Transformations of Graphs

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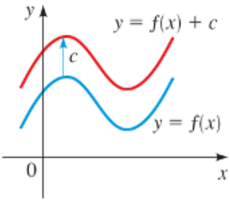
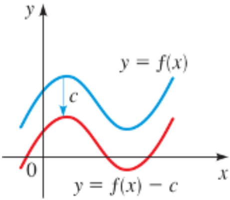
Vertical Shifting

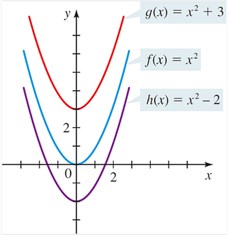
Vertical Shifts Of Graphs:

Suppose $c > 0$,

1. To graph $y = f(x) + c$, shift the graph of $y = f(x)$ upward c units.
2. To graph $y = f(x) - c$, shift the graph of $y = f(x)$ downward c units.







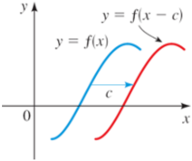
Horizontal Shifting

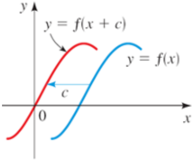
Horizontal Shifts Of Graphs:

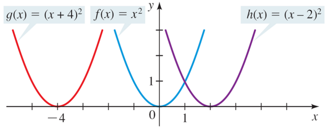
Suppose $c > 0$,

1. To graph $y = f(x - c)$,, shift the graph of $y = f(x)$ to the right c units.
   The value of $f(x - c)$ at x is the same as the value of $f(x)$ at $x - c$.

2. To graph $y = f(x + c)$,, shift the graph of $y = f(x)$ to the left c units.
   The value of $f(x + c)$ at x is the same as the value of $f(x)$ at $x + c$.





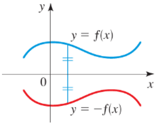


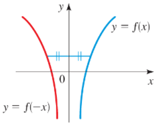
Reflecting Graphs

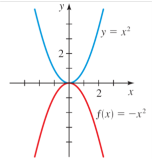
Reflecting Graphs :

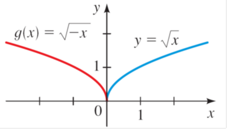
1. To graph $y = -f(x)$, reflect the graph of $y = f(x)$ in the $x-$ axis.

2. To graph $y = f(-x)$, reflect the graph of $y = f(x)$ in the $y-$ axis.









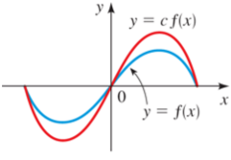
Vertical Stretching and Shrinking

Vertical Stretching And Shrinking Of Graphs :

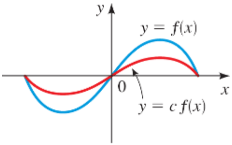
To graph $y = cf(x), c > 0$:

1. If $c > 1$, stretch the graph of $y = f(x)$ vertically by a factor of c.

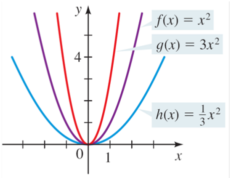
2. If $0 < c < 1$. shrink the graph of $y = f(x)$ vertically by a factor of c.



$c > 1$



$0 < c < 1$

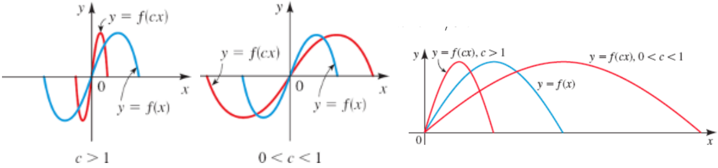


Horizontal Stretching and Shrinking

Horizontal Shrinking And Stretching Of Graphs :
To graph $y = f(cx), c > 0$:

1. If $c > 1$, shrink the graph of $y = f(x)$ horizontally by a factor of $\frac{1}{c}$.

2. If $0 < c < 1$. stretch the graph of $y = f(x)$ horizontally by a factor of $\frac{1}{c}$.



Strategy

Strategy:

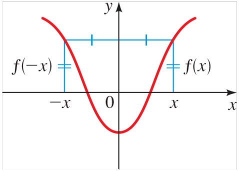
Based on the graph of $y = f(x)$,

- if something changes for the input, i.e. inside the parenthesis (), e.g. $f(2x), f(x+3), f(\frac{1}{2}x - 1)$, then fix a certain y value to check what value for x to take, in the new function , to make the same y value, to determine its transformations horizontally.

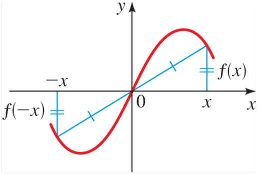
- if something changes for the output, i.e. outside the parenthesis (), e.g. $2f(x), f(x) + 3, \frac{1}{2}f(x) - 1$, then fix a certain x value to check what value for y to take, in the new function, to determine its transformations vertically.

Even and Odd Functions

Even Functions: $f(-x) = f(x)$ for every number x in its domain.
The graph of an even function is symmetric with respect to the y-axis.
eg. $f(x) = x^2$ is even since $f(-x) = (-x)^2 = x^2 = f(x)$



The graph of an even function is symmetric with respect to the y-axis.



The graph of an odd function is symmetric with respect to the origin.

Odd Functions: $f(-x) = -f(x)$ for every number x in its domain.
The graph of an even function is symmetric with respect to the origin.
eg. $f(x) = x^3$ is even since $f(-x) = (-x)^3 = -x^3 = -f(x)$

Example

Example: Sketch the graph of the function
$f(x) = 1 - 2(x-3)^2$