Dynamic Gaussian Graphical Model: coefficient varying model for partial correlation estimation

Meilei Jiang
Department of Statistics and Operations Research
University of North Carolina at Chapel Hill

February 16, 2016

1 Overview of estimation in graphical models

Graphical models are quite useful in many domains to uncover the dependence structure among observed variables. Typically, we consider a p-dimensional multivariate normal distributed random variable $X = (X_1, \cdots, X_p) \sim \mathbb{N}(\mu, \Sigma)$, where p is the number of features. Then a graph of these p features can be constructed based on there conditional dependence structure. More precisely, we can construct a Gaussian graphical model $\mathcal{G} = (V, E)$, where $V = \{1, \cdots, p\}$ is the set of nodes and $E = \{(i,j)|X_i \text{ is conditionally dependent with } X_b, \text{given } X_{V/\{i,j\}}\}$.

Let $\Omega = \Sigma^{-1} = (\omega_{i,j})_{1\le i,j\le p}$ be the precision matrix. Then X_i and X_j are conditionally dependent given other features if and only if $\omega_{ij} = 0$. Therefore, estimating the covariance matrix and precision matrix of X is equivalent to estimate the structure of Gaussian graphical model $\mathcal{G}$. More discussion can be found in [7].

There are lots of literatures discussing about estimating Σ and Ω. Utilizing the idea of LASSO from Tibshirani [**?**], Meinshausen and Bühlmann [8] performed a computationally attractive method for nearest neighborhood selection at each node in the graph. Another nature way is to estimate Σ and Ω is the penalized likelihood approach. Friedman, Hastie nd Tibshirani [4] proposed the graphical lasso. Fan, Feng and Wu [1] studied the penalized likelihood methods with the SCAD penalty and the adaptive LASSO penalty. Cai, Liu and Luo [**?**] performed a constrained l_1 minimization approach to estimate sparse precision matrix (CLIME).

In practice, dynamic graphical model are very attractive. We can consider dynamic graphical model under the context of varying coefficient model [5, 2, 3]. Zhou, Laffery and Wasserman [9] developed a nonparametric framework for estimating time varying graphical model for estimating time varying graphical structure for multivariate Gaussian distributions $X^t \sim \mathbb{N}(0, \Sigma(t))$ using l_1 regularization method. Zhou's model assumed that the observations X^t are independent and changed smoothly. Kolar and Xing [6] showed the model selection consistency for the procedure proposed in Zhou et al. [9] and for the modified neighborhood selection procedure of Meinshausen and Bühlmann [8].

1.1 Nearest neighbor selection approach

1.2 Penalized likelihood approach

1.3 Constrained $l1$ Minimization Approach

1.4 time-varying graph and coefficient varying models

2 Joint estimation of multiple graphs

2.1 Stationary graphs with spacial-temporal dependence: GEMINI

2.2 Time varying graph estimation through smoothed covariance matrix

2.3 Fused Graphical Lasso

2.4 Smoothed Kendall's tau correlation matrix

References

[1] Jianqing Fan, Yang Feng, and Yichao Wu. Network exploration via the adaptive lasso and scad penalties. *The annals of applied statistics*, 3(2):521, 2009.

[2] Jianqing Fan and Wenyang Zhang. Statistical estimation in varying coefficient models. *Annals of Statistics*, pages 1491–1518, 1999.

[3] Jianqing Fan and Wenyang Zhang. Statistical methods with varying coefficient models. *Statistics and its Interface*, 1(1):179, 2008.

[4] Jerome Friedman, Trevor Hastie, and Robert Tibshirani. Sparse inverse covariance estimation with the graphical lasso. *Biostatistics*, 9(3):432–441, 2008.

[5] Trevor Hastie and Robert Tibshirani. Varying-coefficient models. *Journal of the Royal Statistical Society. Series B (Methodological)*, pages 757–796, 1993.

[6] Mladen Kolar and Eric P Xing. On time varying undirected graphs. 2011.

[7] Steffen L Lauritzen. *Graphical models.* Clarendon Press, 1996.

[8] Nicolai Meinshausen and Peter Bühlmann. High-dimensional graphs and variable selection with the lasso. *The annals of statistics*, pages 1436–1462, 2006.

[9] Shuheng Zhou, John Lafferty, and Larry Wasserman. Time varying undirected graphs. *Machine Learning*, 80(2-3):295–319, 2010.