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Indian Institute of Information Technology Vadodara
MA 101: Introduction to Discrete Mathematics
Midsemester Examination

Maximum Marks: 40 Time: 2 hrs

1) Each question carries 5 marks.
2) Do not write anything, except your name and id on the question paper.
3) Write down answers in sequence.

1. Show that $\exists x P(x) \wedge \exists x Q(x)$ and $\exists x (P(x) \wedge Q(x))$ are not logically equivalent.

2. Show that these statements are equivalent. (i) $3x + 2$ is an even integer (ii) $x + 5$ is an odd integer (iii) x^2 is an even integer.

3. Find prime factorization of 385, and the remainder when 6^{2024} is divided by 385.

4. Let $f : X \to Y$ and $g : Y \to Z$ be functions. If f and $f \circ g$ are one-to-one, does it follow that g is one-to-one? Justify your answer.

5. Let $\mathbb{N} = \{1, 2, 3, \ldots\}$. What is the relation between cardinalities of $\mathbb{N}^3$ and $\mathbb{N}^2$? Give justification for your answer.

6. Use resolution principal to check validity of the argument given below. Use propositional variables and define them at the begining.
   Either Heena attended the meeting or Heena was not invited. If the boss wanted Heena at the meeting, then she was invited. Heena did not attend the meeting. If the boss did not want Heena there, and the boss did not invite her there, then she is going to be fired. Therefore, Heena is going to be fired.

7. Find all solutions, if any, to the system of congruences

$$\begin{aligned} x &\equiv 5 \pmod 6 \\ x &\equiv 3 \pmod{10} \\ x &\equiv 8 \pmod{15} \end{aligned}$$

8. Let $A = \{f : \mathbb{N} \to \{0, 1\} \mid f \text{ is a function }\}$. Is A countable, finite? Give justification.