Check slowly varying - $f(x) = (x \ln(1+x))^\alpha$
Section 5 - Home Work 4

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Problem 1

Show that if we set $f(x) = \ln(\ln(e + x))$ then check that is this function slowly varying or not.Embrechts et al. (1997):

Solve 1

To determine if the function $f(x) = \ln(\ln(e + x))$ is a slowly varying function, we use the definition of a slowly varying function. A function $L(x)$ is slowly varying at infinity if for all $a > 0$:

$$\lim_{x\to\infty} \frac{L(ax)}{L(x)} = 1$$

Let's apply this to $f(x) = \ln(\ln(e + x))$:

1. **Substitute ax into the function:**

$$f(ax) = \ln(\ln(e + ax))$$

2. **Form the ratio $\frac{f(ax)}{f(x)}$:**

$$\frac{\ln(\ln(e + ax))}{\ln(\ln(e + x))}$$

3. **Take the limit as x approaches infinity:**

$$\lim_{x\to\infty} \frac{\ln(\ln(e + ax))}{\ln(\ln(e + x))}$$

To evaluate this limit, consider the behavior of the logarithmic functions for large x. For large x, $\ln(e + x) \approx \ln(x)$. Thus, we can approximate:

$$\ln(\ln(e + ax)) \approx \ln(\ln(ax)) = \ln(\ln(a) + \ln(x))$$

For large x, $\ln(a)$ becomes negligible compared to $\ln(x)$, so:

$$\ln(\ln(a) + \ln(x)) \approx \ln(\ln(x))$$

Therefore:

$$\frac{\ln(\ln(e + ax))}{\ln(\ln(e + x))} \approx \frac{\ln(\ln(x))}{\ln(\ln(x))} = 1$$

Thus:

$$\lim_{x\to\infty} \frac{\ln(\ln(e + ax))}{\ln(\ln(e + x))} = 1$$

Since this limit equals 1 for any $a > 0$, $f(x) = \ln(\ln(e + x))$ is indeed a slowly varying function. Wikipedia (2024); Smit (1991); Definitions.net (2024); Gihman (2024); YouTube (2024c,a,b); Symbolab (2024); LibreTexts (2024); Steele (2024)

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