I: 15, 8 ; II: 6 ; III: 4, 12 ; IV: 14

I. 15. $f(x) = \sqrt{x}$ ar $g(x) = \ln x$

$$\lim_{n\to\infty} \frac{(\sqrt{x})'}{(\ln x)'} = \lim_{n\to\infty} \frac{\frac{1}{2\sqrt{x}}}{\frac{1}{x}} = \lim_{n\to\infty} \frac{(x)'}{(2\sqrt{x})'} = \lim_{n\to\infty} \frac{1}{\frac{1}{\sqrt{x}}} = 1 \lim_{n\to\infty} \sqrt{x} = [\infty]$$

$\sqrt{x} = \Omega(\ln x)$

$\ln x = O(\sqrt{x})$ Ats.: $\sqrt{x}$ kyla greičiau, taigi lėtesnė

I. 8. $f(x) = (2x^3 - 3) / (3x^4 + x^3 - 2x^2 - 1)$ ir $g(x) = \sqrt{x^4} = x^2$

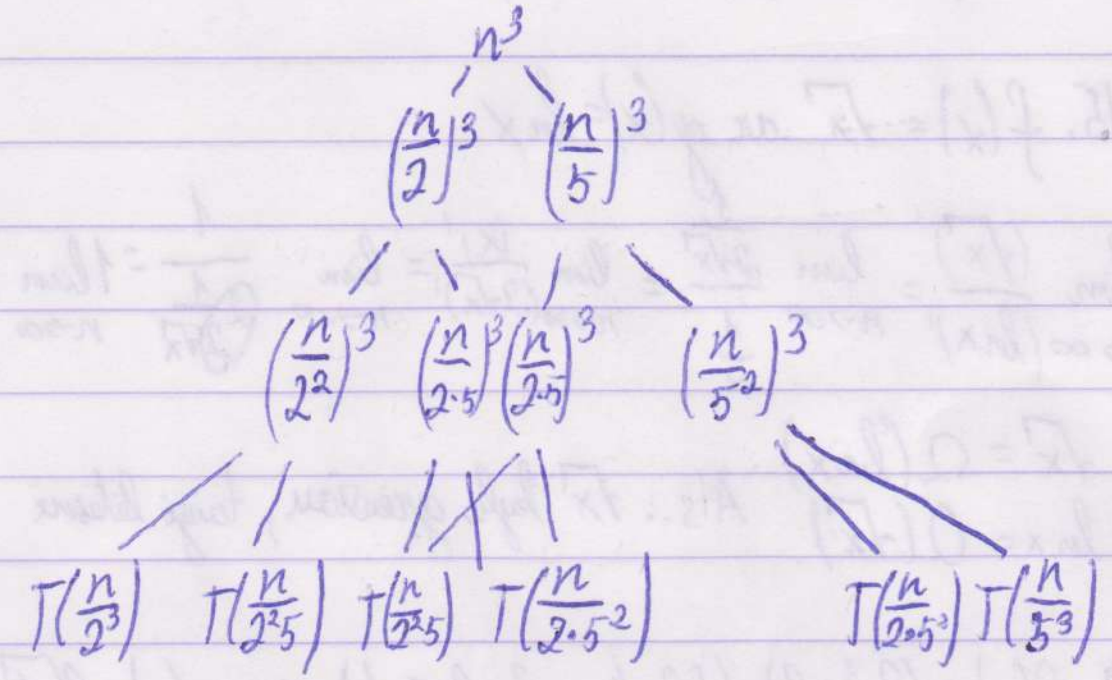
$$\lim_{n\to\infty} \frac{\frac{2x^3-3}{3x^4+x^3-2x^2-1}}{x^2} = \frac{2x^3-3}{3x^4+x^3-2x^2-1} \cdot \frac{1}{x^2} = \frac{2x^3-3}{3x^6+x^3-2x^4-x^2}$$

$$= \lim_{n\to\infty} \frac{\frac{2x^3}{x^6} - \frac{3}{x^6}}{\frac{3x^6}{x^6} + \frac{x^5}{x^6} - \frac{2x^4}{x^6} - \frac{x^2}{x^6}} = \lim_{n\to\infty} \frac{\frac{2}{x^3} - \frac{3}{x^6}}{3 + \frac{1}{x} - \frac{2}{x^2} - \frac{1}{x^4}} = \left[\frac{0}{3}\right] = [0]$$

$x^2 = \Omega(f(x))$

$f(x) = O(x^2)$

II. 6. $T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{3}\right) + n^3$



$\left(\frac{1}{2^3} + \frac{1}{5^3}\right)^i = \left(\frac{133}{1000}\right)^i$ $\quad h(\log_2 n) > h(\log_5 n)$

toigi $h = \log_2 n$

$$T(n) = n^3 \sum_{i=0}^{h} \left(\frac{133}{1000}\right)^i < n^3 \sum_{i=0}^{\infty} \left(\frac{133}{1000}\right)^i = \frac{n^3}{1-\frac{133}{1000}} = \frac{1000}{867} n^2$$

$$T(n) = n^3 \sum_{i=0}^{h} \left(\frac{133}{1000}\right)^i \geq n^3 \sum_{i=0}^{\lfloor \log_5 n \rfloor} \left(\frac{133}{1000}\right)^i = n^3 \frac{\left(\frac{133}{1000}\right)^{\lfloor \log_5 n \rfloor + 1} - 1}{\frac{133}{1000} - 1}$$

$$= \frac{1000}{867} n^3 \left(1 - \left(\frac{133}{1000}\right)^{\lfloor \log_5 n \rfloor + 1}\right)$$

$$1 - \left(\frac{133}{1000}\right)^{\lfloor \log_5 n \rfloor + 1} \geq \frac{867}{1000}, \ \forall n > 0$$

$$T(n) = \Theta(n^3), \ n^3 \leq T(n) \leq \frac{1000}{867} n^3 \quad \forall n > 0$$

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{5}\right) + n^3$$

$$T(n) = O(n \log_2 n)\ ?$$

$$T(n) = O(n \log_2 n) \leq \underbrace{cn \log_2 n}_{cg(n)}, \quad f(n) \leq cn \log_2 n$$

$$\exists c > 0, \quad \forall n \geq n_0$$

$$T\left(\frac{n}{2}\right) = c\,\frac{n}{2} \log_2 \frac{n}{2}$$

$$T\left(\frac{n}{5}\right) = c\,\frac{n}{5} \log_2 \frac{n}{5}$$

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{5}\right) + n^3 \leq c\frac{n}{2} \log_2 \frac{n}{2} + c\,\frac{n}{5} \log_2 \frac{n}{5} + n^3$$

$$\leq \frac{cn}{2}\left(\log_2 n - \log_2 2\right) + \frac{cn}{5}\left(\log_2 n - \log_2 5\right) + n^3$$

$$\leq \underline{\frac{cn}{2} \log_2 n} - \frac{cn}{2} \underset{\downarrow\ 1}{\log_2 2} + \underline{\frac{cn}{5} \log_2 n} - \frac{cn}{5} \log_2 5 + n^3$$

$$\leq \frac{7cn}{10} \log_2 n - \frac{cn}{2} \log_2 2 - \frac{cn}{5} \log_2 5 + n^3 \leq cn \log_2 n$$

$$-\frac{cn}{2} - \frac{cn}{5} \log_2 5 + n^3 \leq \frac{7cn}{10} \log_2 n$$

$$-\frac{cn}{2} - \frac{cn}{5} \log_2 5 - \frac{3cn}{10} \log_2 n \leq -n^3 \ \Big|: -n$$

$$\frac{c}{2} 1 + \frac{c}{5} \log_2 5 + \frac{3c}{10} \log_2 n \geq n^2$$

$$c\left(\frac{1}{2} + \frac{\log_2 5}{5} + \frac{3 \log_2 n}{10}\right) \geq n^2$$

$$c \geq \frac{n^2}{\frac{1}{2} + \frac{\log_2 5}{5} + \frac{3 \log_2 n}{10}};$$

$n = 0$ - sprendinio nėra

$n = 1 = 1{,}037$

$n = 2 = \cancel{3{,}1\ldots}\ 3{,}164$

$n = 3 = 6{,}251$

Sprendinys tenkinamas visiems $n > 0$

III 4. $T(n) = 2^n T(n/2) + n^n$

Neįmanoma išspręst - a - ne konstanta.

12. $T(n) = 3T(n/2) + n \quad a=3 \quad b=2 \quad f(n)=n$

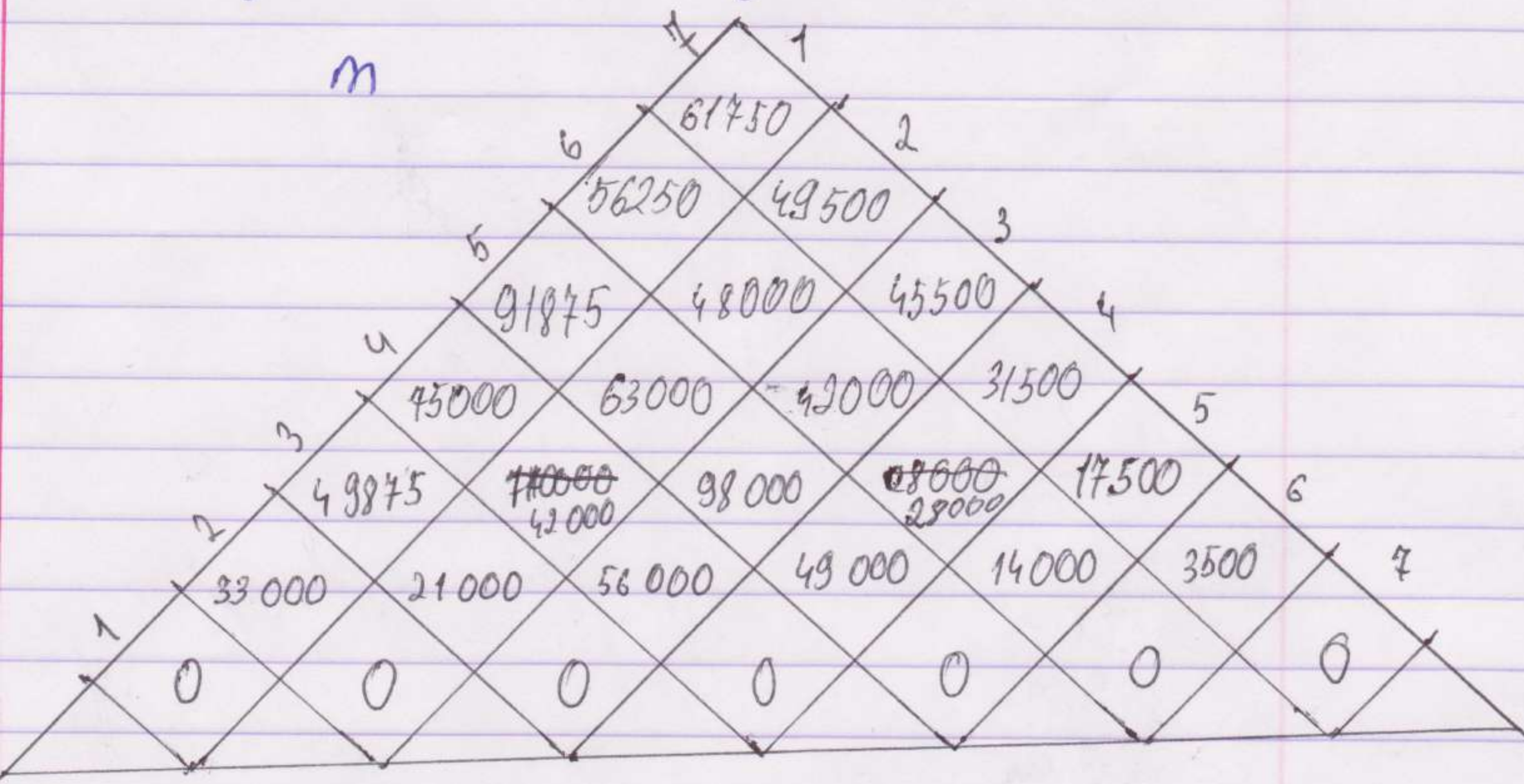
$n^{\log_2 3} > n \quad f(n) = O(n^{\log_2 3 - \varepsilon}) \quad \frac{n}{n^{\log_2 3 - \varepsilon}} = n^{1-(1,6-\varepsilon)} = n^{1-1,6+\varepsilon}$

* $\log_2 3 \approx 1,6$ $\quad \varepsilon = 0,6 \quad \varepsilon > 0$, tai $T(n) = \Theta(n^{\log_2 3})$

IV 14. $A1(55\times15)$; $A2(15\times40)$; $A3(40\times35)$; $A4(35\times40)$ $A5(40\times35)$
$A6(35\times10)$; $A7(10\times10)$

$P = [55, 15, 40, 35, 40, 35, 10, 10] \quad S[i,j] = k$
$\quad P_0 \; P_1 \; P_2 \; P_3 \; P_4 \; P_5 \; P_6 \; P_7$

$m[i,j] = m[i,k] + m[k+1,j] + P_{i-1} P_k P_j$



$m[i,j]$, kai $i = j = 0$
$m[1,1] = 0 \quad m[2,2] = 0 \; \ldots$

$m[i,j]$, kai $j-i=1 \Rightarrow \underbrace{m[i,i]+m[j,j]}_{0}+P_{i-1}P_kP_j$

$m[1,2]=P_0P_1P_2=55\cdot 15\cdot 40=33000$
$m[2,3]=P_1P_2P_3=15\cdot 40\cdot 35=21000$
$m[3,4]=P_2P_3P_4=40\cdot 35\cdot 40=56000$
$m[4,5]=P_3P_4P_5=35\cdot 40\cdot 35=49000$
$m[5,6]=P_4P_5P_6=40\cdot 35\cdot 10=14000$
$m[6,7]=P_5P_6P_7=35\cdot 10\cdot 10=3500$

$m[i,j]$ kai $j-i=2 \Rightarrow \min\big(m[i,i]+m[i+1,j]+P_{i-1}P_kP_j,$
$m[i,i+1]+m[j,j]+P_{i-1}P_kP_j\big)$

$m[1,3]=\underset{0}{m[1,1]}+\underset{21000}{m[2,3]}+P_0P_1P_3=49875$

$m[1,3]=\underset{33000}{m[1,2]}+\underset{0}{m[3,3]}+P_0P_2P_3=77000+33000=110000$

$\min(49875, 110000)=49875$

$m[2,4]=m[3,4]+P_1P_2P_4=\cancel{80000}$
$m[2,4]=m[2,3]+P_1P_3P_4=42000$
$m[3,5]=m[4,5]+P_2P_3P_5=98000$
$m[3,5]=m[3,4]+P_2P_4P_5=\cancel{112000}$
$m[4,6]=m[5,6]+P_3P_4P_6=28000$
$m[4,6]=m[4,5]+P_3P_5P_6=\cancel{61250}$
$m[5,7]=m[6,7]+P_4P_5P_7=17500$
$m[5,7]=m[5,6]+P_4P_6P_7=\cancel{18000}$

$m[1,4] = m[1,1] + m[2,4] + P_0 P_1 P_4 = 75000$

$m[1,4] = m[1,2] + m[2,4] + P_0 P_2 P_4 = \cancel{177000}$

$m[1,4] = m[1,3] + m[4,4] + P_0 P_3 P_4 = \cancel{126875}$

$m[2,5] = m[2,2] + m[3,5] + P_1 P_2 P_5 = \cancel{119000}$

$m[2,5] = m[2,3] + m[4,5] + P_1 P_3 P_5 = \cancel{88375}$

$m[2,5] = m[2,4] + m[5,5] + P_1 P_4 P_5 = 63000$

$m[3,6] = m[3,3] + m[4,6] + P_2 P_3 P_6 = 42000$

$m[3,6] = m[3,4] + m[5,6] + P_2 P_4 P_6 = \cancel{86000}$

$m[3,6] = m[3,5] + m[6,6] + P_2 P_5 P_6 = \cancel{112000}$

$m[4,7] = m[4,4] + m[5,7] + P_3 P_4 P_7 = 31500$

$m[4,7] = m[4,5] + m[6,7] + P_3 P_5 P_7 = \cancel{64750}$

$m[4,7] = m[4,6] + m[7,7] + P_3 P_6 P_7 = 31500$

$m[1,5] = m[1,1] + m[2,5] + P_0 P_1 P_5 = 28875 + 63000 = 91875$

$m[1,5] = m[1,2] + m[3,5] + P_0 P_2 P_5 = \cancel{208000}$

$m[1,5] = m[1,3] + m[4,5] + P_0 P_3 P_5 = \cancel{166250}$

$m[1,5] = m[1,4] + m[5,5] + P_0 P_4 P_5 = \cancel{135200}$

$m[2,6] = m[2,2] + m[3,6] + P_1 P_2 P_6 = 48000$

$m[2,6] = m[2,3] + m[4,6] + P_1 P_3 P_6 = \cancel{54250}$

$m[2,6] = m[2,4] + m[5,6] + P_1 P_4 P_6 = \cancel{62000}$

$m[2,6] = m[2,5] + m[6,6] + P_1 P_5 P_6 = \cancel{68250}$

$m[3,7] = m[3,3] + m[4,7] + P_2 P_3 P_7 = 45500$

$m[3,7] = m[3,4] + m[5,7] + P_2 P_4 P_7 = \cancel{89500}$

$m[3,7] = m[3,5] + m[6,7] + P_2 P_5 P_7 = \cancel{115500}$

$m[3,7] = m[3,6] + m[7,7] + P_2 P_6 P_7 = \cancel{46000}$

$m[1,6] = P_0 P_1 P_6 \ldots = 56250$

$m[1,6] = P_0 P_2 P_6 \ldots = \cancel{71000}\ \cancel{97000}$

$m[1,6] = P_0 P_3 P_6 \ldots = \cancel{97125}$

$m[1,6] = P_0 P_4 P_6 \ldots = \cancel{111000}$

$m[1,6] = P_0 P_5 P_6 \ldots = \cancel{67250}$

$m[2,7] = P_1 P_2 P_7 \ldots = \cancel{51500}$

$m[2,7] = P_1 P_3 P_7 \ldots = \cancel{57750}$

$m[2,7] = P_1 P_4 P_7 \ldots = \cancel{65500}$

$m[2,7] = P_1 P_5 P_7 \ldots = \cancel{71750}$

$m[2,7] = P_1 P_6 P_7 \ldots = 49560$

$m[1,7] = P_0 P_1 P_7 \cdots = \cancel{64500}$

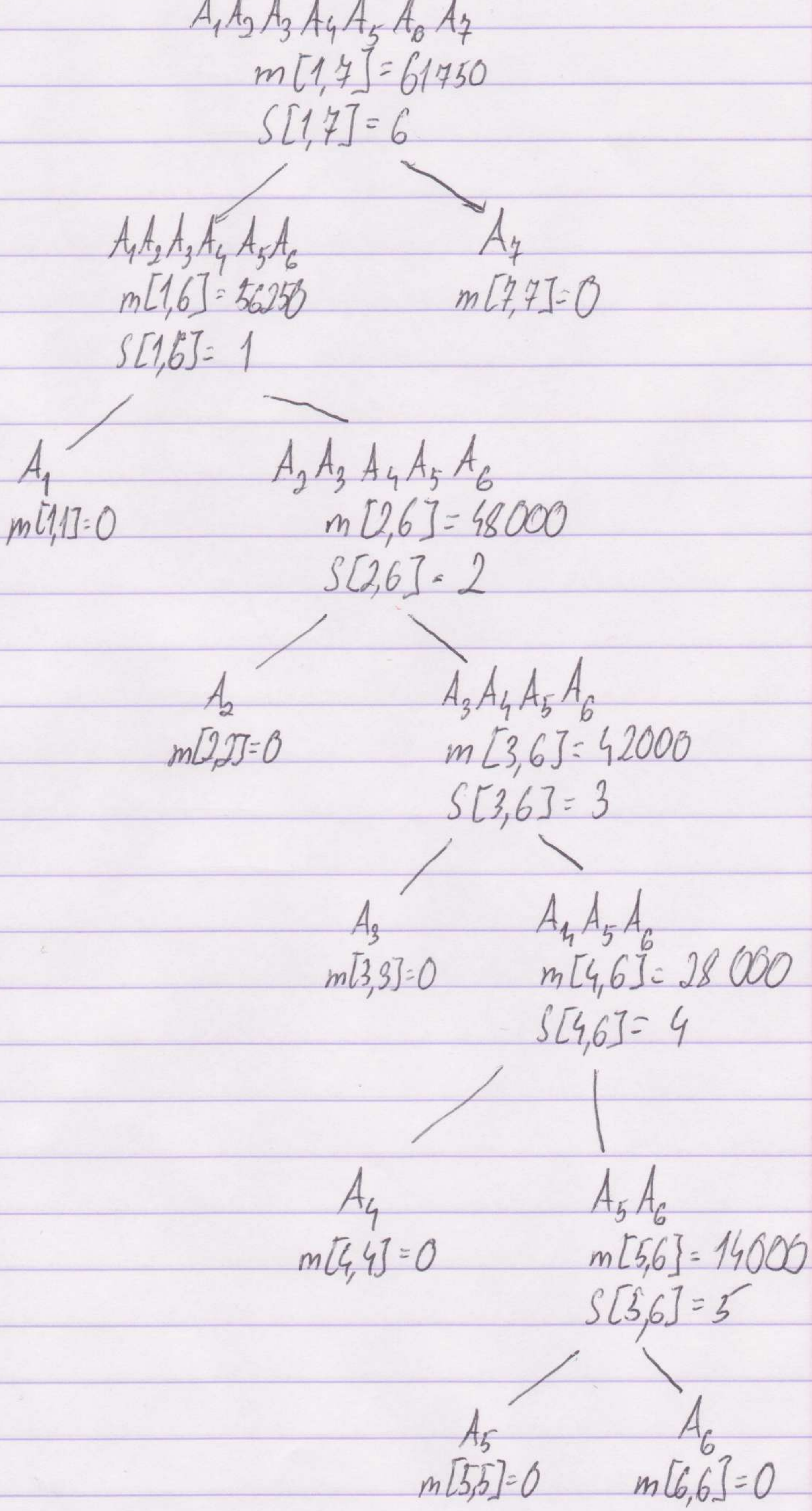
$m[1,7] = P_0 P_2 P_7 \cdots = \cancel{106500}$

$m[1,7] = P_0 P_3 P_7 \cdots = \cancel{100025}$

$m[1,7] = P_0 P_4 P_7 \cdots = \cancel{114500}$

$m[1,7] = P_0 P_5 P_7 \cdots = \cancel{114625}$

$m[1,7] = P_0 P_6 P_7 \cdots = 61750$



$A_1(A_2(A_3(A_4(A_5A_6)))) A_7$