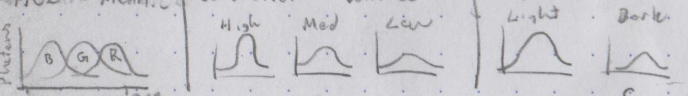
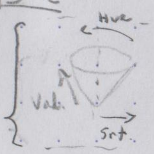
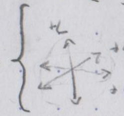
Color

Hue ~ Mean | Saturation ~ Variance | Brightness ~ Area

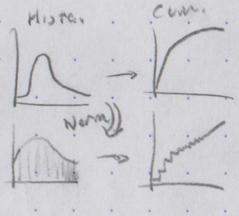


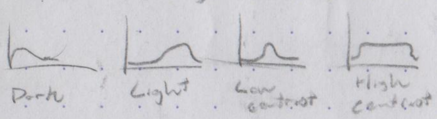
- RGB: Easy for devices, not perceptual
- HSV: Hue, Sat., Value (Bright), intuitive
- L*a*b: Perceptually uniform





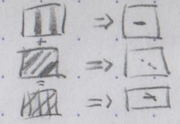
- Histogram Normalization
  - → Want linear cumulative contrast
  - → $S = T(r)$, T is cumulative histo.
  - → Map to y-axis





- Aliasing: Due to undersampling
  - → Fix 1: Sample more; Twice per period of highest Frequency
  - → Fix 2: Remove high freq., less "wiggly"
    - ↳ Low-pass filter
- Filters:
  - Linear: Blur/sharpen imgs, bass/treble for sound
    - ↳ $\text{filter}(f+g) \equiv \text{filter}(f) + \text{filter}(g)$
    - ↳ Shift Invariant:
  - Cross-Corr.: $H \otimes F = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]\, F[i+u, j+v]$
  - Convolution: $H * F$; Same but flipped
    - ↳ Flipped horizontally + vertically, thus commutative + associative
    - ↳ Full (sum), Same (size), Valid (f,g)
  - Gaussian: Low-pass filter, removes high freq.
    - ↳ convolve twice w/width σ ⟹ once w/width $= \sqrt{2}\,\sigma$
  - Edge: $M_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$ gives vertical lines
- Image Pyramids: Repeated filter → subsample
  - → Good for search at many scales
  - → 4/3 size of original img (maximally)
  - Gaussian: Low-pass images
  - Laplacian: Sub-band images
    - → $LS(i,j) = \underbrace{G(i,j)}_{\text{Gauss of mask}} \cdot L_A(i,j) + (1 - G(i,j))\, L_B(i,j)$

Frequency



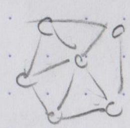
Fourier of conv. of 2 funcs is product of their Fourier transforms.

- → Conv. in spatial ≡ Product in freq. | $F(g * h) = F(g) \cdot F(h)$
- → Gaus of σ in spatial ≡ Gaus. size $1/\sigma$
- Image Sharpening: $\underbrace{f}_{\text{img}} + \alpha(f - \underbrace{f * g}_{\text{blur, low-pass}}) = (1+\alpha) f - \alpha (f * g)$
  $= f * ((1+\alpha)\underbrace{e}_{\text{unit impulse}} - \alpha g)$
  - → Low-pass, High-pass
- Blending window: want smooth + no ghosting
  - → Avoid seams: w = size of biggest feature
  - → Avoid ghosting: w = 2 * size of smallest feature
  - → In freq: Largest freq ≤ 2 * smallest freq.
- JPEG: 8×8 block size
  - → Larger → nicer smooth regions
  - → Smaller → Faster, corr. between neighbors

Template Matching

1.) Zero-mean filter: Fast, no great
2.) SSD: Next fastest, sensitive to overall intensity
  - ↳ Fails w/ color changes
3.) NCC: Slowest, but invariant to local area contrast/intensity

- Delaunay Triang: Minimizes large angles
- Harris: $R = \frac{\det M}{\operatorname{Tr} M} = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$
  - ↳ Significant change in x + y ∝ large λ_1 & λ_2
  - $\lambda_1 >> \lambda_2$ ⟹ vert. edge



- ANMS: Fixed # pts distributed evenly, sort by radius to next
- Feature Desc: Look at fine patches, Find most sim SSD(p_1, p_2) ≤ thresh.
- Lowe's Trick: Look at those with only one nearest neighbor

Transformations

- Linear: origin, lines, parallel go to themselves
  - → Rot: $\begin{bmatrix} \cos & -\sin \\ \sin & \cos \end{bmatrix}$ | → Shear: $\begin{bmatrix} 1 & s_x \\ s_y & 1 \end{bmatrix}$, $s = \tan\theta$
  - → Scale: $\begin{bmatrix} w_1 & 0 \\ 0 & w_2 \end{bmatrix}$
  - → Rigid: Trans. + rot
    - ↳ Ex. flip, DoF = 3
  - → Similarity: Rigid + scale, DoF = 4

θ	0	30	45	60	90
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\pi/2$
cos	1	$\sqrt{3}/2$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\sqrt{3}/2$	1

- Affine: Linear transform + translate
  - ↳ origin ≠ origin
  - ↳ DoF = 6

$$\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix}$$

Homography

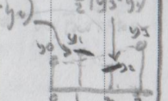
- Projective: Affine + proj. warp
  - origin ≠ .., parallel ≠ .., ratio ≠ ..
  - line = line,
  - ↳ DoF = 8

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix}$$

- Ransac
  - →

Splines | deg n defined by n+1 pts

Catmull-Rom: $\frac{1}{2}(y_2 - y_0)$, $\frac{1}{2}(y_3 - y_1)$

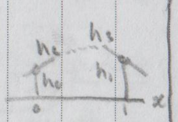


- Match derivative w/ neighbors then Hermite interp.

Cubic Hermite: Given p_0, p_0', p_1, p_1'

Find cubic poly of form $at^3 + bt^2 + ct + d$

$$\underline{h} = \begin{bmatrix} p_0 \\ p_1 \\ p_0' \\ p_1' \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 \end{bmatrix} \underbrace{\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}}_{\underline{x}}$$



Basis for cubic polys

$$\underline{x} = \underbrace{\begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}}_{Q} \underline{h} \quad \therefore P(t) = [t^3\ t^2\ t^1\ 1]\,\underline{x}$$

$$= [H_0 \cdots H_3]\,\underline{h}$$

For
$H_0(t) = 2t^3 - 3t^2 + 1$
$H_1(t) = -2t^3 + 3t^2$
$H_2(t) = t^3 - 2t^2 + t$
$H_3(t) = t^3 - t^2$

Hermite Basis Funcs For cubic poly

- Updates propogate locally (one pt effects two splines)
- Discontinuous 2^{nd}-ord deriv at control pts

de Casteljau (Bezier curve) $b^{(n)}(t) = \sum_{j=0}^{n} \underset{\text{control pt}}{b_j} B_j^{(n)}(t)$ $\quad \binom{n}{j} t^j (1-t)^{n-j}$

$b_i' = \text{lerp}(b_i, b_{i+1}, t)$
$= s b_i + t b_{i+1}$

$$[s\ t] \begin{bmatrix} s & t & 0 \\ 0 & s & t \end{bmatrix} \begin{bmatrix} s & t & 0 & 0 \\ 0 & s & t & 0 \\ 0 & 0 & s & t \end{bmatrix} \cdots$$

For $s = 1 - t$ | # Interp pts $= \sum_{i=1}^{n-1} i$

Russian Roulette | Continue w.p. p_{rr} o/w terminate

- Geometric dist, expected depth is $1/1-p_{rr}$ term prob

↳ Scale value $1/p_{rr}$

$\mathbb{E}[X_{rr}] = p_{rr}\,\mathbb{E}\left[\frac{X}{p_{rr}}\right] + (1-p_{rr})\,\mathbb{E}[0]$ (unbias)

Mesh | Loop: 4x # triangles

Catmull-Clark Subdivision:

1. Add vertex to centers of each face
2. Add vertices to middle of each edge
3. Connect all new nodes

↳ Non-quad faces ~~become~~ lead to extraordinary pts

- Meshes homeomorphic to spheres must have some extraordinary pt (node w/ 6+ degree)
- Edge flips can improve (sometimes) degrees

$\frac{h_0}{z_0 - f} = \frac{h_i}{f}$

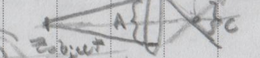
$\therefore \frac{h_0}{f} = \frac{h_i}{z_i - f}$

$m = \frac{h_i}{h_0} = \frac{z_i}{z_0}$

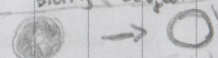
Focused z_{image}

Gauss. lens Eqn: $f^{-1} = z_i^{-1} + z_0^{-1}$

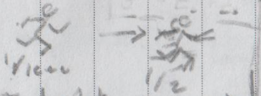
$\frac{C}{A} = \frac{|z_s - z_i|}{z_i}$ opt. int = f, Blurry



Camera | $A = \frac{f}{2}$

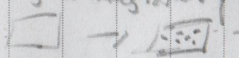


- Aperture: F32, 22, 16, 11, 8, 5.6, 4, 2.8, 2, 1.4
  ↳ Fine/Focused ↳ blurry bg
  ↳ Changes F-stop
- Shutter:



Exposure = Time × Irradiance × Gain

  ↳ Duration light integrated
- Gain:



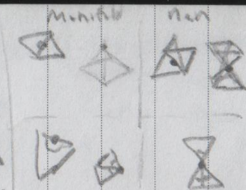
  ↳ Amplification at sensor to digital values
- Decreasing focal len ⟺ increase FOV → $2\arctan\left(\frac{h}{2f}\right)$
- F-number $N = F/D$ For Diameter of aperture, focal length
- Can't focus at objects closer than the lens focal length

▫ $z_i > z_0$ ≡ Magnified, $z = 2f$

▫ $z_i = z_0$ ≡ 1:1 macro imaging

Manifold Properties

- Edge connects 2 faces
- Edge composed of 2 nodes
- Face ≡ Ring of edges / nodes



- $\#F - \#E + \#V = 2$

▫ All nodes of face:

```
HE* h = F->HE;
do {
  process(h->vertex);
  h = h->next;
} while(h != F->HE);
```

▫ All Edges around Node

```
HE* h = v->HE;
do {
  F(h->edge);
  h = h->twin->next;
} while(h != V->HE);
```

Monte Carlo

Numerical integration err $\sim 1/N^{1/d}$ (curse of dim) while random sampling $\sim 1/\sqrt{N}$

MCI: $F_N = \frac{1}{N}\sum_{i=1}^{N} \frac{f(x_i)}{p(x_i)}$

For $x_i \sim p(x)$

Unbias pf

$\mathbb{E}[F_N] = \frac{1}{N}\sum \mathbb{E}\left[\frac{f(\cdot)}{p(\cdot)}\right]$

$\int_a^b p(\cdot) \cdot \frac{f(\cdot)}{p(\cdot)}$

Same as OG int. $\left\{ = \int_a^b f(x)\,dx \right.$

$\text{Var}(F_N) = \frac{1}{N^2}\sum \underbrace{\text{Var}(Y_i)}_{N \cdot \text{Var}(Y)}$

$= \text{Var}\left(\frac{f(\cdot)}{p(\cdot)}\right)/N$

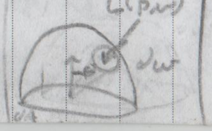
↳ Scales linearly w/ N

Ex) Direct lighting (irradiance)

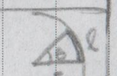
$F_N = 0$

For each N samples: sample $w_i \sim U$, compute Irradiance

$E(p) \approx F_N += \frac{2\pi}{N} L_i \cos\theta_i$



$\theta = \frac{l}{r}$



↳ circle $\theta = 2\pi$

$\Omega = \frac{A}{r^2}$

↳ sphere $\Omega = 4\pi$

$dA = (r\,d\theta)(r\sin\theta\,d\phi)$
$= r^2 \sin\theta\,d\theta\,d\phi$

$d\omega = \frac{dA}{r^2} = \sin\theta\,d\theta\,d\phi$

Rendering Eqn

$L_o(p,\omega_o) = L_e(p,\omega_o) + \int_{H^2} f_r(p, \omega_i \to \omega_o)\,\underbrace{L_o(tr(p,\omega_i), -\omega_i)}_{L_i(p',\omega_i)}\cos\theta\,d\omega_i$

tr ≡ 1st scene intersect along this ray

Ex) $\int_0^\pi f(x)\,dx$ For $N=1$, $X_i \in [0,\pi]$ For $X_i \sim \sin(x)$

$= [x = \pi y]$

$\int_0^1 f(y) \cdot \pi\,dy \longrightarrow \frac{f(x)}{\sin(x)} \cdot \pi \cdot \frac{1}{2}$

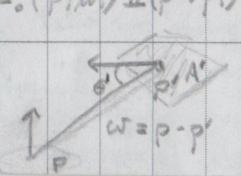
Normal: $1 = \int_0^\pi c \cdot f(x) \Rightarrow c = 2$

Ex) $p(\omega_i) = \frac{1}{\pi}\cos\theta$

$\int_{H^2} L(\omega)\cos\theta\,d\omega \longrightarrow \frac{\pi}{N}\sum_{i=1}^{N} L(\omega_i)$

Ex) Sampling light area

$E(p) \approx \frac{A'}{N}\sum_{i=1}^{N} L_o(p_i', \omega_i)\,\mathbb{1}(p \to p_i')\,\frac{\cos\theta_i \cos\theta_i'}{|p - p_i'|^2}$



$\frac{\int_a^b\int_c^d g(x,y)\,dx\,dy}{N} \sum_{i}^{N} \frac{h(x_i,y_i)}{g(x_i,y_i)}$

Radiometry:

$\frac{dQ}{dt}$ | Energy, Q ↳ lumen·sec

- Flux (power), Φ. Energy emit/reflect (transmit/received) per unit of time
  ↳ lumen [candela·sr] / watts
- Intensity, $I = \frac{d\Phi}{d\omega}$. Power per unit SA
  ↳ Candela [$\frac{\text{watts}}{\text{lumen/sr}}$] emit by pt-light, $1/r^2$ scaling
- Irradiance, $E = \frac{d\Phi}{dA}$. Power per unit area
  ↳ lux [$\frac{\text{watt}}{\text{lumen/m}^2}$] incident at a surface point
  ↳ Lambert cosine law
- Radiance, $L = \frac{dI}{dA\cos\theta}$. Power transport by surface per unit SA per unit proj area
  ↳ nit
  [candela/m²] / [watt/sr m²] $= \frac{d^2\Phi}{d\omega\,dA\cos\theta}$
  - ▫ Incident surface: known $L(p,\omega) = \frac{dE(p)}{d\omega\cos\theta}$ } Light hitting surface from ray
  - ▫ Exiting surface: given $L(p,\omega) = \frac{dI(p,\omega)}{dA\cos\theta}$ } Area light emit along given ray
- Reflection Eqn.:

$L_r(p,\omega_r) = \int_{H^2} f_r(p, \omega_i \to \omega_r)\,L_i(p,\omega_i)\cos\theta\,d\omega_i$

Frequency
- Depends on geo. + view transforms and texture coord fctn

- Nyquist Thm: No aliasing from freq. less than Nyquist freq.
- Nyquist Freq $= \frac{1}{2} f_{sampling}$
- Nyquist Rate $\Rightarrow f_{sampling} > 2 f_{signal}$

Ex| 6 fan blades, 120 FPS camera, fastest RPM? 10 rot/sec as $120 > 2(6 \cdot x)$

- Low-pass filters: Attenuate high freqs ↳ Ex Box ||
- Supersampling: Avg. $n \times n$ samples per pixel; approx. 1-pixel box filter ⇒ samples higher freq then downsample
- Increasing exposure (longer shutter time) introduces motion blur ⟺ less aliasing

Jitter Sampling: rand(0,1) $\cdot 2d$

SuperSample Pos. $= (x + d + (2i\,d),\ y + d + 2j\,d)$

Framebuffer $[h \cdot width + w]$ For $d = 1/2N$

MipMap
Store pyramid of downsamples, just 4/3 of OG space

- Level k is $1/(2^k)^2$ of OG size
- Determining level: edge

map: $(x,y) \to (u,v)_{00}$, $(x+1,y) \to (u,v)_{10}$, $(x,y+1) \to (u,v)_{01}$

$$\left(\frac{du}{dx}, \frac{dv}{dx}\right) = (u,v)_{10} - (u,v)_{00}$$

$$\left(\frac{du}{dy}, \frac{dv}{dy}\right) = (u,v)_{01} - (u,v)_{00}$$

$$D = \log_2 \left[\max \left\{ \sqrt{\left(\frac{du}{dx}\right)^2 + \left(\frac{dv}{dx}\right)^2},\ \sqrt{\left(\frac{du}{dy}\right)^2 + \left(\frac{dv}{dy}\right)^2} \right\} \right]$$

↳ Interpolate between $\lfloor D \rfloor$ and $\lceil D \rceil$ e.g. w/ trilinear filtering

- ☐ Ideally 1:1 pixel-texel sampling rates
- Mag. – many pixels per texel
- Min. – one pixel per many texel (e.g. hard sampled)
- ☐ Anisotropic: e.g. mip map but variations in x/y distortion

Transforms
Rotation CCW $= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

scale $= \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$ ↳ $s_x = -1$ flip over y-axis

shear$_y$ $= \begin{bmatrix} 1 & 0 \\ s_y & 1 \end{bmatrix}$, $\tan\phi \leftarrow s_y$

Perspective $= \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix}$ ↳ Affine ⟺ $0 = g = h$; Linear + translation

☐ Recall: Chain R → L product. Not commutative

Coordinate System

$$F = \begin{bmatrix} u_x & v_x & o_x \\ u_y & v_y & o_y \\ 0 & 0 & 1 \end{bmatrix}$$

u, v unit vectors

Ex| $\frac{1}{2}$ ...

$$\begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & 1 \\ [1] & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \to \begin{pmatrix} x \cdot \frac{d}{z} \\ y \cdot \frac{d}{z} \\ d \end{pmatrix}$$

Perspective
- Div motivates homogenous coords

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix} \Leftrightarrow \begin{bmatrix} x\frac{d}{z} \\ y\frac{d}{z} \\ d \\ 1 \end{bmatrix}$$

Img Plane $z = d$

- Obj. coords apply modelling transforms
- World (scene) coords apply viewing transforms
- Camera coords apply perspective + homo. div.
- Screen transforms maps NDC → screen coords

Ex| 400 × 300 px img shown on 1200 × 1200 canvas
$$\frac{du}{dx} = \frac{4u}{18},\ \frac{dv}{dy} = \frac{3v}{12},\quad \frac{du}{dy} = \frac{dv}{dx} = 0$$

Texture Filtering
Bilinear: Inter $\propto$ area

$F(u,v) = \text{lerp}(t,\ \text{lerp}(s, u_{00}, u_{10}),\ \text{lerp}(s, u_{01}, u_{11}))$

Trilinear: Linear interp. between two series of bilinear interp.

Inverse Sampling
Sample $\xi \sim U[0,1]$

Solve for $F^{-1}(\xi) = x$, or choose $X = x_i$ s.t. $F_{i-1} < \xi \leq F_i$ via B.S.

Ex| $p_2(x) = \lambda \exp(-\lambda x)$

$$F_2(x) = \int_0^x p(x)\,dx = \lambda\left[\tfrac{1}{\lambda}\exp(-\lambda x)\right]_0^x = 1 - \exp(-\lambda x)$$

$x = 1 - \exp(-\lambda F^{-1}(\cdot))$

$-\lambda F^{-1}(\cdot) = -\lg(1-x)$

$F^{-1}(x) = -\lg(1-x)/\lambda$

Barycentric
$$\alpha = \frac{L_{BC}(x,y)}{L_{BC}(x_A, y_A)}$$

- If any $\{\alpha, \beta, \gamma\} < 0 \Rightarrow$ pt outside
  ↳ $\alpha + \beta + \gamma = 1 \Leftrightarrow$ pt on same plane

Ex| C $(0, .3, .7)$, B $(0, .7, .3)$, A $(.9, .05, .05)$, $(.45, .1, .45)$

world space coords: v, e = eye, up, w = viewing dir

$$c2w = \begin{bmatrix} r & u & v & e \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$w2c = \begin{bmatrix} r & 0 \\ u & 0 \\ -v & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I_{3\times3} & -e \\ 0 & 1 \end{bmatrix}$$

Box Int.:

$t_{min} = \max(\min_i \{t_{i,1}, t_{i,2}\})$

$t_{max} = \min(\max_i \{\,''\,\})$

y, z: $p'_x - o_x / d_x$

Shading
Blinn-Phong: Per-pixel local shading given view dir., surface norm/param, light dir.

- Diffuse: Light uniform scatter, indep view dir
  $$L_d = k_d (I/r^2) \max(0, \underbrace{n \cdot l}_{\cos\theta})$$
  (I: illum.) ↳ Matte
- Specular: Intensity $\propto$ viewing dir.
  $$L_s = k_s (I/r^2) \max(0, \underbrace{n \cdot h}_{\cos\alpha})^p$$
  h = bisector(v, I) $= v + I / \|v + I\|$

  $p \to \infty \Rightarrow$ more concentrated
- Ambient: Indep., const. shadow/color
  $$L_a = k_a \cdot I_a$$

Rescale PDF to sum to 1

$$1 = \int_\Omega c \cdot p(x) = \int_0^2 c \cdot x^2 = \frac{cx^3}{3}\Big|_0^2 \Rightarrow c = \frac{3}{8}$$

∴ use $\frac{3}{8} p(x)$ in CDF calc

Ex| $\cos\theta$ → $\cos^2\theta = \frac{1}{\pi}\cos\theta$

$L(p) = V(p) \cdot N = -(x - x_0)(y_1 - y_0) + (y - y_0)(x_1 - x_0)$

$$N = \begin{bmatrix} -(y_1 - y_0) \\ (x_1 - x_0) \end{bmatrix} = \text{perp}(T)$$

$V = p - p_0$, $T = p_1 - p_0$

Plane Int.: check $0 \leq t < \infty$

$t = (p' - o) \cdot N / d \cdot N$

Sphere Int.:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $a = d \cdot d$

$b = 2(o - c) \cdot d$

$c = (o - c) \cdot (o - c) - R^2$

Consider real, positive roots

Vertex Processing ↓ (vertex stream) Triangle Processing ↓ (triangle stream) Rasterization ↓ (frag. stream) Fragment Processing ↓ (shaded frags) Framebuffer calc ↓ (display)

Material Reflections:
- Reflections: Light incident on surface leaves on $[\lambda]$ w/o freq. shift

Ideal Specular | Ideal Diffuse | Glossy Specular | Retro-reflect

- BRDF (Bidirectional Reflectance Distribution Fctn) Models light reflected into each outgoing dir ω_r from each incoming dir.

$$f_r(\omega_i \to \omega_r) = \frac{dL_r(\omega_r)}{dE_i(\omega_i)} = \frac{dL_r(\omega_r)}{L_i(\omega_i)\cos\theta_i\, d\omega_i}$$

Differential exiting rad. ; Differential incoming irradiance

Depth of Field: $= 2x^2 NC / f^2$ (dist. to subject; F-number; circle of conf.)

Physics

$F = \sum m_i a_i$; $F_{a \to b} = -F_{b \to a}$

Spring: $F_{a \to b} = K_s \frac{b-a}{\|b-a\|}(\|b-a\| - \ell)$

dampening: $F_d = -K_d\left((\dot{b}-\dot{a}) \cdot \frac{b-a}{\|b-a\|}\right)\frac{b-a}{\|b-a\|}$

↳ different resolution of meshes cause diff spring behavior

Particles: $F_g = G \cdot \frac{m_1 m_2}{d^2}$

Numerical Methods

Euler's: $\underbrace{x_{t+\Delta t} = x_t + \Delta t\, \dot{x}_t}_{\text{Explicit}}$

$\dot{x}_{t+\Delta t} = \dot{x}_t + \Delta t\, \ddot{x}_t$

- Error accumulates
- Diverges quickly, proportional to Δt

Implicit: $t+\Delta t$ — use future deriv. at curr calc.

Avg. Euler: Add $\underbrace{\frac{(\Delta t)^2}{2}\ddot{x}_t}_{\text{Quad term of Taylor Expansion}}$ to $x_{t+\Delta t}$

can be made ↑ uncond. stable

Implicit E.: $x_{t+\Delta t} = x_t + \Delta t\, \dot{x}_{t+\Delta t}$

Verlet: Constrain pos/velocity, dissipating energy ⇒ stabilizing

Signal-Noise-Ratio $:= \mu / \sigma$ ← std

$\simeq \sqrt{\text{amount light}}$ (dB) $= 20 \log_{10}\left(\frac{\mu}{\sigma}\right)$

- Ex: 100:1 → 200:1 improvement of $20(\log 200 - \log 100) = +6$ dB
- Model photon arrivals as in Geee, so arrival times are Poisson: $\sigma = \sqrt{\lambda}$

[If λ events occur in time int., the Probability k events occur instead is $p(k;\lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$]

- Shot noise: $SNR = \frac{\mu}{\sigma} = \frac{\lambda}{\sqrt{\lambda}} = \sqrt{\lambda}$

↳ Fundamental noise; # photons

- Ex λ: 10,000 → 1000 ≡ $\sqrt{10}$, +10 dB

1 stop ≡ 2x photons ≡ $\sqrt{2}$, +3 dB

Animation

- Forward Kinematics: Ex Animator determines angle, computer determines position

↳ Animation described with $\theta(t)$

- Inverse Kinematics: Given end pts, find intermediary angles

↳ Hard! Many feasible solns

Camera Sensors

- Photoelectric effect: Light's electromagnetic radiation causes emission of electrons from surface it hits; causing "electron hole", acting as signal for camera sensors.

↳ Quantum efficiency: $QE := \frac{\#\text{ electrons}}{\#\text{ photons}}$

Not all photons provide electron

- Human vis. ~15%; smartphone cam. ~60%

- Color sensors mosaiced; green most common as Human Vis most sensitive to green spectrum (given by luminosity efficiency curve)

↳ Must de-mosaic to color in the 2/3 of unknown information

↳ Can also be applied to HDR to interpolate images of various exposures

- Pixel Fill factor: % sensor area that integrates incoming light

↳ Optimize w/per-pixel microlense

↳ Optical cross talk also improved, leading to less washed out img; motivated back-side illumination (BSI)

Color

Defn. visual experience/sense arising from seeing light of varying spectral distributions

- Spectral Power Dist.: Amount of light present at each wavelength [watts/nm]

↳ Vary across lightbulb types

- Superposition (wave)

- Tristimulus Theory of Color: Any color composed of 3 basis colors as summation of 3, or sum of 2 and subtract other

↳ Color vision is rank 3

↳ S/M/L-cone cells for B/G/R each have difference response for different wavelengths

- S least common, lower mag. resp.
- L most ", high " "

↳ Metamers: Two different spectra (∞-dim signal) that map to same (S,M,L)-response; crucial to color reproduction

$$\begin{bmatrix} S \\ M \\ L \end{bmatrix}_{disp} = \begin{bmatrix} - & r_S & - \\ - & r_M & - \\ - & r_L & - \end{bmatrix}_{\lambda} \underbrace{\begin{bmatrix} | & | & | \\ s_R & s_G & s_B \\ | & | & | \end{bmatrix}}_{s_{disp}} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

$$\begin{bmatrix} S \\ M \\ L \end{bmatrix}_{real} = \begin{bmatrix} - & r_S & - \\ - & r_M & - \\ - & r_L & - \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix} \qquad s_{disp} = [s_R \; s_G \; s_B] \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} r_S \cdot s_R & r_S \cdot s_G & r_S \cdot s_B \\ r_M \cdot s_R & r_M \cdot s_G & r_M \cdot s_B \\ r_L \cdot s_R & r_L \cdot s_G & r_L \cdot s_B \end{bmatrix}^{-1} \begin{bmatrix} S \\ M \\ L \end{bmatrix}_{real}$$

Noise Sources:

1) Dark current: Electrons dislodged due to random thermals

↳ includes shot noise;

2) Electron leaks: manufacturing defects,

↳ changes over time

3) Read Noise: Thermal noise in readout circuitry

- Avg. frames ⇒ SNR + $\sqrt{\#\text{ frames}}$ (neglecting read noise); can also downsize too

Gamuts:

1) sRGB / Apple P3: For displays

2) CIE: LMS basis that span all observable colors

- Matching Fctns strictly positive
- $Y \simeq$ luminance, brightness absent color

$= \int \underbrace{\Phi(\lambda)}_{\text{radiance}} \underbrace{V(\lambda)}_{\text{visual luminosity; perceived brightness}} d\lambda$

- Chromaticity

$x = \frac{X}{X+Y+Z}$, $y = \frac{Y}{X+Y+Z}$, $z = \frac{Z}{X+Y+Z}$

↳ Typically standardize: $x+y+z = 1$, usually (x, y, Y)

↳ White at $(x,y) = (1/3, 1/3)$, saturation increases at edges

Color Spaces:

- Hue-Saturation-Value:
  - H: "kind", dominant wavelength
  - S: "colorfulness", purity
  - V: "brightness", luminance

- L*a*b: Perceptually Uniform
  - Defined by CIE basis, non-linear warping
  - Formulas for Hue/Chroma/Lightness
  - l_2-loss ≡ difference between colors, perceptually

JPEG | Exploits Human Visual System

Low-Freq content predominant in images of real-world

Our Vision sys. is:

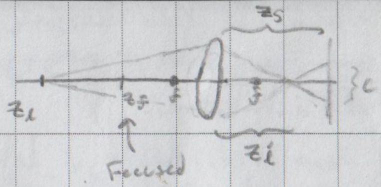
1) Less sensitive to chromaticity than lum.
2) Less sensitive to high-Freq. errors

Encode w/ Y'CbCr, ~ like L*a*b*

↳ Luma (lightness) / Blue-Yellow / Red-green

↳ Downsample chroma channels; For each we take 8x8 blk patches:

- compute DCT
- Quantize (prioritizing low-Freqs)
- Reorder + Huffman Encode



$$f^{-1} = z_s^{-1} + z_f^{-1}$$

$$f^{-1} = z_i^{-1} + z_\ell^{-1}$$

VR | 3D Visual Cues:

- Stereo: diff perspective views in L/R eyes; send 2 diff imgs
- Parallax (user motion): diff views as moving via head tracking coupled to perspective rendering
- Eyepiece Lens: Creates wide FoV while placing focal plane 'close to ∞'
  → Tricks eye into more relaxed state
- Accomodation: Changing in optical power of lens (eye) to focus at diff distances
- Vergence: Rotation of eye in its socket to ensure projection of obj centered on retina
- Conflict: Depth cues differ
  - Eyes accomodate to far distance to (ensure img on screen's focused)
  - Converge in attempt to fuse stereoscopic imgs of object up close

  ↳ Could be avoided w/ lightfield display
- Foveated Rendering: Track users' gaze and lower resolution further away from gazing pt
- Planar projection distorts high FoV, solved w/ lens-matched shading to render four quadrants/planes
- Judder: Eye moving to track moving obj, Img lags eye motion, smearing strobing effect

  ↳ Higher FPS / low-persistence display

Spectrometer: Determine emission spectra of R/G/B

- Turn on just R pixels
- Measure power at every wavelength
- Obtain S_R, repeat

Monochromator: Determine color matching fctn of display

- Color match displays w/ single wavelength light
- Repeat for all visible wavelengths giving $\bar{r}/\bar{g}/\bar{b}$ (row vectors repr. color match fctns)

Reprojection:

- Decouple high-qual (slow) rendering of frames from fast reprojection before displaying
- HQ img uses then-current head-tracking (possibly stale at render time)
- Reproj needs img + depth maps, possibly motion derivatives to warp to very latest head-track data

Light-Field camera:

4D light fields capture radiance along each ray