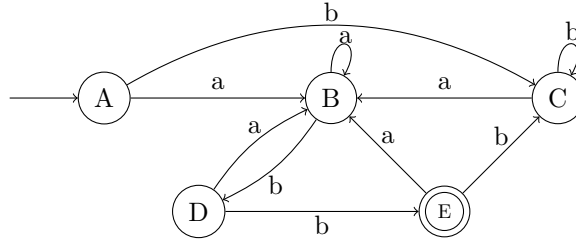
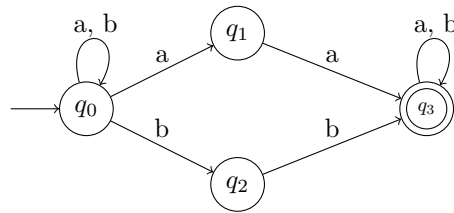


The equivalent DFA is



14. Construct a regular grammar for the RE,  $L = (a + b)^*(aa + bb)(a + b)^*$ .

**Solution:** The NFA for the RE is



There are four states in the FA. So, in the regular grammar, there are four non-terminals. Let us take them as A (for  $q_0$ ), B (for  $q_1$ ), C (for  $q_2$ ), and D (for  $q_3$ ).

Now, we have to construct the production rules of the grammar.

For the state  $q_0$ , the production rules are

$$A \rightarrow aA, A \rightarrow bA, A \rightarrow aB, A \rightarrow bC.$$

For the state  $q_1$ , the production rules are

$$B \rightarrow aD, B \rightarrow a \text{ (as D is the final state).}$$

For the state  $q_2$ , the production rules are

$$C \rightarrow bD, C \rightarrow b \text{ (as D is the final state).}$$

For the state  $q_3$ , the production rules are

$$D \rightarrow aD, D \rightarrow bD, D \rightarrow a/b.$$

The grammar =  $\{V_N, \Sigma, P, S\}$

$$V_N = A, B, C, D \quad \Sigma = a, b$$

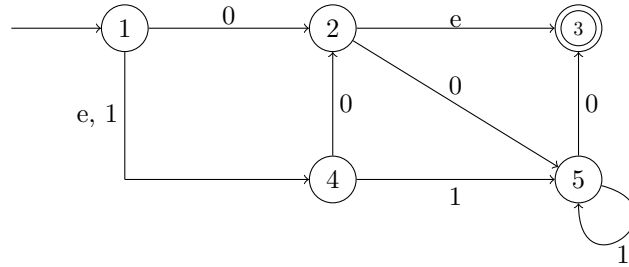
$$P : A \rightarrow aA/bA/aB/bC$$

$$B \rightarrow aD/a$$

$$C \rightarrow bD/b$$

$$D \rightarrow aD/bD/a/b.$$

15. Consider the given FA and construct the smallest DFA which accepts the same language. Draw an RE and the grammar that generates it. [UPTU 2002]



**Solution:** This problem can be solved by the  $\in$ -closure method.

$$\in\text{-closure}(1) = \{1, 4\} \quad \in\text{-closure}(3) = \{3\} \quad \in\text{-closure}(5) = \{5\}$$

$$\in\text{-closure}(2) = \{2, 3\} \quad \in\text{-closure}(4) = \{4\}$$

As 1 is the beginning state, start with  $\in\text{-closure}(1) = 1, 4$ . Let us rename it as A. A is still unmarked.

Then, construct a  $\delta'$  function for the new unmarked state A for inputs 0 and 1.

$$\begin{aligned} \delta'(A, 0) &= \in\text{-closure}(\delta(A, 0)) & \delta'(A, 1) &= \in\text{-closure}(\delta(A, 1)) \\ &= \in\text{-closure}(\delta((1, 4), 0)) & &= \in\text{-closure}(\delta((1, 4), 1)) \\ &= \in\text{-closure}(2, 2) & &= \in\text{-closure}(4, 5) \\ &= \{2, 3\} & &= \in\text{-closure}(4) \cup \in\text{-closure}(5) = \{4, 5\} \end{aligned}$$

It is a new state. Mark it as B.

$$\begin{aligned} \delta'(B, 0) &= \in\text{-closure}(\delta(B, 0)) & \delta'(B, 1) &= \in\text{-closure}(\delta(B, 1)) \\ &= \in\text{-closure}(\delta((2, 3), 0)) & &= \in\text{-closure}(\delta((2, 3), 1)) \\ &= \in\text{-closure}(5) & &= \in\text{-closure}(\emptyset) \\ &= 5 & &= \emptyset \end{aligned}$$

It is a new state. Mark it as D.

$$\begin{aligned} \delta'(C, 0) &= \in\text{-closure}(\delta(C, 0)) & \delta'(C, 1) &= \in\text{-closure}(\delta(C, 1)) \\ &= \in\text{-closure}(\delta((4, 5), 0)) & &= \in\text{-closure}(\delta((4, 5), 1)) \\ &= \in\text{-closure}(2, 3) & &= \in\text{-closure}(5) \\ &= B & &= 5 = D \end{aligned}$$

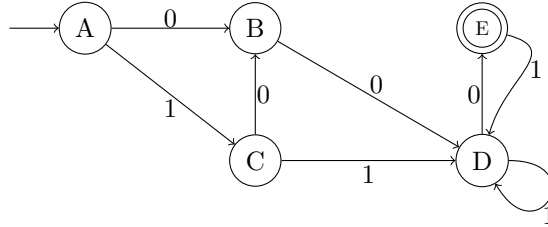
$$\begin{aligned} \delta'(D, 0) &= \in\text{-closure}(\delta(D, 0)) & \delta'(D, 1) &= \in\text{-closure}(\delta(D, 1)) \\ &= \in\text{-closure}(\delta((5), 0)) & &= \in\text{-closure}(\delta(5), 1) \\ &= \in\text{-closure}(3) & &= \in\text{-closure}(5) \\ &= 3 & &= 5 = D \end{aligned}$$

It is a new state. Mark it as E.

$$\begin{aligned} \delta'(E, 0) &= \in\text{-closure}(\delta(E, 0)) & \delta'(E, 1) &= \in\text{-closure}(\delta(E, 1)) \\ &= \in\text{-closure}(\delta(3), 0) & &= \in\text{-closure}(\delta(5), 1) \\ &= \emptyset & &= \in\text{-closure}(5) \\ & & &= 5 = D \end{aligned}$$

The beginning state is A, and the final state is E.

The transitional diagram of the DFA is



The RE is constructed using the Arden's theorem.

$$\begin{aligned}
 A &= \wedge \\
 B &= 0A + 0C \\
 C &= 1A \\
 D &= 0B + 1C + 1D + 1E \\
 E &= 0D
 \end{aligned}$$

Replacing A in B and C, we get

$$\begin{aligned}
 B &= 0 + 0C \\
 C &= 1.
 \end{aligned}$$

Replacing C in B, we get  $B = 0 + 01$ .

Replacing the new value of B, C, and E in D, we get

$$\begin{aligned}
 D &= 0(0 + 01) + 11 + 1D + 10D \\
 B &= 0(0 + 01) + 11 + D(1 + 10).
 \end{aligned}$$

It is in the format of  $R = Q + RP$ , where  $Q = (0 + (0 + 01) + 11)$ ,  $P = (1 + 10)$ .

The solution is  $R = QP^*$ .

$$D = (0 + (0 + 01) + 11) (1 + 10)^*$$

Replacing the value of D in E, we get  $E = 0(0 + (0 + 01) + 11) (1 + 10)^*$ .

As E is the final state, the RE accepted by the FA is

$$0(0 + (0 + 01) + 11) (1 + 10)^*.$$

The regular grammar is constructed as follows taking each state into account.

There are five states in the FA. So, in the regular grammar, there are five non-terminals.

Let us take them as A, B, C, D, and E according to the name of the states.

Now we have to construct the production rules of the grammar.

For the state A, the production rules are

$$A \rightarrow 0B, A \rightarrow 1C.$$

For the state B, the production rules are

$$B \rightarrow 0D$$

For the state C, the production rules are

$$C \rightarrow 0B \quad C \rightarrow 1D.$$

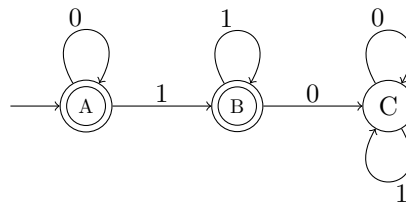
For the state D, the production rules are

$$D \rightarrow 1D \quad D \rightarrow 0E \quad D \rightarrow 0.$$

For the state E, the production rules are

$$E \rightarrow 1D.$$

16. Find the RE recognized by the finite state automaton of the following figure.  
[GATE 1994]



**Solution:** The equation for the FA is

$$A = 0A + \Lambda \quad (1)$$

$$B = 1A + 1B \quad (2)$$

$$C = 0B + 0C + 1C \quad (3)$$

Solving the equation (1) using the Arden's theorem, we get  $A = \Lambda 0^* = 0^*$ .

Putting the value of A in equation (2), we get

$$B = 10^* + 1B.$$

Using the Arden's theorem, we get

$$B = 10^*1^*.$$

Both A and B are final states, and thus the string accepted by the FA is

$$0^* + 10^*1^*$$

$$= 0^* (\Lambda + 11^*) = 0^*1^* \text{ (as } \Lambda + RR^* = R^*).$$

### Multiple Choice Questions

- The machine format of regular expression is: a) Finite automata b) Push down automata c) Turing machine d) All of the above
- The regular expression is accepted by: a) Finite automata b) Push down automata c) Turing machine d) All of the above
- The language of all words with at least 2 a's can be described by the regular expression: a)  $(ab)^*a$  b)  $(a + b)^*ab^*a(a + b)^*$  c)  $b^*ab^*a(a + b)^*$  d) All of the above
- The set of all strings of 0, 1 having exactly two 0's is: a)  $1^*01^*01^*$  b)  $0 + 1)^*1$  c)  $11 + 0^*$  d)  $00 + 11^*$