## Worksheet: Limits to Infinity

Note: for all the problems, identify the limit as a number,  $-\infty$ ,  $\infty$  or DNE (does not exist)

1. Please keep in mind the list of functions in order of their rate of growth – quickest to slowest:

Factorial (x!), Exponential ( $4^x$ ,  $e^x$ ), Algebraic ( $x^4$ ,  $x^{0.5}$ ), Logarithm ( $log_2x$ , lnx)

Then find the following limits quickly without showing any work:

(a) 
$$\lim_{x\to\infty} \frac{x^3}{x!} =$$

(g) 
$$\lim_{n\to\infty} \frac{5^n+2}{n^9+100} =$$

(g) 
$$\lim_{n\to\infty} \frac{5^n+2}{n^9+100} =$$
 (m)  $\lim_{x\to\infty} \frac{x^8+\ln x}{3x^8+2} =$ 

(b) 
$$\lim_{x\to\infty} \frac{\sqrt[3]{x^4 + 2x^3 + 1}}{\ln x + 3} =$$
 (h)  $\lim_{n\to\infty} \frac{n^4 + 10\ln n}{4\sqrt{n^8 + 3n + 100}} =$  (n)  $\lim_{x\to\infty} \frac{x^{10} + 2x^2 + 10}{32x^6 + 12} =$ 

(h) 
$$\lim_{n\to\infty} \frac{n^4+10lnn}{4\sqrt{n^8+3n+100}} =$$

(n) 
$$\lim_{x\to\infty} \frac{x^{10}+2x^2+10}{32x^6+12} =$$

(c) 
$$\lim_{x\to\infty} \frac{x^{10}+4x^7+10}{32x^{12}+10} =$$

(c) 
$$\lim_{n\to\infty} \frac{x^{10} + 4x^7 + 10}{32x^{12} + 10} =$$
 (i)  $\lim_{n\to\infty} \frac{-n^4 + 3n^2}{19n^3 + 3n^2 - 10} =$  (o)  $\lim_{n\to\infty} \frac{n^5 - 10n^7 + lnn}{n^3 + 8n^7} =$ 

(o) 
$$\lim_{n\to\infty} \frac{n^5 - 10n^7 + lnn}{n^3 + 8n^7} =$$

(d) 
$$\lim_{x\to\infty} \frac{5x+1}{x^2-3x+4}$$

(d) 
$$\lim_{x\to\infty} \frac{5x+1}{x^2-3x+4}$$
 (j)  $\lim_{x\to-\infty} \frac{x^{120}+20}{-x^{100}+e^x} =$  (p)  $\lim_{x\to\infty} \frac{e^x}{x^3}$ 

(p) 
$$\lim_{x\to\infty} \frac{e^x}{x^3}$$

(e) 
$$\lim_{x\to-\infty} \frac{2^x}{x^2}$$

(k) 
$$\lim_{n \to -\infty} \frac{5^n + n^2}{-e^n + 1} =$$

(k) 
$$\lim_{n \to -\infty} \frac{5^n + n^2}{-e^n + 1} =$$
 (q)  $\lim_{x \to -\infty} (-3x^3 - 4x + 5)$ 

(f) 
$$\lim_{x\to\infty} \frac{2^x}{x^2}$$

(l) 
$$\lim_{x\to-\infty} \frac{2x^3-2x+5}{13x^3-5x+13} =$$

(l) 
$$\lim_{x\to-\infty} \frac{2x^3 - 2x + 5}{13x^3 - 5x + 13} =$$
 (r)  $\lim_{n\to\infty} \frac{n^5 - 10n^7 + lnn}{n^3 + 8n^7} =$ 

2. Practice some common limits without showing the work:

(a) 
$$\lim_{x\to-\infty} e^x =$$

(d) 
$$\lim_{x\to-\infty} e^{-x} =$$

(g) 
$$\lim_{x\to-\infty} e^{\frac{1}{x}} =$$

(b) 
$$\lim_{x\to 0} e^x =$$

(e) 
$$\lim_{x\to 0+} e^{\frac{1}{x}} =$$

(h) 
$$\lim_{x\to\infty} e^{\frac{1}{x}} =$$

(c) 
$$\lim_{x\to-\infty} x \ln x$$

(f) 
$$\lim_{x\to 0^-} e^{\frac{1}{x}} =$$

(i) 
$$\lim_{x\to\infty} \frac{1}{e^{-x}} =$$

3. Practice more common limits without showing the work:

(a) 
$$\lim_{x\to\infty} \ln x =$$

(e) 
$$\lim_{x\to\infty} \sin x =$$

(i) 
$$\lim_{x\to\infty} \sin(\frac{1}{x}) =$$

(b) 
$$\lim_{x\to 0^+} \ln x =$$

(f) 
$$\lim_{x\to\infty} (\sin x)e^{-x} =$$
 (j)  $\lim_{x\to\infty} x \ln \frac{1}{x} =$ 

(j) 
$$\lim_{x\to\infty} x \ln \frac{1}{x} =$$

(c) 
$$\lim_{x\to-\infty} (e^{3x} - e^{2x}) =$$
 (g)  $\lim_{x\to\infty} x \sin x =$ 

(g) 
$$\lim_{x\to\infty} x \sin x =$$

$$(k) \lim_{x \to -\infty} x e^{\frac{1}{x}} =$$

(d) 
$$\lim_{x \to \infty} \sin(\frac{x^2}{2x^2 + x}) =$$
 (h)  $\lim_{x \to \infty} \frac{1}{x} \sin x =$  (l)  $\lim_{x \to 0^-} x e^{\frac{1}{x}} =$ 

(h) 
$$\lim_{x\to\infty} \frac{1}{x} \sin x =$$

(l) 
$$\lim_{x\to 0^-} xe^{\frac{1}{x}} =$$

4. Show all you work for the following questions (limits at discontinuity and infinity)

(a) 
$$\lim_{x\to 0} \frac{\sin(3x)}{4x}$$

(d) 
$$\lim_{x\to 1} \frac{5x^4 - 4x^2 - 1}{10 - x - 9x^3}$$

(b) 
$$\lim_{x\to\infty} x^{\frac{1}{x}}$$

(e) 
$$\lim_{x\to -1} \frac{\sqrt{x+4}-3}{x+1}$$

(c) 
$$\lim_{x\to\infty} \left(\frac{1}{x}\right)^{\frac{1}{x}}$$

(f) 
$$\lim_{x\to\infty} \left(\frac{x}{x+1}\right)^x$$