In the name of God

Producer:

Mehrab Atighi

Subject:

The accpetance-rejection methods in R

For Generating the standatd normal random variable.

Date:

12/6/2020

Supervisor:

Dr.Seyed Noorullah Mousavi

Isuue:

Generate a random number with a standard normal distribution and use the rejection and acceptance algorithm method.

Solve:

First we write the algorithm in theory, then we run the algorithm in the software.

For generate a standard normal random variable such az $Z \sim N(0,1)$ d

We know that the standard normal distribution probability density function is:

$$f_Z(z) = \frac{2}{\sqrt{2\pi}} e^{\frac{-z^2}{2}} \quad for \;\; 0 < z < \infty$$

because:

If we desire an $X \sim N(\mu, \sigma 2)$, then we can express it as $X = \sigma Z + \mu$, where Z denotes a random variable with the $N(0,1)$ distribution. Thus it suffices to find an algorithm for generating $Z \sim N(0,1)$. Moreover, if we can generate from the absolute value, $|Z|$ is non-negative and has density.

If we set this

$$g(z) = e^{-z} \quad for \;\; 0 < z < \infty$$

According to the rejection and acceptance algorithm, we make the following fraction and have:

$$\frac{f(z)}{g(z)} = \sqrt{2/\pi} e^{\frac{z-z^2}{2}}$$

We know that the best value for the a is Maximum value of the above relationship:

$$a = Max\left(\frac{f(z)}{g(z)}\right)$$

According to the exponential in the above relationship we can see that the maximizes of $\frac{z-z^2}{2}$ occurs in z=1.

if we calculate it and put the values in above relationship we have:

$$a = Max\left(\frac{f(z)}{g(z)}\right) = \frac{f(1)}{g(1)} = \sqrt{2e/\pi} \approx 1.32$$

so:

$$\frac{f(z)}{cg(z)} = \frac{\sqrt{2/\pi} e^{\frac{z-z^2}{2}}}{\sqrt{2e/\pi}} = \exp\left\{z - \frac{z^2}{2} - \frac{1}{2}\right\} = \exp\left\{-\frac{(z-1)^2}{2}\right\}$$

Step 1: Generate Y, an exponentioal random variable with rate 1.

Step 2: Generate a random number U.

Step 3: If $U \leq \exp\{-\frac{(Z-1)^2}{2}\}$, then set X=Z ;else Go to step 1.

We know that:

$$U \leq \exp\left\{-\frac{(Z-1)^2}{2}\right\} \Leftrightarrow -\log(U) \geq \left(\frac{(Z-1)^2}{2}\right)$$

That: $-\log(U) \sim \exp(1)$

It means that we Have two Step:

Step 1:Generate Z1 and Z2 I.I.D exponetional with rate 1.

Step 2:If $Z2 \geq \left(\frac{(Z1-1)^2}{2}\right)$, $set\ X = Z1$,else return to Step 1.

We know that :

by computing $Z2 - \frac{(Z1-1)^2}{2} > 0$
we can also generate an exponential random variable (independent of X) having rate 1.

Step 1: Generate *Z1*,*Z2* ,two exponential random variable with rate 1.

Step 2:If $Z2 - \frac{(Z1-1)^2}{2} > 0$, set $y = Z2 - \frac{(Z1-1)^2}{2} > 0$,else go to step 1.

Step 3 Generate U a random number then: $Z = \begin{cases} Z1 & if \quad U > 1/2 \\ -Z1 & if \quad U \leq 1/2 \end{cases}$

then $Z \sim N(0,1)$.

Now we will show this solve in R program:

We repeat this alg 10^6 and we can see the mean and variance of the $Z_i \quad i = 1,2,\ldots,10^6$

```
> rm(list = ls())
>   Z<-c()
>   for(i in 1:10^6){
+   y1<-rexp(1,rate = 1)
+   y2<-rexp(1,rate = 1)
+   while(y2-((y1-1)^2)/2<=0){
+     y1<-rexp(1,rate = 1)
+     y2<-rexp(1,rate = 1)
+   }
+   y<-y2-((y1-1)^2)/2
+   u<-runif(1)
+   if(u>1/2){
+     z<- -y1
+
+   }else{
+     z<- y1
+   }
+
+ Z[i]<-z}
>   mean(Z)
[1] -0.001371392
>   var(Z)
[1] 0.9989004
```

So we can see that our Z have standard normal distrubtion with mean=0 and variance=1.

End.