



**Kauno technologijos universitetas**

Informatikos fakultetas, programų inžinerijos katedra

## **P170B400 Algoritmų sudarymas ir analizė**

Namų darbai

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Projekto autorius

**IFF-8/11**

Akademinei grupei

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Ex 16

$$f(n) = n^n \sqrt{n} \quad \text{or} \quad g(n) = n!$$

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \lim_{n \rightarrow \infty} \frac{n^n \sqrt{n}}{n!} = \lim_{n \rightarrow \infty} \frac{n \cdot n^{\frac{1}{2}}}{n!} = \lim_{n \rightarrow \infty} \frac{n^{\frac{3}{2}}}{n!} = \frac{\infty}{\infty} = \text{L'Hôpital's rule}$$

$$= \lim_{n \rightarrow \infty} \frac{n^{\frac{3}{2}} \cdot \frac{1}{2} n^{-\frac{1}{2}}}{1 \cdot 2 \cdot 3 \cdots n} = \lim_{n \rightarrow \infty} \frac{(2x^{\frac{3}{2} + \frac{1}{2}} + x^{\frac{3}{2} - \frac{1}{2}} + 2x^{\frac{3}{2} + 1} \cdot \ln(x))^{\frac{1}{2}}}{1} \Rightarrow \infty$$

$$\text{Ans: } n^n \sqrt{n} = \Omega(n!)$$

Ex 2

$$f(x) = x + e^{2x} \quad g(x) = 5x$$

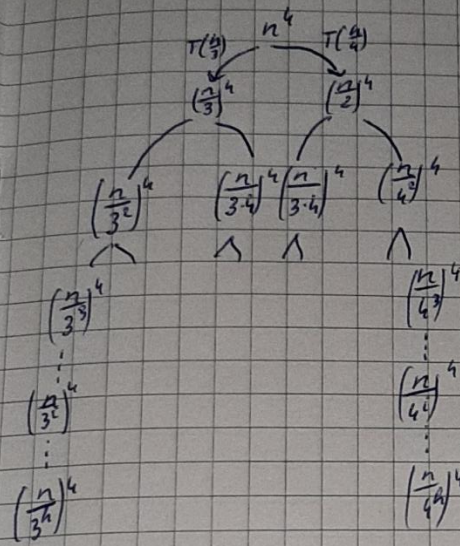
$$\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = \lim_{x \rightarrow \infty} \frac{x + e^{2x}}{5x} \neq \frac{\infty}{\infty} = \frac{\infty}{\infty} = \lim_{x \rightarrow \infty} \frac{(x + e^{2x})'}{(5x)'} =$$

$$= \lim_{x \rightarrow \infty} \frac{1 + 2e^{2x}}{5} \Rightarrow \infty$$

$$\text{Ans: } x + e^{2x} = \Omega(5x)$$

$$T(n) = T\left(\frac{n}{4}\right) + T\left(\frac{n}{3}\right) + n^4$$

II gr 12



$$T\left(\frac{n}{3}\right) = T\left(\frac{n}{4^2}\right) + T\left(\frac{n}{3^2}\right) + \left(\frac{n}{3}\right)^4$$

$$T\left(\frac{n}{4}\right) = T\left(\frac{n}{4^2}\right) + T\left(\frac{n}{3 \cdot 4}\right) + \left(\frac{n}{4}\right)^4$$

$$T\left(\frac{n}{3^2}\right) = T\left(\frac{n}{4 \cdot 3^2}\right) + T\left(\frac{n}{3^3}\right) + \left(\frac{n}{3^2}\right)^4$$

$$T\left(\frac{n}{3 \cdot 4}\right) = T\left(\frac{n}{4^2 \cdot 3}\right) + T\left(\frac{n}{4 \cdot 3^2}\right) + \left(\frac{n}{4 \cdot 3}\right)^4$$

$$T\left(\frac{n}{4^2}\right) = T\left(\frac{n}{4^3}\right) + T\left(\frac{n}{3 \cdot 4^2}\right) + \left(\frac{n}{4^2}\right)^4$$

$$\frac{n^4}{3^{4h}} = 1 \quad n^4 = 81^h \quad h = \log_3 n$$

$$\frac{n^4}{4^{4h}} = 1 \quad n^4 = 256^h$$

$$\log_{256} (256^h) = \log_{256} (n^4) \quad h = \frac{1}{8} \log_2 (n^4)$$

$$h = \log_2 (n^4) \quad h = \frac{1}{2} \log_2 (n)$$

$$\left(\frac{n}{3}\right)^4 + \left(\frac{n}{4}\right)^4 = n^4 \left(\frac{1}{81} + \frac{1}{256}\right) = n^4 \left(\frac{1}{3^4} + \frac{1}{4^4}\right)$$

$$\frac{n^4}{3^8} + 2 \frac{n^4}{12^4} + \frac{n^4}{4^8} = n^4 \left(\frac{1}{3^8} + \frac{2}{12^4} + \frac{1}{4^8}\right) = n^4 \left(\frac{1}{3^8} + \frac{1}{4^8}\right) = n^4 \left(\frac{1}{3^4} + \frac{1}{4^4}\right)^2$$

$$S = \sum_{i=1}^h q^i = \frac{1}{1-q}$$

$$T(n) = T\left(\frac{n}{4}\right) + T\left(\frac{n}{3}\right) + n^4 = n^4 \left(\frac{1}{3^4} + \frac{1}{4^4}\right) + n^4 \left(\frac{1}{3^4} + \frac{1}{4^4}\right)^2 + \dots + u =$$

$$= \sum_{i=1}^{\infty} n^4 \left(\frac{1}{3^4} + \frac{1}{4^4}\right)^i = n^4 \left(\frac{1}{1 - \left(\frac{1}{3^4} + \frac{1}{4^4}\right)}\right) = O(n^4)$$

III gr 4

$$T(n) = 2^n T\left(\frac{n}{2}\right) + n^n$$

$$a = 2^n; b = 2; f(n) = n^n$$

$$n^{\log_2 2^n} \boxed{n^n}$$

$$f(n) = \Theta(n^{\log_2 2^n})$$

$$n^n \boxed{n^n}$$

$$T(n) = \Theta(n^{\log_2 2^n} \log_2 n) = \Theta(n^n \log_2 n)$$



III gr 19

$$T(n) = 64T\left(\frac{n}{8}\right) + n^2 \log n \quad a=64, b=8, f(n) = -n^2 \log n$$

$$f(n) = O(n^{\log_8 64 - \epsilon}) \quad n^{\log_8 64} \square -n^2 \log n$$

$$n^2 \xrightarrow{1/2} -n \log n$$

$$\frac{f(n)}{O(n^{\log_8 64 - \epsilon})} = \frac{-n \log n}{n^{2-\epsilon}} =$$

$$= \frac{-\log n}{n^{2-\epsilon}} = \frac{-\log n}{n^{-\epsilon}} = -n^{\epsilon} \log n$$

$$-n^{\epsilon} \log n$$

IV gr 17

$A_1$ $50 \times 15$	7	0					
$A_2$ $15 \times 40$	6	20000	0				
$A_3$ $40 \times 35$	5	60000	20000	0			
$A_4$ $35 \times 40$	4	116000	52500	35000	0		
$A_5$ $40 \times 25$	3	180000	132000	112000	56000	0	
$A_6$ $25 \times 20$	2	204000	144000	127000	80000	24000	0
$A_7$ $20 \times 40$	1	310000	245000	220000	140000	100000	30000
	7	6	5	4	3	2	1

$$\left. \begin{array}{l} A_1 \ 50 \times 15 \\ A_2 \ 15 \times 40 \\ A_3 \ 40 \times 35 \\ A_4 \ 35 \times 40 \\ A_5 \ 40 \times 25 \\ A_6 \ 25 \times 20 \\ A_7 \ 20 \times 40 \end{array} \right\} \begin{array}{l} A_1 \cdot A_2 = 50 \times 15 \times 40 = 30000 \\ A_2 \cdot A_3 = 15 \times 40 \times 35 = 21000 \\ A_3 \cdot A_4 = 40 \times 35 \times 40 = 56000 \\ A_4 \cdot A_5 = 35 \times 40 \times 25 = 35000 \\ A_5 \cdot A_6 = 40 \times 25 \times 20 = 20000 \\ A_6 \cdot A_7 = 25 \times 20 \times 40 = 20000 \end{array}$$

$$\left. \begin{array}{l} A_1 \ 50 \times 15 \\ A_2 \ 15 \times 40 \\ A_3 \ 40 \times 35 \end{array} \right\} \begin{array}{l} A_1 \times (A_2 \times A_3) = 50 \times 15 \times 35 + 21000 = 47250 \\ A_2 \times (A_1 \times A_3) = 50 \times 40 \times 35 + 30000 = 100000 \end{array} \left. \vphantom{\begin{array}{l} A_1 \\ A_2 \\ A_3 \end{array}} \right\} \text{max } 100000$$

$$\left. \begin{array}{l} A_2 \ 15 \times 40 \\ A_3 \ 40 \times 35 \\ A_4 \ 35 \times 40 \end{array} \right\} \begin{array}{l} A_2 \times (A_3 \times A_4) = 15 \times 40 \times 40 + 56000 = 80000 \\ A_3 \times (A_2 \times A_4) = 15 \times 35 \times 40 + 21000 = 42000 \end{array} \left. \vphantom{\begin{array}{l} A_2 \\ A_3 \\ A_4 \end{array}} \right\} \text{max } 80000$$

$$\left. \begin{array}{l} A_3 \ 40 \times 35 \\ A_4 \ 35 \times 40 \\ A_5 \ 40 \times 25 \\ A_6 \ 25 \times 20 \end{array} \right\} \begin{array}{l} A_3 \times (A_4 \times A_5) = 40 \times 35 \times 25 + 35000 = 70000 \\ A_4 \times (A_3 \times A_5) = 40 \times 40 \times 25 + 56000 = 112000 \end{array} \left. \vphantom{\begin{array}{l} A_3 \\ A_4 \\ A_5 \\ A_6 \end{array}} \right\} \text{max } 112000$$

$$\left. \begin{array}{l} A_4 \ 35 \times 40 \\ A_5 \ 40 \times 25 \\ A_6 \ 25 \times 20 \end{array} \right\} \begin{array}{l} A_4 \times (A_5 \times A_6) = 40 \times 40 \times 20 + 80000 = 160000 \\ A_5 \times (A_4 \times A_6) = 40 \times 25 \times 20 + 35000 = 45000 \end{array} \left. \vphantom{\begin{array}{l} A_4 \\ A_5 \\ A_6 \end{array}} \right\} \text{max } 160000$$



$$\left. \begin{array}{l} A_4 \quad 35 \times 40 \\ A_5 \quad 40 \times 25 \\ A_6 \quad 25 \times 20 \end{array} \right\} A_6 \times (A_4 \times A_5) = 35 \times 25 \times 20 + 35000 = 52500$$

$$\begin{array}{l}
 A_6 \quad 25 \times 20 \\
 A_7 \quad 20 \times 40 \\
 A_5 \quad 40 \times 25 \\
 A_6 \quad 25 \times 20 \\
 A_7 \quad 20 \times 40
 \end{array}
 \left\{
 \begin{array}{l}
 A_5 \times (A_6 \times A_7) = 40 \times 25 \times 40 + 20000 = 60000 \\
 A_7 \times (A_5 \times A_6) = 40 \times 20 \times 40 + 20000 = 52000
 \end{array}
 \right\} \text{max } 60000$$

$$\begin{aligned} 1. & A_1 \times A_2 \times A_3 \times A_4 = 50 \times 15 \times 40 + 10000 = 11000 \\ 2. & (A_1 \times A_2) \times (A_3 \times A_4) = 50 \times 40 \times 40 + 30000 + 56000 = 166000 \\ 3. & (A_1 \times A_2 \times A_3) \times A_4 = 50 \times 35 \times 40 + 100000 = 110000 \end{aligned}$$

$$\begin{aligned} A_3 \times A_4 \times A_5 \times A_6 &= 40 \times 35 \times 20 + 52500 = 80500 \\ (A_3 \times A_4) \times (A_5 \times A_6) &= 40 \times 40 \times 20 + 56000 + 20000 = 108000 \\ (A_3 \times A_4 \times A_5) \times A_6 &= 40 \times 25 \times 20 + 112000 = 132000 \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \max 132000$$

$$\left. \begin{aligned} A_2 \times A_6 \times A_7 \times A_8 \times A_7 &= 40 \times 35 \times 40 + 16.000 = 17.200 \\ A_3 \times A_6 \times (A_5 \times A_7) &= 40 \times 40 \times 40 + 56.000 + 60.000 = 112.000 \\ A_3 \times A_6 \times A_7 \times (A_5 \times A_7) &= 40 \times 25 \times 40 + 112.000 + 20.000 = 172.000 \\ A_3 \times A_6 \times A_5 \times A_6 \times A_7 &= 40 \times 20 \times 40 + 132.000 = 164.000 \end{aligned} \right\} \rightarrow 2 \times 180.000$$

$$\begin{aligned} A_2 \times (A_3 \times A_4 \times A_5 \times A_6) &= 15 \times 40 \times 20 + 132000 = 144000 \\ A_2 \times A_4 \times (A_3 \times A_5 \times A_6) &= 15 \times 35 \times 20 + 21000 + 52500 = 84000 \\ A_4 \times A_3 \times A_5 \times (A_2 \times A_6) &= 15 \times 40 \times 20 + 80000 + 20000 = 142000 \\ A_4 \times A_3 \times A_6 \times A_5 &= 15 \times 25 \times 20 + 121000 = 134500 \end{aligned} \quad \left. \vphantom{\begin{aligned} A_2 \times (A_3 \times A_4 \times A_5 \times A_6) \\ A_2 \times A_4 \times (A_3 \times A_5 \times A_6) \\ A_4 \times A_3 \times A_5 \times (A_2 \times A_6) \\ A_4 \times A_3 \times A_6 \times A_5 \end{aligned}} \right\} \max 144000$$

$$\begin{aligned} A_1 \times (A_2 \times A_3 \times A_4 \times A_5 \times A_6) &= 50 \times 15 \times 20 + 144000 = 159000 \\ (A_1 \times A_2) \times (A_3 \times A_4 \times A_5 \times A_6) &= 50 \times 40 \times 20 + 30000 + 132000 = 202000 \\ (A_1 \times A_2 \times A_3) \times (A_4 \times A_5 \times A_6) &= 50 \times 35 \times 20 + 100000 + 52500 = 187500 \\ (A_1 \times A_2 \times A_3 \times A_4) \times (A_5 \times A_6) &= 50 \times 40 \times 20 + 170000 + 80000 = 230000 \\ (A_1 \times A_2 \times A_3 \times A_4 \times A_5) \times A_6 &= 50 \times 25 \times 20 + 220000 = 245000 \end{aligned} \quad \left. \begin{array}{l} \text{max} \\ 245000 \end{array} \right\}$$

$$\left. \begin{aligned} A_2 \times (A_3 \times A_4 \times A_5 \times A_6 \times A_7) &= 15 \times 40 \times 40 + 180.000 = 204.000 \\ (A_2 \times A_3) \times (A_4 \times A_5 \times A_6 \times A_7) &= 15 \times 35 \times 40 + 210.000 + 116.000 = 158.000 \\ (A_2 \times A_3 \times A_4) \times (A_5 \times A_6 \times A_7) &= 15 \times 40 \times 40 + 100.000 + 60.000 = 164.000 \\ (A_2 \times A_3 \times A_4 \times A_5) \times (A_6 \times A_7) &= 15 \times 25 \times 40 + 127.000 + 20.000 = 162.000 \\ A_2 \times A_3 \times A_4 \times A_5 \times A_6 \times A_7 &= 15 \times 20 \times 40 + 144.000 = 156.000 \end{aligned} \right\} \begin{array}{l} \text{max} \\ 204.000 \end{array}$$

$$\left. \begin{aligned} A_1 \times A_1 \times A_3 \times A_4 \times A_5 \times A_6 \times A_7 &= 50 \times 15 \times 40 + 204000 = 234000 \\ A_1 \times A_3 \times A_3 \times A_4 \times A_5 \times A_6 \times A_7 &= 50 \times 40 \times 40 + 30000 + 180000 = 290000 \\ A_1 \times A_2 \times A_3 \times A_4 \times A_5 \times A_6 \times A_7 &= 50 \times 35 \times 40 + 100000 + 116000 = 286000 \\ A_1 \times A_2 \times A_3 \times A_6 \times A_5 \times A_4 \times A_7 &= 50 \times 40 \times 40 + 100000 + 60000 = 310000 \\ A_1 \times A_2 \times A_4 \times A_3 \times A_5 \times A_6 \times A_7 &= 50 \times 25 \times 40 + 220000 + 20000 = 290000 \\ A_1 \times A_3 \times A_4 \times A_5 \times A_6 \times A_2 \times A_7 &= 50 \times 20 \times 40 + 245000 = 285000 \end{aligned} \right\} \max \quad 310000$$