Team: Instant Math

Connection of discrete optimal control with problems of intersectoral balance

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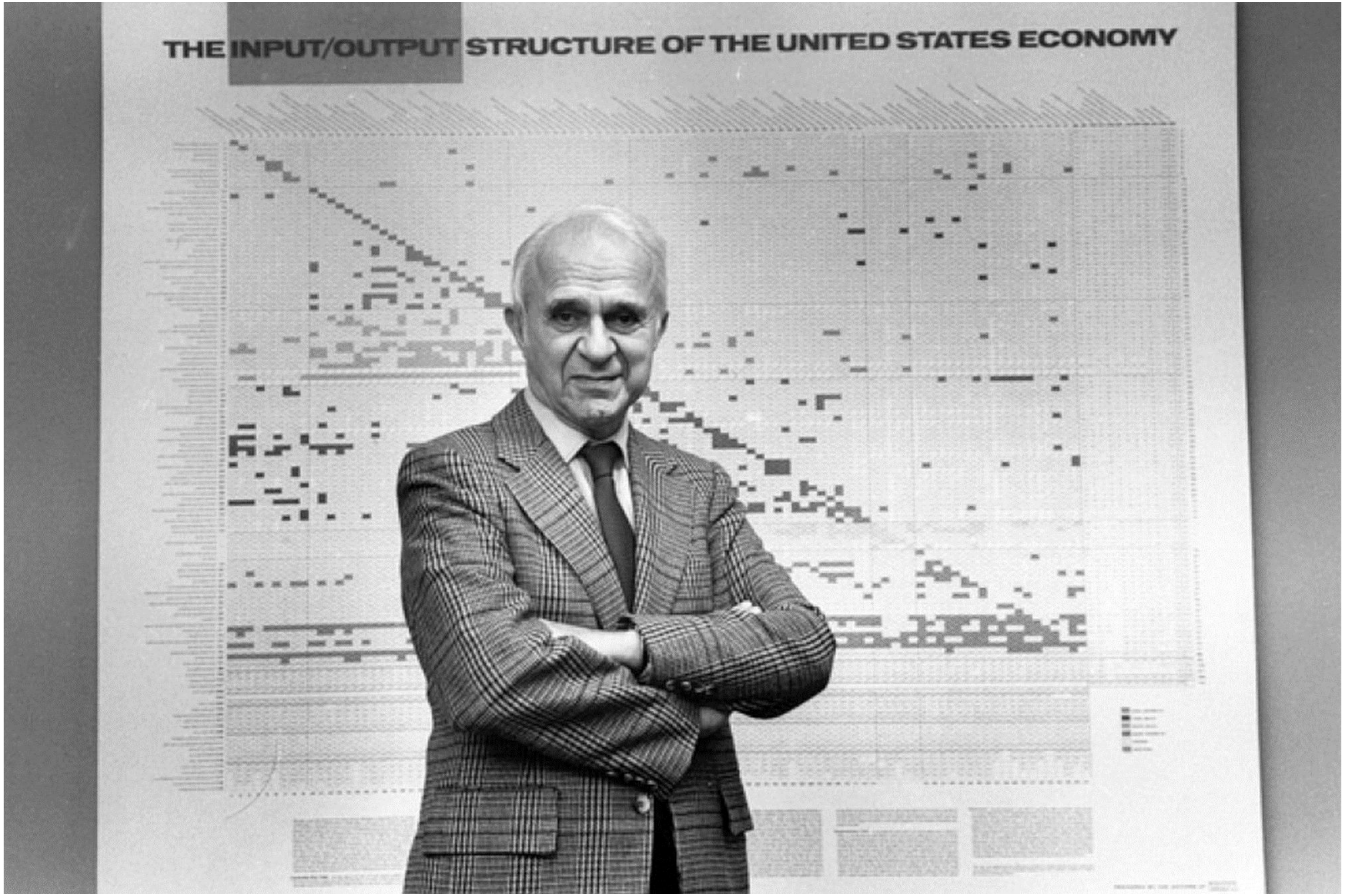
Overview

- **Brief explanation of Leontiev's model**
- **Problem statement**
- **Possible Solution**
- **Experiments and results**
- **The Pros and Cons of Idea (Summary)**

Short history (additionally)

- In the 1936s, V.V. Leontiev applied the method of analysis of intersectoral relations with the involvement of the apparatus of linear algebra to study the US economy. The method became known as input-output.
- In 1959, the Central Statistical Bureau of the USSR developed the world's first intersectoral balance sheet in physical terms (for 157 products) and an intersectoral balance sheet in value terms (for 83 industries)
- In the 1970s and 1980s, in the USSR, on the basis of data from intersectoral balances, more complex intersectoral models and model complexes were developed, which were used in forecast calculations and were partly included in the technology of national economic planning.



Vasily Vasilievich Leontiev (1905-1999)

Brief explanation of Leontiev's model

The economy is broken down into separate production units - a pure industry. It is believed that one net article produces only one product, different industries produce different products. We single out pure individuals and number them and their products.
Leontiev observed the time series of the output of industries:

$x_i(t)$ - gross output of the i-th industry in the period of time t.

$z_{ij}(t)$ - the amount of products of the i-th industry, which is used as raw materials by the j-th industry in the period of time t.

Leontiev analyzed the behavior of the sequence $\{x_i(t), z_{ij}(t)\}_{t=0}^{T}$

$\frac{z_{ij}(t)}{x_i(t)} \approx a_{ij}$ - ratio weakly depends on time => a_{ij} - can be considered constants

$A = \|a_{ij}\| \in \mathbb{R}^{n \times n}$ - Leontief direct cost matrix

$w_i(t)$ - final output of the i-th industry in the time period t

Brief explanation of Leontiev's model

$$x = \begin{pmatrix} x_1 \\ x_2 \\ \dots \\ x_k \\ \dots \\ x_n \end{pmatrix}$$

$$x_k = (a_{k1}x_1 + a_{k2}x_2 + \dots + a_{kk}x_k + \dots + a_{kn}x_n) + y_k$$

$$x = Ax + y$$

$$(I - A)x = y$$

$$x = (I - A)^{-1}y$$

We will assume that the costs are made on the previous time step. We get dynamic model:

,

$$x(t) = Ax(t+1) + y(t+1)$$

Example: *An economy has the two industries R and S. The current consumption is given by the table*

	consumption		
	R	S	external
Industry R production	50	50	20
Industry S production	60	40	100

Assume the new external demand is 100 units of R and 100 units of S. Determine the new production levels.

Solution: *The total production is 120 units for R and 200 units for S. We obtain* $X = \begin{pmatrix} 120 \\ 200 \end{pmatrix}$, $B = \begin{pmatrix} 20 \\ 100 \end{pmatrix}$, $A = \begin{pmatrix} \frac{50}{120} & \frac{50}{200} \\ \frac{60}{120} & \frac{40}{200} \end{pmatrix}$, *and* $B' = \begin{pmatrix} 100 \\ 100 \end{pmatrix}$. *The solution is*

$$X' = (I_2 - A)^{-1}B' = \frac{1}{41}\begin{pmatrix} 96 & 30 \\ 60 & 70 \end{pmatrix}\begin{pmatrix} 100 \\ 100 \end{pmatrix} = \begin{pmatrix} 307.3 \\ 317.0 \end{pmatrix}.$$

The new production levels are 307.3 and 317.0 for R and S, respectively.

SOCIETY 5.0. (Relevance of Liontief's model)

Problem statement

MANUFACTURING INDUSTRIES	CONSUMING INDUSTRIES					FINAL PRODUCTS	GROSS OUTPUT
	1	*2*	*3*	...	*n*		
1 2 3 ... n	x_{11} x_{21} x_{31} ... x_{n1}	x_{12} x_{22} x_{32} ... x_{n2}	x_{13} x_{23} x_{33} ... x_{n3}	... ... I ...	x_{1n} x_{2n} x_{3n} ... x_{nn}	Y_1 Y_2 Y_3 II Y_n	X_1 X_2 X_3 ... X_n
DEPRECIATION SALARY NET INCOME	c_1 v_1 m_1	c_2 v_2 m_2	c_3 v_3 m_3	... III ...	c_n v_n m_n	IV	
GROSS OUTPUT	X_1	X_2	X_3	...	X_n		$\sum_{i=1}^{n} X_i = \sum_{j=1}^{n} X_j$

Problem statement

A linear equation system is compiled in a similar table; as a result of the solution, we get the vector of the gross product, in which it is put into production for the country's enterprise. The volume of gross output that needs to be produced to obtain Y - the final product in the right quantity.

Problem statement

Матрица межотраслевого баланса

Квадратная матрица, содержащая коэффициенты производства продукции во всех отраслях экономики. На пересечении строк и столбцов находятся коэффициенты использования комплектующих для производства продукции. Например, если для производства единицы продукта №1 требуется 0.05 единиц продукта №2, то в ячейке [1,2] стоит число 0.05.

	Молот, шт	Кузнец, чел.ч	Железо, кг	Ведро, шт	Яйцо куриное, шт	Птицевод, чел.ч	Курица, шт	Молоко, л	Доярка, чел.ч	Корова, шт	Сено, кг	Вода, л	Зерно, кг	Яблоко, кг	Садовод, чел.ч	Яблоня, шт	Лестница, шт	Томат, кг	Овощевод, чел.ч	Лейка, шт	Лопата, шт	Коса, шт	Хлебороб, чел.ч	Семена, кг
Молот, шт	0	8	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Кузнец, чел.ч	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Железо, кг	0	0.05	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Ведро, шт	0.01	8	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Яйцо куриное, шт	0	0	0	0.0002	0	0.1	0.0025	0	0	0	0	0.1	0.05	0	0	0	0	0	0	0	0	0	0	0
Птицевод, чел.ч	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Курица, шт	0	0	0	0	0	10	0	0	0	0	0	10	10	0	0	0	0	0	0	0	0	0	0	0
Молоко, л	0	0	0	0.001	0	0	0	0	0.1	0.0001	1	2	0	0	0	0	0	0	0	0	0	0	0	0
Доярка, чел.ч	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Корова, шт	0	0	0	0	0	0	0	0	50	0	0	100	11	0	0	0	0	0	0	0	0	0	0	0
Сено, кг	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.001	0.3	0
Вода, л	0	0	0	0.0005	0	0	0	0	0.015	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Зерно, кг	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.001	1	0
Яблоко, кг	0	0	0	0.002	0	0	0	0	0	0	0	12	0	0	0.3	0.0003	0.002	0	0	0	0	0	0	0
Садовод, чел.ч	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Яблоня, шт	0	0	0	0.5	0	0	0	0	0	0	0	100	0	0	150	0	0	0	0	0	0.5	0	0	0
Лестница, шт	0.1	16	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Томат, кг	0	0	0	0.0005	0	0	0	0	0	0	0	7	0	0	0	0	0	0	0.3	0.0005	0.0005	0	0	0
Овощевод, чел.ч	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Лейка, шт	0.01	8	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Лопата, шт	0.02	8	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Коса, шт	0.01	8	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Хлебороб, чел.ч	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Семена, кг	0	0	0	0.001	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0

https://ogasdemo.ru/ipogv/gosplan/mob/?today=01.01.2022&date=01.01.2023&submit=Y

Problem statement

Now it is in the size 24x24. But in reality it is much more. For example, a billion on a billion. A square matrix containing the coefficient of production in all sectors of the economy. At the intersection of the rows of columns is the coefficient of the use of components for the production of products.

- The solution of the system of linear assignments of the intersectoral balance is a scientific computational problem, because the intersectoral balance table on the scale of the country's economy has tens and hundreds of millions of rows and columns.
- The solution of such a system of linear equations by well-known mathematical methods of Jordan-Gauss or Cramer may require such large computational resources that even with the current level of development of computer technology, they may not be sufficient to obtain planning results in an acceptable time frame.

Problem statement

For example, to solve a System of linear algebraic equations intersectoral balance of 100x100 million variables using the Jordan-Gauss method, which has a computational complexity of the order of $O(N^3)$, it may take up to 10^{28} operations, which even on the most powerful supercomputer in the world with a computing power of about 10^{18} operations per second may take about 317 years.

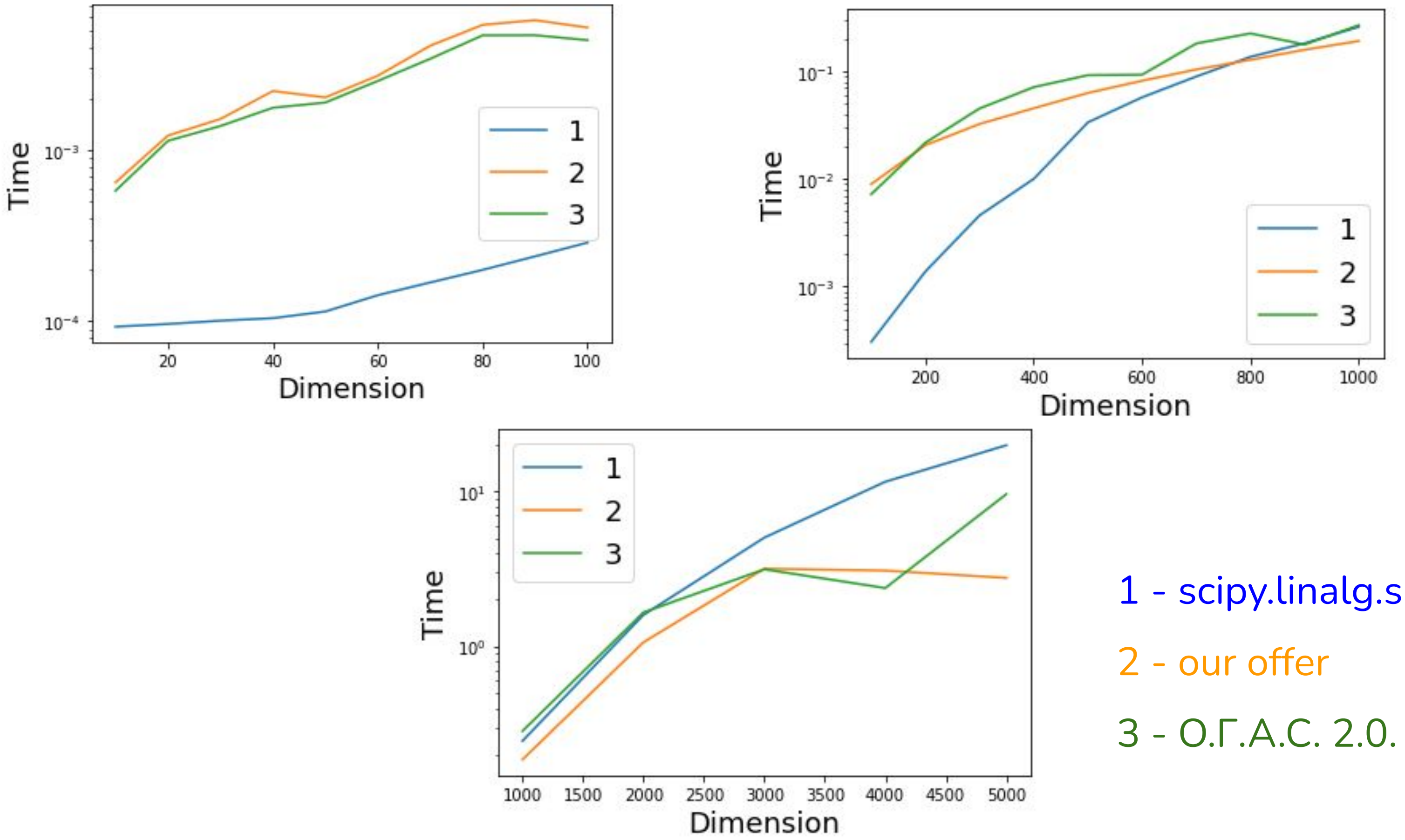
Possible Solution

- It should be taken into account that the intersectoral balance matrix is sparse, it consists mainly of zeros.
- Therefore, the solution of such a sparse system of linear equations by the mathematical methods of Jordan-Gauss or Cramer is not optimal in terms of computer time.
- Use sparse matrix recursive algorithms that allows solving such a system of linear equations with a much smaller number of operations and in much less time than the Jordan-Gauss method allows (in this case, the computational complexity can be less than $O(N^2)$ and depends on the combination of the values of the system of equations and on the required accuracy of the final result) .

Possible Solution

https://ogasdemo.ru/ipogv/gosplan

Experiments and results

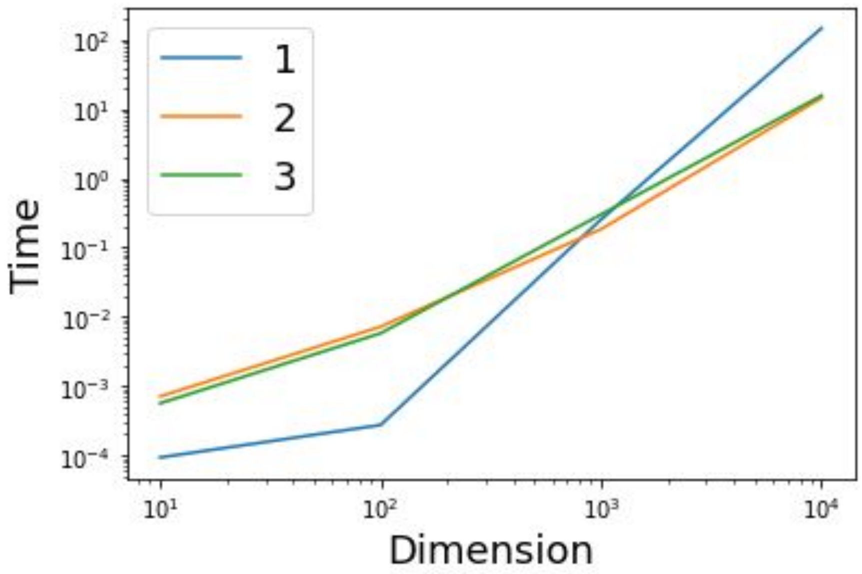


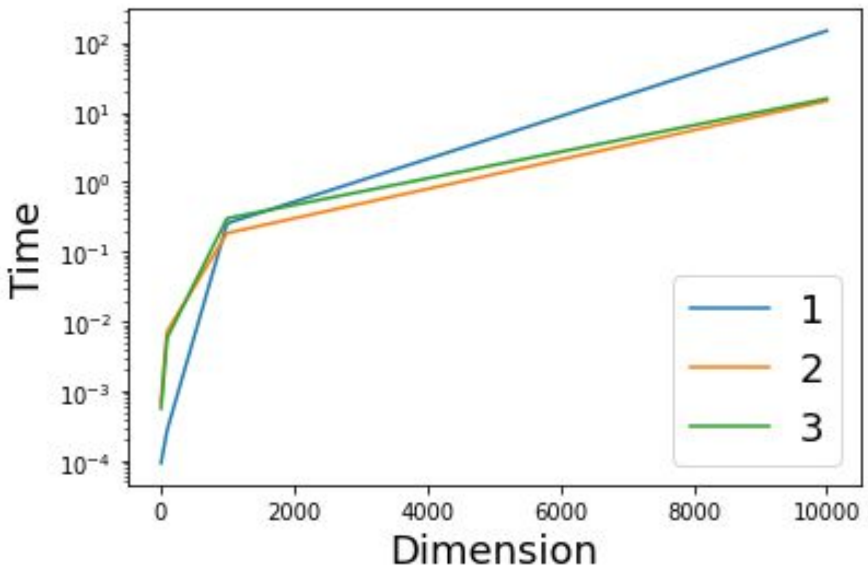
1 - scipy.linalg.spsolve

2 - our offer

3 - О.Г.А.С. 2.0.

Experiments and results





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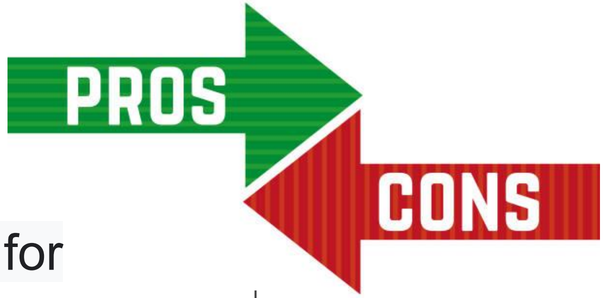
The Pros and Cons of Idea (Summary)



- The idea is optimal for medium-sized matrices
- Optimal use for autonomous firms and companies
- Optimal use for farms in which is engaged in business

- Our idea is not optimal for very large size matrix
- Not optimal for country scale economics use

Thank you for attention!