Implementation of Chebyshev Collocation method

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$$T_d \times \frac{dZ}{dt} = \frac{\partial^2 Z}{\partial x^2} = 4 \times D^2 \times Z \qquad 0 \le x \le 1 \qquad \frac{\partial Z}{\partial x} = 0\,;\ at\ x = 0 \qquad \frac{\partial Z}{\partial x} = T_d \times \frac{i(t)}{Q}\ ;\ at\ x = 1$$

$$T_d \times \begin{pmatrix} \frac{dZ_0}{dt} \\ \frac{dZ_1}{dt} \\ .. \\ .. \\ \frac{dZ_N}{dt} \end{pmatrix} = \mathrm{D} \times \mathrm{D} \times \mathrm{Z} = \begin{pmatrix} D_{0,0} & \cdots & D_{0,,N-1} & D_{0,N} \\ D_{1,0} & \cdots & D_{1,N-1} & D_{1,N} \\ \cdots & \cdots & \cdots & \cdots \\ D_{N-1,0} & \cdots & D_{N-1,N-1} & D_{N-1,N} \\ D_{N,0} & \cdots & D_{N,N-1} & D_{N,N} \end{pmatrix} \times \begin{pmatrix} D_{0,0} & \cdots & D_{0,,N-1} & D_{0,N} \\ D_{1,0} & \cdots & D_{1,N-1} & D_{1,N} \\ \cdots & \cdots & \cdots & \cdots \\ D_{N-1,0} & \cdots & D_{N-1,N-1} & D_{N-1,N} \\ D_{N,0} & \cdots & D_{N,N-1} & D_{N,N} \end{pmatrix} \times \begin{pmatrix} Z_0 \\ Z_1 \\ .. \\ .. \\ .. \\ .. \\ Z_N \end{pmatrix} = \begin{pmatrix} D_{0,0} & \cdots & D_{0,,N-1} & D_{0,N} \\ D_{1,0} & \cdots & D_{1,N-1} & D_{1,N} \\ \cdots & \cdots & \cdots & \cdots \\ D_{N-1,0} & \cdots & D_{N-1,N-1} & D_{N-1,N} \\ D_{N,0} & \cdots & D_{N,N-1} & D_{N,N} \end{pmatrix} \times \begin{pmatrix} \frac{dZ_0}{dx} \\ \frac{dZ_1}{dx} \\ .. \\ \frac{dZ_{N-1}}{dx} \\ \frac{dZ_N}{dx} \end{pmatrix}$$

$$\begin{pmatrix} D_{0,0} & \cdots & D_{0,,N-1} & D_{0,N} \\ D_{1,0} & \cdots & D_{1,N-1} & D_{1,N} \\ \cdots & \cdots & \cdots & \cdots \\ D_{N-1,0} & \cdots & D_{N-1,N-1} & D_{N-1,N} \\ D_{N,0} & \cdots & D_{N,N-1} & D_{N,N} \end{pmatrix} \times \begin{pmatrix} Z_0 \\ Z_1 \\ .. \\ .. \\ .. \\ .. \\ Z_N \end{pmatrix} = \begin{pmatrix} \frac{dZ_0}{dx} \\ \frac{dZ_1}{dx} \\ .. \\ \frac{dZ_{N-1}}{dx} \\ \frac{dZ_N}{dx} \end{pmatrix} = \begin{pmatrix} 0 & \cdots & 0 & 0 & 0 \\ D_{1,0} & \cdots & D_{1,N-1} & D_{1,N} & D_{1,N} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ D_{N-1,0} & \cdots & D_{N-1,N-1} & D_{N-1,N-1} & D_{N-1,N} \\ 0 & \cdots & 0 & 0 & 0 \end{pmatrix} \times \begin{pmatrix} Z_0 \\ Z_1 \\ .. \\ .. \\ .. \\ .. \\ Z_N \end{pmatrix} + \begin{pmatrix} \frac{dZ_0}{dx} \\ 0 \\ .. \\ 0 \\ \frac{dZ_N}{dx} \end{pmatrix}$$

$$T_d \times \begin{pmatrix} \frac{dZ_0}{dt} \\ \frac{dZ_1}{dt} \\ .. \\ .. \\ \frac{dZ_N}{dt} \end{pmatrix} = \mathrm{D} \times \Bigg[\underbrace{\begin{pmatrix} 0 & \cdots & 0 & 0 & 0 \\ D_{1,0} & \cdots & D_{1,N-1} & D_{1,N} & D_{1,N} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ D_{N-1,0} & \cdots & D_{N-1,N-1} & D_{N-1,N-1} & D_{N-1,N} \\ 0 & \cdots & 0 & 0 & 0 \end{pmatrix}}_{D_x} \times \begin{pmatrix} Z_0(t) \\ Z_1(t) \\ .. \\ .. \\ .. \\ .. \\ Z_N(t) \end{pmatrix} + \begin{pmatrix} \frac{dZ_0}{dx} \\ 0 \\ .. \\ 0 \\ \frac{dZ_N}{dx} \end{pmatrix} \Bigg] = \underbrace{\mathrm{D} \times D_x}_{D_m} \times \begin{pmatrix} Z_0(t) \\ Z_1(t) \\ .. \\ .. \\ .. \\ .. \\ Z_N(t) \end{pmatrix} + \mathrm{D} \times \begin{pmatrix} \frac{dZ_0}{dx} \\ 0 \\ .. \\ 0 \\ \frac{dZ_N}{dx} \end{pmatrix} = D_m \times \begin{pmatrix} Z_0(t) \\ Z_1(t) \\ .. \\ .. \\ .. \\ .. \\ Z_N(t) \end{pmatrix} + \mathrm{D} \times \begin{pmatrix} \frac{dZ_0}{dx} \\ 0 \\ .. \\ 0 \\ \frac{dZ_N}{dx} \end{pmatrix}$$

$$T_d \times \underbrace{\begin{pmatrix} \frac{dZ_0}{dt} \\ \frac{dZ_1}{dt} \\ .. \\ .. \\ \frac{dZ_N}{dt} \end{pmatrix}}_{\frac{dZ}{dt}} = D_m \times \underbrace{\begin{pmatrix} Z_0(t) \\ Z_1(t) \\ .. \\ .. \\ .. \\ .. \\ Z_N(t) \end{pmatrix}}_{Z(t)} \underbrace{+ \mathrm{D} \times \begin{pmatrix} \frac{dZ_0}{dx} \\ 0 \\ .. \\ 0 \\ \frac{dZ_N}{dx} \end{pmatrix}}_{\mathrm{A}} \Rightarrow T_d \times \frac{dZ}{dt} = D_m \times Z + \mathrm{A} \Rightarrow T_d \times \frac{Z(t)-Z(t-1)}{\Delta t} = D_m \times Z(t) + \mathrm{A} \Rightarrow Z(t) = [\mathrm{eye}(N+1) - D_m \times \frac{\Delta t}{T_D}] \times [\mathrm{A} \times \frac{\Delta t}{T_D} + Z(t-1)]$$