7MR10040: Medical Robotics: Theory and Applications
Semester 1

Assignment 1

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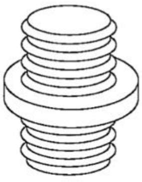
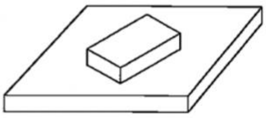
Question 1



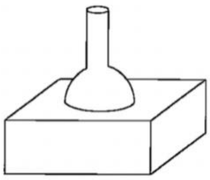
Screw

1 Degree of Freedom: The screw can move up and rotate but the two movements depend on each other



Planar

3 Degrees of Freedom: It can do all movements on the surface (up/down, left/right, rotate) and they are not dependent on each other



Spherical

3 Degrees of Freedom: It can rotate in any direction

Question 2

Grübler's equation (according to the tutorial): $Dof = 6 * (N - 1) - \sum j_b$

Where **DoF** = Degrees of Freedom, **N** = number of links and $\sum \boldsymbol{jb}$ is the degrees of freedom blocked by each joint

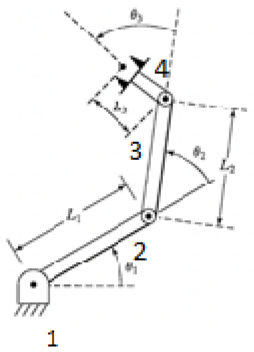


Figure 1: The numbers indicate the links

$N = 4$

$J1 = 5, J2 = 5, J3 = 5$

=> $DoF = 6 * (4 - 1) - 5 - 5 - 5 = 18 - 15 = 3$

=> The mechanism has **3 Degrees of Freedom**

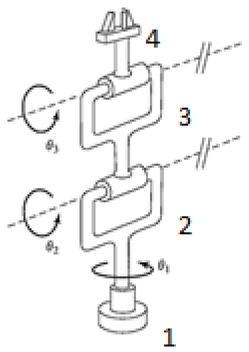


Figure 2: The numbers indicate the links

$N = 4$

$J1 = 5, J2 = 5, J3 = 5$

=> $DoF = 6 * (4 - 1) - 5 - 5 - 5 = 18 - 15 = 3$

=> The mechanism has **3 Degrees of Freedom**

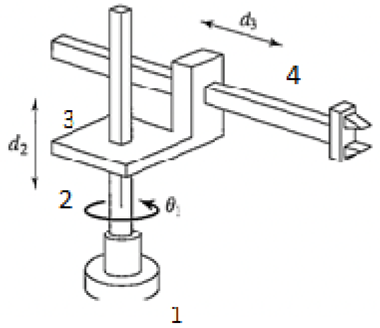


Figure 3: The numbers indicate the links

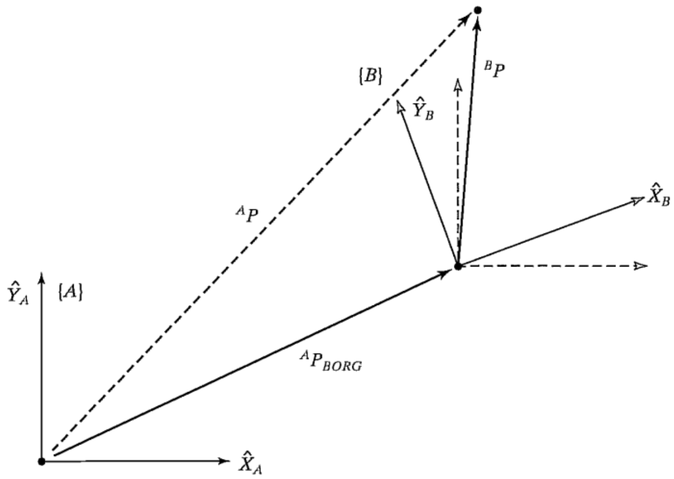
$N = 4$

$J1 = 5, J2 = 5, J3 = 5$

$=> DoF = 6 * (4 - 1) - 5 - 5 - 5 = 18 - 15 = 3$

=> The mechanism has **3 Degrees of Freedom**

Question 3



$P^A = {}^{A}_{B}H * P^A$, where H^A_B $is\ the\ Homogeneous\ Transformation\ Matrix$

$$ {}^{A}_{B}H = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & {}^{A}_{B}T_x \\ \sin(\theta) & \cos(\theta) & {}^{A}_{B}T_y \\ 0 & 0 & 1 \end{pmatrix}, P^B = \begin{pmatrix} X^B \\ Y^B \\ 1 \end{pmatrix} \ and\ P^A = \begin{pmatrix} X^A \\ Y^A \\ 1 \end{pmatrix} $$

$$ => \begin{pmatrix} X^A \\ Y^B \\ 1 \end{pmatrix} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & {}^{A}_{B}T_x \\ \sin(\theta) & \cos(\theta) & {}^{A}_{B}T_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X^B \\ Y^B \\ 1 \end{pmatrix} $$

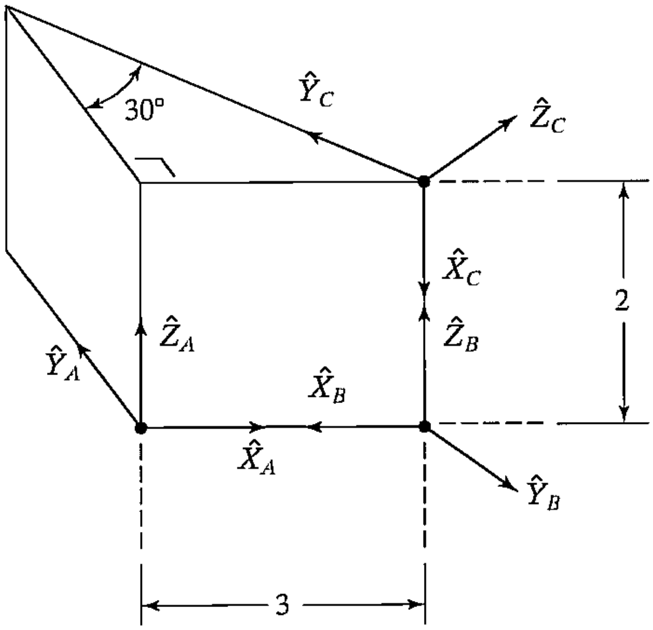
$$ = \begin{pmatrix} \cos(30) & -\sin(30) & 10 \\ \sin(30) & \cos(30) & 5 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 7 \\ 1 \end{pmatrix} $$

$$ = \begin{pmatrix} 3\cos(30) - 7\sin(30) \\ 3\sin(30) + 7\cos(30) \\ 1 \end{pmatrix} $$

$$ = \begin{pmatrix} \frac{3\sqrt{3} + 13}{2} \\ \frac{7\sqrt{3} + 13}{2} \\ 1 \end{pmatrix} $$

$$ => \ P^A = \begin{pmatrix} \frac{3\sqrt{3} + 13}{2} \\ \frac{7\sqrt{3} + 13}{2} \\ 1 \end{pmatrix} $$

Question 4



(i) For ${}^A_BT$, we rotate 180 on the z-axis and translate -3 units along the x-axis. Therefore, using:

the z-axis rotation matrix: $R = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

and the 3d transformation matrix: $\begin{pmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$${}^A_BT = R_{z,\theta} * (Transformation\ matrix)$$

$$=> {}^A_BT = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos(180) & -\sin(180) & 0 & 0 \\ \sin(180) & \cos(180) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(ii) For ${}^{A}_{C}T$, we translate 3 units along the x-axis, translate 2 units along the z-axis, rotate 90 degrees on the y-axis and then rotate -30 degrees on the x-axis. Therefore, using:

The x-axis rotation matrix: $R_{x,\theta} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) & 0 \\ 0 & \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$,

the y-axis rotation matrix: : $R_{y,\varphi} = \begin{pmatrix} \cos(\varphi) & 0 & \sin(\varphi) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\varphi) & 0 & \cos(\varphi) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

and the transformation matrix: $\begin{pmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$${}^{A}_{C}T = (Transformation\ matrix) * R_{y_{,\varphi}} * R_{x,\theta}$$

$$=> {}^{A}_{C}T = \begin{pmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos(\varphi) & 0 & \sin(\varphi) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\varphi) & 0 & \cos(\varphi) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) & 0 \\ 0 & \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos(90) & 0 & \sin(90) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(90) & 0 & \cos(90) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(-30) & -\sin(-30) & 0 \\ 0 & \sin(-30) & \cos(-30) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(-30) & -\sin(-30) & 0 \\ 0 & \sin(-30) & \cos(-30) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} & 3 \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(iii) For ${}^{B}_{C}T$, we translate 2 units on the z-axis, rotate 90 degrees on the y-axis and then rotate -210 degrees on the x-axis. Therefore, using:

The x-axis rotation matrix: $R_{x,\theta} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) & 0 \\ 0 & \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$,

the y-axis rotation matrix: : $R_{y,\varphi} = \begin{pmatrix} \cos(\varphi) & 0 & \sin(\varphi) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\varphi) & 0 & \cos(\varphi) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

and the transformation matrix: $\begin{pmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$${}^{B}_{C}T = (Transformation\ matrix) * R_{y_{,\varphi}} * R_{x,\theta}$$

$$=> {}^{B}_{C}T = \begin{pmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos(\varphi) & 0 & \sin(\varphi) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\varphi) & 0 & \cos(\varphi) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) & 0 \\ 0 & \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos(90) & 0 & \sin(90) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(90) & 0 & \cos(90) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(-210) & -\sin(-210) & 0 \\ 0 & \sin(-210) & \cos(-210) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(-210) & -\sin(-210) & 0 \\ 0 & \sin(-210) & \cos(-210) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1/2 & -\frac{\sqrt{3}}{2} & 0 \\ 0 & -\frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ -1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(iv) For ${}_{A}^{C}T$, we rotate 30 degrees on the x-axis, rotate -90 degrees on the y-axis, we translate -3 units along the x-axis and translate -2 units on the z-axis. Therefore, using:

The x-axis rotation matrix: $R_{x,\theta} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) & 0 \\ 0 & \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$,

the y-axis rotation matrix: : $R_{y,\varphi} = \begin{pmatrix} \cos(\varphi) & 0 & \sin(\varphi) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\varphi) & 0 & \cos(\varphi) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

and the transformation matrix: $\begin{pmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$${}_{A}^{C}T = R_{x,\theta} * R_{y_{,\varphi}} * (Transformation\ matrix)$$

$$=> {}_{A}^{C}T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) & 0 \\ 0 & \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos(\varphi) & 0 & \sin(\varphi) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\varphi) & 0 & \cos(\varphi) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

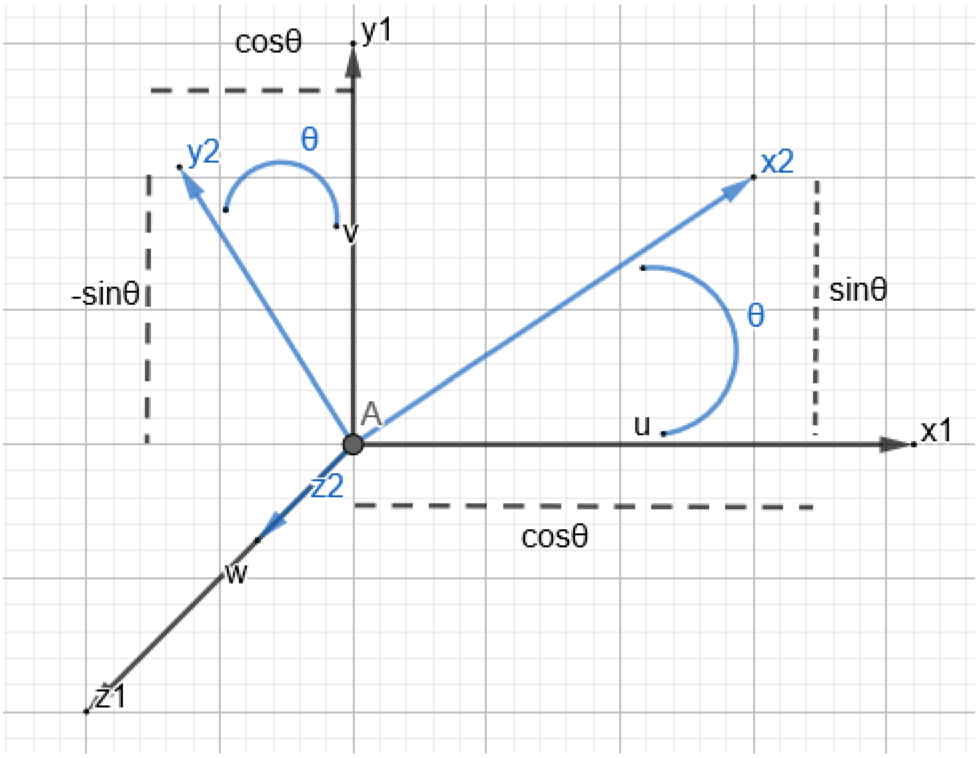
$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(30) & -\sin(30) & 0 \\ 0 & \sin(30) & \cos(30) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos(-90) & 0 & \sin(-90) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(-90) & 0 & \cos(-90) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & -1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & -1 & 2 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & \frac{3}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & -\frac{3\sqrt{3}}{2} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Question 5

For the z-axis:



$$U_0^1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} => U_0^2 = \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix}$$

$$V_0^1 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} => V_0^2 = \begin{pmatrix} \cos(\theta) \\ -\sin(\theta) \\ 0 \end{pmatrix}$$

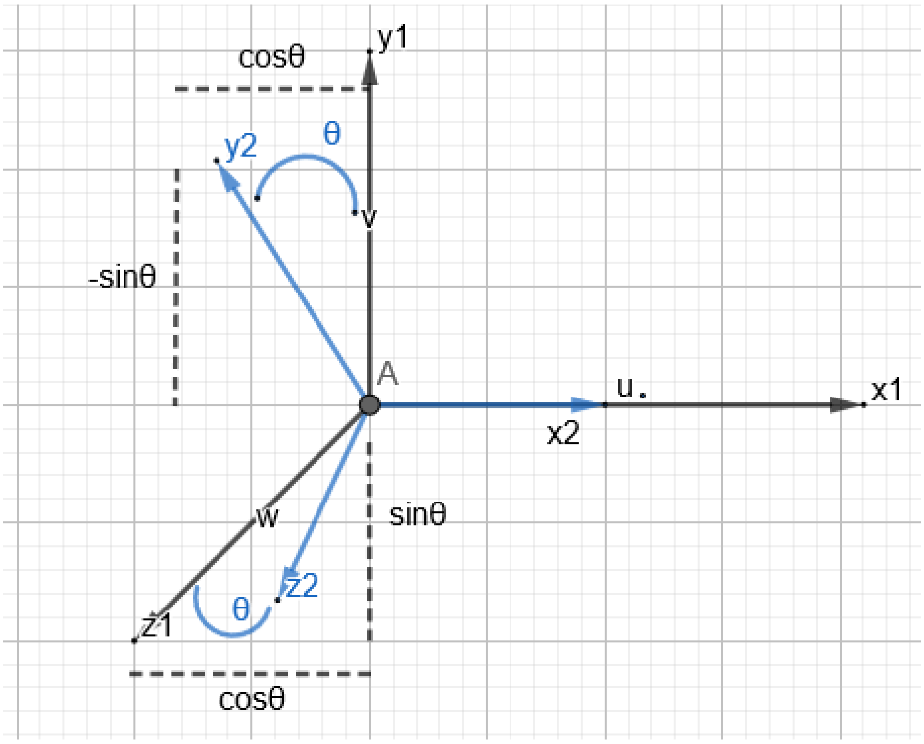
$$W_0^1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} => W_0^2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$=> \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}^1 = x^1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + y^1 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + z^1 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$=> \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}^2 = x^1 \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix} + y^1 \begin{pmatrix} \cos(\theta) \\ -\sin(\theta) \\ 0 \end{pmatrix} + z^1 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$=> R = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

For the x-axis



$$U_0^1 = \begin{pmatrix}1\\0\\0\end{pmatrix} => U_0^2 = \begin{pmatrix}1\\0\\0\end{pmatrix}$$

$$V_0^1 = \begin{pmatrix}0\\1\\0\end{pmatrix} => V_0^2 = \begin{pmatrix}0\\\cos(\theta)\\\sin(\theta)\end{pmatrix}$$

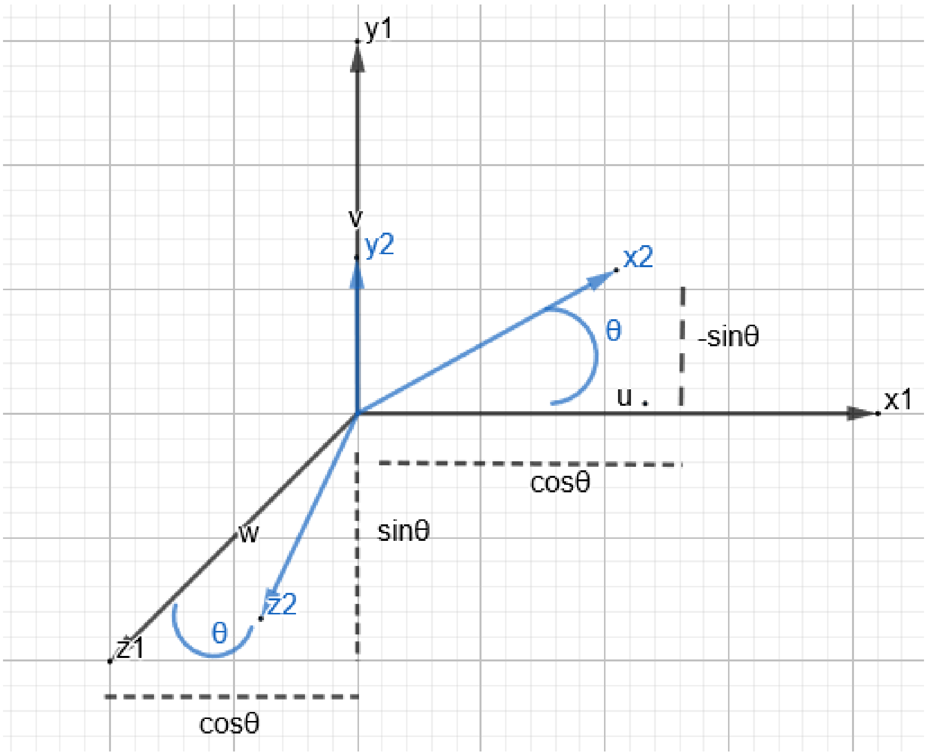
$$W_0^1 = \begin{pmatrix}0\\0\\1\end{pmatrix} => \ W_0^2 = \begin{pmatrix}0\\-\sin(\theta)\\\cos(\theta)\end{pmatrix}$$

$$=> \begin{pmatrix}X\\Y\\Z\end{pmatrix}^1 = x^1\begin{pmatrix}1\\0\\0\end{pmatrix} + y^1\begin{pmatrix}0\\1\\0\end{pmatrix} + z^1\begin{pmatrix}0\\0\\1\end{pmatrix}$$

$$=> \begin{pmatrix}X\\Y\\Z\end{pmatrix}^2 = x^1\begin{pmatrix}1\\0\\0\end{pmatrix} + y^1\begin{pmatrix}0\\\cos(\theta)\\\sin(\theta)\end{pmatrix} + z^1\begin{pmatrix}0\\-\sin(\theta)\\\cos(\theta)\end{pmatrix}$$

$$=> R = \begin{pmatrix}1 & 0 & 0\\0 & \cos(\theta) & -\sin(\theta)\\0 & \sin(\theta) & \cos(\theta)\end{pmatrix}$$

For the y-axis:



$$U_0^1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} => U_0^2 = \begin{pmatrix} \cos(\theta) \\ 0 \\ -\sin(\theta) \end{pmatrix}$$

$$V_0^1 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} => V_0^2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$W_0^1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} => \ W_0^2 = \begin{pmatrix} \sin(\theta) \\ 0 \\ \cos(\theta) \end{pmatrix}$$

$$=> \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}^1 = x^1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + y^1 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + z^1 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$=> \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}^2 = x^1 \begin{pmatrix} \cos(\theta) \\ 0 \\ -\sin(\theta) \end{pmatrix} + y^1 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + z^1 \begin{pmatrix} \sin(\theta) \\ 0 \\ \cos(\theta) \end{pmatrix}$$

$$=> R = \begin{pmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{pmatrix}$$