شرط:

$2\gamma_1^2 - |\gamma_1| < \gamma_2 < |\gamma_1|$

$2(0.38)^2 - |0.38| < -0.18 < |0.38|$

$-0.0912 < -0.18 < 0.38$

بازهم در شرط صدق نکرد ولی ادامه می‌دهیم:

	$k=0$	$k=1$	$k=2$
γ_k	1	+0.38	-0.18
ϕ_{kk}	0.38	-0.03	0.089

$$\hat{\alpha} = \frac{\gamma_2}{\gamma_1} = \frac{-0.18}{0.38} = -0.4736$$

$$\frac{(1-\hat{\alpha}\hat{\beta})(\hat{\alpha}-\hat{\beta})}{1+\hat{\beta}^2-2\hat{\alpha}\hat{\beta}} = 0.38 \iff (1+0.47\hat{\beta})(0.47-\hat{\beta}) = 0.38 + 0.38\hat{\beta}^2 + 0.35\hat{\beta}$$

$$\iff 0.47 - \hat{\beta} + 0.2209\hat{\beta} - 0.47\hat{\beta}^2 - 0.35\hat{\beta} - 0.38\hat{\beta}^2 - 0.38 = 0$$

$$0.87\hat{\beta}^2 - 1.12\hat{\beta} + 0.09 = 0$$

$a = 0.87$ $\quad \Delta = b^2 - 4ac$

$b = -1.12$

$c = 0.09$ $\quad \Delta = 0.9412 \to \sqrt{\Delta} = 0.9701576$

$$x_1, x_2 = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$\frac{1.12 + 0.9701}{1.74} = 1.2012 \checkmark$$

$$\frac{1.12 - 0.9701}{1.74} = 0.086149 \quad 6GE$$

$$\hat{\alpha} = \frac{\gamma_2}{\gamma_1} = -0.4736, \quad \hat{\beta} = 1.2012, \quad \hat{\sigma}_z^2 = \quad , \bar{X} = 0.4314$$

$$\hat{\sigma}_z^2 = C_0(1-\alpha^2)\left[\frac{1}{1+\hat{\beta}^2+2\hat{\alpha}\hat{\beta}}\right] = (2.06)(1+0.47^2)\left[\cdots\right] = 1.927078$$

$$C_0 = \frac{1}{N}\sum_{t=1}^{N}(X_t - \bar{X})^2 = 2.06$$

$S.E = 1.388182$

فرض صفر بدان معنا پذیرفته شد.

$$Z = \frac{\bar{X}}{S.E(\bar{X})} = \frac{0.43175}{1.388182} = 0.31080 \to H_0 \text{ accept} \to \mu = 0$$

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۱) برای پیش بینی داریم که:

$$\hat{x}_t(L) = \mu + \alpha^L (x_t - \mu) - \alpha^{L-1} \theta Z_t \quad 1 \leq L \leq 1$$

$$\hat{x}_t(L) = \alpha_1 \hat{x}_t(L-1) + \dots + \alpha_p \hat{x}_t(L-p) + \theta_0$$

$$\Rightarrow ARMA(1,1) \rightarrow X_t = \underset{\text{میانگین}}{0} + (-0.4736)(x_{t-1}) + Z_{t-1} + Z_t$$

$$\hookrightarrow Z_t \sim N(0, 1.92)$$

$$\cancel{L \geq 2 \Rightarrow \hat{x}_t(L) = 0 + (-0.4736)\, x_{t-1} + Z_{t-1}}$$

$$L = 1 \rightarrow \hat{x}_t(1) = \mu + (-0.4736)^1 (x_t - \mu) - 1 \times \theta \times Z_t = \underline{-0.4736\, x_t - \theta Z_t}$$

با توجه به مقدار اولیه x_t می توانیم پیش بینی را انجام بدهیم.

$$L = 2 \dots 5 \rightarrow \hat{x}_t(2) = -0.4736\, \hat{x}_t(1)$$

بازگشتی

$$L = 5 \rightarrow \hat{x}_t(5) = -0.4736\, \hat{x}_t(4) \dots$$

بازگشتی خواهد بود

$$L = 10 \dots 100 \rightarrow \text{بزرگ است } L \rightarrow \hat{x}_t(L) \xrightarrow[\text{ثابت}]{\text{میل}} \mu = 0 \rightarrow \hat{x}_t(100) = \hat{x}_t(10) = 0$$

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سوال 3

هست $AR(1)$

$X_t = 0.2 X_{t-1} + Z_t$

$\alpha = 0.2 \to \alpha > 0 \to \rho_k = \alpha^{|k|}$ و

$\Rightarrow \gamma_{(k)} = \sigma_x^2 \alpha^{|k|}$

$\sigma_Z^2 = (1-\alpha^2)\sigma_x^2 = (1-0.04)\sigma_x^2$

$$\Rightarrow f_{(\omega)} = \sigma_Z^2 \times \frac{1}{\pi \times [1 - 2(0.2)\cos(\omega) + 0.04]}$$

بنابراین

$\Rightarrow [1 - 0.2B] X_t = Z_t \to X_t = \frac{Z_t}{1-0.2B}$

اگر if $\Theta(B) = \frac{1}{1-0.2B} \to X_t = \Theta(B) Z_t$ و

حال فرآیندمان به یک فرآیند MA نامتناهی تبدیل شده و طبق جزوه داریم که:

$\Gamma(B) = \sigma_Z^2 \Theta(B)\Theta(B^{-1}) \Rightarrow \Gamma(B) = \sigma_Z^2 \times \frac{1}{1-0.2B} \times \frac{1}{1-0.2B^{-1}}$

$\sigma_Z^2 = 0.96\,\sigma_x^2$

$$\Leftrightarrow \Gamma(B) = \frac{0.96}{(1-0.2B)(1-0.2B^{-1})}$$