Check slowly varying - $f(x) = x^\alpha \ln(1+x)$
Section 5 - Home Work 5

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Problem 1

Show that if we set $f(x) = x^\alpha \ln(1+x)$ then check that is this function slowly varying or not.Embrechts et al. (1997):

Solve 1

To determine if the function $f(x) = x^\alpha \ln(1+x)$ is a slowly varying function, we use the definition of a slowly varying function. A function $L(x)$ is slowly varying at infinity if for all $a > 0$:

$$\lim_{x\to\infty} \frac{L(ax)}{L(x)} = 1$$

Let's apply this to $f(x) = x^\alpha \ln(1+x)$:

1. **Substitute ax into the function:**

$$f(ax) = (ax)^\alpha \ln(1+ax)$$

2. **Form the ratio $\frac{f(ax)}{f(x)}$:**

$$\frac{(ax)^\alpha \ln(1+ax)}{x^\alpha \ln(1+x)} = a^\alpha \frac{\ln(1+ax)}{\ln(1+x)}$$

3. **Take the limit as x approaches infinity:**

$$\lim_{x\to\infty} a^\alpha \frac{\ln(1+ax)}{\ln(1+x)}$$

To evaluate this limit, consider the behavior of the logarithmic function for large x. For large x, $\ln(1+x) \approx \ln(x)$. Thus, we can approximate:

$$\frac{\ln(1+ax)}{\ln(1+x)} \approx \frac{\ln(ax)}{\ln(x)} = \frac{\ln(a)+\ln(x)}{\ln(x)} = 1 + \frac{\ln(a)}{\ln(x)}$$

As x approaches infinity, $\frac{\ln(a)}{\ln(x)}$ approaches 0. Therefore:

$$\lim_{x\to\infty} \frac{\ln(1+ax)}{\ln(1+x)} = 1$$

Thus:

$$\lim_{x\to\infty} a^\alpha \frac{\ln(1+ax)}{\ln(1+x)} = a^\alpha$$

Since $a^\alpha \neq 1$ for $\alpha \neq 0$, $f(x) = x^\alpha \ln(1+x)$ is **not** a slowly varying function unless $\alpha = 0$.

> I hope this helps! If you have any more questions or need further clarification, feel free to ask. Wikipedia (2024); Smit (1991); Definitions.net (2024); Gihman (2024); YouTube (2024c,a,b); Symbolab (2024); LibreTexts (2024); **?**); arXiv (2021)

References

arXiv (2021). arxiv:2103.00114v1 [math.pr] 27 feb 2021.

Definitions.net (2024). What does slowly varying function mean?

Embrechts, P., Klüppelberg, C., and Mikosch, T. (1997). *Modelling extremal events*, volume 33 of *Applications of Mathematics (New York)*. Springer-Verlag, Berlin. For insurance and finance.

Gihman, I. (2024). Slowly varying functions in the complex plane. *Mathematics of the USSR-Izvestiya*.

LibreTexts (2024). 3.9: Derivatives of ln, general exponential and log functions and logarithmic differentiation.

Smit, J. D. (1991). Regularly varying functions and pareto-type distributions. In *Proceedings of the Second International Conference on Economic and Financial Modelling*, pages 95–114.

Symbolab (2024). Derivative calculator.

Wikipedia (2024). Slowly varying function.

YouTube (2024a). Derivatives of exponential functions.

YouTube (2024b). Derivatives of exponential functions and logarithmic differentiation calculus.

YouTube (2024c). The exponential function e and the natural log ln.