Hermite interpolasjon

15. februar 2022 09.20

Oppgave 4.6.4

$(0,0)$ $\left(\frac{\pi}{2}, 1\right)$ $(\pi, 0)$

Skal finne = konstruere et polynom av grad ≤ 2 som interpolerer disse punktene.

$$p(x) = ax^2 + bx + c$$

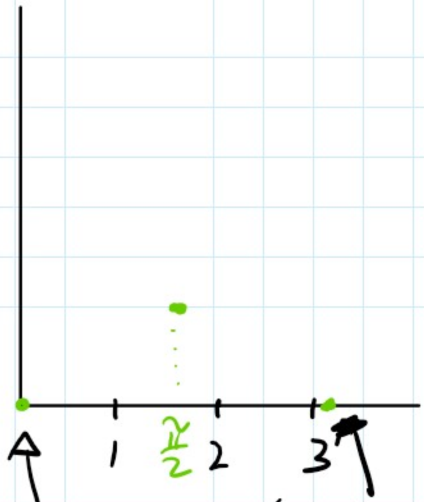
$$\Downarrow$$

Tre ligninger

$$0 = a \cdot 0^2 + b \cdot 0 + c$$

$$1 = a \cdot \left(\frac{\pi}{2}\right)^2 + b \cdot \frac{\pi}{2} + c$$

$$0 = a \cdot \pi^2 + b \cdot \pi + c$$



nullpunkter

Matriseform

$$\begin{array}{ccc} x^2 & x^1 & x^0 \end{array}$$

$$\begin{bmatrix} 0 & 0 & 1 \\ \frac{\pi^2}{4} & \frac{\pi}{2} & 1 \\ \pi^2 & \pi & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$A \quad x \quad = \quad b$$

$$A^{-1} = \begin{bmatrix} \frac{2}{\pi^2} & -\frac{4}{\pi^2} & \frac{2}{\pi^2} \\ -\frac{3}{\pi} & \frac{4}{\pi} & -\frac{1}{\pi} \\ 1 & 0 & 0 \end{bmatrix}$$

$$x = A^{-1} b = \begin{bmatrix} -\frac{4}{\pi^2} \\ \frac{4}{\pi} \\ 0 \end{bmatrix} \begin{matrix} = a \\ = b \\ = c \end{matrix}$$

$$\begin{bmatrix} 0 \end{bmatrix} = c$$

$$p(x) = -\frac{4}{\pi^2}x^2 + \frac{4}{\pi}x$$

- alternativ utregning

$p(x) = ax^2+bx+c$ standardform

$p(x) = a(x-x_0)(x-x_1)$ faktorisert,

hvor x_0 og x_1 er nullpunkter,

dvs $p(x_0) = 0$ og $p(x_1) = 0$

sette inn $x = \frac{\pi}{2}$ i faktorisert form

Da skal venstresiden bli 1

$$1 = a\left(\frac{\pi}{2} - 0\right)\left(\frac{\pi}{2} - \pi\right)$$

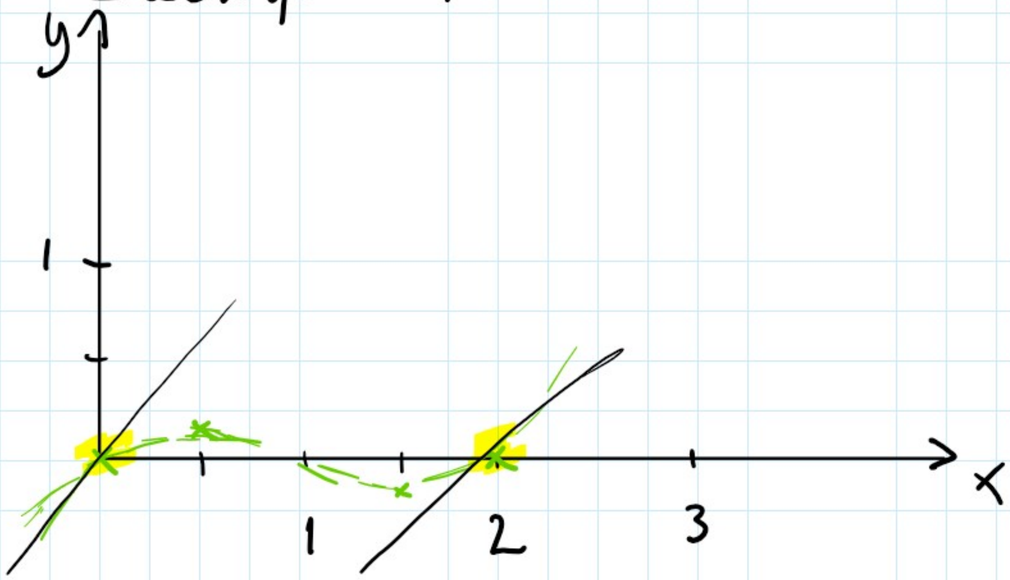
$$1 = a \cdot \frac{\pi}{2}\left(-\frac{\pi}{2}\right)$$

$$1 = a \cdot -\frac{\pi^2}{4}$$

$$\underline{\underline{a = -\frac{4}{\pi^2}}}$$

Kubisk Hermite interpolasjon

Eksempel:



"vanlig" $(0,0)$ $\left(\frac{1}{2}, \frac{1}{8}\right)$ $\left(\frac{3}{2}, -\frac{1}{8}\right)$, $(2,0)$

- kan konstruere kubisk polynom som interpolerer disse punktene.

Hermite:

ønsker derivert i $x_0 = 0$ skal være 2

ønsker —"— $x_1 = 2$ —"— 2

Med notasjon som i 4.7.1:

$x_0 = 0$ $\quad$ $x_1 = 2$

$y_0 = 0$ $\quad$ $y_1 = 0$

$y_0 = 0$ $\quad$ $y_1 = 0$

$y_0' = 2$ $\quad$ $y_1' = 2$

Standard form og derivert

$$p(x) = ax^3 + bx^2 + cx + d$$

$$p'(x) = 3ax^2 + 2bx + c$$

Setter inn

$$0 = a \cdot 0^3 + b \cdot 0^2 + c \cdot 0 + d \quad (i)$$

$$0 = a \cdot 2^3 + b \cdot 2^2 + c \cdot 2 + d \quad (ii)$$

$$2 = 3a \cdot 0^2 + 2b \cdot 0 + c \quad (iii)$$

$$2 = 3a \cdot 2^2 + 2b \cdot 2 + c \quad (iv)$$

Matrise form

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 8 & 4 & 2 & 1 \\ 0 & 0 & 1 & 0 \\ 12 & 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 2 \end{bmatrix}$$

$Ax = b$

Problem å finne den inverse.

Løser ligningssystemet manuelt:

(i) gir $d=0$

(iii) gir $c=2$

Da blir

(ii) $8a+4b+2\cdot 2=0$

(iv) $12a+4b+2=2$

$b=-3a$

innsatt i (ii)

$8a-12a+4=0$

$a=1$

$b=-3$

Vi har da $a=1, b=-3, c=2, d=0$

$p(x)=x^3-3x^2+2x$

$=x(x-1)(x-2)$