FINSEARCH - FINANCE CLUB IOPTION PRICE MODEL AND THEIR ACCURACY

What is an Option?

An option in finance refers to a contract granting its holder the right but not the obligation to buy or sell an underlying asset at a predetermined price (known as the strike price) on or before a specified expiration date. The underlying asset can be a stock, a bond, a commodity, a currency, or even a financial index.

There are two main types of options:

Call Option: A call option gives the holder the right to buy the underlying asset at the strike price within the specified time frame. If the asset's market price rises above the strike price, the holder can exercise the option and buy the asset at the lower strike price, profiting from the difference.

Put Option: A put option gives the holder the right to sell the underlying asset at the strike price within the specified time frame. If the asset's market price falls below the strike price, the holder can exercise the option and sell the asset at the higher strike price, again profiting from the difference.

Options are commonly used by investors and traders for various purposes, including speculation, hedging, and income generation. Here are a few key points about options:

Hedging: Options can be used to hedge against the risk of a decline in the price of an asset. For example, a company that owns a stock might buy a put option on that stock to protect itself from a decline in the stock price.

Speculation: Options can be used to speculate on the future price of an asset. For example, an investor might buy a call option on a stock if they believe the stock price will go up.

Income generation: Options can be used to generate income. For example, an investor might sell a put option on a stock if they are confident that the stock price will not go below a certain level.

Expiration date: Options have a limited lifespan and expire on a specific date. Once the option reaches its expiration date, it becomes worthless if not exercised.

Premium: To acquire an option, the buyer pays the price known as the premium to the seller. The premium represents the option's cost and varies depending on factors such as the underlying asset's price, the strike price, and the time remaining until expiration.

Exercise: If the option holder decides to use their right to buy or sell the underlying asset, they "exercise" the option. Exercising a call option means buying the asset at the strike price, while exercising a put option means selling the asset at the strike price.

Option seller (writer): On the other side of the trade is an option seller (sometimes called the writer). The seller receives the premium from the buyer and must be ready to fulfill their obligation if the option is exercised by the buyer.

Here are some additional terms related to options:

Theoretical value: The value of an option based on a mathematical model

Implied volatility is the volatility of the underlying asset that is implied by the price of the option.

Delta: The sensitivity of the option price to changes in the price of the underlying asset

Gamma: The sensitivity of delta to changes in the price of the underlying asset

Theta: The rate at which the option price decays over time.

Vega: The sensitivity of the option price to changes in implied volatility

The Basics of Option Prices :

The price of an option is determined by a number of factors, including:
The current price of the underlying asset. The higher the price of the underlying asset, the more valuable the option will be.

The strike price. The strike price is the price at which the option can be exercised. If the strike price is below the underlying asset's current price, the option will be in-the-money, and the price will be higher. If the strike price is above the underlying asset's current price, the option will be out-of-the-money, and the option's price will be lower.

The time to expiration. The longer the time to expiration, the more valuable the option will be. This is because there is more time for the underlying asset's price to move in a favorable direction.

Volatility. Volatility is a measure of how much the price of the underlying asset fluctuates. The higher the volatility, the more valuable the option will be. This is because there is a greater chance that the price of the underlying asset will move in a favorable direction.

Interest rates. Interest rates affect the price of options by discounting the future value of the option. Higher interest rates will lower the price of options.
The price of an option is also affected by the risk-free rate, which is the rate of return that an investor could earn on a risk-free investment. The risk-free rate is used to discount the future value of the option.
The price of an option is also affected by the dividends that are paid by the underlying asset. Dividends reduce the value of call options because they reduce the amount of money the option holder will earn if they exercise the option.

The price of an option is a complex function of all of these factors. There are a number of mathematical models that can be used to calculate the theoretical value of an option. However, an option's actual price may differ from the theoretical value due to market sentiment and liquidity.

Here are some additional terms related to option prices:

Intrinsic value: The intrinsic value of an option is the amount of money the option holder would earn if they exercised the option immediately.

Time value: The time value of an option is the difference between the option price and its intrinsic value.

Premium: The price paid for an option.

Theoretical value: The value of an option based on a mathematical model.

Implied volatility: The volatility of the underlying asset implied by the option's price.

A Monte Carlo simulation is a mathematical technique that uses repeated random sampling to obtain numerical results. It is a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results. The underlying concept is to use randomness to solve problems that might be deterministic in principle.
Monte Carlo simulations are used in a wide variety of fields, including:
Finance: Monte Carlo simulations are used to price options, estimate the risk of investments, and test the performance of trading strategies.

Engineering: Monte Carlo simulations are used to design products and systems and analyze structures' reliability.

Physics: Monte Carlo simulations are used to study the behavior of particles and to simulate the evolution of systems.

Business: Monte Carlo simulations are used to make decisions under uncertainty and to forecast future outcomes.

In finance, Monte Carlo simulations are used to price options, estimate the risk of investments, and test the performance of trading strategies. For example, a Monte Carlo simulation could be used to estimate the value of a call option on a stock. The simulation would start by generating a random sequence of stock prices. The option value would then be calculated for each stock price in the sequence. The final value of the option would be the average of the option values calculated for all of the stock prices in the sequence.

In engineering, Monte Carlo simulations are used to design products and systems and to analyze the reliability of structures. For example, a Monte Carlo simulation could be used to design a bridge. The simulation would start by generating a random sequence of loads. The bridge design would then be tested for each load in the sequence. The final bridge design would be the design that passed the most loads in the sequence.

In physics, Monte Carlo simulations are used to study the behavior of particles and to simulate the evolution of systems. For example, a Monte Carlo simulation could be used to study the behavior of neutrons in a nuclear reactor. The simulation would start by generating a random sequence of neutron trajectories. The neutron trajectories would then be tracked through the reactor. The final result of the simulation would be the distribution of neutrons in the reactor.

In business, Monte Carlo simulations are used to make decisions under uncertainty and to forecast future outcomes. For example, a Monte Carlo simulation could be used to forecast the sales of a new product. The simulation would start by generating a random sequence of demand levels. The sales of the new product would then be calculated for each demand level in the sequence. The final forecast of sales would be the average of the sales forecasts calculated for all of the demand levels in the sequence.

Monte Carlo simulations are a powerful tool that can be used to solve a wide variety of problems. However, they are not without their limitations. One limitation is that they can be computationally expensive. Another limitation is that they can be sensitive to the random number generator used.

Option Pricing Using Monte Carlo Simulations -

How Does Monte Carlo Simulation Work?

The simulation produces a large number of possible outcomes along with their probabilities. In summary, it's used to simulate realistic scenarios (stock prices, option prices, probabilities).

Note: Monte Carlo simulations can get computationally expensive and slow depending on the number of generated scenarios.

Monte Carlo Simulation of Pricing a European Call Option

Start with the famous Black-Scholes-Merton formula.

$$dS_t = rS_t dt + \sigma S_t dZ_t$$

S(t) = Stock price at time *t* , r = Risk-free rate, σ = Volatility, Z(t) = Brownian motion

Using Euler Discretization Scheme to solve the stochastic equation above. The solution is given by the expression:

$$S_t = S_{t-\Delta t} \exp\left(\left(r - \frac{1}{2}\sigma^2\right)\Delta t + \sigma\sqrt{\Delta t}\, z_t\right)$$

Now log both sides and simplify the given equation we get,

$$\log S_t = \log S_{t-\Delta t} + \left(r - \frac{1}{2}\sigma^2\right)\Delta t + \sigma\sqrt{\Delta t}\, z_t$$

Now using Python (the **vectorization process** using the **NumPy** package in Python would easily ingest the log version of the solution above).

Monte Carlo Implementation in Python

Monte Carlo Implementation in Python

In [20]:
```
import math
from numpy import *
from time import time
```

In [68]:
```

random.seed(10000)
t0 = time()
# Parameters
S0 = 200.; K = 105.; T = 1.0; r = 0.05; sigma = 0.2
M = 25; dt = T / M; I = 250000
```

In [69]:
```
# Simulate I paths with M time steps
S = S0 * exp(cumsum((r - 0.5 * sigma ** 2) * dt
+ sigma * math.sqrt(dt)
* random.standard_normal((M + 1, I)), axis=0))
```

In [70]:
```
# sum instead of cumsum would also do

S[0] = S0
# Calculating the Monte Carlo estimator
C0 = math.exp(-r * T) * sum(maximum(S[-1] - K, 0)) / I
# Results output
tnp2 = time() - t0
```

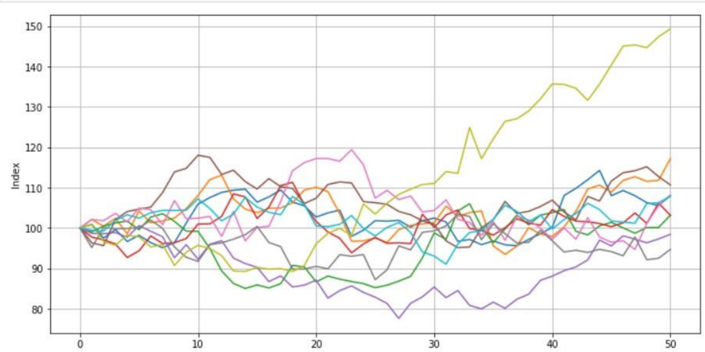
In [71]:
```
print('The European Option Value is: ', C0)
print('The Execution Time is: ',tnp2)
```

```
The European Option Value is:  100.52469984646264
The Execution Time is:  0.9434139728546143
```

Graphical Visualization

GRID

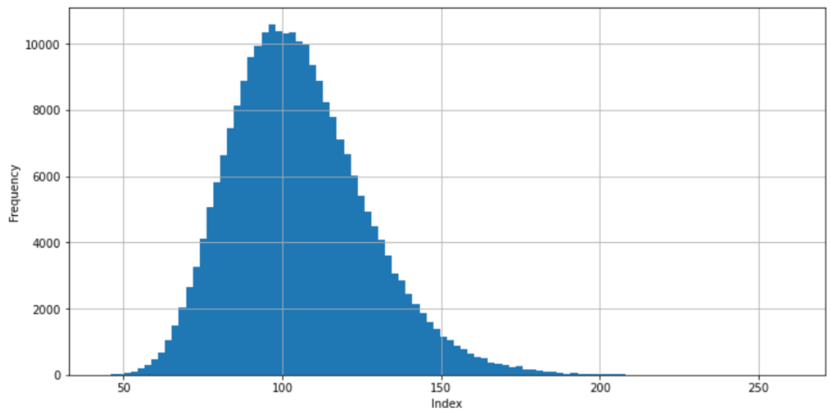
In [26]:
```
import matplotlib.pyplot as plt
plt.plot(S[:, :10])
plt.grid(True)
plt.xlabel('Step')
plt.ylabel('Index')
plt.show()
```



HISTOGRAM

In [34]:
```
plt.rcParams["figure.figsize"] = (12,6)
plt.hist(S[-1], bins=100)
plt.grid(True)
plt.xlabel('Index')
plt.ylabel('Frequency')
```

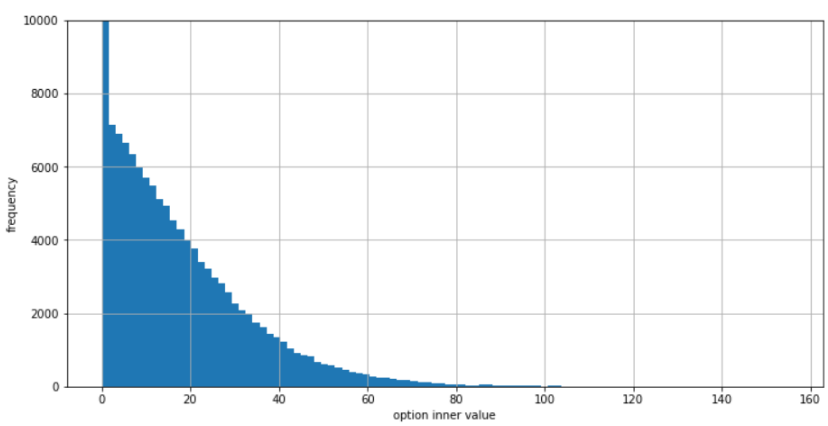
Out[34]: Text(0, 0.5, 'Frequency')



HISTOGRAM

In [38]:
```
import numpy as np
plt.rcParams["figure.figsize"] = (12,6)
plt.hist(np.maximum(S[-1] - K, 0), bins=100)
plt.grid(True)
plt.xlabel('option inner value')
plt.ylabel('frequency')
plt.ylim(0, 10000)
```

Out[38]: (0.0, 10000.0)



The Binomial Model

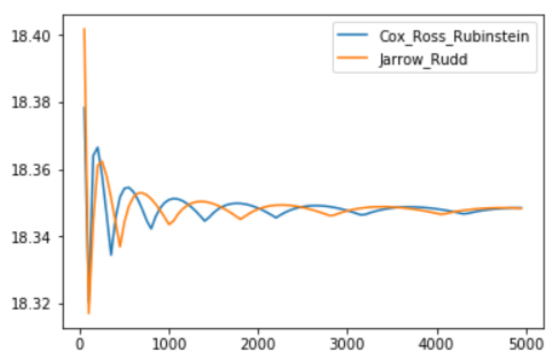
The binomial model is a mathematical model used to price options. It works by dividing the time to expiration of the option into a number of discrete time steps, and then calculating the possible prices of the underlying asset at each step. The model then uses these prices to calculate the value of the option at each step.
The binomial model is a relatively simple model, and it is easy to understand and implement. However, it is also a less accurate model than the Black Scholes model. This is because the binomial model assumes that the price of the underlying asset can only move up or down by a fixed amount at each time step. In reality, the price of the underlying asset can move by any amount, so the binomial model is not a perfect representation of reality.

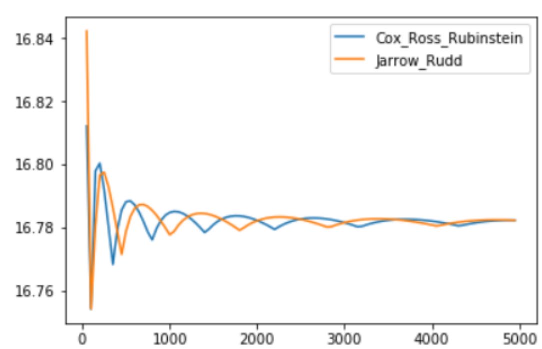
Binomial Model Implementation in Python

```
What is the current stock price? 100
What is the strike price? 100
What is the expiration date of the options? (mm-dd-yyyy) 8-26-2023
What is the continuously compounding risk-free interest rate in percentage(%)? 5
What is the volatility in percentage(%)? 30
```

	Symbol	Input
Underlying price	S	100.000000
Strike price	K	100.000000
Time to maturity	T	0.019178
Risk-free interest rate	r	5.000000
Volatility	sigma	30.000000

	Option	Price
Cox-Ross-Rubinstein	Call	1.704461
Cox-Ross-Rubinstein	Put	1.608616
Jarrow-Rudd	Call	1.704577
Jarrow-Rudd	Put	1.608733





The Black Scholes Model

The Black Scholes model is a more complex mathematical model than the binomial model. It uses a stochastic differential equation to model the price of the underlying asset. This means that the Black Scholes model can take into account the fact that the price of the underlying asset can move by any amount at any time.
The Black Scholes model is a more accurate model than the binomial model, but it is also more difficult to understand and implement. The Black Scholes model also requires more input parameters, such as the volatility of the underlying asset.

Back Scholes Model Implementation in Python

```
What is the current stock price? 100
What is the strike price? 100
What is the expiration date of the options? (mm-dd-yyyy) 8-24-2023
What is the continuously compounding risk-free interest rate in percentage(%)? 5
What is the volatility in percentage(%)? 30
```

	Symbol	Input
Underlying price	S	100.000000
Strike price	K	100.000000
Time to maturity	T	0.013699
Risk-free interest rate	r	5.000000
Volatility	sigma	30.000000

	Call	Put
Price	4.112199	1.569736
delta	0.520269	-0.479731
gamma	0.055516	0.055516
vega	0.142972	0.142972
rho	0.061698	-0.063795
theta	-0.206861	-0.011946

```
[['Price', 'S', 'K', 'T', 'r'], ['8.0', '100.0', '100.0', '0.0136986301369863', '0.05']]
 accuracy: 84.13%
CI = 95.023%
```

Comparison of the Two Models

The binomial model and the Black Scholes model are both used to price options. However, the binomial model is a simpler model that is easier to understand and implement. The Black Scholes model is a more complex model that is more accurate.

Which Model to Use?

The choice of which model to use depends on the specific situation. If **accuracy** is the most important factor, then the Black Scholes model should be used. However, if **simplicity and ease of implementation** are more important, then the binomial model can be used.

In general, the binomial model is a good choice for pricing options when the underlying asset is not very volatile. The Black Scholes model is a good choice for pricing options when the underlying asset is very volatile.

Qualitative Differences:
Here are some qualitative differences between the binomial model and the Black Scholes model:

The binomial model is a <u>discrete-time model</u>, while the Black Scholes model is a <u>continuous-time model</u>.
The binomial model assumes that the <u>price of the underlying asset can only move up or down by a fixed amount at each time step</u>, while the Black Scholes model does not make this assumption.
The binomial model is <u>easier to understand</u> and implement than the Black Scholes model.
The binomial model is <u>less accurate</u> than the Black Scholes model.

Conclusion:
The binomial model and the Black Scholes model are both used to price options. The binomial model is a simpler model that is easier to understand and implement, but it is also less accurate. The Black Scholes model is a more complex model that is more accurate, but it is also more difficult to understand and implement. The choice of which model to use depends on the specific situation.