PreCalculus-Limit at Infinity (Learning Target LF)

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"Guess"

"Guess" the limit works well sometimes and can be tried before applying any specific technique.

Example

Find $\lim_{x\to\infty} e^{\frac{1}{x}}$

Solution: take x values very big, eg. $x = 1000, 10000, 100000, \cdots$, we can see that the power $\frac{1}{x} \to 0$, so the limit is like e^0, which is 1.

Similarly, by plugging in some values tending to $+\infty$ or $-\infty$, we can get

Note : $\operatorname{sgn}(a) = 1$ if $a > 0$ and $\operatorname{sgn}(a) = -1$ if $a < 0$.

1. $\lim_{x\to\infty} \mathbf{e}^x = \infty$ & $\lim_{x\to-\infty} \mathbf{e}^x = 0$
2. $\lim_{x\to\infty} \ln(x) = \infty$ & $\lim_{x\to 0^+} \ln(x) = -\infty$
3. If $r > 0$ then $\lim_{x\to\infty} \frac{b}{x^r} = 0$
4. If $r > 0$ and x^r is real for negative x then $\lim_{x\to-\infty} \frac{b}{x^r} = 0$
5. n even : $\lim_{x\to\pm\infty} x^n = \infty$
6. n odd : $\lim_{x\to\infty} x^n = \infty$ & $\lim_{x\to-\infty} x^n = -\infty$
7. n even : $\lim_{x\to\pm\infty} a x^n + \cdots + b x + c = \operatorname{sgn}(a)\infty$
8. n odd : $\lim_{x\to\infty} a x^n + \cdots + b x + c = \operatorname{sgn}(a)\infty$
9. n odd : $\lim_{x\to-\infty} a x^n + \cdots + c x + d = -\operatorname{sgn}(a)\infty$

The type of $\pm\frac{\infty}{\infty}$

Technique 1: Compare the growth speed:
when $x \to \infty$, here is a list of functions in order of their rate of growth to $+\infty$, quickest to slowest:

$$x!, \cdots, 4^x, 3^x, e^x, 1.5^x, \cdots, x^4, x^3, x^2, x * logx, x, logx, \cdots, 3, 2, 1$$

by category, it is

$$factorial \gg exponential \gg algebraic \gg logarithmic \gg constant$$

Then $\lim\limits_{x\to\pm\infty} \frac{n(x)}{d(x)} = \lim\limits_{x\to\pm\infty} \frac{\text{dominant term of } n(x)}{\text{dominant term of } d(x)}$,

Examples
$\lim\limits_{x\to\infty} \frac{4^x - x^7 + 2x}{x! + x^10 - 1} = \lim\limits_{x\to\infty} \frac{4^x}{x!} =$? (soon)

Case 1: If the numerator

Examples

Note: sometimes you need to do some algebraic manipulation to apply the above techniques.

Find $\lim_{x \to 0^+} (x * \ln x)$

Find $\lim_{x \to 2} \left(\frac{4}{x^2 - 4} - \frac{1}{x-2} \right)$