9.3 $$\frac{n_{He}}{n_{He^+} n_e} = \frac{g_{He}}{g_{He^+} g_e} \left(\frac{m_{He}}{m_{He^+} m_e}\right)^{3/2} \left(\frac{kT}{2\pi\hbar^2}\right)^{-3/2} e^{Q/kT}$$

$\frac{g_{He}}{g_{He^+} g_e} = \frac{1}{4}$

$\frac{m_{He}}{m_{He^+} m_e} = \frac{2m_p + 2m_n + 2m_e}{(2m_p + 2m_n + m_e)m_e} \approx \frac{2m_p + 2m_n}{(2m_p + 2m_n)m_e} \approx \frac{1}{m_e}$

$$\rightarrow \frac{n_{He}}{n_{He^+} n_e} \propto \frac{1}{4} m_e^{-3/2} \Rightarrow S_{He^+}(T,\eta) = \frac{1}{4} S_H(T,\eta)$$

$$S_{He}(T,\eta) = \frac{1}{4} \times 3.84\eta \left(\frac{kT}{m_e c^2}\right)^{3/2} e^{Q/kT}$$

$$X = \frac{-1 + \sqrt{1+4S}}{2S} = \frac{1}{2}$$

$$-1 + \sqrt{1+4S} = S$$

$$\sqrt{1+4S} = S+1$$

$$1 + 4S = S^2 + 2S + 1$$

$$S^2 = 2S$$

$$\boxed{S = 2}$$

$\eta = 5.5 \times 10^{-10}$ $\quad Q = 24.6\,eV$ $\quad k = 8.6 \times 10^{-5}\,eV/K$

$m_e c^2 = .511 \times 10^6\,eV$

$$S = 2 = \frac{3.84}{4}(5.5 \times 10^{-10}) \left(\frac{8.6 \times 10^{-5}\,eV/K \cdot T}{.511 \times 10^6\,eV}\right)^{3/2} e^{\frac{24.6\,eV}{8.6 \times 10^{-5}\,\frac{eV}{K}\,T}}$$

$$\rightarrow \boxed{T = 6715\,K}$$

9.5

$$\tau = \int_{t_*}^{t_0} n_e(t)\,\sigma_e\,c\,dt = 1$$

n scales with volume, or a^3, so $n_e(t) = \frac{n_0}{a^3}$

$a(t) = \left(\frac{t}{t_0}\right)^{\frac{2}{3+3w}} = \left(\frac{t}{t_0}\right)^{2/3}$ for matter-dominated U ($w=0$)

$$\rightarrow n_e(t) = \frac{n_0}{\left(\left(t/t_0\right)^{2/3}\right)^3} = \frac{n_0 t_0^2}{t^2}$$

$$\tau = n_0 t_0^2 \sigma_e c \int_{t_*}^{t_0} t^{-2}\,dt$$

$$-t^{-1}\Big|_{t_*}^{t_0} = \frac{1}{t_*} - \frac{1}{t_0}$$

$$\tau = n_0 t_0^2 \sigma_e c\left[\frac{1}{t_*} - \frac{1}{t_0}\right] = 1$$

$H_0 = 70\ \mathrm{km/s/Mpc} \times \frac{1\ \mathrm{Mpc}}{3.09\times10^{19}\ \mathrm{km}} = 2.27\times10^{-18}\ \mathrm{s^{-1}}$

$$\frac{1}{t_*} = \frac{1}{n_0 t_0^2 \sigma_e c} + \frac{1}{t_0}$$

$$t_0 = \frac{2}{3(1+w)H_0} = \frac{2}{3H_0} \quad \text{for matter-dominated}$$

$$= 2.94\times10^{17}\ \mathrm{s}$$

$$\rightarrow t_* = \left[\left[(.22\ \mathrm{m^{-3}})(2.94\times10^{17}\ \mathrm{s})^2(6.65\times10^{-29}\ \mathrm{m^2})(3\times10^8\ \mathrm{m/s})\right]^{-1} + \left[2.94\times10^{17}\ \mathrm{s}\right]^{-1}\right]^{-1}$$

$$\boxed{t_* = 3.7\times10^{14}\ \mathrm{s} = 1.2\times10^7\ \mathrm{yr}}$$

$$1+z = \left(\frac{t_0}{t}\right)^{\frac{2}{3+3w}} \rightarrow z = \left(\frac{t_0}{t}\right)^{2/3} - 1 = \left(\frac{2.94\cdot10^{17}\ \mathrm{s}}{3.7\times10^{14}\ \mathrm{s}}\right)^{2/3} - 1 = \boxed{84.8}$$