Your Name:	ID #:
Solution	

Note: for all the problems, identify the limit as a number, $-\infty$, ∞ or DNE (does not exist)

1. Please keep in mind the list of functions in order of their rate of growth – quickest to slowest:

   Factorial(x!), **Exponential** (4^x, e^x), **Algebraic** (x^4, $x^{0.5}$), **Logarithm** (log_2x, lnx)

   Then find the following limits quickly without showing any work:

   (a) $\lim_{x\to\infty}\frac{x^3}{x!} = 0$

   (b) $\lim_{x\to\infty}\frac{\sqrt[3]{x^4+2x^3+1}}{lnx+3} = \infty$

   (c) $\lim_{x\to\infty}\frac{x^{10}+4x^7+10}{32x^{12}+10} = 0$

   (d) $\lim_{x\to\infty}\frac{5x+1}{x^2-3x+4} = 0$

   (e) $\lim_{x\to-\infty}\frac{2^x}{x^2} = 0$

   (f) $\lim_{x\to\infty}\frac{2^x}{x^2} = \infty$

   (g) $\lim_{n\to\infty}\frac{5^n+2}{n^9+100} = \infty$

   (h) $\lim_{n\to\infty}\frac{n^4+10lnn}{4\sqrt{n^8+3n+100}} = \frac{1}{4}$

   (i) $\lim_{n\to\infty}\frac{-n^4+3n^2}{19n^3+3n^2-10} = -\infty$

   (j) $\lim_{x\to-\infty}\frac{x^{120}+20}{-x^{100}+e^x} = -\infty$
   if $x\to+\infty$, then 0

   (k) $\lim_{n\to-\infty}\frac{5^n+n^2}{-e^n+1} = \infty$
   if $n\to+\infty$, then $-\infty$

   (l) $\lim_{x\to-\infty}\frac{2x^3-2x+5}{13x^3-5x+13} = \frac{2}{13}$

   (m) $\lim_{x\to\infty}\frac{x^8+lnx}{3x^8+2} = \frac{1}{3}$

   (n) $\lim_{x\to\infty}\frac{x^{10}+2x^2+10}{32x^6+12} = \infty$

   (o) $\lim_{n\to\infty}\frac{n^5-10n^7+lnn}{n^3+8n^7} = 0$

   (p) $\lim_{x\to\infty}\frac{e^x}{x^3} = \infty$

   (q) $\lim_{x\to-\infty}(-3x^3-4x+5) = \infty$

   (r) $\lim_{n\to\infty}\frac{n^5-10n^7+lnn}{n^3+8n^7} = \infty$

2. Practice some common limits without showing the work:

   (a) $\lim_{x\to-\infty}e^x = 0$

   (b) $\lim_{x\to0}e^x = e^0 = 1$

   (c) $\lim_{x\to-\infty}xlnx$ "No sense"
   if $x\to+\infty$
   then ∞

   (d) $\lim_{x\to-\infty}e^{-x} = \infty$

   (e) $\lim_{x\to0+}e^{\frac{1}{x}} = \infty$

   (f) $\lim_{x\to0-}e^{\frac{1}{x}} = 0$

   (g) $\lim_{x\to-\infty}e^{\frac{1}{x}} = e^0 = 1$

   (h) $\lim_{x\to\infty}e^{\frac{1}{x}} = e^0 = 1$

   (i) $\lim_{x\to\infty}\frac{1}{e^{-x}} = \infty$

3. Practice more common limits without showing the work:

(a) $\lim_{x\to\infty} lnx = \infty$

(b) $\lim_{x\to 0^+} lnx = -\infty$

(c) $\lim_{x\to-\infty}(e^{3x} - e^{2x}) = 0$

if $x \to +\infty$, then ∞

(d) $\lim_{x\to\infty} \sin(\frac{x^2}{2x^2+x}) =$ $\sin(\frac{1}{2})$

(e) $\lim_{x\to\infty} \sin x =$ DNE

(f) $\lim_{x\to\infty}(\sin x)e^{-x} = 0$

(g) $\lim_{x\to\infty} x \sin x =$ DNE

(h) $\lim_{x\to\infty} \frac{1}{x}\sin x = 0$

if $x \to 0$, then 1

(i) $\lim_{x\to\infty} \sin(\frac{1}{x}) = \sin 0 = 0$

(j) $\lim_{x\to\infty} x \ln\frac{1}{x} = -\infty$

(k) $\lim_{x\to-\infty} xe^{\frac{1}{x}} = -\infty$

if $x \to +\infty$, then $+\infty$

(l) $\lim_{x\to 0^-} xe^{\frac{1}{x}} = 0$

4. Show all you work for the following questions.

(a) $\lim_{x\to 0} \frac{sin(3x)}{4x}$

$= \frac{3}{4}\lim_{x\to 0}\frac{\sin(3x)}{3x}$

$= \frac{3}{4}\cdot 1$

$= \boxed{\frac{3}{4}}$

or

$\overset{L\cdot H}{=} \lim_{x\to 0}\frac{3\cos(3x)}{4}$

$= \boxed{\frac{3}{4}}$

(b) $\lim_{x\to\infty} x^{\frac{1}{x}}$ " ∞^0 "

$y = x^{\frac{1}{x}} \Rightarrow \ln y = \ln x^{\frac{1}{x}} \Rightarrow \ln y = \frac{1}{x}\ln x$

$\lim_{x\to\infty}\ln y = \lim_{x\to\infty}\frac{\ln x}{x} \overset{L\cdot H}{=} \lim_{x\to\infty}\frac{1}{x} = 0$

$\Rightarrow \lim_{x\to\infty} x^{\frac{1}{x}} = \lim_{y\to\infty} y = e^0 = \boxed{1}$

(c) $\lim_{x\to\infty}(\frac{1}{x})^{\frac{1}{x}}$

" 0^0 "

let $y = (\frac{1}{x})^{\frac{1}{x}}$ then $\ln y = \ln(\frac{1}{x})^{\frac{1}{x}} = \frac{1}{x}\ln\frac{1}{x}$

$\ln y = \frac{1}{x}\ln(x^{-1}) = -\frac{1}{x}\ln x = -\frac{\ln x}{x}$

$\lim_{x\to\infty}\ln y = \lim_{x\to\infty}(-\frac{\ln x}{x}) \overset{L\cdot H}{=} 0$

meanwhile, $\lim_{y\to\infty}\ln y = \ln(\lim_{x\to\infty} y) = 0$

$\Rightarrow \lim_{x\to\infty} y = \lim_{x\to\infty}(\frac{1}{x})^{\frac{1}{x}} = e^0 = \boxed{1}$

(d) $\lim_{x\to 1} \frac{5x^4-4x^2-1}{10-x-9x^3}$ " $\frac{0}{0}$ "

$\overset{L\cdot H}{=} \lim_{x\to 1}\frac{20x^3 - 8x}{-1-27x^2}$

$= \frac{20-8}{-1-27} = \boxed{-\frac{3}{7}}$

(e) $\lim_{x\to -1} \frac{\sqrt{x+4}-3}{x+1}$

$= \boxed{DNE}$ since

$\lim_{x\to(-1)^+}\frac{\sqrt{x+4}-3}{x+1} = -\infty$

but $\lim_{x\to(-1)^-}\frac{\sqrt{x+4}-3}{x+1} = +\infty$

(f) $\lim_{x\to\infty}(\frac{x}{x+1})^x$ " 1^∞ "

$y = (\frac{x}{x+1})^x \Rightarrow \ln y = x\ln(\frac{x}{x+1})$

$\lim_{x\to\infty} x\ln\frac{x}{x+1} = \lim_{x\to\infty}\frac{\ln(1-\frac{1}{x+1})}{\frac{1}{x}}$

$\overset{L\cdot H}{=} \lim_{x\to\infty}\frac{\frac{1}{1-\frac{1}{x+1}}\cdot(1-\frac{1}{x+1})'}{(\frac{1}{x})'}$

$= \lim_{x\to\infty}\frac{1}{1-\frac{1}{x+1}}\cdot\frac{1}{(x+1)^2}\cdot(-x^2) = -1$

$\Rightarrow \lim_{x\to\infty} y = \boxed{e^{-1}}$