# 第12周作业

## 1、

### 7.1

设状态集$S=\{0,1,2,3\}$, 事件$X_t=k$表示$t$时刻距离此前(含$t$时刻)最近一次到达的距离

$$P_{00}=0.2$$
$$P_{01}=0.8$$
$$P_{12}=1$$
$$P_{20}=P(k=3|k\geq 3)=\frac{0.3}{0.3+0.5}=\frac{5}{8}$$
$$P_{23}=1-\frac{5}{8}=\frac{3}{8}$$
$$P_{30}=1$$

### 7.2

不是，假设$m>1$, 当老鼠在瓷砖1处时，$X_N=L$，$P(X_{N+1}=L|X_N=L)=1$；而当瓷砖在$m$处时，$P(X_{N+1}=L|X_N=L)=0.5$

### 7.3

不是，当苍蝇处于状态2时，$P(Y_{N+1}=2|Y_N=1)=0.3$；当苍蝇处于状态1时，$P(Y_{N+1}=2|Y_N=1)=0$

### 7.4

(a)

$$S=\{0,1,2,3,\cdots\}$$
$$X_t\text{表示当时间为}t\text{时，蜘蛛与苍蝇间的距离}$$
$$P_{00}=1,P_{10}=0.7,P_{11}=0.3$$
$$P_{ii}=0.3,P_{i(i-1)}=0.4,P_{i(i-2)}=0.3$$

(b)

$$P_{00}^{(n)}=1$$
$$P_{ii}^{(n)}=0.3^n$$
$$\sum_{n=0}^{+\infty}P_{00}^{(n)}=+\infty$$

$$\sum_{n=0}^{+\infty} P_{ii}^{(n)} = \frac{10}{7}\text{有限}$$

所以0状态是常返的，其他状态是非常返的

**2、**

$$\text{是}Markov\text{链}$$
$$P_{01} = 1$$
$$P_{10} = \frac{1}{9}, P_{11} = \frac{4}{9}, P_{12} = \frac{4}{9}$$
$$P_{21} = \frac{4}{9}, P_{22} = \frac{4}{9}, P_{23} = \frac{1}{9}$$
$$P_{32} = 1$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{1}{9} & \frac{4}{9} & \frac{4}{9} & 0 \\ 0 & \frac{4}{9} & \frac{4}{9} & \frac{1}{9} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

**3、**

$$\text{记}r\text{表示下雨，}s\text{表示晴天}$$
$$S = \{rrr, rrs, rsr, rss, srr, srs, ssr, sss\}$$
$$P(rrr \to rrs) = 0.2, P(rrr \to rrr) = 0.8$$
$$P(rrs \to rsr) = 0.4, P(rrs \to rss) = 0.6$$
$$\cdots$$
$$P(sss \to sss) = 0.8, P(sss, ssr) = 0.2$$

**4、**

$$X_2 = \begin{pmatrix} 0.25 & 0.25 & 0.5 \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$$
$$X_2 = (\frac{3}{8} \;\; \frac{1}{6} \;\; \frac{11}{24})$$

$$X_3 = \begin{pmatrix} \frac{3}{8} & \frac{1}{6} & \frac{11}{24} \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$$
$$X_3 = (0.417 \;\; 0.181 \;\; 0.402)$$

$$E(X_3) = X_3 \cdot (0\ \ 1\ \ 2) = 0.985$$

**5、**

$$\begin{gathered}\text{由状态}i\text{可达到状态}j\text{，可知存在中间状态}a_1, a_2, \cdots, a_n (\forall a_n \neq i, j) \\ \text{使得状态转移} i \to a_1 \to a_2 \to \cdots \to a_n \to j \\ \text{对}1 \leq k < l \leq n, \text{若}a_k = a_l, \text{则删去}a_{k+1}, a_{k+2}, \cdots, a_l\text{不会对可达性产生影响} \\ \text{所以}a_n\text{各不相同，且不等于}i, j\text{，因此状态}i\text{可以在}m\text{步内到达状态}j.\end{gathered}$$

**6、**

$$\begin{gathered}\text{反证：假设}\exists k, P_{ij}^{(k)} = \varepsilon > 0, \\ i\text{常返} \iff \sum_{i=0}^{+\infty} P_{ii}^{(n)} = +\infty\end{gathered}$$

$$\begin{aligned}&\text{当}1 \leq n \leq k-1, P_{ii} \leq 1 \\ &\text{当}k \leq n \leq 2k-1, P_{ii} \leq 1-\varepsilon (\text{因为经过}k\text{步有}\varepsilon\text{概率转移到}j, \text{但}j\text{无法回到}i) \\ &\text{当}2k \leq n \leq 3k-1, P_{ii} \leq (1-\varepsilon)^2 \\ &\cdots \\ &\text{对由}k\text{个概率组成的每一组，}\sum_{i=0}^{+\infty} P_{ii}^{(n)} \leq k(1 + (1-\varepsilon + (1-\varepsilon)^2) + \cdots) = \frac{k}{\varepsilon}\end{aligned}$$

**7、**

(1)

$$\begin{gathered}P_{00} = \frac{2}{9}, P_{01} = \frac{7}{9} \\ P_{10} = \frac{1}{2}, P_{11} = \frac{1}{2} \\ \text{转移矩阵}M = \begin{bmatrix}\frac{2}{9} & \frac{7}{9} \\ \frac{1}{2} & \frac{1}{2}\end{bmatrix}\end{gathered}$$

(2)

$$\begin{gathered}\text{当前状态}X_0 = [1\ \ 0] \\ X_1 = X_0 M = [\frac{2}{9}\ \ \frac{7}{9}] \\ X_2 = X_1 M, X3 = X_2 M \approx [0.378\ \ 0.622]\end{gathered}$$

**8、**

(1)

$$
\begin{gathered}
p=0.25\text{时，（位置，时间）}=(0.0,106) \\
p=0.5\text{时，（位置，时间）}=(0.0,2658) \\
p=0.75\text{时，（位置，时间）}=(100.0,100)
\end{gathered}
$$

(2)

| p | 0.25 | 0.5 | 0.75 |
|---|---|---|---|
| time_avg | 99.834 | 2473.896 | 99.826 |
| zero_proportion | 1.0 | 0.482 | 0.0 |

```
import numpy as np

def random_walk(N=100, p=0.5):
    pos = N / 2
    cnt = 0
    while(1):
        cnt = cnt + 1
        rand = np.random.uniform(0, 1)
        if rand < p:
            pos = pos + 1
        else:
            pos = pos - 1
        if pos == 0 or pos == N:
            return (cnt, pos)

print(random_walk(p=0.25))
print(random_walk(p=0.5))
print(random_walk(p=0.75))

result = [[], [], []]
for i in range(1000):
    result[0].append(random_walk(p=0.25))
    result[1].append(random_walk(p=0.5))
    result[2].append(random_walk(p=0.75))

time_avg = [0, 0, 0]
zeros = [0, 0, 0]
for i in range(3):
    time_avg[i] = sum(a for a, _ in result[i]) / len(result[i])
    zeros[i] = sum(1 for _, b in result[i] if b == 0) / 1000

print(f'time_avg={time_avg}')
print(f'zeros={zeros}')
```