Draft version July 28, 2020
Typeset using LaTeX **modern** style in AASTeX62

Interpretable Gaussian Processes for Stellar Light Curves

Rodrigo Luger[1,2,*]

[1] *Center for Computational Astrophysics, Flatiron Institute, New York, NY*
[2] *Virtual Planetary Laboratory, University of Washington, Seattle, WA*

Keywords: methods: analytic

1. THE SPOT EXPANSION

We adopt the following expression for the spherical harmonic coefficient of degree l and order m in the expansion of a spot at $\theta = \varphi = 0$:

$$y_{lm}(\delta, r, \theta = 0, \varphi = 0) = \begin{cases} 1 - \dfrac{\delta c r}{2(1 + cr)} & l = m = 0 \\ \\ -\dfrac{\delta c r\,(2 + cr)}{2\sqrt{2l+1}(1 + cr)^{l+1}} & l > 0, m = 0 \\ \\ 0 & m \neq 0 \end{cases} \tag{1}$$

where $\delta \in [-\infty, 1]$ is the fractional decrease in the brightness at the center of the spot and $r \in [0, 1]$ is a normalized spot radius. The quantity c is a normalization constant for the radius (see below).

The expression in Equation (1) is convenient because it satisfies three important properties:

1. The surface intensity monotonically increases away from the spot center
2. The surface intensity at the spot center is $1 - \delta$
3. The surface intensity at the antipode of the spot center is unity

rluger@flatironinstitute.org
* Flatiron Fellow

These properties may be demonstrated by considering the expression for the surface intensity at a given point (θ, φ):

$$I(\theta,\varphi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} y_{lm} Y_{lm}(\theta,\varphi) \tag{2}$$

$$= \sum_{l=0}^{\infty} y_{l0} \sqrt{2l+1} P_l(\cos\theta) \tag{3}$$

where P_l is the Legendre polynomial of degree l and we have implicitly assumed a normalization such that the integral of our expansion over the unit sphere is 4π. Combining this with Equation (1) and rearranging, we may write

$$I(\theta,\varphi) = 1 + \frac{\delta cr}{2} - \frac{\delta cr\,(2+cr)}{2(1+cr)} \sum_{l=0}^{\infty} \left(\frac{1}{1+cr}\right)^l P_l(\cos\theta)\,. \tag{4}$$

The summation in Equation (4) has a closed-form expression in terms of the generating function of the Legendre polynomials:

$$\sum_{l=0}^{\infty} t^l P_l(\cos\theta) = \frac{1}{\sqrt{1+t^2-2t\cos\theta}}\,, \tag{5}$$

so we may express the intensity in the fairly simple form

$$I(\theta,\varphi) = A - \frac{B}{\sqrt{C-\cos\theta}}\,, \tag{6}$$

where

$$A = 1 + \frac{\delta cr}{2} \tag{7}$$

$$B = \delta cr(2+cr)\sqrt{\frac{1}{8+8cr}} \tag{8}$$

$$C = \frac{1+(1+cr)^2}{2+2cr} \tag{9}$$

are positive constants.

Differentiating Equation (6) with respect to θ, we have

$$\frac{\mathrm{d}I(\theta,\varphi)}{\mathrm{d}\theta} = -\frac{B\sin\theta}{2(C-\cos\theta)^{\frac{3}{2}}}\,, \tag{10}$$

which is zero only for $\theta = 0$ (for which $I(\theta,\varphi)$ is minimized) and $\theta = \pi$ (for which it is maximized). The intensity therefore increases monotonically from the spot center to the antipode, as stated in **1**. The value at the minimum is

$$I_{\min} = A - \frac{B}{\sqrt{C-1}} \tag{11}$$

$$= 1 - \delta\,, \tag{12}$$

as stated in **2**, and the value at the maximum is

$$I_{\max} = A - \frac{B}{\sqrt{C+1}} \tag{13}$$

$$= 1\,, \tag{14}$$

as stated in **3**.