# Check slowly varying - $f(x) = (x \ln(1+x))^{\alpha}$
Section 5 - Home Work 3

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**Problem 1**

Show that if we set $f(x) = (x \ln(1+x))^{\alpha}$ then check that is this function slowly varying or not.Embrechts et al. (1997):

**Solve 1**

To determine if the function $f(x) = (x \ln(1+x))^{\alpha}$ is a slowly varying function, we need to use the definition of a slowly varying function. A function $L(x)$ is slowly varying at infinity if for all $a > 0$:

$$\lim_{x \to \infty} \frac{L(ax)}{L(x)} = 1$$

Let's apply this to $f(x) = (x \ln(1+x))^{\alpha}$:

1. **Substitute $ax$ into the function:**

$$f(ax) = (ax \ln(1+ax))^{\alpha}$$

2. **Form the ratio $\frac{f(ax)}{f(x)}$:**

$$\frac{(ax \ln(1+ax))^{\alpha}}{(x \ln(1+x))^{\alpha}} = \left(\frac{ax \ln(1+ax)}{x \ln(1+x)}\right)^{\alpha} = \left(a \frac{\ln(1+ax)}{\ln(1+x)}\right)^{\alpha}$$

3. **Take the limit as $x$ approaches infinity:**

$$\lim_{x \to \infty} \left(a \frac{\ln(1+ax)}{\ln(1+x)}\right)^{\alpha}$$

To evaluate this limit, consider the behavior of the logarithmic function for large $x$. For large $x$, $\ln(1+x) \approx \ln(x)$. Thus, we can approximate:

$$\frac{\ln(1+ax)}{\ln(1+x)} \approx \frac{\ln(ax)}{\ln(x)} = \frac{\ln(a) + \ln(x)}{\ln(x)} = 1 + \frac{\ln(a)}{\ln(x)}$$

As $x$ approaches infinity, $\frac{\ln(a)}{\ln(x)}$ approaches 0. Therefore:

$$\lim_{x \to \infty} \frac{\ln(1+ax)}{\ln(1+x)} = 1$$

Thus:

$$\lim_{x \to \infty} \left(a \frac{\ln(1+ax)}{\ln(1+x)}\right)^{\alpha} = a^{\alpha}$$

Since $a^{\alpha} \neq 1$ for $\alpha \neq 0$, $f(x) = (x \ln(1+x))^{\alpha}$ is **not** a slowly varying function unless $\alpha = 0$. Wikipedia (2024); Springer (2024b); Definitions.net (2024); JSTOR (2024); Symbolab (2024); Calculator (2024); Springer (2024a); arXiv (2021); LibreTexts (2024); is Fun (2024); Wiki (2024)

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