

PreCalculus-Graph Logarithmic Functions (Learning Targets GL)

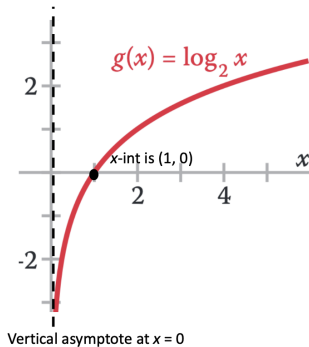
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Logarithmic Functions

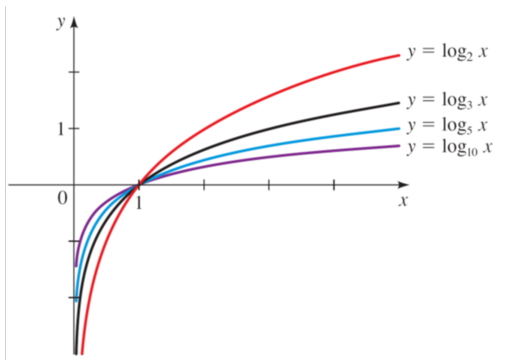
- The function $f(x) = \log_b x$ is called the **logarithmic function with base b** .
- The base b must be a positive number and $b \neq 1$.
- It is often useful to express a logarithmic function in its equivalent exponential form:

$$y = \log_b x \leftrightarrow x = b^y$$



Graphs of the family of Logarithmic Functions

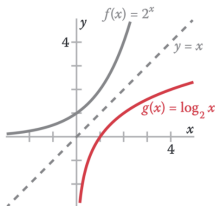
The figure below shows the graphs of the family of logarithmic functions with bases 2, 3, 5, and 10.



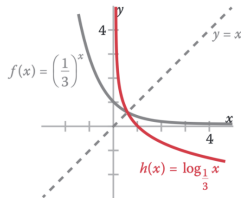
A family of logarithmic functions

Logarithmic Functions: Graph Inverses

Since $y = b^x$ and $y = \log_b x$ are inverses, their graphs are symmetric along the line $y = x$.

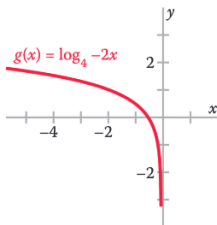
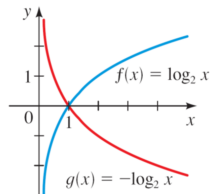
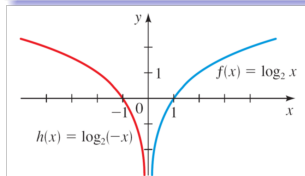


$f(x) = \log_2 x$ is increasing because it is the inverse of an exponential growth function.

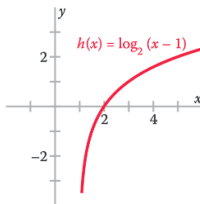


$f(x) = \log_{\frac{1}{3}} x$ is decreasing because it is the inverse of an exponential decay function.

Graph by Transformations



$g(x)$ is a reflection of $f(x) = \log_4 x$ along the y-axis
and horizontal compression by a factor of $\frac{1}{2}$



$h(x)$ is the horizontal translation of $f(x) = \log_2 x$
one unit to the right

Natural and Common Logarithmic Functions

Natural Logarithm:

The logarithm with base e is called the **natural logarithm** and is denoted by \ln :

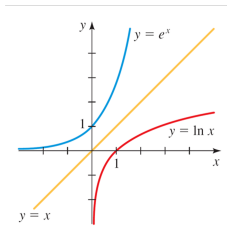
$$\ln x = \log_e x$$

$y = \ln x$ is the inverse function of $y = e^x$, $\ln x = y \leftrightarrow y = e^x$

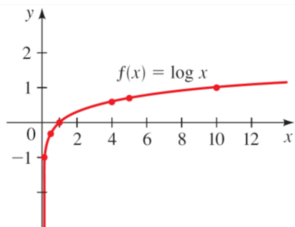
Common Logarithm:

The logarithm with base 10 is called the **common logarithm** and is denoted by omitting the base:

$$\log x = \log_{10} x$$



Graph of natural exponential function and the natural logarithmic function



Example

Example: Graph $h(x) = e^{x+2}$ and $g(x) = e^{x+2}$

