PreCalculus-Graph Logarithmic Functions (Learning Targets GL)

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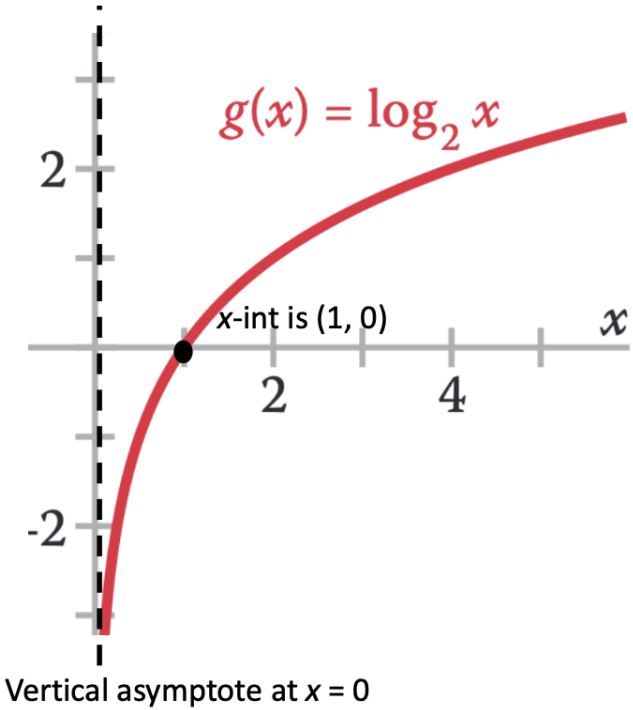
Logarithmic Functions

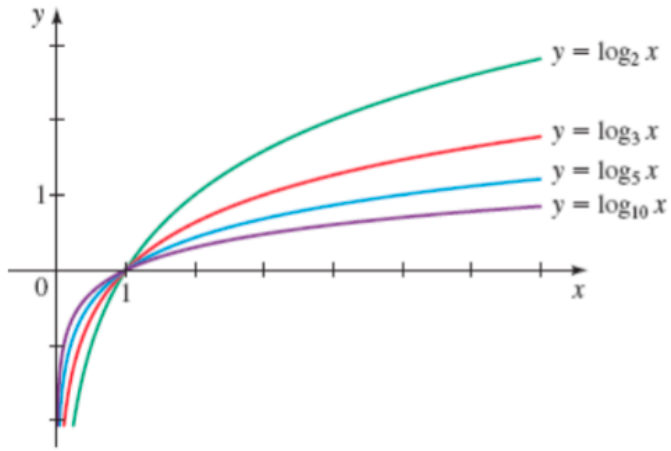
- The function $f(x) = \log_b x$ is called the **logarithmic function with base b**.
- The base b must be a positive number and $b \neq 1$.
- It is often useful to express a logarithmic function in its equivalent exponential form:

$$y = \log_b x \quad \leftrightarrow \quad x = b^y$$

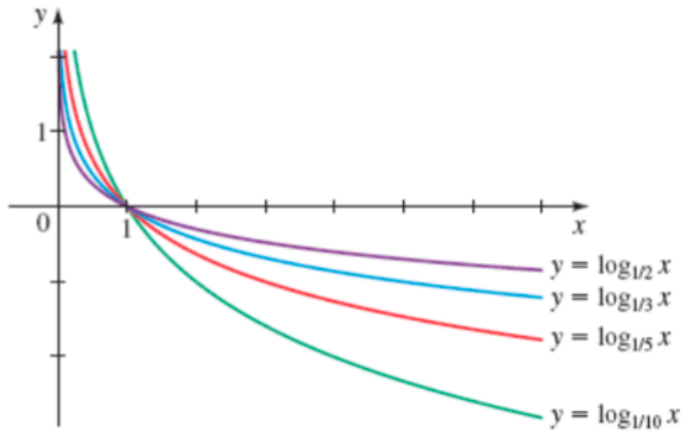


Graphs of the family of Logarithmic Functions

The figure below shows the graphs of the family of logarithmic functions with bases > 1 and < 1



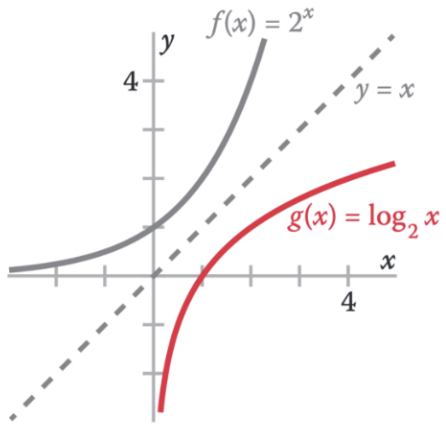
(a) $y = \log_a x$ for $a = 2, 3, 5, 10$



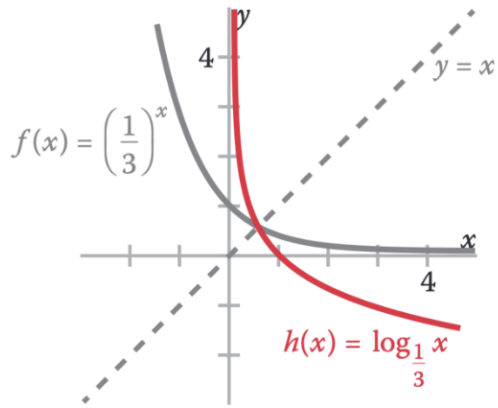
(b) $y = \log_a x$ for $a = \frac{1}{2}, \frac{1}{3}, \frac{1}{5}, \frac{1}{10}$

Logarithmic Functions: Graph Inverses

Since $y = b^x$ and $y = \log_b x$ are inverses, their graphs are symmetric along the line $y = x$.

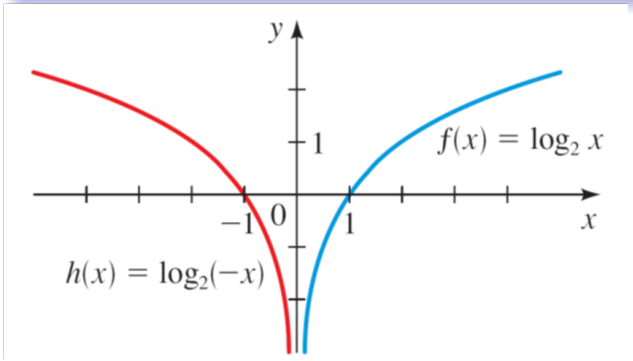


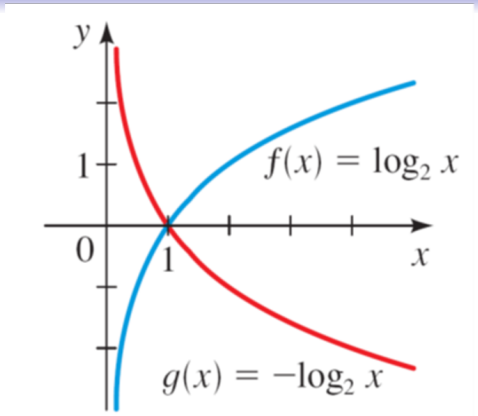
$f(x) = \log_2 x$ is increasing because it is the inverse of an exponential growth function.

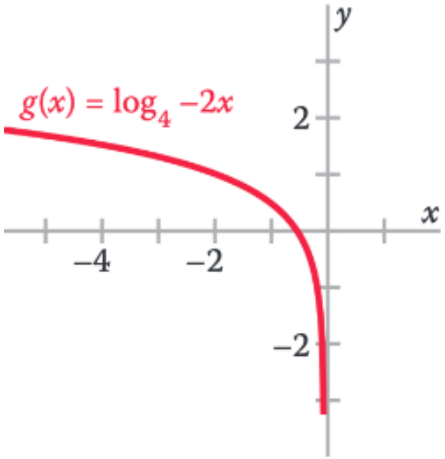


$f(x) = \log_{\frac{1}{3}} x$ is decreasing because it is the inverse of an exponential decay function.

Graph by Transformations

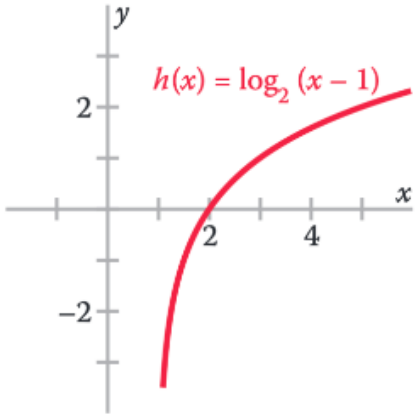






$g(x)$ is a reflection of $f(x) = log_4 x$ along the y-axis

and horizontal compression by a factor of $\frac{1}{2}$



$h(x)$ is the horizontal translation of $f(x) = log_2 x$ one unit to the right

Natural and Common Logarithmic Functions

Natural Logarithm:

The logarithm with base e is called the **natural logarithm** and is denoted by ln:

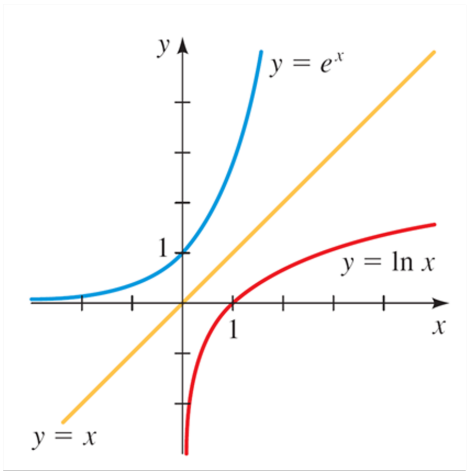
$$lnx = log_e x$$

$y = \ln x$ is the inverse function of $y = e^x$, $\ln x = y \leftrightarrow y = e^x$

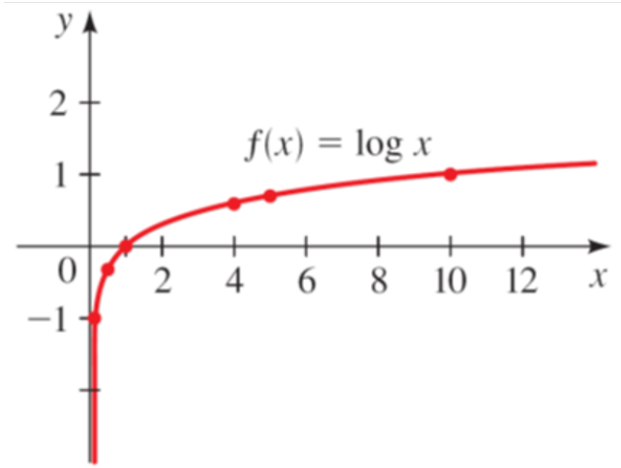
Common Logarithm:

The logarithm with base 10 is called the **common logarithm** and is denoted by omitting the base:

$$logx = log_{10} x$$



Graph of natural exponential function and the natural logarithmic function



Example

Example: Given the graph of $f(x) = \log_2 x$, graph $h(x) = \log_2(x - 2)$ and $g(x) = \log_2 x - 2$