Can Use The Unit Circle
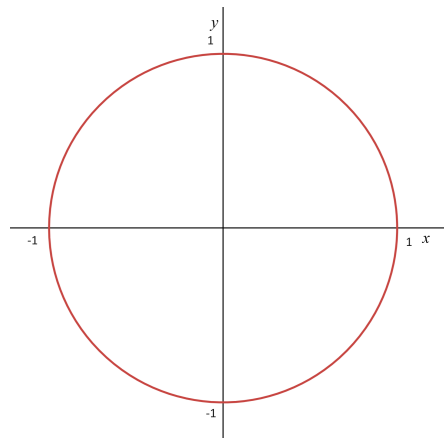
APMA Faculty
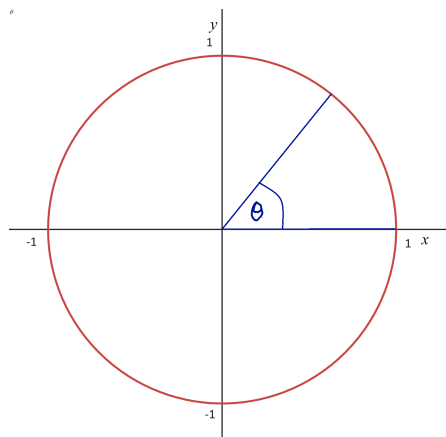University of Virginia

July 27, 2024

What is the Unit Circle?

The unit circle is a circle with radius $= 1$, that is drawn in the xy-plane, with the center at the origin.
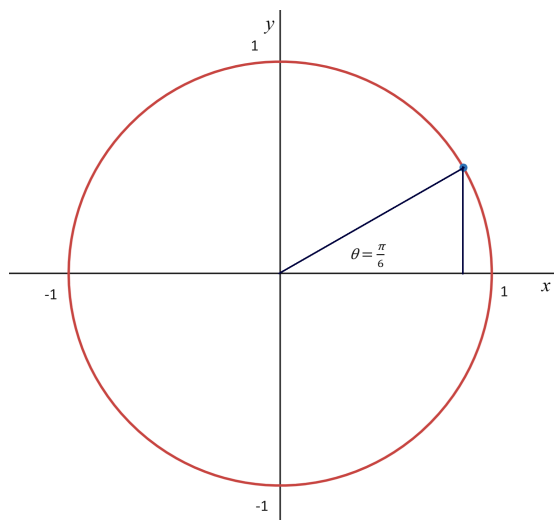
Angles in the Unit Circle

To remember trig function values, especially the values of sine and cosine, it is very helpful to picture the angle in the unit circle with the initial side on the positive $x - axis$.
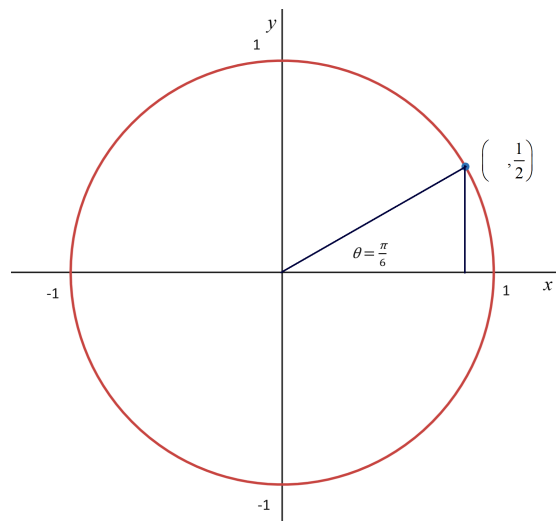
The First Special Angle

There are some angles for which we must know the trig function values. The first such angle is $\frac{\pi}{6}$.
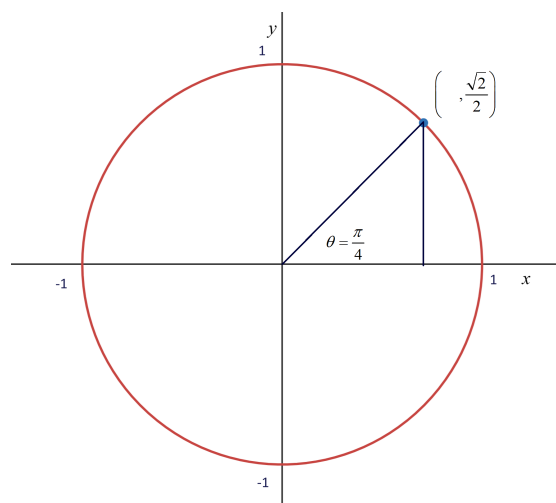
Vertical Coordinate

Observe the right triangle in the figure. The length of the hypotenuse is 1. So the y-coordinate of the point at $\frac{\pi}{6}$ radians is $\sin\frac{\pi}{6}$. That y-coordinate is $\frac{1}{2}$. Thus, $\sin\frac{\pi}{6} = \frac{1}{2}$.
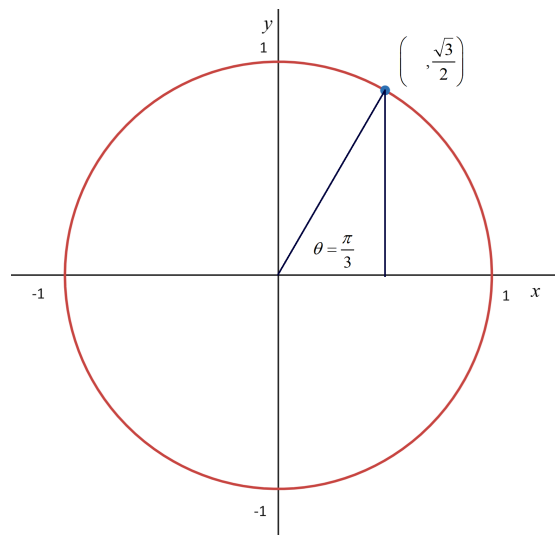
Another Special Angle

Similarly, $\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$.

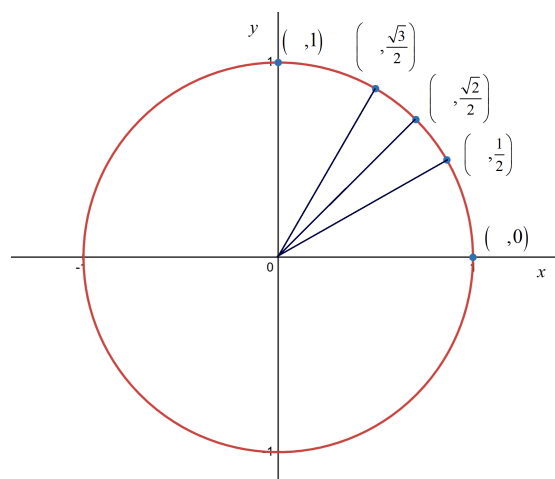# A Third Special Angle

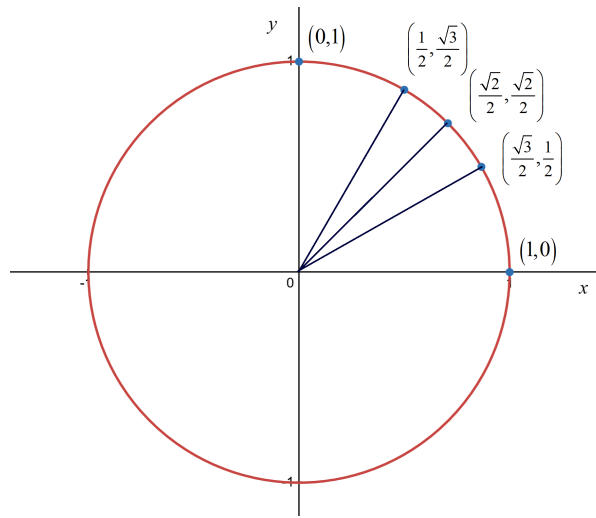Similarly, $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$.

Climbing Up the Unit Circle

The Horizontal Coordinate

$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$
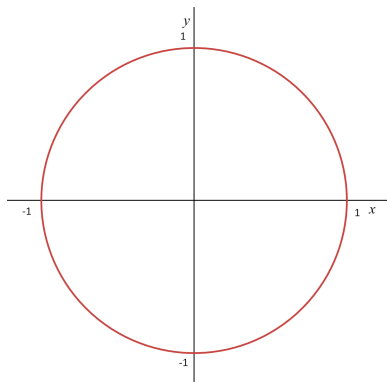
$\cos \frac{\pi}{3} = \frac{1}{2}$

Other Quadrants

The signs of the horizontal and vertical coordinates determine the signs of Cosine and Sine. So the quadrant determines the sign.
The reference angle provides the magnitude of cosine and sine.
The reference angle is the angle to the x-axis.
For example, what is $\cos \frac{5\pi}{3}$? What is $\sin \frac{17\pi}{6}$? Use the reference angle and the quadrant to determine these.

Shifting angles

What if we wish to know the coordinates of a point on the unit circle that is opposite to point (a, b) where (a, b) is at rotation θ? That is, what are the coordinates of the point at rotation $\theta + \pi$?