Reading: Review your notes from Linear Algebra. $u_0(t)$ denotes the unit step. $\delta(t)$ denotes the Dirac delta.

Problems:

1. Sketch each of the following continuous-time signals:

1. $x(t) = u_0(t-1)$. This is example of unit step shifted to the right by 1.
2. $x(t) = \text{rect}(t) := u_0(t) - u_0(t-1)$.
3. $x(t) = (t-1)\,\text{rect}(t)$.
4. $x(t) = t\,e^{-2t}u_0(t)$.
5. $x(t) = e^{-2|t|}$.

2. Sketch each of the following discrete-time signals:

1. $x[n] = u_0[n-1]$. This is example of unit step shifted to the right by 1.
2. $x[n] = u_0[n] - u_0[n-2]$.
3. $x[n] = nu_0[n] - nu_0[n-2]$.
4. $x[n] = a^n u_0[n]$ for $a = \frac{1}{2}$. Discuss the difference when $|a| < 1$ and $|a| > 1$.
5. $x[n] = \frac{1}{2^{|n|}}$.

3. In $\mathbb{R}^2$, depict the following vectors

$$v_a = \begin{bmatrix} -2 \\ 1 \end{bmatrix}, \quad v_b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad v_c = \begin{bmatrix} 0 \\ -2 \end{bmatrix},$$

Consider the following matrix

$$W = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

Review your matrix-vector multiplication and compute Wv_a, Wv_b and Wv_c. Next, compute $W(v_a + v_b + 2v_c)$. Do this using two methods:

1. First compute $(v_a + v_b + 2v_c)$ and multiplying it by the matrix W.
2. First recall the fact that $W(v_a + v_b + 2v_c) = Wv_a + Wv_b + 2Wv_c$ and use the result from your prior computation.

In the 2D plane comprised of vectors $x \in \mathbb{R}^2$, find and depict the regions where

$$Wx < 0, \qquad Wx > 0, \qquad Wx = 0$$

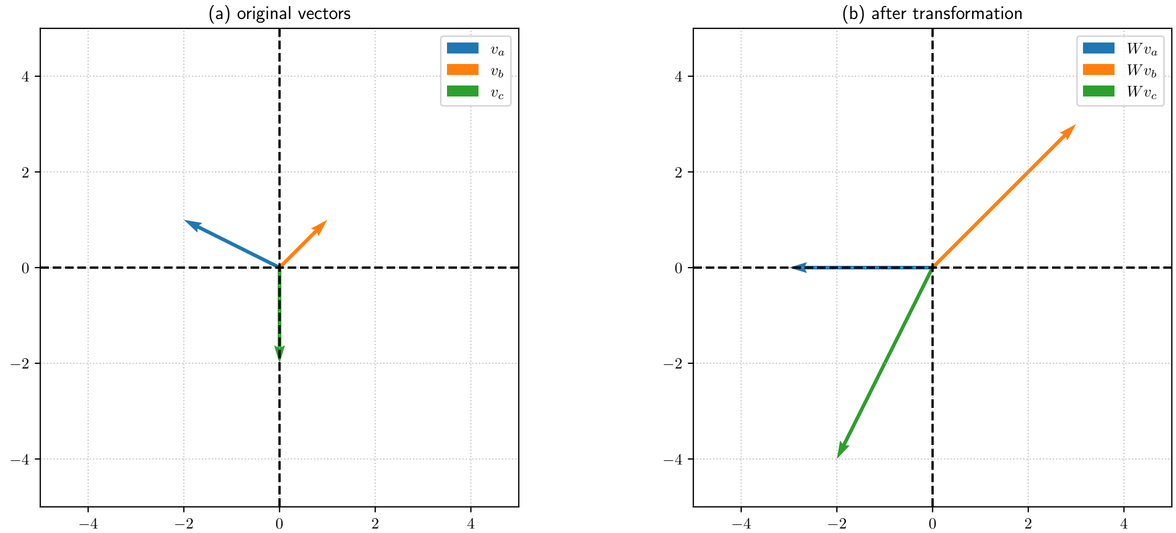

Figure 1: Example of a linear transformation of vectors in $\mathbb{R}^2$

4. [Coding Problem] Implement and visualize a linear transformation of vectors through python.

This problem is designed to help you understand how vectors are transformed by matrix and how to represent these transformations graphically. An example Python code is provided and can be accessed on GitHub as well. The code can be run to visualize the transformation of vectors in $\mathbb{R}^2$ through a matrix M defined below:

$$M = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

As part of this exercise, you are required to first explain what is plotted in the demo code.

Secondly, modify M to the following two matrices and plot the transformed vectors:

$$M_2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, M_3 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix},$$

and discuss the effect of these matrices on the vectors v_a, v_b, and v_c.