

1. Solve for  $x$ .

(a)  $\sqrt{2x+5} + 5 = x$

**Solution:**

$$\begin{aligned}\sqrt{2x+5} + 5 = x &\implies \sqrt{2x+5} = x - 5 \\ \implies 2x + 5 &= (x - 5)^2 = x^2 - 10x + 25 \\ \implies x^2 - 12x + 20 &= 0 \\ \implies (x - 10)(x - 2) &= 0 \\ \implies x &= 2, 10\end{aligned}$$

By inspection, only  $\boxed{x = 10}$  satisfies the original equation.

(b)  $\sqrt{x-3} + \sqrt{x-6} = \sqrt{x+2}$

**Solution:**

$$\begin{aligned}\sqrt{x-3} + \sqrt{x-6} = \sqrt{x+2} &\implies x - 3 + 2(\sqrt{x-3})(\sqrt{x-6}) + x - 6 = x + 2 \\ \implies 2\sqrt{(x-3)(x-6)} &= 11 - x \\ \implies 4(x-3)(x-6) &= (11-x)^2 \\ \implies 4(x^2 - 9x + 18) &= 121 - 22x + x^2 \\ \implies 3x^2 - 14x - 49 &= 0 \\ \implies (3x + 7)(x - 7) &= 0 \\ \implies x &= -\frac{7}{3}, 7\end{aligned}$$

By inspection, only  $\boxed{x = 7}$  satisfies the original equation.

(c)  $\frac{\frac{1}{1+x}}{1 - \left(\frac{1}{1+x}\right)} = 7$

**Solution:**

$$\frac{\frac{1}{1+x}}{\left(\frac{1+x}{1+x}\right) - \left(\frac{1}{1+x}\right)} = 7 \implies \frac{\frac{1}{1+x}}{\frac{x}{1+x}} = 7 \implies \left(\frac{1}{1+x}\right) \left(\frac{1+x}{x}\right) = 7$$

$$\implies \frac{1}{x} = 7 \implies \boxed{x = \frac{1}{7}}$$

$$(d) \frac{1}{x-2} + \frac{6}{x+2} - \frac{7}{x} = \frac{5}{2}$$

**Solution:** Finding a common denominator for all fractions,

$$\frac{x(x+2) + 6x(x-2) - 7(x-2)(x+2)}{x(x-2)(x+2)} = \frac{5x(x-2)(x+2)}{2x(x-2)(x+2)}$$

$$\implies -10x + 28 = \frac{5}{2}x^3 - 10x$$

$$\implies \frac{5}{2}x^3 = 28$$

$$\implies \boxed{x = \sqrt[3]{56/5}}$$

2. Simplify the expressions.

$$(a) \frac{4x}{\sqrt{2x} - \sqrt{x}}$$

**Solution:**

$$\begin{aligned} \frac{4x}{\sqrt{2x} - \sqrt{x}} \left( \frac{\sqrt{2x} + \sqrt{x}}{\sqrt{2x} + \sqrt{x}} \right) &= \frac{4x(\sqrt{2x} + \sqrt{x})}{2x - x} \\ &= \frac{4x(\sqrt{2x} + \sqrt{x})}{x} \\ &= \boxed{4(\sqrt{2x} + \sqrt{x})} \end{aligned}$$

$$(b) \frac{4 - \frac{1}{x+5}}{\frac{7}{x+2} - \frac{3}{x+3}}$$

**Solution:**

$$\begin{aligned} \frac{4 - \frac{1}{x+5}}{\frac{7}{x+2} - \frac{3}{x+3}} &= \frac{\frac{4(x+5) - 1}{x+5}}{\frac{7(x+3) - 3(x+2)}{(x+2)(x+3)}} \\ &= \left( \frac{4x+19}{x+5} \right) \left( \frac{x^2+5x+6}{4x+15} \right) \\ &= \boxed{\frac{(4x+19)(x^2+5x+6)}{(x+5)(4x+15)}} \end{aligned}$$

We could obviously continue by multiplying out the numerator and denominator.

3. Provide some choice of numbers  $x, y, z$  such that

$$\sqrt{x^2 + y^2 + z^2} \neq x + y + z.$$

**Solution:**

Choose  $x = y = z = 1$  and we have  $\sqrt{3}$  vs. 3.