ME 360, Fall 2024 **PROBLEM SET 6** **Due at 3PM Sunday, Oct 18**

Reading: Course notes.

Problems:

1. Obtain and identify (on the F axis) the digital frequency for the following signals:

1. $x(t) = \cos(2\pi(100)t)$ sampled at 450 Hz
2. $x(t) = \sin(2\pi(200)t - \frac{\pi}{4})$ sampled at 300 Hz
3. $x(t) = \cos(2\pi(200)t) + \sin(2\pi(1100)t)$ sampled at 300 Hz
4. $x(t) = \cos(2\pi(12)t + \frac{\pi}{4}) + \sin(2\pi(1212)t - \frac{\pi}{4})$ sampled at 120 Hz

For each example, write the sampled signal as a sum of digital harmonic signals.

2. Express the following signals using a digital frequency $|F| < 0.5$:

1. $x[n] = \cos(\frac{4n\pi}{3})$
2. $x[n] = \cos(\frac{4n\pi}{7}) + \sin(\frac{8n\pi}{7})$

3. Short questions:

1. A signal $x(t)$ is made up of sum of pure cosines with unit amplitude and the following frequencies:

   $$10, 40, 200, 220, 240, 260, 300, 320, 380, 400 \text{ Hz}$$

   If the signal is sampled at $S_F = 140$ Hz, which components, if any, will show aliasing? What is the minimum sampling rate that will allow avoid aliasing?

2. Show that

   $$\cos(2\pi F_0 n) = \cos(2\pi(F_0 + k)n)$$

   for all integer values of k. Relate this property to aliasing for the spinner example.

3. True or False. A digital harmonic signal $x[n] = \cos(2\pi F_0 n)$ is periodic as a function of n.
4. True or False. A continuous harmonic signal $x(t) = \cos(2\pi f_0 t)$ is periodic as a function of t.

4. Python programming. This question explores the idea of aliasing which happens when sampling a signal. If the frequency of the original signal is f, then any sampling done at a frequency greater than $2f$ successfully recovers the original signal. If the sampling frequency is less than $2f$, the frequency of the sampled signal is incorrectly obtained.

The task is to modify the provided codes (which simulates an analytical sine wave of f =30 Hz, samples it at 100 Hz and plots both the original signal and the sample reconstruction) to sample the 30 Hz sine wave at various different frequencies, namely 100 Hz, 70 Hz, 61 Hz, 60 Hz, 50 Hz, 40 Hz, 30 Hz, and 15 Hz. For each sampling, make a plot of the obtained output, and conclude whether aliasing is happening or not.

There are two versions of the provided codes uploaded on GitHub, one titled `problem4_colab.py` which can be used by those who prefer to use Google colab, and the other titled `problem4_local.py` which can be used by those who prefer to use python locally on their computer.