Homework for Computational Fluid Dynamics of Active Fluids

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(Dated: January 23, 2021)

The transport equation for the local particle concentration c from Ref. [6] of the notes is

$$\frac{\partial c}{\partial t^*} + \mathbf{u}^* \cdot \boldsymbol{\nabla}^* c = d\boldsymbol{\nabla}^{*2} c - U_0 \boldsymbol{\nabla} \cdot \mathbf{m} \qquad (1)$$

where $\mathbf{u}$ is the velocity, d is the diffusion coefficient, U_0 the swimming velocity and $\mathbf{m}$ is the polarization which captures the mean direction of swimming.

1. Non-dimensionalise equation 1 by using L as length scale and T as time scale. You have to introduce the swimming Peclet number $Pe_s = \frac{U_0 T}{L}$ and the parameter $\Lambda = \frac{d}{U_0^2 T}$.

2. Assuming that the polarization field is zero, and $\mathbf{u}$ is given by the Taylor-Green vortex $\mathbf{u}(u, v) = (\frac{1}{2}\cos x \sin z; \frac{1}{2}\sin x \cos z]$ discretise the non-dimensional version of equation 1 in a squared domain $(0, 2\pi)$x$(0, 2\pi)$. Please use central differencing for both advection and diffusion term and Euler implicit/explicit as temporal scheme. Find the spatio-temporal evolution of the concentration c by choosing your favorite programming language (comparing the two different temporal schemes you used to discretise). Set $\Lambda = 0.1$ and $Pe_s = \sqrt{10}$. Use a uniform grid and choose the optimal values for the number of grid points $N_x = N_y = N$ and the time step Δt. The boundary condition are periodic in both directions and the initial condition is $c(x, z, 0) = (\cos x)^2$.

3. Let's assume that the polarization vector is $\mathbf{m}(m_x, m_y) = (\cos z \sin x; \sin z \cos x)$, for (i) $\Lambda = 0.1$ $Pe_s = \sqrt{10}$ and (ii) $\Lambda = 10$ and $Pe_s = \sqrt{\frac{1}{10}}$. Discretise the non-dimensional version of equation 1 using the same spatial discretization used in question 2 and choosing your preferred temporal scheme. Show the spatio-temporal evolution of the concentration c by choosing your favorite programming language. Use a uniform grid and choose the optimal values for the number of grid points $N_x = N_y = N$ and the time step Δt. The boundary condition are periodic in both directions and the initial condition is $c(x, z, 0) = (\cos x)^2$.