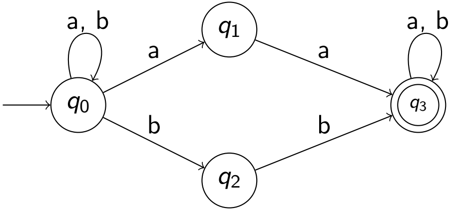
14. Construct a regular grammar for the RE, L = (a + b)*(aa + bb)(a + b)*.
Solution: The NFA for the RE is



There are four states in the FA. So, in the regular grammar, there are four non-terminals. Let us take them as A (for q_0), B (for q_1), C (for q_2), and D (for q_3).
Now, we have to construct the production rules of the grammar.
For the state q_0, the production rules are

$$A \rightarrow aA,\ A \rightarrow bA,\ A \rightarrow aB,\ A \rightarrow bC.$$

For the state q_1, the production rules are

$B \rightarrow aD$, $B \rightarrow a$ (as D is the final state).

For the state q_2, the production rules are

$C \rightarrow b\,D$, $C \rightarrow b$ (as D is the final state).

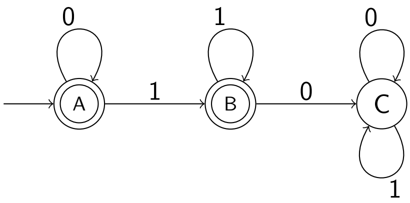
For the state q_3, the production rules are

$$D \rightarrow aD,\ D \rightarrow bD,\ D \rightarrow a/\ b.$$

The grammar $= \{V_N, \Sigma, P, S\}$

$$V_N = \{A, B, C, D\}\ \Sigma = \{a, b\}$$
$$P : A \rightarrow aA/bA/aB/bC$$
$$B \rightarrow aD/a$$
$$C \rightarrow bD/b$$
$$D \rightarrow aD/bD/a/b.$$

16. Find the RE recognized by the finite state automaton of the following figure. [GATE 1994]



Solution: The equation for the FA is

$$A = 0A + \wedge \quad (1)$$
$$B = 1A + 1B \quad (2)$$
$$C = 0B + 0C + 1C \quad (3)$$

Solving the equation (1) using the Arden's theorem, we get $A = \wedge 0^* = 0^*$.
Putting the value of A in equation (2), we get

$$B = 10^* + 1B.$$

Using the Arden's theorem, we get

$$B = 10^*1^*.$$

Both A and B are final states, and thus the string accepted by the FA is

$$0^* + 10^*1^*$$
$$= 0^* (\wedge + 11^*) = 0^*1^* \text{ (as } \wedge + RR^* = R^*).$$