Knows And Can Use Important Trig Identities

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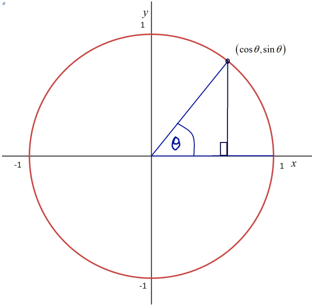
Pythagorean Identity

The most important trigonometric identity is the Pythagorean Identity. Many of the other identities can be derived from the Pythagorean Identity:

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$



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Double-Angle Formulas

Here are the double angle formulas that you should know:

$$\boxed{\sin 2\theta = 2\sin\theta\cos\theta}$$

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$$\tan 2\theta = \frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$$

$\tan 2\theta = \frac{2\frac{\sin\theta}{\cos\theta}}{1 - \frac{\sin^2\theta}{\cos^2\theta}}$ (Divide numerator and denominator by $\cos^2\theta$.)

$$\boxed{\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}}$$

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These forms are used extensively in Calculus II to aid in integration.

Even and Odd

Finally, it is helpful to remember that Sine is an odd function and Cosine is an even function:

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

Summary

The boxed identities in these notes are the ones that it is most important for you to know and memorize. In a pinch, some of these identities can be derived from the others.

There are other identities like the sum formulas and the product formulas. It is helpful to be aware of these, but we do not expect you to memorize them.

Example

If $\sec\theta = 2$ with $0 < \theta < \frac{\pi}{2}$, then $\tan\theta =$

Practice

Use identities to answer these questions. Try to work without consulting your notes. We will use a triangle as a shortcut to the Pythagorean identities.

If $\cos\theta = \frac{1}{3}$ with $0 < \theta < \frac{\pi}{2}$, then $\sin\theta =$

If $\tan\theta = -\frac{2}{3}$ with $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, then $\cos\theta =$

If $\sin\theta = -\frac{2}{3}$ with $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, then $\sin 2\theta =$

If $\sin\theta = \frac{5}{6}$, then $\sin(-\theta) =$