In the name of God

Producer:
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Subject:
Make confidence interval for the antegral solving with montecarlo method …

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Issue: For the answer of the following integral, which is to be solved by the Ment Carlo method, we obtain four confidence intervals with the number of iterations and the confidence intervals mentioned below.

$$I = \int_0^{10} e^{x^2}\, dx$$

$$we\ need\ to\ 4\ confidence\ interval\ with: \begin{cases} n = 100\ ,95\% \\ n = 50000\ ,95\% \end{cases}, \begin{cases} n = 100\ ,99\% \\ n = 50000\ ,99\% \end{cases}$$

Solve:we know that if $U \sim uniform(a,b)$ $\quad ; f(u)_U = \frac{1}{b-a}$

$$I = \int_0^{10} e^{x^2}\, dx = 10I = \int_0^{10} \frac{e^{x^2}}{10}\, dx$$

We put: $g(x) = e^{x^2}$ $\quad , U \sim uniform(0{,}10);\ f(u)_U = \frac{1}{10-0} = \frac{1}{10}$

$$I = 10 \int_0^{10} g(x) * f(x)\, dx \overset{M.c}{\Longrightarrow} I = 10E[g(u)] = 10 * \lim_{n\to\infty} \frac{\sum_{i=1}^{n} g(u_i)}{n}$$

Given the above, we have:

Step1 :for n=100,50000,100,50000 times we solve this integral by the method of Mont Carlo and save the result.

Step2 : for each n in step1 we have a confidence interval(According to the central limit theorem):

$$P\left(\hat{\theta}_n - Z_{1-a\backslash 2}\sqrt{\frac{\sigma^2}{n}} \le \theta < \hat{\theta}_n + Z_{1-a\backslash 2}\sqrt{\frac{\sigma^2}{n}}\right) \cong 1 - \alpha$$

$$\hat{\theta}_n = (I_1, I_2, \dots, I_n), \qquad \hat{\sigma}_n^2 = \sum_{i=1}^{n} \frac{\left(I_i - \hat{\theta}_n\right)^2}{n-1}$$

We know that:

$$L(I): lower\ band\ of\ our\ confidence\ interval\ , U(I): is\ Upper\ band\ of\ our\ confidence\ interval$$

$$[L(I), U(I)] = \left[\hat{\theta}_n - Z_{1-a\backslash 2}\sqrt{\frac{\sigma^2}{n}}\ ,\ \hat{\theta}_n + Z_{1-a\backslash 2}\sqrt{\frac{\sigma^2}{n}}\right]$$

Given the above content for simulation we have:

```
> rm(list=ls())
> memory.limit(size = 999999999)
[1] 1e+09
> n=c(100,50000,100,50000)
> Lower.band<-vector(length = 4)
>   Upper.band<-vector(length = 4)
>   L<-c()
>   z=abs(c(qnorm(0.05),qnorm(0.05),qnorm(0.01),qnorm(0.01)))
>   for(w in 1:4){
+   theta.hat<-vector(length = n[w])
+   N=10^4
+   for(j in 1:n[w]){
+     g<-vector(length = N)
+     for(i in 1:N){
+       u<-runif(1,0,10)
+       g[i]<-exp(u^2)
+     }
+     theta.hat[j]<-10*mean(g)}
+   sigma2<-sd(theta.hat)
+   theta.hat.mean<-mean(theta.hat)
+   Lower.band[w]<-theta.hat.mean-z[w]*(sigma2/sqrt(n[w]))
+   Upper.band[w]<-theta.hat.mean+z[w]*(sigma2/sqrt(n[w]))
+   L[w]<-Upper.band[w]-Lower.band[w]}
>   for(w in 1 :4){
+   print(paste("the confidence interval is:","(",Lower.band,Upper.band,")","
with alpha=",1-pnorm(z[w]),"repeat=",n[w]))
+ print(paste("the long of our confidence is:",L[w]))}
[1] "the confidence interval is: ( 1.33997484517119e+42 1.38645426827967e+42)
with alpha= 0.05 repeat= 100"
[2] "the confidence interval is: ( 1.34947774896461e+42 1.35144530991326e+42)
with alpha= 0.05 repeat= 100"
[3] "the confidence interval is: ( 1.31821579229368e+42 1.37644378031682e+42)
with alpha= 0.05 repeat= 100"
[4] "the confidence interval is: ( 1.34830343206368e+42 1.35108050430998e+42)
with alpha= 0.05 repeat= 100"
[1] "the long of our confidence is: 4.64794231084848e+40"
[1] "the confidence interval is: ( 1.33997484517119e+42 1.38645426827967e+42)
with alpha= 0.05 repeat= 50000"
[2] "the confidence interval is: ( 1.34947774896461e+42 1.35144530991326e+42)
with alpha= 0.05 repeat= 50000"
[3] "the confidence interval is: ( 1.31821579229368e+42 1.37644378031682e+42)
with alpha= 0.05 repeat= 50000"
[4] "the confidence interval is: ( 1.34830343206368e+42 1.35108050430998e+42)
with alpha= 0.05 repeat= 50000"
[1] "the long of our confidence is: 1.96756094864751e+39"
[1] "the confidence interval is: ( 1.33997484517119e+42 1.38645426827967e+42)
with alpha= 0.01 repeat= 100"
[2] "the confidence interval is: ( 1.34947774896461e+42 1.35144530991326e+42)
with alpha= 0.01 repeat= 100"
[3] "the confidence interval is: ( 1.31821579229368e+42 1.37644378031682e+42)
with alpha= 0.01 repeat= 100"
[4] "the confidence interval is: ( 1.34830343206368e+42 1.35108050430998e+42)
with alpha= 0.01 repeat= 100"
[1] "the long of our confidence is: 5.8227988023134e+40"
[1] "the confidence interval is: ( 1.33997484517119e+42 1.38645426827967e+42)
with alpha= 0.01 repeat= 50000"
[2] "the confidence interval is: ( 1.34947774896461e+42 1.35144530991326e+42)
with alpha= 0.01 repeat= 50000"
[3] "the confidence interval is: ( 1.31821579229368e+42 1.37644378031682e+42)
with alpha= 0.01 repeat= 50000"
[4] "the confidence interval is: ( 1.34830343206368e+42 1.35108050430998e+42)
with alpha= 0.01 repeat= 50000"
[1] "the long of our confidence is: 2.77707224630273e+39
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