Linnaeus university

Linear algebra for engineers: Computer assignment

Contributors:

Earmyas Measho Gebre
Melat Haile
Philomina Ejegi Ede

Course code <1MA133>
Supervisor < Stefan Adolfsson>
Discipline < Mathematics>
Semester <fall 2021>

1. Adaptation of a circle to data

a. Set up the overdetermined system of equations Ac = b to be solved to find c = (c1, c2, c3) T . Note! You should not try to solve the system, just write it down.

```
% Question [i]
a=[-4 3 1 ;0 2 1;1 12 1;5 6 1];
 A=a'*a;
 d=[25 4 145 61];
 b=a'*d';
 inv(A)*b;
```

b. Construct the matrices A and b (as you determined above) in Matlab. Matrices are made using square brackets.

```
% Question  [ii]
c=inv(A)*b;
```

c. Solve the indefinite system of equations Ac = b with Matlab's command \ to get c.

```
% Question [iii]
 c=A\b;
```

d. From c = (c1, c2, c3) T you can find the radius r and the center of the circle (p, q). Note, use Matlab's notation to extract individual elements from matrices. That is, do not write of the values in decimal form.

```
% Question [iv]
 c1=c(1);
 round(c1)
 c2=c(2)
  round(c2)
 c3=c(3)
  round(c3)
 p=c1/2
  round(p)
 q=c2/2
  round(q)
 r=sqrt(c3+p^2+q^2)
 round(r)
```

e. Create a vector v with 101 elements starting at 0 and rising up to 2π (i.e. with steps 2π/100). You should now draw the circle (x, y) = (c1, c2)+(r cos(v), r sin(v)). Useful command is plot.

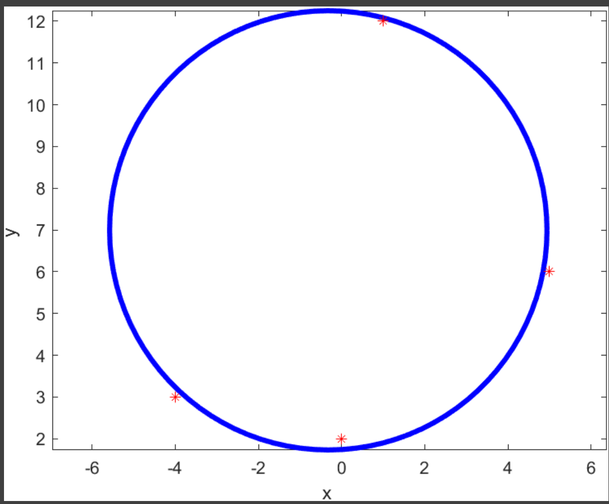
```
% Question (v)
v=(0:2*pi/100:2*pi);
% (x, y) = (p, q)+(r cos(v), r sin(v))
x=p+r*cos(v);
y=q+r*sin(v);
plot(x,y,'b','LineWidth',3.0)
```

f.

```
% Question vi
hold on
plot(-4,3,'r*',0,2,'r*',1,12,'r*',5,6,'r*','LineWidth',0.002)
```

g.

```
% Question vii
ylabel ('y ')
xlabel ('x ')
axis equal
```



h.

1.

$$\left.\begin{aligned} -4C_1 + 3C_2 + C_3 &= 25 \\ 0C_1 + 2C_2 + C_3 &= 4 \\ 1C_1 + 12C_2 + C_3 &= 145 \\ 5C_1 + 6C_2 + C_3 &= 61 \end{aligned}\right\}$$

$$\underbrace{\begin{pmatrix} -4 & 3 & 1 \\ 0 & 2 & 1 \\ 1 & 12 & 1 \\ 5 & 6 & 1 \end{pmatrix}}_{A} \begin{pmatrix} C_1 \\ C_2 \\ C_3 \end{pmatrix} = \underbrace{\begin{pmatrix} 25 \\ 4 \\ 145 \\ 61 \end{pmatrix}}_{b}$$

$$A^T \cdot A = \underset{3\times 4}{\begin{pmatrix} -4 & 0 & 1 & 5 \\ 3 & 2 & 12 & 6 \\ 1 & 1 & 1 & 1 \end{pmatrix}} \underset{4\times 3}{\begin{pmatrix} -4 & 3 & 1 \\ 0 & 2 & 1 \\ 1 & 12 & 1 \\ 5 & 6 & 1 \end{pmatrix}} = \begin{pmatrix} 42 & 30 & 2 \\ 30 & 193 & 23 \\ 2 & 23 & 4 \end{pmatrix}$$

$$A^T \cdot b = \begin{pmatrix} -4 & 0 & 1 & 5 \\ 3 & 2 & 12 & 6 \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 25 \\ 4 \\ 145 \\ 61 \end{pmatrix} = \begin{pmatrix} 350 \\ 2189 \\ 235 \end{pmatrix}$$

$$(A^T \cdot A)^{-1} = \left(\begin{array}{ccc|ccc} 42 & 30 & 2 & 1 & 0 & 0 \\ 30 & 193 & 23 & 0 & 1 & 0 \\ 2 & 23 & 4 & 0 & 0 & 1 \end{array}\right) \; R_1/42$$

$$= \left(\begin{array}{ccc|ccc} 1 & 5/7 & 1/21 & 1/42 & 0 & 0 \\ 30 & 193 & 23 & 0 & 1 & 0 \\ 2 & 23 & 4 & 0 & 0 & 1 \end{array}\right) = \left(\begin{array}{ccc|ccc} 1 & 5/7 & 1/21 & 1/42 & 0 & 0 \\ 0 & 1201/7 & 151/7 & -5/7 & 1 & 0 \\ 0 & 151/7 & 82/21 & -1/21 & 0 & 1 \end{array}\right) \; \frac{R_2}{\frac{1201}{7}}$$

$$= \left(\begin{array}{ccc|ccc} 1 & 5/7 & 1/21 & 1/42 & 0 & 0 \\ 0 & 1 & 151/1201 & -5/1201 & 7/1201 & 0 \\ 0 & 151/7 & 82/21 & -1/21 & 0 & 1 \end{array}\right) = \left(\begin{array}{ccc|ccc} 1 & 0 & \frac{-152}{3603} & \frac{193}{7206} & \frac{-5}{1201} & 0 \\ 0 & 1 & \frac{151}{1201} & \frac{-5}{1201} & \frac{7}{1201} & 0 \\ 0 & 0 & \frac{4297}{3603} & \frac{152}{3603} & \frac{-151}{1201} & 1 \end{array}\right)$$

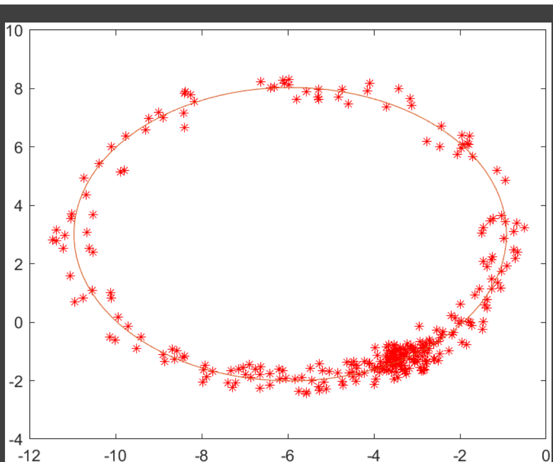
$$= \left(\begin{array}{ccc|ccc} 1 & 0 & \frac{-152}{3603} & \frac{193}{7206} & \frac{-5}{1201} & 0 \\ 0 & 1 & \frac{151}{1201} & \frac{-5}{1201} & \frac{7}{1201} & 0 \\ 0 & 0 & 1 & \frac{152}{4297} & \frac{-453}{4297} & \frac{3603}{4297} \end{array}\right) \; \frac{R_3}{4297/3603} = \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{243}{8594} & \frac{-37}{4297} & \frac{152}{4297} \\ 0 & 1 & 0 & \frac{-37}{4297} & \frac{82}{4297} & \frac{-453}{4297} \\ 0 & 0 & 1 & \frac{152}{4297} & \frac{-453}{4297} & \frac{3603}{4292} \end{array}\right)$$

2. Adapting a circle to a larger amount of data

a.

```
load("cirkel300.mat")
plot(X,Y,'R*')
b=X.^2+Y.^2
f=ones(300)
f=f(:,1)
A =[X Y f]
c=inv(A'*A)*A'*b
%plot(P,Q)
hold on
c1=c(1);
round(c1)
c2=c(2)
round(c2)
c3=c(3)
round(c3)
p=c1/2
round(p)
q=c2/2
round(q)
r=sqrt(c3+p^2+q^2)
round(r)
alpha =linspace(0,2*pi,100)
l=p+r*cos(alpha)
k=q+r*sin(alpha)
plot(l,k)
```

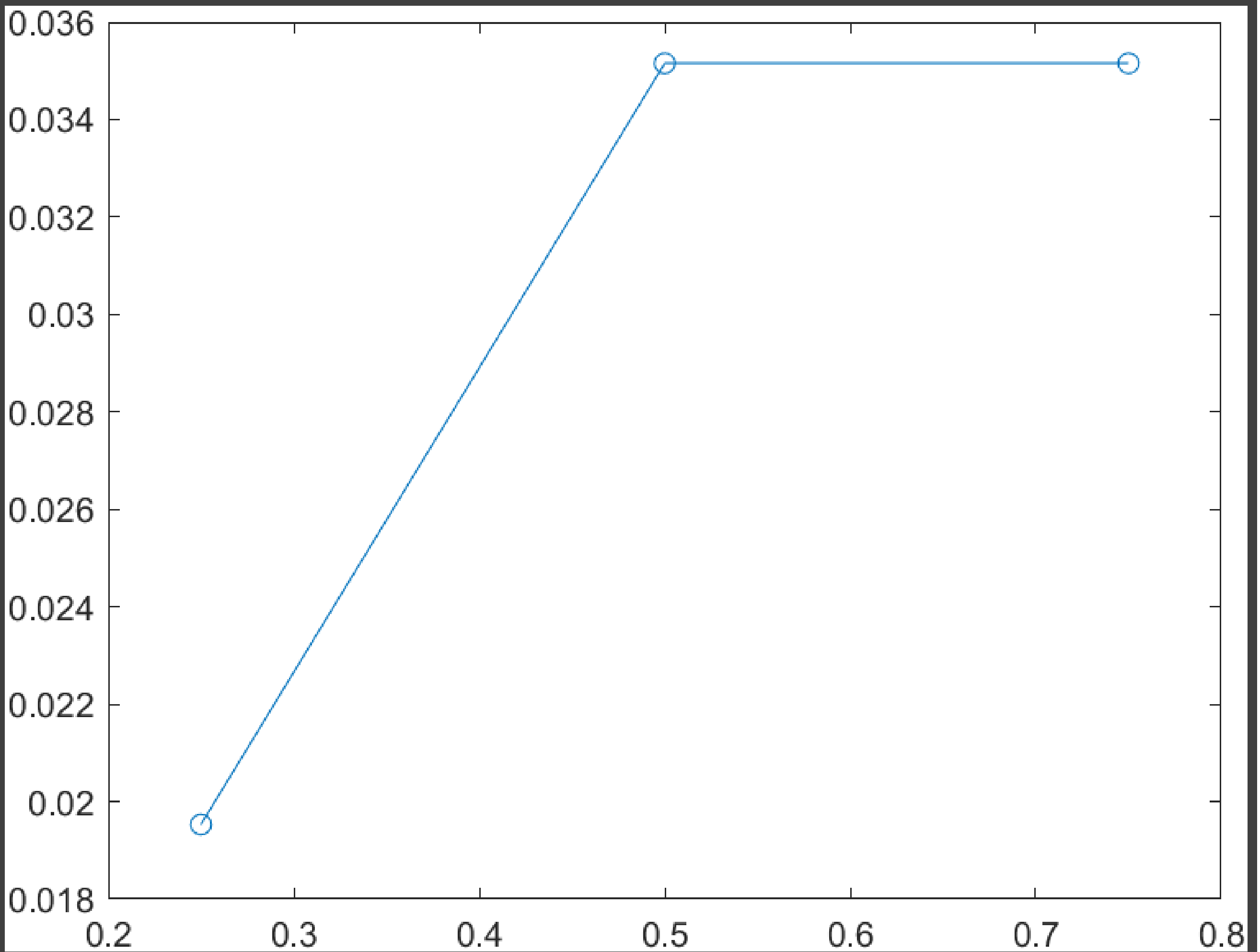
b.



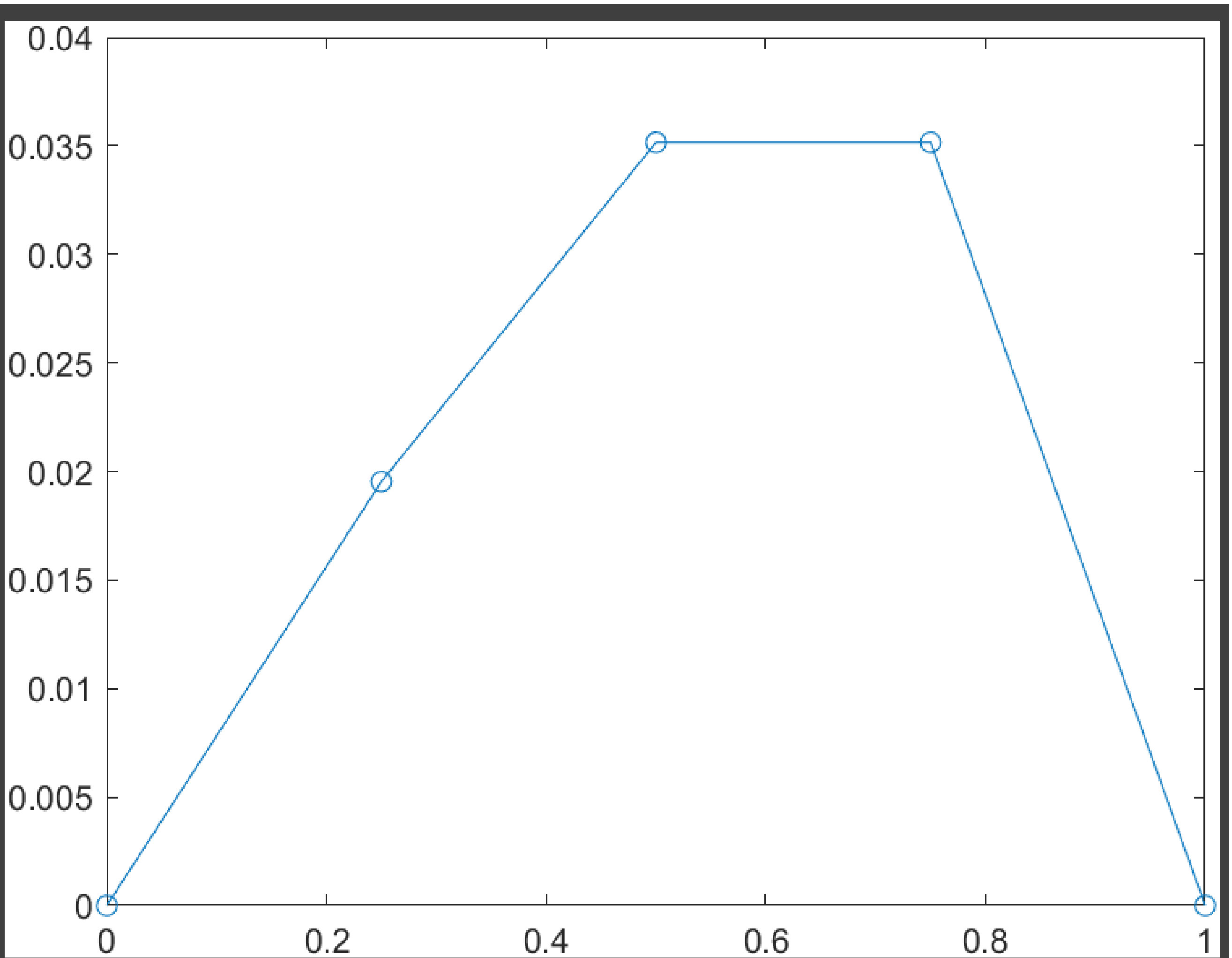
3. Solve systems of any size.

a.

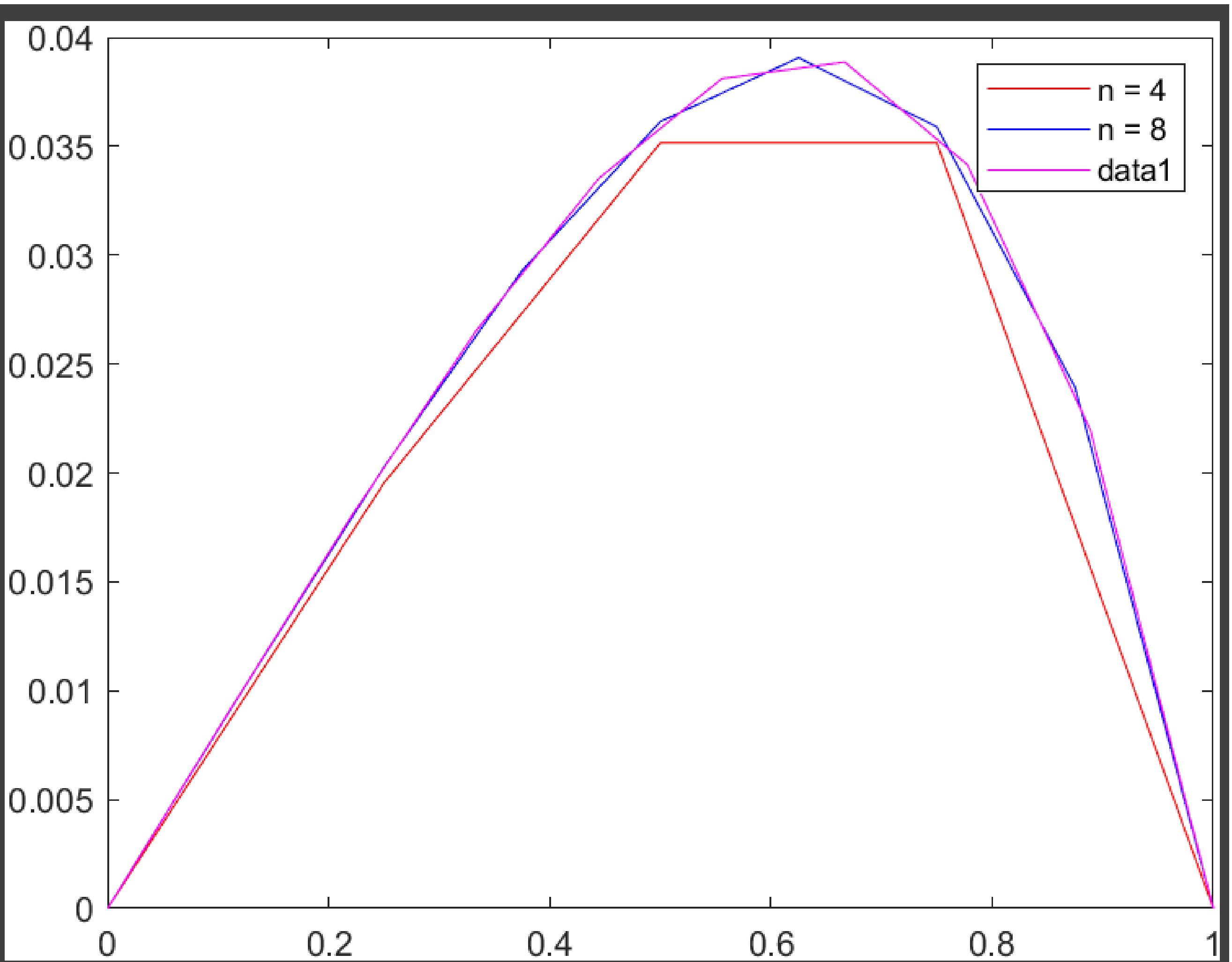
```
A = 16*[2 -1 0;-1 2 -1;0 -1 2];
x = (1/4)*[1;2;3]
f = (1/16)*[1;4;9];

y = mldivide(A,f)

plot(x,y,'-o')
```




```
y_ = [0;y;0];
x_ = [0;x;1];
plot(x_,y_, '-o')
```




```
anySizeSystem(4, 'r');
hold on
anySizeSystem(8, 'b');
legend('n = 4', 'n = 8');
n = input('Provide an integer n: ')
anySizeSystem(n,'m');
hold off
```




```
function anySizeSystem(n,s)

   % Create matrix A
   v = 2.* ones(length(n),1);
   v1 = 2.* ones(length(n),1);
   for i = 1:n-1
      v(i) = 2;
      v1(i) = -1;
   end
   v1(end)=[];
   A = (n^2)*(diag(v1,-1) + diag(v,0) + diag(v1,1));

   % Find x and f
   x = (1/n)*((1:n-1).');
   f = (1/n^2)*(((1:n-1).').^2);

   % Find y
   y = mldivide(A,f);

   % Find y_ and x_
   x_ = [0;x;1];
   y_ = [0;y;0];

   % Plot Graph
   plot(x_,y_, s)
end
```

4. Draw a plane in 3D.

a. u

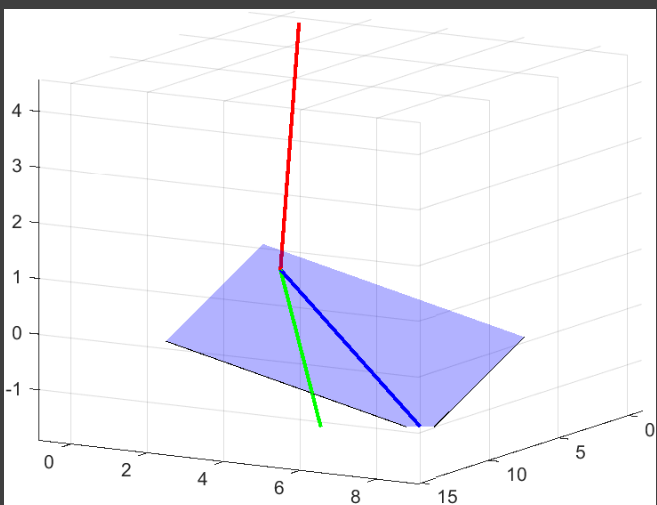
```
u=[3,3,-1]
v=[2,4,-1]
n=cross(u,v)
—-------------------------
w =
   1    1    6
```

b.

```
from the equation of a plane a(x-xo)+b(y-yo)+c(z-zo)+d=0  ⇒
w = (a,b,c)=(1,1,6)
origin=(0,0,0)
1(x-0)+1(y-0)+6(z-0)=0  ⇒ the equation of the plane is  x+y+6z=0
since the point p1, p2,p3,p4  belongs to the plane. they must make the equation true.
p1=(6,6,z1) =  x+y+6z=0  ⇒  6+6+6(z1)=0  ⇒ z1 = -2
p2=(6,-6,z2)=x+y+6z=0  ⇒    6-6+6(z2)=0  ⇒  z2 = 0
p3=(-6,-6,z3) =x+y+6z=0  ⇒   -6-6+6(z3)=0  ⇒ z3 = 2
p4=(-6,6,z4) =x+y+6z=0  ⇒   -6+6+6(z4)=0   ⇒ z4 = 0
 the coordinates are p1=(6,6,-2) , p2 =(6,-6,0),  p3=(-6,-6,2) , p4=(-6,6,0)
```

c.

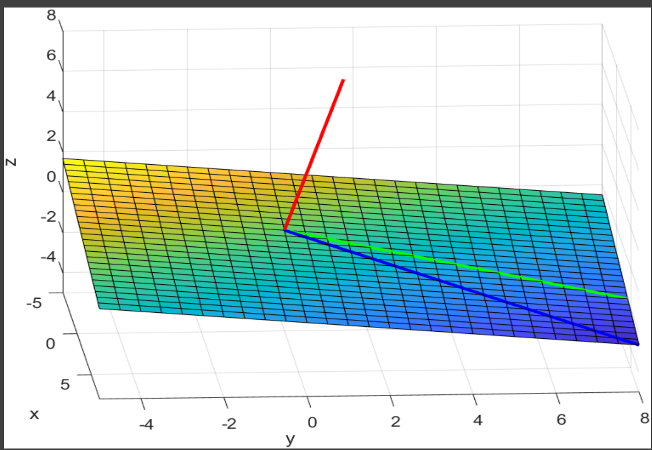
```
p=[6 6 -2;6 -6 0;-6 -6 2;-6 6 0]
x=p(:,1)
y=p(:,2)
z=p(:,3)
fill3(x,y,z,'b','facealpha',0.4)
hold on
plot3(v1x,v1y,v1z,'b','linew',2)
hold on
plot3(v2x,v2y,v2z,'g','linew',2)
box
```



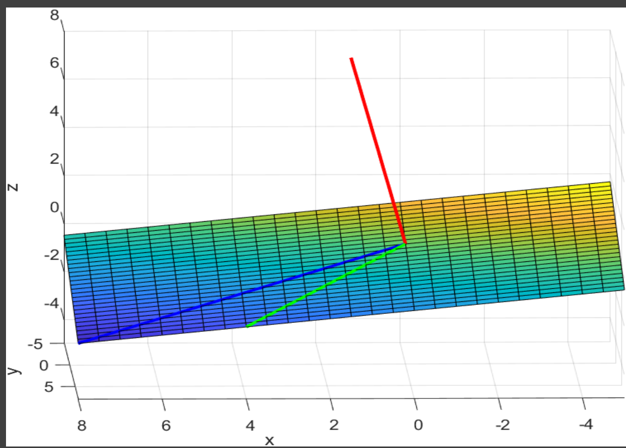
d.

```
hold on; grid on; rotate3d on;
```

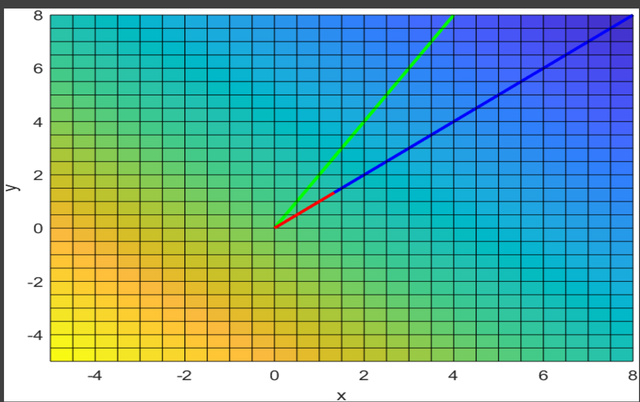
```
xlim([-5 8]);ylim([-5 8]);zlim([-5 8]);
xlabel('x');ylabel('y');zlabel('z');
u=[3 3 -1]
v=[2 4 -1]
t=linspace(0,100)
v1x=3*t; v1y=3*t;  v1z=-1*t;
v2x=2*t; v2y=4*t;  v2z=-1*t;
plot3(v1x,v1y,v1z,'b','linew',2)
plot3(v2x,v2y,v2z,'g','linew',2)

%from the equation of a plane a(x-xo)+b(y-yo)+c(z-zo)=0
n=n/norm(n)
nx=n(1)*t; ny=n(2)*t; nz=n(3)*t
plot3(nx,ny,nz,'r','linew',2)
[x,y]=meshgrid(-5:0.5:8)
z=(-x-y)./6
surf(x,y,z)
```



view([1,0,0])



view ([0,1,0])



view ([0,0,1])

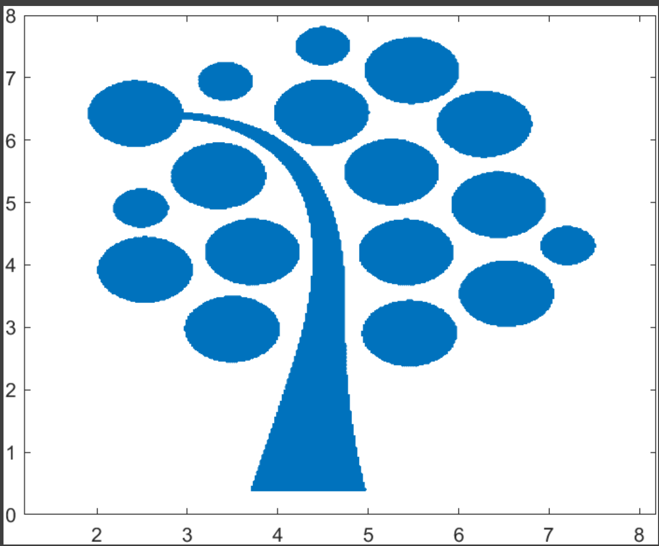
5. Linear Transformations of a Tree

a.

```
load('lnu.mat')
```

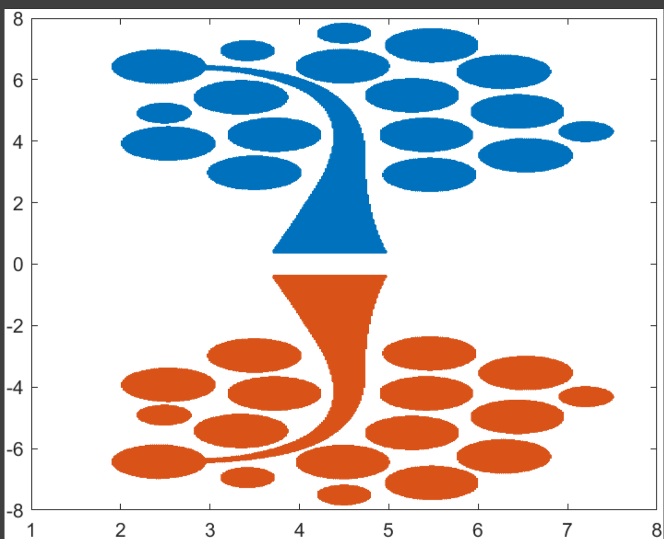
b.

```
xcoordinat_of_xy=xy(1,:)
ycoordinat_of_xy=xy(2,:)
plot(xcoordinat_of_xy,ycoordinat_of_xy,'.')
```



c.

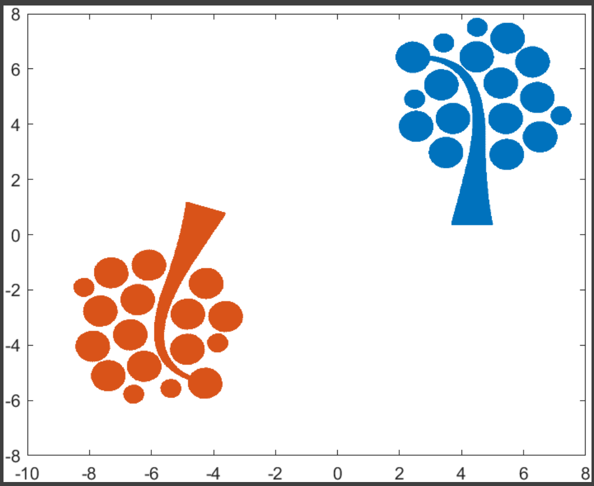
```
transformation=s*xy
hold on
x_of_transformation=transformation(1,:)
y_of_transformation=transformation(2,:)
plot(x_of_transformation,y_of_transformation,'.')
hold off
```



d.

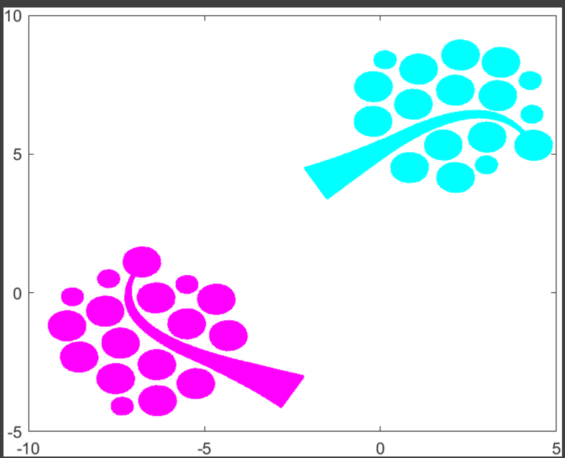
```
plot(xcoordinat_of_xy,ycoordinat_of_xy,'.')
theta=-60;
alpha= -2/3*pi
ro=[cos(theta) -sin(theta);sin(theta) cos(theta)]
rota=[cos(alpha) -sin(alpha);sin(alpha) cos(alpha)]
```

```
q=ro*xy
hold on
plot(q(1,:),q(2,:),'.')
```



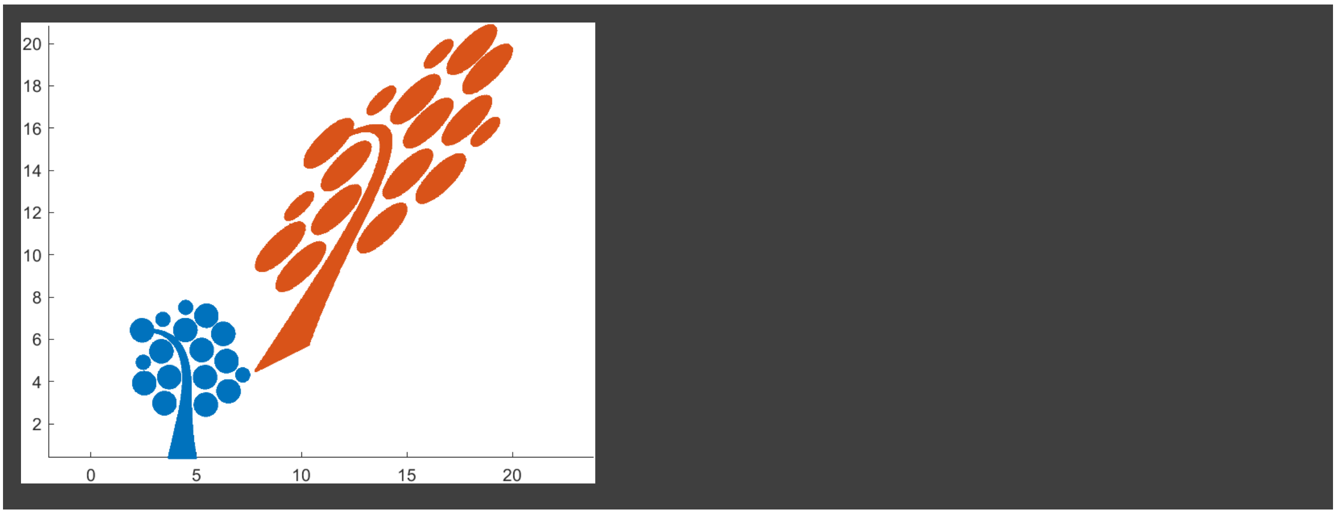
e.

```
reflect=[1 0;
   0 -1]
xyreflect=reflect*xy
vi1=rota*xyreflect
plot(vi1(1,:),vi1(2,:),'.','markeredgecolor','m')
hold on
piover3=rota*xy
vi2=reflect*piover3
plot(vi2(1,:),vi2(2,:),'.','markeredgecolor','c')
```



f.

```
hold on
xcoordinat_of_xy=xy(1,:);
ycoordinat_of_xy=xy(2,:);
plot(xcoordinat_of_xy,ycoordinat_of_xy,'.')
k=input('insert the value of k = ')
M=1/2*[1+k k-1;k-1 1+k]
S=M*xy
hold on
plot(S(1,:),S(2,:),'.')
axis equal
```



20
18
16
14
12
10
8
6
4
2
0
5
10
15
20