I gr 16 | I gr 2 | II gr 12 | III gr 13 | III gr 19 | IV gr 4 — Arenas Švenčionis C8926

III gr 12 | II gr 13 | II gr 19

I gr 16

$f(x) = \sqrt[2]{x^4} \qquad g(x) = x + e^{3x}$

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{\sqrt[2]{x^4}}{x+e^{3x}} = \lim_{x\to\infty} \frac{x^{\frac{4}{2}}}{x+e^{3x}} = \lim_{x\to\infty} \frac{x^2}{x+e^{3x}} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to\infty} \left(\frac{x^2}{x+e^{3x}}\right)' =$$

$$= \lim_{x\to\infty} \frac{2x}{1+3e^{3x}} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to\infty} \left(\frac{2x}{1+3e^{3x}}\right)' = \lim_{x\to\infty} \frac{2}{9e^{3x}} = \frac{2}{\infty} = 0$$

$x + e^{3x} = \Omega(\sqrt[2]{x^4})$ arba $\sqrt[2]{x^4} = O(x+e^{3x})$

I gr. 2

$f(x) = e^x \qquad g(x) = \frac{(e^x + e^{-x})}{2}$

$$\lim_{x\to\infty} \frac{e^x}{\frac{(e^x+e^{-x})}{2}} = \lim_{x\to\infty} \frac{2e^x}{e^x+e^{-x}} = \lim_{x\to\infty} \frac{2e^x}{e^x+\frac{1}{e^x}} = \lim_{x\to\infty} \frac{2e^x}{e^x+\frac{e^x}{(e^x)^2}} = \lim_{x\to\infty} \frac{2e^x}{e^x(1+\frac{1}{e^x})} =$$

$$= \lim_{x\to\infty} \frac{2}{1+\frac{1}{e^x}} = \frac{2}{1+0} = 2$$

$e^x = \Theta\left(\frac{e^x+e^{-x}}{2}\right)$

II gr 12

$T(n) = 3T(\sqrt[4]{n}) + n\log_2 n$

$n = 2^m;\ m = \log_2 n$

$T(2^m) = 3T(\sqrt[4]{2^m}) + 2^m \log_2 2^m$

$T(2^m) = 3T(2^{\frac{m}{4}}) + 2^m m$

$S(m) = T(2^m)$

$S(m) = \cancel{3} 3S(\frac{m}{4}) + 2^m m$

$a = 3 \quad b = 4 \quad f(m) = 2^m m$

$m^{\log_4 3} < 2^m m \qquad \cancel{f(m) = \Omega(m^{\log_4 3} \log_2 m)}$

$f(m) = \Omega(m^{\log_4 3 + \varepsilon})$

$\cancel{\frac{2^m m}{m^{\log_4 3+\varepsilon}}} \qquad a(f(\frac{n}{b})) \leq cf(n) \qquad \cancel{3(\frac{2^m m}{4}) \leq c2^m m}$

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$a\,(f(\frac{n}{b})) \le c\,f(n)$

$3(2^{\frac{m}{4}} \cdot \frac{m}{4}) \le c\, 2^m m$

$\frac{3}{4} \cdot 2^{\frac{m}{4}} \cdot \cancel{m} \le c\, 2^m \cancel{m}$

$\frac{3}{4} \le c\, \cancel{2^{\frac{3}{4}}}\, 2^{\frac{3}{4}m} \quad m=4 \quad c \ge \frac{3}{32}$

ats.: $S(m)=\Theta(2^m m) \Rightarrow T(n)=\Theta(2^{\log_2 n}\log_2 n)=\Theta(n\log_2 n)$

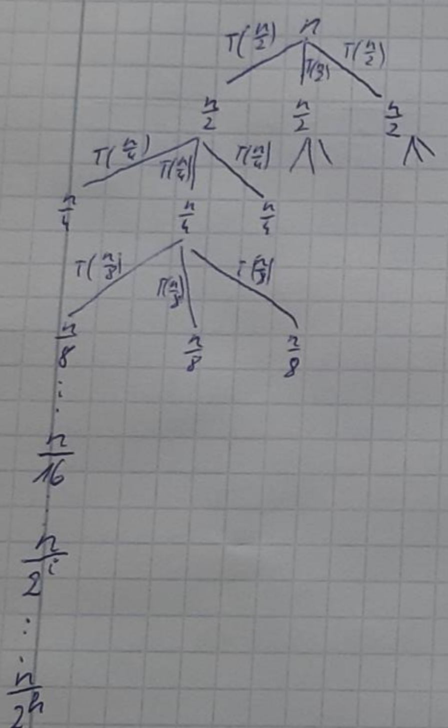
III gr 13

$T(n)=3T(\frac{n}{2})+n$

$T(\frac{n}{2})=3T(\frac{n}{4})+\frac{n}{2}$

$T(\frac{n}{4})=3T(\frac{n}{8})+\frac{n}{4}$

$T(\frac{n}{8})=3T(\frac{n}{16})+\frac{n}{8}$



$\frac{n}{2^h}=1 \quad n=2^h$

$h=\log_2 n$

$$\left.\begin{aligned} 3\left(\frac{n}{2}\right) &= \frac{3n}{2} \\ 9\left(\frac{n}{4}\right) &= \frac{3^2 n}{2^2} \\ 27\left(\frac{n}{8}\right) &= \frac{3^3 n}{2^3} \\ &\vdots \end{aligned}\right\} \frac{3^i n}{2^i} \quad q=\frac{3}{2}$$

$$T(n)=3T\left(\frac{n}{2}\right)+n=\frac{3n}{2}+\frac{3^2 n}{2^2}+\frac{3^3 n}{2^3}+\dots+n=$$

$$=\sum_{i=1}^{h}\frac{3^i n}{2^i}+n=\sum_{i=1}^{\log_2 n}\frac{3^i n}{2}+n=\left[S=\sum_{i=1}^{n} q_i=\frac{1-q^i}{1-q}\right]=$$

$$=n\left(\frac{1-\left(\frac{3}{2}\right)^{\log_2 n}}{1-\frac{3}{2}}\right)+n=-2n\left(1-\left(\frac{3}{2}\right)^{\log_2 n}\right)+n=$$

$$=-2n\left(1-n^{\log_2 \frac{3}{2}}\right)+n=-2n\left(1-n^{\log_2 3-1}\right)+n=$$

$$=-2n\left(1-\frac{n^{\log_2 3}}{n^1}\right)+n=O(n)$$

~~III gr 19~~ ~~$T(n) = 3T(\frac{n}{2}) + n$~~

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II gr 19

~~$T(n) = 2\sqrt{}$~~

$T(n) = 2T\left(\frac{\sqrt{n^2}}{3}\right) + n \log_2 n$

$n = 2^m$; $m = \log_2 n$

$T(2^m) = 2T\left(\frac{\sqrt{2^{2m}}}{3}\right) + 2^m \log_2 2^m$

$T(2^m) = 2T\left(\frac{2^{\frac{2m}{2}}}{3}\right) + 2^m m$

$T(2^m) = 2T\left(\frac{2^m}{3}\right) + 2^m m$

$S(m) = T(2^m)$

$S(m) = 2T\left(\frac{m}{3}\right) + 2^m m$

$a = 2 \quad b = 3 \quad f(m) = 2^m m$

$m^{\log_3 2} < 2^m m$

$f(m) = \Omega\left(m^{\log_3 2 + \varepsilon}\right)$

$a\left(f\left(\frac{n}{b}\right)\right) \leq c f(n)$

$2\left(2^{\frac{m}{3}} \cdot \frac{m}{3}\right) \leq c\, 2^m m$

$\frac{2}{3} \cdot 2^{\frac{m}{3}} \cdot \cancel{m} \leq c\, 2^m \cancel{m}$

$\frac{2}{3} \leq c\, 2^{\frac{2m}{3}} \qquad m = 3 \quad c \geq \frac{2}{12} \qquad (m = \log_2 n)$

Ats.: $S(m) = \Theta(2^m m) \Rightarrow T(n) \in (2^{\log_2 n} \log_2 n) = \Theta(n \log_2 n)$

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IV gr 4

AAA kaina	AAA kiekis	BB kaina	BB kiekis
$T_{BB}(n,m)$	1	c_1	1
c_1	1	c_1	$n-m+1$
$T_{AAA}(m, m+p)$	1	c_1	$n-m$
$T_{AAA}(m+p, m+2p)$	1	c_1	1
c_1	$n-m+2$		
c_1	$n-m+1$		
$T_{AAA}(n-p, n)$	1		

$T_{BB}(n,m) = 2c_1 + (n-m+1)c_1 + (n-m)c_1$

$T_{AAA}(m,n) = T_{BB}(n-m) + c_1(n-m+2) + c_1(n-m+1) + c_1 + T_{AAA}(m, m+p) + T_{AAA}(m+p, m+2p) +$
$+ T_{AAA}(n-p, n)$

$k = n-m \qquad T(m,n) = T(k)$

$T(k) = 3c_1 + 2c_1(k+1) + c_1(k+2) + c_1 k + T(m+p-m) + T((m+2p)-(m+p)) +$
$+ T(n-(n+p)) = 3c_1 + 2c_1(k+1) + c_1(k+2) + c_1 k + T(p) + T(p) + T(p) =$
$= 3T(p) + 3c_1 + 2c_1(k+1) + c_1(k+2) + c_1 k = 3T(p) + 3c_1 + 2c_1 k + 2c_1 +$
$+ c_1 k + 2c_1 + c_1 k = 3T(p) + 4c_1 k + 7c_1 \Rightarrow p = \frac{k}{2}$

$\Rightarrow 3T(\frac{k}{2}) + 4c_1 k + 7c_1$

$a = 3 \quad b = 2 \quad f(k) = 4c_1 k + 7c_1$

$k^{\log_2 3} \boxed{>} 4c_1 k + 7c_1$

$f(n) = O(n^{\log_2 3 - \varepsilon})$

$\frac{4c_1 k + 7c_1}{k^{\log_2 3 - \varepsilon}} \qquad n^{1-\log_2 3+\varepsilon} \qquad \varepsilon = \log_2 3 - 1 \quad \varepsilon > 0$

$T(k) = \Theta(k^{\log_2 3}) \qquad k = n-m$

$T(n-m) = \Theta((n-m)^{\log_2 3})$