Inequalities with Absolute Values

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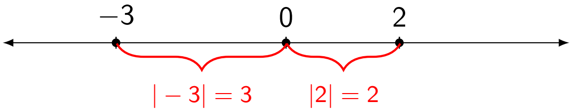
Definition

Definition. $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}.$

For example, $|5| = 5$, but $|-5| = -(-5) = 5$.

Intuition: The absolute value of x is the distance between the number x and 0 on a number line.



Facts

Crucial Facts.

$$|x| = a \implies x = \pm a$$
$$|x| < a \implies -a < x < a$$
$$|x| > a \implies x < -a \quad \text{or} \quad x > a$$

Finally, $\sqrt{x^2} = |x|$. For example, $\sqrt{(-6)^2} = \sqrt{36} = 6$.

Absolute Value Less Than

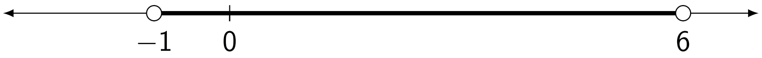
Example. *Solve for x: $|2x-5|<7$.*

<u>Solution</u>.

$$|2x-5|<7 \implies -7<2x-5<7$$

This means $-7<2x-5$ **and** $2x-5<7$:

$$\implies -1<x \quad \text{and} \quad x<6$$



In interval form, the solution is $\boxed{(-1,6)}$.

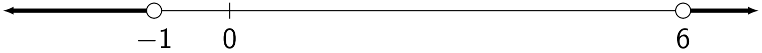
Absolute Value Greater Than

Example. *Solve for x:* $|2x - 5| > 7$.

<u>Solution</u>.

$$|2x-5| > 7 \;\Rightarrow\; 2x-5 < -7 \quad \textbf{or} \quad 2x-5 > 7$$

$$\Rightarrow x < -1 \quad \text{or} \quad x > 6.$$



In interval form, the answer is $\boxed{(-\infty, -1) \cup (6, \infty)}$.