Reading: Course notes.

Problems:

1. Obtain and identify (on the F axis) the digital frequency for the following signals:

1. $x(t) = \cos(2\pi(100)t)$ sampled at 450 Hz
2. $x(t) = \sin(2\pi(200)t - \frac{\pi}{4})$ sampled at 300 Hz
3. $x(t) = \cos(2\pi(200)t) + \sin(2\pi(1100)t)$ sampled at 300 Hz
4. $x(t) = \cos(2\pi(12)t + \frac{\pi}{4}) + \sin(2\pi(1212)t - \frac{\pi}{4})$ sampled at 120 Hz

For each example, write the sampled signal as a sum of digital harmonic signals.

2. Express the following signals using a digital frequency $|F| < 0.5$:

1. $x[n] = \cos(\frac{4n\pi}{3})$
2. $x[n] = \cos(\frac{4n\pi}{7}) + \sin(\frac{8n\pi}{7})$

3. Short questions:

1. A signal $x(t)$ is made up of sum of pure cosines with unit amplitude and the following frequencies:

   $$10, 40, 200, 220, 240, 260, 300, 320, 380, 400 \text{ Hz}$$

   If the signal is sampled at $S_F = 140$ Hz, which components, if any, will show aliasing? What is the minimum sampling rate that will allow avoid aliasing?

2. Show that

   $$\cos(2\pi F_0 n) = \cos(2\pi(F_0 + k)n)$$

   for all integer values of k. Relate this property to aliasing for the spinner example.

3. True or False. A digital harmonic signal $x[n] = \cos(2\pi F_0 n)$ is periodic as a function of n.

4. True or False. A continuous harmonic signal $x(t) = \cos(2\pi f_0 t)$ is periodic as a function of t.

4. Python programming. For this problem, two versions of codes have been uploaded on GitHub, one titled `problem4_colab.py` for use with Google colab, and the other titled `problem4_local.py` for use with python installed locally on your computer. These codes implement sampling, plotting, and re-construction of a f_0 =30 Hz sine wave at the sampling frequency of $S_F = 100$ Hz.

As part of this homework assignment, your task is to modify these codes to sample the $f_0 = 30$ Hz sine wave at different sampling frequencies, namely, 100 Hz, 70 Hz, 61 Hz, 60 Hz, 50 Hz, 40 Hz, 30 Hz, and 15 Hz. For each sampling frequency, create a plot of the output, and deduce whether aliasing has occured or not. Turn in your plots.