Indian Institute of Information Technology Vadodara
MA 101: Introduction to Discrete Mathematics
Endsemester Examination

Maximum Marks: 50   Time: 3 hrs

1) Each question carries 5 marks.
2) Do not write anything, except your name and id on the question paper.
3) Write down answers in sequence.

1. Write down final answer. Justification is not required.

   (a) Find the maximum number of edges in a simple graph with 6 vertices.

   (b) Find GCD of $3^{20} - 1$, $3^{30} - 1$.

   (c) Find total number of non-isomorphic simple graphs with three vertices

   (d) How many equivalence relations are there on a set $\{a, b, c, d\}$ with three equivalence classes?

   (e) Give an example of two uncountable sets A and B such that $A - B$ is

      i) $A - B$ is countably infinite.

      ii) $A - B$ is uncountable.

      iii) A and B have different cardinalities.

2. Determine whether the directed graph shown below can be drawn with a pen in a continuous motion without lifting the pen or retracing part of the picture. If yes then give a path. Follow the directions while drawing. Is it strongly connected? Give justification.
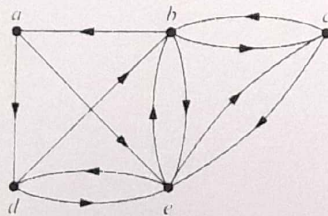
3. How many bit strings of length 10 contain at least three 1s and at least three 0s ? Give justification.

4. Obtain the conjunctive normal form and disjunctive normal form of the formula $(p \leftrightarrow q) \leftrightarrow r$.

5. Prove by mathematical induction that 21 divides $4^{n+1} + 5^{2n-1}$ whenever n is a positive integer.

6. Use generating functions to determine the number of different ways 12 identical action figures can be given to five children so that each child receives at most three action figures.

7. Let R be the relation on the set of ordered pairs of positive integers such that $((a, b), (c, d)) \in R$ iff $a + d = b + c$. Show that R is an equivalence relation. What is the equivalence class of $(1, 2)$?

8. Define a relation R on $\mathbb{N} \times \mathbb{N}$ such that it becomes well-ordered set. Is it also a lattice? State well ordering principle and use it to prove that $\sqrt{2}$ is irrational.

9. Give a bijective function from $\mathbb{N} \times \mathbb{N}$ to $\mathbb{N}$. Prove its bijectivity.

10. Find total number of squares and rectangles in a $n * n$ grid of squares given below.