Truth table

even parity

A	B	C	D	F	
0	0	0	0	0	
0	0	0	1	1	m_1
0	0	1	0	1	m_2
0	0	1	1	0	
0	1	0	0	1	m_4
0	1	0	1	0	
0	1	1	0	0	
0	1	1	1	1	m_7
1	0	0	0	1	m_8
1	0	0	1	0	
1	0	1	0	0	
1	0	1	1	1	m_{11}
1	1	0	0	0	
1	1	0	1	1	m_{13}
1	1	1	0	1	m_{14}
1	1	1	1	0	

Karnaugh map

CD \ AB			A	
	0	4 1	12	8 1
	1 1	5	13 1	9
C	3	7 1	15	11 1
C	2 1	6	14 1	10
		B	B	

D (rows 2–3)

$$AB C\bar{D} + AB\bar{C}D + A\bar{B}CD + A\bar{B}\bar{C}\bar{D} + \bar{A}BCD + \bar{A}B\bar{C}\bar{D}$$

$$+ \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D$$

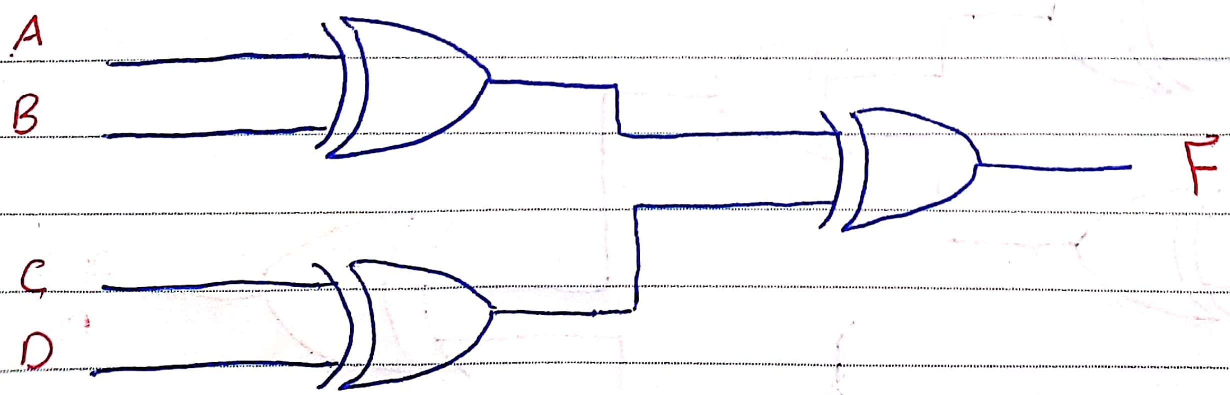
$$= AB(C \oplus D) + A\bar{B}(CD + \bar{C}\bar{D}) + \bar{A}B(CD + \bar{C}\bar{D}) + \bar{A}\bar{B}(C \oplus D)$$

$$= AB(C \oplus D) + CD(A \oplus B) + \bar{C}\bar{D}(A \oplus B) + \bar{A}\bar{B}(C \oplus D)$$

$$(A \oplus B)(CD + \bar{C}\bar{D}) + (C \oplus D)(AB + \bar{A}\bar{B})$$

$$= (A \oplus B) \oplus (C \oplus D) = F \text{ (even parity)}$$

Of course, it was possible to write this form the beginning.



expand the circuit to include the odd parity as well.

even ↑

F	$even/\overline{odd}$	q
0	0	1
0	1	0
1	0	0
1	1	1

expand the circuit to include the odd parity as well.

even ↑

F	$odd/\overline{even}$	q
0	0	0
0	1	1
1	0	1
1	1	0

$\Rightarrow q = F \oplus odd$