In the name of God

Producer:
Mehrab Atighi

Subject:
Normal-standard probability estimate with Monte.Carlo method

Date:
12/29/2020

Supervisor:
Dr.Seyed Noorullah Mousavi

Issue : If we are interested in the tail probability $\Pr(X > 20)$ when $X \sim N(0, 1)$, simulating from a $N(0, 1)$ distribution does not work. Express
the probability as an integral and use an obvious change of variable to rewrite
this integral as an expectation under a $U(0, 1/20)$ distribution. Deduce a Monte Carlo approximation to $\Pr(X > 20)$ along with an error assessment.

Solve:

$$if\ X \sim Normal(0,1) \Rightarrow f_X(x) = \frac{e^{-\frac{x^2}{2}}}{\sqrt{2*\pi}}$$

$$P(x > 20) = P(20 < x < \infty) = P\left(\frac{1}{20} > \frac{1}{x} > \frac{1}{\infty}\right)$$

$$\Leftrightarrow P\left(0 < \frac{1}{x} < \frac{1}{20}\right) \overset{u=\frac{1}{x}}{\Longleftrightarrow} P\left(0 < u < \frac{1}{20}\right)$$

So we know that the u value is between 0 and 1/20 , so we can say that:

$$U \sim uniform\left(0\,, \frac{1}{20}\right)\ , g_U(u) = \frac{1}{0 - \frac{1}{20}} = 20$$

And we know that $u = \frac{1}{x}, so \quad du = -\frac{1}{x^2}dx \overset{x=\frac{1}{u}}{\Longleftrightarrow} du = -u^2 dx \Leftrightarrow dx = \frac{du}{-u^2}$

So:

$$P(20 > x) = \frac{1}{2\pi}\int_{20}^{\infty} \frac{e^{-\frac{x^2}{2}}}{\sqrt{2*\pi}}\, dx \overset{u=\frac{1}{x}}{\Longleftrightarrow} P\left(\frac{1}{20} < u\right)$$

$$= \int_0^{\frac{1}{20}} \frac{1}{u^2\sqrt{2\pi}} e^{-\frac{1}{2u^2}} du$$

$$= \frac{1}{20}\int_0^{\frac{1}{20}} \frac{1}{u^2\sqrt{2\pi}} e^{-\frac{1}{2u^2}} * 20du = \frac{1}{20}\int_0^{\frac{1}{20}} h_X(x) * g_U(u) du$$

$$\overset{M.c}{\Longrightarrow} \frac{1}{20} * E_g[h_X(u)] = \frac{1}{20} * \lim_{n\to\infty} \frac{\sum_{i=1}^{n} h_X(u_i)}{n}$$

We know that of we want to calculate this probabity we have:

$$P(x > 20) \xLeftrightarrow[\text{in Normal distrubtion}]{\text{Property of symmetry}} P(x < -20)$$

```
> pnorm(-20)
[1] 2.753624e-89
```

And the Monte-Carlo estimate method for this probability is :

```
> N=10^6
> u<-runif(N,min = 0 , max = 1/20)
> fx<-function(u){exp(20/(-2*u^2))/u^2 * sqrt(2*pi)}
> E<-c()
> for(i in 1:N){
+   E[i]<-fx(u[i])
+ }
> print(1/20*mean(E))
[1] 0
```

Conculsion: we understand that our probability is very small and we can say its 0.