Wiktor Kostka

4.1 Równanie transportu ciepła

$$-\frac{d}{dx}\left(k(x)\frac{du(x)}{dx}\right) = 100x^2$$

$$u(2) = -20$$

$$\frac{du(0)}{dx} + u(0) = 20$$

$$k(x) = \begin{cases} 1 & \text{dla } x \in [0,1] \\ 2x & \text{dla } x \in (1,2] \end{cases}$$

Gdzie u to poszukiwana funkcja

$$[0,2] \ni x \mapsto u(x) \in \mathbb{R}$$

$$-\frac{d}{dx}\left(k(x)\frac{du(x)}{dx}\right) = 100x^2$$

$$-k(x)u''(x) = 100x^2$$

War. brzegowe:

$$\begin{cases} u(2) = -20 \leftarrow \text{war. Dirichleta} \\ u'(0) + u(0) = 20 \leftarrow \text{war. Robina} \end{cases}$$

$$\Downarrow$$

$$u'(0) = 20 - u(0)$$

$$k(x) = \begin{cases} 1 & \text{dla } x \in \langle 0, 1\rangle \\ 2x & \text{dla } x \in (1, 2\rangle \end{cases}$$

$$[0,2] \ni x \longmapsto u(x) \in \mathbb{R}$$

$$\Omega = \langle 0, 2 \rangle$$

$$V = \{ f \in H^1 : \underset{\uparrow}{f(2) = 0} \}, \quad v \in V$$

brzeg z war. Dirichleta

$$-k(x)u''(x) = 100x^2 \quad / \cdot v(x)$$

$$-k(x)u''(x)v(x) = 100x^2 v(x) \quad / \int_\Omega$$

$$-\int_0^2 k(x)u''(x)v(x)\,dx = \int_0^2 100x^2 v(x)\,dx = (*)$$

$$-\int_0^2 k(x)v(x)u''(x)\,dx = \left| \begin{array}{ll} a = k(x)v(x) & b = u'(x) \\ a' = k(x)v'(x) & b' = u''(x) \end{array} \right|$$

$$= -k(x)v(x)u'(x) \Big|_0^2 + \int_0^2 k(x)v'(x)u'(x)\,dx =$$

$$= \underbrace{-k(2)v(2)u'(2)}_{=0} + k(0)v(0)u'(0)$$

$v(2)=0$ $\qquad u'(0) = 20 - u(0)$

$$+ \int_0^2 k(x)v'(x)u'(x)\,dx =$$

$$= k(0)v(0)\big(20 - u(0)\big) + \int_0^2 k(x)v'(x)u'(x)\,dx =$$

$$= 20k(0)v(0) - k(0)v(0)u(0) + \int_0^2 k(x)v'(x)u'(x)\,dx$$

$k(0) = 1$!

$$(*) = 20v(0) - v(0)u(0) + \int_0^2 k(x)v'(x)u'(x)\,dx$$

$$= \int_0^2 100x^2 v(x)\,dx$$

$$-v(0)u(0) + \int_0^2 k(x)v'(x)u'(x)\,dx =$$

$$= \int_0^2 100x^2 v(x)\,dx - 20v(0)$$

SFORMUŁOWANIE WARIACYJNE

Dla :

$$B(u,v) = -u(0)v(0) + \int_0^2 k(x)v'(x)u'(x)dx$$

$$L(v) = \int_0^2 100x^2 v(x)dx - 20v(0)$$

dla $u \in H^1(0,2)$, $u(2) = -20$

$\forall v \in H^1(0,2)$, $v(2) = 0$

$$\underbrace{B(u,v) = L(v)}$$