Test functions for optimization

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In applied mathematics, **test functions**, known as **artificial landscapes**, are useful to evaluate characteristics of optimization algorithms, such as:

- Convergence rate.
- Precision.
- Robustness.
- General performance.

Here some test functions are presented with the aim of giving an idea about the different situations that optimization algorithms have to face when coping with these kinds of problems. In the first part, some objective functions for single-objective optimization cases are presented. In the second part, test functions with their respective Pareto fronts for multi-objective optimization problems (MOP) are given.

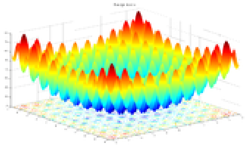
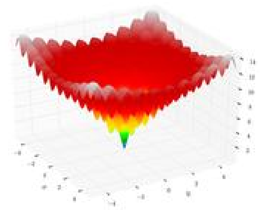
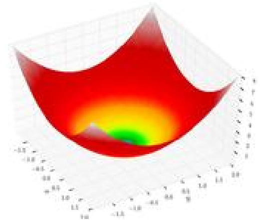
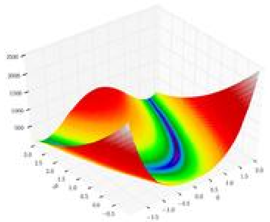
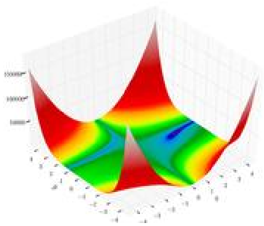
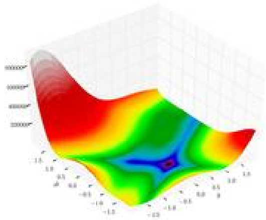
The artificial landscapes presented herein for single-objective optimization problems are taken from Bäck,[1] Haupt et al.[2] and from Rody Oldenhuis software.[3] Given the number of problems (55 in total), just a few are presented here. The complete list of test functions is found on the Mathworks website.[4]

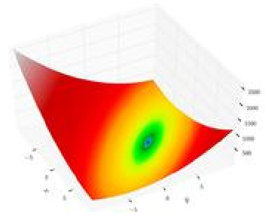
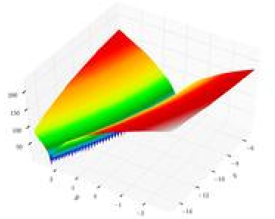
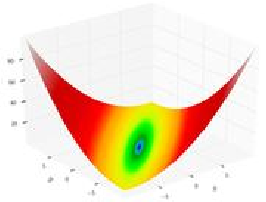
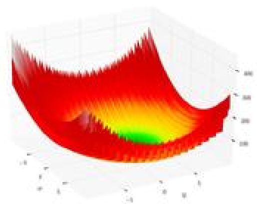
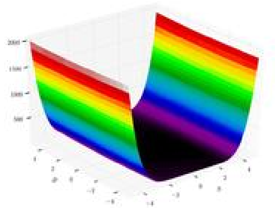
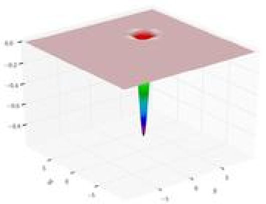
The test functions used to evaluate the algorithms for MOP were taken from Deb,[5] Binh et al.[6] and Binh.[7] You can download the software developed by Deb,[8] which implements the NSGA-II procedure with GAs, or the program posted on Internet,[9] which implements the NSGA-II procedure with ES.

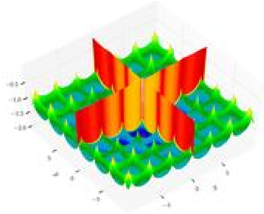
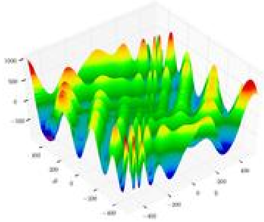
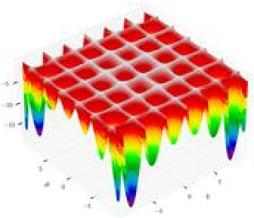
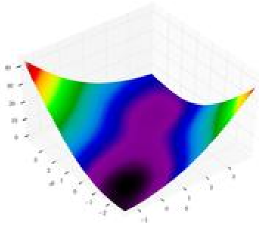
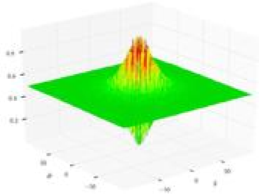
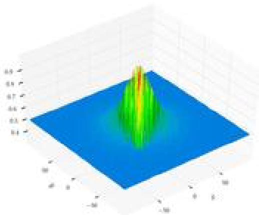
Just a general form of the equation, a plot of the objective function, boundaries of the object variables and the coordinates of global minima are given herein.

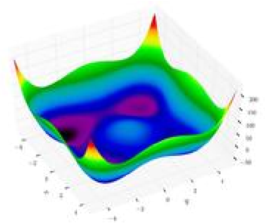
Contents

Test functions for single-objective optimization

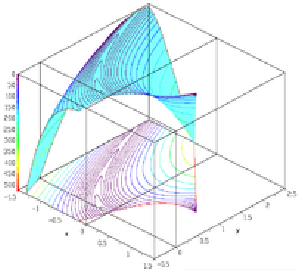
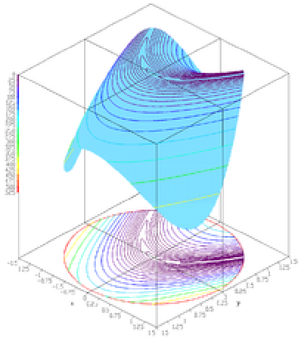
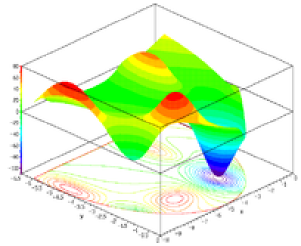
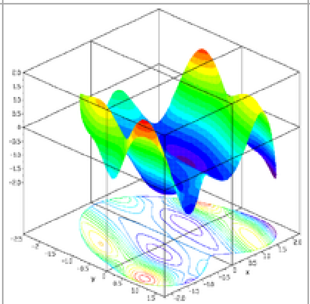
Name	Plot	Formula	Global minimum	Search domain
Rastrigin function		$f(\mathbf{x}) = An + \sum_{i=1}^{n} \left[x_i^2 - A\cos(2\pi x_i)\right]$ where: $A = 10$	$f(0,0) = 0$	$-5.12 \le x, y \le 5.12$
Ackley's function		$f(x,y) = -20\exp\left[-0.2\sqrt{0.5\left(x^2+y^2\right)}\right]$ $-\exp\left[0.5\left(\cos 2\pi x + \cos 2\pi y\right)\right] + e + 20$	$f(0,0) = 0$	$-5 \le x, y \le 5$
Sphere function		$f(\boldsymbol{x}) = \sum_{i=1}^{n} x_i^2$	$f(x_1, \dots, x_n) = f(0, \dots, 0) = 0$	$-\infty \le x_i \le \infty$, $1 \le i \le n$
Rosenbrock function		$f(\boldsymbol{x}) = \sum_{i=1}^{n-1} \left[100\left(x_{i+1} - x_i^2\right)^2 + \left(x_i - 1\right)^2\right]$	$\text{Min} = \begin{cases} n=2 & \rightarrow \quad f(1,1)=0, \\ n=3 & \rightarrow \quad f(1,1,1)=0, \\ n>3 & \rightarrow \quad f(\underbrace{1,\dots,1}_{n \text{ times}})=0 \end{cases}$	$-\infty \le x_i \le \infty$, $1 \le i \le n$
Beale's function		$f(x,y) = \left(1.5 - x + xy\right)^2 + \left(2.25 - x + xy^2\right)^2$ $+\left(2.625 - x + xy^3\right)^2$	$f(3, 0.5) = 0$	$-4.5 \le x, y \le 4.5$
Goldstein–Price function		$f(x,y) = \left[1 + \left(x+y+1\right)^2\left(19 - 14x + 3x^2 - 14y + 6xy + 3y^2\right)\right]$ $\left[30 + \left(2x - 3y\right)^2\left(18 - 32x + 12x^2 + 48y - 36xy + 27y^2\right)\right]$	$f(0,-1) = 3$	$-2 \le x, y \le 2$
Booth's function		$f(x,y) = \left(x + 2y - 7\right)^2 + \left(2x + y - 5\right)^2$	$f(1,3) = 0$	$-10 \le x, y \le 10$

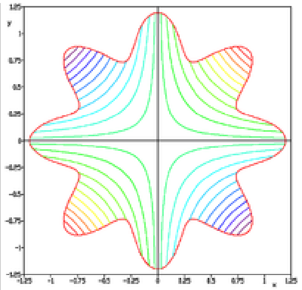
				
Bukin function N.6		$f(x,y) = 100\sqrt{\left\lvert y - 0.01x^{2}\right\rvert} + 0.01\left\lvert x+10 \right\rvert.$	$f(-10,1) = 0$	$-15\le x \le -5$, $-3\le y \le 3$
Matyas function		$f(x,y) = 0.26 \left(x^{2} + y^{2}\right) - 0.48 xy$	$f(0,0) = 0$	$-10\le x,y \le 10$
Lévi function N.13		$f(x,y) = \sin^{2} 3\pi x + \left(x-1\right)^{2}\left(1+\sin^{2} 3\pi y\right)$ $+\left(y-1\right)^{2}\left(1+\sin^{2} 2\pi y\right)$	$f(1,1) = 0$	$-10\le x,y \le 10$
Three-hump camel function		$f(x,y) = 2x^{2} - 1.05x^{4} + \frac{x^{6}}{6} + xy + y^{2}$	$f(0,0) = 0$	$-5\le x,y \le 5$
Easom function		$f(x,y) = -\cos \left(x\right)\cos \left(y\right) \exp\left(-\left(\left(x-\pi\right)^{2} + \left(y-\pi\right)^{2}\right)\right)$	$f(\pi , \pi) = -1$	$-100\le x,y \le 100$
Cross-in-tray function				$-10\le x,y \le 10$

		$f(x,y) = -0.0001\left[\left\lvert \sin x \sin y \exp\left(\left\lvert 100 - \frac{\sqrt{x^2+y^2}}{\pi}\right\rvert\right)\right\rvert + 1\right]^{0.1}$	$\text{Min} = \begin{cases} f(1.34941, -1.34941) & = -2.06261 \\ f(1.34941, 1.34941) & = -2.06261 \\ f(-1.34941, 1.34941) & = -2.06261 \\ f(-1.34941, -1.34941) & = -2.06261 \end{cases}$	
Eggholder function		$f(x,y) = -(y+47)\sin\sqrt{\left\lvert \frac{x}{2} + (y+47)\right\rvert} - x\sin\sqrt{\lvert x-(y+47)\rvert}$	$f(512, 404.2319) = -959.6407$	$-512 \le x, y \le 512$
Hölder table function		$f(x,y) = -\left\lvert \sin x \cos y \exp\left(\left\lvert 1 - \frac{\sqrt{x^2+y^2}}{\pi}\right\rvert\right)\right\rvert$	$\text{Min} = \begin{cases} f(8.05502, 9.66459) & = -19.2085 \\ f(-8.05502, 9.66459) & = -19.2085 \\ f(8.05502, -9.66459) & = -19.2085 \\ f(-8.05502, -9.66459) & = -19.2085 \end{cases}$	$-10 \le x, y \le 10$
McCormick function		$f(x,y) = \sin(x+y) + (x-y)^2 - 1.5x + 2.5y + 1$	$f(-0.54719, -1.54719) = -1.9133$	$-1.5 \le x \le 4,$ $-3 \le y \le 4$
Schaffer function N. 2		$f(x,y) = 0.5 + \frac{\sin^2(x^2-y^2) - 0.5}{[1 + 0.001(x^2+y^2)]^2}$	$f(0,0) = 0$	$-100 \le x, y \le 100$
Schaffer function N. 4		$f(x,y) = 0.5 + \frac{\cos^2[\sin(\lvert x^2-y^2\rvert)] - 0.5}{[1 + 0.001(x^2+y^2)]^2}$	$f(0, 1.25313) = 0.292579$	$-100 \le x, y \le 100$
Styblinski–Tang				$-5 \le x_i \le 5,$

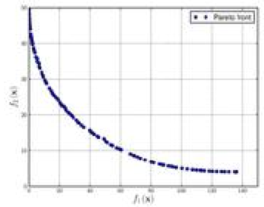
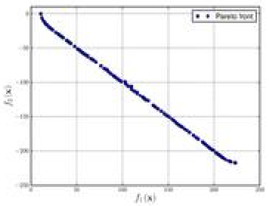
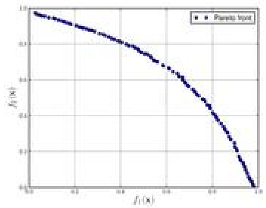
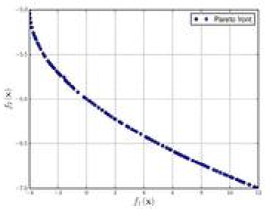
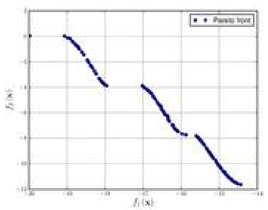
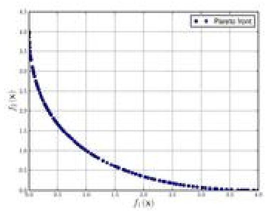
function		$f(\boldsymbol{x}) = \frac{\sum_{i=1}^{n} x_i^4 - 16x_i^2 + 5x_i}{2}$	$-39.16617n < f(\underbrace{-2.903534, \ldots, -2.903534}_{n \text{ times}}) < -39.16616n$	$1 \le i \le n$..

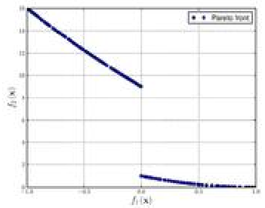
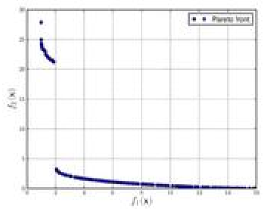
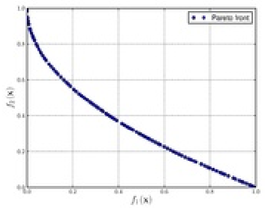
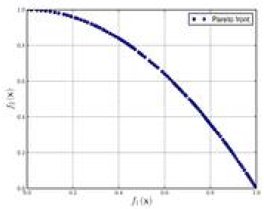
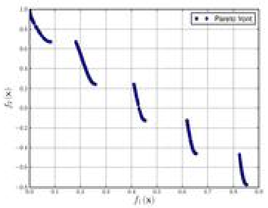
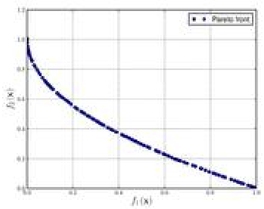
Test functions for constrained optimization

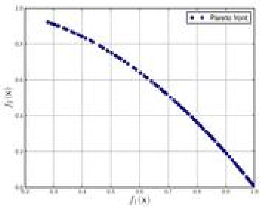
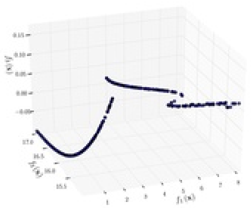
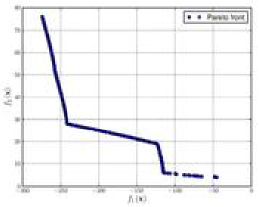
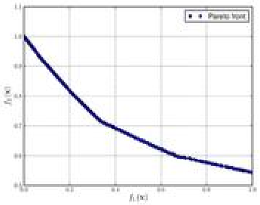
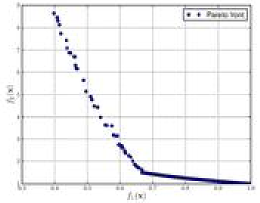
Name	Plot	Formula	Global minimum	Search domain
Rosenbrock function constrained with a cubic and a line[10]		$f(x,y) = (1-x)^2 + 100(y-x^2)^2,$ subjected to: $(x-1)^3 - y + 1 < 0 \text{ and } x + y - 2 < 0$	$f(1.0, 1.0) = 0$	$-1.5 \le x \le 1.5,$ $-0.5 \le y \le 2.5$
Rosenbrock function constrained to a disk[11]		$f(x,y) = (1-x)^2 + 100(y-x^2)^2,$ subjected to: $x^2 + y^2 < 2$	$f(1.0, 1.0) = 0$	$-1.5 \le x \le 1.5,$ $-1.5 \le y \le 1.5$
Mishra's Bird function - constrained[12]		$f(x,y) = \sin y \exp(1-\cos x)^2 + \cos x \exp(1-\sin y)^2 + (x-y)^2,$ subjected to: $(x+5)^2 + (y+5)^2 < 25$	$f(-3.1302468, -1.5821422) = -106.7645367$	$-10 \le x \le 0,$ $-6.5 \le y \le 0$
Townsend function[13]		$f(x,y) = -[\cos((x-0.1)y)]^2 - x\sin(3x+y),$ subjected to: $x^2 + y^2 < [2\cos t - 0.5\cos 2t - 0.25\cos 3t - 0.125\cos 4t]^2 + [2\sin t]^2$ where: $t = \mathrm{Atan2}(y/x)$	$f(2.0052938, 1.1944509) = 2.0239884$	$-2.25 \le x \le 2.5,$ $-2.5 \le y \le 1.75$
Simionescu function[14]		$f(x,y) = 0.1xy,$ subjected to: $x^2 + y^2 \le \left[r_T + r_S \cos\left(n \arctan \frac{x}{y}\right)\right]^2$ where: $r_T = 1, r_S = 0.2 \text{ and } n = 8$	$f(\pm 0.85586214, \mp 0.85586214) = -0.072625$	$-1.25 \le x, y \le 1.25$

Test functions for multi-objective optimization

Name	Plot	Functions	Constraints	Search domain
Binh and Korn function:		$\text{Minimize} = \begin{cases} f_1(x,y) & = 4x^2 + 4y^2 \\ f_2(x,y) & = (x-5)^2 + (y-5)^2 \end{cases}$	$\text{s.t.} = \begin{cases} g_1(x,y) & = (x-5)^2 + y^2 \leq 25 \\ g_2(x,y) & = (x-8)^2 + (y+3)^2 \geq 7.7 \end{cases}$	$0 \leq x \leq 5$, $0 \leq y \leq 3$
Chakong and Haimes function:		$\text{Minimize} = \begin{cases} f_1(x,y) & = 2 + (x-2)^2 + (y-1)^2 \\ f_2(x,y) & = 9x - (y-1)^2 \end{cases}$	$\text{s.t.} = \begin{cases} g_1(x,y) & = x^2 + y^2 \leq 225 \\ g_2(x,y) & = x - 3y + 10 \leq 0 \end{cases}$	$-20 \leq x, y \leq 20$
Fonseca and Fleming function:[15]		$\text{Minimize} = \begin{cases} f_1(\boldsymbol{x}) & = 1 - \exp\left[-\sum_{i=1}^{n}\left(x_i - \frac{1}{\sqrt{n}}\right)^2\right] \\ f_2(\boldsymbol{x}) & = 1 - \exp\left[-\sum_{i=1}^{n}\left(x_i + \frac{1}{\sqrt{n}}\right)^2\right] \end{cases}$		$-4 \leq x_i \leq 4$, $1 \leq i \leq n$
Test function 4:[7]		$\text{Minimize} = \begin{cases} f_1(x,y) & = x^2 - y \\ f_2(x,y) & = -0.5x - y - 1 \end{cases}$	$\text{s.t.} = \begin{cases} g_1(x,y) & = 6.5 - \frac{x}{6} - y \geq 0 \\ g_2(x,y) & = 7.5 - 0.5x - y \geq 0 \\ g_3(x,y) & = 30 - 5x - y \geq 0 \end{cases}$	$-7 \leq x, y \leq 4$
Kursawe function:[16]		$\text{Minimize} = \begin{cases} f_1(\boldsymbol{x}) & = \sum_{i=1}^{2}\left[-10\exp\left(-0.2\sqrt{x_i^2 + x_{i+1}^2}\right)\right] \\ f_2(\boldsymbol{x}) & = \sum_{i=1}^{3}\left[\lvert x_i\rvert^{0.8} + 5\sin\left(x_i^3\right)\right] \end{cases}$		$-5 \leq x_i \leq 5$, $1 \leq i \leq 3$.
Schaffer function N. 1:[17]		$\text{Minimize} = \begin{cases} f_1(x) & = x^2 \\ f_2(x) & = (x-2)^2 \end{cases}$		$-A \leq x \leq A$. Values of A from 10 to 10^5 have been used successfully. Higher values of A increase the difficulty of the problem.

Schaffer function N. 2:		$\text{Minimize} = \begin{cases} f_1(x) & = \begin{cases} -x, & \text{if } x \le 1 \\ x-2, & \text{if } 1 < x \le 3 \\ 4-x, & \text{if } 3 < x \le 4 \\ x-4, & \text{if } x > 4 \end{cases} \\ f_2(x) & = (x-5)^2 \end{cases}$		$-5 \le x \le 10$.
Poloni's two objective function:		$\text{Minimize} = \begin{cases} f_1(x,y) & = \left[1 + (A_1 - B_1(x,y))^2 + (A_2 - B_2(x,y))^2\right] \\ f_2(x,y) & = (x+3)^2 + (y+1)^2 \end{cases}$ $\text{where} = \begin{cases} A_1 & = 0.5\sin(1) - 2\cos(1) + \sin(2) - 1.5\cos(2) \\ A_2 & = 1.5\sin(1) - \cos(1) + 2\sin(2) - 0.5\cos(2) \\ B_1(x,y) & = 0.5\sin(x) - 2\cos(x) + \sin(y) - 1.5\cos(y) \\ B_2(x,y) & = 1.5\sin(x) - \cos(x) + 2\sin(y) - 0.5\cos(y) \end{cases}$		$-\pi \le x, y \le \pi$
Zitzler–Deb–Thiele's function N. 1:		$\text{Minimize} = \begin{cases} f_1(\boldsymbol{x}) & = x_1 \\ f_2(\boldsymbol{x}) & = g(\boldsymbol{x})\, h(f_1(\boldsymbol{x}), g(\boldsymbol{x})) \\ g(\boldsymbol{x}) & = 1 + \frac{9}{29}\sum_{i=2}^{30} x_i \\ h(f_1(\boldsymbol{x}), g(\boldsymbol{x})) & = 1 - \sqrt{\frac{f_1(\boldsymbol{x})}{g(\boldsymbol{x})}} \end{cases}$		$0 \le x_i \le 1$, $1 \le i \le 30$.
Zitzler–Deb–Thiele's function N. 2:		$\text{Minimize} = \begin{cases} f_1(\boldsymbol{x}) & = x_1 \\ f_2(\boldsymbol{x}) & = g(\boldsymbol{x})\, h(f_1(\boldsymbol{x}), g(\boldsymbol{x})) \\ g(\boldsymbol{x}) & = 1 + \frac{9}{29}\sum_{i=2}^{30} x_i \\ h(f_1(\boldsymbol{x}), g(\boldsymbol{x})) & = 1 - \left(\frac{f_1(\boldsymbol{x})}{g(\boldsymbol{x})}\right)^2 \end{cases}$		$0 \le x_i \le 1$, $1 \le i \le 30$.
Zitzler–Deb–Thiele's function N. 3:		$\text{Minimize} = \begin{cases} f_1(\boldsymbol{x}) & = x_1 \\ f_2(\boldsymbol{x}) & = g(\boldsymbol{x})\, h(f_1(\boldsymbol{x}), g(\boldsymbol{x})) \\ g(\boldsymbol{x}) & = 1 + \frac{9}{29}\sum_{i=2}^{30} x_i \\ h(f_1(\boldsymbol{x}), g(\boldsymbol{x})) & = 1 - \sqrt{\frac{f_1(\boldsymbol{x})}{g(\boldsymbol{x})}} - \left(\frac{f_1(\boldsymbol{x})}{g(\boldsymbol{x})}\right)\sin(10\pi f_1(\boldsymbol{x})) \end{cases}$		$0 \le x_i \le 1$, $1 \le i \le 30$.
Zitzler–Deb–Thiele's function N. 4:		$\text{Minimize} = \begin{cases} f_1(\boldsymbol{x}) & = x_1 \\ f_2(\boldsymbol{x}) & = g(\boldsymbol{x})\, h(f_1(\boldsymbol{x}), g(\boldsymbol{x})) \\ g(\boldsymbol{x}) & = 91 + \sum_{i=2}^{10}\left(x_i^2 - 10\cos(4\pi x_i)\right) \\ h(f_1(\boldsymbol{x}), g(\boldsymbol{x})) & = 1 - \sqrt{\frac{f_1(\boldsymbol{x})}{g(\boldsymbol{x})}} \end{cases}$		$0 \le x_1 \le 1$, $-5 \le x_i \le 5$, $2 \le i \le 10$

Zitzler–Deb–Thiele's function N. 6:		$\text{Minimize} = \begin{cases} f_{1}\left(\boldsymbol{x}\right) & = 1 - \exp(-4x_{1})\sin^{6}(6\pi x_{1}) \\ f_{2}\left(\boldsymbol{x}\right) & = g\left(\boldsymbol{x}\right) h\left(f_{1}\left(\boldsymbol{x}\right), g\left(\boldsymbol{x}\right)\right) \\ g\left(\boldsymbol{x}\right) & = 1 + 9\left[\frac{\sum_{i=2}^{10} x_{i}}{9}\right]^{0.25} \\ h\left(f_{1}\left(\boldsymbol{x}\right), g\left(\boldsymbol{x}\right)\right) & = 1 - \left(\frac{f_{1}(\boldsymbol{x})}{g(\boldsymbol{x})}\right)^{2} \end{cases}$		$0 \le x_{i} \le 1$, $1 \le i \le 10$.
Viennet function:		$\text{Minimize} = \begin{cases} f_{1}\left(x, y\right) & = 0.5\left(x^{2} + y^{2}\right) + \sin\left(x^{2} + y^{2}\right) \\ f_{2}\left(x, y\right) & = \frac{(3x - 2y + 4)^{2}}{8} + \frac{(x - y + 1)^{2}}{27} + 15 \\ f_{3}\left(x, y\right) & = \frac{1}{x^{2} + y^{2} + 1} - 1.1\exp\left(-\left(x^{2} + y^{2}\right)\right) \end{cases}$		$-3 \le x, y \le 3$.
Osyczka and Kundu function:		$\text{Minimize} = \begin{cases} f_{1}\left(\boldsymbol{x}\right) & = -25(x_{1} - 2)^{2} - (x_{2} - 2)^{2} - (x_{3} - 1)^{2} - (x_{4} - 4)^{2} - (x_{5} - 1)^{2} \\ f_{2}\left(\boldsymbol{x}\right) & = \sum_{i=1}^{6} x_{i}^{2} \end{cases}$	$\text{s.t.} = \begin{cases} g_{1}\left(\boldsymbol{x}\right) & = x_{1} + x_{2} - 2 \ge 0 \\ g_{2}\left(\boldsymbol{x}\right) & = 6 - x_{1} - x_{2} \ge 0 \\ g_{3}\left(\boldsymbol{x}\right) & = 2 - x_{2} + x_{1} \ge 0 \\ g_{4}\left(\boldsymbol{x}\right) & = 2 - x_{1} + 3x_{2} \ge 0 \\ g_{5}\left(\boldsymbol{x}\right) & = 4 - (x_{3} - 3)^{2} - x_{4} \ge 0 \\ g_{6}\left(\boldsymbol{x}\right) & = (x_{5} - 3)^{2} + x_{6} - 4 \ge 0 \end{cases}$	$0 \le x_{1}, x_{2}, x_{6} \le 10$, $1 \le x_{3}, x_{5} \le 5$, $0 \le x_{4} \le 6$.
CTP1 function (2 variables):[5]		$\text{Minimize} = \begin{cases} f_{1}\left(x, y\right) & = x \\ f_{2}\left(x, y\right) & = (1 + y)\exp\left(-\frac{x}{1 + y}\right) \end{cases}$	$\text{s.t.} = \begin{cases} g_{1}\left(x, y\right) & = \frac{f_{2}(x, y)}{0.858\exp\left(-0.541 f_{1}(x, y)\right)} \ge 1 \\ g_{1}\left(x, y\right) & = \frac{f_{2}(x, y)}{0.728\exp\left(-0.295 f_{1}(x, y)\right)} \ge 1 \end{cases}$	$0 \le x, y \le 1$.
Constr-Ex problem:[5]		$\text{Minimize} = \begin{cases} f_{1}\left(x, y\right) & = x \\ f_{2}\left(x, y\right) & = \frac{1 + y}{x} \end{cases}$	$\text{s.t.} = \begin{cases} g_{1}\left(x, y\right) & = y + 9x \ge 6 \\ g_{1}\left(x, y\right) & = -y + 9x \ge 1 \end{cases}$	$0.1 \le x \le 1$, $0 \le y \le 5$

See also

- Himmelblau's function
- Rastrigin function
- Rosenbrock function
- Shekel function

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External links

- Virtual Library of Simulation Experiments: Test Functions and Datasets (http://www.sfu.ca/~ssurjano/index.html)