Equations with Absolute Values

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Definitions

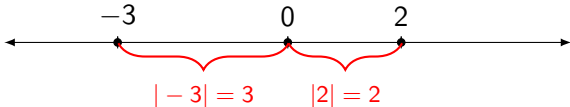**Definition.** $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$.

For example, $|5| = 5$, but $|-5| = -(-5) = 5$.

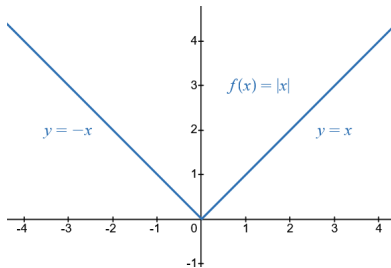*Intuition*: The absolute value of x is the distance between the number x and 0 on a number line.

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Graph of $f(x) = |x|$:

Equation with Absolute Value

Example. *Solve for x: $|2x - 5| = 7$.*

$$|2x - 5| = 7 \implies 2x - 5 = \pm 7$$

$$\implies \begin{cases} 2x - 5 = 7 \implies x = 6 \\ 2x - 5 = -7 \implies x = -1 \end{cases}$$

The two solutions are then $\boxed{x = 6, -1.}$

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Two Absolute Values

Example. *Solve $|x + 1| = 4|3x - 8|$.*

$$|x + 1| = 4|3x - 8| \implies \frac{|x + 1|}{|3x - 8|} = 4$$

$$\implies \left| \frac{x + 1}{3x - 8} \right| = 4$$

$$\implies \frac{x + 1}{3x - 8} = \pm 4.$$

Then $x + 1 = 4(3x - 8)$ *or* $x + 1 = -4(3x - 8)$.

$$\implies \boxed{x = 3, \ \frac{31}{13}.}$$

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Equation with $\sqrt{x^2}$

Recall that $\sqrt{x^2} = |x|$. For example, $\sqrt{(-6)^2} = \sqrt{36} = 6$.

Example.

Solve for u: $\quad \dfrac{18}{u} = \sqrt{x^2} - 2$.

$$\frac{u}{18} = \frac{1}{|x| - 2} \implies \boxed{u = \frac{18}{|x| - 2}}$$