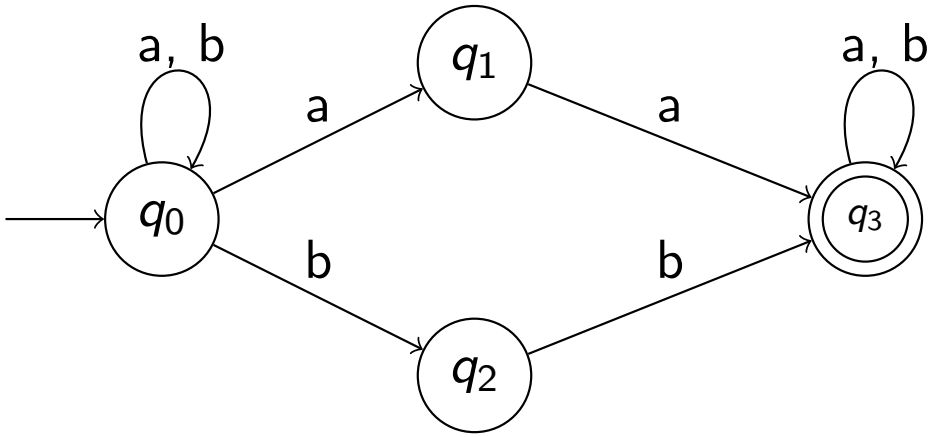
14. Construct a regular grammar for the RE, L = (a + b)*(aa + bb)(a + b)*.
Solution: The NFA for the RE is



There are four states in the FA. So, in the regular grammar, there are four non-terminals. Let us take them as A (for q_0), B (for q_1), C (for q_2), and D (for q_3).

Now, we have to construct the production rules of the grammar.

For the state q_0, the production rules are

$$A \to aA,\ A \to bA,\ A \to aB,\ A \to bC.$$

For the state q_1, the production rules are

$$B \to aD,\ B \to a \text{ (as D is the final state).}$$

For the state q_2, the production rules are

$$C \to b\ D,\ C \to b \text{ (as D is the final state).}$$

For the state q_3, the production rules are

$$D \rightarrow aD,\ D \rightarrow bD,\ D \rightarrow a/\ b.$$

The grammar $= \{V_N, \Sigma, P, S\}$

$V_N = A, B, C, D\ \Sigma = a, b$

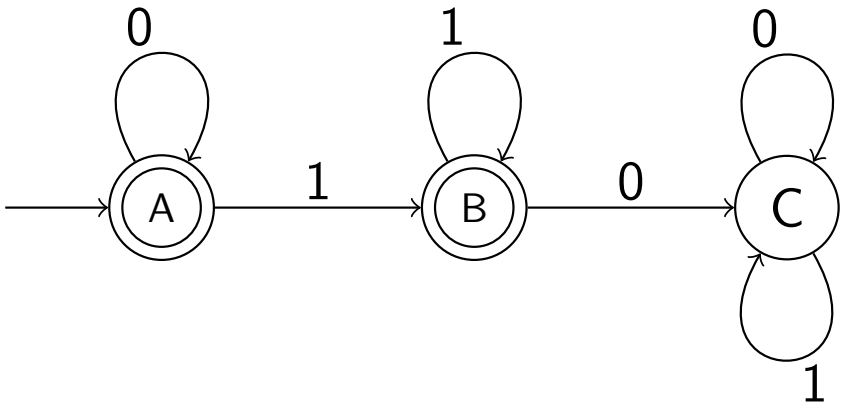
$P : A \rightarrow aA/bA/aB/bC$

$B \rightarrow aD/a$

$C \rightarrow bD/b$

$D \rightarrow aD/bD/a/b.$

16. Find the RE recognized by the finite state automaton of the following figure. [GATE 1994]



Solution: The equation for the FA is

$$A = 0A + \wedge \quad (1)$$
$$B = 1A + 1B \quad (2)$$
$$C = 0B + 0C + 1C \quad (3)$$

Solving the equation (1) using the Arden's theorem, we get $A = \wedge 0^* = 0^*$.
Putting the value of A in equation (2), we get

$$B = 10^* + 1B.$$

Using the Arden's theorem, we get

$$B = 10^*1^*.$$

Both A and B are final states, and thus the string accepted by the FA is

$$0^* + 10^*1^*$$
$$= 0^* (\wedge + 11^*) = 0^*1^* \text{ (as } \wedge + RR^* = R^*).$$