PreCalculus-Graph Power Functions (Learning Targets GP)

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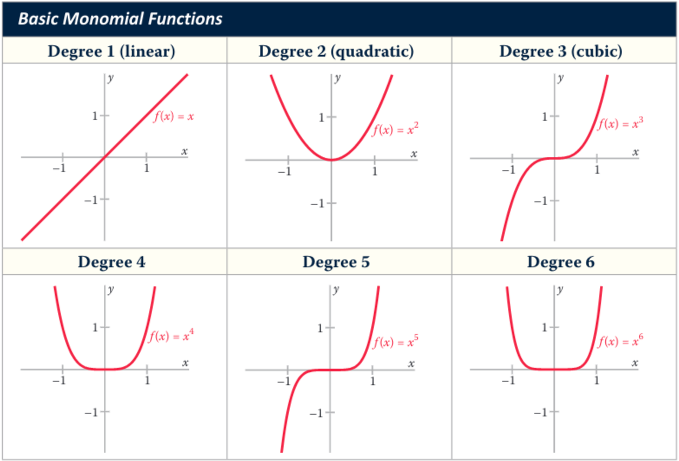
Basic Monomial Functions



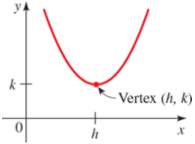
Quadratic Functions

A quadratic function $f(x) = ax^2 + bx + c$ can be expressed in the standard form

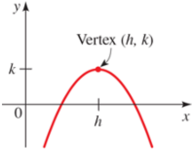
$$f(x) = a(x-h)^2 + k$$

by completing the square, and the discriminant is $D = b^2 - 4ac$.

1. $a > 0$, opens upward
2. $a < 0$, open downward
3. $D > 0$, two $x - intercepts$
4. $D < 0$, no $x - intercept$
5. $D = 0$, one $x - intercept$
6. vertex (h, k)
7. symmetric axis: $x = h = -\frac{b}{2a}$
8. $k = f(-\frac{b}{2a})$



$f(x) = a(x-h)^2 + k, a > 0$

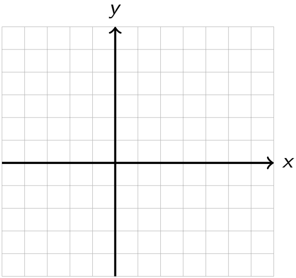


$f(x) = a(x-h)^2 + k, a < 0$

Way 1 to graph $f(x) = ax^2 + bx + c$

- S1 check a to determine whether it opens upward or downward
- S2 find the symmetric axis: $x = -\frac{b}{2a}$
- S3 compute the vertex $(h, k) = (-\frac{b}{2a}, f(-\frac{b}{2a}))$
- S4 find all $x - intercepts$ and $y - intercepts$
- S5 sketch the parabola

Example: Graph $f(x) = 2x^2 - 12x + 13$



Way 2 to graph $f(x) = ax^2 + bx + c$

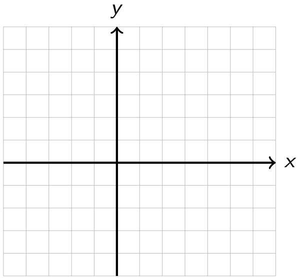
S1 find the standard form $f(x) = a(x-h)^2 + k$

S2 graph $f(x) = x^2$ first

S3 Apply transformations:

$$x^2 \xrightarrow[\text{horizontally}]{\text{shift}} (x-h)^2 \xrightarrow[\text{vertically}]{\text{stretch(shrink)}} a(x-h)^2 \xrightarrow[\text{vertically}]{\text{shift}} a(x-h)^2 + k$$

Redo Example: Graph $f(x) = 2x^2 - 12x + 13$



Polynomial Functions

Define a polynomial function of degree n:

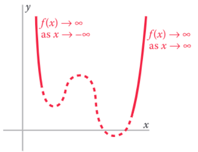
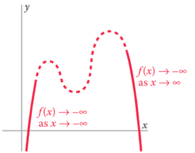
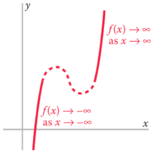
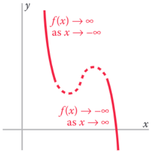
$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where $a_n \neq 0$ and n is nonnegative integer.

- **coefficients:** $a_0, a_1, \cdots, a_n$
- **constant coefficient** or constant term: a_0
- **leading coefficient:** a_n
- **leading term:** $a_n x^n$, which is also called **dominant term**

End Behavior Of Polynomials

End Behavior Of Polynomials:
The end behavior of polynomials $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ is determined by the degree n and the sign of the leading coefficient a_n, as indicated in the following graphs

	$a_n > 0$	$a_n < 0$
n is even	 *If the degree is even and the leading coefficient is positive, then $f(x) \to \infty$ at the left and right ends.*	 *If the degree is even and the leading coefficient is negative, then $f(x) \to -\infty$ at the left and right ends.*
n is odd	 *If the degree is odd and the leading coefficient is positive, then $f(x) \to -\infty$ at the left end and $f(x) \to \infty$ at the right end.*	 *If the degree is odd and the leading coefficient is negative, then $f(x) \to \infty$ at the left end and $f(x) \to -\infty$ at the right end.*

Guidelines for Graphing Polynomial Functions

Guidelines for Graphing Polynomial Functions:

The graph of a polynomial function is a smooth curve; that is, it has no corners or sharp points.

- **Zeros.** Factor the polynomial to find all its real zeros: these are the x-intercepts of the graph.
- **Test Points.** Make a table of values for the polynomial. Include test points to determine whether the graph of the polynomial lies above or below the x-axis on the intervals determined by the zeros. Include the y-intercept in the table.
- **End Behavior.** Determine the end behavior of the polynomial.
- **Graph.** Plot the intercepts and other points you found in the table. Sketch a smooth curve that passes through these points and exhibits the required end behavior.

Example

Example: Graph $p(x) = -2x^4 - x^3 + 3x^2$