General claim arrival process proof

Section 6 - Home Work 1

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Problem 1

Consider equation (1.35) with $F \in S$. Fix $t > 0$, and suppose that the sequence $(p_t(n))$ satisfies:

$$\sum_{n=0}^{\infty}(1+\epsilon)^n p_t(n) < \infty$$

for some $\epsilon > 0$. We need to show that $G_t \in S$ and that

$$\overline{G_t}(x) \sim EN(t)\overline{F}(x), \quad x \to \infty.$$

Embrechts et al. (1997):

Solve 1

Cramér-Lundberg Theorem for Large Claims I:
Theorem (Cramér-Lundberg I): In a risk process where the claim size distribution F has a subexponential tail, the ruin probability satisfies:

$$\psi(u) \sim \rho^{-1}\overline{F_I}(u) \text{ as } u \to \infty,$$

where ρ is the adjustment coefficient, and $\overline{F_I}(u)$ represents the tail distribution of the integrated claim size.

Proof Using Cramér-Lundberg Theorem

1. *Subexponential Assumption:* Given $F \in S$, we know that F is subexponential. This implies for all $t > 0$:

$$\overline{F * F}(x) \sim 2\overline{F}(x).$$

2. *Transformation of G_t:* Define G_t through the mixture transformation with **weights** $p_t(n)$:

$$\overline{G_t}(x) = \sum_{n=0}^{\infty} p_t(n)\overline{F^{*n}}(x).$$

3. *Applying Cramér-Lundberg:* From the Cramér-Lundberg theorem, for large claims, the tail distribution of the aggregate claims can be approximated by:

$$\psi(u) \sim \rho^{-1}\overline{F_I}(u) \text{ as } u \to \infty.$$

4. *Simplifying the Asymptotic Behavior:* Using the properties of subexponential distributions and the decay condition on $p_t(n)$:

$$\overline{G_t}(x) = \sum_{n=0}^{\infty} p_t(n)\overline{F^{*n}}(x) \sim EN(t)\overline{F}(x).$$

5. *Result:* Therefore, by the property of subexponentiality:

$$\overline{G_t}(x) \sim EN(t)\overline{F}(x), \quad x \to \infty.$$

This shows that $G_t \in S$ and completes the proof of Theorem 1.3.9.
Schmidli (2018); Mandjes and Boxma (2023); Springer (2023)

References

Embrechts, P., Klüppelberg, C., and Mikosch, T. (1997). *Modelling extremal events*, volume 33 of *Applications of Mathematics (New York)*. Springer-Verlag, Berlin. For insurance and finance.

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