Lcm sum 1 to n:

Sum = lcm(1,n) + lcm(2,n) + lcm(3,n) + ……………. + lcm(n,n).

We know lcm(n,n)=n

Sum - n = lcm(1,n) + lcm(1,n) + lcm(1,n) + ……. + lcm(n-1,n)..................eqn1

Reverse eqn1,

Sum - n = lcm(n-1,n) + lcm(n-2,n) +.............. lcm(1,n) …………………...eqn2

X = lcm(a,n) + lcm(n-a,n)

= an/ gcd(a,n) + (n-a)n/gcd(n-a,n)

Gcd lemma gcd(a,n) ==gcd(n-a,n)

= an/gcd(a,n) + (n-a)n/gcd(a,n)

= n^2/gcd(a,n) ……………………….eqn3

Add eqn1 and eqn2 then we get,

2(Sum - n) = n^2/gcd(1,n) + n^2/gcd(2,n) + n^2/gcd(3,n)...........n^2/gcd(n-1,n)

2(sum - n) = $\sum_{i=1}^{n-1} \frac{n^2}{gcd(i,n)}$

2(sum - n) = $n \sum_{i=1}^{n-1} \frac{n}{gcd(i,n)}$

We know gcd(i,n) = d .

Now we only need to find how many times $\frac{n}{d}$ should be added. Remember that $\frac{n}{d}$ is also a divisor of n.

So, how we count the number of times...see, gcd(i,n) = d then gcd(i/d,n/d) = 1.

So, phi(n/d) gives us the result.

2(sum - n) = $n(\sum_{d|n}(\Phi(d) \ * d) - 1)$

2(sum - n) = $n \sum_{d|n}(\Phi(d) \ * d) - n$

2sum - n = $n \sum_{d|n}(\Phi(d) \ * d)$

2sum = $n \sum_{d|n}(\Phi(d) \ * d) \ + \ n$

2sum = $n(\sum_{d|n}(\Phi(d) \ * d) \ + \ 1)$

sum = $\frac{n}{2} (\sum_{d|n}(\Phi(d) \ * d) \ + \ 1)$

Reference : forthright48