

Your Name:

ID #:

### Worksheet: Limits at Infinity

**Note:** for all the problems, identify the limit as a number,  $-\infty$ ,  $\infty$  or DNE (does not exist)

1. Please keep in mind the list of functions in order of their rate of growth – quickest to slowest:

**Factorial**(  $x!$  ), **Exponential** (  $4^x, e^x$  ), **Algebraic** (  $x^4, x^{0.5}$  ), **Logarithm** (  $\log_2 x, \ln x$  )

Then find the following limits quickly without showing any work:

(a)  $\lim_{x \rightarrow \infty} \frac{x^3}{x!} =$

(g)  $\lim_{n \rightarrow \infty} \frac{5^n + 2}{n^9 + 100} =$

(m)  $\lim_{x \rightarrow \infty} \frac{x^8 + \ln x}{3x^8 + 2} =$

(b)  $\lim_{x \rightarrow \infty} \frac{\sqrt[3]{x^4 + 2x^3 + 1}}{\ln x + 3} =$

(h)  $\lim_{n \rightarrow \infty} \frac{n^4 + 10 \ln n}{4\sqrt{n^8 + 3n + 100}} =$

(n)  $\lim_{x \rightarrow \infty} \frac{x^{10} + 2x^2 + 10}{32x^6 + 12} =$

(c)  $\lim_{x \rightarrow \infty} \frac{x^{10} + 4x^7 + 10}{32x^{12} + 10} =$

(i)  $\lim_{n \rightarrow \infty} \frac{-n^4 + 3n^2}{19n^3 + 3n^2 - 10} =$

(o)  $\lim_{n \rightarrow \infty} \frac{n^5 - 10n^7 + \ln n}{n^3 + 8n^7} =$

(d)  $\lim_{x \rightarrow \infty} \frac{5x + 1}{x^2 - 3x + 4} =$

(j)  $\lim_{x \rightarrow -\infty} \frac{x^{120} + 20}{-x^{100} + e^x} =$

(p)  $\lim_{x \rightarrow \infty} \frac{e^x}{x^3} =$

(e)  $\lim_{x \rightarrow -\infty} \frac{2^x}{x^2} =$

(k)  $\lim_{n \rightarrow -\infty} \frac{5^n + n^2}{-e^n + 1} =$

(q)  $\lim_{x \rightarrow -\infty} (-3x^3 - 4x + 5) =$

(f)  $\lim_{x \rightarrow \infty} \frac{2^x}{x^2} =$

(l)  $\lim_{x \rightarrow -\infty} \frac{2x^3 - 2x + 5}{13x^3 - 5x + 13} =$

(r)  $\lim_{n \rightarrow \infty} \frac{n^5 - 10n^7 + \ln n}{n^3 + 8n^7} =$

2. Practice some common limits without showing the work:

(a)  $\lim_{x \rightarrow -\infty} e^x =$

(d)  $\lim_{x \rightarrow -\infty} e^{-x} =$

(g)  $\lim_{x \rightarrow -\infty} e^{\frac{1}{x}} =$

(b)  $\lim_{x \rightarrow 0} e^x =$

(e)  $\lim_{x \rightarrow 0^+} e^{\frac{1}{x}} =$

(h)  $\lim_{x \rightarrow \infty} e^{\frac{1}{x}} =$

(c)  $\lim_{x \rightarrow -\infty} x \ln x$

(f)  $\lim_{x \rightarrow 0^-} e^{\frac{1}{x}} =$

(i)  $\lim_{x \rightarrow \infty} \frac{1}{e^{-x}} =$

3. Practice more common limits without showing the work:

$$(a) \lim_{x \rightarrow \infty} \ln x = \quad (e) \lim_{x \rightarrow \infty} \sin x = \quad (i) \lim_{x \rightarrow \infty} \sin\left(\frac{1}{x}\right) =$$

$$(b) \lim_{x \rightarrow 0^+} \ln x = \quad (f) \lim_{x \rightarrow \infty} (\sin x)e^{-x} = \quad (j) \lim_{x \rightarrow \infty} x \ln \frac{1}{x} =$$

$$(c) \lim_{x \rightarrow -\infty} (e^{3x} - e^{2x}) = \quad (g) \lim_{x \rightarrow \infty} x \sin x = \quad (k) \lim_{x \rightarrow -\infty} xe^{\frac{1}{x}} =$$

$$(d) \lim_{x \rightarrow \infty} \sin\left(\frac{x^2}{2x^2+x}\right) = \quad (h) \lim_{x \rightarrow \infty} \frac{1}{x} \sin x = \quad (l) \lim_{x \rightarrow 0^-} xe^{\frac{1}{x}} =$$

4. Show all your work for the following questions (limits at discontinuity and infinity)

$$(a) \lim_{x \rightarrow 0} \frac{\sin(3x)}{4x}$$

$$(d) \lim_{x \rightarrow 1} \frac{5x^4 - 4x^2 - 1}{10 - x - 9x^3}$$

$$(b) \lim_{x \rightarrow \infty} x^{\frac{1}{x}}$$

$$(e) \lim_{x \rightarrow -1} \frac{\sqrt{x+4} - 3}{x+1}$$

$$(c) \lim_{x \rightarrow \infty} \left(\frac{1}{x}\right)^{\frac{1}{x}}$$

$$(f) \lim_{x \rightarrow \infty} \left(\frac{x}{x+1}\right)^x$$