

vi_and_pi

August 11, 2019

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[1]: ### MDP Value Iteration and Policy Iteration

import numpy as np
import gym
import time
from lake_envs import *

np.set_printoptions(precision=3)

"""
For policy_evaluation, policy_improvement, policy_iteration and
↳value_iteration,
the parameters P, nS, nA, gamma are defined as follows:

P: nested dictionary
From gym.core.Environment
For each pair of states in [1, nS] and actions in [1, nA], P[state][action]
↳is a
tuple of the form (probability, nextstate, reward, terminal) where
- probability: float
the probability of transitioning from "state" to "nextstate" with "action"
- nextstate: int
denotes the state we transition to (in range [0, nS - 1])
- reward: int
either 0 or 1, the reward for transitioning from "state" to
"nextstate" with "action"
- terminal: bool
True when "nextstate" is a terminal state (hole or goal), False otherwise
nS: int
number of states in the environment
nA: int
number of actions in the environment
gamma: float
Discount factor. Number in range [0, 1)
"""
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[1]: '\nFor policy_evaluation, policy_improvement, policy_iteration and value_iteration,\nthe parameters P, nS, nA, gamma are defined as follows:\n\n P: nested dictionary\n From gym.core.Environment\n For each pair of states in [1, nS] and actions in [1, nA], P[state][action] is a\n tuple of the form (probability, nextstate, reward, terminal) where\n - probability: float\n the probability of transitioning from "state" to "nextstate" with "action"\n - nextstate: int\n denotes the state we transition to (in range [0, nS - 1])\n - reward: int\n either 0 or 1, the reward for transitioning from "state" to\n "nextstate" with "action"\n - terminal: bool\n True when "nextstate" is a terminal state (hole or goal), False otherwise\n nS: int\n number of states in the environment\n nA: int\n number of actions in the environment\n gamma: float\n Discount factor. Number in range [0, 1)\n'

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[2]: def policy_evaluation(P, nS, nA, policy, gamma=0.9, tol=1e-3):

    """Evaluate the value function from a given policy.
    Parameters
    -----
    P, nS, nA, gamma:
        defined at beginning of file
    policy: np.array[nS]
        The policy to evaluate. Maps states to actions.
    tol: float
        Terminate policy evaluation when
        max |value_function(s) - prev_value_function(s)| < tol
    Returns
    -----
    value_function: np.ndarray[nS]
        The value function of the given policy, where value_function[s] is
        the value of state s
    """

    V_prime, V = np.zeros(nS), np.zeros(nS)

    #####
    # YOUR IMPLEMENTATION HERE #
    while True:
        V = V_prime
        V_prime = np.zeros(nS)
        for s in range(nS):
            for pr, new_s, reward, _ in P[s][ policy[s] ]:
                V_prime[s] += pr*reward + gamma*pr*V[new_s]
            if np.max(V_prime-V) < tol:
                break
        value_function = V_prime
    #####
    return value_function
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[3]: def policy_improvement(P, nS, nA, value_from_policy, policy, gamma=0.9):

    """Given the value function from policy improve the policy.

    Parameters
    -----
    P, nS, nA, gamma:
        defined at beginning of file
    value_from_policy: np.ndarray
        The value calculated from the policy
    policy: np.array
        The previous policy.

    Returns
    -----
    new_policy: np.ndarray[nS]
        An array of integers. Each integer is the optimal action to take
        in that state according to the environment dynamics and the
        given value function.
    """

    new_policy = np.zeros(nS, dtype='int')
    value = np.zeros(nA)

    #####
    # YOUR IMPLEMENTATION HERE

    for s in range(nS):
        for a in range(nA):
            for pr, new_s, reward, _ in P[s][a]:
                value[a] += pr*reward + gamma*pr*value_from_policy[new_s]
            new_policy[s] = np.argmax(value)
        value = np.zeros(nA)

    #####
    return new_policy

def policy_iteration(P, nS, nA, gamma=0.9, tol=10e-3):
    """Runs policy iteration.

    You should call the policy_evaluation() and policy_improvement() methods to
    implement this method.

    Parameters
    -----
    P, nS, nA, gamma:
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        defined at beginning of file
    tol: float
        tol parameter used in policy_evaluation()
    Returns:
    -----
    value_function: np.ndarray[nS]
    policy: np.ndarray[nS]
    """

    policy = np.random.randint(nA, size=nS, dtype = int)

    #####
    # YOUR IMPLEMENTATION HERE #

    while True:
        value = policy_evaluation(P, nS, nA, policy, gamma, tol)
        policy_improved = policy_improvement(P, nS, nA, value, policy, gamma)
        if all(policy_improved == policy):
            break
        policy = policy_improved
    #####
    policy = policy_improved
    value_function = policy_evaluation(P, nS, nA, policy_improved, gamma, tol)
    return value_function, policy

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[4]: def value_iteration(P, nS, nA, gamma=0.9, tol=1e-3):
    """
    Learn value function and policy by using value iteration method for a given
    gamma and environment.

    Parameters:
    -----
    P, nS, nA, gamma:
        defined at beginning of file
    tol: float
        Terminate value iteration when
            max |value_function(s) - prev_value_function(s)| < tol
    Returns:
    -----
    value_function: np.ndarray[nS]
    policy: np.ndarray[nS]
    """

    policy = np.random.randint(nA, size=nS)
    V_prime, V = np.zeros(nS), np.zeros(nS)
    value = np.zeros(nA)

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#####
# YOUR IMPLEMENTATION HERE #

while True:
    V = V_prime
    for s in range(nS):
        for a in range(nA):
            for pr, new_s, reward, _ in P[s][a]:
                value[a] += pr*reward + gamma*pr*V_prime[new_s]
            policy[s] = np.argmax(value)
            value = np.zeros(nA)
        V_prime = policy_evaluation(P, nS, nA, policy, gamma, tol)
    if np.max(V_prime-V) < tol:
        break
value_function = V_prime

#####
return value_function, policy
```

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[5]: def render_single(env, policy, max_steps=100):
    """
    This function does not need to be modified
    Renders policy once on environment. Watch your agent play!

    Parameters
    -----
    env: gym.core.Environment
        Environment to play on. Must have nS, nA, and P as
        attributes.
    Policy: np.array of shape [env.nS]
        The action to take at a given state
    """

    episode_reward = 0
    ob = env.reset()
    for t in range(max_steps):
        env.render()
        time.sleep(0.25)
        a = policy[ob]
        ob, rew, done, _ = env.step(a)
        episode_reward += rew
        if done:
            break
        env.render();
    if not done:
        print("The agent didn't reach a terminal state in {} steps.".
        →format(max_steps))
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else:
    print("Episode reward: %f" % episode_reward)

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[8]: # Edit below to run policy and value iteration on different environments and
      # visualize the resulting policies in action!
      # You may change the parameters in the functions below
      if __name__ == "__main__":

          # comment/uncomment these lines to switch between deterministic/stochastic
          # environments
          # env = gym.make("Deterministic-4x4-FrozenLake-v0")
          env = gym.make("Deterministic-8x8-FrozenLake-v0")
          # env = gym.make("Stochastic-4x4-FrozenLake-v0")

          print("\n" + "-"*25 + "\nBeginning Policy Iteration\n" + "-"*25)

          V_pi, p_pi = policy_iteration(env.P, env.nS, env.nA, gamma=0.9, tol=1e-3)
          render_single(env, p_pi, 50)

          print("\n" + "-"*25 + "\nBeginning Value Iteration\n" + "-"*25)

          V_vi, p_vi = value_iteration(env.P, env.nS, env.nA, gamma=0.9, tol=1e-3)
          render_single(env, p_vi, 50)

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Beginning Policy Iteration

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Episode reward: 1.000000

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Beginning Value Iteration
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Episode reward: 1.000000

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