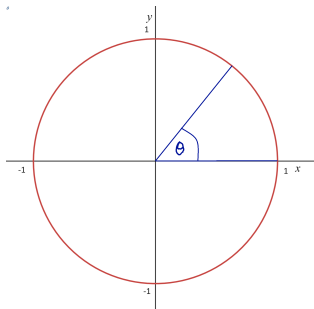# Can Quickly Determine a Trig Function Value

APMA Faculty
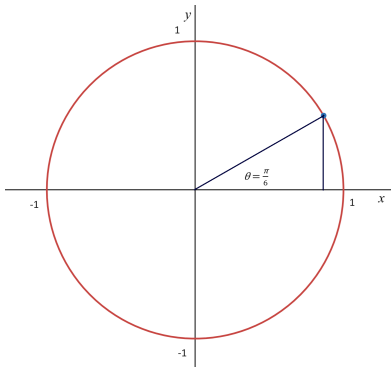University of Virginia

July 30, 2024

Unit Circle

To remember trig function values, especially the values of sine
and cosine, it is very helpful to picture the angle in the unit
circle with the initial side on the positive $x - axis$. Work
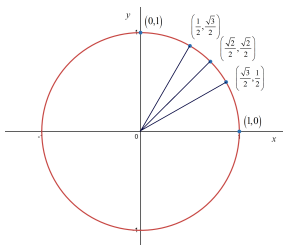through the "Can Use the Unit Circle" unit before proceeding
with this unit.

Tangent

Once you are able to quickly determine the values of Sine and Cosine, you can find Tangent because $\tan \theta = \frac{\sin \theta}{\cos \theta}$.

$\tan \frac{\pi}{6} = \frac{\sin \frac{\pi}{6}}{\cos \frac{\pi}{6}} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

Other Angles

Similarly,

$$\tan\frac{\pi}{4} = \frac{\sin\frac{\pi}{4}}{\cos\frac{\pi}{4}} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1.$$

$$\tan\frac{\pi}{3} = \frac{\sin\frac{\pi}{3}}{\cos\frac{\pi}{3}} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}.$$
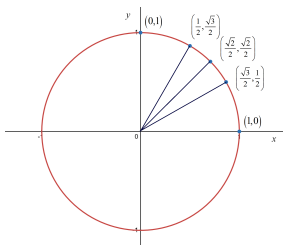
Other Angles

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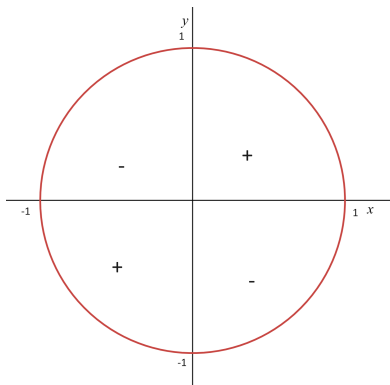
And

$$\tan 0 = \frac{\sin 0}{\cos 0} = \frac{0}{1} = 0.$$

$$\tan \frac{\pi}{2} = \frac{\sin \frac{\pi}{2}}{\cos \frac{\pi}{2}} = \frac{1}{0} \text{ (Undefined)}$$

Sign of Tangent

For angles in the other quadrants, determine the reference angle (the angle to the x-axis). The reference angle determines the magnitude. The quadrant determines the sign of Tangent:

Secant, Cosecant, Cotangent

Determine Secant, Cosecant, and Cotangent by taking the reciprocal of Cosine, Sine, and Tangent, respectively:

$\sec \theta = \frac{1}{\cos \theta}$

$\csc \theta = \frac{1}{\sin \theta}$

$\cot \theta = \frac{1}{\tan \theta}$

A few Examples

Try these:

$\tan \frac{2\pi}{3} =$

$\sec \frac{\pi}{4} =$

$\cot \frac{4\pi}{3} =$

$\csc \frac{5\pi}{6} =$

$\sec \frac{11\pi}{3} =$

$\csc \frac{\pi}{4} =$

$\cot \frac{7\pi}{6} =$