Exam

Friday, December 2, 2022 1:00 PM

1. Question (25 points)

Consider the demand for x_1, x_2 bundle in consumer against which x_1 and x_2 are perfect complements, where the ratio with the fixed quantity of one unit of x_1 to 2 units of x_2. Creating a good product bundle (x_1, x_2) which is sold. The price of the bundle is $p_1 + 2p_2$ and the consumer's income is m. Suppose the consumer's utility function is $u(x_1, x_2) = \min\{x_1, x_2\}$. Suppose the consumer has preferences $\succ$ on the set of consumption bundles $X = \mathbb{R}^2_+$ that are represented by the utility function u. Suppose the consumer's preferences satisfy the assumptions of the utility maximization problem. Suppose p_1, p_2, m are fixed. Show the consumer's demand function.

(a) Suppose the prices in a competitive market are given by setting the revenue of the publishing company from goods.

(b) Suppose the consumer's utility function is not as perfectly linear. Assume that each product consumer has consumed utility, where the total income of the consumer is m and the price of the bundle is $p_1 + 2p_2$.

(c) Identify the Nash equilibrium in your diagram and calculate it explicitly. What is the equilibrium price of the product bundle?

(d) Assume now that the consumer has utility of whether x_1 has two functions is negatively affected. $u(x_1, x_2) = \min\{x_1, x_2\}$. Assume the same budget constraint. Suppose the consumer's utility function is $u(x_1, x_2) = \min\{x_1, x_2\}$. Suppose the consumer's preferences satisfy the assumptions of the utility maximization problem. Suppose p_1, p_2, m are fixed. Show the consumer's demand function. Show your calculation. Make sure to show that the Nash equilibrium is unique. Suppose the consumer's utility function is $u(x_1, x_2) = \min\{x_1, x_2\}$. Suppose the consumer's preferences satisfy the assumptions of the utility maximization problem. Suppose p_1, p_2, m are fixed. Show the consumer's demand function.

(a) Proper utility

Consumer U: $\min\{x_1, 2x_2\}$

Type of consumer

$U(x_1, x_2) = \min\{x_1, 2x_2\}$

$\frac{\partial U}{\partial x_1} = \begin{cases} 1 & \text{if } x_1 < 2x_2 \\ 0 & \text{if } x_1 > 2x_2 \end{cases}$

$\frac{\partial U}{\partial x_2} = \begin{cases} 2 & \text{if } x_1 < 2x_2 \\ 0 & \text{if } x_1 > 2x_2 \end{cases}$

$\frac{\partial^2 U}{\partial x_1^2} = 0$

$\frac{\partial^2 U}{\partial x_2^2} = 0$

$\frac{\partial^2 U}{\partial x_1 \partial x_2} = 0$

Type in current U

$U_1(x_1) = \frac{\partial U}{\partial x_1} = 0$

$U_2(x_2) = \frac{\partial U}{\partial x_2} = 0$

$\frac{\partial^2 U}{\partial x_1^2} = 0$

$\frac{\partial^2 U}{\partial x_2^2} = 0$

$\frac{\partial^2 U}{\partial x_1 \partial x_2} = 0$

(b) Income than consumer

Income U: $\min\{\frac{x_1}{2}, x_2\}$ $\frac{\partial U}{\partial x_1} = \frac{1}{2}$ $\frac{\partial U}{\partial x_2} = 1$

$\frac{\partial^2 U}{\partial x_1^2} = 0$

$\frac{\partial^2 U}{\partial x_2^2} = 0$

$\frac{\partial^2 U}{\partial x_1 \partial x_2} = 0$

$\frac{\partial U}{\partial x_1} = \frac{1}{2}$

$\frac{\partial U}{\partial x_2} = 1$

Let $\frac{x_1}{2} = x_2$

$x_1 = 2x_2$

Budget constraint:

$U(x_1, x_2) = \frac{x_1}{2} + x_2$

Consumer income budget: $p_1 x_1 + p_2 x_2 = m$

$x_1 = \frac{m - p_2 x_2}{p_1}$

$U(x_2) = \frac{m - p_2 x_2}{2p_1} + x_2$

$\frac{\partial U}{\partial x_2} = -\frac{p_2}{2p_1} + 1$

$\frac{\partial U}{\partial x_2} = 0$

$-\frac{p_2}{2p_1} + 1 = 0$

$\frac{p_2}{2p_1} = 1$

$p_2 = 2p_1$

Equilibrium price is indifferent

(c) Consumer than equilibrium

$x_1 = \frac{m - p_2 x_2}{p_1}$

Substitution $U(x_2) = \frac{m - p_2 x_2}{2p_1} + x_2$

$\frac{\partial U}{\partial x_2} = -\frac{p_2}{2p_1} + 1$

Substitution $U(x_2) = \frac{m - p_2 x_2}{2p_1} + x_2$

$\frac{\partial U}{\partial x_2} = -\frac{(-p_2)(x_2)}{2p_1} + 1$

$x_2 = \frac{m}{p_2}$

$x_1 = \frac{m - p_2 \cdot \frac{m}{p_2}}{p_1}$

$x_1 = \frac{m - m}{p_1}$

$x_1 = 0$

$x_1 = 0$ because this is equivalent to income (break income is being shared)

Equilibrium are x_1 and x_2 $x_1 = 0$

$x_2 = \frac{m}{p_2}$

Consumer income budget per consumer U

$x_1 = \frac{m - p_2 x_2}{p_1}$

$x_1 = \frac{m - p_2 \cdot \frac{m}{p_2}}{p_1}$

$x_1 = \frac{m - m}{p_1}$

$x_1 = 0$

$x_1 = 0$ because this is equivalent to income (break income is being shared)

$x_2 = \frac{m}{p_2}$

Consumer income budget per consumer U

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$x_1 = \frac{m - m}{p_1}$

$x_1 = 0$

$x_1 = 0$

$x_2 = \frac{m}{p_2}$

(d) $U(x_1, x_2)$

$U(x_1, x_2)$

Proof Budget per consumer U

$U = \min\{x_1, x_2\}$

$U = \min\{x_1, 2x_2\}$

$U = \min\{x_1, x_2\}$

Proof $\frac{\partial U}{\partial x_1} = \frac{\partial U}{\partial x_2}$

$\frac{\partial U}{\partial x_1} = \frac{\partial U}{\partial x_2}$

$\frac{\partial U}{\partial x_1} = \frac{\partial U}{\partial x_2}$

Budget income U

$U = \min\{x_1, 2x_2\}$

$U = \min\{x_1, x_2\}$

$U = \min\{x_1, x_2\