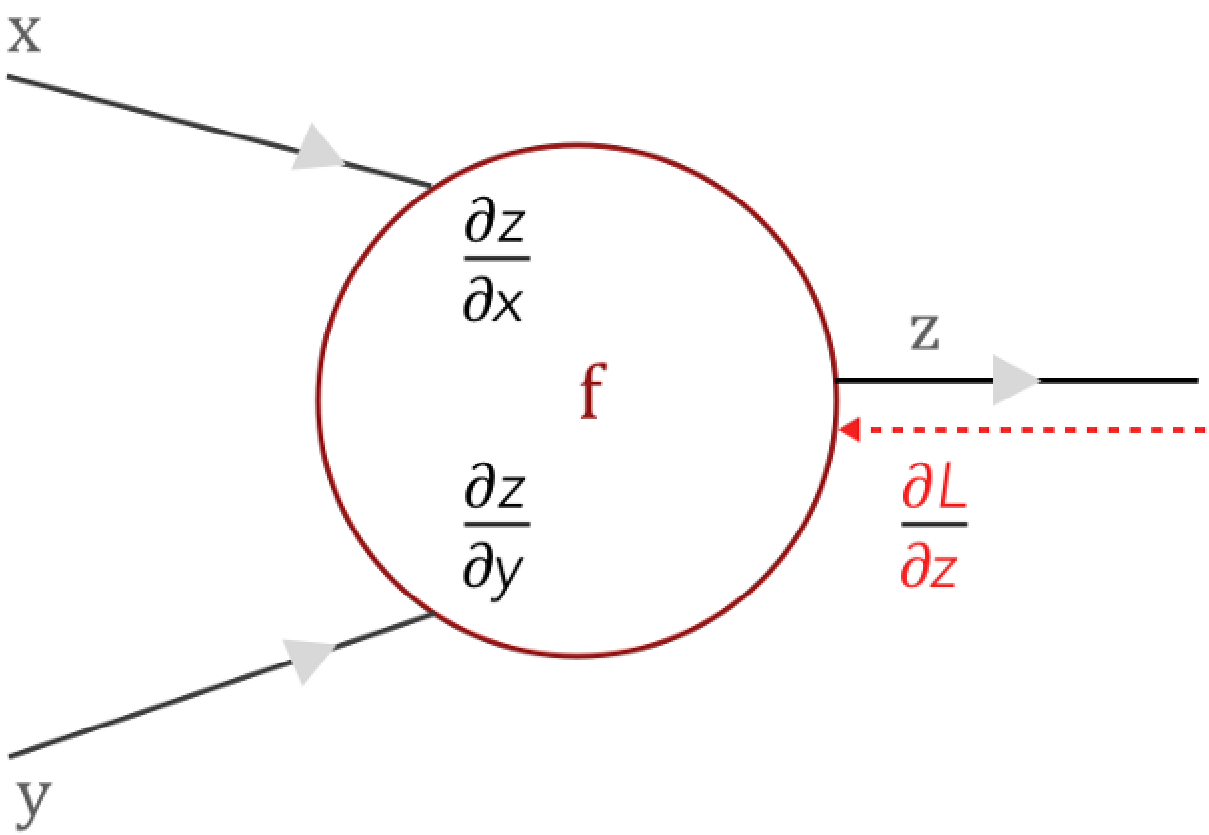
Exercise 1 :

So the propagation in convolutional networks is calculated in a different way. We will explain it step by step.

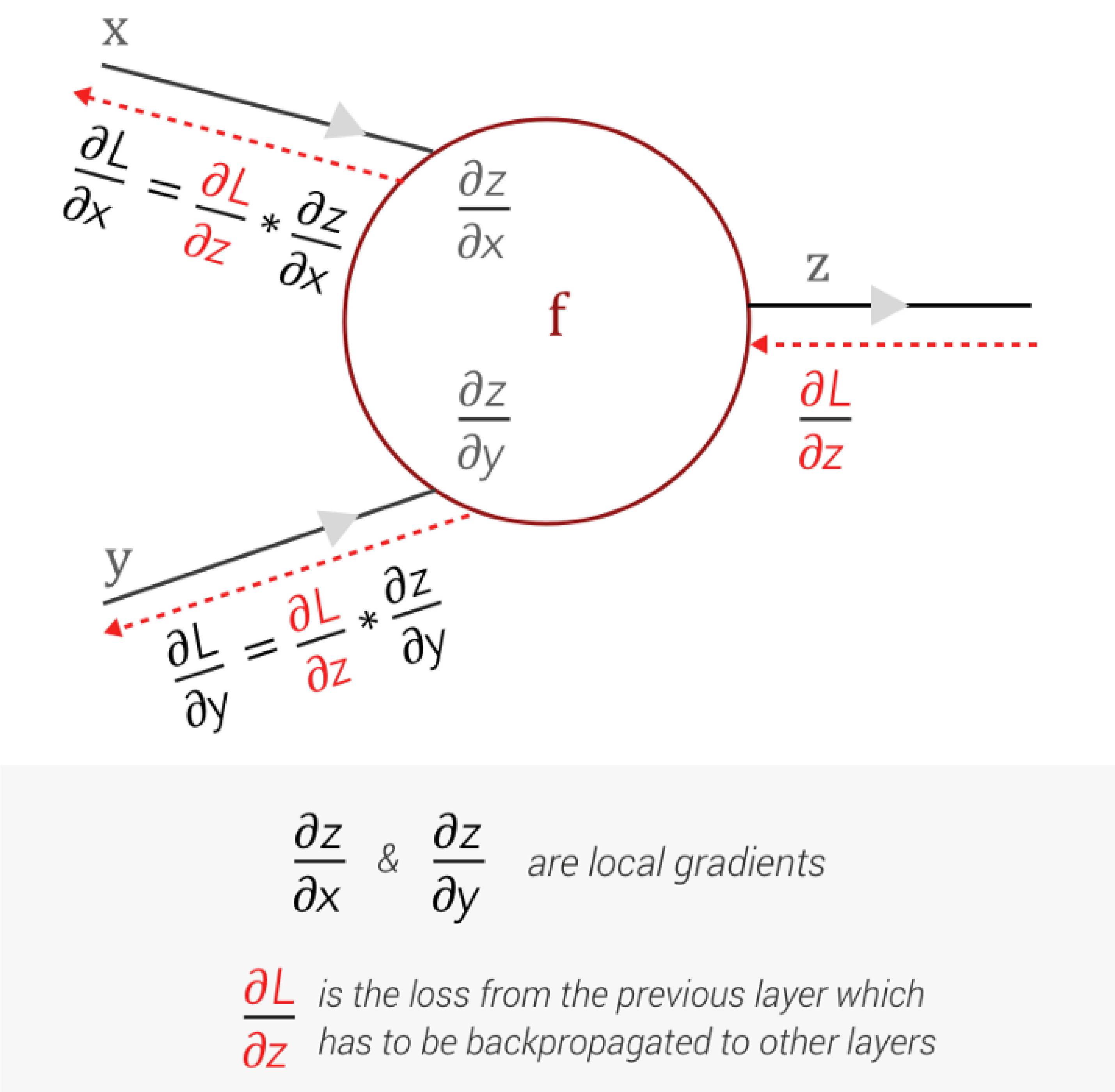


$\frac{\partial z}{\partial x}$ & $\frac{\partial z}{\partial y}$ *are local gradients*

$\frac{\partial L}{\partial z}$ *is the loss from the previous layer which has to be backpropagated to other layers*

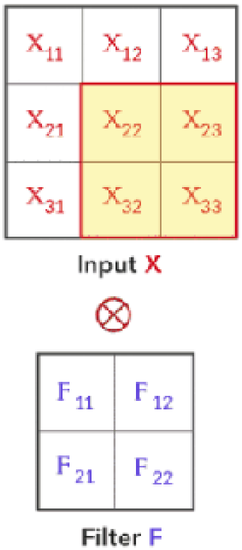
After doing the forward propagation and reaching the final output, we need to update the weights according to the loss function(L).

For this purpose, we calculate the local gradient loss for z, x, and y values, which can either be calculated directly or by using the chain rule.



So, what is the convolutional neural network like now?

Assume that the function f in the previous image is the convolution between the input X and the filter F. which produce O output.



X_{11}	X_{12}	X_{13}
X_{21}	$X_{22}F_{11}$	$X_{23}F_{12}$
X_{31}	$X_{32}F_{21}$	$X_{33}F_{22}$

$$O_{11} = X_{11}F_{11} + X_{12}F_{12} + X_{21}F_{21} + X_{22}F_{22}$$

$$O_{12} = X_{12}F_{11} + X_{13}F_{12} + X_{22}F_{21} + X_{23}F_{22}$$

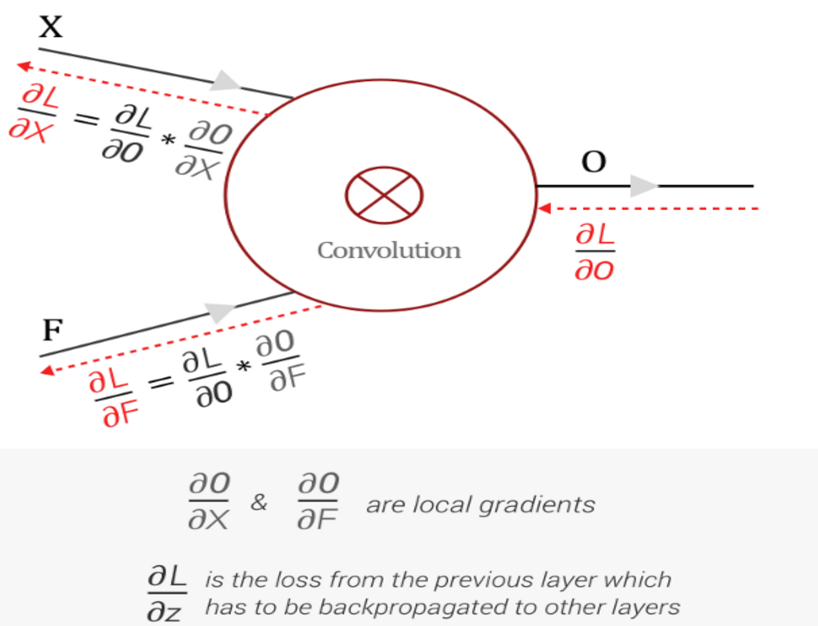
$$O_{21} = X_{21}F_{11} + X_{22}F_{12} + X_{31}F_{21} + X_{32}F_{22}$$

$$O_{22} = X_{22}F_{11} + X_{23}F_{12} + X_{32}F_{21} + X_{33}F_{22}$$

We did the forward and got the O output, now it's time to go backward.

We calculate the loss gradient according to the O output from the next layer in the backward pass.

Now, using calculations and chain rule, we can calculate backward.



we can calculate **∂L/∂X** and **∂L/∂F.**

By using local gradients, we can calculate the gradient of O relative to X and F, and finally, the gradient of the loss function can be calculated for all three of these.

Finally, by calculating these, we can update the weights which are applied as a filter in the form of weight sharing in the input. By using weight sharing, the parameters needed to train the network are also reduced and the accuracy of the network is also increased.

$$F_{\text{updated}} = F - \alpha \frac{\partial L}{\partial F}$$

First we should find local gradient and then calculate **∂L/∂F** using chain rule.

Local Gradients $\longrightarrow$ A

$$O_{11} = X_{11}F_{11} + X_{12}F_{12} + X_{21}F_{21} + X_{22}F_{22}$$

Finding derivatives with respect to F_{11}, F_{12}, F_{21} and F_{22}

$$\frac{\partial O_{11}}{\partial F_{11}} = X_{11} \quad \frac{\partial O_{11}}{\partial F_{12}} = X_{12} \quad \frac{\partial O_{11}}{\partial F_{21}} = X_{21} \quad \frac{\partial O_{11}}{\partial F_{22}} = X_{22}$$

Similarly, we can find the local gradients for O_{12}, O_{21} and O_{22}

$$\frac{\partial L}{\partial F} = \frac{\partial L}{\partial O} * \frac{\partial O}{\partial F}$$

Gradient to update Filter F

Loss Gradient from previous layer

Local Gradients

To simplify calculations, we calculate **∂L/∂F** for all Fi and finally replace **∂O/∂F** for each Fi with its corresponding Xi.

(According to equation A)

For every element of F

$$\frac{\partial L}{\partial F_i} = \sum_{k=1}^{M} \frac{\partial L}{\partial O_k} * \frac{\partial O_k}{\partial F_i}$$

$$\frac{\partial L}{\partial F_{11}} = \frac{\partial L}{\partial O_{11}} * X_{11} + \frac{\partial L}{\partial O_{12}} * X_{12} + \frac{\partial L}{\partial O_{21}} * X_{21} + \frac{\partial L}{\partial O_{22}} * X_{22}$$

$$\frac{\partial L}{\partial F_{12}} = \frac{\partial L}{\partial O_{11}} * X_{12} + \frac{\partial L}{\partial O_{12}} * X_{13} + \frac{\partial L}{\partial O_{21}} * X_{22} + \frac{\partial L}{\partial O_{22}} * X_{23}$$

$$\frac{\partial L}{\partial F_{21}} = \frac{\partial L}{\partial O_{11}} * X_{21} + \frac{\partial L}{\partial O_{12}} * X_{22} + \frac{\partial L}{\partial O_{21}} * X_{31} + \frac{\partial L}{\partial O_{22}} * X_{32}$$

$$\frac{\partial L}{\partial F_{22}} = \frac{\partial L}{\partial O_{11}} * X_{22} + \frac{\partial L}{\partial O_{12}} * X_{23} + \frac{\partial L}{\partial O_{21}} * X_{32} + \frac{\partial L}{\partial O_{22}} * X_{33}$$

This itself is nothing but a convolution between X and ∂L/∂O

$$\begin{bmatrix} \frac{\partial L}{\partial F_{11}} & \frac{\partial L}{\partial F_{12}} \\ \frac{\partial L}{\partial F_{21}} & \frac{\partial L}{\partial F_{22}} \end{bmatrix} = \text{Convolution}\left(\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{bmatrix}, \begin{bmatrix} \frac{\partial L}{\partial O_{11}} & \frac{\partial L}{\partial O_{12}} \\ \frac{\partial L}{\partial O_{21}} & \frac{\partial L}{\partial O_{22}} \end{bmatrix} \right)$$

where

$$\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{bmatrix} = \text{Input } X \qquad \begin{bmatrix} \frac{\partial L}{\partial O_{11}} & \frac{\partial L}{\partial O_{12}} \\ \frac{\partial L}{\partial O_{21}} & \frac{\partial L}{\partial O_{22}} \end{bmatrix} = \frac{\partial L}{\partial O} \text{ Loss gradient from previous layer}$$

$\partial L/\partial F$ is nothing but the convolution between Input X and Loss Gradient from the next layer $\partial L/\partial O$