$$D : \frac{1}{(2\pi\alpha)^{3/2}} e^{-\frac{1}{2\alpha^2}(x^2+y^2+z^2)}$$

$$= \left(\frac{1}{\sqrt{2\pi\alpha}} e^{-\frac{x^2}{2\alpha^2}}\right)\left(\qquad\right)\left(\quad\right)$$

$$-\frac{x}{\alpha^2} e^{-\frac{x^2}{2\alpha^2}}$$

$$\lim_{\alpha \to 0}\left[\frac{1}{\sqrt{2\pi\alpha}} e^{\frac{-x^2}{2\alpha^2}}\right] \qquad \boxed{-\frac{e^{-x^2/2\alpha^2}}{\alpha^2} x^2 - - -}$$

↳ All other values for $e^{-f(x)}$ where $x \neq 0$ vanish.

Q3

(i) $Q = 4\pi \int f(r)\, r^2 \delta(r - R)\, dr$ $\qquad$ $\rho(r) = f(r)\delta(r-R) = f(R)\delta(r-R)$

volume

$$q = 4\pi R^2 f(R) \qquad \therefore \qquad \rho(r) = \frac{Q}{4\pi R^2}\delta(r-R)$$

(ii)

$$\lambda L = \int f(r)\,\delta(r-R)\, r\, dr\, d\phi\, dz$$

$$\lambda L = L(2\pi) f(R) R$$

$$f(r) = \frac{\lambda}{2\pi R} \qquad \therefore \quad \rho(r) = \frac{\lambda}{2\pi R}\delta(r-R)$$

(iii)

$$\rho = f(r)\,\delta(z)\,g(\theta)$$

$$Q = \int f(r)\,\delta(z)\,g(\theta)\, r\, dr\, d\theta\, dz$$

$$= \int_0^{2\pi}\int_0^{r} f(r)\, g(\theta)\, r\, dr\, d\theta$$

$$Q = \underbrace{f(r)g(\theta)}\ \pi R^2$$

we move them out of integral as charge distribution is uniform.

$$g(\theta) = 1, \quad f(r) = \frac{Q}{\pi r^2}$$

$$\rho(r) = \frac{Q}{\pi r^2}\,\delta(z)$$

d) in spherical coordinates

$$\rho(r) = f(r)\, g(\phi)\, \delta(\theta - \pi/2)$$

$$Q = g(\phi)\int_0^{R}\int_0^{2\pi} f(r)\, r^2\, dr\, d\phi$$

We know that the charge distribution is uniform over the surface.

$\therefore$ we move the path out leaving the area element inside:-

$$q = g(\phi) f(r) r \left[\pi R^2\right]$$

$$\therefore \rho = \frac{q}{\pi R^2} \frac{1}{r} \delta(\theta - \pi/2)$$

Q $\nabla^2 \phi = \rho/\epsilon_0$.

∇^2 in spherical coordinates:-

Laplacian:-

$$\frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r}$$

$$= \frac{q}{4\pi\epsilon_0}\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \left[-\alpha e^{-\alpha r}\left[\frac{1}{r} + \frac{\alpha}{2}\right] + e^{-\alpha r}\left[-\frac{1}{r^2}\right]\right]\right]$$

$$\frac{q}{4\pi\epsilon_0 r^2}\left[\frac{\partial}{\partial r}\left[\frac{-\alpha r^2 e^{-\alpha r}}{r} - \frac{\alpha^2 e^{-\alpha r} r^2}{2} - e^{-\alpha r} r^2 \frac{\partial}{\partial r}\left(\frac{1}{r}\right)\right]\right]$$

$$\left[-\alpha e^{-\alpha r} + \alpha^2 r e^{-\alpha r} - r\alpha^2 e^{-\alpha r} + \frac{\alpha^3 e^{-\alpha r} r^2}{2} + \left(\alpha r^2 e^{-\alpha r} - 2r e^{-\alpha r}\right)\frac{\partial}{\partial r}\left(\frac{1}{r}\right) - 4\pi\delta(r)\right]$$

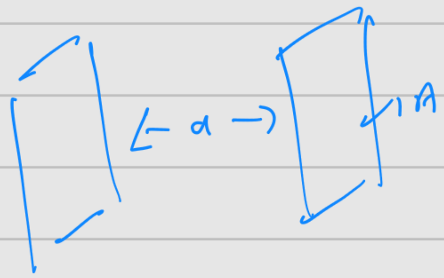
Solve further.

↳ but we see the appearance of the delta fn :

We get $\rho(x) = \underbrace{q\,\delta(x)}_{\text{nucleus at the centre}} - \underbrace{\frac{q\alpha^3}{8\pi} e^{-\alpha r}}_{\text{electron distribution around the nucleus}}$

Q 6

(i)



$\vec{E}_1 = \frac{Q}{2\epsilon A}$, $\vec{E}_2 = \frac{-Q}{2\epsilon A}$

$E_{tot} = \frac{Q}{\epsilon A}$

Potential $= \int E\,dx = \frac{Qd}{\epsilon A}$

Ratio of Q / Potential $= C = \frac{\epsilon A}{d}$

(ii)

→ conducting spheres.

$E = \frac{Q}{4\pi\epsilon_0 a}$ by Gauss law

$\Rightarrow$ Potential : $\dfrac{Q}{4\pi\epsilon_0 a} - \dfrac{Q}{4\pi\epsilon_0 b} = \dfrac{Q(b-a)}{4\pi\epsilon_0 ab}$

$$C = \frac{4\pi\epsilon_0 ab}{b-a}$$

(c) :-

$\rightarrow$ infinite cylinders.

$$E = \frac{Q}{2\pi r L\epsilon_0}$$

Potential : $\dfrac{Q}{2\pi\epsilon_0 L} \ln\left(\dfrac{b}{a}\right)$

capacitance = $2\pi\epsilon_0 L \ln^{-1}\left(b/a\right)$

Q1.6 :-

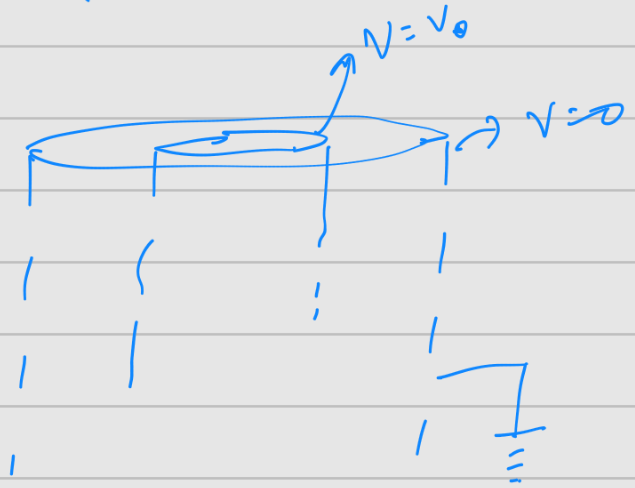
Parallel Cylinder:-

$$E = \frac{Q}{2\pi r L \epsilon_0}$$ (case where conductors hold fixed charges)

$$\therefore \text{Force} = \frac{Q^2}{2\pi r L \epsilon_0}$$

For fixed potential difference.

We can ground one of the conductors & keep the other at a potential V



Electric field due to inner conductor:-

$$\vec{E} = \frac{Q}{2\pi r L \epsilon_0} = \frac{Q'}{2\pi r L \epsilon_0}$$

$$\therefore Q = Q'$$

$\therefore$ Force remains the same.

Q12 :-

$$\int_V (\rho\phi' - \rho'\phi)\, d^3x = \int_S (\sigma'\phi - \sigma\phi')\, d^2x$$

$$\epsilon_0 \int_V (\phi'\nabla^2\phi - \phi\nabla^2\phi')\, d^3x$$

$$=$$

$$\nabla(\phi'\nabla\phi - \phi\nabla\phi')$$

$$= \cancel{\nabla\phi'\nabla\phi} + \phi'\nabla^2\phi - \cancel{\nabla\phi\nabla\phi} - \phi\nabla^2\phi'$$

$$\therefore \epsilon_0 \int_S (\phi'\nabla\phi - \phi\nabla\phi')\, d^2x$$

At the surface

$$n\cdot(\vec{E}_1 - \vec{E}_2) = \sigma/\epsilon_0 \qquad \nabla\phi = -E$$

$\nabla\phi$ across the surface normal $= -n\cdot(\vec{E}_1 - \vec{E}_2)$

$$= \epsilon_0 \int_S (\rho' \phi - \rho \phi') d^2x$$

Q1.14:-

Electrostatic Green's fn:-

$$\nabla'^2 \left(\frac{1}{|x - x'|} \right) = -4\pi \delta(x - x')$$

$$= \nabla^2 G(x, x') = -4\pi \delta(x - x')$$

Green's Theorem:-

$$\phi(x) = \frac{1}{4\pi\epsilon_0} \int_V \rho(x') G(x, x') d^3x'$$

$$+ \frac{1}{4\pi} \oint_S G(x, x') \frac{\partial \phi}{\partial n'} - \phi(x') \frac{\partial G(x, x')}{\partial n'}$$

General :-

$$\int_V (\phi \nabla^2 \psi - \psi \nabla^2 \phi)\, d^3x = \oint_S \left[\phi \frac{\partial \psi}{\partial n} - \psi \frac{\partial \phi}{\partial n} \right] da$$

$$\int_V \left(G(x,y)\, \delta(x'-y) - G(x',y)\, \delta(x-y) \right) d^3y$$

$$G(x,x') - G(x',x) = \oint_S G(x,y) \frac{\partial G(x',y)}{\partial n} - G(x',y) \frac{\partial G(x,y)}{n}$$

$$= -\frac{1}{S} \oint G(x',y) + \frac{1}{S} \oint G(x,y)$$

a) For dirichlet Boundary conditions :-

$$G(x,x') - G(x',x) = -\oint_S G(x',x) \frac{\partial G(x,x'}{\partial n} d^2x'$$

$$G(x',x) - G(x,x') = -\oint G(x,x') \frac{\partial G(x',x)}{\partial n} da$$

$$= \oint -\frac{\partial}{\partial n} \left(G(x,x')\, G(x',x) \right) da$$

$$= \oint -\frac{\partial}{\partial n}(0)\, da = 0$$

b) For

$$G(x, x') - G(x', x) = F(x) - F(x')$$

$$\cancel{G(x, x') - F(x) = G(x', x) - F(x)}$$

$\downarrow$

symmetric in x'

c)

Solution for potential in Neumann:-

$$\Phi(x) = \langle \Phi \rangle_S + \frac{1}{4\pi \epsilon_0} \int_V \rho(x') G_N(x, x') d^3x' + \frac{1}{4\pi} \oint \frac{\partial \Phi}{\partial n'} G_N \, da'$$

$\swarrow$

$$\Delta \Phi = \frac{F(x)}{4\pi} \left[\int_V \frac{\rho(x')}{\epsilon_0} d^3x' + \oint \frac{\partial \Phi}{\partial n'} da' \right]$$

$$= \frac{F(x)}{4\pi} \left[\int_V \frac{\rho(x')}{\epsilon_0} d^3x' + \oint \vec{\nabla} \Phi \cdot d\vec{a} \right]$$

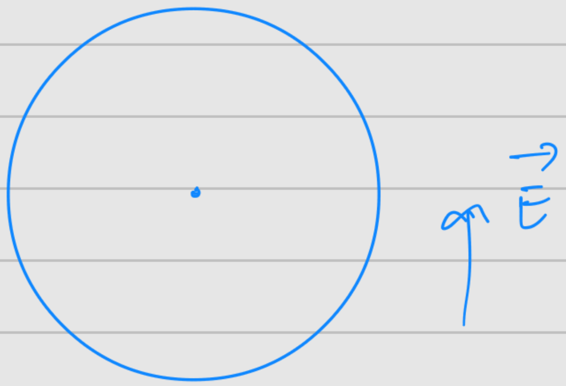
$$= \frac{F(x)}{4\pi}\left[\int_V \frac{\rho(x')}{\epsilon_0} d^3x - \oint \frac{\rho(x')}{\epsilon_0} d^3x\right] = 0$$

Q1 Problems. 1) :-

(a)

$$W = \int \frac{q^2 a d}{4\pi\epsilon_0 (a^2-d^2)^2} dd$$

Q2.7. :-



$$V_{out} = \frac{A_1}{r^{l+1}} P_1(\cos\theta) - E r \cos\theta$$

$$V_{ins} = \frac{q}{4\pi\epsilon_0 R}$$

$$\vec{E} = 3 e E \cos\theta$$

$$\frac{A}{R^2} = ER$$

$$A = ER^3$$

$$V_{out} = \frac{ER^3}{r^2}\cos\theta - Er\cos\theta$$

$$\Rightarrow \vec{E} = (\epsilon_0)\left[2E\cos\theta + E\cos\theta\right]$$

$$\sigma = 3\epsilon_0 E\cos\theta$$

electrostatic pressure $= \dfrac{9\epsilon_0^2 E^2 \cos\theta}{2\epsilon_0}$

$$= \frac{9}{2}\epsilon_0 E^2 \cos^2\theta$$

$$F = \int dF_z = \int_{\text{surface}} \frac{9}{2}\epsilon_0 E^2 R^2 \cos^3\theta \, \sin\theta \, d\theta \, d\phi$$

$$= \frac{9}{2}\epsilon_0 E^2 R^2 \int_{\text{surface}} \sin\theta \cos^3\theta \, d\theta \, d\phi$$

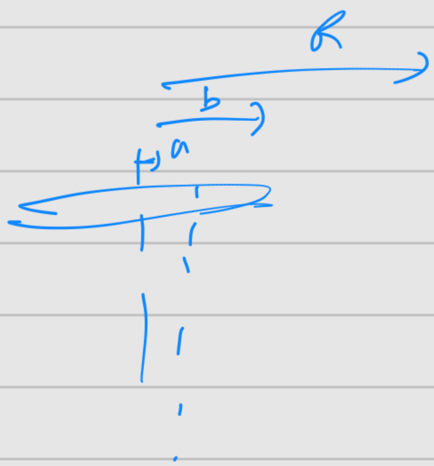
$$= 9\pi\epsilon_0 E^2 R^2 \int_0^{\pi/2} \sin\theta \cos^3 d\theta$$

$$= 9\pi\epsilon_0 E^2 R^2 \left[-\frac{\cos^4\theta}{4} \Big|_0^{\pi/2} \right]$$

$$= \epsilon_0 \pi \left(\frac{3}{2} ER\right)^2$$

b) If total charge on shell is Q :-

Q2-11



a) $\vec{E} = \dfrac{\lambda}{2\pi \epsilon_0 r}$

$$d' = \sqrt{r^2 + a^2 - 2ar\cos\theta}$$

$$d = \sqrt{r^2 + b^2 - 2br\cos\theta}$$

$\Rightarrow$

$$\lambda' \ln\left(\frac{r^2 + a^2 - 2ar\cos\theta}{a^2}\right) + \lambda \ln\left(\frac{r^2 + b^2 - 2br\cos\theta}{b^2}\right) = 0$$

$$\left(\frac{r^2 + a^2 - 2ar\cos\theta}{a^2}\right)^{\lambda'} = \left(\frac{r^2 + b^2 - 2br\cos\theta}{b^2}\right)^{-\lambda}$$

$$\left[1 + \frac{r^2}{a^2} - \frac{2r}{a}\cos\theta\right]^{\lambda'} = \left[1 + \frac{r^2}{b^2} - \frac{2r}{b}\cos\theta\right]^{-\lambda}$$

$$\lambda'\left(\frac{r^2}{a^2} - \frac{2r}{a}\cos\theta\right) + \frac{\lambda'(\lambda'-1)}{2}\left[\frac{4r^2}{a^2}\cos\theta\right]$$

$$= \lambda\left[\frac{r^2}{b^2} - \frac{2r}{b}\cos\theta\right]$$

$$+ \frac{\lambda(\lambda-1)}{2}\left[\frac{4r^2}{b^2}\cos\theta\right]$$

$$\frac{\lambda'(\lambda'-1)}{a^2} = \frac{\lambda(\lambda-1)}{b^2} \qquad \left[\frac{\lambda}{a} = \frac{\lambda'}{b}\right]$$

$$\left[\frac{\lambda}{a^2} = \frac{\lambda'}{b^2}\right.$$

$\underline{\underline{Q2.13}}$:-

$$V(r,\phi) = \sum_l A_l r^l P_l(\cos\theta)$$

$$\sum_l A_l b^l P_l(\cos\theta), \theta \in [0,\pi] = V_1$$

$$\sum_l A_l b^l P_l(\cos\theta) \theta \in [\pi, 2\pi] = V_2$$