7MR10040: Medical Robotics: Theory and Applications
Semester 1

Assignment 2

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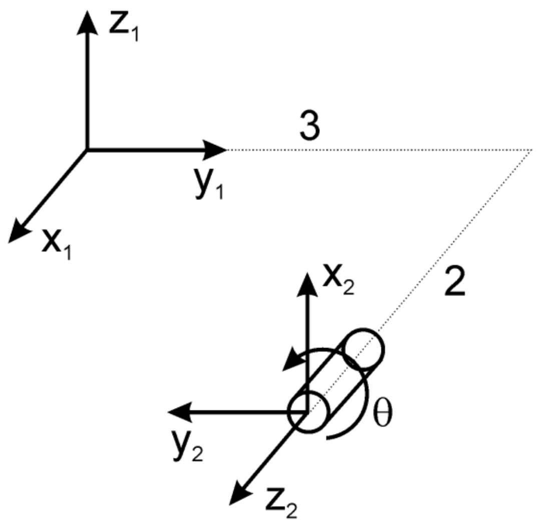
Question 1



(i)Find initial homogenous transformation g12(0) for frame 2 w.r.t. frame 1.

We translate 2 units along the x-axis, translate 3 units along the y-axis, rotate 180 degrees on the z-axis and then rotate -90 degrees on the y-axis. Therefore, using:

The z-axis rotation matrix: $R_{z,\theta} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$,

the y-axis rotation matrix: : $R_{y,\varphi} = \begin{pmatrix} \cos(\varphi) & 0 & \sin(\varphi) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\varphi) & 0 & \cos(\varphi) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

and the transformation matrix: $\begin{pmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$

=> ${}_2^1g = {}_2^1T * {}_2^1R_{z,\theta} * {}_2^1R_{y,\varphi}$

$$=> \ {}^1_2g = \begin{pmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos(\varphi) & 0 & \sin(\varphi) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\varphi) & 0 & \cos(\varphi) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$=> \ {}^1_2g$$

$$= \begin{pmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos(180) & -\sin(180) & 0 & 0 \\ \sin(180) & \cos(180) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos(-90) & 0 & \sin(-90) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(-90) & 0 & \cos(-90) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 0 & 0 & -2 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos(-90) & 0 & \sin(-90) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(-90) & 0 & \cos(-90) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 1 & -2 \\ 0 & -1 & 0 & 3 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(ii) Write the screw parameters for the motion of frame 2 w.r.t. frame 1.

ω = (1,0,0)

q = (0,3,0)

Pitch: h = 0 as there is no translational motion

Axis: l = $\frac{\omega x v}{||\omega||^2} + \lambda\omega = q + \lambda\omega = \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \ since\ \omega \neq 0$

Magnitude: M = || ω|| = 1 since $M = ||\omega|| = 1 \ since\ \omega \neq 0$

(iii) Express the corresponding twist parameters.

$$\xi = \begin{pmatrix} -\omega x q \\ \omega \end{pmatrix} = \begin{pmatrix} v \\ \omega \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -3 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$where \quad -\omega x q = -\breve{\omega} x q = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix}$$

(iv) Find the general homogenous transformation for frame 2 w.r.t. frame 1

$${}^{1}_{2}g(\vartheta) = e^{\xi\vartheta} * {}^{1}_{2}g(0)$$

$$= \begin{pmatrix} e^{\breve{\omega}\vartheta} & \left(I - e^{\breve{\omega}\vartheta}\right) * (\omega \mathrm{x} \mathrm{v}) \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 3 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where:

$$\boldsymbol{e^{\breve{\omega}\vartheta}} = I + \breve{\omega} * \sin(\vartheta) + \breve{\omega}^2(1 - \cos(\vartheta)$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\sin(\vartheta) \\ 0 & \sin(\vartheta) & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \cos(\vartheta)$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\sin(\vartheta) \\ 0 & \sin(\vartheta) & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & \cos(\vartheta) - 1 & 0 \\ 0 & 0 & \cos(\vartheta) - 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\vartheta) & -\sin(\vartheta) \\ 0 & \sin(\vartheta) & \cos(\vartheta) \end{pmatrix},$$

$$\boldsymbol{\omega \mathrm{x} \mathrm{v}} = \breve{\omega}\mathrm{v} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix},$$

$$\left(\boldsymbol{I} - \boldsymbol{e^{\breve{\omega}\vartheta}}\right) * (\boldsymbol{\omega \mathrm{x} \mathrm{v}}) = (\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\vartheta) & -\sin(\vartheta) \\ 0 & \sin(\vartheta) & \cos(\vartheta) \end{pmatrix}) * \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 - \cos(\vartheta) & \sin(\vartheta) \\ 0 & -\sin(\vartheta) & 1 - \cos(\vartheta) \end{pmatrix} \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}$$

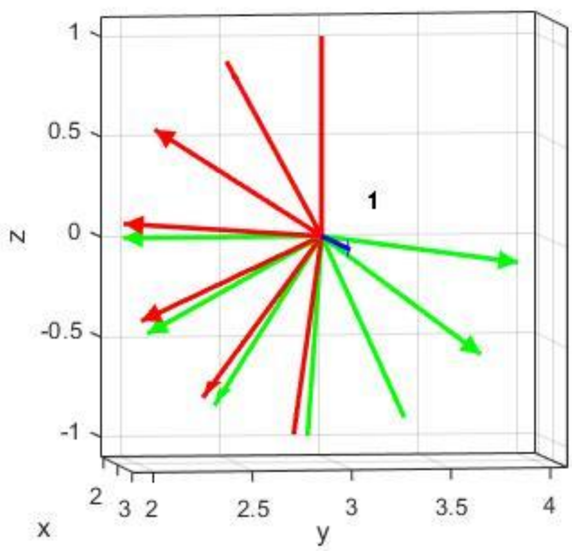
$$= \begin{pmatrix} 0 \\ 3 - 3\cos(\vartheta) \\ -3\sin(\vartheta) \end{pmatrix}$$

$$=> \ e^{\xi\vartheta} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\vartheta) & -\sin(\vartheta) & 3 - 3\cos(\vartheta) \\ 0 & \sin(\vartheta) & \cos(\vartheta) & -3\sin(\vartheta) \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

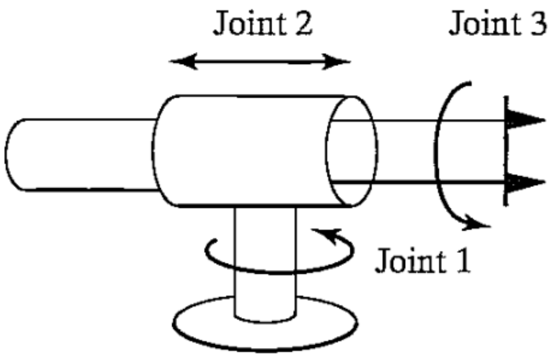
$$=> \ {}^{1}_{2}g(\vartheta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\vartheta) & -\sin(\vartheta) & 3 - 3\cos(\vartheta) \\ 0 & \sin(\vartheta) & \cos(\vartheta) & -3\sin(\vartheta) \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & -2 \\ 0 & -1 & 0 & 3 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

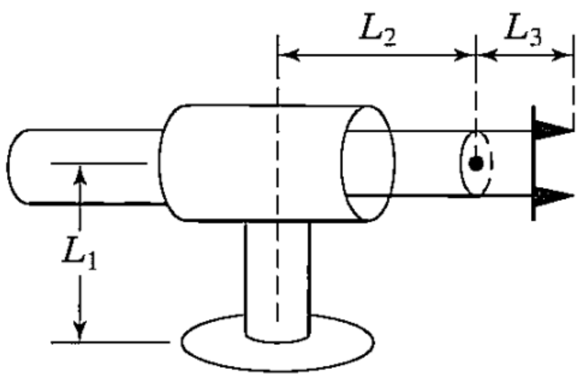
$$= \begin{pmatrix} 0 & 0 & 1 & -2 \\ -\sin(\vartheta) & -\sin(\vartheta) & 0 & 3\sin(\vartheta) - 3\cos(\vartheta) + 3 \\ \cos(\vartheta) & -\sin(\vartheta) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(v) Use Matlab to plot the 3D path of the following points in frame 2 for θ = [0,3], p₁ = [1,0,0], p₂ =[0,1,0], p₃ = [0,0,1].



Question 2

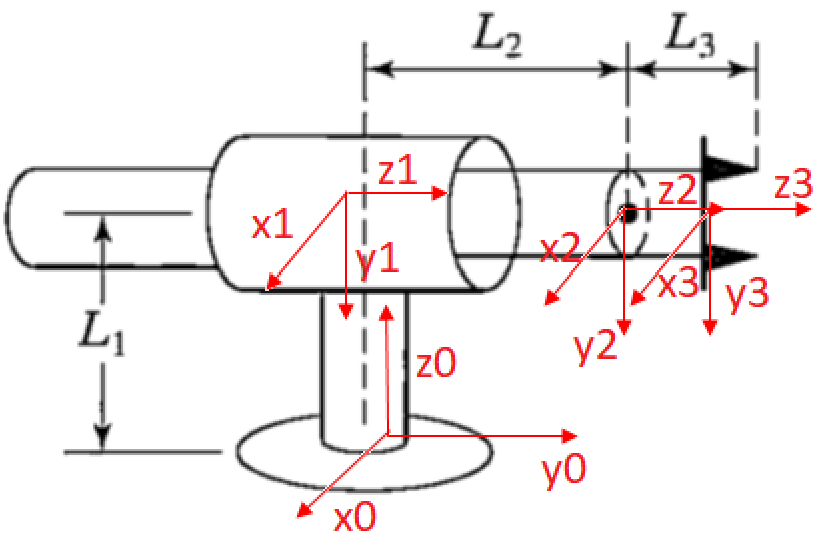




(i) What are the link DH parameters of the 3DOF robots in the figures?

i	θ_i	d_i	a_i	α_i
1	θ_1	L_1	0	-90
2	0	L_2	0	0
3	θ_3	$L_2 + L_3$	0	0

(ii) Draw the link-frame assignments based on the DH parameters.



(iii) Use the DH parameters to compute the individual transformations for each link.

To find the transformation matrix at each link we use:

$$T = \begin{pmatrix} \cos(\theta_i) & -\sin(\theta_i)\cos(\alpha_i) & \sin(\theta_i)\sin(\alpha_i) & a_i\cos(\theta_i) \\ \sin(\theta_i) & \cos(\theta_i)\cos(\alpha_i) & -\cos(\theta_i)\sin(\alpha_i) & a_i\sin(\theta_i) \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & d_i \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$ {}^{0}_{1}T = \begin{pmatrix} \cos(\theta_1) & 0 & -\sin(\theta_1) & 0 \\ \sin(\theta_1) & 0 & \cos(\theta_1) & 0 \\ 0 & -1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{1}_{2}T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{2}_{3}T = \begin{pmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & 0 \\ \sin(\theta_3) & \cos(\theta_3) & 0 & 0 \\ 0 & 0 & 1 & L_2 + L_3 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

And if we want to find out ${}^{0}_{3}T$:

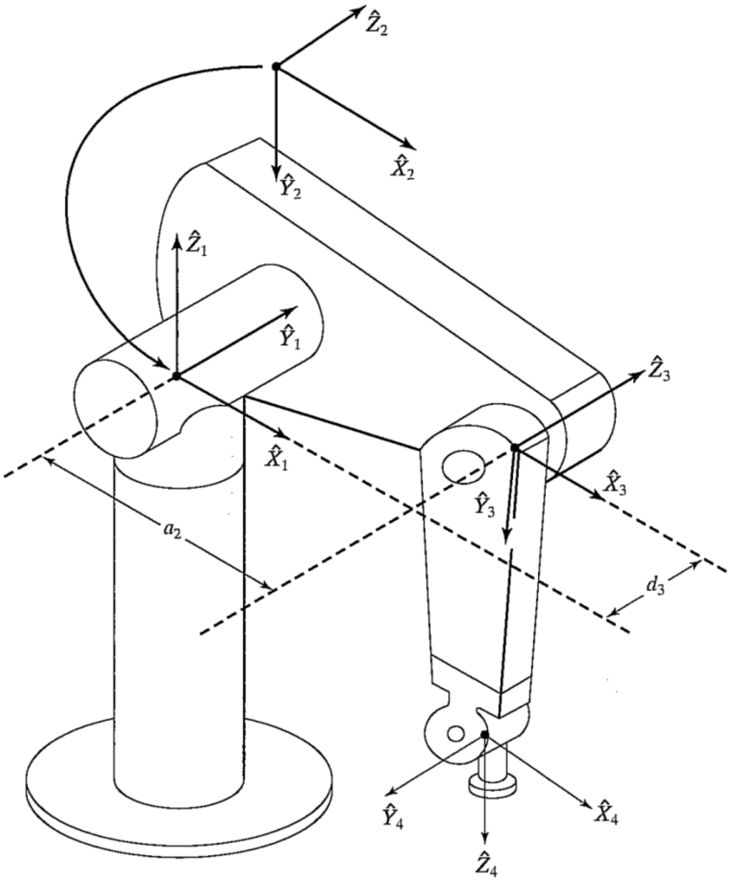
=> ${}^{0}_{3}T = {}^{0}_{1}T * {}^{1}_{2}T * {}^{2}_{3}T$

$$ = \begin{pmatrix} \cos(\theta_1) & 0 & -\sin(\theta_1) & 0 \\ \sin(\theta_1) & 0 & \cos(\theta_1) & 0 \\ 0 & -1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & 0 \\ \sin(\theta_3) & \cos(\theta_3) & 0 & 0 \\ 0 & 0 & 1 & L_2 + L_3 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ = \begin{pmatrix} \cos(\theta_1) & 0 & -\sin(\theta_1) & -L_2\sin(\theta_1) \\ \sin(\theta_1) & 0 & \cos(\theta_1) & L_2\cos(\theta_1) \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & 0 \\ \sin(\theta_3) & \cos(\theta_3) & 0 & 0 \\ 0 & 0 & 1 & L_2 + L_3 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ = \begin{pmatrix} \cos(\theta_1)\cos(\theta_3) & -\cos(\theta_1)\sin(\theta_3) & -\sin(\theta_1) & -\sin(\theta_1) * (L_2 + L_3) - L_2\sin(\theta_1) \\ \sin(\theta_1)\cos(\theta_3) & -\sin(\theta_1)\sin(\theta_3) & \cos(\theta_1) & \cos(\theta_1) * (L_2 + L_3) + L_2\cos(\theta_1) \\ -\sin(\theta_3) & -\cos(\theta_3) & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

Question 3



(i) Using DH parameters, find the forward kinematics (transformation matrix w.r.t. base frame) for the tool frame (manipulator tip).

Using the modified parameters (John J. Craig, Introduction to Robotics), we get:

i	θ_i	d_i	a_{i-1}	α_{i-1}
1	θ_1	0	0	0
2	θ_2	0	0	-90
3	θ_3	d_3	a_2	0
4	θ_4	d_4	a_3	-90
5	θ_5	0	0	90
6	θ_6	0	0	-90

To find the transformation matrix at each point we use:

$$\mathrm{H} = \begin{pmatrix} \cos(\theta_i) & -\sin(\theta_i)\cos(\alpha_i) & \sin(\theta_i)\sin(\alpha_i) & a_i\cos(\theta_i) \\ \sin(\theta_i) & \cos(\theta_i)\cos(\alpha_i) & -\cos(\theta_i)\sin(\alpha_i) & a_i\sin(\theta_i) \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & d_i \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$=> \; {}^0_6T = {}^0_1T * {}^1_2T * {}^2_3T * {}^3_4T * {}^4_5T * {}^5_6T$$

$$= \begin{pmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin(\theta_2) & -\cos(\theta_2) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & a_2 \\ 0 & 0 & 0 & 0 \\ \sin(\theta_3) & \cos(\theta_3) & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \cos(\theta_4) & -\sin(\theta_4) & 0 & a_3 \\ 0 & 0 & 1 & d_4 \\ -\sin(\theta_4) & -\cos(\theta_4) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos(\theta_5) & -\sin(\theta_5) & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin(\theta_5) & \cos(\theta_5) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos(\theta_6) & -\sin(\theta_6) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin(\theta_6) & -\cos(\theta_6) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$Where$:

ci stands for $\cos(\theta_i)$ and si stands for $\sin(\theta_i)$,

cij stands for $\cos(\theta_i + \theta_j) = ci * cj - si * sj$

sij stands for $\sin(\theta_i + \theta_j) = ci * sj + si * cj$

$r_{11} = \mathrm{c1} * [\mathrm{c23} * (\mathrm{c4} * \mathrm{c5} * \mathrm{c6} - \mathrm{s4} * \mathrm{s6}) - \mathrm{s23} * \mathrm{s5} * \mathrm{s6}] + \mathrm{s1}(\mathrm{s4} * \mathrm{c5} * \mathrm{c6} + \mathrm{c4} * \mathrm{s6})$

$r_{21} = \mathrm{s1} * [\mathrm{c23} * (\mathrm{c4} * \mathrm{c5} * \mathrm{c6} - \mathrm{s4} * \mathrm{s6}) - \mathrm{s23} * \mathrm{s5} * \mathrm{s6}] - \mathrm{c1} * (\mathrm{s4} * \mathrm{c5} * \mathrm{c6} + \mathrm{c4} * \mathrm{s6})$

$r_{31} = -s23 * (c4 * c5 * c6 - s4 * s6) - c23 * s5 * c6$

$r_{12} = \mathrm{c1} * [\mathrm{c23} * (-\mathrm{c4} * \mathrm{c5} * \mathrm{c6} - \mathrm{s4} * \mathrm{c6}) + \mathrm{s23} * \mathrm{s5} * \mathrm{s6}] + \mathrm{s1} * (\mathrm{c4} * \mathrm{c6} - \mathrm{s4} * \mathrm{c5} * \mathrm{s6})$

$r_{22} = \mathrm{s1} * [\mathrm{c23} * (-\mathrm{c4} * \mathrm{c5} * \mathrm{c6} + \mathrm{s4} * \mathrm{s6}) + \mathrm{s23} * \mathrm{s5} * \mathrm{s6}] - \mathrm{c1} * (\mathrm{c4} * \mathrm{c6} - \mathrm{s4} * \mathrm{c5} * \mathrm{s6})$

$r_{31} = -s23 * (-c4 * c5 * c6 - s4 * c6) + c23 * s5 * c6$

$r_{13} = -c1 * (c23 * c4 * s5 + s23 * c5) - s1 * s4 * s5$

$r_{23} = -s1 * (c23 * c4 * s5 + s23 * c5) + c1 * s4 * s5$

$r_{33} = s23 * c4 * s5 - c23 * c5$

$r_x = c1 * [a2 * c2 + a3 * c23 - d4 * s23] - d3s1$

$r_y = s1 * [a2 * c2 + a3 * c23 - d4 * s23] + d3s1$

$r_z = -a3 * s23 - a2 * s2 - d4 * s23$

(ii) Assume all the lengths are unit, write a Matlab script to plot the tip 3D motion if the joints rotate synchronously from 0 to π

```
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%
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%
% http://www.petercorke.com


% Create the links according to Puma 560 DH Parameters using the modified
% parameters
L1 = Link('d', 0, 'a', 0, 'alpha', 0, 'modified')
L2 = Link('d', 0, 'a', 0, 'alpha', -pi/2, 'modified')
L3 = Link('d', 1, 'a', 1, 'alpha', 0, 'modified')
L4 = Link('d', 1, 'a', 1, 'alpha', -pi/2, 'modified')
L5 = Link('d', 0, 'a', 0, 'alpha', pi/2, 'modified')
L6 = Link('d', 0, 'a', 0, 'alpha', -pi/2, 'modified')
L7 = Link('d', 0, 'a', 0, 'alpha', -pi, 'modified') % This is only for visual-
isation purposes


% Add the links to the robot
bot = SerialLink([L1 L2 L3 L4 L5 L6 L7], 'name', 'Puma 560')


% To keep track where the tip has been
trailLine = {'r', 'LineWidth', 2}


%  rotate all the links from zero to pi
for i=0:0.3:pi
   bot.fkine([i i i i i i 0])
   bot.plot([i i i i i i 0],'trail', trailLine, 'tilesize', 0.6)
end
```
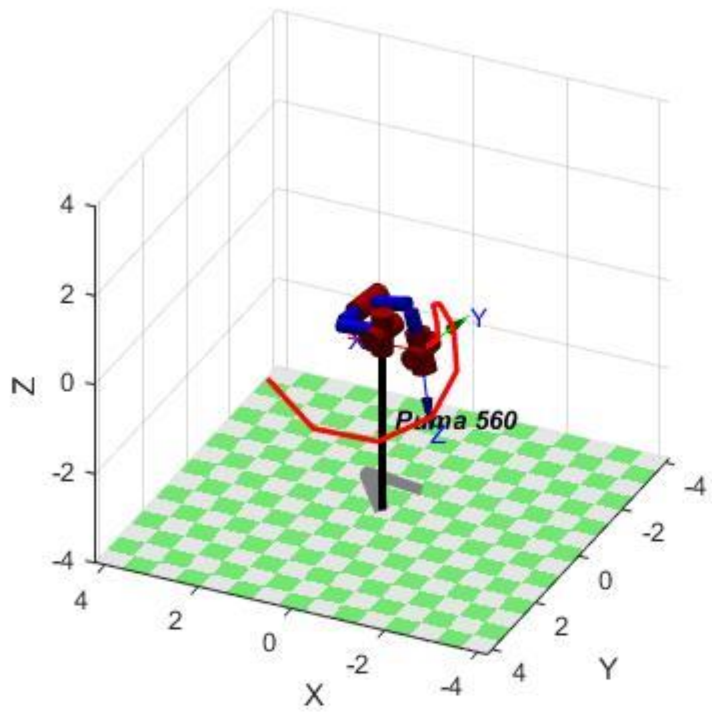

The script above gives as the following graph:



DH Parameter transformation matrix script:

This was mainly used to sanity check any manual multiplications done:

```
% Calculate the transformation matrix using DH Parameters
function [T] = tMatrix(theta, d, l, a)

T = [cos(theta) -sin(theta)*cosd(a) sin(theta)*sind(a) l*cosd(theta);
     sin(theta) cos(theta)*cosd(a) -cos(theta)*sind(a) l*sin(theta);
         0            sind(a)           cosd(a)         d;
         0              0                0            1;
    ];


End

% Example:
syms th1 L1

A = tMatrix(th1, 0, 0, 0)
B = tMatrix(0, L1, 0, 90)


AB = A*B
```