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# Glass formation and crystallization in $\text{Li}_2\text{O}-\text{Na}_2\text{O}-\text{K}_2\text{O}-\text{SiO}_2$

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## Abstract

Glass-forming regions in the  $\text{Li}_2\text{O}-\text{Na}_2\text{O}-m\text{SiO}_2$  ( $m = 2$  or  $1.5$ ) and  $\text{Li}_2\text{O}-\text{Na}_2\text{O}-\text{K}_2\text{O}-m\text{SiO}_2$  ( $m = 2$  or  $1.5$ ) systems were determined with various cooling rates. Viscosity and liquidus temperature were measured. Eutectics were observed in these systems. The critical cooling rate shows a minimum and the liquidus viscosity shows a maximum. It was found that  $\text{Li}_2\text{O}-\text{Na}_2\text{O}-m\text{SiO}_2$  showed more extensive glass-forming ability than  $\text{Li}_2\text{O}-m\text{SiO}_2$ ,  $\text{Na}_2\text{O}-m\text{SiO}_2$  or  $\text{K}_2\text{O}-m\text{SiO}_2$ .  $\text{Li}_2\text{O}-\text{Na}_2\text{O}-\text{K}_2\text{O}-m\text{SiO}_2$  shows an even larger glass-forming ability than  $\text{Li}_2\text{O}-\text{Na}_2\text{O}-m\text{SiO}_2$ . The significant role of mixing entropy or liquidus viscosity in the glass-forming kinetics of mixed alkali silicate systems is recognized.

## 1. Introduction

The glass-forming tendency of a melt is related to the viscosity at the liquidus (liquidus viscosity). Ota and Kunugi proposed a glass-forming criterion of a multi-component system in terms of the liquidus viscosity [1] and showed qualitative agreement between the compositional dependence of the critical cooling rates and liquidus viscosities for  $\text{Li}_2\text{O}-\text{B}_2\text{O}_3$  [2],  $\text{Na}_2\text{O}-\text{B}_2\text{O}_3$  [3–6],  $\text{Li}_2\text{O}-\text{SiO}_2$  [7],  $\text{Na}_2\text{O}-\text{GeO}_2$  [8],  $\text{NaPO}_3-\text{MnCl}_2$  [9],  $\text{ZnCl}_2-\text{PbCl}_2-\text{KCl}$  [10] and  $\text{Bi}_2\text{O}_3-\text{CuO}-\text{BaO}$  [11]. Havermans et al. [12] reported the critical cooling rate for  $\text{Na}_2\text{O}-\text{K}_2\text{O}-\text{SiO}_2$  and single alkali systems,  $\text{Li}_2\text{O}-\text{SiO}_2$ ,  $\text{Na}_2\text{O}-\text{SiO}_2$  and  $\text{K}_2\text{O}-\text{SiO}_2$ . They showed that mixed alkali systems exhibit a lower critical cooling rate than either of the single alkali systems. Mazurin [13] interpreted the compositional dependency of the critical cooling

rate of the  $\text{M}_2\text{O}-\text{SiO}_2$  system ( $M = \text{Li}, \text{Na}$  and  $\text{K}$ ) by Havermans et al. [12] in term of the liquidus viscosity after the proposition of Ota and Kunugi [1]. Theoretically, the critical cooling rate,  $Q^*$ , is a function of the liquidus viscosity,  $\eta_L$ , the activation energy for viscous flow,  $E$ , and the fusion entropy  $\Delta S_f$  [6,14] etc. (See Eq. (9) of the Appendix). It is obvious from Eq. (7) in the Appendix that  $Q^*$  decreases, i.e., glass formation is favored, with increasing  $\eta_L$ ,  $E/T_L$ ,  $\Delta S_f$ , flow unit,  $V$ , and Fulcher's constant,  $T_0/T_L$ , and with decreasing  $T_L$ . It is expected that the eutectic composition is favored for glass formation because, even if the isothermal viscosity,  $\eta_T$ , and  $E$  change monotonically with composition,  $\eta_L$  and  $E/T_L$  terms will rise at the eutectic composition where  $T_L$  falls to a minimum.  $\Delta S_f$  may assume a maximum at the eutectic composition. It seems that the viscosity-related terms ( $\eta_L$  and  $E/T_L$ ) and the entropy-related terms play a decisive role in the glass-forming kinetics. In mixed alkali systems the mixing entropy of the alkali ions will contribute to increase the fusion entropy. The increase of  $\Delta S_f$ ,

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consequently, will lower the liquidus temperature. Thus enhanced glass formation is foreseen in the mixed alkali systems. In the present study, the glass-forming tendency of mixed alkali silicate,  $(1-x)\text{Li}_2\text{O}-(x)\text{Na}_2\text{O}-m\text{SiO}_2$  ( $m = 2$  or  $1.5$ ) and  $(1-y)(0.4\text{Li}_2\text{O} \cdot 0.6\text{Na}_2\text{O})-(y)\text{K}_2\text{O}-m\text{SiO}_2$  ( $m = 2$  or  $1.5$ ) was studied.

## 2. Experimental

Glass samples for critical cooling rate determination were prepared by melt quenching using reagent-grade alkali carbonate ( $\text{M}_2\text{CO}_3$ ;  $\text{M} = \text{Li}, \text{Na}$  and  $\text{K}$ ) and quartz powder ( $\text{SiO}_2$ ). The composition was  $x = 0-1.0$  in the series  $(1-x)\text{Li}_2\text{O}-(x)\text{Na}_2\text{O}-m\text{SiO}_2$  ( $m = 2$  or  $1.5$ ) and  $y = 0-1.0$  in the series  $(1-y)(0.4\text{Li}_2\text{O} \cdot 0.6\text{Na}_2\text{O})-(y)\text{K}_2\text{O}-m\text{SiO}_2$  ( $m = 2$  or  $1.5$ ). The  $0.4\text{Li}_2\text{O} \cdot 0.6\text{Na}_2\text{O} \cdot 2\text{SiO}_2$  composition is a eutectic composition of the  $\text{Li}_2\text{O}-\text{Na}_2\text{O}-2\text{SiO}_2$  system (see Fig. 3(a) below). Batches were melted in a platinum crucible at  $1300-1550^\circ\text{C}$  for 30–60 min, and the molten glass was cast onto an iron plate. 10 g of glass in a Pt crucible was cooled to below  $T_g$  at various cooling rates,  $Q_1-Q_{11}$ , listed in Table 1.  $Q$  is the mean cooling rate between the liquidus temperature and  $T_g$ .  $Q$  ranged from  $1.6 \times 10^{-4}$  K/s ( $Q_1$ ) by programmed furnace cooling to 59 K/s ( $Q_{11}$ ) by plate press cooling. Table 1 shows

that 69 days are required to cool the  $1000^\circ\text{C}$  temperature gap with  $Q_1$  and only 17 s with  $Q_{11}$ .

Possible devitrification was inspected on the surface as well as in the interior of the quenched specimen with optical microscopy. The devitrified phases were identified by X-ray diffraction (XRD) with a polarizing microscope.

$T_L$  was determined by the quenching method, whereby 100 mg of powdered glass sample, wrapped in a platinum foil was kept in an oven at various temperatures for at least 100–200 h. After being dropped into a carbon tetrachloride bath, the solidified specimen was checked for devitrification by optical microscopy.

The viscosity of the molten glass was measured in the temperature range of  $370-1400^\circ\text{C}$ . Three types of viscometer were employed according to the viscosity range: a counter balancing viscometer in the range  $\eta = 0.1-10^3$  Pa s; a penetration viscometer [15] in the range  $\eta = 10^3-10^5$  Pa s; and a beam-bending viscometer [15] in the range  $\eta = 10^5-10^{12}$  Pa s.

$T_g$  was determined from dilatometry with a heating rate of 3 K/min.  $T_g$  is regarded as the temperature at which the viscosity, reaches  $10^{12}$  Pa s.

## 3. Results

The variation of glass-forming regions with cooling rates,  $Q_1-Q_{11}$ , is illustrated in Fig. 1(a) for the  $\text{Li}_2\text{O}-\text{Na}_2\text{O}-\text{SiO}_2$  system, and Fig. 1(b) for the  $0.4\text{Li}_2\text{O} \cdot 0.6\text{Na}_2\text{O}-\text{K}_2\text{O}-2\text{SiO}_2$  system. The solid line indicates the critical cooling rate,  $Q^*$ , for glass formation. Results are summarized in Table 2(a)–(d).

In the  $(1-x)\text{Li}_2\text{O}-(x)\text{Na}_2\text{O}-2\text{SiO}_2$  series (Fig. 1(a)),  $Q^*$  decreases from 1 K/s at the composition  $x = 0$  to a minimum  $5 \times 10^{-4}$  K/s at the composition  $x = 0.8-0.9$ , and then increases to  $1.7 \times 10^{-3}$  K/s at the composition  $x = 1.0$ . In the  $(1-y)(0.4\text{Li}_2\text{O} \cdot 0.6\text{Na}_2\text{O})-(y)\text{K}_2\text{O}-2\text{SiO}_2$  series (Fig. 1(b)), most of the compositions do not show crystallization with the slowest cooling rate,  $Q_1$ , except at the compositions  $y = 0$  and  $0.9-1.0$ .  $Q^*$  is  $1.2 \times 10^{-3}$  K/s in the  $y = 0-0.1$  range, and increases from  $3.2 \times 10^{-4}$  K/s at the composition  $y = 0.9$  to  $1.6 \times 10^{-3}$  K/s at the composition  $y = 1.0$ . Figs. 2(a) and (b) indicate the glass-forming regions and

Table 1

Cooling methods and mean-cooling rates,  $Q_1-Q_{11}$ , employed to determine the critical cooling rate for glass formation; 10 g melt in a platinum crucible of 20 cm<sup>3</sup> capacity was used for each cooling

	Method	Cooling rate, $\log(Q/\text{K s}^{-1})$	Time to cool (1000°C)
$Q_1$	Programmed cool	−3.78	69 days
$Q_2$	Programmed cool	−3.48	35 days
$Q_3$	Programmed cool	−3.08	14 days
$Q_4$	Programmed cool	−2.60	5 days
$Q_5$	Programmed cool	−2.38	3 days
$Q_6$	Programmed cool	−2.00	1 day
$Q_7$	Programmed cool	−1.48	8 h
$Q_8$	Furnace cool	−1.09	3 h
$Q_9$	Crucible air cool	0.11	13 min
$Q_{10}$	Air cool on iron plate	1.20	1 min
$Q_{11}$	Crucible water cool	1.77	17 s

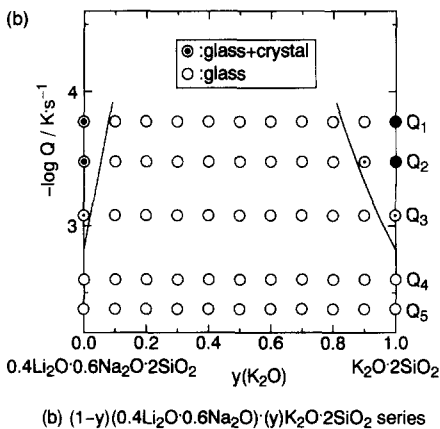
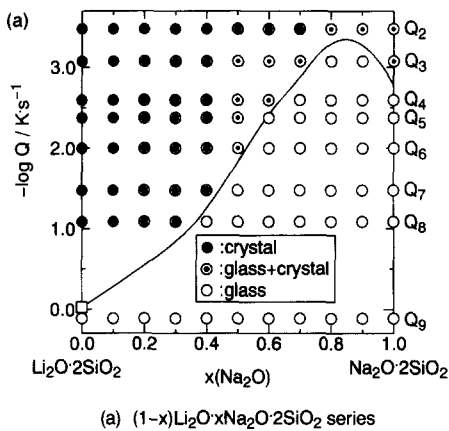


Fig. 1. Glass-forming regions at various cooling rates and critical cooling rate (solid line) in the  $(1-x)\text{Li}_2\text{O}-(x)\text{Na}_2\text{O}-2\text{SiO}_2$  series (a) and  $(1-y)(0.4\text{Li}_2\text{O} \cdot 0.6\text{Na}_2\text{O})-(y)\text{K}_2\text{O}-2\text{SiO}_2$  series (b). Open circles (○) are glass, closed circles (●) crystal, and double circles (⊙) partially crystallized glass. Inner portion indicates the crystallized portion. Square (□) indicates data by Ota et al. [7] for the critical cooling rate in the  $\text{Li}_2\text{O} \cdot 2\text{SiO}_2$  composition.

critical cooling rate for the  $\text{Li}_2\text{O}-\text{Na}_2\text{O}-1.5\text{SiO}_2$  and  $0.4\text{Li}_2\text{O} \cdot 0.6\text{Na}_2\text{O}-\text{K}_2\text{O}-1.5\text{SiO}_2$  systems. Fig. 2(a) indicates that  $Q^*$  decreases from  $4.6 \times 10^1$  K/s at  $x = 0$  to a minimum of  $1 \times 10^{-2}$  K/s at the composition  $x = 0.8-0.9$ , and then increases to  $5.2 \times 10^{-2}$  K/s at  $x = 1.0$ . In Fig. 2(b)  $Q^*$  is  $5.2 \times 10^{-2}$  K/s at  $y = 0$  and is  $5.1 \times 10^{-2}$  K/s at  $y = 1.0$ .  $Q^*$  becomes less than  $10^{-4}$  K/s at the composition  $y = 0.4-0.6$ .

The variation of liquidus temperature,  $T_L$ , in the  $\text{Li}_2\text{O}-\text{Na}_2\text{O}-\text{K}_2\text{O}-m\text{SiO}_2$  system, ( $m = 2$  or  $1.5$ ), is listed in Table 2(a)–(d) and shown in Figs. 3 and

4. The maximum error in the measurements is  $\pm 5^\circ\text{C}$ . Fig. 3(a) indicates that  $T_L$  decreases from  $1040^\circ\text{C}$  at the composition  $x = 0$  to an apparent eutectic  $755^\circ\text{C}$  at the composition  $x = 0.6$ , and then increases to  $860^\circ\text{C}$  at  $x = 1.0$ . The phase diagrams of the  $\text{Li}_2\text{O} \cdot 2\text{SiO}_2-\text{Na}_2\text{O} \cdot 2\text{SiO}_2$  system by Kracek [16] and West [17] show eutectics at  $x = 0.7$  and  $x = 0.65$ ,

Table 2

Critical cooling rate,  $Q^*$  ( $\text{K s}^{-1}$ ), liquidus temperature,  $T_L$  ( $^\circ\text{C}$ ), glass transition temperature,  $T_g$  ( $^\circ\text{C}$ ) and liquidus viscosity,  $\eta_L$  (Pas) of the  $(1-x)\text{Li}_2\text{O}-(x)\text{Na}_2\text{O}-2\text{SiO}_2$  series (a),  $(1-y)(0.4\text{Li}_2\text{O} \cdot 0.6\text{Na}_2\text{O})-(y)\text{K}_2\text{O}-2\text{SiO}_2$  series (b),  $(1-x)\text{Li}_2\text{O}-(x)\text{Na}_2\text{O}-1.5\text{SiO}_2$  series (c) and  $(1-y)(0.4\text{Li}_2\text{O} \cdot 0.6\text{Na}_2\text{O})-(y)\text{K}_2\text{O}-1.5\text{SiO}_2$  series (d)

$x, y$	$-\log(Q^* / \text{K s}^{-1})$	$T_L$ ( $^\circ\text{C}$ )	$T_g$ ( $^\circ\text{C}$ )	$\log(\eta_L / \text{Pa s})$
(a) $(1-x)\text{Li}_2\text{O}-(x)\text{Na}_2\text{O}-2\text{SiO}_2$ series				
0.0	0.00	1043	437	1.34
0.1	0.86	1013	413	1.37
0.2	1.00	985	405	1.48
0.3	1.24	937	390	1.63
0.4	2.09	885	387	1.89
0.5	2.50	822	383	2.21
0.6	2.91	744	383	3.05
0.7	3.30	778	382	3.02
0.8	3.28	808	384	2.73
0.9	2.94	836	401	2.77
1.0	2.76	862	435	2.83
(b) $(1-y)(0.4\text{Li}_2\text{O} \cdot 0.6\text{Na}_2\text{O})-(y)\text{K}_2\text{O}-2\text{SiO}_2$ series				
0.0	2.91	744	383	3.05
0.1	—	718	375	3.35
0.2	—	674	380	4.03
0.3	—	648	380	4.38
0.4	—	643	380	4.53
0.5	—	650	385	4.56
0.6	—	732	384	3.87
0.7	—	825	398	3.27
0.8	—	881	386	2.99
0.9	3.47	935	430	2.78
1.0	2.89	1036 <sup>a</sup>	484	2.31
(c) $(1-x)\text{Li}_2\text{O}-(x)\text{Na}_2\text{O}-1.5\text{SiO}_2$ series				
0.0	−1.66	926	400	0.48
0.1	−1.52	861	373	0.60
0.2	−1.30	804	353	0.77
0.3	−0.97	725	354	0.86
0.4	−0.47	806	355	1.15
0.5	0.23	899	367	1.60
0.6	1.28	965	360	2.23
0.7	2.00	1022	374	3.11
0.8	2.00	1074	378	2.55
0.9	2.03	1112	393	2.28
1.0	1.28	1144	421	2.07

Table 2 (continued)

$x, y$	$-\log(Q^* / \text{K s}^{-1})$	$T_L$ (°C)	$T_g$ (°C)	$\log(\eta_L / \text{Pa s})$
(d) $(1-y)(0.4\text{Li}_2\text{O} \cdot 0.6\text{Na}_2\text{O})-(y)\text{K}_2\text{O}-1.5\text{SiO}_2$				
0.0	1.28	806	355	2.23
0.1	1.86	772	332	2.47
0.2	2.52	742	325	2.76
0.3	3.40	704	323	3.25
0.4	—	639	322	3.99
0.5	—	705	342	3.35
0.6	—	803	342	2.69
0.7	3.42	866	351	2.43
0.8	3.14	913	375	2.20
0.9	2.15	955	399	2.22
1.0	1.29	996	460	2.25

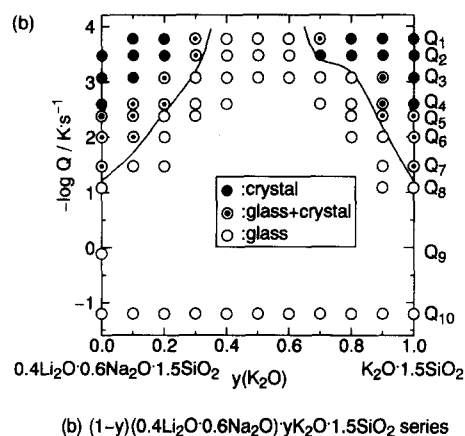
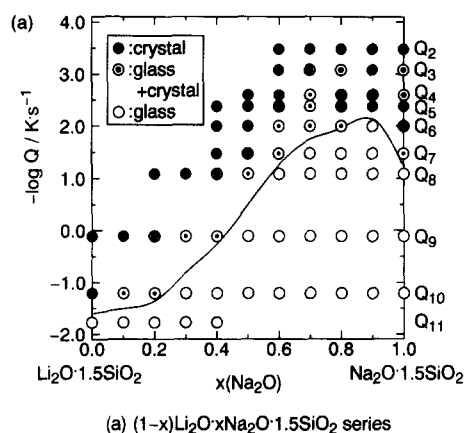


Fig. 2. Glass-forming regions and critical cooling rate (solid line) in the  $(1-x)\text{Li}_2\text{O}-(x)\text{Na}_2\text{O}-1.5\text{SiO}_2$  series (a) and  $(1-y)(0.4\text{Li}_2\text{O}-0.6\text{Na}_2\text{O})-(y)\text{K}_2\text{O}-1.5\text{SiO}_2$  series (b). Open circles (○) denote glass, closed circles (●) crystal, and double circles (⊙) partially crystallized glass. Inner portion of the double circles indicates the crystallized portion.

respectively. Fig. 3(b) shows that there is a quasi-eutectic  $650^\circ\text{C}$  at the composition  $y = 0.4-0.5$ . Fig. 4(a) shows that  $T_L$  decreases from  $1140^\circ\text{C}$  at the composition  $x = 0$  to eutectic of  $700^\circ\text{C}$  at the composition  $x = 0.7$ , and then increases to  $920^\circ\text{C}$  at the composition  $x = 1.0$ . Fig. 4(b) shows that  $T_L$  decreases from  $800^\circ\text{C}$  at  $y = 0$  to a quasi-eutectic  $620^\circ\text{C}$  at the composition  $y = 0.4-0.5$ , and increases to  $700^\circ\text{C}$  at the composition  $y = 1.0$ .

Viscosity isotherms of the  $\text{Li}_2\text{O}-\text{Na}_2\text{O}-\text{K}_2\text{O}-m\text{SiO}_2$  ( $m = 2$  or  $1.5$ ) systems are summarized in a temperature range of  $370-1300^\circ\text{C}$  in Table 3(a)–(b)

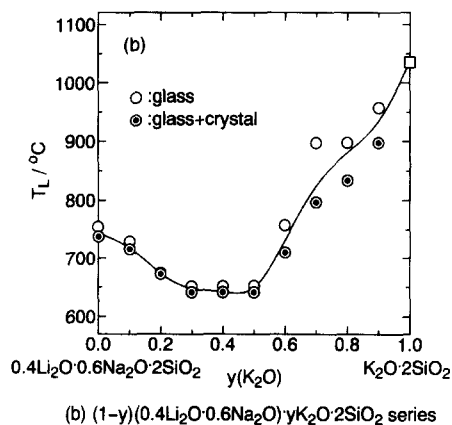
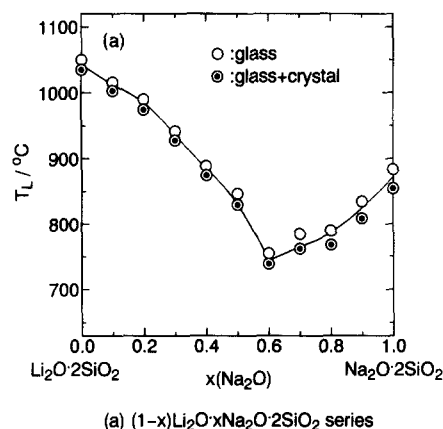


Fig. 3. Liquidus temperatures (solid line) of the  $(1-x)\text{Li}_2\text{O}-(x)\text{Na}_2\text{O}-2\text{SiO}_2$  series (a) and  $(1-y)(0.4\text{Li}_2\text{O}-0.6\text{Na}_2\text{O})-(y)\text{K}_2\text{O}-2\text{SiO}_2$  series (b). Open circles (○) denote glass and double circles (⊙) partially crystallized glass. Square (□) indicates the data of Kracek [16].

Table 3

Viscosity isotherms of the  $\text{Li}_2\text{O}-\text{Na}_2\text{O}-\text{K}_2\text{O}-m\text{SiO}_2$  glass ( $m = 2$  or  $1.5$ )(a)  $(1-x)\text{Li}_2\text{O}-(x)\text{Na}_2\text{O}-2\text{SiO}_2$  series [ $\log(\eta_L/\text{Pa s})$ ]

$x$ :	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$T(^{\circ}\text{C})$											
380	—	—	—	—	—	—	11.81	—	—	—	—
400	—	—	—	11.89	11.67	11.53	11.32	11.38	11.60	12.05	—
420	—	11.94	11.68	11.46	10.54	10.46	10.13	10.33	10.64	11.64	—
440	11.91	11.26	10.64	10.35	9.36	9.24	9.10 <sup>a</sup>	9.26 <sup>a</sup>	9.42	10.58	11.78
460	11.35	10.26	9.57	9.40 <sup>a</sup>	8.49 <sup>a</sup>	8.43 <sup>a</sup>	8.40 <sup>a</sup>	8.51 <sup>a</sup>	8.68 <sup>a</sup>	9.32	11.35
480	10.38	9.34 <sup>a</sup>	8.79 <sup>a</sup>	8.73 <sup>a</sup>	7.73 <sup>a</sup>	7.74 <sup>a</sup>	7.78 <sup>a</sup>	7.85 <sup>a</sup>	8.00 <sup>a</sup>	8.62 <sup>a</sup>	10.27
500	9.35 <sup>a</sup>	8.66 <sup>a</sup>	8.11 <sup>a</sup>	8.11 <sup>a</sup>	7.07 <sup>a</sup>	7.06	7.03	7.46	7.20	8.00 <sup>a</sup>	9.46 <sup>a</sup>
550	7.75 <sup>a</sup>	7.21 <sup>a</sup>	6.70 <sup>a</sup>	6.79 <sup>a</sup>	5.76 <sup>a</sup>	5.56	5.80	6.14	5.88	6.70 <sup>a</sup>	8.08 <sup>a</sup>
600	6.50 <sup>a</sup>	6.05 <sup>a</sup>	5.59 <sup>a</sup>	5.71 <sup>a</sup>	4.78 <sup>a</sup>	4.86 <sup>a</sup>	4.84	5.10 <sup>a</sup>	5.22 <sup>a</sup>	5.68 <sup>a</sup>	6.94 <sup>a</sup>
700	4.64 <sup>a</sup>	4.32 <sup>a</sup>	3.98 <sup>a</sup>	4.04 <sup>a</sup>	3.41 <sup>a</sup>	3.42 <sup>a</sup>	3.56	3.73 <sup>a</sup>	3.83 <sup>a</sup>	4.18 <sup>a</sup>	5.17 <sup>a</sup>
800	3.33 <sup>a</sup>	3.08 <sup>a</sup>	2.85 <sup>a</sup>	2.81 <sup>a</sup>	2.49 <sup>a</sup>	2.42 <sup>a</sup>	2.69 <sup>a</sup>	2.78 <sup>a</sup>	2.86 <sup>a</sup>	3.12 <sup>a</sup>	3.86 <sup>a</sup>
900	2.36 <sup>a</sup>	2.15 <sup>a</sup>	2.02 <sup>a</sup>	1.88	1.84	1.77	1.94	2.08 <sup>a</sup>	2.15 <sup>a</sup>	2.35 <sup>a</sup>	2.86 <sup>a</sup>
1000	1.61 <sup>a</sup>	1.45	1.39	1.39	1.37	1.35	1.48	1.54	1.61	1.78	2.06
1100	1.11	1.01	0.98	0.95	0.98	0.99	1.08	1.13	1.21	1.33	1.60
1200	0.76	0.65	0.65	0.61	0.66	0.69	0.73	0.78	0.84	0.95	1.20
1300	0.47	0.37	0.38	0.30	0.39	0.43	0.44	0.45	0.55	0.65	0.85

(b)  $(1-y)(0.4\text{Li}_2\text{O} \cdot 0.6\text{Na}_2\text{O})-(y)\text{K}_2\text{O}-2\text{SiO}_2$  series [ $\log(\eta_L/\text{Pa s})$ ]

$y$ :	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$T(^{\circ}\text{C})$											
370	—	—	12.02	11.98	11.96	—	—	—	—	—	—
380	11.81	11.83	11.82	11.75	11.81	12.04	—	—	—	—	—
400	11.32	11.02	10.80	10.92	11.02	11.47	11.79	12.06	12.00	—	—
420	10.13	9.94	9.95	9.87	9.97	10.52	11.01	11.56	11.64	—	—
440	9.10 <sup>a</sup>	8.82 <sup>a</sup>	8.89 <sup>a</sup>	8.74 <sup>a</sup>	8.83 <sup>a</sup>	9.48	10.04	10.62	11.17	11.84	—
460	8.40 <sup>a</sup>	8.09 <sup>a</sup>	8.22 <sup>a</sup>	8.07 <sup>a</sup>	8.17 <sup>a</sup>	8.80 <sup>a</sup>	9.20 <sup>a</sup>	9.66	10.20	11.37	12.04
480	7.78 <sup>a</sup>	7.44 <sup>a</sup>	7.63 <sup>a</sup>	7.47 <sup>a</sup>	7.58 <sup>a</sup>	8.20 <sup>a</sup>	8.58 <sup>a</sup>	9.02 <sup>a</sup>	9.41 <sup>a</sup>	10.46	11.86
500	7.22 <sup>a</sup>	6.87 <sup>a</sup>	7.10 <sup>a</sup>	6.94 <sup>a</sup>	7.05 <sup>a</sup>	7.65 <sup>a</sup>	8.02 <sup>a</sup>	8.43 <sup>a</sup>	8.88 <sup>a</sup>	9.32	11.53
550	6.04 <sup>a</sup>	5.71 <sup>a</sup>	5.97 <sup>a</sup>	5.83 <sup>a</sup>	5.95 <sup>a</sup>	6.48 <sup>a</sup>	6.82 <sup>a</sup>	7.19 <sup>a</sup>	7.70 <sup>a</sup>	7.91 <sup>a</sup>	9.57
600	5.09 <sup>a</sup>	4.82 <sup>a</sup>	5.08 <sup>a</sup>	4.96 <sup>a</sup>	5.08 <sup>a</sup>	5.54 <sup>a</sup>	5.85 <sup>a</sup>	6.18 <sup>a</sup>	6.71 <sup>a</sup>	6.77 <sup>a</sup>	8.43 <sup>a</sup>
700	3.69 <sup>a</sup>	3.52 <sup>a</sup>	3.75 <sup>a</sup>	3.68 <sup>a</sup>	3.78 <sup>a</sup>	4.11 <sup>a</sup>	4.37 <sup>a</sup>	4.65 <sup>a</sup>	5.12 <sup>a</sup>	5.08 <sup>a</sup>	6.55 <sup>a</sup>
800	2.69 <sup>a</sup>	2.64 <sup>a</sup>	2.80 <sup>a</sup>	2.78 <sup>a</sup>	2.86 <sup>a</sup>	3.07 <sup>a</sup>	3.29 <sup>a</sup>	3.53 <sup>a</sup>	3.90 <sup>a</sup>	3.87 <sup>a</sup>	5.04 <sup>a</sup>
900	1.94	1.99	2.09	2.12 <sup>a</sup>	2.18	2.29 <sup>a</sup>	2.48 <sup>a</sup>	2.69 <sup>a</sup>	2.94 <sup>a</sup>	2.97 <sup>a</sup>	3.81 <sup>a</sup>
1000	1.48	1.54	1.60	1.62	1.68	1.71	1.88	2.03 <sup>a</sup>	2.16 <sup>a</sup>	2.27 <sup>a</sup>	2.78 <sup>a</sup>
1100	1.08	1.15	1.20	1.22	1.27	1.30	1.42	1.52	1.59	1.72	1.92
1200	0.73	0.79	0.84	0.86	0.90	0.95	1.03	1.09	1.16	1.27	1.41
1300	0.44	0.47	0.52	0.54	0.57	0.62	0.68	0.72	0.79	0.87	0.96

(c)  $(1-x)\text{Li}_2\text{O}-(x)\text{Na}_2\text{O}-1.5\text{SiO}_2$  series [ $\log(\eta_L/\text{Pa s})$ ]

$x$ :	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$T(^{\circ}\text{C})$											
600	5.39 <sup>a</sup>	4.83 <sup>a</sup>	4.83 <sup>a</sup>	4.34 <sup>a</sup>	4.22 <sup>a</sup>	4.52 <sup>a</sup>	4.49 <sup>a</sup>	4.70 <sup>a</sup>	4.79 <sup>a</sup>	5.26 <sup>a</sup>	5.81 <sup>a</sup>
700	3.71 <sup>a</sup>	3.37 <sup>a</sup>	3.41 <sup>a</sup>	3.03 <sup>a</sup>	2.97 <sup>a</sup>	3.20 <sup>a</sup>	3.22 <sup>a</sup>	3.40 <sup>a</sup>	3.50 <sup>a</sup>	3.84 <sup>a</sup>	4.24 <sup>a</sup>
800	2.57 <sup>a</sup>	2.37 <sup>a</sup>	2.42 <sup>a</sup>	2.13 <sup>a</sup>	2.11 <sup>a</sup>	2.28 <sup>a</sup>	2.32	2.47	2.57 <sup>a</sup>	2.81 <sup>a</sup>	3.13 <sup>a</sup>
900	1.76 <sup>a</sup>	1.64 <sup>a</sup>	1.68 <sup>a</sup>	1.47 <sup>a</sup>	1.47 <sup>a</sup>	1.59	1.64	1.77	1.86	2.03	2.29
1000	1.14 <sup>a</sup>	1.09 <sup>a</sup>	1.11 <sup>a</sup>	0.97	0.98	1.06	1.12	1.22	1.31	1.42	1.63
1100	0.66 <sup>a</sup>	0.65	0.66	0.58	0.60	0.65	0.70	0.78	0.86	0.93	1.11
1200	0.27	0.30	0.30	0.26	0.29	0.31	0.36	0.42	0.50	0.53	0.68
1300	−0.04	0.01	−0.01	0.01	0.03	0.03	0.08	0.12	0.19	0.19	0.32

Table 3 (continued)

(d) $(1-y)(0.4\text{Li}_2\text{O} \cdot 0.6\text{Na}_2\text{O})-(y)\text{K}_2\text{O}-1.5\text{SiO}_2$ series [ $\log(\eta_L/\text{Pa s})$ ]											
$y$ :	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$T(^{\circ}\text{C})$											
600	4.49 <sup>a</sup>	4.35 <sup>a</sup>	4.32 <sup>a</sup>	4.46 <sup>a</sup>	4.45 <sup>a</sup>	4.67 <sup>a</sup>	4.91 <sup>a</sup>	5.30 <sup>a</sup>	5.68 <sup>a</sup>	6.31 <sup>a</sup>	7.59 <sup>a</sup>
700	3.22 <sup>a</sup>	3.18 <sup>a</sup>	3.18 <sup>a</sup>	3.30 <sup>a</sup>	3.29 <sup>a</sup>	3.42 <sup>a</sup>	3.66 <sup>a</sup>	3.96 <sup>a</sup>	4.23 <sup>a</sup>	4.74 <sup>a</sup>	5.62 <sup>a</sup>
800	2.32 <sup>a</sup>	2.34 <sup>a</sup>	2.35 <sup>a</sup>	2.44 <sup>a</sup>	2.44 <sup>a</sup>	2.52 <sup>a</sup>	2.75 <sup>a</sup>	2.97 <sup>a</sup>	3.16 <sup>a</sup>	3.57 <sup>a</sup>	4.19 <sup>a</sup>
900	1.64	1.70	1.72	1.79	1.79	1.83	2.05	2.21	2.34	2.67 <sup>a</sup>	3.09 <sup>a</sup>
1000	1.12	1.19	1.23	1.27	1.28	1.29	1.50	1.60	1.68	1.95	2.23
1100	0.70	0.79	0.83	0.85	0.86	0.85	1.05	1.11	1.15	1.37	1.54
1200	0.36	0.46	0.50	0.50	0.51	0.49	0.68	0.70	0.71	0.88	0.97
1300	0.08	0.18	0.22	0.21	0.22	0.19	0.37	0.36	0.34	0.47	0.49

<sup>a</sup> Interpolated values calculated from  $T_g$  and high-temperature viscosity values of the  $(1-x)\text{Li}_2\text{O}-(x)\text{Na}_2\text{O}-2\text{SiO}_2$  series (a),  $(1-y)(0.4\text{Li}_2\text{O} \cdot 0.6\text{Na}_2\text{O})-(y)\text{K}_2\text{O}-2\text{SiO}_2$  series (b),  $(1-x)\text{Li}_2\text{O}-(x)\text{Na}_2\text{O}-1.5\text{SiO}_2$  series (c) and  $(1-y)(0.4\text{Li}_2\text{O} \cdot 0.6\text{Na}_2\text{O})-(y)\text{K}_2\text{O}-1.5\text{SiO}_2$  series (d).

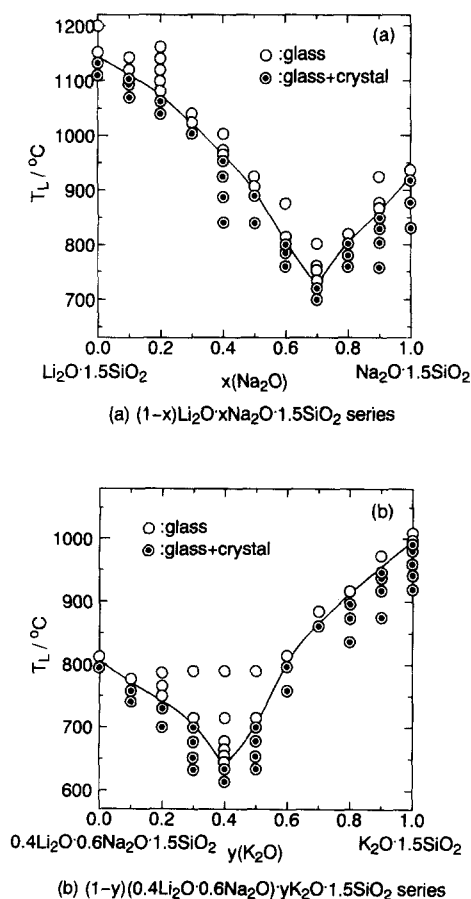


Fig. 4. Liquidus temperature (solid line) of the  $(1-x)\text{Li}_2\text{O}-(x)\text{Na}_2\text{O}-1.5\text{SiO}_2$  series (a) and  $(1-y)(0.4\text{Li}_2\text{O} \cdot 0.6\text{Na}_2\text{O})-(y)\text{K}_2\text{O}-1.5\text{SiO}_2$  series (b). Open circles (○) denote glass and double circles (⊙) partially crystallized glass.

and in a range of 600–1300°C in Table 3(c)–(d). Interpolated viscosity values at 440–1000°C were estimated by the Fulcher type viscosity equation  $\log \eta_L = A + B/(T - T_0)$ , using  $T_g$  and high-temperature viscosity data. Fig. 5(a) exhibits a shallow viscosity minimum at middle compositions around  $x = 0.5$  in the 1000–1300°C range, and a deep minimum in the 420–600°C range. Fig. 5(b) shows that the isothermal viscosity increases progressively with increase of  $\text{K}_2\text{O}$  content in the 900–1300°C range, but a viscosity minimum appears in the 400–600°C range. Figs. 6(a) and (b) show that there is a viscosity minimum in the 600–900°C range, but the minimum disappears in the 1200–1300°C range.

The variations of  $T_g$  with composition in the  $\text{Li}_2\text{O}-\text{Na}_2\text{O}-\text{K}_2\text{O}-m\text{SiO}_2$  systems ( $m = 1.5$  or  $2$ ) are summarized in Table 2(a)–(d) and shown in Figs. 7(a) and (b). Fig. 7(a) shows that  $T_g$  becomes minimum at the composition  $x = 0.7-0.8$  for  $m = 2$ , and the composition  $x = 0.2-0.3$  for  $m = 1.5$ . Fig. 7(b) shows that  $T_g$  increases continuously with an increase of  $\text{K}_2\text{O}$  in the  $(1-y)(0.4\text{Li}_2\text{O} \cdot 0.6\text{Na}_2\text{O})-(y)\text{K}_2\text{O}-2\text{SiO}_2$  series, and becomes a minimum at the composition  $y = 0.2-0.4$  composition in the  $(1-y)(0.4\text{Li}_2\text{O} \cdot 0.6\text{Na}_2\text{O})-(y)\text{K}_2\text{O}-1.5\text{SiO}_2$  series.

Liquidus viscosities,  $\eta_L$ , were obtained from  $T_L$  and the Fulcher type viscosity–temperature curve.  $\eta_L$  values are summarized in Table 2(a)–(d). The variation of  $\eta_L$  of the  $\text{Li}_2\text{O}-\text{Na}_2\text{O}-\text{K}_2\text{O}-2\text{SiO}_2$  and the  $\text{Li}_2\text{O}-\text{Na}_2\text{O}-\text{K}_2\text{O}-1.5\text{SiO}_2$  systems are shown in Figs. 8(a), (b) and 9(a), (b). Fig. 8(a) shows a  $\eta_L$  maximum,  $\log(\eta_L/\text{Pa s}) = 3.2$  at the

eutectic composition  $x = 0.6$ . Fig. 8(b) shows a  $\eta_L$  maximum,  $\log(\eta_L/\text{Pa s}) = 4.56$  at the eutectic composition  $y = 0.5$ . Fig. 9(a) shows a  $\eta_L$  maximum,

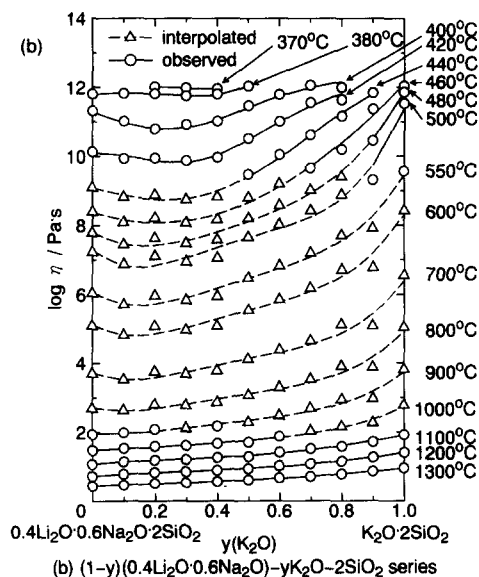
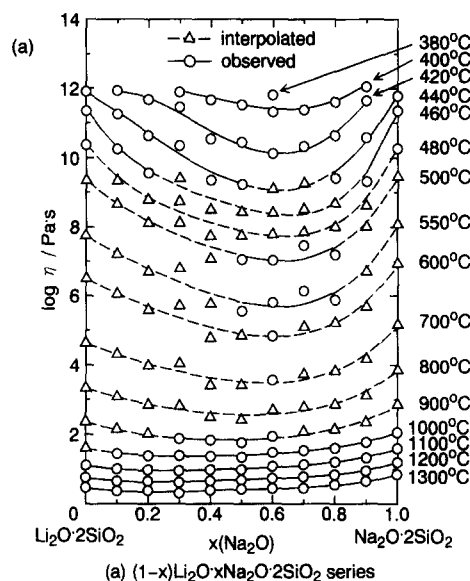


Fig. 5. Isothermal viscosities in the temperature range 370–1300°C of the  $(1-x)\text{Li}_2\text{O} \cdot (x)\text{Na}_2\text{O} \cdot 2\text{SiO}_2$  series (a) and  $(1-y)(0.4\text{Li}_2\text{O} \cdot 0.6\text{Na}_2\text{O}) \cdot (y)\text{K}_2\text{O} \cdot 2\text{SiO}_2$  series (b). Open circles (○) and solid line indicate the observed values, and triangles (Δ) and broken line interpolated values from  $T_g$  and high-temperature viscosity values.

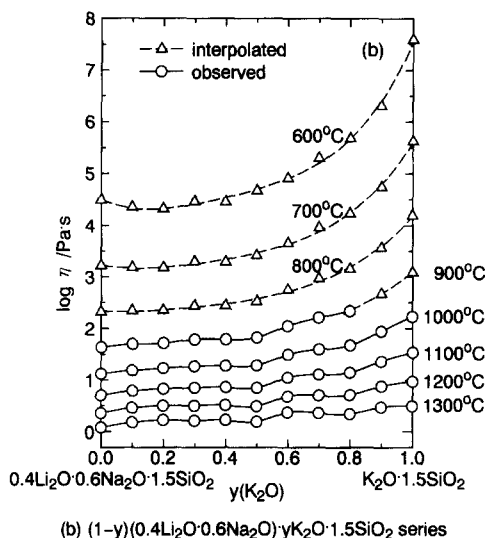
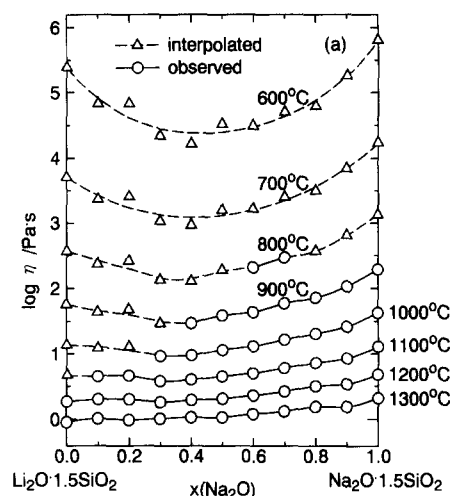


Fig. 6. Isothermal viscosities in the temperature range 600–1300°C of the  $(1-x)\text{Li}_2\text{O} \cdot (x)\text{Na}_2\text{O} \cdot 1.5\text{SiO}_2$  series (a) and  $(1-y)(0.4\text{Li}_2\text{O} \cdot 0.6\text{Na}_2\text{O}) \cdot (y)\text{K}_2\text{O} \cdot 1.5\text{SiO}_2$  series (b). Open circles (○) and solid line indicate the observed values, and triangles (Δ) and broken line interpolated values from  $T_g$  and high-temperature viscosity values.

$\log(\eta_L/\text{Pa s}) = 3.2$ , at the eutectic composition  $x = 0.7$ . Fig. 9(b) shows a  $\eta_L$  maximum,  $\log(\eta_L/\text{Pa s}) = 4.0$ , at the eutectic composition  $y = 0.4$ .

$\log \eta_L$  values are plotted against  $\log Q^*$  values in Fig. 10. A linear relationship is observed with a

slope of approximately  $d \log \eta_L / d \log Q^* = -0.5$ . This is consistent with the general relationship, that is, a greater liquidus viscosity corresponds to a smaller critical cooling rate and vice versa. The broken line indicates a slope  $-0.5$ , and the  $\eta_L$ - $Q^*$  relationship is expressed as

$$\log \eta_L = -0.5 \log Q^* + 1.5 \pm 0.5.$$

#### 4. Discussion

The fusion entropy,  $\Delta S_f$ , consists of the configurational entropy,  $\Delta S_{\text{conf}}$ , and the thermal entropy,  $\Delta S_{\text{th}}$ .  $\Delta S_{\text{conf}}$  includes the mixing entropy  $\Delta S_m$  which

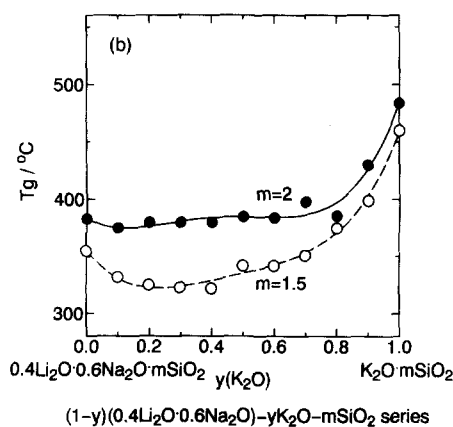
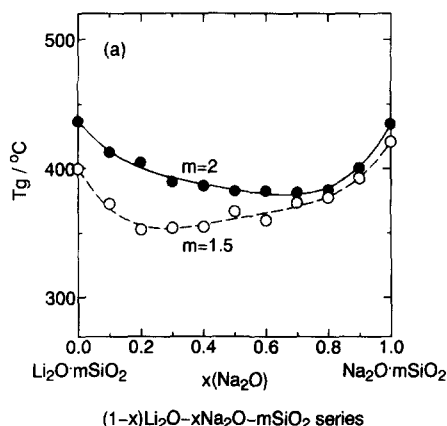


Fig. 7. Glass transition temperature,  $T_g$ , of the  $(1-x)\text{Li}_2\text{O}-(x)\text{Na}_2\text{O}-m\text{SiO}_2$  series (a) and  $(1-y)(0.4\text{Li}_2\text{O} \cdot 0.6\text{Na}_2\text{O})-(y)\text{K}_2\text{O}-m\text{SiO}_2$  series (b). Closed circle (●) is for  $m=2$  and open circle (○) for  $m=1.5$ .

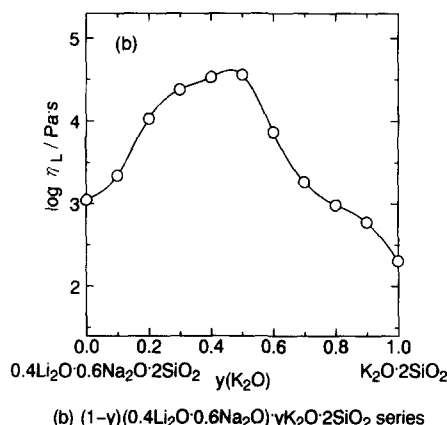
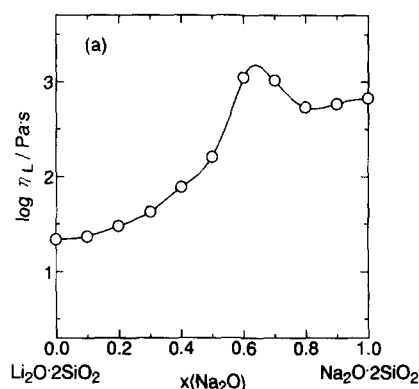


Fig. 8. Liquidus viscosity,  $\eta_L$ , of the  $(1-x)\text{Li}_2\text{O}-(x)\text{Na}_2\text{O}-2\text{SiO}_2$  series (a) and  $(1-y)(0.4\text{Li}_2\text{O} \cdot 0.6\text{Na}_2\text{O})-(y)\text{K}_2\text{O}-2\text{SiO}_2$  series (b).

arises from the mixed alkali ions. In the mixed alkali system, if the compositional dependence of  $\Delta S_{\text{conf}}$  is affected by  $\Delta S_m$ , the compositional dependence of  $\Delta S_f$  is affected by  $\Delta S_m$ .  $T_L$  is determined from the fusion enthalpy  $\Delta H_f$  and  $\Delta S_f$  when  $\Delta H_f$  changes monotonically with composition, then a  $T_L$  minimum should result when  $\Delta S_m$  assumes a maximum.  $\Delta S_m$  can be calculated according to the regular solution model assuming certain glass constituents. Glass constituents may be deduced from the primarily precipitated compounds in the glass. By heat treatments ( $600^\circ\text{C}$ ,  $650^\circ\text{C}$  for 2–20 h) of the  $\text{Li}_2\text{O}-\text{Na}_2\text{O}-2\text{SiO}_2$  glasses,  $\text{Li}_2\text{O} \cdot 2\text{SiO}_2$  crystal along with  $\text{Li}_2\text{O} \cdot \text{SiO}_2$  crystal precipitated in the  $\text{Li}_2\text{O}$ -rich compositions, and  $\text{Na}_2\text{O} \cdot 2\text{SiO}_2$  crystal in the



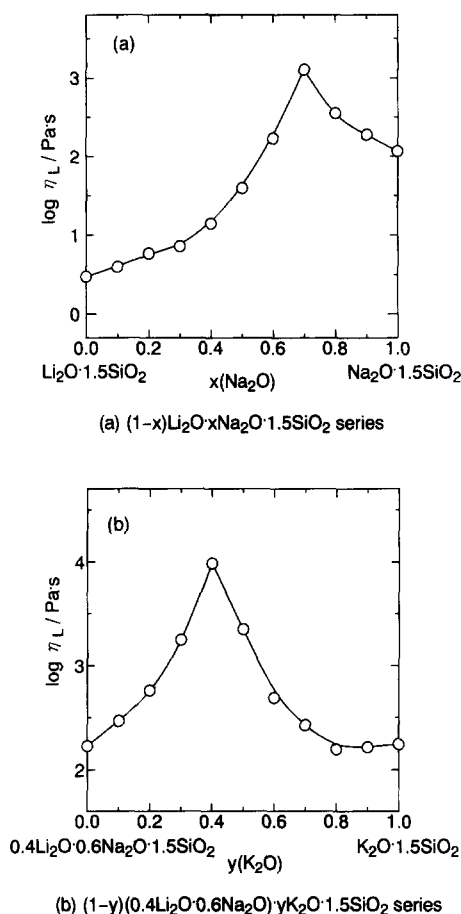


Fig. 9. Liquidus viscosity,  $\eta_L$ , of the  $(1-x)\text{Li}_2\text{O}-(x)\text{Na}_2\text{O}-1.5\text{SiO}_2$  series (a) and  $(1-y)(0.4\text{Li}_2\text{O} \cdot 0.6\text{Na}_2\text{O})-(y)\text{K}_2\text{O}-1.5\text{SiO}_2$  series (b).

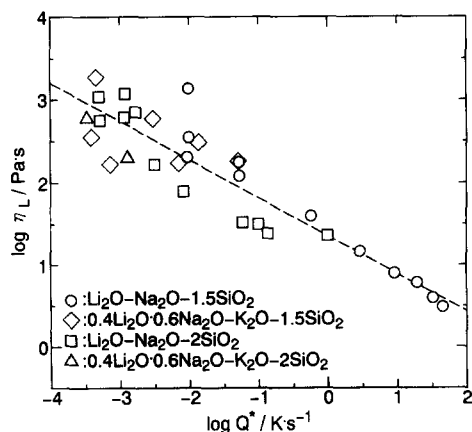


Fig. 10. Relationship between  $\eta_L$  and  $Q^*$  for glass formation in the  $\text{Li}_2\text{O}-\text{Na}_2\text{O}-\text{K}_2\text{O}-m\text{SiO}_2$  system ( $m = 2$  or  $1.5$ ).

$\text{Na}_2\text{O}$ -rich compositions [18]. No phase change was observed in the primarily deposited phases during prolonged heating. On cooling the  $\text{Li}_2\text{O}-\text{Na}_2\text{O}-1.5\text{SiO}_2$  melts,  $\text{Li}_2\text{O} \cdot \text{SiO}_2$ , and  $\text{Na}_2\text{O} \cdot \text{SiO}_2$  crystals together with unknown crystals precipitated in the  $\text{Li}_2\text{O}$ -rich compositions, and  $\text{Na}_2\text{O} \cdot \text{SiO}_2$  and  $\text{Na}_2\text{O} \cdot 2\text{SiO}_2$  crystals together with a  $\text{Li}_2\text{O} \cdot \text{SiO}_2$  crystal in the  $\text{Na}_2\text{O}$ -rich compositions. On the other hand, on cooling the  $0.4\text{Li}_2\text{O} \cdot 0.6\text{Na}_2\text{O}-\text{K}_2\text{O}-1.5\text{SiO}_2$  melts,  $\text{Li}_2\text{O} \cdot \text{SiO}_2$ ,  $\text{Na}_2\text{O} \cdot \text{SiO}_2$  and  $\text{Na}_2\text{O} \cdot 2\text{SiO}_2$  crystals precipitated in the compositions poor in  $\text{K}_2\text{O}$ , and a  $\text{K}_2\text{O} \cdot 2\text{SiO}_2$  crystal and an unknown crystal in the compositions rich in  $\text{K}_2\text{O}$ . It is speculated from these crystals that glass constituents are  $\text{M}_2\text{O} \cdot 2\text{SiO}_2$  and  $\text{M}_2\text{O} \cdot \text{SiO}_2$  ( $\text{M} = \text{Li}, \text{Na}$  and  $\text{K}$ ) components. However, the exact concentration of the  $\text{M}_2\text{O} \cdot 2\text{SiO}_2$  and  $\text{M}_2\text{O} \cdot \text{SiO}_2$  components cannot be estimated. To a first approximation, therefore, the glass constituents are assumed to be  $\text{Li}_2\text{O}-m\text{SiO}_2$ ,  $\text{Na}_2\text{O}-m\text{SiO}_2$  and  $\text{K}_2\text{O}-m\text{SiO}_2$  units ( $m = 1.5$  or  $2$ ) in the  $(1-x)\text{Li}_2\text{O}-(x)\text{Na}_2\text{O}-m\text{SiO}_2$  and  $(1-y)(0.4\text{Li}_2\text{O} \cdot 0.6\text{Na}_2\text{O})-(y)\text{K}_2\text{O}-m\text{SiO}_2$  series. Then the mixing entropy,  $\Delta S_m$ , among the  $\text{Li}_2\text{O}-m\text{SiO}_2$  and  $\text{Na}_2\text{O}-m\text{SiO}_2$  units may be given as

$$\Delta S_m = -R\{x \ln x + (1-x) \ln(1-x)\}, \quad (1)$$

where  $R$  is the gas constant and  $x$  is the molar fraction of the  $\text{Na}_2\text{O}-m\text{SiO}_2$  unit ( $m = 1.5$  or  $2$ ). Eq. (1) produces a maximum value  $R \ln 2$  at the composition  $x = 0.5$ . If the compositional dependence of  $\Delta S_m$  very strongly affects the compositional dependence of  $\Delta S_f$ , the  $(1-x)\text{Li}_2\text{O}-(x)\text{Na}_2\text{O}-m\text{SiO}_2$  series should show a eutectic at  $x = 0.5$ . In fact, a eutectic is observed at  $x = 0.6$  ( $m = 2$ ) and at  $x = 0.7$  ( $m = 1.5$ ), even though  $x = 0.6$  is somewhat larger than the predicted composition  $x = 0.5$  for  $m = 2$ . For the  $(1-y)(0.4\text{Li}_2\text{O} \cdot 0.6\text{Na}_2\text{O})-(y)\text{K}_2\text{O}-m\text{SiO}_2$  series,  $\Delta S_m$  among the  $\text{Li}_2\text{O}-m\text{SiO}_2$ ,  $\text{Na}_2\text{O}-m\text{SiO}_2$  and  $\text{K}_2\text{O}-m\text{SiO}_2$  units may be given as

$$\Delta S_m = -R\{0.4(1-y) \ln 0.4(1-y) + 0.6(1-y) \ln 0.6(1-y) + y \ln y\}, \quad (2)$$

where  $y$  is the molar fraction of the  $\text{K}_2\text{O}-m\text{SiO}_2$  unit. Eq. (2) gives a maximum value of  $1.09R$  at  $y = 0.34$ . In fact, a eutectic is observed at  $y = 0.4-0.5$ , which is somewhat larger than the predicted one, though the gaps between the theoretical and

observed eutectic compositions should be due to the oversimplification of the structural constituents assumed and the ambiguity of their concentrations.

## 5. Conclusion

The critical cooling rate corresponds to the liquidus viscosity in the mixed alkali silicate systems. It was found that mixed alkali silicate  $\text{Li}_2\text{O}-\text{Na}_2\text{O}-m\text{SiO}_2$  shows a larger glass-forming ability than the single alkali silicate  $\text{Li}_2\text{O}-m\text{SiO}_2$ ,  $\text{Na}_2\text{O}-m\text{SiO}_2$  or  $\text{K}_2\text{O}-m\text{SiO}_2$ . The quasi-ternary mixed alkali  $\text{Li}_2\text{O}-\text{Na}_2\text{O}-\text{K}_2\text{O}-m\text{SiO}_2$  shows an even larger glass-forming ability than  $\text{Li}_2\text{O}-\text{Na}_2\text{O}-m\text{SiO}_2$ . The significant role of  $\Delta S_m$  or  $\eta_L$  in glass-forming kinetics was verified. Based on the present results we predict that the more numerous the components incorporated, the more easily will a glass result, provided that the components are mutually miscible.

## Appendix

The theoretical background for the relationship between critical cooling rate and liquidus viscosity is introduced [6]. The crystal portion,  $X(0)$ , in the cooled sample must be smaller than the  $10^{-4}$ – $10^{-6}$  level when the melt is cooled at the critical cooling rate,  $Q^*$ .  $Q^*$  may be calculated by Eq. (3), where  $X(0)$  assumes a value of the order of  $10^{-4}$ – $10^{-6}$ :

$$X(0) = 4\pi/3Q^{*4} \int I \left( \int U dT \right)^3 dT, \quad (3)$$

where  $I$  is the steady-state, homogeneous nucleation rate [6] and  $U$  is the crystal growth rate [6] given by Eqs. (4) and (5), respectively:

$$I = (32\alpha^2 n_0 RT) / (V\eta T_r^2 \Delta T_r^2) \times \exp(-16\alpha^3 \Delta S_f / 3RT_r^3 \Delta T_r^2 - \Delta S_f / RT_r) \times \{1 - \exp(T_r \Delta T_r^2 \Delta S_f / 2\alpha R)\} \quad (4)$$

$$U = (2RT / \eta N\pi a^2) \exp(-\Delta S_f / RT_r) \times \{1 - \exp(-T_r \Delta T_r^2 \Delta S_f / 2\alpha R)\}, \quad (5)$$

where  $n_0$  is the number of flow units per unit volume,  $N$  is Avogadro's number,  $R$  is the gas constant,  $\Delta S_f$  is the fusion entropy per mole of flow

units,  $\eta$  is the viscosity,  $V$  is the molar volume of flow units,  $a$  is the diameter of a flow unit,  $T_r = T/T_L$  is the reduced temperature with respect to the liquidus temperature,  $T_L$ , and  $\Delta T_r = 1 - T_r$  is the reduced undercooling.  $\Delta S_f$  and  $V$  come into Eqs. (4) and (5) through an empirical relationship between the surface tension,  $\sigma$ , and the molar heat of fusion,  $\Delta H_f$ , with a constant  $\alpha = 0.3$ – $0.5$  as given by Eq. (6):

$$\sigma = \Delta H_f / N^{1/3} V^{2/3}, \\ = \alpha T_L \Delta S_f / N^{1/3} V^{2/3}. \quad (6)$$

Eqs. (4) and (5) are different from the existing equations [19–21] in several respects:

(a) the energy barrier,  $\Delta G_c$ , for a flow unit to jump over the liquid–crystal boundary was assumed as  $\Delta G_c = \Delta G_d + T_L \Delta S_f$ , where  $\Delta G_d$  is the activation energy for diffusion or viscous flow and  $T_L \Delta S_f$  is the entropy barrier over the liquid–crystal interface. The probability for a flow unit to jump over the liquid–crystal interface is proportional to  $\exp(-\Delta S_f / RT_r)$ . The  $\exp(-\Delta S_f / RT_r)$  term is included in Eqs. (4) and (5).

(b)  $\Delta S_f$  is taken as a variable since the magnitude of  $\Delta S_f$  is dependent on the size of the flow unit.

The temperature dependence of the viscosity was assumed to be Fulcher type, Eq. (7), with constants  $A$ ,  $B$  and  $T_0$ :

$$\eta = A \exp(B/T - T_0) \quad (7)$$

Using the liquidus viscosity,  $\eta_L$ , activation energy for viscous flow,  $E = R d \ln \eta / d(1/T)$  at  $T_L$ , Eq. (7) is described in the form:

$$\eta = \eta_L \exp E(1 - T_r)(1 - T_0/T_L) / RT_L(T_r - T_0/T_L). \quad (8)$$

Numerical calculation of Eq. (3) showed that  $Q^*$  can be expressed, to a first approximation, as a linear function of the liquid parameters  $\eta_L$ ,  $E/T_L$ ,  $T_0/T_L$ ,  $\Delta S_f$ ,  $T_L$  and  $V$  for  $\Delta S_f$  values greater than  $0.5R$  as shown in Eq. (9):

$$\log Q^* (\text{K/s}) \\ = -\log \eta_L (\text{Pa s}) - 0.014E/T_L (\text{J/K mol}) \\ - 3T_0/T_L - 9.1\alpha^3 \Delta S_f (\text{J/K mol}) \\ + 2 \log T_L (\text{K}) - \log V (\text{m}^3/\text{mol}) \\ - 0.25 \log X(0) - 2 \pm 0.5, \quad (9)$$

where  $X(0) = 10^{-4}$ – $10^{-6}$ .

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