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Final Project AAE 555

A lamina is made of carbon fiber with $E_1 = 276$ GPa, $E_2 = 19.5$ GPa, $\nu_{12} = 0.28$, $\nu_{23} = 0.70$ $G_{12} = 70$ GPa and epoxy with $E = 4.76$ GPa, $\nu = 0.37$.

1. Predict the effective properties of the lamina with respect to the fiber volume fraction of 0.5, 0.6, and 0.7 using the hybrid rule of mixture (ROM).
2. Assume a $[\pm 45/0/90]s$ laminate made of composite layers with the lamina of fiber volume fraction equal to 0.5. The effective lamina properties are obtained using the hybrid ROM results). The thickness of each layer is 0.127 mm. The laminate is subject to tensile load N_{22}. Assumes that the lamina fails according to the Tsai-Wu failure criterion and once a layer is failed then the stiffness matrix of that ply is degraded to be zero. Compute the ultimate failure load N22. The strength parameters are given in Table 1.
3. Design a laminate with initial failure load $N_{22} \geq 4 * 10^5$ N/m. The design options are
    a. Layup schemes: $[\pm 45/0/90]_s$, $[0/30/60/90]_s$, $[(\pm 45)_2]_s$, $[0_2 90_2]_s$
    b. Fiber volume fraction: 0.5, 0.6, 0.7

    Assumes the lamina fails according to the Tsai-Wu failure criterion. Which option(s) (the combination of layup scheme and fiber volume fraction) can achieve the design objective?

Table 1 Different fiber volume fractions strength parameters

Vf	X	Xp	Y	Yp	S	R
0.5	2.1E+09	2.1E+09	6.2E+07	2.1E+08	1.0E+08	1.0E+08
0.6	2.4E+09	2.4E+09	6.8E+07	2.3E+08	1.1E+08	1.1E+08
0.7	2.5E+09	2.5E+09	7.1E+07	2.4E+08	1.2E+08	1.2E+08

*unit: Pa

Question 1

A lamina is made of carbon fiber with $E_1 = 276$ GPa, $E_2 = 19.5$ GPa, $\nu_{12} = 0.28$, $\nu_{23} = 0.70$
$G_{12} = 70$ GPa and epoxy with $E = 4.76$ GPa, $\nu = 0.37$.

1. Predict the effective properties of the lamina with respect to the fiber volume fraction of 0.5, 0.6, and 0.7 using the hybrid rule of mixture (ROM).

The material properties are

In[1]:=
$$\text{material} = \{\text{e1f} \to 276 * 10^9,\ \text{e2f} \to 19.5 * 10^9,\ \nu\text{12f} \to 0.28,\ \nu\text{23f} \to 0.7,\ \text{g12f} \to 70 * 10^9,\ \text{em} \to 4.76 * 10^9,\ \nu\text{m} \to 0.37\};$$

In[2]:=
$$\text{sf} = \begin{pmatrix} \frac{1}{\text{e1f}} & -\frac{\nu\text{12f}}{\text{e1f}} & -\frac{\nu\text{12f}}{\text{e1f}} & 0 & 0 & 0 \\ -\frac{\nu\text{12f}}{\text{e1f}} & \frac{1}{\text{e2f}} & -\frac{\nu\text{23f}}{\text{e2f}} & 0 & 0 & 0 \\ -\frac{\nu\text{12f}}{\text{e1f}} & -\frac{\nu\text{23f}}{\text{e2f}} & \frac{1}{\text{e2f}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2\,(1+\nu\text{23f})}{\text{e2f}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\text{g12f}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\text{g12f}} \end{pmatrix} \text{ // Simplify;}$$

Stiffness matrix for transversely isotropic fiber is

In[3]:=
```
cf = Inverse[sf] // Simplify;
```

The compliance matrix of isotropic matrix is

In[4]:=
$$\text{sm} = \begin{pmatrix} \frac{1}{\text{em}} & -\frac{\nu\text{m}}{\text{em}} & -\frac{\nu\text{m}}{\text{em}} & 0 & 0 & 0 \\ -\frac{\nu\text{m}}{\text{em}} & \frac{1}{\text{em}} & -\frac{\nu\text{m}}{\text{em}} & 0 & 0 & 0 \\ -\frac{\nu\text{m}}{\text{em}} & -\frac{\nu\text{m}}{\text{em}} & \frac{1}{\text{em}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2\,(1+\nu\text{m})}{\text{em}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2\,(1+\nu\text{m})}{\text{em}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2\,(1+\nu\text{m})}{\text{em}} \end{pmatrix} \text{ // Simplify;}$$

The stiffness matrix of isotropic matrix is

In[5]:=
```
cm = Inverse[sm] // Simplify;
```

According to hrbrid rule of mixtures, S^H can be calculated as follows

In[6]:=
```
ϵ = {ϵ11, ϵ22, ϵ33, γ23, γ13, γ12};
σ = {σ11, σ22, σ33, σ23, σ13, σ12};
```

In[8]:=
$$s = \begin{pmatrix} \frac{1}{e1} & -\frac{v12}{e1} & -\frac{v12}{e1} & 0 & 0 & 0 \\ -\frac{v12}{e1} & \frac{1}{e2} & -\frac{v23}{e2} & 0 & 0 & 0 \\ -\frac{v12}{e1} & -\frac{v23}{e2} & \frac{1}{e2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2\,(1+v23)}{e2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{g12} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{g12} \end{pmatrix};$$

In[9]:=
```
ϵtemp = s.σ;
```

In[10]:=
```
equations = Table[ϵtemp[[i]] == ϵ[[i]], {i, 1, 6}];
sol = Solve[equations, {σ11, ϵ22, ϵ33, γ23, γ13, γ12}][[1]];
ϵH = {σ11, ϵ22, ϵ33, γ23, γ13, γ12} /. sol;
σH = {ϵ11, σ22, σ33, σ23, σ13, σ12};
sH = Table[Coefficient[ϵH[[i]], σH[[j]]], {i, 1, 6}, {j, 1, 6}] // Simplify;
```

In[15]:=
```
% // TraditionalForm
```

Out[15]//TraditionalForm=
$$\begin{pmatrix} e1 & \nu 12 & \nu 12 & 0 & 0 & 0 \\ -\nu 12 & \frac{1}{e2} - \frac{\nu 12^2}{e1} & -\frac{\nu 12^2}{e1} - \frac{\nu 23}{e2} & 0 & 0 & 0 \\ -\nu 12 & -\frac{\nu 12^2}{e1} - \frac{\nu 23}{e2} & \frac{1}{e2} - \frac{\nu 12^2}{e1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2\,(\nu 23+1)}{e2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{g12} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{g12} \end{pmatrix}$$

S^H matrix for fiber

In[16]:=
```
fiber = {e1 → e1f, e2 → e2f, ν12 → ν12f, ν23 → ν23f, g12 → g12f};
```

In[17]:=
```
sHf = sH /. fiber // Simplify;
```

In[18]:=
```
% // TraditionalForm
```

Out[18]//TraditionalForm=
$$\begin{pmatrix} e1f & \nu 12f & \nu 12f & 0 & 0 & 0 \\ -\nu 12f & \frac{1}{e2f} - \frac{\nu 12f^2}{e1f} & -\frac{\nu 12f^2}{e1f} - \frac{\nu 23f}{e2f} & 0 & 0 & 0 \\ -\nu 12f & -\frac{\nu 12f^2}{e1f} - \frac{\nu 23f}{e2f} & \frac{1}{e2f} - \frac{\nu 12f^2}{e1f} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2\,(\nu 23f+1)}{e2f} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{g12f} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{g12f} \end{pmatrix}$$

S^H matrix for fiber

In[19]:=
```
matrix = {e1 → em, e2 → em, ν12 → νm, ν23 → νm, g12 → gm};
```

In[20]:=
```
sHm = sH /. matrix // Simplify;
```

In[21]:= `% // TraditionalForm`

Out[21]//TraditionalForm=

$$\begin{pmatrix} em & \nu m & \nu m & 0 & 0 & 0 \\ -\nu m & \frac{1-\nu m^2}{em} & -\frac{\nu m(\nu m+1)}{em} & 0 & 0 & 0 \\ -\nu m & -\frac{\nu m(\nu m+1)}{em} & \frac{1-\nu m^2}{em} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2(\nu m+1)}{em} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{gm} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{gm} \end{pmatrix}$$

S^{H*} matrix is

In[22]:= `sHs = sHf * vf + sHm * (1 - vf) // Simplify;`

In[23]:= `% // TraditionalForm`

Out[23]//TraditionalForm=

$$\begin{pmatrix} e1f\,vf + em(-vf) + em & \nu m + vf(\nu 12f - \nu m) & \nu m + vf(\nu 12f - \nu m) & 0 \\ vf(\nu m - \nu 12f) - \nu m & vf\left(\frac{1}{e2f} - \frac{\nu 12f^2}{e1f}\right) + \frac{(\nu m^2-1)(vf-1)}{em} & -\frac{\nu 12f^2\,vf}{e1f} - \frac{\nu 23f\,vf}{e2f} + \frac{\nu m(\nu m+1)(vf-1)}{em} & 0 \\ vf(\nu m - \nu 12f) - \nu m & -\frac{\nu 12f^2\,vf}{e1f} - \frac{\nu 23f\,vf}{e2f} + \frac{\nu m(\nu m+1)(vf-1)}{em} & vf\left(\frac{1}{e2f} - \frac{\nu 12f^2}{e1f}\right) + \frac{(\nu m^2-1)(vf-1)}{em} & 0 \\ 0 & 0 & 0 & \frac{2(\nu 23f+1)\,vf}{e2f} - \frac{2(\nu m+1)(vf-1)}{em} \\ 0 & 0 & 0 & 0 & \frac{vf}{g12f} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Effective engineering constants are calculated from Hybrid ROM are

In[24]:= `E1Hs = vf * e1f + (1 - vf) * em // Simplify;`

In[25]:= `ν12Hs = vf * ν12f + (1 - vf) * νm // Simplify;`

In[26]:= $E2Hs = \dfrac{1}{\frac{vf}{e2f} + \frac{1-vf}{em} - \frac{vf*(1-vf)*(em*\nu 12f - e1f*\nu m)^2}{e1f*em*(e1f*vf+em\,(1-vf))}}$ `// Simplify;`

In[27]:= $\nu 23Hs = E2Hs\left(vf\left(\frac{\nu 23f}{e2f} + \frac{(\nu 12f)^2}{e1f}\right) + \frac{(1-vf)*\nu m}{em}(1+\nu m) - \frac{(\nu 12Hs)^2}{E1Hs}\right)$ `// Simplify;`

In[28]:= $G12Hs = \dfrac{1}{\frac{vf}{g12f} + \frac{(1-vf)}{gm}}$ `// Simplify;`

According to hybrid ROM, effective engineering constants are as following. They are given as a function of volume fraction

In[29]:= `E1HsP = E1Hs /. material // N // Simplify`

Out[29]= $4.76 \times 10^9 + 2.7124 \times 10^{11}\, vf$

In[30]:=
```
E2HsP = E2Hs /. material // N // Simplify
```

Out[30]=
$$\frac{1.34686\times10^{8}+7.67485\times10^{9}\,\text{vf}}{0.0282954+1.37219\,\text{vf}-1.\,\text{vf}^{2}}$$

In[31]:=
```
ν12HsP = ν12Hs /. material // N // Simplify
```

Out[31]= $0.37-0.09\,\text{vf}$

In[32]:=
```
ν23HsP = ν23Hs /. material // N // Simplify
```

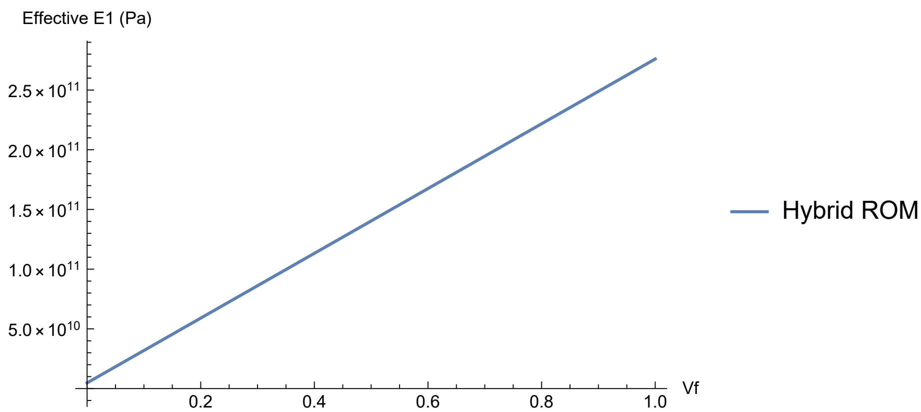
Out[32]=
$$\frac{0.0104693+0.809722\,\text{vf}-0.539849\,\text{vf}^{2}}{0.0282954+1.37219\,\text{vf}-1.\,\text{vf}^{2}}$$

In[33]:=
$$\text{G12HsP} = \text{G12Hs} \,/.\, \left\{\text{gm} \to \frac{\text{em}}{2\,(1+\nu\text{m})}\right\} \,/.\, \text{material} \,//\, \text{N} \,//\, \text{Simplify}$$

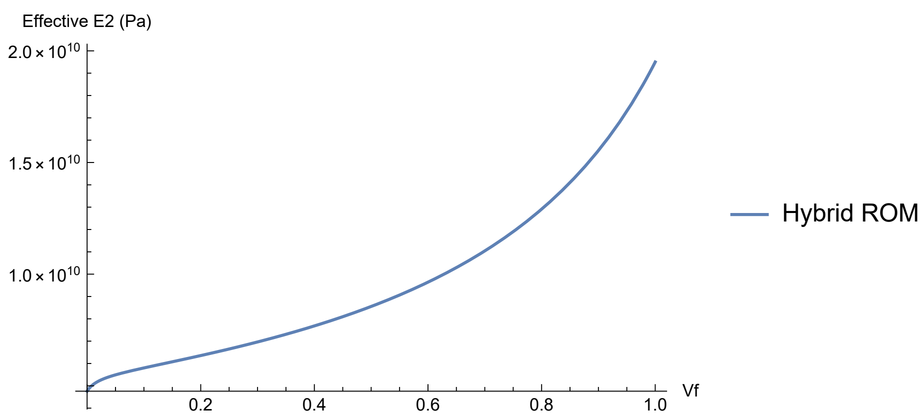
Out[33]=
$$\frac{1.78144\times10^{9}}{1.02545-1.\,\text{vf}}$$

Plots and predictions

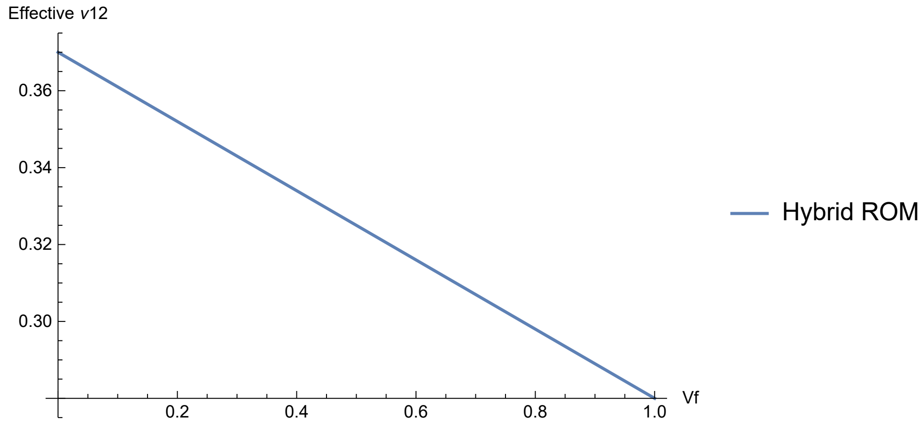
In[34]:=
```
Plot[{E1HsP}, {vf, 0, 1}, PlotLegends → {"Hybrid ROM"},
 AxesLabel → {"Vf", "Effective E1 (Pa)"}, PlotStyle → {Automatic}]
```

Out[34]=


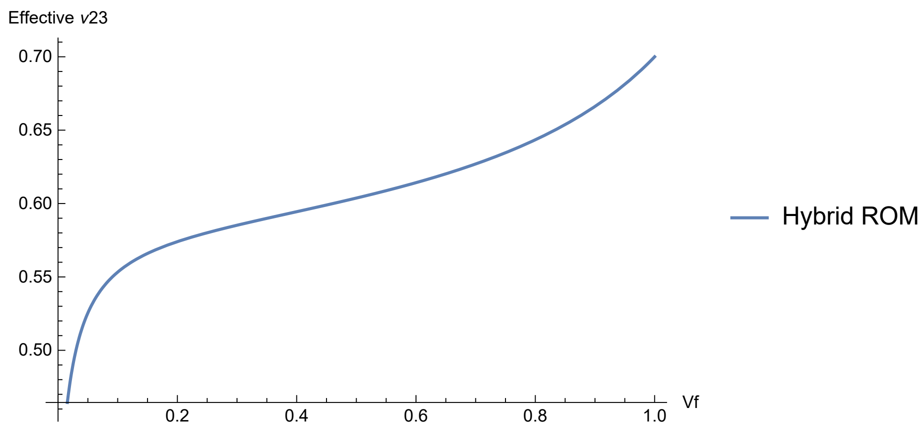
In[35]:=
```
Plot[{E2HsP}, {vf, 0, 1}, PlotLegends → {"Hybrid ROM"},
 AxesLabel → {"Vf", "Effective E2 (Pa)"}, PlotStyle → {Automatic}]
```

Out[35]=


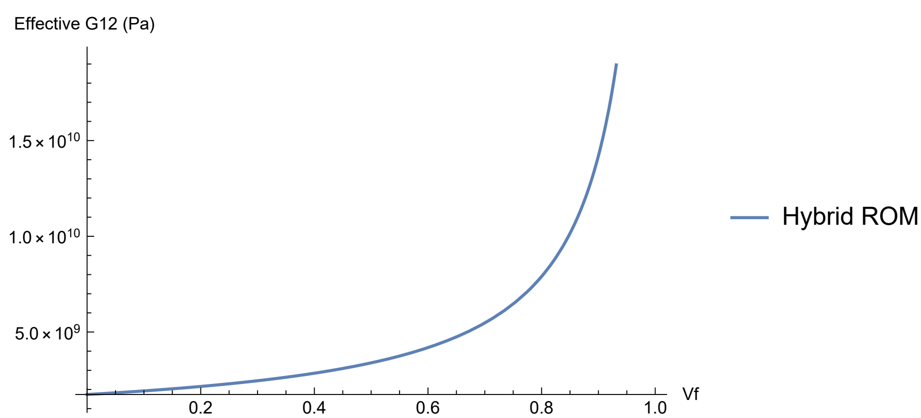
In[36]:=
```
Plot[{ν12HsP}, {vf, 0, 1}, PlotLegends → {"Hybrid ROM"},
 AxesLabel → {"Vf", "Effective ν12"}, PlotStyle → {Automatic}]
```

Out[36]=


In[37]:=
```
Plot[{ν23HsP}, {vf, 0, 1}, PlotLegends → {"Hybrid ROM"},
 AxesLabel → {"Vf", "Effective ν23"}, PlotStyle → {Automatic}]
```

Out[37]=


In[38]:=
```
Plot[{G12HsP}, {vf, 0, 1}, PlotLegends → {"Hybrid ROM"},
 AxesLabel → {"Vf", "Effective G12 (Pa) "}, PlotStyle → {Automatic}]
```

Out[38]=


Effective Properties for Volume fraction vf=0.5

In[39]:=
```
E1HsP1 = E1HsP "Pa" /. vf → 0.5
```
Out[39]= 1.4038×10^{11} Pa

In[40]:=
```
E2HsP1 = E2HsP "Pa" /. vf → 0.5
```
Out[40]= 8.55336×10^{9} Pa

In[41]:=
```
ν12HsP1 = ν12HsP /. vf → 0.5
```
Out[41]= 0.325

In[42]:=
```
ν23HsP1 = ν23HsP /. vf → 0.5
```
Out[42]= 0.603731

In[43]:=
```
G12HsP1 = G12HsP "Pa" /. vf → 0.5
```
Out[43]= 3.39031×10^{9} Pa

Effective Properties for Volume fraction vf=0.6

In[44]:=
```
E1HsP2 = E1HsP  "Pa" /. vf → 0.6
```
Out[44]= 1.67504×10^{11} Pa

In[45]:=
```
E2HsP2 = E2HsP "Pa" /. vf → 0.6
```
Out[45]= 9.64094×10^{9} Pa

In[46]:=
```
ν12HsP2 = ν12HsP /. vf → 0.6
```
Out[46]= 0.316

In[47]:=
```
ν23HsP2 = ν23HsP /. vf → 0.6
```
Out[47]= 0.614218

In[48]:=
```
G12HsP2 = G12HsP "Pa" /. vf → 0.6
```
Out[48]= 4.18719×10^{9} Pa

Effective Properties for Volume fraction vf=0.7

In[49]:=
```
E1HsP3 = E1HsP "Pa" /. vf → 0.7
```
Out[49]= 1.94628×10^{11} Pa

In[50]:=
```
E2HsP3 = E2HsP "Pa" /. vf → 0.7
```
Out[50]= 1.104×10^{10} Pa

In[51]:=
```
ν12HsP3 = ν12HsP /. vf → 0.7
```
Out[51]= 0.307

```
In[52]:= ν23HsP3 = ν23HsP /. vf → 0.7
Out[52]= 0.626963

In[53]:= G12HsP3 = G12HsP "Pa" /. vf → 0.7
Out[53]= 5.47378 × 10^9 Pa
```

Material Properties corresponding to volume fractions from question 1

Volume Fraction 0.5

E_1= 1.4038 * 10^{11} Pa ; E_2=8.55 * 10^9 Pa ; v_{12}=0.325

V_{23}=0.603731 ; G_{12}= 3.39031 * 10^9

Volume Fraction 0.6

E_1= 1.67504 * 10^{11} Pa ; E_2=9.64094 * 10^9 Pa ; v_{12}=0.316

V_{23}=0.614218 ; G_{12}=4.18719* 10^9

Volume Fraction 0.7

E_1= 1.94628 * 10^{11} Pa ; E_2=11.04* 10^9 Pa ; v_{12}=0.307

V_{23}=0.626963 ; G_{12}=5.47378 * 10^9

Question 2

2. Assume a $[\pm 45/0/90]s$ laminate made of composite layers with the lamina of fiber volume fraction equal to 0.5. The effective lamina properties are obtained using the hybrid ROM results). The thickness of each layer is 0.127 mm. The laminate is subject to tensile load N_{22}. Assumes that the lamina fails according to the Tsai-Wu failure criterion and once a layer is failed then the stiffness matrix of that ply is degraded to be zero. Compute the ultimate failure load N22. The strength parameters are given in Table 1.

Material Properties are taken from the result of the previous problem for 0.5 volume fraction

```
In[∘]:= laminaConstants = {E1 → 1.4038 * 10^11, E2 → 8.55336 * 10^9, G12 → 3.39031 * 10^9, ν12 → 0.325,
          ν23 → 0.603731, X → 2.1 * 10^9, XP → 2.1 * 10^9, Y → 6.2 * 10^7, YP → 2.1 * 10^8, S → 1 * 10^8};

In[∘]:= t = 127/1000 * 10^-3;

In[∘]:= n = 8;

In[∘]:= Nn = {0, N22, 0};

In[∘]:= M = {0, 0, 0};

In[∘]:= angles = {45, -45, 0, 90, 90, 0, -45, 45};

In[∘]:= ti = Table[t, {i, 1, n}];

In[∘]:= bzi = Table[-1/2 (n + 1) t + i t, {i, 1, n}];

In[∘]:= layercoord = Table[{bzi[[i]] - 1/2 ti[[i]], bzi[[i]] + 1/2 ti[[i]]}, {i, 1, n}];

In[∘]:= laminaConstantsN = Table[laminaConstants, {i, 1, 8}];
```

$$R\sigma e = \begin{pmatrix} c^2 & s^2 & -2\,c\,s \\ s^2 & c^2 & 2\,c\,s \\ c\,s & -c\,s & c^2 - s^2 \end{pmatrix};$$

$$Sep = \begin{pmatrix} \frac{1}{E1} & -\frac{\nu 12}{E1} & 0 \\ -\frac{\nu 12}{E1} & \frac{1}{E2} & 0 \\ 0 & 0 & \frac{1}{G12} \end{pmatrix};$$

```
In[∘]:= Q = Simplify[Inverse[Sep]];

In[∘]:= Qθ[i_] :=
         Rσe.Q.Transpose[Rσe] /. {s → Sin[θ Degree], c → Cos[θ Degree]} /. θ → angles[[i]] /.
          laminaConstantsN[[i]]
```

```
In[•]:= A = Sum[Qθ[i] × ti[[i]], {i, 1, n}] // N
Out[•]= {{5.95441 × 10^7, 1.94467 × 10^7, 0.}, {1.94467 × 10^7, 5.95441 × 10^7, 0.}, {0., 0., 2.00487 × 10^7}}

In[•]:= B = Sum[Qθ[i] × ti[[i]] × bzi[[i]], {i, 1, n}]
Out[•]= {{0., 0., 0.}, {0., 0., 0.}, {0., 0., 0.}}

In[•]:= Dd = Sum[Qθ[i] (ti[[i]] bzi[[i]]^2 + ti[[i]]^3 / 12), {i, 1, n}] // N
Out[•]= {{4.59439, 2.74407, 0.815342}, {2.74407, 3.50727, 0.815342}, {0.815342, 0.815342, 2.79585}}

In[•]:= ϵ = Inverse[A].Nn // Chop
Out[•]= {-6.1398 × 10^-9 N22, 1.87995 × 10^-8 N22, 0}

In[•]:= κ = Inverse[Dd].M // Chop
Out[•]= {0, 0, 0}

In[•]:= σe = Table[Inverse[Rσe].Qθ[i].(ϵ + x3 κ) /. {s → Sin[θ Degree], c → Cos[θ Degree]} /.
            θ → angles[[i]], {i, 1, n}]
Out[•]= {{912.05 N22, 72.2021 N22, 84.552 N22}, {912.05 N22, 72.2021 N22, -84.552 N22},
         {0. - 814.89 N22, 0. + 144.662 N22, 0.}, {0. + 2638.99 N22, 0. - 0.257956 N22, 0.},
         {0. + 2638.99 N22, 0. - 0.257956 N22, 0.}, {0. - 814.89 N22, 0. + 144.662 N22, 0.},
         {912.05 N22, 72.2021 N22, -84.552 N22}, {912.05 N22, 72.2021 N22, 84.552 N22}}
```

In[•]:= TW[σ_] :=

$$\left(\frac{1}{X}-\frac{1}{XP}\right)*\sigma[[1]]+\left(\frac{1}{Y}-\frac{1}{YP}\right)*\sigma[[2]]+\frac{\sigma[[1]]^2}{X*XP}+\frac{\sigma[[2]]^2}{Y*YP}+\left(\frac{\sigma[[3]]}{S}\right)^2-\frac{\sigma[[1]]\times\sigma[[2]]}{X^2};$$

```
In[•]:= sol = Table[Solve[(TW[σe[[i]]] /. laminaConstants) == 1, N22], {i, 1, n}]
Out[•]= {{{N22 → -1.25493 × 10^6}, {N22 → 618 204.}}, {{N22 → -1.25493 × 10^6}, {N22 → 618 204.}},
         {{N22 → -1.33969 × 10^6}, {N22 → 418 264.}}, {{N22 → -794 791.}, {N22 → 796 648.}},
         {{N22 → -794 791.}, {N22 → 796 648.}}, {{N22 → -1.33969 × 10^6}, {N22 → 418 264.}},
         {{N22 → -1.25493 × 10^6}, {N22 → 618 204.}}, {{N22 → -1.25493 × 10^6}, {N22 → 618 204.}}}

In[•]:= % // TraditionalForm
Out[•]//TraditionalForm=
```

$$\begin{pmatrix} \{N22 \to -1.25493\times10^6\} & \{N22 \to 618\,204.\} \\ \{N22 \to -1.25493\times10^6\} & \{N22 \to 618\,204.\} \\ \{N22 \to -1.33969\times10^6\} & \{N22 \to 418\,264.\} \\ \{N22 \to -794\,791.\} & \{N22 \to 796\,648.\} \\ \{N22 \to -794\,791.\} & \{N22 \to 796\,648.\} \\ \{N22 \to -1.33969\times10^6\} & \{N22 \to 418\,264.\} \\ \{N22 \to -1.25493\times10^6\} & \{N22 \to 618\,204.\} \\ \{N22 \to -1.25493\times10^6\} & \{N22 \to 618\,204.\} \end{pmatrix}$$

```
In[•]:=

In[•]:= Min[Abs[Values[sol]]]
Out[•]= 418 264.
```

In[∘]:=
```
N1W = N22 /. sol[[3, 2]]
```
Out[∘]= 418 264.

In[∘]:=
```
ϵ1W = ϵ[[2]] /. N22 → N1W
```
Out[∘]= 0.00786315

In[∘]:=
```
Modes[σ_] := If[σ[[1]] / X > σ[[2]] / Y, Fiber, Matrix]
Modes[σe[[1]]] /. laminaConstants /. sol[[3, 2]]
```
Out[∘]= Matrix

From the above results, we can tell the 3rd and 6th layer will fail first as the tensile fiber mode yield N11= 418264 N/m.
We will degrade these two layers and recalculate the failure load

In[∘]:=
```
laminaConstantsN[[3]] = {E1 → 1.4038 * 10^11, E2 → 0, G12 → 0,
    ν12 → 0.325, ν23 → 0.603731, X → 2.1 * 10^9, Y → 6.2 * 10^7, S → 1 * 10^8};
laminaConstantsN[[6]] = {E1 → 1.4038 * 10^11, E2 → 0, G12 → 0, ν12 → 0.325,
    ν23 → 0.603731, X → 2.1 * 10^9, Y → 6.2 * 10^7, S → 1 * 10^8};
```

In[∘]:=
```
A = Sum[Qθ[i] × ti[[i]], {i, 1, n}] // N
```
Out[∘]= $\{\{5.93131\times10^{7}, 1.87361\times10^{7}, 0.\}, \{1.87361\times10^{7}, 5.73575\times10^{7}, 0.\}, \{0., 0., 1.91875\times10^{7}\}\}$

In[∘]:=
```
ϵ = Inverse[A].Nn // Chop
```
Out[∘]= $\{-6.14094\times10^{-9}\,\text{N22}, 1.94405\times10^{-8}\,\text{N22}, 0\}$

In[∘]:=
```
σe = Table[Inverse[Rσe].Qθ[i].(ϵ + x3 κ) /. {s → Sin[θ Degree], c → Cos[θ Degree]} /.
    θ → angles[[i]], {i, 1, n}]
```
Out[∘]= {{958.147 N22, 75.8514 N22, 86.729 N22}, {958.147 N22, 75.8514 N22, -86.729 N22},
 {0. - 862.066 N22, 0., 0.}, {0. + 2729.55 N22, 0. + 1.5256 N22, 0.},
 {0. + 2729.55 N22, 0. + 1.5256 N22, 0.}, {0. - 862.066 N22, 0., 0.},
 {958.147 N22, 75.8514 N22, -86.729 N22}, {958.147 N22, 75.8514 N22, 86.729 N22}}

In[∘]:=
```
sol = Table[Solve[(TW[σe[[i]]] /. laminaConstants) == 1, N22], {i, 1, n}]
```
Out[∘]= $\{\{\{\text{N22}\to-1.21575\times10^{6}\}, \{\text{N22}\to593559.\}\}, \{\{\text{N22}\to-1.21575\times10^{6}\}, \{\text{N22}\to593559.\}\},$
$\{\{\text{N22}\to-2.43601\times10^{6}\}, \{\text{N22}\to2.43601\times10^{6}\}\}, \{\{\text{N22}\to-774683.\}, \{\text{N22}\to764414.\}\},$
$\{\{\text{N22}\to-774683.\}, \{\text{N22}\to764414.\}\}, \{\{\text{N22}\to-2.43601\times10^{6}\}, \{\text{N22}\to2.43601\times10^{6}\}\},$
$\{\{\text{N22}\to-1.21575\times10^{6}\}, \{\text{N22}\to593559.\}\}, \{\{\text{N22}\to-1.21575\times10^{6}\}, \{\text{N22}\to593559.\}\}\}$

In[∘]:= `% // TraditionalForm`

Out[∘]//TraditionalForm=

$$\begin{pmatrix} \{N22 \to -1.21575\times 10^6\} & \{N22 \to 593\,559.\} \\ \{N22 \to -1.21575\times 10^6\} & \{N22 \to 593\,559.\} \\ \{N22 \to -2.43601\times 10^6\} & \{N22 \to 2.43601\times 10^6\} \\ \{N22 \to -774\,683.\} & \{N22 \to 764\,414.\} \\ \{N22 \to -774\,683.\} & \{N22 \to 764\,414.\} \\ \{N22 \to -2.43601\times 10^6\} & \{N22 \to 2.43601\times 10^6\} \\ \{N22 \to -1.21575\times 10^6\} & \{N22 \to 593\,559.\} \\ \{N22 \to -1.21575\times 10^6\} & \{N22 \to 593\,559.\} \end{pmatrix}$$

In[∘]:= `Min[Abs[Values[sol]]]`

Out[∘]= 593 559.

In[∘]:= `N2W = N22 /. sol[[1, 2]]`

Out[∘]= 593 559.

In[∘]:= `ϵ2W = ϵ[[2]] /. N22 → N2W`

Out[∘]= 0.0115391

In[∘]:= `Modes[σe[[3]]] /. laminaConstants /. sol[[1, 2]]`

Out[∘]= Matrix

From the above results, we can tell the 1st, 2nd 7th and the 8th layer will fail as the tensile fiber mode yield N11= 593559 N/m.
We will degrade these four layers and recalculate the failure load

In[∘]:=
```
laminaConstantsN[[1]] = {E1 → 1.4038 * 10^11, E2 → 0,
    G12 → 0, ν12 → 0, ν23 → 0.603731, X → 2.1 * 10^9, Y → 6.2 * 10^7, S → 1 * 10^8};
laminaConstantsN[[2]] = {E1 → 1.4038 * 10^11, E2 → 0, G12 → 0, ν12 → 0,
    ν23 → 0.603731, X → 2.1 * 10^9, Y → 6.2 * 10^7, S → 1 * 10^8};
laminaConstantsN[[7]] = {E1 → 1.4038 * 10^11, E2 → 0, G12 → 0, ν12 → 0,
    ν23 → 0.603731, X → 2.1 * 10^9, Y → 6.2 * 10^7, S → 1 * 10^8};
laminaConstantsN[[8]] = {E1 → 1.4038 * 10^11, E2 → 0, G12 → 0, ν12 → 0,
    ν23 → 0.603731, X → 2.1 * 10^9, Y → 6.2 * 10^7, S → 1 * 10^8};
```

In[∘]:= `A = (Sum[Qθ[i] × ti[[i]], {i, 1, n}] // N)`

Out[∘]= $\{\{5.56714\times 10^7, 1.85389\times 10^7, 0.\}, \{1.85389\times 10^7, 5.37157\times 10^7, 0.\}, \{0., 0., 1.86894\times 10^7\}\}$

In[∘]:= `ϵ = Inverse[A].Nn // Chop`

Out[∘]= $\{-7.00443\times 10^{-9}\,N22, 2.1034\times 10^{-8}\,N22, 0\}$

In[∘]:=
```
σe = Table[Inverse[Rσe].Qθ[i].(ϵ + x3 κ) /. {s → Sin[θ Degree], c → Cos[θ Degree]} /.
    θ → angles[[i]], {i, 1, n}]
```

Out[∘]=
```
{{984.732 N22, 0., 0.}, {984.732 N22, 0., 0.}, {0. - 983.282 N22, 0., 0.},
 {0. + 2952.28 N22, 0. - 1.4497 N22, 0.}, {0. + 2952.28 N22, 0. - 1.4497 N22, 0.},
 {0. - 983.282 N22, 0., 0.}, {984.732 N22, 0., 0.}, {984.732 N22, 0., 0.}}
```

In[∘]:=
```
sol = Table[Solve[(TW[σe[[i]]] /. laminaConstants) == 1, N22], {i, 1, n}]
```

Out[∘]=
$$\{\{\{N22 \to -2.13256\times10^{6}\}, \{N22 \to 2.13256\times10^{6}\}\}, \{\{N22 \to -2.13256\times10^{6}\}, \{N22 \to 2.13256\times10^{6}\}\}, \{\{N22 \to -2.1357\times10^{6}\}, \{N22 \to 2.1357\times10^{6}\}\}, \{\{N22 \to -706958.\}, \{N22 \to 715291.\}\}, \{\{N22 \to -706958.\}, \{N22 \to 715291.\}\}, \{\{N22 \to -2.1357\times10^{6}\}, \{N22 \to 2.1357\times10^{6}\}\}, \{\{N22 \to -2.13256\times10^{6}\}, \{N22 \to 2.13256\times10^{6}\}\}, \{\{N22 \to -2.13256\times10^{6}\}, \{N22 \to 2.13256\times10^{6}\}\}\}$$

In[∘]:=
```
% // TraditionalForm
```

Out[∘]//TraditionalForm=
$$\begin{pmatrix} \{N22 \to -2.13256\times10^{6}\} & \{N22 \to 2.13256\times10^{6}\} \\ \{N22 \to -2.13256\times10^{6}\} & \{N22 \to 2.13256\times10^{6}\} \\ \{N22 \to -2.1357\times10^{6}\} & \{N22 \to 2.1357\times10^{6}\} \\ \{N22 \to -706958.\} & \{N22 \to 715291.\} \\ \{N22 \to -706958.\} & \{N22 \to 715291.\} \\ \{N22 \to -2.1357\times10^{6}\} & \{N22 \to 2.1357\times10^{6}\} \\ \{N22 \to -2.13256\times10^{6}\} & \{N22 \to 2.13256\times10^{6}\} \\ \{N22 \to -2.13256\times10^{6}\} & \{N22 \to 2.13256\times10^{6}\} \end{pmatrix}$$

In[∘]:=
```
N3W = N22 /. sol[[4, 2]]
```

Out[∘]= 715 291.

In[∘]:=
```
ϵ3W = ϵ[[2]] /. N22 → N3W
```

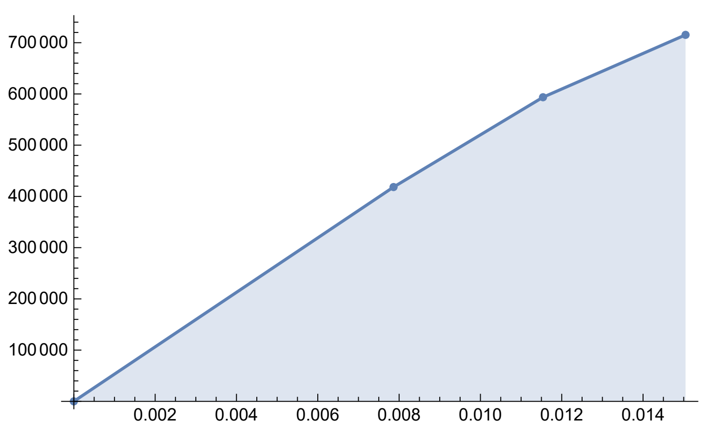
Out[∘]= 0.0150454

In[∘]:=
```
Modes[σe[[4]]] /. laminaConstants /. sol[[4, 2]]
```

Out[∘]= Fiber

In[∘]:=
```
p2 = ListPlot[{{0, 0}, {ϵ1W, N1W}, {ϵ2W, N2W}, {ϵ3W, N3W}},
   Joined → True, Mesh → All, Filling → Axis]
```

Out[∘]=



The Plot above is the Progressive failure analysis of laminate Under N22. The ultimate failure N is 715291 N/m according to Tsai Wu failure criteria

Question 3

3. Design a laminate with initial failure load $N_{22} \geq 4 * 10^5$ N/m. The design options are
   a. Layup schemes: $[\pm 45/0/90]_s$, $[0/30/60/90]_s$, $[(\pm 45)_2]_s$, $[0_2 90_2]_s$
   b. Fiber volume fraction: 0.5, 0.6, 0.7

   Assumes the lamina fails according to the Tsai-Wu failure criterion. Which option(s) (the combination of layup scheme and fiber volume fraction) can achieve the design objective?

Table 1 Different fiber volume fractions strength parameters

Vf	X	Xp	Y	Yp	S	R
0.5	2.1E+09	2.1E+09	6.2E+07	2.1E+08	1.0E+08	1.0E+08
0.6	2.4E+09	2.4E+09	6.8E+07	2.3E+08	1.1E+08	1.1E+08
0.7	2.5E+09	2.5E+09	7.1E+07	2.4E+08	1.2E+08	1.2E+08

*unit: Pa

Summary of Solution to Question 3

Layup	Volume Fraction	Initial Failure Load (N/m)	Design objective achieved
$[\pm 45/0/90]_s$	0.5	418264	**Yes**
	0.6	483318	**Yes**
	0.7	513030	**Yes**
$[0/30/60/90]_s$	0.5	333592	**No**
	0.6	383233	**No**
	0.7	408611	**Yes**
$[(\pm 45)_2]_s$	0.5	184963	**No**
	0.6	204558	**No**
	0.7	222583	**No**
$[0_2 90_2]_s$	0.5	554238	**Yes**
	0.6	640790	**Yes**
	0.7	677905	**Yes**

Detailed Mathematica Solutions attached below

Question 3

3. Design a laminate with initial failure load $N_{22} \geq 4 * 10^5$ N/m. The design options are

   a. Layup schemes: $[\pm 45/0/90]_s$, $[0/30/60/90]_s$, $[(\pm 45)_2]_s$, $[0_2 90_2]_s$

   b. Fiber volume fraction: 0.5, 0.6, 0.7

Assumes the lamina fails according to the Tsai-Wu failure criterion. Which option(s) (the combination of layup scheme and fiber volume fraction) can achieve the design objective?

Table 1 Different fiber volume fractions strength parameters

Vf	X	Xp	Y	Yp	S	R
0.5	2.1E+09	2.1E+09	6.2E+07	2.1E+08	1.0E+08	1.0E+08
0.6	2.4E+09	2.4E+09	6.8E+07	2.3E+08	1.1E+08	1.1E+08
0.7	2.5E+09	2.5E+09	7.1E+07	2.4E+08	1.2E+08	1.2E+08

*unit: Pa

Design option 1 : [±45/0/90]s Vf=0.5

```
In[1]:= laminaConstants = {E1 → 1.4038 * 10^11, E2 → 8.55336 * 10^9, G12 → 3.39031 * 10^9, ν12 → 0.325,
          ν23 → 0.603731, X → 2.1 * 10^9, XP → 2.1 * 10^9, Y → 6.2 * 10^7, YP → 2.1 * 10^8, S → 1 * 10^8};
```

In[2]:= $t = \frac{127}{1000} * 10^{-3};$

```
In[3]:= n = 8;

In[4]:= Nn = {0, Ny, 0};

In[5]:= M = {0, 0, 0};

In[6]:= angles = {45, -45, 0, 90, 90, 0, -45, 45};

In[7]:= ti = Table[t, {i, 1, n}];
```

In[8]:= $\text{bzi} = \text{Table}\left[-\frac{1}{2}(n+1)\,t + i\,t, \{i, 1, n\}\right];$

In[9]:= $\text{layercoord} = \text{Table}\left[\left\{\text{bzi}[[i]] - \frac{1}{2}\text{ti}[[i]], \text{bzi}[[i]] + \frac{1}{2}\text{ti}[[i]]\right\}, \{i, 1, n\}\right];$

In[10]:= $R\sigma e = \begin{pmatrix} c^2 & s^2 & -2\,c\,s \\ s^2 & c^2 & 2\,c\,s \\ c\,s & -c\,s & c^2 - s^2 \end{pmatrix};$

In[11]:= $\text{Sep} = \begin{pmatrix} \frac{1}{E1} & -\frac{\nu 12}{E1} & 0 \\ -\frac{\nu 12}{E1} & \frac{1}{E2} & 0 \\ 0 & 0 & \frac{1}{G12} \end{pmatrix}$ /. laminaConstants;

```
In[12]:= Q = Simplify[Inverse[Sep]];

In[13]:= Qθ[i_] := Rσe.Q.Transpose[Rσe] /. {s → Sin[θ Degree], c → Cos[θ Degree]} /. θ → angles[[i]]
```

```
In[14]:= A = Sum[Qθ[i] × ti[[i]], {i, 1, n}] // N
Out[14]= {{5.95441 × 10^7, 1.94467 × 10^7, 0.}, {1.94467 × 10^7, 5.95441 × 10^7, 0.}, {0., 0., 2.00487 × 10^7}}

In[15]:= B = Sum[Qθ[i] × ti[[i]] × bzi[[i]], {i, 1, n}]
Out[15]= {{0., 0., 0.}, {0., 0., 0.}, {0., 0., 0.}}

In[16]:= Dd = Sum[Qθ[i] (ti[[i]] bzi[[i]]^2 + ti[[i]]^3 / 12), {i, 1, n}] // N
Out[16]= {{4.59439, 2.74407, 0.815342}, {2.74407, 3.50727, 0.815342}, {0.815342, 0.815342, 2.79585}}

In[17]:= ϵ = Inverse[A].Nn // Chop
Out[17]= {-6.1398 × 10^-9 Ny, 1.87995 × 10^-8 Ny, 0}

In[18]:= κ = Inverse[Dd].M // Chop
Out[18]= {0, 0, 0}

In[19]:= σe = Table[Inverse[Rσe].Qθ[i].(ϵ + x3 κ) /. {s → Sin[θ Degree], c → Cos[θ Degree]} /.
            θ → angles[[i]], {i, 1, n}]
Out[19]= {{912.05 Ny, 72.2021 Ny, 84.552 Ny}, {912.05 Ny, 72.2021 Ny, -84.552 Ny},
          {-814.89 Ny, 144.662 Ny, 0.}, {2638.99 Ny, -0.257956 Ny, 0.},
          {2638.99 Ny, -0.257956 Ny, 0.}, {-814.89 Ny, 144.662 Ny, 0.},
          {912.05 Ny, 72.2021 Ny, -84.552 Ny}, {912.05 Ny, 72.2021 Ny, 84.552 Ny}}

In[20]:= TW[σ_] :=
          (1/X - 1/XP) σ[[1]] + (1/Y - 1/YP) σ[[2]] + σ[[1]]^2/(X XP) + σ[[2]]^2/(Y YP) + (1/S σ[[3]])^2 - 1/X^2 σ[[1]] × σ[[2]];

In[21]:= sol = Table[Solve[(TW[σe[[i]]] /. laminaConstants) == 1, Ny], {i, 1, n}]
Out[21]= {{{Ny → -1.25493 × 10^6}, {Ny → 618204.}}, {{Ny → -1.25493 × 10^6}, {Ny → 618204.}},
          {{Ny → -1.33969 × 10^6}, {Ny → 418264.}}, {{Ny → -794791.}, {Ny → 796648.}},
          {{Ny → -794791.}, {Ny → 796648.}}, {{Ny → -1.33969 × 10^6}, {Ny → 418264.}},
          {{Ny → -1.25493 × 10^6}, {Ny → 618204.}}, {{Ny → -1.25493 × 10^6}, {Ny → 618204.}}}

In[22]:= % // TraditionalForm
```

Out[22]//TraditionalForm=

$$\begin{pmatrix} \{Ny \to -1.25493\times 10^6\} & \{Ny \to 618\,204.\} \\ \{Ny \to -1.25493\times 10^6\} & \{Ny \to 618\,204.\} \\ \{Ny \to -1.33969\times 10^6\} & \{Ny \to 418\,264.\} \\ \{Ny \to -794\,791.\} & \{Ny \to 796\,648.\} \\ \{Ny \to -794\,791.\} & \{Ny \to 796\,648.\} \\ \{Ny \to -1.33969\times 10^6\} & \{Ny \to 418\,264.\} \\ \{Ny \to -1.25493\times 10^6\} & \{Ny \to 618\,204.\} \\ \{Ny \to -1.25493\times 10^6\} & \{Ny \to 618\,204.\} \end{pmatrix}$$

For the Tsai-Wu failure criterion, the result indicates that max N22= 418264 N/m for layer 3 and 6. (orientation 0^o) . The failure load N22≥ 400000 N/m
The initial failure load is higher that the given failure load. Thus, design

objective is achieved

Design option 2 : [±45/0/90]s   Vf=0.6

In[23]:= $\text{laminaConstants} = \{E1 \to 1.67504 * 10^{11},\ E2 \to 9.64094 * 10^{9},\ G12 \to 4.18719 * 10^{9},\ \nu 12 \to 0.316,\ \nu 23 \to 0.614218,\ X \to 2.4 * 10^{9},\ XP \to 2.4 * 10^{9},\ Y \to 6.8 * 10^{7},\ YP \to 2.3 * 10^{8},\ S \to 1.1 * 10^{8}\};$

In[24]:= $t = \frac{127}{1000} * 10^{-3};$

In[25]:= `n = 8;`

In[26]:= `Nn = {0, Ny, 0};`

In[27]:= `M = {0, 0, 0};`

In[28]:= `angles = {45, -45, 0, 90, 90, 0, -45, 45};`

In[29]:= `ti = Table[t, {i, 1, n}];`

In[30]:= $\text{bzi} = \text{Table}\left[-\frac{1}{2}(n+1)\,t + i\,t,\ \{i, 1, n\}\right];$

In[31]:= $\text{layercoord} = \text{Table}\left[\left\{\text{bzi}[[i]] - \frac{1}{2}\text{ti}[[i]],\ \text{bzi}[[i]] + \frac{1}{2}\text{ti}[[i]]\right\},\ \{i, 1, n\}\right];$

In[32]:= $R\sigma e = \begin{pmatrix} c^2 & s^2 & -2\,c\,s \\ s^2 & c^2 & 2\,c\,s \\ c\,s & -c\,s & c^2 - s^2 \end{pmatrix};$

In[33]:= $\text{Sep} = \begin{pmatrix} \frac{1}{E1} & -\frac{\nu 12}{E1} & 0 \\ -\frac{\nu 12}{E1} & \frac{1}{E2} & 0 \\ 0 & 0 & \frac{1}{G12} \end{pmatrix} \text{/. laminaConstants}$

Out[33]= $\{\{5.97001 \times 10^{-12},\ -1.88652 \times 10^{-12},\ 0\},\ \{-1.88652 \times 10^{-12},\ 1.03724 \times 10^{-10},\ 0\},\ \{0,\ 0,\ 2.38824 \times 10^{-10}\}\}$

In[34]:= `Q = Simplify[Inverse[Sep]];`

In[35]:= `Qθ[i_] := Rσe.Q.Transpose[Rσe] /. {s → Sin[θ Degree], c → Cos[θ Degree]} /. θ → angles[[i]]`

In[36]:= `A = Sum[Qθ[i] × ti[[i]], {i, 1, n}] // N`

Out[36]= $\{\{7.07878 \times 10^{7},\ 2.28352 \times 10^{7},\ 0.\},\ \{2.28352 \times 10^{7},\ 7.07878 \times 10^{7},\ 0.\},\ \{0.,\ 0.,\ 2.39763 \times 10^{7}\}\}$

In[37]:= `B = Sum[Qθ[i] × ti[[i]] × bzi[[i]], {i, 1, n}]`

Out[37]= `{{0., 0., 0.}, {0., 0., 0.}, {0., 0., 0.}}`

In[38]:= $\text{Dd} = \text{Sum}\left[Q\theta[i]\left(\text{ti}[[i]]\,\text{bzi}[[i]]^2 + \text{ti}[[i]]^3 / 12\right),\ \{i, 1, n\}\right] \text{ // N}$

Out[38]= `{{5.46733, 3.23671, 0.9757}, {3.23671, 4.1664, 0.9757}, {0.9757, 0.9757, 3.33486}}`

In[39]:= `ϵ = Inverse[A].Nn // Chop`

Out[39]= $\{-5.08641 \times 10^{-9}\ \text{Ny},\ 1.57676 \times 10^{-8}\ \text{Ny},\ 0\}$

In[40]:=
```
κ = Inverse[Dd].M // Chop
```
Out[40]=
```
{0, 0, 0}
```

In[41]:=
```
σe = Table[Inverse[Rσe].Qθ[i].(ϵ + x3 κ) /. {s → Sin[θ Degree], c → Cos[θ Degree]} /.
    θ → angles[[i]], {i, 1, n}]
```
Out[41]=
```
{{916.102 Ny, 68.15 Ny, 87.3195 Ny}, {916.102 Ny, 68.15 Ny, -87.3195 Ny},
 {-808.606 Ny, 137.307 Ny, 0.}, {2640.81 Ny, -1.00718 Ny, 0.},
 {2640.81 Ny, -1.00718 Ny, 0.}, {-808.606 Ny, 137.307 Ny, 0.},
 {916.102 Ny, 68.15 Ny, -87.3195 Ny}, {916.102 Ny, 68.15 Ny, 87.3195 Ny}}
```

In[42]:= TW[σ_] :=

$$\left(\frac{1}{X}-\frac{1}{XP}\right)\sigma[[1]]+\left(\frac{1}{Y}-\frac{1}{YP}\right)\sigma[[2]]+\frac{\sigma[[1]]^2}{X\,XP}+\frac{\sigma[[2]]^2}{Y\,YP}+\left(\frac{1}{S}\sigma[[3]]\right)^2-\frac{1}{X^2}\sigma[[1]]\times\sigma[[2]];$$

In[43]:=
```
sol = Table[Solve[(TW[σe[[i]]] /. laminaConstants) == 1, Ny], {i, 1, n}]
```
Out[43]= $\{\{\{Ny\to -1.35808\times10^6\},\{Ny\to 693369.\}\},\{\{Ny\to -1.35808\times10^6\},\{Ny\to 693369.\}\},$
$\{\{Ny\to -1.54608\times10^6\},\{Ny\to 483318.\}\},\{\{Ny\to -904318.\},\{Ny\to 912931.\}\},$
$\{\{Ny\to -904318.\},\{Ny\to 912931.\}\},\{\{Ny\to -1.54608\times10^6\},\{Ny\to 483318.\}\},$
$\{\{Ny\to -1.35808\times10^6\},\{Ny\to 693369.\}\},\{\{Ny\to -1.35808\times10^6\},\{Ny\to 693369.\}\}\}$

In[44]:=
```
% // TraditionalForm
```
Out[44]//TraditionalForm=

$$\begin{pmatrix} \{Ny\to -1.35808\times10^6\} & \{Ny\to 693369.\} \\ \{Ny\to -1.35808\times10^6\} & \{Ny\to 693369.\} \\ \{Ny\to -1.54608\times10^6\} & \{Ny\to 483318.\} \\ \{Ny\to -904318.\} & \{Ny\to 912931.\} \\ \{Ny\to -904318.\} & \{Ny\to 912931.\} \\ \{Ny\to -1.54608\times10^6\} & \{Ny\to 483318.\} \\ \{Ny\to -1.35808\times10^6\} & \{Ny\to 693369.\} \\ \{Ny\to -1.35808\times10^6\} & \{Ny\to 693369.\} \end{pmatrix}$$

For the Tsai-Wu failure criterion, the result indicates that max N22= N_{22}=483318 N/m for layer 3 and 6. (orientation 0^o) . The failure load N22⩾ 400000 N/m
The initial failure load is higher that the given failure load. Thus, design objective is achieved

Design option 3 : [±45/0/90]s   Vf=0.7

In[45]:= laminaConstants = $\{E1 \to 1.94628 * 10^{11}, E2 \to 11.04 * 10^{9}, G12 \to 5.47378 * 10^{9}, \nu12 \to 0.307, \nu23 \to 0.626963, X \to 2.5 * 10^{9}, XP \to 2.5 * 10^{9}, Y \to 7.1 * 10^{7}, YP \to 2.4 * 10^{8}, S \to 1.2 * 10^{8}\}$;

In[46]:= t = $\frac{127}{1000} * 10^{-3}$;

In[47]:= n = 8;

In[48]:= Nn = {0, Ny, 0};

In[49]:= M = {0, 0, 0};

In[50]:= angles = {45, -45, 0, 90, 90, 0, -45, 45};

In[51]:= ti = Table[t, {i, 1, n}];

In[52]:= bzi = Table$\left[-\frac{1}{2}(n+1)\, t + i\, t, \{i, 1, n\}\right]$;

In[53]:= layercoord = Table$\left[\left\{bzi[[i]] - \frac{1}{2} ti[[i]], bzi[[i]] + \frac{1}{2} ti[[i]]\right\}, \{i, 1, n\}\right]$;

In[54]:= Rσe = $\begin{pmatrix} c^2 & s^2 & -2\,c\,s \\ s^2 & c^2 & 2\,c\,s \\ c\,s & -c\,s & c^2 - s^2 \end{pmatrix}$;

In[55]:= Sep = $\begin{pmatrix} \frac{1}{E1} & -\frac{\nu12}{E1} & 0 \\ -\frac{\nu12}{E1} & \frac{1}{E2} & 0 \\ 0 & 0 & \frac{1}{G12} \end{pmatrix}$ /. laminaConstants;

In[56]:= Q = Simplify[Inverse[Sep]];

In[57]:= Qθ[i_] := Rσe.Q.Transpose[Rσe] /. {s → Sin[θ Degree], c → Cos[θ Degree]} /. θ → angles[[i]]

In[58]:= A = Sum[Qθ[i] × ti[[i]], {i, 1, n}] // N

Out[58]= $\{\{8.24269 \times 10^{7}, 2.60761 \times 10^{7}, 0.\}, \{2.60761 \times 10^{7}, 8.24269 \times 10^{7}, 0.\}, \{0., 0., 2.81754 \times 10^{7}\}\}$

In[59]:= B = Sum[Qθ[i] × ti[[i]] × bzi[[i]], {i, 1, n}]

Out[59]= {{0., 0., 0.}, {0., 0., 0.}, {0., 0., 0.}}

In[60]:= Dd = Sum$\left[Q\theta[i]\left(ti[[i]]\, bzi[[i]]^2 + ti[[i]]^3 / 12\right), \{i, 1, n\}\right]$ // N

Out[60]= {{6.38766, 3.70206, 1.13424}, {3.70206, 4.87534, 1.13424}, {1.13424, 1.13424, 3.88265}}

In[61]:= ϵ = Inverse[A].Nn // Chop

Out[61]= $\{-4.26482 \times 10^{-9}\, Ny, 1.34812 \times 10^{-8}\, Ny, 0\}$

In[62]:= κ = Inverse[Dd].M // Chop

Out[62]= {0, 0, 0}

In[63]:=
```
σe = Table[Inverse[Rσe].Qθ[i].(ϵ + x3 κ) /. {s → Sin[θ Degree], c → Cos[θ Degree]} /.
    θ → angles[[i]], {i, 1, n}]
```

Out[63]=
```
{{917.402 Ny, 66.85 Ny, 97.1376 Ny}, {917.402 Ny, 66.85 Ny, -97.1376 Ny},
 {-788.577 Ny, 135.1 Ny, 0.}, {2623.38 Ny, -1.39964 Ny, 0.},
 {2623.38 Ny, -1.39964 Ny, 0.}, {-788.577 Ny, 135.1 Ny, 0.},
 {917.402 Ny, 66.85 Ny, -97.1376 Ny}, {917.402 Ny, 66.85 Ny, 97.1376 Ny}}
```

In[64]:= TW[σ_] :=

$$\left(\frac{1}{X}-\frac{1}{XP}\right)\sigma[[1]]+\left(\frac{1}{Y}-\frac{1}{YP}\right)\sigma[[2]]+\frac{\sigma[[1]]^2}{X\,XP}+\frac{\sigma[[2]]^2}{Y\,YP}+\left(\frac{1}{S}\sigma[[3]]\right)^2-\frac{1}{X^2}\sigma[[1]]\times\sigma[[2]];$$

In[65]:=
```
sol = Table[Solve[(TW[σe[[i]]] /. laminaConstants) == 1, Ny], {i, 1, n}]
```

Out[65]= $\{\{\{Ny \to -1.34784\times10^6\}, \{Ny \to 711775.\}\}, \{\{Ny \to -1.34784\times10^6\}, \{Ny \to 711775.\}\}, \{\{Ny \to -1.64121\times10^6\}, \{Ny \to 513030.\}\}, \{\{Ny \to -946386.\}, \{Ny \to 958985.\}\}, \{\{Ny \to -946386.\}, \{Ny \to 958985.\}\}, \{\{Ny \to -1.64121\times10^6\}, \{Ny \to 513030.\}\}, \{\{Ny \to -1.34784\times10^6\}, \{Ny \to 711775.\}\}, \{\{Ny \to -1.34784\times10^6\}, \{Ny \to 711775.\}\}\}$

In[66]:= % // TraditionalForm

Out[66]//TraditionalForm=

$$\begin{pmatrix} \{Ny \to -1.34784\times10^6\} & \{Ny \to 711775.\} \\ \{Ny \to -1.34784\times10^6\} & \{Ny \to 711775.\} \\ \{Ny \to -1.64121\times10^6\} & \{Ny \to 513030.\} \\ \{Ny \to -946386.\} & \{Ny \to 958985.\} \\ \{Ny \to -946386.\} & \{Ny \to 958985.\} \\ \{Ny \to -1.64121\times10^6\} & \{Ny \to 513030.\} \\ \{Ny \to -1.34784\times10^6\} & \{Ny \to 711775.\} \\ \{Ny \to -1.34784\times10^6\} & \{Ny \to 711775.\} \end{pmatrix}$$

For the Tsai-Wu failure criterion, the result indicates that max N22= N_{22}=513030 N/m for layer 3 and 6. (orientation 0^o) . The failure load N22⩾ 400000 N/m
The initial failure load is higher that the given failure load. Thus, design objective is achieved

Design option 4 : [0/30/60/90]s Vf=0.5

```
In[67]:= laminaConstants = {E1 → 1.4038 * 10^11, E2 → 8.55336 * 10^9, G12 → 3.39031 * 10^9, ν12 → 0.325,
           ν23 → 0.603731, X → 2.1 * 10^9, XP → 2.1 * 10^9, Y → 6.2 * 10^7, YP → 2.1 * 10^8, S → 1 * 10^8};

In[68]:= t = 127/1000 * 10^-3;

In[69]:= n = 8;

In[70]:= Nn = {0, Ny, 0};

In[71]:= M = {0, 0, 0};

In[72]:= angles = {0, 30, 60, 90, 90, 60, 30, 0};

In[73]:= ti = Table[t, {i, 1, n}];

In[74]:= bzi = Table[-1/2 (n + 1) t + i t, {i, 1, n}];

In[75]:= layercoord = Table[{bzi[[i]] - 1/2 ti[[i]], bzi[[i]] + 1/2 ti[[i]]}, {i, 1, n}];
```

$$\text{In[76]:= } R\sigma e = \begin{pmatrix} c^2 & s^2 & -2\,c\,s \\ s^2 & c^2 & 2\,c\,s \\ c\,s & -c\,s & c^2 - s^2 \end{pmatrix};$$

$$\text{In[77]:= } Sep = \begin{pmatrix} \frac{1}{E1} & -\frac{\nu 12}{E1} & 0 \\ -\frac{\nu 12}{E1} & \frac{1}{E2} & 0 \\ 0 & 0 & \frac{1}{G12} \end{pmatrix} \text{ /. laminaConstants;}$$

```
In[78]:= Q = Simplify[Inverse[Sep]];

In[79]:= Qθ[i_] := Rσe.Q.Transpose[Rσe] /. {s → Sin[θ Degree], c → Cos[θ Degree]} /. θ → angles[[i]]

In[80]:= A = Sum[Qθ[i] × ti[[i]], {i, 1, n}] // N

Out[80]= {{6.36951 × 10^7, 1.52957 × 10^7, 1.45929 × 10^7},
          {1.52957 × 10^7, 6.36951 × 10^7, 1.45929 × 10^7}, {1.45929 × 10^7, 1.45929 × 10^7, 1.58976 × 10^7}}

In[81]:= B = Sum[Qθ[i] × ti[[i]] × bzi[[i]], {i, 1, n}]

Out[81]= {{0., 0., 0.}, {0., 0., 0.}, {0., 0., 0.}}

In[82]:= Dd = Sum[Qθ[i] (ti[[i]] bzi[[i]]^2 + ti[[i]]^3 / 12), {i, 1, n}] // N

Out[82]= {{9.48492, 1.1149, 1.25186}, {1.1149, 1.87507, 0.788003}, {1.25186, 0.788003, 1.16668}}

In[83]:= ϵ = Inverse[A].Nn // Chop

Out[83]= {-7.52225 × 10^-10 Ny, 1.99092 × 10^-8 Ny, -1.75847 × 10^-8 Ny}

In[84]:= κ = Inverse[Dd].M // Chop

Out[84]= {0, 0, 0}
```

In[85]:=
```
σe = Table[Inverse[Rσe].Qθ[i].(ϵ + x3 κ) /. {s → Sin[θ Degree], c → Cos[θ Degree]} /.
    θ → angles[[i]], {i, 1, n}]
```

Out[85]=
```
{{0. - 50.5785 Ny, 0. + 169.289 Ny, 0. - 59.6176 Ny}, {-389.751 Ny, 183.52 Ny, 30.855 Ny},
 {1040.96 Ny, 123.489 Ny, 90.4726 Ny}, {0. + 2810.85 Ny, 0. + 49.2271 Ny, 0. + 59.6176 Ny},
 {0. + 2810.85 Ny, 0. + 49.2271 Ny, 0. + 59.6176 Ny}, {1040.96 Ny, 123.489 Ny, 90.4726 Ny},
 {-389.751 Ny, 183.52 Ny, 30.855 Ny}, {0. - 50.5785 Ny, 0. + 169.289 Ny, 0. - 59.6176 Ny}}
```

In[86]:= TW[σ_] :=

$$\left(\frac{1}{X}-\frac{1}{XP}\right)\sigma[[1]]+\left(\frac{1}{Y}-\frac{1}{YP}\right)\sigma[[2]]+\frac{\sigma[[1]]^2}{X\,XP}+\frac{\sigma[[2]]^2}{Y\,YP}+\left(\frac{1}{S}\sigma[[3]]\right)^2-\frac{1}{X^2}\sigma[[1]]\times\sigma[[2]];$$

In[87]:=
```
sol = Table[Solve[(TW[σe[[i]]] /. laminaConstants) == 1, Ny], {i, 1, n}]
```

Out[87]=
```
{{{Ny → -1.10545 × 10^6}, {Ny → 353490.}}, {{Ny → -1.09699 × 10^6}, {Ny → 333592.}},
 {{Ny → -1.06271 × 10^6}, {Ny → 426493.}}, {{Ny → -791797.}, {Ny → 548690.}},
 {{Ny → -791797.}, {Ny → 548690.}}, {{Ny → -1.06271 × 10^6}, {Ny → 426493.}},
 {{Ny → -1.09699 × 10^6}, {Ny → 333592.}}, {{Ny → -1.10545 × 10^6}, {Ny → 353490.}}}
```

In[88]:= % // TraditionalForm

Out[88]//TraditionalForm=

$$\begin{pmatrix} \{Ny \to -1.10545\times10^6\} & \{Ny \to 353490.\} \\ \{Ny \to -1.09699\times10^6\} & \{Ny \to 333592.\} \\ \{Ny \to -1.06271\times10^6\} & \{Ny \to 426493.\} \\ \{Ny \to -791797.\} & \{Ny \to 548690.\} \\ \{Ny \to -791797.\} & \{Ny \to 548690.\} \\ \{Ny \to -1.06271\times10^6\} & \{Ny \to 426493.\} \\ \{Ny \to -1.09699\times10^6\} & \{Ny \to 333592.\} \\ \{Ny \to -1.10545\times10^6\} & \{Ny \to 353490.\} \end{pmatrix}$$

The 0^o layers will fail in tension when N_{22}=353490 N/m
The 30^o layers will fail in tension when N_{22}=333592 N/m
The 60^o layers will fail in tension when N_{22}=426493 N/m
The 90^o layers will fail in tension when N_{22}=548690 N/m

For the Tsai-Wu failure criterion, the result indicates that max N22= N_{22}=333592 N/m for layer 2 and 7. (orientation 30^o) . The failure load N22⩾ 400000 N/m
The initial failure load is lower that the given failure load. Thus, design objective is not achieved

Design option 5 : [0/30/60/90]s   Vf=0.6

In[89]:= laminaConstants = {E1 → 1.67504 * 10^{11}, E2 → 9.64094 * 10^{9}, G12 → 4.18719 * 10^{9}, ν12 → 0.316, ν23 → 0.614218, X → 2.4 * 10^{9}, XP → 2.4 * 10^{9}, Y → 6.8 * 10^{7}, YP → 2.3 * 10^{8}, S → 1.1 * 10^{8}};

In[90]:= $t = \frac{127}{1000} * 10^{-3}$;

In[91]:= n = 8;

In[92]:= Nn = {0, Ny, 0};

In[93]:= M = {0, 0, 0};

In[94]:= angles = {0, 30, 60, 90, 90, 60, 30, 0};

In[95]:= ti = Table[t, {i, 1, n}];

In[96]:= bzi = Table$\left[-\frac{1}{2}(n+1)\,t + i\,t, \{i, 1, n\}\right]$;

In[97]:= layercoord = Table$\left[\left\{bzi[[i]] - \frac{1}{2} ti[[i]], bzi[[i]] + \frac{1}{2} ti[[i]]\right\}, \{i, 1, n\}\right]$;

In[98]:= $R\sigma e = \begin{pmatrix} c^2 & s^2 & -2\,c\,s \\ s^2 & c^2 & 2\,c\,s \\ c\,s & -c\,s & c^2 - s^2 \end{pmatrix}$;

In[99]:= $Sep = \begin{pmatrix} \frac{1}{E1} & -\frac{\nu12}{E1} & 0 \\ -\frac{\nu12}{E1} & \frac{1}{E2} & 0 \\ 0 & 0 & \frac{1}{G12} \end{pmatrix}$ /. laminaConstants;

In[100]:= Q = Simplify[Inverse[Sep]];

In[101]:= Qθ[i_] := Rσe.Q.Transpose[Rσe] /. {s → Sin[θ Degree], c → Cos[θ Degree]} /. θ → angles[[i]]

In[102]:= A = Sum[Qθ[i] × ti[[i]], {i, 1, n}] // N

Out[102]= {{7.57183 × 10^{7}, 1.79047 × 10^{7}, 1.7463 × 10^{7}}, {1.79047 × 10^{7}, 7.57183 × 10^{7}, 1.7463 × 10^{7}}, {1.7463 × 10^{7}, 1.7463 × 10^{7}, 1.90457 × 10^{7}}}

In[103]:= B = Sum[Qθ[i] × ti[[i]] × bzi[[i]], {i, 1, n}]

Out[103]= {{0., 0., 0.}, {0., 0., 0.}, {0., 0., 0.}}

In[104]:= Dd = Sum$\left[Q\theta[i]\left(ti[[i]]\,bzi[[i]]^2 + ti[[i]]^3/12\right), \{i, 1, n\}\right]$ // N

Out[104]= {{11.3052, 1.30162, 1.49601}, {1.30162, 2.19869, 0.945047}, {1.49601, 0.945047, 1.39977}}

In[105]:= ϵ = Inverse[A].Nn // Chop

Out[105]= {−5.31546 × 10^{-10} Ny, 1.67654 × 10^{-8} Ny, −1.48848 × 10^{-8} Ny}

In[106]:= ϰ = Inverse[Dd].M // Chop

Out[106]= {0, 0, 0}

In[107]:=
```
σe = Table[Inverse[Rσe].Qθ[i].(ϵ + x3 κ) /. {s → Sin[θ Degree], c → Cos[θ Degree]} /.
    θ → angles[[i]], {i, 1, n}]
```

Out[107]=
```
{{0. - 38.1791 Ny, 0. + 160.94 Ny, 0. - 62.3255 Ny}, {-389.02 Ny, 175.008 Ny, 31.5598 Ny},
 {1041.51 Ny, 117.647 Ny, 93.8853 Ny}, {0. + 2822.88 Ny, 0. + 46.2176 Ny, 0. + 62.3255 Ny},
 {0. + 2822.88 Ny, 0. + 46.2176 Ny, 0. + 62.3255 Ny}, {1041.51 Ny, 117.647 Ny, 93.8853 Ny},
 {-389.02 Ny, 175.008 Ny, 31.5598 Ny}, {0. - 38.1791 Ny, 0. + 160.94 Ny, 0. - 62.3255 Ny}}
```

In[108]:= TW[σ_] :=

$$\left(\frac{1}{X}-\frac{1}{XP}\right)\sigma[[1]]+\left(\frac{1}{Y}-\frac{1}{YP}\right)\sigma[[2]]+\frac{\sigma[[1]]^2}{X\,XP}+\frac{\sigma[[2]]^2}{Y\,YP}+\left(\frac{1}{S}\sigma[[3]]\right)^2-\frac{1}{X^2}\sigma[[1]]\times\sigma[[2]];$$

In[109]:=
```
sol = Table[Solve[(TW[σe[[i]]] /. laminaConstants) == 1, Ny], {i, 1, n}]
```

Out[109]=
$\{\{\{Ny\to -1.24769\times 10^6\}, \{Ny\to 405102.\}\}, \{\{Ny\to -1.25528\times 10^6\}, \{Ny\to 383233.\}\},$
$\{\{Ny\to -1.16607\times 10^6\}, \{Ny\to 481656.\}\}, \{\{Ny\to -884799.\}, \{Ny\to 621533.\}\},$
$\{\{Ny\to -884799.\}, \{Ny\to 621533.\}\}, \{\{Ny\to -1.16607\times 10^6\}, \{Ny\to 481656.\}\},$
$\{\{Ny\to -1.25528\times 10^6\}, \{Ny\to 383233.\}\}, \{\{Ny\to -1.24769\times 10^6\}, \{Ny\to 405102.\}\}\}$

In[110]:=
```
% // TraditionalForm
```

Out[110]//TraditionalForm=

$$\begin{pmatrix} \{Ny\to -1.24769\times 10^6\} & \{Ny\to 405102.\} \\ \{Ny\to -1.25528\times 10^6\} & \{Ny\to 383233.\} \\ \{Ny\to -1.16607\times 10^6\} & \{Ny\to 481656.\} \\ \{Ny\to -884799.\} & \{Ny\to 621533.\} \\ \{Ny\to -884799.\} & \{Ny\to 621533.\} \\ \{Ny\to -1.16607\times 10^6\} & \{Ny\to 481656.\} \\ \{Ny\to -1.25528\times 10^6\} & \{Ny\to 383233.\} \\ \{Ny\to -1.24769\times 10^6\} & \{Ny\to 405102.\} \end{pmatrix}$$

The 0^o layer will fail in tension when N_{22}=405102 N/m
The 30^o layer will fail in tension when N_{22}=383233 N/m
The 60^o layer will fail in tension when N_{22}=481656 N/m
The 90^o layer will fail in tension when N_{22}=621533 N/m

For the Tsai-Wu failure criterion, the result indicates that max N22= N_{22}=383233 N/m for layer 2 and 7. (orientation 30^o) . The failure load N22⩾ 400000 N/m
The initial failure load is lower that the given failure load. Thus, design objective is not achieved

Design option 6 : [0/30/60/90]s   Vf=0.7

In[111]:= laminaConstants = $\{E1 \to 1.94628 * 10^{11}, E2 \to 11.04 * 10^{9}, G12 \to 5.47378 * 10^{9}, \nu 12 \to 0.307, \nu 23 \to 0.626963, X \to 2.5 * 10^{9}, XP \to 2.5 * 10^{9}, Y \to 7.1 * 10^{7}, YP \to 2.4 * 10^{8}, S \to 1.2 * 10^{8}\}$;

In[112]:= $t = \frac{127}{1000} * 10^{-3}$;

In[113]:= n = 8;

In[114]:= Nn = {0, Ny, 0};

In[115]:= M = {0, 0, 0};

In[116]:= angles = {0, 30, 60, 90, 90, 60, 30, 0};

In[117]:= ti = Table[t, {i, 1, n}];

In[118]:= bzi = Table$\left[-\frac{1}{2}(n+1)\,t + i\,t, \{i, 1, n\}\right]$;

In[119]:= layercoord = Table$\left[\left\{bzi[[i]] - \frac{1}{2}ti[[i]], bzi[[i]] + \frac{1}{2}ti[[i]]\right\}, \{i, 1, n\}\right]$;

In[120]:= $R\sigma e = \begin{pmatrix} c^2 & s^2 & -2\,c\,s \\ s^2 & c^2 & 2\,c\,s \\ c\,s & -c\,s & c^2 - s^2 \end{pmatrix}$;

In[121]:= $Sep = \begin{pmatrix} \frac{1}{E1} & -\frac{\nu 12}{E1} & 0 \\ -\frac{\nu 12}{E1} & \frac{1}{E2} & 0 \\ 0 & 0 & \frac{1}{G12} \end{pmatrix}$ /. laminaConstants;

In[122]:= Q = Simplify[Inverse[Sep]];

In[123]:= Qθ[i_] := Rσe.Q.Transpose[Rσe] /. {s → Sin[θ Degree], c → Cos[θ Degree]} /. θ → angles[[i]]

In[124]:= A = Sum[Qθ[i] × ti[[i]], {i, 1, n}] // N

Out[124]= $\{\{8.80804 \times 10^{7}, 2.04225 \times 10^{7}, 2.03005 \times 10^{7}\}, \{2.04225 \times 10^{7}, 8.80804 \times 10^{7}, 2.03005 \times 10^{7}\}, \{2.03005 \times 10^{7}, 2.03005 \times 10^{7}, 2.25219 \times 10^{7}\}\}$

In[125]:= B = Sum[Qθ[i] × ti[[i]] × bzi[[i]], {i, 1, n}]

Out[125]= {{0., 0., 0.}, {0., 0., 0.}, {0., 0., 0.}}

In[126]:= Dd = Sum$\left[Q\theta[i]\left(ti[[i]]\,bzi[[i]]^2 + ti[[i]]^3/12\right), \{i, 1, n\}\right]$ // N

Out[126]= {{13.1435, 1.48322, 1.73472}, {1.48322, 2.55723, 1.10297}, {1.73472, 1.10297, 1.66381}}

In[127]:= ϵ = Inverse[A].Nn // Chop

Out[127]= $\{-4.36655 \times 10^{-10}\,Ny, 1.43436 \times 10^{-8}\,Ny, -1.25353 \times 10^{-8}\,Ny\}$

In[128]:= κ = Inverse[Dd].M // Chop

Out[128]= {0, 0, 0}

In[129]:=
```
σe = Table[Inverse[Rσe].Qθ[i].(ϵ + x3 κ) /. {s → Sin[θ Degree], c → Cos[θ Degree]} /.
    θ → angles[[i]], {i, 1, n}]
```

Out[129]=
```
{{0. - 36.5663 Ny, 0. + 157.717 Ny, 0. - 68.6153 Ny}, {-369.738 Ny, 171.046 Ny, 35.7572 Ny},
 {1051.14 Ny, 114.202 Ny, 104.372 Ny}, {0. + 2805.18 Ny, 0. + 44.0292 Ny, 0. + 68.6153 Ny},
 {0. + 2805.18 Ny, 0. + 44.0292 Ny, 0. + 68.6153 Ny}, {1051.14 Ny, 114.202 Ny, 104.372 Ny},
 {-369.738 Ny, 171.046 Ny, 35.7572 Ny}, {0. - 36.5663 Ny, 0. + 157.717 Ny, 0. - 68.6153 Ny}}
```

In[130]:= TW[σ_] :=

$$\left(\frac{1}{X}-\frac{1}{XP}\right)\sigma[[1]]+\left(\frac{1}{Y}-\frac{1}{YP}\right)\sigma[[2]]+\frac{\sigma[[1]]^2}{X\,XP}+\frac{\sigma[[2]]^2}{Y\,YP}+\left(\frac{1}{S}\sigma[[3]]\right)^2-\frac{1}{X^2}\sigma[[1]]\times\sigma[[2]];$$

In[131]:=
```
sol = Table[Solve[(TW[σe[[i]]] /. laminaConstants) == 1, Ny], {i, 1, n}]
```

Out[131]=

$$\{\{\{Ny\to-1.30388\times10^{6}\},\{Ny\to428973.\}\},\{\{Ny\to-1.33171\times10^{6}\},\{Ny\to408611.\}\},\{\{Ny\to-1.17931\times10^{6}\},\{Ny\to504900.\}\},\{\{Ny\to-912349.\},\{Ny\to652424.\}\},\{\{Ny\to-912349.\},\{Ny\to652424.\}\},\{\{Ny\to-1.17931\times10^{6}\},\{Ny\to504900.\}\},\{\{Ny\to-1.33171\times10^{6}\},\{Ny\to408611.\}\},\{\{Ny\to-1.30388\times10^{6}\},\{Ny\to428973.\}\}\}$$

In[132]:= % // TraditionalForm

Out[132]//TraditionalForm=

$$\begin{pmatrix}\{Ny\to-1.30388\times10^{6}\} & \{Ny\to428973.\}\\ \{Ny\to-1.33171\times10^{6}\} & \{Ny\to408611.\}\\ \{Ny\to-1.17931\times10^{6}\} & \{Ny\to504900.\}\\ \{Ny\to-912349.\} & \{Ny\to652424.\}\\ \{Ny\to-912349.\} & \{Ny\to652424.\}\\ \{Ny\to-1.17931\times10^{6}\} & \{Ny\to504900.\}\\ \{Ny\to-1.33171\times10^{6}\} & \{Ny\to408611.\}\\ \{Ny\to-1.30388\times10^{6}\} & \{Ny\to428973.\}\end{pmatrix}$$

The 0^o layer will fail in tension when N_{22}=428973 N/m
The 30^o layer will fail in tension when N_{22}=408611 N/m
The 60^o layer will fail in tension when N_{22}=504900 N/m
The 90^o layer will fail in tension when N_{22}=652424 N/m

For the Tsai-Wu failure criterion, the result indicates that max N22= N_{22}=408611 N/m for layer 2 and 7. (orientation 30^o) . The failure load N22≥ 400000 N/m
The initial failure load is higher that the given failure load. Thus, design objective is achieved

Design option 7 : $[(\pm 45)_2]_s$ Vf=0.5

```
In[1043]:= laminaConstants = {E1 → 1.4038 * 10^11, E2 → 8.55336 * 10^9, G12 → 3.39031 * 10^9, ν12 → 0.325,
    ν23 → 0.603731, X → 2.1 * 10^9, XP → 2.1 * 10^9, Y → 6.2 * 10^7, YP → 2.1 * 10^8, S → 1 * 10^8};

In[1044]:= t = 127/1000 * 10^-3;

In[1045]:= n = 8;

In[1046]:= Nn = {0, Ny, 0};

In[1047]:= M = {0, 0, 0};

In[1048]:= angles = {45, -45, 45, -45, -45, 45, -45, 45};

In[1049]:= ti = Table[t, {i, 1, n}];

In[1050]:= bzi = Table[-1/2 (n + 1) t + i t, {i, 1, n}];

In[1051]:= layercoord = Table[{bzi[[i]] - 1/2 ti[[i]], bzi[[i]] + 1/2 ti[[i]]}, {i, 1, n}];
```

$$\text{In[1052]:= } R\sigma e = \begin{pmatrix} c^2 & s^2 & -2\,c\,s \\ s^2 & c^2 & 2\,c\,s \\ c\,s & -c\,s & c^2 - s^2 \end{pmatrix};$$

$$\text{In[1053]:= } Sep = \begin{pmatrix} \frac{1}{E1} & -\frac{\nu 12}{E1} & 0 \\ -\frac{\nu 12}{E1} & \frac{1}{E2} & 0 \\ 0 & 0 & \frac{1}{G12} \end{pmatrix} \text{ /. laminaConstants;}$$

```
In[1054]:= Q = Simplify[Inverse[Sep]];

In[1055]:= Qθ[i_] := Rσe.Q.Transpose[Rσe] /. {s → Sin[θ Degree], c → Cos[θ Degree]} /. θ → angles[[i]]

In[1056]:= A = Sum[Qθ[i] × ti[[i]], {i, 1, n}] // N
Out[•]= {{4.294 × 10^7, 3.60509 × 10^7, 0.}, {3.60509 × 10^7, 4.294 × 10^7, 0.}, {0., 0., 3.66528 × 10^7}}

In[1057]:= B = Sum[Qθ[i] × ti[[i]] × bzi[[i]], {i, 1, n}]
Out[•]= {{0., 0., 0.}, {0., 0., 0.}, {0., 0., 0.}}

In[1058]:= Dd = Sum[Qθ[i] (ti[[i]] bzi[[i]]^2 + ti[[i]]^3 / 12), {i, 1, n}] // N
Out[•]= {{3.69375, 3.10114, 1.08712}, {3.10114, 3.69375, 1.08712}, {1.08712, 1.08712, 3.15292}}

In[1059]:= ϵ = Inverse[A].Nn // Chop
Out[•]= {-6.62485 × 10^-8 Ny, 7.89082 × 10^-8 Ny, 0}

In[1060]:= κ = Inverse[Dd].M // Chop
Out[•]= {0, 0, 0}
```

In[1061]:=
```
σe = Table[Inverse[Rσe].Qθ[i].(ϵ + x3 κ) /. {s → Sin[θ Degree], c → Cos[θ Degree]} /.
    θ → angles[[i]], {i, 1, n}]
```

Out[∘]=
```
{{912.05 Ny, 72.2021 Ny, 492.126 Ny}, {912.05 Ny, 72.2021 Ny, -492.126 Ny},
 {912.05 Ny, 72.2021 Ny, 492.126 Ny}, {912.05 Ny, 72.2021 Ny, -492.126 Ny},
 {912.05 Ny, 72.2021 Ny, -492.126 Ny}, {912.05 Ny, 72.2021 Ny, 492.126 Ny},
 {912.05 Ny, 72.2021 Ny, -492.126 Ny}, {912.05 Ny, 72.2021 Ny, 492.126 Ny}}
```

In[∘]:=
```
% // TraditionalForm
```

Out[∘]//TraditionalForm=

$$\begin{pmatrix} 912.05\,\text{Ny} & 72.2021\,\text{Ny} & 492.126\,\text{Ny} \\ 912.05\,\text{Ny} & 72.2021\,\text{Ny} & -492.126\,\text{Ny} \\ 912.05\,\text{Ny} & 72.2021\,\text{Ny} & 492.126\,\text{Ny} \\ 912.05\,\text{Ny} & 72.2021\,\text{Ny} & -492.126\,\text{Ny} \\ 912.05\,\text{Ny} & 72.2021\,\text{Ny} & -492.126\,\text{Ny} \\ 912.05\,\text{Ny} & 72.2021\,\text{Ny} & 492.126\,\text{Ny} \\ 912.05\,\text{Ny} & 72.2021\,\text{Ny} & -492.126\,\text{Ny} \\ 912.05\,\text{Ny} & 72.2021\,\text{Ny} & 492.126\,\text{Ny} \end{pmatrix}$$

In[1063]:=
```
TW[σ_] :=
```

$$\left(\frac{1}{X} - \frac{1}{XP}\right) \sigma[[1]] + \left(\frac{1}{Y} - \frac{1}{YP}\right) \sigma[[2]] + \frac{\sigma[[1]]^2}{X\,XP} + \frac{\sigma[[2]]^2}{Y\,YP} + \left(\frac{1}{S} \sigma[[3]]\right)^2 - \frac{1}{X^2} \sigma[[1]] \times \sigma[[2]]$$

In[1064]:=
```
sol = Table[Solve[(TW[σe[[i]]] /. laminaConstants) == 1, Ny], {i, 1, n}]
```

Out[∘]=
```
{{{Ny → -218066.}, {Ny → 184963.}}, {{Ny → -218066.}, {Ny → 184963.}},
 {{Ny → -218066.}, {Ny → 184963.}}, {{Ny → -218066.}, {Ny → 184963.}},
 {{Ny → -218066.}, {Ny → 184963.}}, {{Ny → -218066.}, {Ny → 184963.}},
 {{Ny → -218066.}, {Ny → 184963.}}, {{Ny → -218066.}, {Ny → 184963.}}}
```

In[1065]:=
```
% // TraditionalForm
```

Out[∘]//TraditionalForm=

$$\begin{pmatrix} \{\text{Ny} \to -218066.\} & \{\text{Ny} \to 184963.\} \\ \{\text{Ny} \to -218066.\} & \{\text{Ny} \to 184963.\} \\ \{\text{Ny} \to -218066.\} & \{\text{Ny} \to 184963.\} \\ \{\text{Ny} \to -218066.\} & \{\text{Ny} \to 184963.\} \\ \{\text{Ny} \to -218066.\} & \{\text{Ny} \to 184963.\} \\ \{\text{Ny} \to -218066.\} & \{\text{Ny} \to 184963.\} \\ \{\text{Ny} \to -218066.\} & \{\text{Ny} \to 184963.\} \\ \{\text{Ny} \to -218066.\} & \{\text{Ny} \to 184963.\} \end{pmatrix}$$

The 45^o layer will fail in tension when N_{22}=184963 N/m
The -45^o layer will fail in tension when N_{22}=184963 N/m

For the Tsai-Wu failure criterion, the result indicates that max N22= N_{22}= 184963 N/m for all layers. The failure load N22⩾ 400000 N/m
The initial failure load is lower that the given failure load. Thus, design objective is not achieved

Design option 8 : $[(\pm 45)_2]_s$ Vf=0.6

In[1066]:= laminaConstants = $\{E1 \to 1.67504 * 10^{11}, E2 \to 9.64094 * 10^9, G12 \to 4.18719 * 10^9, \nu12 \to 0.316, \nu23 \to 0.614218, X \to 2.4 * 10^9, XP \to 2.4 * 10^9, Y \to 6.8 * 10^7, YP \to 2.3 * 10^8, S \to 1.1 * 10^8\}$;

In[1067]:= $t = \frac{127}{1000} * 10^{-3}$;

In[1068]:= n = 8;

In[1069]:= Nn = {0, Ny, 0};

In[1070]:= M = {0, 0, 0};

In[1071]:= angles = {45, -45, 45, -45, -45, 45, -45, 45};

In[1072]:= ti = Table[t, {i, 1, n}];

In[1073]:= bzi = Table$\left[-\frac{1}{2}(n+1)\, t + i\, t, \{i, 1, n\}\right]$;

In[1074]:= layercoord = Table$\left[\left\{bzi[[i]] - \frac{1}{2} ti[[i]], bzi[[i]] + \frac{1}{2} ti[[i]]\right\}, \{i, 1, n\}\right]$;

In[1075]:= $R\sigma e = \begin{pmatrix} c^2 & s^2 & -2\,c\,s \\ s^2 & c^2 & 2\,c\,s \\ c\,s & -c\,s & c^2 - s^2 \end{pmatrix}$;

In[1076]:= $Sep = \begin{pmatrix} \frac{1}{E1} & -\frac{\nu12}{E1} & 0 \\ -\frac{\nu12}{E1} & \frac{1}{E2} & 0 \\ 0 & 0 & \frac{1}{G12} \end{pmatrix}$ /. laminaConstants;

In[1077]:= Q = Simplify[Inverse[Sep]];

In[1078]:= Qθ[i_] := Rσe.Q.Transpose[Rσe] /. {s → Sin[θ Degree], c → Cos[θ Degree]} /. θ → angles[[i]]

In[1079]:= A = Sum[Qθ[i] × ti[[i]], {i, 1, n}] // N

Out[•]= $\{\{5.10657 \times 10^7, 4.25573 \times 10^7, 0.\}, \{4.25573 \times 10^7, 5.10657 \times 10^7, 0.\}, \{0., 0., 4.36983 \times 10^7\}\}$

In[1080]:= B = Sum[Qθ[i] × ti[[i]] × bzi[[i]], {i, 1, n}]

Out[•]= {{0., 0., 0.}, {0., 0., 0.}, {0., 0., 0.}}

In[1081]:= Dd = Sum$\left[Q\theta[i] \left(ti[[i]]\, bzi[[i]]^2 + ti[[i]]^3 / 12\right), \{i, 1, n\}\right]$ // N

Out[•]= {{4.39274, 3.66084, 1.30093}, {3.66084, 4.39274, 1.30093}, {1.30093, 1.30093, 3.75899}}

In[1082]:= ϵ = Inverse[A].Nn // Chop

Out[•]= $\{-5.34251 \times 10^{-8}\, Ny, 6.41062 \times 10^{-8}\, Ny, 0\}$

In[1083]:= κ = Inverse[Dd].M // Chop

Out[•]= {0, 0, 0}

In[1084]:=
```
σe = Table[Inverse[Rσe].Qθ[i].(ϵ + x3 κ) /. {s → Sin[θ Degree], c → Cos[θ Degree]} /.
    θ → angles[[i]], {i, 1, n}]
```

Out[•]=
```
{{916.102 Ny, 68.15 Ny, 492.126 Ny}, {916.102 Ny, 68.15 Ny, -492.126 Ny},
 {916.102 Ny, 68.15 Ny, 492.126 Ny}, {916.102 Ny, 68.15 Ny, -492.126 Ny},
 {916.102 Ny, 68.15 Ny, -492.126 Ny}, {916.102 Ny, 68.15 Ny, 492.126 Ny},
 {916.102 Ny, 68.15 Ny, -492.126 Ny}, {916.102 Ny, 68.15 Ny, 492.126 Ny}}
```

In[1085]:= TW[σ_] :=

$$\left(\frac{1}{X}-\frac{1}{XP}\right)\sigma[[1]]+\left(\frac{1}{Y}-\frac{1}{YP}\right)\sigma[[2]]+\frac{\sigma[[1]]^2}{X\,XP}+\frac{\sigma[[2]]^2}{Y\,YP}+\left(\frac{1}{S}\sigma[[3]]\right)^2-\frac{1}{X^2}\sigma[[1]]\times\sigma[[2]];$$

In[1086]:=
```
sol = Table[Solve[(TW[σe[[i]]] /. laminaConstants) == 1, Ny], {i, 1, n}]
```

Out[•]=
```
{{{Ny → -239081.}, {Ny → 204558.}}, {{Ny → -239081.}, {Ny → 204558.}},
 {{Ny → -239081.}, {Ny → 204558.}}, {{Ny → -239081.}, {Ny → 204558.}},
 {{Ny → -239081.}, {Ny → 204558.}}, {{Ny → -239081.}, {Ny → 204558.}},
 {{Ny → -239081.}, {Ny → 204558.}}, {{Ny → -239081.}, {Ny → 204558.}}}
```

In[1087]:=
```
% // TraditionalForm
```

Out[•]//TraditionalForm=

$$\begin{pmatrix} \{Ny\to-239081.\} & \{Ny\to 204558.\} \\ \{Ny\to-239081.\} & \{Ny\to 204558.\} \\ \{Ny\to-239081.\} & \{Ny\to 204558.\} \\ \{Ny\to-239081.\} & \{Ny\to 204558.\} \\ \{Ny\to-239081.\} & \{Ny\to 204558.\} \\ \{Ny\to-239081.\} & \{Ny\to 204558.\} \\ \{Ny\to-239081.\} & \{Ny\to 204558.\} \\ \{Ny\to-239081.\} & \{Ny\to 204558.\} \end{pmatrix}$$

The 45^o layer will fail in tension when N_{22}=204558 N/m
The -45^o layer will fail in tension when N_{22}=204558 N/m

For the Tsai-Wu failure criterion, the result indicates that max N22= N_{22}= 204558 N/m for all layers. The failure load N22⩾ 400000 N/m
The initial failure load is lower that the given failure load. Thus, design objective is not achieved

Design option 9 : $[(\pm 45)_2]_s$ Vf=0.7

In[1088]:= $\text{laminaConstants} = \{E1 \to 1.94628 * 10^{11},\ E2 \to 11.04 * 10^{9},\ G12 \to 5.47378 * 10^{9},\ \nu 12 \to 0.307,\ \nu 23 \to 0.626963,\ X \to 2.5 * 10^{9},\ XP \to 2.5 * 10^{9},\ Y \to 7.1 * 10^{7},\ YP \to 2.4 * 10^{8},\ S \to 1.2 * 10^{8}\};$

In[1089]:= $t = \frac{127}{1000} * 10^{-3};$

In[1090]:= `n = 8;`

In[1091]:= `Nn = {0, Ny, 0};`

In[1092]:= `M = {0, 0, 0};`

In[1093]:= `angles = {45, -45, 45, -45, -45, 45, -45, 45};`

In[1094]:= `ti = Table[t, {i, 1, n}];`

In[1095]:= $\text{bzi} = \text{Table}\left[-\frac{1}{2}(n+1)\, t + i\, t,\ \{i, 1, n\}\right];$

In[1096]:= $\text{layercoord} = \text{Table}\left[\left\{\text{bzi}[[i]] - \frac{1}{2}\text{ti}[[i]],\ \text{bzi}[[i]] + \frac{1}{2}\text{ti}[[i]]\right\},\ \{i, 1, n\}\right];$

In[1097]:= $R\sigma e = \begin{pmatrix} c^2 & s^2 & -2\, c\, s \\ s^2 & c^2 & 2\, c\, s \\ c\, s & -c\, s & c^2 - s^2 \end{pmatrix};$

In[1098]:= $\text{Sep} = \begin{pmatrix} \frac{1}{E1} & -\frac{\nu 12}{E1} & 0 \\ -\frac{\nu 12}{E1} & \frac{1}{E2} & 0 \\ 0 & 0 & \frac{1}{G12} \end{pmatrix} /.\ \text{laminaConstants};$

In[1099]:= `Q = Simplify[Inverse[Sep]];`

In[1100]:= $Q\theta[i\_] := R\sigma e.Q.\text{Transpose}[R\sigma e]\ /.\ \{s \to \text{Sin}[\theta\ \text{Degree}],\ c \to \text{Cos}[\theta\ \text{Degree}]\}\ /.\ \theta \to \text{angles}[[i]]$

In[1101]:= $A = \text{Sum}[Q\theta[i] \times \text{ti}[[i]],\ \{i, 1, n\}]\ //\ N$

Out[ ]= $\{\{5.98128 \times 10^{7},\ 4.86901 \times 10^{7},\ 0.\},\ \{4.86901 \times 10^{7},\ 5.98128 \times 10^{7},\ 0.\},\ \{0.,\ 0.,\ 5.07894 \times 10^{7}\}\}$

In[1102]:= $B = \text{Sum}[Q\theta[i] \times \text{ti}[[i]] \times \text{bzi}[[i]],\ \{i, 1, n\}]$

Out[ ]= `{{0., 0., 0.}, {0., 0., 0.}, {0., 0., 0.}}`

In[1103]:= $Dd = \text{Sum}\left[Q\theta[i]\left(\text{ti}[[i]]\ \text{bzi}[[i]]^2 + \text{ti}[[i]]^3 / 12\right),\ \{i, 1, n\}\right]\ //\ N$

Out[ ]= `{{5.14518, 4.18839, 1.51232}, {4.18839, 5.14518, 1.51232}, {1.51232, 1.51232, 4.36898}}`

In[1104]:= $\epsilon = \text{Inverse}[A].\text{Nn}\ //\ \text{Chop}$

Out[ ]= $\{-4.03449 \times 10^{-8}\ Ny,\ 4.95612 \times 10^{-8}\ Ny,\ 0\}$

In[1105]:= $\kappa = \text{Inverse}[Dd].M\ //\ \text{Chop}$

Out[ ]= `{0, 0, 0}`

```
In[1106]:= σe = Table[Inverse[Rσe].Qθ[i].(ϵ + x3 κ) /. {s → Sin[θ Degree], c → Cos[θ Degree]} /.
     θ → angles[[i]], {i, 1, n}]

Out[•]= {{917.402 Ny, 66.85 Ny, 492.126 Ny}, {917.402 Ny, 66.85 Ny, -492.126 Ny},
  {917.402 Ny, 66.85 Ny, 492.126 Ny}, {917.402 Ny, 66.85 Ny, -492.126 Ny},
  {917.402 Ny, 66.85 Ny, -492.126 Ny}, {917.402 Ny, 66.85 Ny, 492.126 Ny},
  {917.402 Ny, 66.85 Ny, -492.126 Ny}, {917.402 Ny, 66.85 Ny, 492.126 Ny}}
```

In[1107]:= TW[σ_] :=

$$\left(\frac{1}{X}-\frac{1}{XP}\right)\sigma[[1]]+\left(\frac{1}{Y}-\frac{1}{YP}\right)\sigma[[2]]+\frac{\sigma[[1]]^2}{X\,XP}+\frac{\sigma[[2]]^2}{Y\,YP}+\left(\frac{1}{S}\sigma[[3]]\right)^2-\frac{1}{X^2}\sigma[[1]]\times\sigma[[2]];$$

```
In[1108]:= sol = Table[Solve[(TW[σe[[i]]] /. laminaConstants) == 1, Ny], {i, 1, n}]

Out[•]= {{{Ny → -261117.}, {Ny → 222583.}}, {{Ny → -261117.}, {Ny → 222583.}},
  {{Ny → -261117.}, {Ny → 222583.}}, {{Ny → -261117.}, {Ny → 222583.}},
  {{Ny → -261117.}, {Ny → 222583.}}, {{Ny → -261117.}, {Ny → 222583.}},
  {{Ny → -261117.}, {Ny → 222583.}}, {{Ny → -261117.}, {Ny → 222583.}}}

In[1109]:= % // TraditionalForm
```

Out[•]//TraditionalForm=

$$\begin{pmatrix} \{Ny\to -261\,117.\} & \{Ny\to 222\,583.\} \\ \{Ny\to -261\,117.\} & \{Ny\to 222\,583.\} \\ \{Ny\to -261\,117.\} & \{Ny\to 222\,583.\} \\ \{Ny\to -261\,117.\} & \{Ny\to 222\,583.\} \\ \{Ny\to -261\,117.\} & \{Ny\to 222\,583.\} \\ \{Ny\to -261\,117.\} & \{Ny\to 222\,583.\} \\ \{Ny\to -261\,117.\} & \{Ny\to 222\,583.\} \\ \{Ny\to -261\,117.\} & \{Ny\to 222\,583.\} \end{pmatrix}$$

The 45^o layer will fail in tension when N_{22}=222583 N/m
The -45^o layer will fail in tension when N_{22}=222583 N/m

For the Tsai-Wu failure criterion, the result indicates that max N22= N_{22}= 222583 N/m for all layers. The failure load N22⩾ 400000 N/m
The initial failure load is lower that the given failure load. Thus, design objective is not achieved

Design option 10 : $[(0)_2\,(90)_2]_s$ Vf=0.5

In[177]:= laminaConstants = {E1 → 1.4038 * 10^{11}, E2 → 8.55336 * 10^9, G12 → 3.39031 * 10^9, ν12 → 0.325, ν23 → 0.603731, X → 2.1 * 10^9, XP → 2.1 * 10^9, Y → 6.2 * 10^7, YP → 2.1 * 10^8, S → 1 * 10^8};

In[178]:= t = $\frac{127}{1000}$ * 10^{-3};

In[179]:= n = 8;

In[180]:= Nn = {0, Ny, 0};

In[181]:= M = {0, 0, 0};

In[182]:= angles = {0, 0, 90, 90, 90, 90, 0, 0};

In[183]:= ti = Table[t, {i, 1, n}];

In[184]:= bzi = Table[$-\frac{1}{2}$ (n + 1) t + i t, {i, 1, n}];

In[185]:= layercoord = Table[{bzi[[i]] − $\frac{1}{2}$ ti[[i]], bzi[[i]] + $\frac{1}{2}$ ti[[i]]}, {i, 1, n}];

In[186]:= Rσe = $\begin{pmatrix} c^2 & s^2 & -2\,c\,s \\ s^2 & c^2 & 2\,c\,s \\ c\,s & -c\,s & c^2 - s^2 \end{pmatrix}$;

In[187]:= Sep = $\begin{pmatrix} \frac{1}{E1} & -\frac{\nu12}{E1} & 0 \\ -\frac{\nu12}{E1} & \frac{1}{E2} & 0 \\ 0 & 0 & \frac{1}{G12} \end{pmatrix}$ /. laminaConstants;

In[188]:= Q = Simplify[Inverse[Sep]];

In[189]:= Qθ[i_] := Rσe.Q.Transpose[Rσe] /. {s → Sin[θ Degree], c → Cos[θ Degree]} /. θ → angles[[i]]

In[190]:= A = Sum[Qθ[i] × ti[[i]], {i, 1, n}] // N

Out[190]= {{7.61482 × 10^7, 2.84261 × 10^6, 0.}, {2.84261 × 10^6, 7.61482 × 10^7, 0.}, {0., 0., 3.44455 × 10^6}}

In[191]:= B = Sum[Qθ[i] × ti[[i]] × bzi[[i]], {i, 1, n}]

Out[191]= {{0., 0., 0.}, {0., 0., 0.}, {0., 0., 0.}}

In[192]:= Dd = Sum[Qθ[i] (ti[[i]] bzi[[i]]^2 + ti[[i]]^3 / 12), {i, 1, n}] // N

Out[192]= {{10.8989, 0.244525, 0.}, {0.244525, 2.20188, 0.}, {0., 0., 0.296305}}

In[193]:= ϵ = Inverse[A].Nn // Chop

Out[193]= {−4.90912 × 10^{-10} Ny, 1.31506 × 10^{-8} Ny, 0}

In[194]:= κ = Inverse[Dd].M // Chop

Out[194]= {0, 0, 0}

In[195]:= σe = Table[Inverse[Rσe].Qθ[i].(ϵ + x3 κ) /. {s → Sin[θ Degree], c → Cos[θ Degree]} /. θ → angles[[i]], {i, 1, n}]

Out[195]= {{-32.5673 Ny, 111.837 Ny, 0.}, {-32.5673 Ny, 111.837 Ny, 0.}, {1856.67 Ny, 32.5673 Ny, 0.}, {1856.67 Ny, 32.5673 Ny, 0.}, {1856.67 Ny, 32.5673 Ny, 0.}, {1856.67 Ny, 32.5673 Ny, 0.}, {-32.5673 Ny, 111.837 Ny, 0.}, {-32.5673 Ny, 111.837 Ny, 0.}}

In[196]:= TW[σ_] :=

$$\left(\frac{1}{X}-\frac{1}{XP}\right)\sigma[[1]]+\left(\frac{1}{Y}-\frac{1}{YP}\right)\sigma[[2]]+\frac{\sigma[[1]]^2}{X\,XP}+\frac{\sigma[[2]]^2}{Y\,YP}+\left(\frac{1}{S}\sigma[[3]]\right)^2-\frac{1}{X^2}\sigma[[1]]\times\sigma[[2]];$$

In[197]:= sol = Table[Solve[(TW[σe[[i]]] /. laminaConstants) == 1, Ny], {i, 1, n}]

Out[197]= $\{\{\{Ny \to -1.87613\times10^6\}, \{Ny \to 554238.\}\}, \{\{Ny \to -1.87613\times10^6\}, \{Ny \to 554238.\}\}, \{\{Ny \to -1.32459\times10^6\}, \{Ny \to 888773.\}\}, \{\{Ny \to -1.32459\times10^6\}, \{Ny \to 888773.\}\}, \{\{Ny \to -1.32459\times10^6\}, \{Ny \to 888773.\}\}, \{\{Ny \to -1.32459\times10^6\}, \{Ny \to 888773.\}\}, \{\{Ny \to -1.87613\times10^6\}, \{Ny \to 554238.\}\}, \{\{Ny \to -1.87613\times10^6\}, \{Ny \to 554238.\}\}\}$

In[198]:= % // TraditionalForm

Out[198]//TraditionalForm=

$$\begin{pmatrix}\{Ny\to-1.87613\times10^6\} & \{Ny\to554238.\}\\ \{Ny\to-1.87613\times10^6\} & \{Ny\to554238.\}\\ \{Ny\to-1.32459\times10^6\} & \{Ny\to888773.\}\\ \{Ny\to-1.32459\times10^6\} & \{Ny\to888773.\}\\ \{Ny\to-1.32459\times10^6\} & \{Ny\to888773.\}\\ \{Ny\to-1.32459\times10^6\} & \{Ny\to888773.\}\\ \{Ny\to-1.87613\times10^6\} & \{Ny\to554238.\}\\ \{Ny\to-1.87613\times10^6\} & \{Ny\to554238.\}\end{pmatrix}$$

The 0^o layer will fail in tension when N_{22}=554238 N/m
The 90^o layer will fail in tension when N_{22}=888773 N/m

For the Tsai-Wu failure criterion, the result indicates that max N22= N_{22}= 554238 N/m for layers with 0^o fiber orientation . The failure load N22⩾ 400000 N/m The initial failure load is higher that the given failure load. Thus, design objective is achieved

Design option 11 : $[(0)_2\,(90)_2]_s$ Vf=0.6

```
In[199]:= laminaConstants = {E1 → 1.67504 * 10^11, E2 → 9.64094 * 10^9, G12 → 4.18719 * 10^9, ν12 → 0.316,
    ν23 → 0.614218, X → 2.4 * 10^9, XP → 2.4 * 10^9, Y → 6.8 * 10^7, YP → 2.3 * 10^8, S → 1.1 * 10^8};

In[200]:= t = 127/1000 * 10^-3;

In[201]:= n = 8;

In[202]:= Nn = {0, Ny, 0};

In[203]:= M = {0, 0, 0};

In[204]:= angles = {0, 0, 90, 90, 90, 90, 0, 0};

In[205]:= ti = Table[t, {i, 1, n}];

In[206]:= bzi = Table[-1/2 (n + 1) t + i t, {i, 1, n}];

In[207]:= layercoord = Table[{bzi[[i]] - 1/2 ti[[i]], bzi[[i]] + 1/2 ti[[i]]}, {i, 1, n}];

In[208]:= Rσe = ( c^2   s^2   -2 c s
                  s^2   c^2    2 c s
                  c s  -c s   c^2 - s^2 );

In[209]:= Sep = ( 1/E1      -ν12/E1   0
                  -ν12/E1   1/E2      0
                  0         0         1/G12 ) /. laminaConstants;

In[210]:= Q = Simplify[Inverse[Sep]];

In[211]:= Qθ[i_] := Rσe.Q.Transpose[Rσe] /. {s → Sin[θ Degree], c → Cos[θ Degree]} /. θ → angles[[i]]

In[212]:= A = Sum[Qθ[i] × ti[[i]], {i, 1, n}] // N

Out[212]= {{9.05098 × 10^7, 3.11317 × 10^6, 0.}, {3.11317 × 10^6, 9.05098 × 10^7, 0.}, {0., 0., 4.25419 × 10^6}}

In[213]:= B = Sum[Qθ[i] × ti[[i]] × bzi[[i]], {i, 1, n}]

Out[213]= {{0., 0., 0.}, {0., 0., 0.}, {0., 0., 0.}}

In[214]:= Dd = Sum[Qθ[i] (ti[[i]] bzi[[i]]^2 + ti[[i]]^3 / 12), {i, 1, n}] // N

Out[214]= {{12.9895, 0.267799, 0.}, {0.267799, 2.58204, 0.}, {0., 0., 0.365951}}

In[215]:= ϵ = Inverse[A].Nn // Chop

Out[215]= {-3.80475 × 10^-10 Ny, 1.10616 × 10^-8 Ny, 0}

In[216]:= κ = Inverse[Dd].M // Chop

Out[216]= {0, 0, 0}
```

In[217]:=
```
σe = Table[Inverse[Rσe].Qθ[i].(ϵ + x3 κ) /. {s → Sin[θ Degree], c → Cos[θ Degree]} /.
    θ → angles[[i]], {i, 1, n}]
```

Out[217]=
```
{{-30.2051 Ny, 106.095 Ny, 0.}, {-30.2051 Ny, 106.095 Ny, 0.},
 {1862.41 Ny, 30.2051 Ny, 0.}, {1862.41 Ny, 30.2051 Ny, 0.}, {1862.41 Ny, 30.2051 Ny, 0.},
 {1862.41 Ny, 30.2051 Ny, 0.}, {-30.2051 Ny, 106.095 Ny, 0.}, {-30.2051 Ny, 106.095 Ny, 0.}}
```

In[218]:= TW[σ_] :=

$$\left(\frac{1}{X}-\frac{1}{XP}\right)\sigma[[1]]+\left(\frac{1}{Y}-\frac{1}{YP}\right)\sigma[[2]]+\frac{\sigma[[1]]^2}{X\,XP}+\frac{\sigma[[2]]^2}{Y\,YP}+\left(\frac{1}{S}\sigma[[3]]\right)^2-\frac{1}{X^2}\sigma[[1]]\times\sigma[[2]];$$

In[219]:=
```
sol = Table[Solve[(TW[σe[[i]]] /. laminaConstants) == 1, Ny], {i, 1, n}]
```

Out[219]=
```
{{{Ny → -2.16621 × 10^6}, {Ny → 640790.}}, {{Ny → -2.16621 × 10^6}, {Ny → 640790.}},
 {{Ny → -1.50311 × 10^6}, {Ny → 1.02234 × 10^6}}, {{Ny → -1.50311 × 10^6}, {Ny → 1.02234 × 10^6}},
 {{Ny → -1.50311 × 10^6}, {Ny → 1.02234 × 10^6}}, {{Ny → -1.50311 × 10^6}, {Ny → 1.02234 × 10^6}},
 {{Ny → -2.16621 × 10^6}, {Ny → 640790.}}, {{Ny → -2.16621 × 10^6}, {Ny → 640790.}}}
```

In[220]:= % // TraditionalForm

Out[220]//TraditionalForm=

$$\begin{pmatrix} \{Ny \to -2.16621\times10^6\} & \{Ny \to 640790.\} \\ \{Ny \to -2.16621\times10^6\} & \{Ny \to 640790.\} \\ \{Ny \to -1.50311\times10^6\} & \{Ny \to 1.02234\times10^6\} \\ \{Ny \to -1.50311\times10^6\} & \{Ny \to 1.02234\times10^6\} \\ \{Ny \to -1.50311\times10^6\} & \{Ny \to 1.02234\times10^6\} \\ \{Ny \to -1.50311\times10^6\} & \{Ny \to 1.02234\times10^6\} \\ \{Ny \to -2.16621\times10^6\} & \{Ny \to 640790.\} \\ \{Ny \to -2.16621\times10^6\} & \{Ny \to 640790.\} \end{pmatrix}$$

The 0^o layer will fail in tension when N_{22}=640790 N/m
The 90^o layer will fail in tension when N_{22}=1022340 N/m

For the Tsai-Wu failure criterion, the result indicates that max N22= 640790 N/m for layers with 0^o fiber orientation . The failure load N22⩾ 400000 N/m The initial failure load is higher that the given failure load. Thus, design objective is achieved

Design option 12 : $[(0)_2\,(90)_2]_s$ Vf=0.7

```
In[221]:= laminaConstants = {E1 → 1.94628 * 10^11, E2 → 11.04 * 10^9, G12 → 5.47378 * 10^9, ν12 → 0.307,
             ν23 → 0.626963, X → 2.5 * 10^9, XP → 2.5 * 10^9, Y → 7.1 * 10^7, YP → 2.4 * 10^8, S → 1.2 * 10^8};

In[222]:= t = 127/1000 * 10^-3;

In[223]:= n = 8;

In[224]:= Nn = {0, Ny, 0};

In[225]:= M = {0, 0, 0};

In[226]:= angles = {0, 0, 90, 90, 90, 90, 0, 0};

In[227]:= ti = Table[t, {i, 1, n}];

In[228]:= bzi = Table[-1/2 (n + 1) t + i t, {i, 1, n}];

In[229]:= layercoord = Table[{bzi[[i]] - 1/2 ti[[i]], bzi[[i]] + 1/2 ti[[i]]}, {i, 1, n}];
```

In[230]:= $R\sigma e = \begin{pmatrix} c^2 & s^2 & -2\,c\,s \\ s^2 & c^2 & 2\,c\,s \\ c\,s & -c\,s & c^2 - s^2 \end{pmatrix}$;

In[231]:= $Sep = \begin{pmatrix} \frac{1}{E1} & -\frac{\nu 12}{E1} & 0 \\ -\frac{\nu 12}{E1} & \frac{1}{E2} & 0 \\ 0 & 0 & \frac{1}{G12} \end{pmatrix}$ /. laminaConstants;

```
In[232]:= Q = Simplify[Inverse[Sep]];

In[233]:= Qθ[i_] := Rσe.Q.Transpose[Rσe] /. {s → Sin[θ Degree], c → Cos[θ Degree]} /. θ → angles[[i]]

In[234]:= A = Sum[Qθ[i] × ti[[i]], {i, 1, n}] // N

Out[234]= {{1.05041 × 10^8, 3.46202 × 10^6, 0.}, {3.46202 × 10^6, 1.05041 × 10^8, 0.}, {0., 0., 5.56136 × 10^6}}

In[235]:= B = Sum[Qθ[i] × ti[[i]] × bzi[[i]], {i, 1, n}]

Out[235]= {{0., 0., 0.}, {0., 0., 0.}, {0., 0., 0.}}

In[236]:= Dd = Sum[Qθ[i] (ti[[i]] bzi[[i]]^2 + ti[[i]]^3 / 12), {i, 1, n}] // N

Out[236]= {{15.085, 0.297807, 0.}, {0.297807, 2.98648, 0.}, {0., 0., 0.478396}}

In[237]:= ϵ = Inverse[A].Nn // Chop

Out[237]= {-3.14112 × 10^-10 Ny, 9.53045 × 10^-9 Ny, 0}

In[238]:= ϰ = Inverse[Dd].M // Chop

Out[238]= {0, 0, 0}
```

In[239]:=
```
σe = Table[Inverse[Rσe].Qθ[i].(ϵ + x3 κ) /. {s → Sin[θ Degree], c → Cos[θ Degree]} /.
    θ → angles[[i]], {i, 1, n}]
```

Out[239]=
```
{{-28.9886 Ny, 104.711 Ny, 0.}, {-28.9886 Ny, 104.711 Ny, 0.},
 {1863.79 Ny, 28.9886 Ny, 0.}, {1863.79 Ny, 28.9886 Ny, 0.}, {1863.79 Ny, 28.9886 Ny, 0.},
 {1863.79 Ny, 28.9886 Ny, 0.}, {-28.9886 Ny, 104.711 Ny, 0.}, {-28.9886 Ny, 104.711 Ny, 0.}}
```

In[240]:= TW[σ_] :=

$$\left(\frac{1}{X}-\frac{1}{XP}\right)\sigma[[1]]+\left(\frac{1}{Y}-\frac{1}{YP}\right)\sigma[[2]]+\frac{\sigma[[1]]^2}{X\,XP}+\frac{\sigma[[2]]^2}{Y\,YP}+\left(\frac{1}{S}\sigma[[3]]\right)^2-\frac{1}{X^2}\sigma[[1]]\times\sigma[[2]];$$

In[241]:=
```
sol = Table[Solve[(TW[σe[[i]]] /. laminaConstants) == 1, Ny], {i, 1, n}]
```

Out[241]=
```
{{{Ny → -2.29031 × 10^6}, {Ny → 677905.}}, {{Ny → -2.29031 × 10^6}, {Ny → 677905.}},
 {{Ny → -1.55806 × 10^6}, {Ny → 1.07605 × 10^6}}, {{Ny → -1.55806 × 10^6}, {Ny → 1.07605 × 10^6}},
 {{Ny → -1.55806 × 10^6}, {Ny → 1.07605 × 10^6}}, {{Ny → -1.55806 × 10^6}, {Ny → 1.07605 × 10^6}},
 {{Ny → -2.29031 × 10^6}, {Ny → 677905.}}, {{Ny → -2.29031 × 10^6}, {Ny → 677905.}}}
```

In[242]:=
```
% // TraditionalForm
```

Out[242]//TraditionalForm=

$$\begin{pmatrix}
\{Ny\to -2.29031\times10^6\} & \{Ny\to 677905.\}\\
\{Ny\to -2.29031\times10^6\} & \{Ny\to 677905.\}\\
\{Ny\to -1.55806\times10^6\} & \{Ny\to 1.07605\times10^6\}\\
\{Ny\to -1.55806\times10^6\} & \{Ny\to 1.07605\times10^6\}\\
\{Ny\to -1.55806\times10^6\} & \{Ny\to 1.07605\times10^6\}\\
\{Ny\to -1.55806\times10^6\} & \{Ny\to 1.07605\times10^6\}\\
\{Ny\to -2.29031\times10^6\} & \{Ny\to 677905.\}\\
\{Ny\to -2.29031\times10^6\} & \{Ny\to 677905.\}
\end{pmatrix}$$

The 0^o layer will fail in tension when N_{22}=677905 N/m
The 90^o layer will fail in tension when N_{22}=1076050 N/m

For the Tsai-Wu failure criterion, the result indicates that max N22= 677905 N/m for layers with 0^o fiber orientation . The failure load N22⩾ 400000 N/m The initial failure load is higher that the given failure load. Thus, design objective is achieved