Reading: Class notes; The notation $u_0[n]$ is used to denote the unit step.

Problems:

1. Find the impulse response $h[n]$ of the following difference equation:

$$y[n] = 0.5y[n-1] - u[n]$$

1. Use the method of z-transform
2. Use the method of z-transform to find the response to the input $u[n] = u_0[n]$. Assume $y[-1] = 0$ to solve this problem.
3. Use the method of z-transform to find the response to the input $u[n] = (0.5)^n u_0[n]$. Assume $y[-1] = 0$ to solve this problem.
4. Use the method of z-transform to find the response to the input $u[n] = (0.5)^n u_0[n]$ and $y[-1] = 7$.

2. The transfer function of a discrete-time system is given by

$$H(z) = \frac{z^2}{(z-\frac{1}{2})(z-\frac{1}{3})}$$

with initial conditions $y[-1] = 0$ and $y[-2] = 1$.

1. What is the zero-input response?
2. What is the zero-state response with input $u[n] = \left(\frac{1}{2}\right)^n u_0[n]$ where $u_0[n]$ denotes the unit step.
3. What is the total response with the initial condition indicated in the main part, and the input indicated in the part 2 above? What property did you use to arrive at this conclusion?

3. For a signal $x[n]$, the two-sided z-transform is

$$X(z) = \sum_{n=-\infty}^{n=\infty} x[n]z^{-n}$$

Compute the two-sided z-transform for the following signals:

1. $u[n] = u_0[n]$. Plot the signal.
2. $u[n] = -u_0[-n-1]$. Plot the signal.

Although the z-transform is the same, the region of convergence (ROC) is different. Identify the ROC for each signal.

4. Coding question: we will implement the systems in Problem 1. A code file is uploaded on GitHub which a demo code for the system $y[n] = y[n-1] + u_0[n]$. You need to edit the code file to correctly implement the systems in Problem 1.2, 1.3, 1.4. You need to submit plots of the output y. Make sure the output matches the one you calculated in the solution of each problem. The plotting code is included in the code file.