

4. (a) Find $\lim_{x \rightarrow -2} \frac{x+2}{\sqrt{x+6}-2}$

$$= \lim_{x \rightarrow -2} \frac{(x+2)(\sqrt{x+6}+2)}{(\sqrt{x+6}-2)(\sqrt{x+6}+2)}$$

$$= \lim_{x \rightarrow -2} \frac{(x+2)(\sqrt{x+6}+2)}{(x+6)-4}$$

$$= \lim_{x \rightarrow -2} \frac{(x+2)(\sqrt{x+6}+2)}{x+2}$$

$$= \lim_{x \rightarrow -2} (\sqrt{x+6}+2)$$

$$= \boxed{4}$$

5. (a) Find $\lim_{x \rightarrow 0} \frac{x^2+8\sin(x)}{x}$

$$= \lim_{x \rightarrow 0} \left(\frac{x^2}{x} + 8 \frac{\sin x}{x} \right)$$

$$= \lim_{x \rightarrow 0} x + 8 \lim_{x \rightarrow 0} \frac{\sin x}{x}$$

$$= 0 + 8 \cdot 1$$

$$= \boxed{8}$$

6. (a) Find $\lim_{y \rightarrow 0} \left(\frac{6}{y^2+y} - \frac{6}{y} \right)$

$$= \lim_{y \rightarrow 0} \left(\frac{6}{y(y+1)} - \frac{6(y+1)}{y(y+1)} \right)$$

$$= \lim_{y \rightarrow 0} \frac{6-6y-6}{y(y+1)}$$

$$= \lim_{y \rightarrow 0} \frac{-6y}{y(y+1)}$$

$$= \lim_{y \rightarrow 0} \frac{-6}{y+1}$$

$$= \boxed{-6}$$

(b) Find $\lim_{x \rightarrow -1} \frac{\sqrt{x+10}-3}{x+1}$

$$= \lim_{x \rightarrow -1} \frac{(\sqrt{x+10}-3)(\sqrt{x+10}+3)}{(x+1)(\sqrt{x+10}+3)}$$

$$= \lim_{x \rightarrow -1} \frac{(x+10)-3^2}{(x+1)(\sqrt{x+10}+3)}$$

$$= \lim_{x \rightarrow -1} \frac{x+1}{(x+1)(\sqrt{x+10}+3)}$$

$$= \lim_{x \rightarrow -1} \frac{1}{\sqrt{x+10}+3}$$

$$= \boxed{\frac{1}{6}}$$

(b) Find $\lim_{x \rightarrow 0} \frac{\tan(6x^2)+\sin^2(2x)}{x^2}$

$$= \lim_{x \rightarrow 0} \frac{\tan(6x^2)}{x^2} + \lim_{x \rightarrow 0} \frac{\sin^2(2x)}{x^2}$$

$$= 6 \lim_{x \rightarrow 0} \frac{\tan(6x^2)}{6x^2} + \lim_{x \rightarrow 0} \left[\frac{\sin(2x)}{x} \right]^2$$

$$= 6 + \left[\lim_{x \rightarrow 0} \frac{\sin(2x)}{x} \right]^2$$

$$= 6 + \left[2 \lim_{x \rightarrow 0} \frac{\sin(2x)}{2x} \right]^2$$

$$= 6 + 2^2 = \boxed{10}$$

(b) Find $\lim_{x \rightarrow 0} \frac{10\sin(2x)}{1-\cos(2x)}$

$$= \lim_{x \rightarrow 0} \frac{10\sin(2x)(1+\cos(2x))}{(1-\cos(2x))(1+\cos(2x))}$$

$$= \lim_{x \rightarrow 0} \frac{10\sin(2x)(1+\cos(2x))}{1-\cos^2(2x)}$$

$$= \lim_{x \rightarrow 0} \frac{10\sin(2x)(1+\cos(2x))}{\sin^2(2x)}$$

$$= \lim_{x \rightarrow 0} \frac{10(1+\cos(2x))}{\sin(2x)}$$

$$= \boxed{\infty}$$