Can Graph Trigonometric Functions

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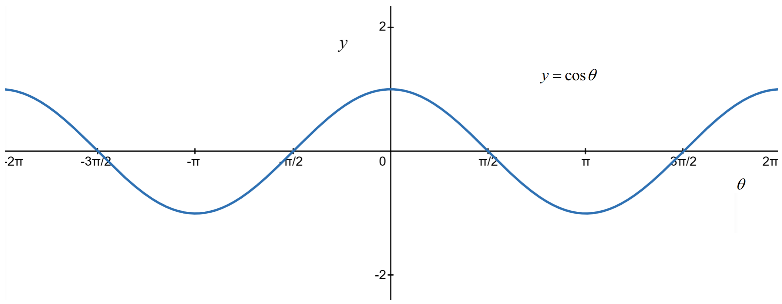
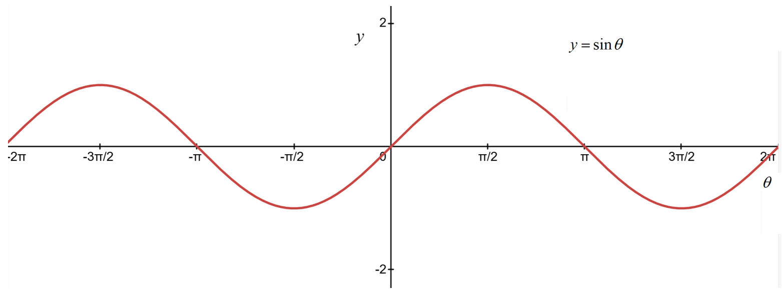
Pre-requisites

Before working through this unit, complete the "Can Use the Unit Circle" unit.

Sine and Cosine

By understanding Sine and Cosine as coordinates on the unit circle, you can see that Sine and Cosine are cyclical and that they are essentially the same function, but shifted by $\frac{\pi}{2}$:





Observations

Sine and Cosine are cyclical (periodic). They repeat with period 2π.

Sine and Cosine are the same graph but shifted by $\pi/2$.

If you remember that $\sin 0 = 0$ and $\cos 0 = 1$, you can distinguish between these graphs.

Note that the graphs have the same shape above the x-axis as below. The midline is the x-axis.

Practice sketching both $y = \sin\theta$ and $y = \cos\theta$.

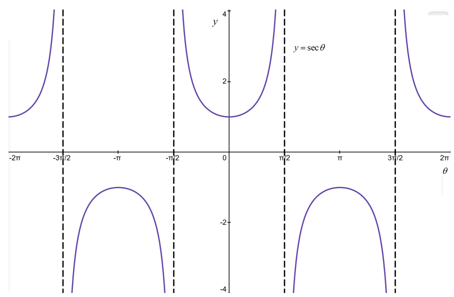
Secant

Because $\sec\theta = \frac{1}{\cos\theta}$ we know that the function is undefined when $\cos\theta = 0$. That is, where $\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$, etc.

We know that $\cos 0 = 1$ and $\cos\pi = -1$, so $\sec 0 = 1$ and $\sec\pi = -1$

Because $\cos\theta$ is periodic with period $= 2\pi$, then $\sec\theta$ is also periodic with period $= 2\pi$.

Here is the graph of $y = \sec\theta$:



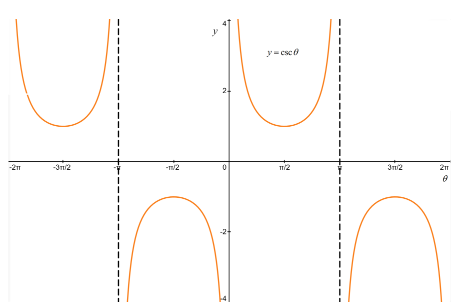
Cosecant

Because $\csc\theta = \frac{1}{\sin\theta}$ we know that the function is undefined when $\sin\theta = 0$. That is, where $\theta = 0, \pi, 2\pi, 3\pi$, etc.

We know that $\sin\frac{\pi}{2} = 1$ and $\sin\frac{3\pi}{2} = -1$, so $\csc\frac{\pi}{2} = 1$ and $\csc\frac{3\pi}{2} = -1$

Because $\sin\theta$ is periodic with period $= 2\pi$, then $\csc\theta$ is also periodic with period $= 2\pi$.

Here is the graph of $y = \csc\theta$:



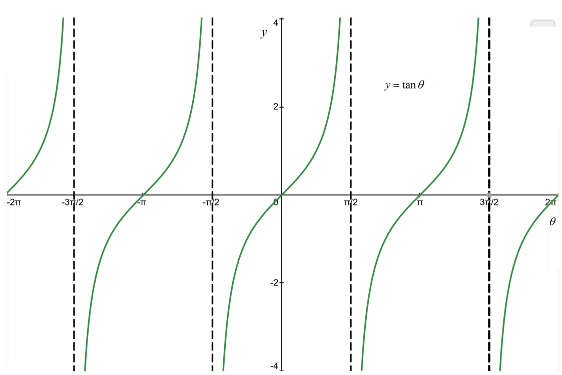
Observation

Sine and Cosine are the same function, but shifted by $\frac{\pi}{2}$, so Cosecant and Secant are the same function, but shifted by $\frac{\pi}{2}$.

Tangent

Because $\tan\theta = \frac{\sin\theta}{\cos\theta}$ we know that the function is undefined when $\cos\theta = 0$. That is, where $\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$, etc.

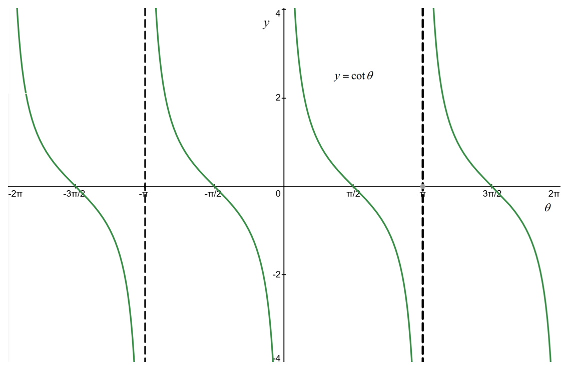
Tangent is positive in quadrants I $(0, \frac{\pi}{2})$ and III $(\pi, \frac{3\pi}{2})$, and negative in quadrants II $(\frac{\pi}{2}, \pi)$ and IV $(\frac{3\pi}{2}, 2\pi)$. Also, $\tan\theta = 0$ at $\theta = 0, \pi, 2\pi$, etc. Here is the graph of $y = \tan\theta$:



Cotangent

Because $\cot\theta = \frac{\cos\theta}{\sin\theta}$ we know that the function is undefined when $\sin\theta = 0$. That is, where $\theta = 0, \pi, 2\pi, 3\pi$, etc.

Cotangent is positive in quadrants I $(0, \frac{\pi}{2})$ and III $(\pi, \frac{3\pi}{2})$, and negative in quadrants II $(\frac{\pi}{2}, \pi)$ and IV $(\frac{3\pi}{2}, 2\pi)$. Also, $\cot\theta = 0$ at $\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$, etc. Here is the graph of $y = \cot\theta$:



Observation

The graphs of Tangent and Cotangent are similar. Knowing where the zeroes and where the asymptotes are will help to sketch the graphs.

Other Graphs

Translations, stretching, and shrinking affect trigonometric graphs in the same way as for other functions. See the "Transformations of Graphs" unit for details.

Practice

Practice by sketching the following:

$y = \sin x$

$y = 2\sin x$

$y = 2\sin 3x$

$y = 2\sin 3x + 1$

More Practice

Sketch the following:

$y = \tan x$

$y = \tan 2x$

$y = \tan(2x + \pi)$