01 Different Derivations (Lab)

Consider again grammar `G` . Give two *different* derivations of `Dave runs`.

First derivation:

S → NP VP →PN VP →Dave V →Dave runs

Second derivation: S →NP VP →PN VP →PN V →PN runs →Dave runs

02 Equivalent Languages (Lab)

Let `G ' = (T, N, P, S)` with `T = {a}`, `N = {S}` productions `P`:

```
S →
S → Sa
```

Prove that `L(G ') = {a  | n   0}`, which is the same as `L(G )` from the course notes.

We prove this formally by inclusion in both directions. By definition of `L(G )`,

`{   T* | S    }   {a  | n   0}`

means that for every `T*` that is derivable from `S` there exists an `n   0` such that `= a` . We show this by induction over the length of derivations.

- *Base.* Suppose is derived directly from `S` by `S` , which leaves only `=` according to the first production. Then `= a` .
- *Step.* Suppose is derived from `S` in multiple steps, which leaves only `S` `Sa  *` according to the second production. Then must be `a` to be derivable from `Sa`. As the derivation `S  *` is shorter than the derivation of `S  *` , by induction assumption there exists an `n` such that `= a` . Therefore `= a = a a = a` 1.

The inclusion in the other direction means that every `a` for `n   0` can be derived from `S`:

`{a  | n   0}   {   T* | S    }`

We show this by induction over `n`.

- *Base.* obviously `a  =` can be generated by the first production, `S` .
- *Step.* Suppose `a` can be generated, `S` `a` . We need to show that `a` 1 can be generated as well. This follows from `S` `Sa` `a a = a` 1.

Thus, we can conclude `L(G )' = {a  | n   0}`.

03 Derivation in Copy Language (Lab)

S

$\rightarrow$ aAS

$\rightarrow$ aAbBS

$\rightarrow$ abASb

$\rightarrow$ abAbBSb

$\rightarrow$ abbABSb

$\rightarrow$ abbASbb

$\rightarrow$ abbSabb

$\rightarrow$ abbabb

04 Ambiguity in English (Lab)

A well-known syntactically ambiguous English sentence is `Time flies like an arrow`. Explain the ambiguity!

[Time flies] like [an arrow], i.e the species of time flies happen to like an arrow or

TIme [flies [like [an arrow]]], i.e time flies figurateively very fast like an arrorw.

First interpretation does not semantically make sense.

05 Extending English Grammar (Lab)

Define a new grammar `G '` by extending `G` :

- A verb phrase is a verb phrase followed by a prepositional phrase.
- A prepositional phrase is a preposition followed by a noun phrase.
- Prepositions are `in` and `on`.
- Nouns are `park` and `child`.

Give the full definition of `G '`! Is `G '` regular, context-free, or context-sensitive? Draw the parse tree of `the child eats a banana in the park`! Use draw.io or a similar program to generate an SVG figure and insert that in the cell below.

0=(, , ,)

= (Kevin, Dave, a, the, banana, apple, eats, runs, park, child, in, on)

= , , , , , , , ,

Grammar Definition:

S →NP VP NP →PN NP →D N VP →V VP →VP NP VP →VP PP PP →P NP PN →Kevin PN →Dave D →a D →the P →in P →on N →banana N →apple N →park N →child V →eats V →runs

0 is not regular, but it is context-free.

Derivation: S →NP VP →D N VP PP →the child VP NP P NP →the child VP D N P NP →the child V D N P NP →the child eats a banana P NP →the child eats a banana in D N →the child eats a banana in the park

[[q5.drawio.svg]]

06 Generated Language a bc (Lab)

Prove formally that `L(G ) = {a bc  | n   0}`!

We prove this formally by inclusion in both directions. By definition of $L(G_3)$.

$$\{\chi \in T^* \mid S \Rightarrow^+ \chi\} \subseteq \{a^n bc^n \mid n \geq \theta\}$$

means that for every $\chi \in T^*$ that is derivable from S there exists an $n \geq 0$ such that $\chi = a^n bc^n$. We show this by induction over the length of derivations.

- Base. Suppose χ is derived directly from S by $S \Rightarrow \chi$, which leaves only $b\varepsilon = b$ according to the first production. Then $\chi = a^n bc^n \circ$.
- Step. Suppose χ is derived from S in multiple steps, which leaves only $S \Rightarrow Sabc \Rightarrow^* \chi$ according to the second production. Then χ must be $wabc$ to be derivable from $Sabc$. As the derivation $S \Rightarrow^* w$ is shorter than the derivation of $S \Rightarrow^* \chi$, by induction assumption there exists an n such that $w = a^n bc^n$. Therefore $\chi = wa = a^n bc^n = a^{n+1} bc^{n+1}$.

The inclusion in the other direction means that every $a^n bc^n$ for $n \geq 0$ can be derived from S :

$$\{a^n bc^n \mid n \geq \theta\} \subseteq \{\chi \in T^* \mid S \Rightarrow^+ \chi\}$$

We show this by induction over n.

- Base, obviously $a^\circ bc^\circ = \varepsilon$ can be generated by the first production, $a^+ bc^+ \varepsilon$.
- Step. Suppose $a^n bc^n$ can be generated, $S \Rightarrow^+ a^n bc^n$. We need to show that $a^{n+1} bc^{n+1}$ can be generated as well. This follows from $S \Rightarrow Sa \Rightarrow^+ a^n bc^n a = a^{n+1} bc^{n+1}$.

Thus we can conclude $L(G_3)' = \{a^n bc^n \mid n \geq \theta\}$.