eRm package (Reinhold Hatzinger)

Salvador Garcia

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Contents

1 Introduction

The eRM package uses Conditional Marginal Likelihood as the method of estimation.

IRT tries to find a function that maps a latent variable (sometimes expressed as the ability, attitude or trait of the subject) to the probability of a reaction given this latent variable.

2 The Rasch model (RM)

2.1 The Rasch model

$$P(X_{vi} = 1|\theta_v, \beta_i) = \frac{\exp(\theta_v - \beta_i)}{1 + \exp(\theta_v - \beta_i)} \tag{1}$$

- X is the matrix that the rows represent the persons and the columns the item of the quiz.
- $X_{vi} \in \{0, 1\}$ is the entry (v, i) of the matrix X. This entry is a binary variable that represents the answer to the item i of the person v. 0 means that the person answers incorrectly and 1 that answers correctly.
- $\theta_v \in (-\infty, \infty)$ is the ability or trait of the person v.
- $\beta_i \in [0, \infty)$ is the difficulty of the item i.

It is useful to define two *Raw Scores*. These scores are simple marginalization of X_{vi} with respect to the variables v and i:

- **Raw score per person:** $r_v = \sum_i X_{vi}$
- **Raw score per item:** $s_i = \sum_v X_{vi}$

2.2 Assumptions of the Rasch Model

- **Unidimensionality**
- **Sufficiency**
- **Conditional independence**
- **Monotonicity**

2.3 Item Parameter Estimation

2.3.1 Item parameter estimation

Different methods of estimation for **item parameter estimation**:

Joint Maximum likelihood Estimation (JML) (look for demonstration)

$$L_u = \frac{\exp(\sum_v \theta_v r_v) \exp(\sum_i \beta_i s_i)}{\prod_v \prod_i (1 + \exp(\theta_v - \beta_i)} \tag{2}$$

In the last expression we can find that the sufficient statistics are $r_v = \sum_i X_{vi}$ for θ_v and $s_i = \sum_v X_{vi}$ for β_i.

Problem: inconsistency of the parameters when $n \to \infty$

Marginal Maximum likelihood Estimation (MML)

If we integrate the person parameter, we can marginalize the expression:

$$L_m = \prod_r [\exp(-\sum_i \beta_i s_i)] continue \tag{3}$$

Conditional Maximum likelihood Estimation (CML)

Condition on r_v

$$L_c = \exp() \tag{4}$$

Advantages of CML and MML

2.3.2 Person parameter estimation

Different methods of estimation for **person parameter estimation**:

Maximum Likelihood (ML) and Weighted Maximum Likelihood (WML)

Bayes approach.

3 The ICC Curve

A The eRm package

This is an appendix

LPCM PCM LRSM RSM LLTM