Ben-Gurion University of the Negev | Data Structures | Spring 2016

Assignment 1

מרצה אחראית: הדסה דלטרוף
מתרגלים אחראים : דינה סבטליצקי, דן שולמן

נהלי הגשת עבודה:
-את העבודה יש להגיש בזוגות, הגשה ביחידים עם אישור מיוחד באמצעות הטופס.
-העבודה חייבת להיות מוקלדת (לא בכתב יד), בפורמט PDF.
- הגשת הקובץ למערכת ההגשה.
-שאלות לגבי העבודה יש להעלות בפורום של הקורס או בשעות קבלה של המרצה\המתרגל האחראיים על העבודה.

ענו על השאלות בעבודה בפירוט והראו את חישוביכם.
*ייתכן ולא כל הסעיפים ייבדקו.

Question 1 (*25 pts*):

Prove or disprove:

*In all sections you may assume that f and g are monotonically increasing functions for n>0 and that $\forall n: f(n), g(n) \geq 1$

a. $f(n) = O((f(n))^2)$

b. If $f(n) = O(n)$ then $2^{f(n)} = O(2^n)$

c. $\max(f(n), g(n)) = \Theta(f(n) + g(n))$

d. There exists f such that $f(n) = \Omega(\log n)$ and $(f(n))^2 = O(f(n))$.

e. If $f(g(n)) = O(n)$ and $f(n) = \Omega(n)$, then $g(n) = O(n)$.

Question 2 (*15 pts*):

Choose only 5 sections

Prove the following claims:

a. $(n+1)^5 = O(n^5)$

b. $3n \cdot \log_2 n + 2n = \Omega(n \cdot \log_2 n)$

c. $n \cdot (2^n + 3^{n+1})^2 = O(n \cdot 9^n)$

d. $\log_3 2^n = O(n)$

e. $n^2 - 10n = \Omega(n^2)$

f. $(2^n + 3^{n+1})^2 = O(n * 9^n)$

g. $\log_3 2^n = O(n)$

Question 3 (*28 pts*):

Choose only 7 sections

Solve the following recurrence relations (assume *T(a)=Θ(1)* for any constant *a*). Give your answer in the terms of O and Ω, or Θ if they are the same:

a. $T(n) = T\left(\frac{2n}{3}\right) + T(\frac{n}{3})$

b. $T(n) = 6T\left(\frac{n}{2}\right) + n^3$

c. $T(n) = 5T\left(\frac{n}{2}\right) + n^3\, log_2\, n$

d. $T(n) = cT(\sqrt{n}) + n,\ \ c\ is\ a\ constant > 0$

e. $T(n) = \sqrt{n}\ * T(\sqrt{n}) + nlog_2 n$

f. $T(n) = 2T\left(\frac{n}{2}\right) + n^3$

g. $T(n) = 2T\left(\frac{n}{4}\right) + 1$

h. $T(n) = T(n-1) + n$

i. $T(n) = T\left(n^{\frac{1}{2}}\right) + 1$

j. $T(n) = 2T(n-1) + O(1)$

Question 4 (*12 pts*):

Given $T(n) = 16T\left(\frac{n}{4}\right) + f(n)$, find a function $f(n)$ (if exists) such that the solution of $T(n)$ is equal to the written in the following sections.

Prove your answer for each section (could be different $f(n)$ for each section).

a. $\Theta(n \log n)$

b. $\Theta(n^2)$

c. $\Theta(n^2 \cdot \log n)$

d. $\Theta(n^4)$

Question 5 (*15 pts*):
What is the computational complexity bound (in *Θ)* of the following functions? Explain.

a. **function BubbleSort(A[1..n])**
```
for i←1 to n -1
    for j← n downto i+1
        if A[j-1] > A[j]
            temp =A[j-1];
            A[j-1] = A[j];
            A[j] = temp;
```

b. **function exp(base , power)**
```
if (power = 0)
        return 1
else if (power = 1)
        return base
else
        return base * exp(base, power-1)
```

c. **function exp2(base , power)**
```
if (power = 0)
        return 1
else if (power = 1)
        return base
else if (mod(power, 2) = 0)
        tmp ← exp2(base, power/2)
        return tmp * tmp
else
        return  base * exp2(base, power-1)
```

Question 6 (*5 pts*)

a. Plot the graphs $f_1(n) = n^2, \; g(n) = n^3$ on the same x-y axis
b. Plot the graphs $f_2(n) = 2n^2, g(n) = n^3$ on another x-y axis

(You can use WolframAlpha or Google – by writing plot x^2, x^3 for example)

What is the relations between $f_1(n), f_2(n), g(n)$? (in terms of O , Ω, or Θ).
What is the value of n_0 for (a) and (b) given that c=1? Mark n_0 on the plots. How can compute n_0 mathematically?

c. Find 2 monotonically increasing functions $f_3(n), g_2(n)$ for n>0, with more than one point of intersection, so that $f_3(n) = O(g_2(n))$. Write them down and plot these functions on the same x-y axis.