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at CHAPEL HILL

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Linear Algebra

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1 Introduction

These notes are intended for use in the warm-up camp for incoming UNC STOR and Biostatistics graduate students. Welcome to Carolina!

We assume that you have taken a linear algebra course before and that most of the material in these notes will be a review of what you've already known. If some of the material is unfamiliar, do not be intimidated! We hope you find these notes helpful! If not, you can consult the references listed at the end, or any other textbooks of your choice for more information or another style of presentation (most of the proofs on linear algebra part have been adopted from Strang, the proof of F-test from Montgomery et al, and the proof of bivariate normal density from Bickel and Doksum).

Linear algebra is an important and fundamental math tool for probability, statistics, numerical analysis and operations research. Lots of material in this notes will show up in your future study and research. There will be 9 algebraic classes in total (one class per weekday for two weeks, excluding a day for the university orientation). Each class will last two hours with a short break in between.

Go Tar Heels!