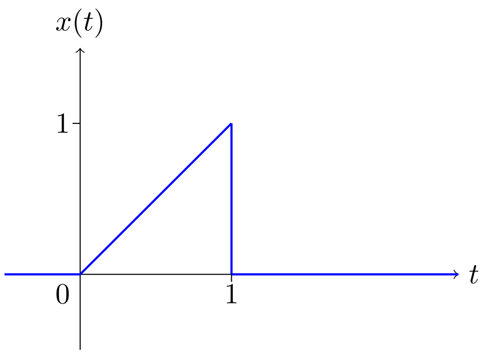
Reading: Class notes; Classification of systems (Sec. 4.1-4.3 of Ambardar).

Problems:

1. Consider the following signal:



Draw $y(t) = x(-2t+1)$. Solve the problem three different ways.

1. Scale first. This means as a first step compute the intermediate signal $u(t) = x(2t)$.
2. Shift first. This means as a first step compute the intermediate signal $u(t) = x(t+1)$.
3. Flip first. This means as a first step compute the intermediate signal $u(t) = x(-t)$.

2. Classify each of he following continuous-time systems in terms of linearity, time invariance, static/dynamic, and causality (u is the input and y is the output):

1. $y(t) = \int_{-\infty}^{t} u(\tau)\cos(t-\tau)\,\mathrm{d}\tau$
2. $y(t) = \int_{-\infty}^{t} \cos(u(\tau))\sin(\tau)\,\mathrm{d}\tau$
3. $y(t) = \int_{-\infty}^{t} u(\tau+1)\cos(t-\tau)\,\mathrm{d}\tau$
4. $y(t) = \int_{-\infty}^{t} u(\tau-\sin(\tau))\,\mathrm{d}\tau$
5. $y(t) = u(t+1) + \sin(tu(t))$
6. $\frac{\mathrm{d}^2}{\mathrm{d}t^2}y(t) + \cos(2t)\frac{\mathrm{d}}{\mathrm{d}t}y(t) = u(t)$
7. $\frac{\mathrm{d}}{\mathrm{d}t}y(t) + 3y(t) = u(t)$
8. $\frac{\mathrm{d}}{\mathrm{d}t}y(t) + ty(t) = u(t)$
9. $\frac{\mathrm{d}}{\mathrm{d}t}y(t) + y^2(t) = u(t)$
10. $\frac{\mathrm{d}^2}{\mathrm{d}t^2}y(t) + 3y(t)\frac{\mathrm{d}}{\mathrm{d}t}y(t) = 2\frac{\mathrm{d}}{\mathrm{d}t}u(t) + u(t)$

3. In each of the discrete-time systems below, $u[n]$ is the input and $y[n]$ is the output. Classify for linearity, time invariance, dynamic, and causality.

1. $y[n] = u[n] + u[n-1] + u[n+1]$
2. $y[n] = u[n] + u[n-1]$
3. $y[n] = \sum_{m=n-2}^{n} \frac{1}{2^{n-m}} u[m] = u[n] + \frac{1}{2}u[n-1] + \frac{1}{4}u[n-2]$
4. $y[n] = \sum_{m=-\infty}^{n} \frac{1}{2^{n-m}} u[m]$
5. $y[n] = u[n] + (n-1)u[n-1] + (n-2)^2 u[n-2]$

4. [Coding Problem] Implement a simple signal processing system using PyTorch.

This problem is designed to help you understand how to use the PyTorch package in Python to generate an input signal, its transformation through a linear system, and the visualization of both input and output signals. This practice is also a combination of concepts in the coding problems of the previous two homeworks. An example Python code is provided and can be accessed on GitHub as well.

You are given a Python script that implements a basic signal processing system (problem 3.2) with input defined as following (similar in homework 1)

$$u[n] = \sin^3(n\Delta t)u_0[n], \quad \Delta t = 0.1, \quad n \in \mathbb{N}$$

The code consists of three main parts: (i) input signal generation, (ii) output signal generation through linear transformation, and (iii) visualization of input and output signals. Your task is to understand the code, modifiy the system to fit problem 3.3 and problem 3.5, and visualize the input and output signals for the two systems, respectively. Submit your plots with proper axis labels, axis limits, tick marks, and legends indicating which time signals are being plotted.

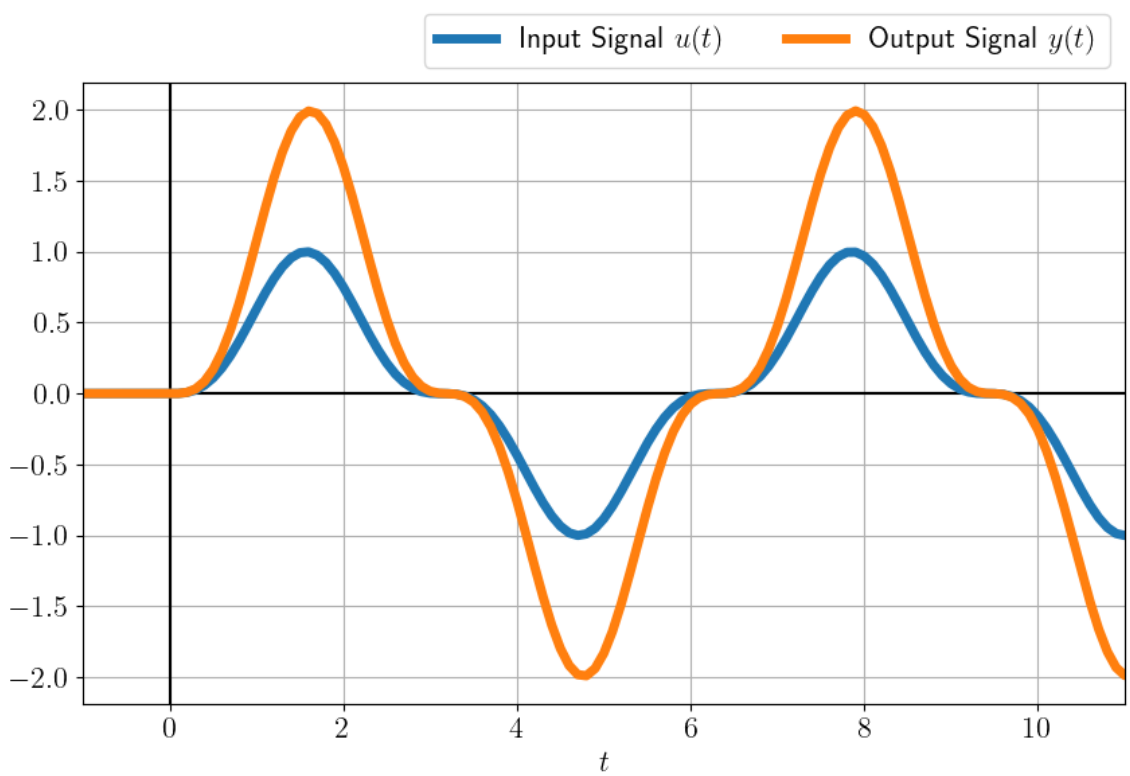


Figure 1: Example plotting of input and output signals in problem 3.2