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Research Statement

Traditionally, most models in macroeconomics (and other fields such as industrial organization) have dealt with either a representative agent or a continuum of agents[1]. Some models take an intermediate position with two (or a few) agents, for example, models of financial frictions (with a borrower and a lender) or models of international business cycles (with a home country and a foreign country). More recently, models with many finite agents are becoming very popular, in large part due to the explosion of available microdata. Unfortunately, dealing with multi-agent models is challenging, as we quickly bump into the curse of dimensionality. As soon as we have more than a few agents, it becomes nearly impossible to solve these models or estimate them with the data.

During my PhD, I have been working on addressing the curse of dimensionality in dynamic economic models. In my work I utilize modern machine learning methods (such as deep learning), high-dimensional probability and statistics, and theoretical economics to provide more reliable solution methods for high-dimensional dynamic economic models. Below I provide a list of my research projects and their contribution. I provide a list of my working papers and work in progress accompanied with a detailed explanation of their contribution.

"Exploiting Symmetry in High-Dimensional Dynamic Programming": Most heterogeneous agent models in macroeconomics have a latent but obvious symmetry. Changing the labels or indices of the agents in the model does not change the outcome of the equilibrium objects. For instance, in general equilibrium models, the Walrasian auctioneer collects all the excess supplies of each agent, aggregates them, and sells them. This aggregation removes the indices of agents in the economy and the solution of the model is invariant under all the permutations of the agents' states. We prove that under this symmetry, the equilibrium objects (such as investment decisions) have a functional representation that can substantially reduce the dimensionality of the problem. In stochastic environments, where agents face idiosyncratic and aggregate shocks, another form of the curse of dimensionality manifests itself, i.e., calculating high-dimensional conditional expectations. Relying on results from high-dimensional probability theory, we show that with high accuracy we can calculate the conditional expectations only by calculating the function of interest with one draw from the space of the idiosyncratic shocks. In probability theory, these results are known as concentration of measures, intuitively they state "A random variable that depends in a Lipschitz way on many independent variables (but not too much on any of them) is essentially constant". We solve a nonlinear version of Lucas and Prescott (1971) investment model, where no closed-form solution exists, for $10,000$ agents. We approximate the investment decision with a deep neural network. However, the theoretical framework we provide in this paper is not limited to deep neural networks and can be applied to other function approximation methods. The results of this paper shows that exploiting symmetry and concentration of measures can be a very promising avenue for solving high-dimensional heterogeneous models with uncertainty such as wealth distribution or spatial models in trade.
Co-authors: Jesús Fernández-Villaverde (UPenn), Jesse Perla (UBC), Arnav Sood (CMU).

"Spooky Boundaries at a Distance: Exploring Transversality and Stability with Deep Learning": Computing the solution of dynamic equilibrium models in economics usually requires economic assumptions that rule out explosive solutions (e.g., transversality or no-bubble conditions). These assumptions are variations of "boundary conditions" at infinity. Without these assumptions, economic models have many (in some cases infinitely many) solutions. Ignoring these boundary conditions can lead to solutions that might seem correct but are not optimal. Unfortunately, these forward-looking boundary conditions are a key limitation when we want to increase the dimensionality of the model (e.g., increasing the state variables of the model). Why do boundary conditions make life difficult?

[1] Such as Aiyagari and Krusell-Smith models.

Because they force us to solve the model over a wide range of possible values of the state variables. For instance, conditions for recursive formulations, where we are looking for a value function and optimal policy function, manifest as requiring accurate solutions for arbitrary values of the state variables, even though one may only care about the solution from a single initial condition. In this paper we show that deep neural networks (deep learning) automatically fulfill these boundary conditions. Rather than jump to complicated models where the state-of-the-art algorithms are heuristic, we carefully analyze deep learning solutions with standard benchmark models: neoclassical growth and linear asset pricing. Both are well understood, have reference numerical solutions, and can be analyzed theoretically. What makes deep learning special is its inherent implicit bias toward a specific class of solutions that matches the class of solutions we are interested in economics. Using recent theoretical advances in modern machine learning literature we provide a theoretical argument for these results. While this paper analyzes benchmarks such as the neoclassical growth model, the results suggest that deep learning may let us calculate accurate transition dynamics with high-dimensional state spaces, and without directly solving for the boundary conditions at infinity. Therefore, deep learning can be very beneficial in studying dynamic problems that are far from their steady states such as growth models, also they can provide better solutions for high-dimensional complex problems such as dynamic climate change models.
Co-authors: Jesús Fernández-Villaverde (UPenn), Sebastián Gómez-Cardona (UBC) Jesse Perla (UBC), Jan Rosa (UBC).

"Optimal Entry Decision with Correlated Variable Cost and Output Price": In models with irrecoverable investment and uncertainty in the output price it is a well-established result that uncertainty increases the output price that a firm starts investment. This paper studies a model of irrecoverable investment (entry) where the variable cost and output price are characterized by two correlated geometric Brownian motions. The numerical results indicate that in the presence of high levels of correlation the impact of uncertainty in output prices is ambiguous and depends on the level of variable cost. Specifically, increasing uncertainty in output prices increases the entry output price for low levels of variable cost and the reverse happens for high levels of variable cost. Therefore, in the presence of high levels of correlation the conventional result does not hold anymore. Moreover, this study establishes that increasing the correlation level decreases the entry output price.
Future direction: In this paper I used a classical collocation method to solve the partial differential equations that determine the value functions and entry decision. As discussed in my other works, the classical collocation methods are not suitable for dimensions higher than two or three. At the moment I am working on designing a reliable deep learning framework to solve these problems. Models of this sort are frequently used in industrial organization, environmental economics and investment friction literature. This framework will help researchers to study richer models that can be used in structural estimations.

Work in progress and future works

Solving Equilibrium Economic Models with Deep Learning": The success of deep learning in a variety of applications is leading economists to explore its potential for *orders of magnitude* increase in the size of the state-space and the complexity of models we can solve. In this paper, we provide a clear mental picture of what deep learning is, how it relates to existing solution methods, how to encode economic insights and domain knowledge, and where the methods are likely to be revolutionary. In answering those questions, we demystify these methods, explain the core concepts with simple, however, insightful examples, and debunk "folk wisdom" commonly held, while elaborating on places where we should proceed with caution. We establish that deep learning as a method of function approximation is not limited to macroeconomic models and are very general. Therefore, they can be very promising in many fields such as industrial organization and spatial models with heterogeneity.
Co-authors: Jesús Fernández-Villaverde (UPenn), Jesse Perla (UBC).