PreCalculus-Graph Exponential Functions (Learning Targets GE)

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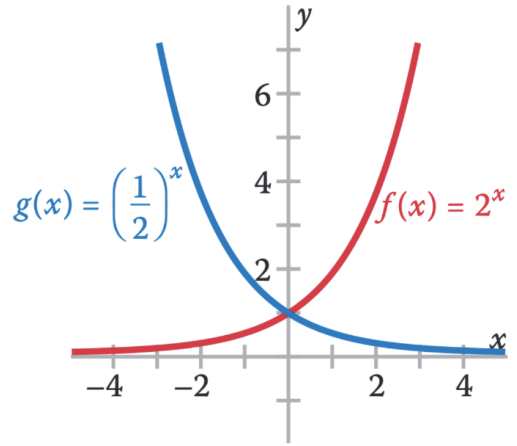
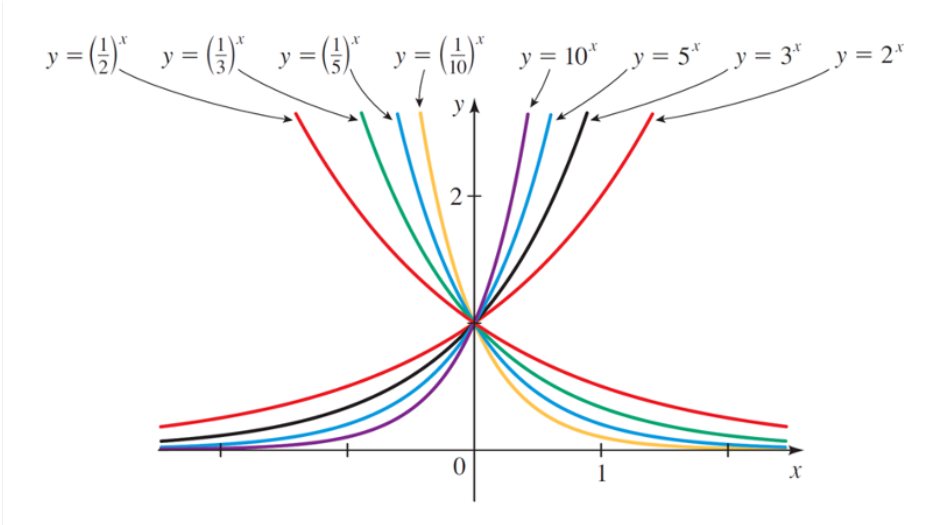
Exponential Functions

- A function of the form $y = b^x$ where b is a real number such that $b > 0$ and $b \neq 1$ is an **exponential function**.

The graph of an exponential function has the following properties:

- A y-intercept at (0, 1)
- A horizontal asymptote at $y = 0$
- Increasing when $b > 1$
- Decreasing when $0 < b < 1$





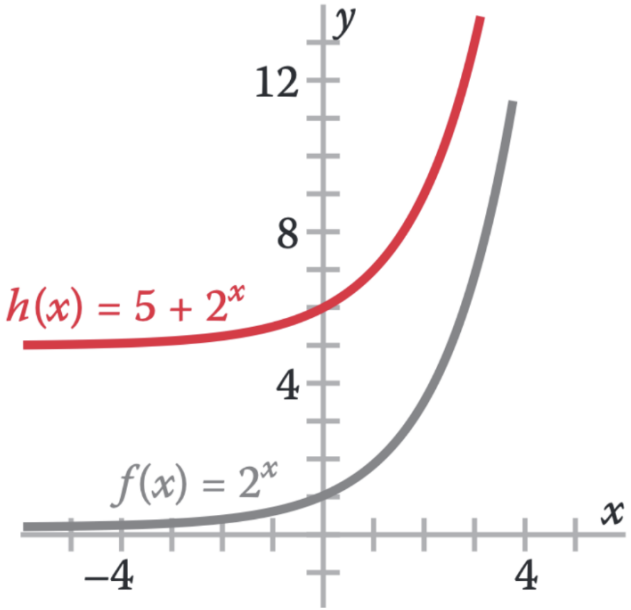
Exponential Functions: Transformations

From the parent function of $y = b^x$, many functions may be graphed using transformations:

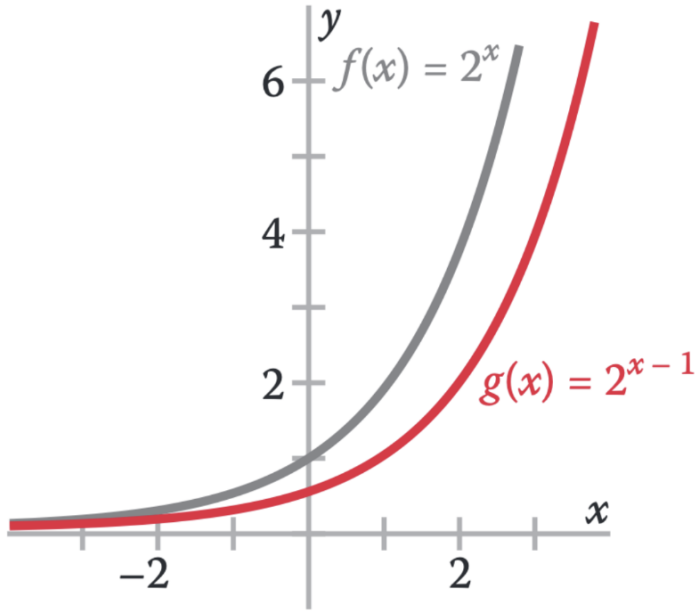
Transformations of Functions	
Vertical translation	$f(x) + k$ or $f(x) - k$
Horizontal translation	$f(x + h)$ or $f(x - h)$
Vertical stretch or compression	$cf(x)$
Reflection over the *x*-axis	$-f(x)$
Reflection over the *y*-axis	$f(-x)$

Exponential Functions: Transformations

Vertical Translation



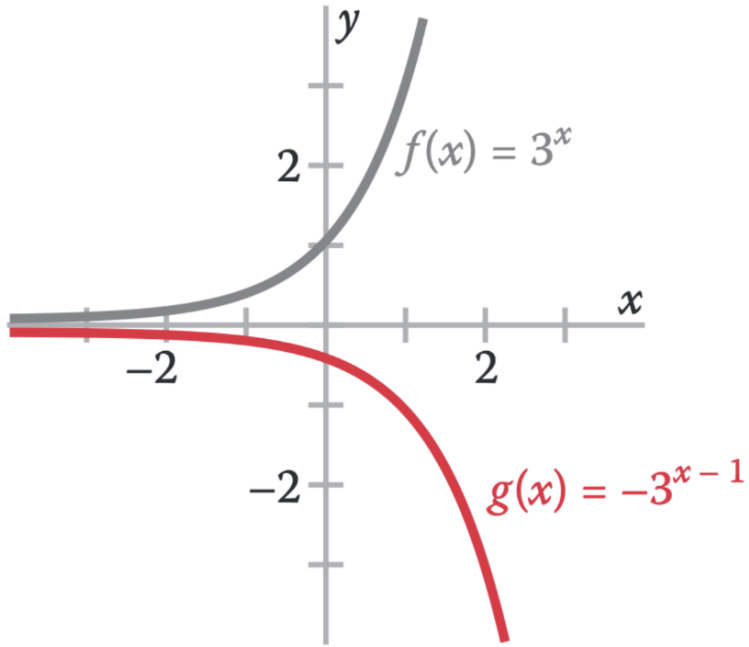
Horizontal Translation



Exponential Functions: Transformations

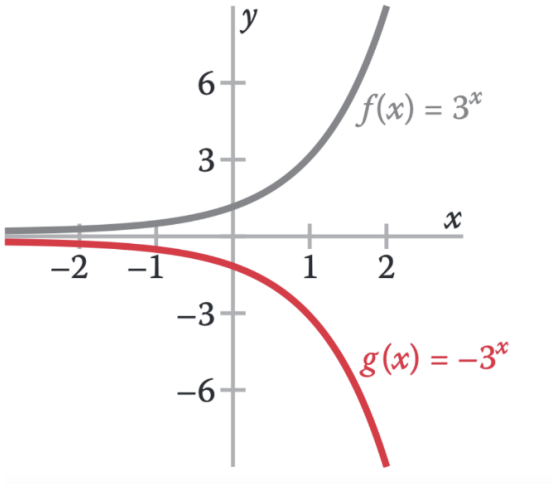
Combinations of Transformations

The function $g(x)$ is obtained from the parent function $f(x)$ by reflecting it over the x-axis and translating it one unit to the right.

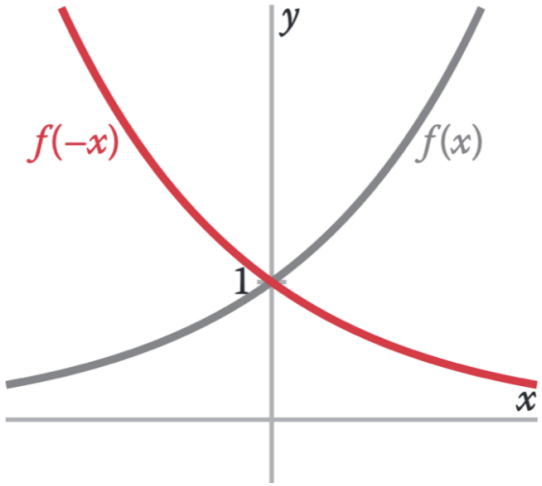


Exponential Functions: Reflections

The graph of $y = -b^x$ is the reflection of $y = b^x$ along the $x-$axis.



The graph of $y = b^{-x}$ is the reflection of $y = b^x$ along the $y-$axis.

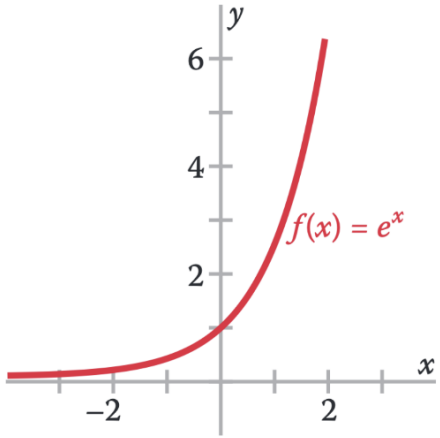


Natural Exponential Functions

The **natural base** $e \approx 2.71828 \ldots$

The number e is irrational, so it cannot be expressed exactly as a decimal, only estimated. Exact values should be in terms of e.

The graph of the parent function $f(x) = e^x$ is an exponential growth function since $e > 1$.



Example

Example: Given the graph of $f(x) = 2^x$, graph $h(x) = 2^{x+2}$ and $g(x) = 2^x - 2$