I worked with: None

A lamina is made of carbon fiber with E_1=276 GPa, E_2=19.5 GPa, ν_{12}= 0.28, ν_{23}= 0.70, G_{12}=70 GPa and epoxy with E=4.76 GPa, ν=0.37.

Predict the effective properties of the lamina with respect to the fiber volume fraction of 0.5, 0.6, and 0.7 using the hybrid rule of mixture (ROM).

Solution:

Procedure:

The effective in-plane engineering constants using hybrid rule of mixture (HROM) are given by the following equations:

$$\boxed{\begin{aligned}
E_1^{H*} &= V^f E_1^f + V^m E^m \\
\nu_{12}^{H*} &= V^f \nu_{12}^f + V^m \nu^m \\
\frac{1}{E_2^{H*}} &= \frac{V^f}{E_2^f} + \frac{V^m}{E^m} - \frac{V^f V^m \left(E^m \nu_{12}^f - E_1^f \nu^m\right)^2}{E_1^f E^m (E_1^f V^f + E^m V^m)} \\
\nu_{23}^{H*} &= E_2^{H*}\left[V^f\left(\frac{\nu_{23}^f}{E_2^f} + \frac{(\nu_{12}^f)^2}{E_1^f}\right) + \frac{V^m \nu^m}{E^m}(1+\nu^m) - \frac{(\nu_{12}^{H*})^2}{E_1^{H*}}\right] \\
\frac{1}{G_{12}^{H*}} &= \frac{V^f}{G_{12}^f} + \frac{V^m}{G^m}
\end{aligned}} \tag{1}$$

Where, $V^m = (1 - V^f)$

Results:

Fiber volume fraction = 0.5

Effective Engineering Constant	**Value**
E_1^{H*}	140.38 GPa
ν_{12}^{H*}	0.33
E_2^{H*}	8.55 GPa
ν_{23}^{H*}	0.60
G_{12}^{H*}	3.39 GPa

Table 1: Effective engineering constants of the lamina computed using HROM for fiber volume fraction ($V_f = 0.5$)

Fiber volume fraction = 0.6

Effective Engineering Constant	**Value**
E_1^{H*}	167.50 GPa
ν_{12}^{H*}	0.32
E_2^{H*}	9.64 GPa
ν_{23}^{H*}	0.61
G_{12}^{H*}	4.19 GPa

Table 2: Effective engineering constants of the lamina computed using HROM for fiber volume fraction ($V_f = 0.6$)

Fiber volume fraction = 0.7

Effective Engineering Constant	**Value**
E_1^{H*}	194.63 GPa
ν_{12}^{H*}	0.31
E_2^{H*}	11.04 GPa
ν_{23}^{H*}	0.63
G_{12}^{H*}	5.47 GPa

Table 3: Effective engineering constants of the lamina computed using HROM for fiber volume fraction ($V_f = 0.7$)

Codes:

- 'HROM.m' - Computes effective engineering constants for a lamina using Hybrid Rule of Mixtures (HROM) taking volume fraction as the input

```matlab
% Compute effective engineering constants using
% HROM

function PROPS = HROM(Vf)

% Input:
% Vf: Fiber volume fraction

% Output:
% PROPS(1) = E1H*;
% PROPS(2) = nu12H*;
% PROPS(3) = E2H*;
% PROPS(4) = nu23H*;
% PROPS(5) = G12H*;

global E1 E2 nu12 nu23 G12
global E nu

Gm = 0.5*E/(1+nu);

Vm = 1-Vf;

E1H   = Vf*E1 + Vm*E;
nu12H = Vf*nu12 + Vm*nu;

%------------------------------------
term1 = Vf/E2; term2 = Vm/E;
term3 = (Vf*Vm*(E*nu12 - E1*nu)^2)/(E1*E*(E1*Vf + E*Vm));

E2H = 1/(term1+term2-term3);
%------------------------------------
term4 = Vf*((nu23/E2)+((nu12^2)/E1));
term5 = Vm*nu*(1+nu)/E;
term6 = nu12H^2/E1H;

nu23H = E2H*(term4+term5-term6);

%------------------------------------
G12H = 1/((Vf/G12)+(Vm/Gm));

PROPS = [E1H, nu12H, E2H, nu23H, G12H];

end
```

- 'Proj_Part1.m'

```
% Part-1: HROM for Lamina Properties

% Fiber (Carbon) Properties
global E1 E2 nu12 nu23 G12
E1 = 276.0*1e9; % in Pa
E2 = 19.5*1e9;  % in Pa
nu12 = 0.28;
nu23 = 0.70;
G12 = 70.0*1e9; % in Pa


% Matrix (Epoxy) Properties
global E nu

E = 4.76*1e9; % in Pa
nu = 0.37;

% Volume fraction of fiber
Vf = [0.5,0.6,0.7];

% Calculate Effective Engineering constants
% using HROM for different Volume Fractions

for i=1:3
    PROPS = HROM(Vf(i));
    fprintf("Fiber Volume Fraction: %0.2f \n",Vf(i))
    fprintf("-------------------------------------- \n")
    fprintf("E1H*   : %0.2f GPa \n", 1e-9*PROPS(1))
    fprintf("nu12H* : %0.2f \n", PROPS(2))
    fprintf("E2H*   : %0.2f GPa \n", 1e-9*PROPS(3))
    fprintf("nu23H* : %0.2f \n", PROPS(4))
    fprintf("G12H*  : %0.2f GPa \n", 1e-9*PROPS(5))
    fprintf("====================================== \n")
    fprintf("\n")
end
```

Assume that a $[\pm 45/0/90]_s$ laminate is made of composite layers with the lamina of fiber volume fraction equal to 0.5. The thickness of each layer is 0.127 mm. The laminate is subject to tensile load N_{22}. Assume that the lamina fails according to the Tsai-Wu failure criterion and once a layer is failed then the stiffness matrix of that ply is degraded to zero. Compute the ultimate failure load N_{22}. The strength parameters are given in the following table.

Vf	**X (MPa)**	**X' (MPa)**	**Y (MPa)**	**Y' (MPa)**	**S (MPa)**	**R (MPa)**
0.5	2100	2100	62	210	100	100
0.6	2400	2400	68	230	110	110
0.7	2500	2500	71	240	120	120

Table 4: Strength parameters of different fiber volume fractions.

Solution:

Procedure:

1. Load in the Lamina properties when the fiber volume fraction V^f is 0.5

Effective Engineering Constant	**Value**
E_1^{H*}	140.38 GPa
ν_{12}^{H*}	0.33
E_2^{H*}	8.55 GPa
ν_{23}^{H*}	0.60
G_{12}^{H*}	3.39 GPa

2. Find [Q] for the lamina in Material Coordinate System

$$[Q] = \begin{bmatrix} 1.4129 & 0.0280 & 0 \\ 0.0280 & 0.0861 & 0 \\ 0 & 0 & 0.0339 \end{bmatrix} * 10^{11} \text{ Pa}$$

3. Find [Q'] for each lamina in the Global Coordinate System

$$[\overline{Q}_{\pm 45}] = \begin{bmatrix} 4.2264 & 3.5483 & \pm 3.3170 \\ 3.5483 & 4.2264 & \pm 3.3170 \\ \pm 3.3170 & \pm 3.3170 & 3.6076 \end{bmatrix} * 10^{10} \text{ Pa}$$

$$[\overline{Q}_0] = \begin{bmatrix} 1.4129 & 0.0280 & 0 \\ 0.0280 & 0.0861 & 0 \\ 0 & 0 & 0.0339 \end{bmatrix} * 10^{11} \text{ Pa}$$

$$[\overline{Q}_{90}] = \begin{bmatrix} 0.0861 & 0.0280 & 0 \\ 0.0280 & 1.4129 & 0 \\ 0 & 0 & 0.0339 \end{bmatrix} * 10^{11} \text{ Pa}$$

4. Find the [A] matrix for Layup sequence: $[\pm 45/0/90]_s$

$$[A] = \begin{bmatrix} 5.954 & 1.945 & 0 \\ 1.945 & 5.954 & 0 \\ 0 & 0 & 2.005 \end{bmatrix} * 10^{7} \text{ Pa.m}$$

5. Find the in-plane strains ϵ

$$\epsilon = \begin{bmatrix} -61.398 \\ 187.99 \\ 0 \end{bmatrix} N_{22} * 10^{-10}$$

6. Compute the stresses in each layer in the material coordinate system in terms of N_{22}

   - Layers $\pm 45°$

$$\sigma_{\pm 45°} = \begin{bmatrix} 912.05 \\ 72.202 \\ \pm 84.552 \end{bmatrix} N_{22}$$

   - Layers $0°$

$$\sigma_{0°} = \begin{bmatrix} -814.89 \\ 144.66 \\ 0 \end{bmatrix} N_{22}$$

   - Layers $90°$

$$\sigma_{90°} = \begin{bmatrix} 2639.0 \\ -0.25795 \\ 0 \end{bmatrix} N_{22}$$

7. Apply the Tsai-Wu failure criteria for each layer and find the least possible positive value for N_{22} among all the layers

$$\boxed{\left(\frac{1}{X} - \frac{1}{X'}\right)\sigma_{11} + \left(\frac{1}{Y} - \frac{1}{Y'}\right)\sigma_{22} + \frac{\sigma_{11}^2}{XX'} + \frac{\sigma_{22}^2}{YY'} - \frac{\sigma_{11}}{X}\frac{\sigma_{22}}{X'} + \left(\frac{\sigma_{12}}{S}\right)^2 = 1}$$

Substitute the σ_{11}, σ_{22}, and σ_{12} values for each lamina in terms of N_{22} and solve for the quadratic in N_{22} and choose the positive root because it is told that the loading is tensile loading.

- For Layer $\pm 45°$ the allowable N_{22} is: 618.20 N/mm
- For Layer $0°$ the allowable N_{22} is: 418.26 N/mm
- For Layer $90°$ the allowable N_{22} is: 796.65 N/mm

The layer with least value of allowable N_{22} would fail, i.e. layers of $0°$ orientation for the tensile load of

$$\boxed{N_{22} = 418.26 \text{ N/mm}} \leftarrow \text{“First Ply Failure”}$$

8. Note down the layer that has failed, degrade its stiffness matrix by making it a null matrix. Now repeat steps 4 to 7 again.

   Degrading the stiffness matrix of the 0° laminas, i.e. $[\overline{Q}_{0^\circ}] = [0]_{3x3}$. With this we compute the degraded $[A]$

$$[A] = \begin{bmatrix} 2.366 & 1.874 & 0 \\ 1.874 & 5.736 & 0 \\ 0 & 0 & 1.919 \end{bmatrix} * 10^7 \text{ Pa.m}$$

   In-plane strains ϵ are now -

$$\epsilon = \begin{bmatrix} -186.272 \\ 235.191 \\ 0 \end{bmatrix} N_{22} * 10^{-10}$$

   Stresses in each layer in the material coordinate system in terms of N_{22}

   - Layers ±45°

$$\sigma_{\pm 45^\circ} = \begin{bmatrix} 352.43 \\ 27.9 \\ \pm 142.89 \end{bmatrix} N_{22}$$

   - Layers 90°

$$\sigma_{90^\circ} = \begin{bmatrix} 3270.9 \\ -94.554 \\ 0 \end{bmatrix} N_{22}$$

   Applying the Tsai-Wu failure criteria as done previously and find allowable N_{22} for each lamina

   - For Layer ±45° the allowable N_{22} is: 615.10 N/mm
   - For Layer 90° the allowable N_{22} is: 754.25 N/mm

   The layer with least value of allowable N_{22} would fail, i.e. layers of ±45° orientation for the tensile load of

$$\boxed{N_{22} = 615.10 \text{ N/mm}} \leftarrow \text{“INTERMEDIATE PLY FAILURE”}$$

9. Again repeat steps 4 to 7 by degrading the stiffness matrices of the laminas with orientation ±45°

   Degraded $[A]$

$$[A] = \begin{bmatrix} 2.187 * 10^6 & 7.107 * 10^5 & 0 \\ 7.107 * 10^5 & 3.589 * 10^7 & 0 \\ 0 & 0 & 8.611 * 10^5 \end{bmatrix} \text{ Pa.m}$$

   In-plane strains ϵ are now -

$$\epsilon = \begin{bmatrix} -91.147 \\ 280.453 \\ 0 \end{bmatrix} N_{22} * 10^{-10}$$

Stresses in the 90° lamina in the material coordinate system in terms of N_{22}

$$\sigma_{90^\circ} = \begin{bmatrix} 3937.0 \\ 0 \\ 0 \end{bmatrix} N_{22}$$

Applying the Tsai-Wu failure criteria as done previously and find allowable N_{22} for the 90° lamina we get the allowable N_{22} is: 533.40 N/mm

$$\boxed{N_{22} = 533.40 \text{ N/mm}} \leftarrow \text{“Final Ply Failure”}$$

Conclusion: It is very interesting to observe that the final ply failure load is lower than the intermediate failure load. This means that the ultimate failure load is not the computed final ply failure load but the intermediate failure load that has been computed.

$$\boxed{N_{22} = 615.10 \text{ N/mm}} \leftarrow \text{“Ultimate Failure Load”}$$

Codes:

- 'TsaiWu.m' - Finds N_{22} (tensile) for the Tsa-Wu failure criteria

```
function f_allow = TsaiWu(strength,stress)
% Solves for N22 (tensile)

% Format: strength = [X; Xdash; Y; Ydash; S; R] % in Pa
X     = strength(1);
Xdash = strength(2);
Y     = strength(3);
Ydash = strength(4);
S     = strength(5);
R     = strength(6);

% Plane Stress Assumption (sigma33=sigma23=sigma13==0)
B = stress(1)*((1/X)-(1/Xdash)) + stress(2)*((1/Y)-(1/Ydash));
A = ((stress(1))^2)/(X*Xdash) + ((stress(2))^2)/(Y*Ydash) - ...
(stress(1)*stress(2))/(X*Xdash) + (stress(3)/S)^2;
C = -1;

syms t real

if A~=0 && B~=0
    eqn = A*t^2 + B*t + C==0;
    sol = solve(eqn,t);
else
    sol = [-Inf,Inf];
end

% Store the positive solution because N22 is in tension
f_allow = max(sol);


end
```

- 'checkFailure.m' - Check which lamina fails and at what load value

```
function checkFailure(f_allow)

[failN22, idx] = min(f_allow);

if idx==1
    fprintf("Lamina-1 (+45) fails at N22 = %0.2f x 10^6 N/m \n",
       failN22*1e-6)
% elseif idx==2
    fprintf("Lamina-2 (-45) fails at N22 = %0.2f x 10^6  N/m \n
       ",failN22*1e-6)
elseif idx==3
```

```
    fprintf("Lamina-3 (0) fails at N22 = %0.2f x 10^6  N/m \n",
       failN22*1e-6)
elseif idx==4
    fprintf("Lamina-4 (90) fails at N22 = %0.2f x 10^6  N/m \n",
       failN22*1e-6)
else
    fprintf("Error! \n")
end

end
```

- 'findA.m' - Find A matrix for symmetrical laminate with 8 layers and thickness equal to 0.127 mm

```
function A = findA(Q1,Q2,Q3,Q4)

% For a symmetric laminate with 8 layers

% Step-3: Find [A] matrix
t    = 0.127*1e-3; %in mm (thickness of ply)

syms x real
z0 = -4*t; z1 = -3*t; z2 = -2*t; z3 = -t;         % in mm
z4 = 0;    z5 = t;  z6 = 2*t; z7 = 3*t; z8 = 4*t; % in mm

% Q1 = Qd_p45;  Q2 = Qd_m45;  Q3 = Qd_0;   Q4 = Qd_90;  % in Pa

[A] = vpa(int(Q1,x,z0,z1),4) + vpa(int(Q2,x,z1,z2),4) + vpa(int(
   Q3,x,z2,z3),4)...
    + vpa(int(Q4,x,z3,z4),4) + vpa(int(Q4,x,z4,z5),4) + ...
      vpa(int(Q3,x,z5,z6),4) + vpa(int(Q2,x,z6,z7),4) + vpa(int(
        Q1,x,z7,z8),4); % N/m

A = vpa(A,4); % N/m

end
```

- 'Proj_Part2.m' - Main MATLAB code file

```
% Project Part-2


% Save Strength Parameters
% Format: [X; Xdash; Y; Ydash; S; R] % in Pa


%     Vf - 0.5, 0.6, 0.7
STRENGTH = [2100, 2400, 2500;
```

```
            2100, 2400, 2500;
              62,   68,   71;
             210,  230,  240;
             100,  110,  120;
             100,  110,  120]*1e6; %in Pa

save("strength.mat","STRENGTH")

%---------------------------------------------------
% Load HROM engineering constants of lamina for
% varying volume fiber fraction
load("consts.mat")

% Lamina Constants for Vf=0.5
E1_lam   = CONSTS(1,1);
nu12_lam = CONSTS(2,1);
E2_lam   = CONSTS(3,1);
nu23_lam = CONSTS(4,1);
G12_lam  = CONSTS(5,1);

nu21_lam = E2_lam*(nu12_lam/E1_lam);

t    = 0.127*1e-3; %in mm (thickness of ply)

%
   -------------------------------------------------------------------

% Step1: Find [Q] for material in the material CSYS
Q11 = E1_lam/(1-nu12_lam*nu21_lam);
Q22 = E2_lam/(1-nu12_lam*nu21_lam);
Q12 = nu12_lam*E2_lam/(1-nu12_lam*nu21_lam);
Q66 = G12_lam;

Q = [Q11, Q12, 0;
    Q12, Q22, 0;
    0, 0, Q66]; %in Pa

%
   -------------------------------------------------------------------

% Step-2: Find Qbar for different angles
[Qd_0]   = findQbar(Q,0);   % in Pa*m
[Qd_p45] = findQbar(Q,45);  % in Pa*m
[Qd_m45] = findQbar(Q,-45); % in Pa*m
[Qd_90]  = findQbar(Q,90);  % in Pa*m

%
```

```
   -----------------------------------------------------------------

% Step-3: Find [A] matrix

Q1 = Qd_p45;  Q2 = Qd_m45;  Q3 = Qd_0;   Q4 = Qd_90;  % in Pa

Qnull = zeros(3,3); %in Pa

A = findA(Q1,Q2,Q3,Q4); %in Pa*m

syms N22 real % tensile
eps = inv(A)*[0,N22,0]';

% (eps/N22)*10^10


%
   -----------------------------------------------------------------

% Step-4: Find Stresses in each layer in the PROBLEM CSYS

% Each column represents layers 1 to 8
% Each row element is sig11, sig22, sig12
stress_prob = sym(zeros(3,8));
Q = sym(zeros(3,3,8));

Q(:,:,1)=Q1; Q(:,:,2)=Q2; Q(:,:,3)=Q3; Q(:,:,4)=Q4;
Q(:,:,5)=Q4; Q(:,:,6)=Q3; Q(:,:,7)=Q2; Q(:,:,8)=Q1;

for i=1:8
stress_prob(:,i) = Q(:,:,i)*eps; %in Pa
end

%
   -----------------------------------------------------------------

% Step-5: Convert stresses from PROBLEM CSYS to MATERIAL CSYS
stress_mat = sym(zeros(3,8));
R = sym(zeros(3,3,8));

R(:,:,1) = Rsig(+45); R(:,:,2) = Rsig(-45); R(:,:,3) = Rsig(0);
     R(:,:,4) = Rsig(90);
R(:,:,5) = Rsig(90);  R(:,:,6) = Rsig(0);   R(:,:,7) = Rsig(-45)
   ; R(:,:,8) = Rsig(+45);

for i=1:8
stress_mat(:,i) = inv(R(:,:,i))*stress_prob(:,i); %in Pa
```

```
end

% Stress in material coordinate system divided by N22
stress_mat_by_N22 = vpa(stress_mat/N22, 5);

%
   -------------------------------------------------------------------

% Step-6: Apply Tsai-Wu Failure Criteria for each layer

% Store allowable N22 for each layer
f_allow=zeros(1,4);

for i=1:4
    % For lamina with Vf=0.5
    f_allow(i) = TsaiWu(STRENGTH(:,1),stress_mat_by_N22(:,i));
end

fprintf("First Ply Failure \n")
checkFailure(f_allow)
f_allow;
fprintf("-------------------------------------------- \n")

%
   -------------------------------------------------------------------

% Step-7: Find degraded [A] after first ply failure

Q1 = Qd_p45;  Q2 = Qd_m45;  Q3 = Qnull;   Q4 = Qd_90;  % in Pa

A = findA(Q1,Q2,Q3,Q4); %in N/m

syms N22 real % tensile
eps = inv(A)*[0,N22,0]';

% (eps/N22)*10^10

%
   -------------------------------------------------------------------

% Step-8: Find Stresses in each layer in the PROBLEM CSYS

% Each column represents layers 1 to 8
% Each row element is sig11, sig22, sig12
stress_prob = sym(zeros(3,8));
Q = sym(zeros(3,3,8));
```

```
Q(:,:,1)=Q1; Q(:,:,2)=Q2; Q(:,:,3)=Q3; Q(:,:,4)=Q4;
Q(:,:,5)=Q4; Q(:,:,6)=Q3; Q(:,:,7)=Q2; Q(:,:,8)=Q1;

for i=1:8
stress_prob(:,i) = Q(:,:,i)*eps;
end

%
   -------------------------------------------------------------------

% Step-9: Convert stresses from PROBLEM CSYS to MATERIAL CSYS
stress_mat = sym(zeros(3,8));
R = sym(zeros(3,3,8));

R(:,:,1) = Rsig(+45); R(:,:,2) = Rsig(-45); R(:,:,3) = Rsig(0);
     R(:,:,4) = Rsig(90);
R(:,:,5) = Rsig(90);  R(:,:,6) = Rsig(0);   R(:,:,7) = Rsig(-45)
   ; R(:,:,8) = Rsig(+45);

for i=1:8
stress_mat(:,i) = inv(R(:,:,i))*stress_prob(:,i); %in Pa
end

% Stress in material coordinate system divided by N22
stress_mat_by_N22 = vpa(stress_mat/N22, 5);

%
   -------------------------------------------------------------------

% Step-10: Apply Tsai-Wu Failure Criteria for each layer

% Store allowable N22 for each layer
f_allow=zeros(1,4);

for i=1:4
    % For lamina with Vf=0.5
    f_allow(i) = TsaiWu(STRENGTH(:,1),stress_mat_by_N22(:,i));
end

fprintf("Second Ply/Plies Failure \n")
checkFailure(f_allow)
f_allow
fprintf("-------------------------------------------- \n")

%
   -------------------------------------------------------------------
```

```
% Step-11: Find degraded [A] after first ply failure

Q1 = Qnull;  Q2 = Qnull;  Q3 = Qnull;   Q4 = Qd_90;  % in Pa

A = findA(Q1,Q2,Q3,Q4) %in N/m

syms N22 real % tensile
eps = inv(A)*[0,N22,0]';

% (eps/N22)*10^10

%
   ------------------------------------------------------------------

% Step-12: Find Stresses in each layer in the PROBLEM CSYS

% Each column represents layers 1 to 8
% Each row element is sig11, sig22, sig12
stress_prob = sym(zeros(3,8));
Q = sym(zeros(3,3,8));

Q(:,:,1)=Q1; Q(:,:,2)=Q2; Q(:,:,3)=Q3; Q(:,:,4)=Q4;
Q(:,:,5)=Q4; Q(:,:,6)=Q3; Q(:,:,7)=Q2; Q(:,:,8)=Q1;

for i=1:8
stress_prob(:,i) = Q(:,:,i)*eps;
end

%
   ------------------------------------------------------------------

% Step-13: Convert stresses from PROBLEM CSYS to MATERIAL CSYS
stress_mat = sym(zeros(3,8));
R = sym(zeros(3,3,8));

R(:,:,1) = Rsig(+45); R(:,:,2) = Rsig(-45); R(:,:,3) = Rsig(0);
     R(:,:,4) = Rsig(90);
R(:,:,5) = Rsig(90);  R(:,:,6) = Rsig(0);   R(:,:,7) = Rsig(-45)
   ; R(:,:,8) = Rsig(+45);

for i=1:8
stress_mat(:,i) = inv(R(:,:,i))*stress_prob(:,i); %in Pa
end

% Stress in material coordinate system divided by N22
stress_mat_by_N22 = vpa(stress_mat/N22, 5)
```

```
%
   ----------------------------------------------------------------------

% Step-14: Apply Tsai-Wu Failure Criteria for each layer

% Store allowable N22 for each layer
f_allow=zeros(1,4);

for i=1:4
    % For lamina with Vf=0.5
    f_allow(i) = TsaiWu(STRENGTH(:,1),stress_mat_by_N22(:,i));
end

fprintf("Third Ply Failure \n")
checkFailure(f_allow)
f_allow
fprintf("------------------------------------------- \n")
```

Design a laminate with an initial failure load of $N_{22} \geq$ 400 N/mm. The designer can select from four different layup schemes ($[\pm45/0/90]_s$, $[0/30/60/90]_s$, $[(\pm45)_s]_s$, $[0_2/90_2]_s$) and three different fiber volume fractions (0.5, 0.6, 0.7). Assume that the lamina fails according to the Tsai-Wu failure criterion. Which option(s) (the combination of layup scheme and fiber volume fraction) can achieve the design objective?

Solution:

Let us define a nomenclature for the ease of referring to the various layup scheme and volume fraction combinations.

"Design-ij": 'i' represents the layup scheme and 'j' represents the fiber volume fraction in the lamina. 'i' goes from 1 to 4, and 'j' can be A, B, or C. Where **'1'** corresponds to layup scheme $[\pm45/0/90]_s$, **'2'** corresponds to layup scheme $[0/30/60/90]_s$, **'3'** corresponds to layup scheme $[(\pm45)_s]_s$, and **'4'** corresponds to layup scheme $[0_2/90_2]_s$. Similarly, **'A'**, **'B'**, and **'C'** correspond to the fiber volume fractions 0.5, 0.6, and 0.7 respectively.

For example, **'Design-2B'**: Laminate with $[0/30/60/90]_s$ layup where the fiber volume fraction of each lamina is 0.6.

Procedure:

1. Following the same procedure as what we did in the previous part of the question, we do the same but now we vary the layup and the fiber volume fraction values and choose the corresponding strength values to conduct the analysis.

2. Finding [A] matrix for each design

   Design-1A

$$[A] = \begin{bmatrix} 5.954 * 10^7 & 1.945 * 10^7 & 0 \\ 1.945 * 10^7 & 5.954 * 10^7 & 0 \\ 0 & 0 & 2.005 * 10^7 \end{bmatrix} \text{ Pa.m}$$

   Design-1B

$$[A] = \begin{bmatrix} 7.079 * 10^7 & 2.284 * 10^7 & 0 \\ 2.284 * 10^7 & 7.079 * 10^7 & 0 \\ 0 & 0 & 2.398 * 10^7 \end{bmatrix} \text{ Pa.m}$$

   Design-1C

$$[A] = \begin{bmatrix} 8.243 * 10^7 & 2.608 * 10^7 & 0 \\ 2.608 * 10^7 & 8.243 * 10^7 & 0 \\ 0 & 0 & 2.818 * 10^7 \end{bmatrix} \text{ Pa.m}$$

Design-2A

$$[A] = \begin{bmatrix} 6.37 * 10^7 & 1.53 * 10^7 & 1.459 * 10^7 \\ 1.53 * 10^7 & 6.37 * 10^7 & 1.459 * 10^7 \\ 1.459 * 10^7 & 1.459 * 10^7 & 1.59 * 10^7 \end{bmatrix} \text{ Pa.m}$$

Design-2B

$$[A] = \begin{bmatrix} 7.572 * 10^7 & 1.79 * 10^7 & 1.746 * 10^7 \\ 1.79 * 10^7 & 7.572 * 10^7 & 1.746 * 10^7 \\ 1.746 * 10^7 & 1.746 * 10^7 & 1.905 * 10^7 \end{bmatrix} \text{ Pa.m}$$

Design-2C

$$[A] = \begin{bmatrix} 8.808 * 10^7 & 2.042 * 10^7 & 2.03 * 10^7 \\ 2.042 * 10^7 & 8.808 * 10^7 & 2.03 * 10^7 \\ 2.03 * 10^7 & 2.03 * 10^7 & 2.252 * 10^7 \end{bmatrix} \text{ Pa.m}$$

Design-3A

$$[A] = \begin{bmatrix} 4.294 * 10^7 & 3.605 * 10^7 & 0 \\ 3.605 * 10^7 & 4.294 * 10^7 & 0 \\ 0 & 0 & 3.665 * 10^7 \end{bmatrix} \text{ Pa.m}$$

Design-3B

$$[A] = \begin{bmatrix} 5.107 * 10^7 & 4.256 * 10^7 & 0 \\ 4.256 * 10^7 & 5.107 * 10^7 & 0 \\ 0 & 0 & 4.37 * 10^7 \end{bmatrix} \text{ Pa.m}$$

Design-3C

$$[A] = \begin{bmatrix} 5.981 * 10^7 & 4.869 * 10^7 & 0 \\ 4.869 * 10^7 & 5.981 * 10^7 & 0 \\ 0 & 0 & 5.079 * 10^7 \end{bmatrix} \text{ Pa.m}$$

Design-4A

$$[A] = \begin{bmatrix} 7.615 * 10^7 & 2.843 * 10^6 & 0 \\ 2.843 * 10^6 & 7.615 * 10^7 & 0 \\ 0 & 0 & 3.445 * 10^6 \end{bmatrix} \text{ Pa.m}$$

Design-4B

$$[A] = \begin{bmatrix} 9.051 * 10^7 & 3.113 * 10^6 & 0 \\ 3.113 * 10^6 & 9.051 * 10^7 & 0 \\ 0 & 0 & 4.254 * 10^6 \end{bmatrix} \text{ Pa.m}$$

Design-4C

$$[A] = \begin{bmatrix} 1.05 * 10^8 & 3.462 * 10^6 & 0 \\ 3.462 * 10^6 & 1.05 * 10^8 & 0 \\ 0 & 0 & 5.561 * 10^6 \end{bmatrix} \text{ Pa.m}$$

3. Finding in plane strains ϵ each design

Design-1A

$$\epsilon = \begin{bmatrix} -61.398 \\ 187.994 \\ 0 \end{bmatrix} * 10^{-10} \; N_{22}$$

Design-1B

$$\epsilon = \begin{bmatrix} -50.864 \\ 157.675 \\ 0 \end{bmatrix} * 10^{-10}\ N_{22}$$

Design-1C

$$\epsilon = \begin{bmatrix} -42.648 \\ 134.812 \\ 0 \end{bmatrix} * 10^{-10}\ N_{22}$$

Design-2A

$$\epsilon = \begin{bmatrix} -7.522 \\ 199.092 \\ -175.847 \end{bmatrix} * 10^{-10}\ N_{22}$$

Design-2B

$$\epsilon = \begin{bmatrix} -5.315 \\ 167.654 \\ -148.848 \end{bmatrix} * 10^{-10}\ N_{22}$$

Design-2C

$$\epsilon = \begin{bmatrix} -4.366 \\ 143.436 \\ -125.353 \end{bmatrix} * 10^{-10}\ N_{22}$$

Design-3A

$$\epsilon = \begin{bmatrix} -662.483 \\ 789.081 \\ 0 \end{bmatrix} * 10^{-10}\ N_{22}$$

Design-3B

$$\epsilon = \begin{bmatrix} -534.250 \\ 641.062 \\ 0 \end{bmatrix} * 10^{-10}\ N_{22}$$

Design-3C

$$\epsilon = \begin{bmatrix} -403.448 \\ 495.6125 \\ 0 \end{bmatrix} * 10^{-10}\ N_{22}$$

Design-4A

$$\epsilon = \begin{bmatrix} -4.909 \\ 131.506 \\ 0 \end{bmatrix} * 10^{-10}\ N_{22}$$

Design-4B

$$\epsilon = \begin{bmatrix} -3.805 \\ 110.616 \\ 0 \end{bmatrix} * 10^{-10}\ N_{22}$$

Design-4C

$$\epsilon = \begin{bmatrix} -3.141 \\ 95.304 \\ 0 \end{bmatrix} * 10^{-10} \; N_{22}$$

4. Compute the stresses in each layer in the material coordinate system in terms of N_{22}

   (a) **Design-1A**

   - Layer-1: 45°

   $$\sigma_\theta = \begin{bmatrix} 912.05 \\ 72.202 \\ 84.552 \end{bmatrix} N_{22}$$

   - Layer-2: −45°

   $$\sigma_\theta = \begin{bmatrix} 912.05 \\ 72.202 \\ -84.552 \end{bmatrix} N_{22}$$

   - Layer-3: 0°

   $$\sigma_\theta = \begin{bmatrix} -814.89 \\ 144.66 \\ 0 \end{bmatrix} N_{22}$$

   - Layer-4: 90°

   $$\sigma_\theta = \begin{bmatrix} 2639.0 \\ -0.25795 \\ 0 \end{bmatrix} N_{22}$$

   (b) **Design-1B**

   - Layer-1: 45°

   $$\sigma_\theta = \begin{bmatrix} 916.1 \\ 68.15 \\ 87.32 \end{bmatrix} N_{22}$$

   - Layer-2: −45°

   $$\sigma_\theta = \begin{bmatrix} 916.1 \\ 68.15 \\ -87.32 \end{bmatrix} N_{22}$$

   - Layer-3: 0°

   $$\sigma_\theta = \begin{bmatrix} -808.61 \\ 137.31 \\ 0 \end{bmatrix} N_{22}$$

   - Layer-4: 90°

   $$\sigma_\theta = \begin{bmatrix} 2640.8 \\ -1.0072 \\ 0 \end{bmatrix} N_{22}$$

   (c) **Design-1C**

   - Layer-1: 45°

   $$\sigma_\theta = \begin{bmatrix} 917.4 \\ 66.85 \\ 97.138 \end{bmatrix} N_{22}$$

- Layer-2: $-45°$

$$\sigma_\theta = \begin{bmatrix} 917.4 \\ 66.85 \\ -97.138 \end{bmatrix} N_{22}$$

- Layer-3: $0°$

$$\sigma_\theta = \begin{bmatrix} -788.58 \\ 135.1 \\ 0 \end{bmatrix} N_{22}$$

- Layer-4: $90°$

$$\sigma_\theta = \begin{bmatrix} 2623.4 \\ -1.3996 \\ 0 \end{bmatrix} N_{22}$$

(d) **Design-2A**

- Layer-1: $0°$

$$\sigma_\theta = \begin{bmatrix} -50.578 \\ 169.29 \\ -59.618 \end{bmatrix} N_{22}$$

- Layer-2: $30°$

$$\sigma_\theta = \begin{bmatrix} -389.75 \\ 183.52 \\ 30.855 \end{bmatrix} N_{22}$$

- Layer-3: $60°$

$$\sigma_\theta = \begin{bmatrix} 1041.0 \\ 123.49 \\ 90.473 \end{bmatrix} N_{22}$$

- Layer-4: $90°$

$$\sigma_\theta = \begin{bmatrix} 2810.8 \\ 49.227 \\ 59.618 \end{bmatrix} N_{22}$$

(e) **Design-2B**

- Layer-1: $0°$

$$\sigma_\theta = \begin{bmatrix} -38.179 \\ 160.94 \\ -62.326 \end{bmatrix} N_{22}$$

- Layer-2: $30°$

$$\sigma_\theta = \begin{bmatrix} -389.02 \\ 175.01 \\ 31.56 \end{bmatrix} N_{22}$$

- Layer-3: $60°$

$$\sigma_\theta = \begin{bmatrix} 1041.5 \\ 117.65 \\ 93.885 \end{bmatrix} N_{22}$$

- Layer-4: 90°

$$\sigma_\theta = \begin{bmatrix} 2822.9 \\ 46.218 \\ 62.326 \end{bmatrix} N_{22}$$

(f) **Design-2C**

- Layer-1: 0°

$$\sigma_\theta = \begin{bmatrix} -36.566 \\ 157.72 \\ -68.615 \end{bmatrix} N_{22}$$

- Layer-2: 30°

$$\sigma_\theta = \begin{bmatrix} -369.74 \\ 171.05 \\ 35.757 \end{bmatrix} N_{22}$$

- Layer-3: 60°

$$\sigma_\theta = \begin{bmatrix} 1051.1 \\ 114.2 \\ 104.37 \end{bmatrix} N_{22}$$

- Layer-4: 90°

$$\sigma_\theta = \begin{bmatrix} 2805.2 \\ 44.029 \\ 68.615 \end{bmatrix} N_{22}$$

(g) **Design-3A**

- Layer-1 and 4: 45°

$$\sigma_\theta = \begin{bmatrix} 912.05 \\ 72.202 \\ 492.13 \end{bmatrix} N_{22}$$

- Layer-2 and 3: −45°

$$\sigma_\theta = \begin{bmatrix} 912.05 \\ 72.202 \\ -492.13 \end{bmatrix} N_{22}$$

(h) **Design-3B**

- Layer-1 and 4: 45°

$$\sigma_\theta = \begin{bmatrix} 916.1 \\ 68.15 \\ 492.13 \end{bmatrix} N_{22}$$

- Layer-2 and 3: −45°

$$\sigma_\theta = \begin{bmatrix} 916.1 \\ 68.15 \\ -492.13 \end{bmatrix} N_{22}$$

(i) **Design-3C**

- Layer-1 and 4: 45°

$$\sigma_\theta = \begin{bmatrix} 917.4 \\ 66.85 \\ 492.13 \end{bmatrix} N_{22}$$

- Layer-2 and 3: $-45°$

$$\sigma_\theta = \begin{bmatrix} 917.4 \\ 66.85 \\ -492.13 \end{bmatrix} N_{22}$$

(j) **Design-4A**

- Layer-1 and 2: $0°$

$$\sigma_\theta = \begin{bmatrix} -32.567 \\ 111.84 \\ 0 \end{bmatrix} N_{22}$$

- Layer-3 and 4: $90°$

$$\sigma_\theta = \begin{bmatrix} 1856.7 \\ 32.567 \\ 0 \end{bmatrix} N_{22}$$

(k) **Design-4B**

- Layer-1 and 2: $0°$

$$\sigma_\theta = \begin{bmatrix} -30.205 \\ 106.09 \\ 0 \end{bmatrix} N_{22}$$

- Layer-3 and 4: $90°$

$$\sigma_\theta = \begin{bmatrix} 1862.4 \\ 30.205 \\ 0 \end{bmatrix} N_{22}$$

(l) **Design-4C**

- Layer-1 and 2: $0°$

$$\sigma_\theta = \begin{bmatrix} -28.989 \\ 104.71 \\ 0 \end{bmatrix} N_{22}$$

- Layer-3 and 4: $90°$

$$\sigma_\theta = \begin{bmatrix} 1863.8 \\ 28.989 \\ 0 \end{bmatrix} N_{22}$$

5. Apply Tsai-Wu failure criteria for each layer and find the lamina that is failing and the load at which it fails

Design-1A:

$$\text{Lamina} = [\pm 45/0/90]_s; \quad \text{Volume Fraction:}0.5$$
$$\text{Initial Failure Tensile Load: } N_{22} = 418.26 \text{ N/mm}$$

Design-1B:

$$\text{Lamina} = [\pm 45/0/90]_s; \quad \text{Volume Fraction:}0.6$$
$$\text{Initial Failure Tensile Load: } N_{22} = 483.32 \text{ N/mm}$$

Design-1C:

$$\text{Lamina} = [\pm 45/0/90]_s; \quad \text{Volume Fraction:}0.7$$
$$\text{Initial Failure Tensile Load: } N_{22} = 513.03 \text{ N/mm}$$

Design-2A:

$$\text{Lamina} = [0/30/60/90]_s; \quad \text{Volume Fraction:}0.5$$
$$\text{Initial Failure Tensile Load: } N_{22} = 333.59 \text{ N/mm}$$

Design-2B:

$$\text{Lamina} = [0/30/60/90]_s; \quad \text{Volume Fraction:}0.6$$
$$\text{Initial Failure Tensile Load: } N_{22} = 383.23 \text{ N/mm}$$

Design-2C:

$$\text{Lamina} = [0/30/60/90]_s; \quad \text{Volume Fraction:}0.7$$
$$\text{Initial Failure Tensile Load: } N_{22} = 408.61 \text{ N/mm}$$

Design-3A:

$$\text{Lamina} = [(\pm 45)_s]_s; \quad \text{Volume Fraction:}0.5$$
$$\text{Initial Failure Tensile Load: } N_{22} = 184.96 \text{ N/mm}$$

Design-3B:

$$\text{Lamina} = [(\pm 45)_s]_s; \quad \text{Volume Fraction:}0.6$$
$$\text{Initial Failure Tensile Load: } N_{22} = 204.56 \text{ N/mm}$$

Design-3C:

$$\text{Lamina} = [(\pm 45)_s]_s; \quad \text{Volume Fraction:}0.7$$
$$\text{Initial Failure Tensile Load: } N_{22} = 222.58 \text{ N/mm}$$

Design-4A:

$$\text{Lamina} = [0_2/90_2]_s; \quad \text{Volume Fraction:}0.5$$
$$\text{Initial Failure Tensile Load: } N_{22} = 554.24 \text{ N/mm}$$

Design-4B:

$$\text{Lamina} = [0_2/90_2]_s; \quad \text{Volume Fraction:}0.6$$
$$\text{Initial Failure Tensile Load: } N_{22} = 640.79 \text{ N/mm}$$

Design-4C:

$$\text{Lamina} = [0_2/90_2]_s; \quad \text{Volume Fraction:}0.7$$
$$\text{Initial Failure Tensile Load: } N_{22} = 677.91 \text{ N/mm}$$

Results:

Layup: $[\pm 45°, 0°, 90°]_s$**; Fiber Volume Fraction: 0.50**

Design - 1A		
	Ply / Plies	N_{22}
First Ply Failure	[0°]	418.26 N/mm
Second Ply Failure	[±45°]	615.10 N/mm
Third Ply Failure	[90°]	533.40 N/mm
Fourth Ply Failure	Laminate has already failed	N/A
Ultimate Load	[±45°]	**615.10 N/mm**

Table 5: Design-1A: Failure loads for Layup:$[45, -45, 0, 90]_s$; Volume fraction: 0.50.

Layup: $[\pm 45°, 0°, 90°]_s$**; Fiber Volume Fraction: 0.60**

Design - 1B		
	Ply / Plies	N_{22}
First Ply Failure	[0°]	483.32 N/mm
Second Ply Failure	[±45°]	664.59 N/mm
Third Ply Failure	[90°]	609.60 N/mm
Fourth Ply Failure	Laminate has already failed	N/A
Ultimate Load	[±45°]	**664.59 N/mm**

Table 6: Design-1B: Failure loads for Layup:$[45, -45, 0, 90]_s$; Volume fraction: 0.60

Layup: $[\pm 45°, 0°, 90°]_s$**; Fiber Volume Fraction: 0.70**

Design - 1C		
	Ply / Plies	N_{22}
First Ply Failure	[0°]	513.03 N/mm
Second Ply Failure	[±45°]	662.09 N/mm
Third Ply Failure	[90°]	635.00 N/mm
Fourth Ply Failure	Laminate has already failed	N/A
Ultimate Load	[±45°]	**662.09 N/mm**

Table 7: Design-1C: Failure loads for Layup:$[45, -45, 0, 90]_s$; Volume fraction: 0.70

Layup: $[0°, 30°, 60°, 90°]_s$**; Fiber Volume Fraction: 0.50**

Design - 2A		
	Ply / Plies	N_{22}
First Ply Failure	[30°]	333.59 N/mm
Second Ply Failure	[0°]	316.19 N/mm
Third Ply Failure	[60°]	445.51 N/mm
Fourth Ply Failure	[90°]	533.40 N/mm
Ultimate Load	[90°]	**533.40 N/mm**

Table 8: Design-2A: Failure loads for Layup:$[0, 30, 60, 90]_s$; Volume fraction: 0.50

Layup: $[0°, 30°, 60°, 90°]_s$**; Fiber Volume Fraction: 0.60**

Design - 2B		
	Ply / Plies	N_{22}
First Ply Failure	[30°]	383.23 N/mm
Second Ply Failure	[0°]	362.35 N/mm
Third Ply Failure	[60°]	488.35 N/mm
Fourth Ply Failure	[90°]	609.60 N/mm
Ultimate Load	[90°]	**609.60 N/mm**

Table 9: Design-2B: Failure loads for Layup:$[0, 30, 60, 90]_s$; Volume fraction: 0.60

Layup: $[0°, 30°, 60°, 90°]_s$**; Fiber Volume Fraction: 0.70**

Design - 2C		
	Ply / Plies	N_{22}
First Ply Failure	[30°]	408.61 N/mm
Second Ply Failure	[0°]	384.71 N/mm
Third Ply Failure	[60°]	494.39 N/mm
Fourth Ply Failure	[90°]	635.00 N/mm
Ultimate Load	[90°]	**635.00 N/mm**

Table 10: Design-2C: Failure loads for Layup:$[0, 30, 60, 90]_s$; Volume fraction: 0.70

Layup: $[(\pm 45°)_s]_s$**; Fiber Volume Fraction: 0.50**

Design - 3A		
	Ply / Plies	N_{22}
First Ply Failure	[±45°]	184.96 N/mm
Second Ply Failure	Laminate failed already	N/A
Third Ply Failure	Laminate failed already	N/A
Fourth Ply Failure	Laminate failed already	N/A
Ultimate Load	[±45°]	**184.96 N/mm**

Table 11: Design-3A: Failure loads for Layup:$[(\pm 45)_s]_s$; Volume fraction: 0.50

Layup: $[(\pm 45°)_s]_s$**; Fiber Volume Fraction: 0.60**

Design - 3B		
	Ply / Plies	N_{22}
First Ply Failure	[±45°]	204.56 N/mm
Second Ply Failure	Laminate failed already	N/A
Third Ply Failure	Laminate failed already	N/A
Fourth Ply Failure	Laminate failed already	N/A
Ultimate Load	[±45°]	**204.56 N/mm**

Table 12: Design-3B: Failure loads for Layup:$[(\pm 45)_s]_s$; Volume fraction: 0.60

Layup: $[(\pm 45°)_s]_s$**; Fiber Volume Fraction: 0.70**

Design - 3C		
	Ply / Plies	N_{22}
First Ply Failure	[±45°]	222.58 N/mm
Second Ply Failure	Laminate failed already	N/A
Third Ply Failure	Laminate failed already	N/A
Fourth Ply Failure	Laminate failed already	N/A
Ultimate Load	[±45°]	**222.58 N/mm**

Table 13: Design-3C: Failure loads for Layup:$[(\pm 45)_s]_s$; Volume fraction: 0.70

Layup: $[0^\circ_2, 90^\circ_2]_s$**; Fiber Volume Fraction: 0.50**

Design - 4A		
	Ply / Plies	N_{22}
First Ply Failure	[0°]	554.24 N/mm
Second Ply Failure	[90°]	1066.80 N/mm
Third Ply Failure	Laminate failed already	N/A
Fourth Ply Failure	Laminate failed already	N/A
Ultimate Load	[90°]	**1066.80 N/mm**

Table 14: Design-4A: Failure loads for Layup:$[0_2/90_2]_s$; Volume fraction: 0.50

Layup: $[0^\circ_2, 90^\circ_2]_s$**; Fiber Volume Fraction: 0.60**

Design - 4B		
	Ply / Plies	N_{22}
First Ply Failure	[0°]	640.79 N/mm
Second Ply Failure	[90°]	1219.20 N/mm
Third Ply Failure	Laminate failed already	N/A
Fourth Ply Failure	Laminate failed already	N/A
Ultimate Load	[90°]	**1219.20 N/mm**

Table 15: Design-4B: Failure loads for Layup:$[0_2/90_2]_s$; Volume fraction: 0.60

Layup: $[0^\circ_2, 90^\circ_2]_s$**; Fiber Volume Fraction: 0.70**

Design - 4C		
	Ply / Plies	N_{22}
First Ply Failure	[0°]	677.91 N/mm
Second Ply Failure	[90°]	1270.00 N/mm
Third Ply Failure	Laminate failed already	N/A
Fourth Ply Failure	Laminate failed already	N/A
Ultimate Load	[90°]	**1270.00 N/mm**

Table 16: Design-4C: Failure loads for Layup:$[0_2/90_2]_s$; Volume fraction: 0.70

Conclusion: Laminate designs **1A** (418.26 N/mm), **1B** (483.32 N/mm), **1C** (513.03 N/mm), **2C** (408.61 N/mm), **4A** (554.24 N/mm), **4B** (640.79 N/mm), and **4C** (677.91 N/mm) have initial failure loads greater than 400 N/mm

Codes:

- 'TsaiWu.m' - Finds the allowable tensile N_{22} load according to the Tsai-Wu Failure criteria

```
function f_allow = TsaiWu(strength,stress)
% Solves for N22 (tensile)

% Format: strength = [X; Xdash; Y; Ydash; S; R] % in Pa
X     = strength(1);
Xdash = strength(2);
Y     = strength(3);
Ydash = strength(4);
S     = strength(5);
R     = strength(6);

% Plane Stress Assumption (sigma33=sigma23=sigma13==0)
B = stress(1)*((1/X)-(1/Xdash)) + stress(2)*((1/Y)-(1/Ydash));
A = ((stress(1))^2)/(X*Xdash) + ((stress(2))^2)/(Y*Ydash) - ...
(stress(1)*stress(2))/(X*Xdash) + (stress(3)/S)^2;
C = -1;

syms t real

if A~=0 && B~=0
    eqn = A*t^2 + B*t + C==0;
    sol = solve(eqn,t);
else
    sol = [-Inf,Inf];
end

% Store the positive solution because N22 is in tension
f_allow = max(sol);


end
```

- 'checkFailureNew.m' - Checks and prints the lamina orientation and the failure load of the lamina that is failing

```
function idx=checkFailureNew(f_allow,LAYUP)

[failN22, idx] = min(f_allow);

if failN22 ~= Inf
if idx==1 && LAYUP(1)==abs(LAYUP(2))
    if abs(LAYUP(2))~=abs(LAYUP(3))
        fprintf("Lamina-1 (%0.2f) fails at N22 = %0.2f x 10^6 N/
           m \n",LAYUP(1),failN22*1e-6)
```

```
        fprintf("Lamina-2 (%0.2f) fails at N22 = %0.2f x 10^6  N
           /m \n",LAYUP(2),failN22*1e-6)
    else
        fprintf("Lamina-1 (%0.2f) fails at N22 = %0.2f x 10^6 N/
           m \n",LAYUP(1),failN22*1e-6)
        fprintf("Lamina-2 (%0.2f) fails at N22 = %0.2f x 10^6  N
           /m \n",LAYUP(2),failN22*1e-6)
        fprintf("Lamina-3 (%0.2f) fails at N22 = %0.2f x 10^6  N
           /m \n",LAYUP(3),failN22*1e-6)
        fprintf("Lamina-4 (%0.2f) fails at N22 = %0.2f x 10^6  N
           /m \n",LAYUP(4),failN22*1e-6)
    end
elseif idx==1 && LAYUP(1)~=abs(LAYUP(2))
    fprintf("Lamina-1 (%0.2f) fails at N22 = %0.2f x 10^6 N/m \n
       ",LAYUP(1),failN22*1e-6)
elseif idx==2
    fprintf("Lamina-2 (%0.2f) fails at N22 = %0.2f x 10^6  N/m \
       n",LAYUP(2),failN22*1e-6)
elseif idx==3
    if abs(LAYUP(3))==abs(LAYUP(4))
        fprintf("Lamina-3 (%0.2f) fails at N22 = %0.2f x 10^6  N
           /m \n",LAYUP(3),failN22*1e-6)
        fprintf("Lamina-4 (%0.2f) fails at N22 = %0.2f x 10^6  N
           /m \n",LAYUP(4),failN22*1e-6)
    else
        fprintf("Lamina-3 (%0.2f) fails at N22 = %0.2f x 10^6  N
           /m \n",LAYUP(3),failN22*1e-6)
    end
elseif idx==4
    fprintf("Lamina-4 (%0.2f) fails at N22 = %0.2f x 10^6  N/m \
       n",LAYUP(4),failN22*1e-6)
else
    fprintf("Error! \n")
end

else
    fprintf("Laminate has already failed \n")
end


end
```

- 'doMyJob.m' - Finds the failure loads for each lamina for a given layup sequence and fiber volume fraction

```
function doMyJob(CONSTS,LAYUP,STRENGTH)
```

```
% INPUT:
% CONSTS   - [E1,nu12,E2,nu23,G12] - Lamina props for varying Vf
    (SI Units)
% LAYUP    - [theta1,theta2,theta3,theta4]s - Angles in degrees
% STRENGTH - [X, Xdash, Y, Ydash, S, R] % in Pa

% Lamina Constants for Vf=0.5
E1_lam   = CONSTS(1); %in Pa
nu12_lam = CONSTS(2);
E2_lam   = CONSTS(3); %in Pa
nu23_lam = CONSTS(4);
G12_lam  = CONSTS(5); %in Pa

nu21_lam = E2_lam*(nu12_lam/E1_lam);

t    = 0.127*1e-3; %in mm (thickness of ply)

% LAYUP(1,:)

%
   --------------------------------------------------------------------

% Step1: Find [Q] for material in the material CSYS
Q11 = E1_lam/(1-nu12_lam*nu21_lam);
Q22 = E2_lam/(1-nu12_lam*nu21_lam);
Q12 = nu12_lam*E2_lam/(1-nu12_lam*nu21_lam);
Q66 = G12_lam;

Q = [Q11, Q12, 0;
    Q12, Q22, 0;
    0, 0, Q66]; %in Pa

%
   --------------------------------------------------------------------

% Step-2: Find Qbar for different angles
Q1_0   = findQbar(Q,LAYUP(1));  % in Pa
Q2_0   = findQbar(Q,LAYUP(2));  % in Pa
Q3_0   = findQbar(Q,LAYUP(3));  % in Pa
Q4_0   = findQbar(Q,LAYUP(4));  % in Pa

Qnull = zeros(3,3); %in Pa

%
   --------------------------------------------------------------------

% Step-3: Find [A] matrix
```

```
Q1 = Q1_0;  Q2 = Q2_0;  Q3 = Q3_0;   Q4 = Q4_0;  % in Pa

A = findA(Q1,Q2,Q3,Q4); %in Pa*m

syms N22 real % tensile
eps = inv(A)*[0,N22,0]';

%
  ---------------------------------------------------------------------

% Step-4: Find Stresses in each layer in the PROBLEM CSYS

% Each column represents layers 1 to 4
% Each row element is sig11, sig22, sig12
stress_prob = sym(zeros(3,4));
Q = sym(zeros(3,3,4));

Q(:,:,1)=Q1; Q(:,:,2)=Q2; Q(:,:,3)=Q3; Q(:,:,4)=Q4;

for i=1:4
stress_prob(:,i) = Q(:,:,i)*eps; %in Pa
end

%
  ---------------------------------------------------------------------

% Step-5: Convert stresses from PROBLEM CSYS to MATERIAL CSYS
stress_mat = sym(zeros(3,4));
R = sym(zeros(3,3,4));

R(:,:,1) = Rsig(LAYUP(1)); R(:,:,2) = Rsig(LAYUP(2));
R(:,:,3) = Rsig(LAYUP(3)); R(:,:,4) = Rsig(LAYUP(4));

for i=1:4
stress_mat(:,i) = inv(R(:,:,i))*stress_prob(:,i); %in Pa
end

% Stress in material coordinate system divided by N22
stress_mat_by_N22 = vpa(stress_mat/N22, 5);


%
  ---------------------------------------------------------------------

% Step-6: Apply Tsai-Wu Failure Criteria for each layer
```

```
% Store allowable N22 for each layer
f_allow=zeros(1,4);

for i=1:4
    % For lamina with Vf=0.5
    f_allow(i) = TsaiWu(STRENGTH(:),stress_mat_by_N22(:,i));
end

fprintf("First Ply/Plies Failure: \n")
idx1 = checkFailureNew(f_allow,LAYUP);
f_allow;
fprintf("------------------------------------------------------- \
   n")

%
   ---------------------------------------------------------------------

% Step-7: Find degraded [A] after first ply failure


% Case-1: When idx=1
% Account for [+-45/0/90]s
if idx1==1 && LAYUP(1)==abs(LAYUP(2))
    Q1=Qnull;
    Q2=Qnull;
    % Account for [(+-45)s]s
    if abs(LAYUP(2))==abs(LAYUP(3))
        Q3=Qnull;
        Q4=Qnull;
    else
        Q3=Q3_0;
        Q4=Q4_0;
    end
else
    Q1 = Qnull;
    Q2 = Q2_0;
    Q3 = Q3_0;
    Q4 = Q4_0;
end

% Case-2: When idx=2
if idx1==2
    Q1 = Q1_0;
    Q2 = Qnull;
    Q3 = Q3_0;
    Q4 = Q4_0;
end
```

```
% Case-3: When idx=3
if idx1==3
    % Account for [0_2/90_2]s
    if LAYUP(3)==LAYUP(4)
        Q1 = Q1_0;
        Q2 = Q2_0;
        Q3 = Qnull;
        Q4 = Qnull;
    else
        Q1 = Q1_0;
        Q2 = Q2_0;
        Q3 = Qnull;
        Q4 = Q4_0;
    end

end

% Case-4: When idx=4
if idx1==4
    Q1 = Q1_0;
    Q2 = Q2_0;
    Q3 = Q3_0;
    Q4 = Qnull;
end

%-------------------
% Q1
% Q2
% Q3
% Q4

A = findA(Q1,Q2,Q3,Q4); %in N/m

syms N22 real % tensile
eps = inv(A)*[0,N22,0]';


%
   ------------------------------------------------------------------

% Step-8: Find degraded [A] after first ply failure
A = findA(Q1,Q2,Q3,Q4); %in N/m

syms N22 real % tensile
eps = inv(A)*[0,N22,0]';
```

```
% (eps/N22)*10^10

%
   ----------------------------------------------------------------------

% Step-8: Find Stresses in each layer in the PROBLEM CSYS

% Each column represents layers 1 to 4
% Each row element is sig11, sig22, sig12
stress_prob = sym(zeros(3,4));
Q = sym(zeros(3,3,4));

Q(:,:,1)=Q1; Q(:,:,2)=Q2; Q(:,:,3)=Q3; Q(:,:,4)=Q4;


for i=1:4
stress_prob(:,i) = Q(:,:,i)*eps;
end

%
   ----------------------------------------------------------------------

% Step-9: Convert stresses from PROBLEM CSYS to MATERIAL CSYS
stress_mat = sym(zeros(3,4));
R = sym(zeros(3,3,4));

R(:,:,1) = Rsig(LAYUP(1)); R(:,:,2) = Rsig(LAYUP(2));
R(:,:,3) = Rsig(LAYUP(3)); R(:,:,4) = Rsig(LAYUP(4));

for i=1:4
stress_mat(:,i) = inv(R(:,:,i))*stress_prob(:,i); %in Pa
end

% Stress in material coordinate system divided by N22
stress_mat_by_N22 = vpa(stress_mat/N22, 5);

%
   ----------------------------------------------------------------------

% Step-10: Apply Tsai-Wu Failure Criteria for each layer

% Store allowable N22 for each layer
f_allow=zeros(1,4);

for i=1:4
    % For lamina with Vf=0.5
    f_allow(i) = TsaiWu(STRENGTH(:,1),stress_mat_by_N22(:,i));
```

```
end

fprintf("Second Ply/Plies Failure: \n")
idx1 = checkFailureNew(f_allow,LAYUP);
f_allow;
fprintf("------------------------------------------------------- \
   n")


%
   -----------------------------------------------------------------------

% Step-11: Find degraded [A] after first ply failure


% Case-1: When idx=1
% Account for [+-45/0/90]s
if idx1==1 && LAYUP(1)==abs(LAYUP(2))
    Q1=Qnull;
    Q2=Qnull;
    % Account for [(+-45)s]s
    if abs(LAYUP(2))==abs(LAYUP(3))
        Q3=Qnull;
        Q4=Qnull;
    %else
    %    Q3=Q3_0;
    %    Q4=Q4_0;
    end
else
    Q1 = Qnull;
    %Q2 = Q2_0;
    %Q3 = Q3_0;
    %Q4 = Q4_0;
end

% Case-2: When idx=2
if idx1==2
    %Q1 = Q1_0;
    Q2 = Qnull;
    %Q3 = Q3_0;
    %Q4 = Q4_0;
end

% Case-3: When idx=3
if idx1==3
    % Account for [0_2/90_2]s
    if LAYUP(3)==LAYUP(4)
```

```
        %Q1 = Q1_0;
        %Q2 = Q2_0;
        Q3 = Qnull;
        Q4 = Qnull;
    else
        %Q1 = Q1_0;
        %Q2 = Q2_0;
        Q3 = Qnull;
        %Q4 = Q4_0;
    end

end

% Case-4: When idx=4
if idx1==4
    %Q1 = Q1_0;
    %Q2 = Q2_0;
    %Q3 = Q3_0;
    Q4 = Qnull;
end

%-------------------
% Q1
% Q2
% Q3
% Q4

A = findA(Q1,Q2,Q3,Q4); %in N/m

syms N22 real % tensile
eps = inv(A)*[0,N22,0]';


%
   -----------------------------------------------------------------

% Step-12: Find Stresses in each layer in the PROBLEM CSYS

% Each column represents layers 1 to 4
% Each row element is sig11, sig22, sig12
stress_prob = sym(zeros(3,4));
Q = sym(zeros(3,3,4));

Q(:,:,1)=Q1; Q(:,:,2)=Q2; Q(:,:,3)=Q3; Q(:,:,4)=Q4;


for i=1:4
```

```
stress_prob(:,i) = Q(:,:,i)*eps;
end

%
   ------------------------------------------------------------------

% Step-13: Convert stresses from PROBLEM CSYS to MATERIAL CSYS
stress_mat = sym(zeros(3,4));
R = sym(zeros(3,3,4));

R(:,:,1) = Rsig(LAYUP(1)); R(:,:,2) = Rsig(LAYUP(2));
R(:,:,3) = Rsig(LAYUP(3)); R(:,:,4) = Rsig(LAYUP(4));

for i=1:4
stress_mat(:,i) = inv(R(:,:,i))*stress_prob(:,i); %in Pa
end

% Stress in material coordinate system divided by N22
stress_mat_by_N22 = vpa(stress_mat/N22, 5);

%
   ------------------------------------------------------------------

% Step-14: Apply Tsai-Wu Failure Criteria for each layer

% Store allowable N22 for each layer
f_allow=zeros(1,4);

for i=1:4
    % For lamina with Vf=0.5
    f_allow(i) = TsaiWu(STRENGTH(:,1),stress_mat_by_N22(:,i));
end

fprintf("Third Ply/Plies Failure: \n")
idx1 = checkFailureNew(f_allow,LAYUP);
f_allow;
fprintf("----------------------------------------------------- \
   n")


%
   ------------------------------------------------------------------

% Step-15: Find degraded [A] after first ply failure


% Case-1: When idx=1
```

```
% Account for [+-45/0/90]s
if idx1==1 && LAYUP(1)==abs(LAYUP(2))
    Q1=Qnull;
    Q2=Qnull;
    % Account for [(+-45)s]s
    if abs(LAYUP(2))==abs(LAYUP(3))
        Q3=Qnull;
        Q4=Qnull;
    %else
    %     Q3=Q3_0;
    %     Q4=Q4_0;
    end
else
    Q1 = Qnull;
    %Q2 = Q2_0;
    %Q3 = Q3_0;
    %Q4 = Q4_0;
end

% Case-2: When idx=2
if idx1==2
    %Q1 = Q1_0;
    Q2 = Qnull;
    %Q3 = Q3_0;
    %Q4 = Q4_0;
end

% Case-3: When idx=3
if idx1==3
    % Account for [0_2/90_2]s
    if LAYUP(3)==LAYUP(4)
        %Q1 = Q1_0;
        %Q2 = Q2_0;
        Q3 = Qnull;
        Q4 = Qnull;
    else
        %Q1 = Q1_0;
        %Q2 = Q2_0;
        Q3 = Qnull;
        %Q4 = Q4_0;
    end

end

% Case-4: When idx=4
if idx1==4
    %Q1 = Q1_0;
```

```
    %Q2 = Q2_0;
    %Q3 = Q3_0;
    Q4 = Qnull;
end

%-------------------
% Q1
% Q2
% Q3
% Q4

A = findA(Q1,Q2,Q3,Q4); %in N/m

syms N22 real % tensile
eps = inv(A)*[0,N22,0]';


%
   --------------------------------------------------------------------

% Step-16: Find Stresses in each layer in the PROBLEM CSYS

% Each column represents layers 1 to 4
% Each row element is sig11, sig22, sig12
stress_prob = sym(zeros(3,4));
Q = sym(zeros(3,3,4));

Q(:,:,1)=Q1; Q(:,:,2)=Q2; Q(:,:,3)=Q3; Q(:,:,4)=Q4;


for i=1:4
stress_prob(:,i) = Q(:,:,i)*eps;
end

%
   --------------------------------------------------------------------

% Step-17: Convert stresses from PROBLEM CSYS to MATERIAL CSYS
stress_mat = sym(zeros(3,4));
R = sym(zeros(3,3,4));

R(:,:,1) = Rsig(LAYUP(1)); R(:,:,2) = Rsig(LAYUP(2));
R(:,:,3) = Rsig(LAYUP(3)); R(:,:,4) = Rsig(LAYUP(4));

for i=1:4
stress_mat(:,i) = inv(R(:,:,i))*stress_prob(:,i); %in Pa
end
```

```
% Stress in material coordinate system divided by N22
stress_mat_by_N22 = vpa(stress_mat/N22, 5);

%
   ----------------------------------------------------------------------

% Step-18: Apply Tsai-Wu Failure Criteria for each layer

% Store allowable N22 for each layer
f_allow=zeros(1,4);

for i=1:4
    % For lamina with Vf=0.5
    f_allow(i) = TsaiWu(STRENGTH(:,1),stress_mat_by_N22(:,i));
end

fprintf("Fourth Ply/Plies Failure: \n")
idx1 = checkFailureNew(f_allow,LAYUP);
f_allow;


end
```

- 'Proj_Part3.m' - MAIN MATLAB file for the 3rd part of the project

```
% Project Part-3:

% Layup-1: [+45/-45/0/90]s
% Layup-2: [0/30/60/90]s
% Layup-3: [+45/-45/-45/+45]s
% Layup-4: [0/0/90/90]s

%----------------------------------------------------
% Load HROM engineering constants of lamina for
% varying volume fiber fraction

% Format: [E1,nu12,E2,nu23,G12]'
load("consts.mat")



%----------------------------------------------------
% Save Strength Parameters
% Format: [X; Xdash; Y; Ydash; S; R] % in MPa
load("strength.mat")

%----------------------------------------------------
```

```
% Save Layup Sequences
LAYUP = [45,-45,  0, 90;  % Layup-1
          0, 30, 60, 90;  % Layup-2
         45,-45,-45, 45;  % Layup-3
          0,  0, 90, 90]; % Layup-4

%---------------------------------------------------

% Design - 1A

% CONSTS and STRENGTH - depend on volume fraction i.e. 'A'
% LAYUP - depends on '1'

vfCode = ["A","B","C"];
vfVals = [0.5,0.6,0.7];

for i=1:4 % Layup Scheme (1,2,3,4)
    for j=1:3 % Volume Fraction (A,B,C)
        fprintf("Design: %.15g %s \n",i,vfCode(j))
        fprintf('Layup: [%s]_s \n', join(string(LAYUP(i,:)), ','
           ));
        fprintf('Volume Fraction: %0.2f \n \n',vfVals(j));
        doMyJob(CONSTS(:,j),LAYUP(i,:),STRENGTH(:,j))
        fprintf
           ("=====================================================
            \n")
    end
end
```