Proof of corollary 1.3.2

Section 5 - Home Work 1

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Problem 1

Proof of bellow relation Embrechts et al. (1997):

(a) Prove the following

(i) If $F(x) = -x^{\alpha}L(x)$ for $\alpha > 0$ and $L \in R_{\rho}$, then for all $n \geq 1$,

$$\bar{F}^{n*}(x) \sim n\bar{F}(x), \quad x \to \infty$$

Solve 1

To solve this problem, I will take the help of inductive proof, which will be discussed further: Suppose now that $X_1, \cdots, X_n$ are iid with df F as in the above corollary Denote the partial sum of $X_1, \cdots, X_n$ by $S_n = X_1 + \cdot + X_n$ and their maximum by $M_n = max(X_1, \cdots, X_n)$. Then for all $n \geq 2$,

$$P(S_n > x) = \bar{F^{n*}}(x),$$

$$P(M_n > x) = \bar{F}^n(x) = \bar{F}(x)\sum_{k=0}^{n-1} F^k(x) \sim n\bar{F}(x)$$

so we start with set n = 2 and we have:

$$P(M_2 > x) = 1 - P(M_2 \leq x) = 1 - P(X_1 < x)P(X_2 < x)$$

$$1 - (F(x))^2 = (1 - F(x))(1 + F(x))$$

$$\bar{F}(x)(1 + F(x)) = \bar{F}(x)\sum_{k=0}^{n-1} F^k(x)$$

and now we should assume that n = k is usable for proofing n = k+1 so we have:

$$P(M_{n+1} > x) = 1 - P(M_{n+1} < x) = 1 - P(M_1 < x)P(M_n < x)$$

$$= 1 - F(x)(1 - P(M_n > x)) = 1 - F(x)(1 - \bar{F}(x)\sum_{k=0}^{n-1} F^k(x))$$

$$= 1 - (F(x) + F(x)\bar{F}(x)\sum_{k=0}^{n-1} F^k(x)) = 1 - F(x) + F(x)\bar{F}(x)\sum_{k=0}^{n-1} F^k(x)$$

$$= \bar{F}(x) + F(x)\bar{F}(x)\sum_{k=0}^{n-1} F^k(x) = \bar{F}(x)[1 + F(x)\sum_{k=0}^{n-1} F^k(x)] = \bar{F}(x)[\sum_{k=0}^{n} F^k(x)]$$

$$\rightarrowtail \lim_{x \to \infty} \bar{F}(x)\sum_{k=0}^{n} F^k(x) = \bar{F}(x) * n$$

Done. so we can say the assumed where n = k is ok was write.

References

Embrechts, P., Klüppelberg, C., and Mikosch, T. (1997). *Modelling extremal events*, volume 33 of *Applications of Mathematics (New York)*. Springer-Verlag, Berlin. For insurance and finance.